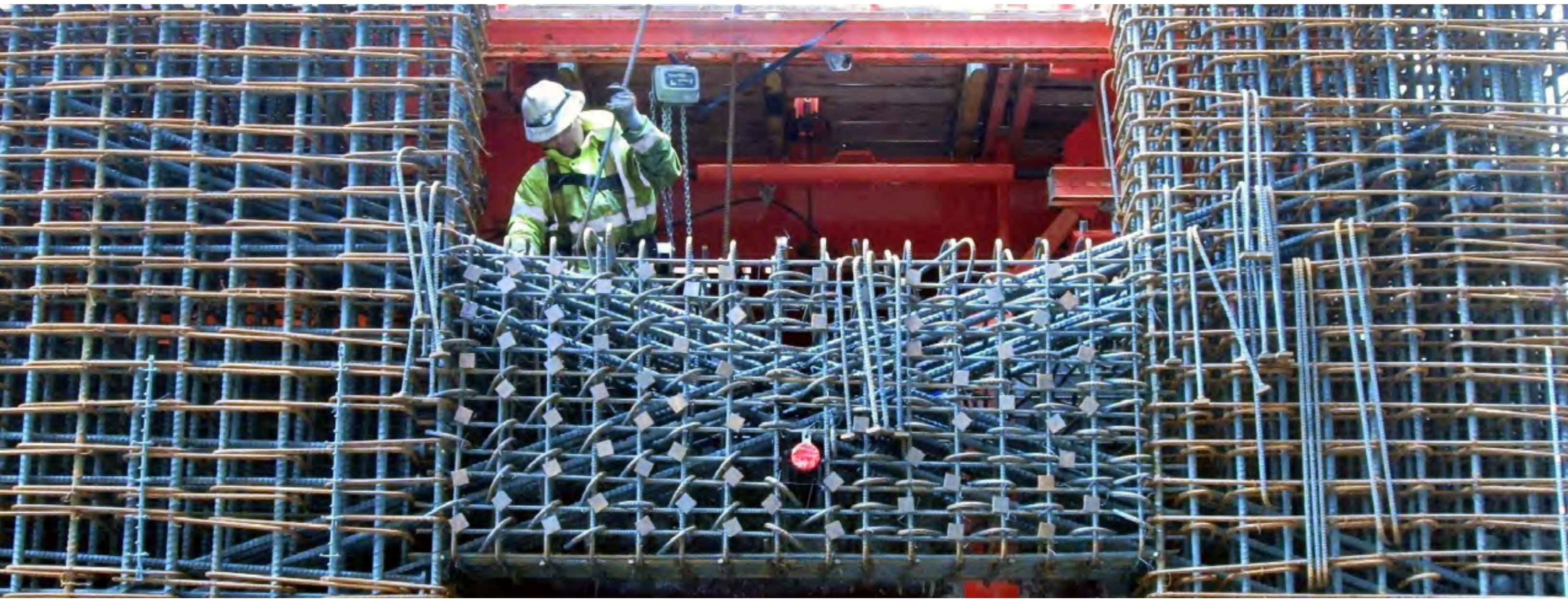
Design Guide on the ACI 318 Building Code Requirements for Structural Concrete



A comprehensive guide to assist design professionals on the design and detailing of reinforced concrete buildings.

Based on ACI 318-19

CRSI Concrete Reinforcing Steel Institute

2020

Publication No:
DG-ACI318-19

ISBN: 978-1-943961-52-8

First Edition 2020, First Printing
Contains errata material, December 2020

Printed in the U.S.A

This publication is intended for the use of professionals competent to evaluate the significance and limitations of its contents and who will accept responsibility for the application of the material it contains. The Concrete Reinforcing Steel Institute reports the foregoing material as a matter of information and, therefore, disclaims any and all responsibility for application of the stated principles or for the accuracy of the sources other than material developed by the Institute.

Founded in 1924, the Concrete Reinforcing Steel Institute (CRSI) is a technical institute and an ANSI-accredited Standards Developing Organization (SDO) that stands as the authoritative resource for information related to steel reinforced concrete construction. Serving the needs of engineers, architects and construction professionals, CRSI offers many industry-trusted technical publications, standards documents, design aids, reference materials and educational opportunities. CRSI Industry members include manufacturers, fabricators, material suppliers and placers of steel reinforcing bars and related products. Our Professional members are involved in the research, design, and construction of steel reinforced concrete. CRSI also has a broad Region Manager network that supports both members and industry professionals and creates awareness among the design/construction community through outreach activities. Together, they form a complete network of industry information and support.

Foreword

The purpose of this Design Guide is to assist in the proper application of the design and detailing requirements in Building Code Requirements for Structural Concrete (ACI 318-19) for cast-in-place concrete buildings with nonprestressed steel reinforcement. Many design aids and worked-out examples are provided to make designing and detailing reinforced concrete members simpler and faster. The goal is to acquire an understanding of the code requirements and to apply them properly and efficiently.

In addition to structural engineers, this Design Guide is a valuable resource for educators, students, individuals studying for licensing exams, and building officials.

Since the first CRSI Design Handbook in 1952, users of CRSI publications have been cooperative in suggesting to the Design Aids Committee and CRSI Staff, many improvements, clarifications and additional design short-cuts. This professional assistance is very helpful, and is appreciated. Comments on this design guide are welcome so that future editions can be further improved. Please direct all comments to Amy Trygestad, P.E., F.ACI, CRSI Vice President of Engineering.

Acknowledgements

Special recognition is due to the following individuals who provided a review of the publication and made insightful comments and suggestions for improvement:

David A. Fanella, S.E., P.E., F.ACI, F.ASCE, F.SEI, Senior Director of Engineering, is the author of this Design Guide.

Amy Trygestad, P.E., F.ACI, Vice President of Engineering

Nathan Westin, P.E., Technical Director

Special recognition is also due to Dave Mounce, Director of Communications, who was in charge of production and layout of this Design Guide. His dedication and expertise is greatly appreciated.

Photo Credits

Front cover image courtesy of Skidmore, Owings & Merrill, SEOR
Project Name: 350 Mission
Client: Kilroy Realty
General Contractor: Webcor Builders

Back cover image courtesy of DeSimone Consulting Engineers
Project Name: Panorama Tower
Client: Florida East Coast Realty (FECR)
General Contractor: Florida East Coast Realty (FECR)

Contents

Chapter 4

One-way Slabs

Chapter 5

Two-way Slabs

Chapter 6
Beams

Chapter 7
Columns

Chapter 8
Walls

Chapter 9
Diaphragms

Chapter 10
Foundations

Chapter 11

Beam-Column and Slab-Column Joints

Chapter 12

Earthquake-Resistant Structures – Overview

Chapter 13
Earthquake-Resistant Structures – SDC B and C

Chapter 14
Earthquake-Resistant Structures – SDC D, E and F

Chapter 1
INTRODUCTION

1.1 Overview

The purpose of this design guide is to assist in the proper application of the provisions in the 2019 edition of *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)* for cast-in-place concrete buildings with nonprestressed steel reinforcing bars (Reference 1; see Appendix A for a list of references in this design guide).

The main goals of this publication are to provide:

- a simplified roadmap that can be used to navigate through the ACI 318 requirements
- step-by-step design procedures and design aids that make designing and detailing reinforced concrete buildings simpler and faster.

The information contained in this publication can be used for reinforced concrete buildings of any size assigned to Seismic Design Categories A through F.

Throughout this publication, emphasis is placed on how to correctly apply the ACI 318 requirements, and explanatory material is provided to supplement the requirements. Readers who are interested in the origins of the provisions can find a wealth of information in the ACI 318 Commentary and accompanying references.

Over 140 worked-out, practical design examples are provided that illustrate the proper application of the ACI 318 requirements. The examples are derived from the four buildings in Section 1.2 of this publication, which range in height from 5 to 30 stories. These buildings include a wide array of gravity and lateral force resisting systems and examples are given throughout the chapters of this design guide that systematically show the design and detailing of the structural members in these systems using the explanatory material and design aids in the first part of the chapter. A number of the examples address certain requirements in ACI 318 not covered in any other reinforced concrete resources. The titles for the examples are purposely very descriptive so users can quickly locate what they are looking for in the table of contents of this publication.

The section index in Appendix C can be used to quickly locate information in this design guide on a specific ACI 318 provision: ACI 318 section numbers are given in the left column of the index and the corresponding chapters and page numbers in this design guide are given in the right column of the index. This method of cross-referencing between ACI 318 and this design guide is a very useful feature for users who are interested in a particular requirement.

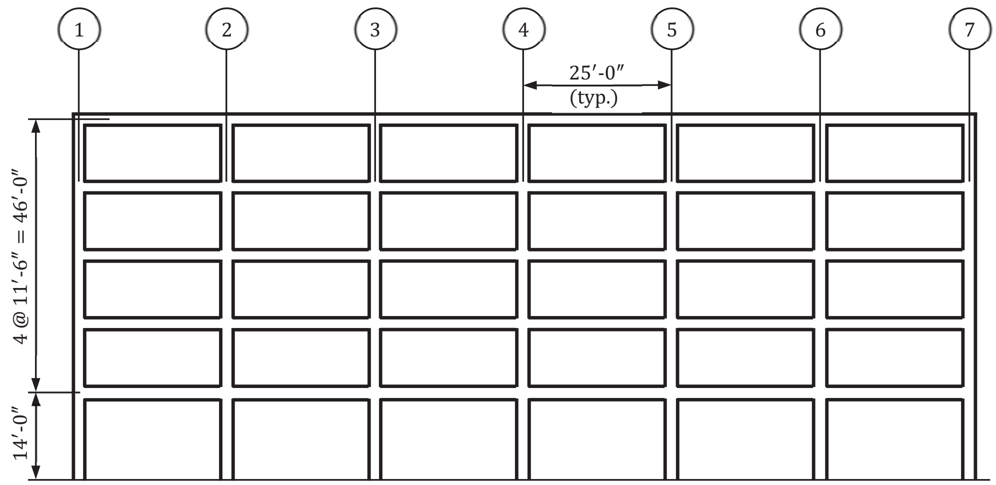
Throughout this publication, section numbers from ACI 318-19 are referenced as illustrated by the following: Section 5.3 of ACI 318-19 is denoted as ACI 5.3. Similarly, section numbers from the 2018 International Building Code (IBC) [Reference 2] and ASCE/SEI 7-16 (Reference 3) are referenced as follows: Section 1613 from the 2018 IBC and Section 12.3 of ASCE/SEI 7-16 are denoted as IBC 1613 and ASCE/SEI 12.3, respectively (Note: The 2014 edition of ACI 318 is referenced in the 2018 IBC, but the information referenced in this design guide is applicable regardless of the edition of the IBC).

The notation used throughout this design guide conforms to the notation given in Section 2.2 of ACI 318. Where additional notation is needed, it is clearly defined where it is first introduced.

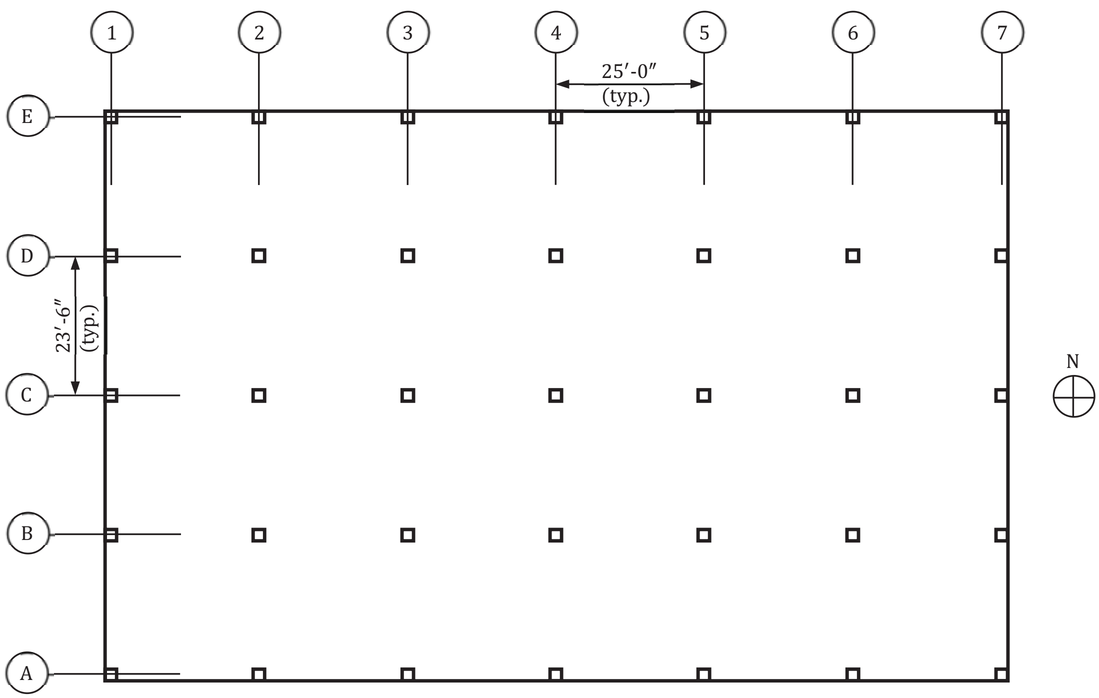
In addition to practicing engineers, this design guide is a valuable resource for educators, students, individuals studying for licensing exams, and building officials.

1.2 Example Buildings

1.2.1 Building #1 – 5-Story Office Building

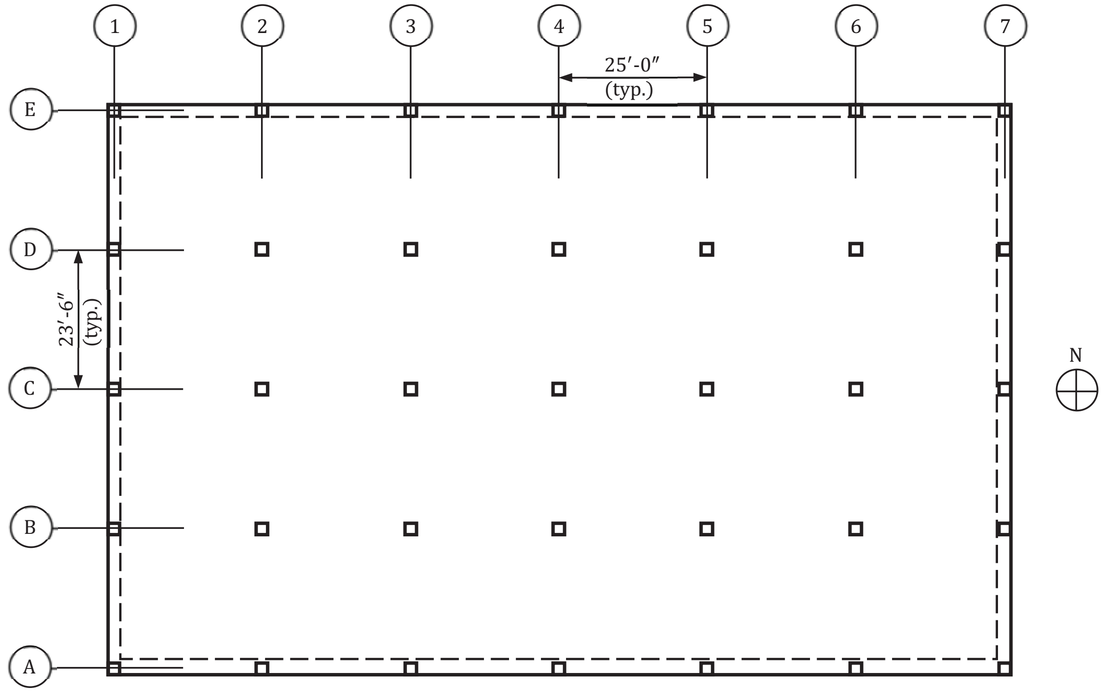


Elevation

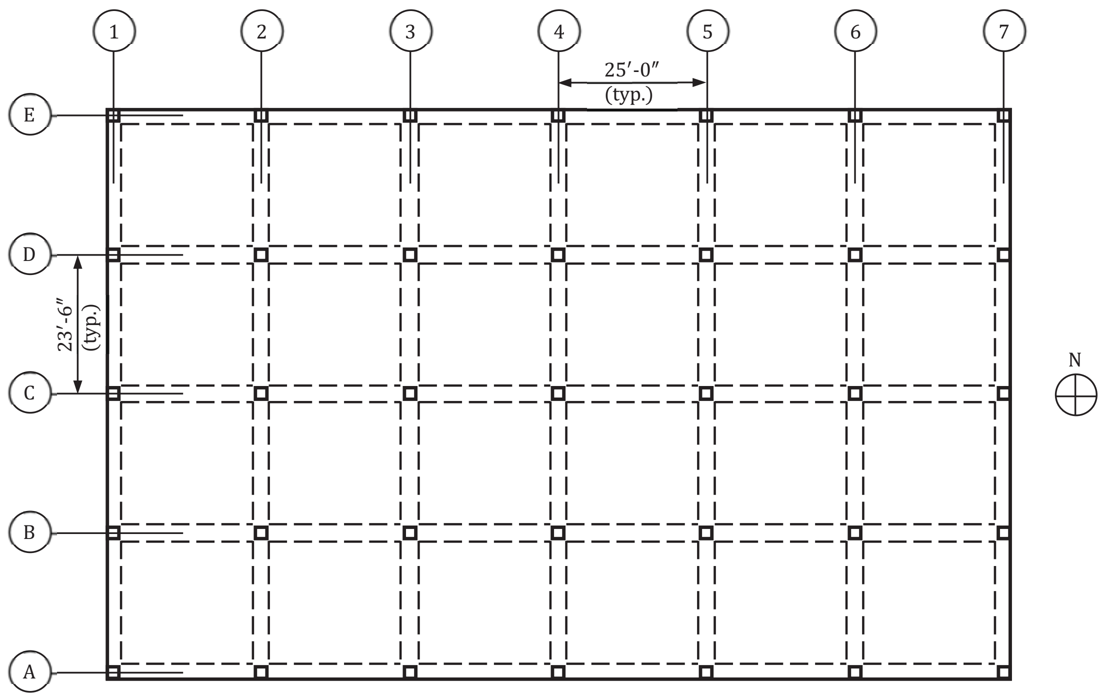


Framing Option A

Figure 1.1 Building #1 – 5-story office building.

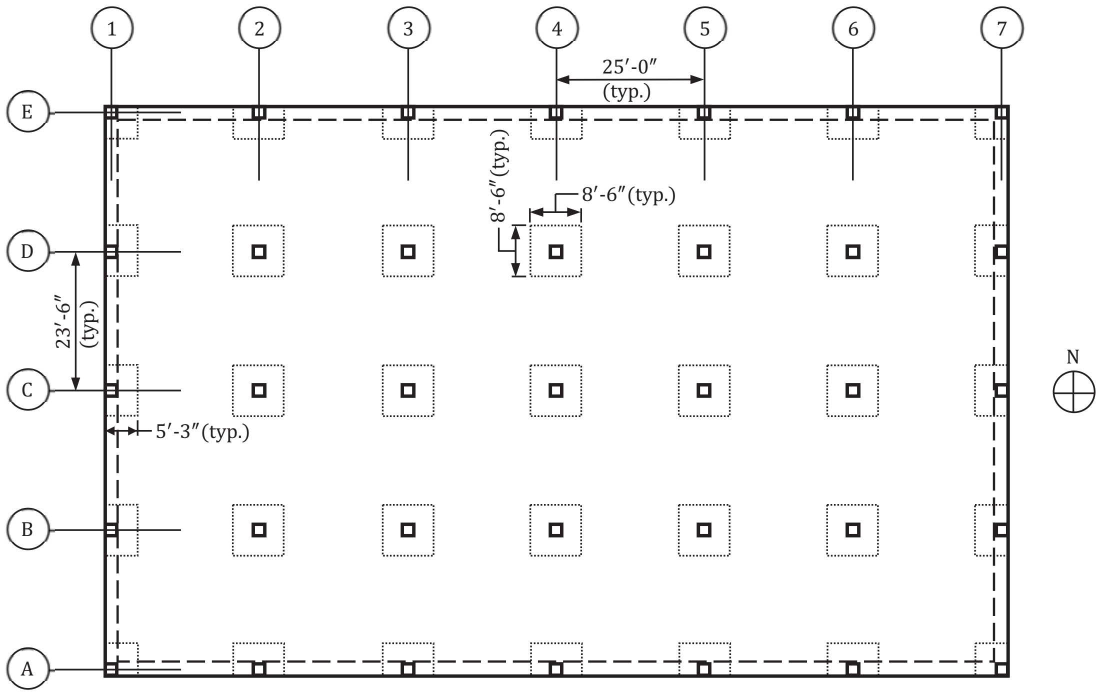


Framing Option B

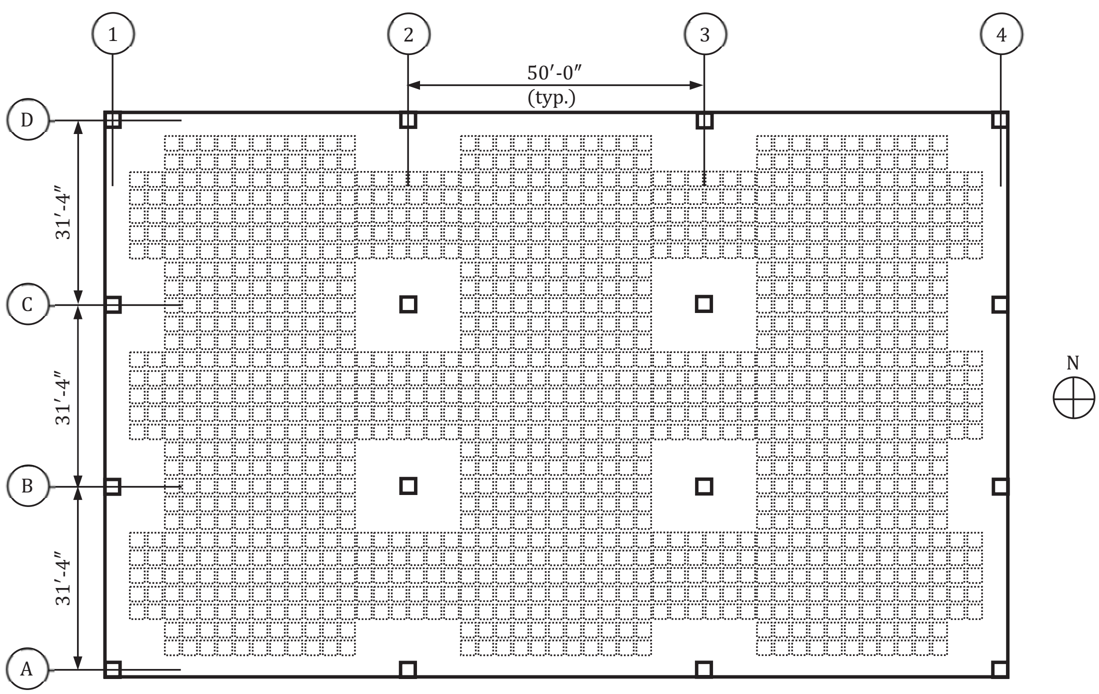


Framing Option C

Figure 1.1 Building #1 – 5-story office building (cont.).



Framing Option D



Framing Option E

Figure 1.1 Building #1 – 5-story office building (cont.).

Table 1.1 Design Information for Building #1

Design Data	
Location	Latitude $= 42.05°$, Longitude $= -88.06°$
Occupancy	Business Group B
Topography	Not situated on a hill, ridge, or escarpment
Exposure category	C
Soil classification	Site Classes C and D (default)
Load Data	
Roof	Superimposed dead load $= 12\ \mathrm{lb/ft^2}$ Live load $= 20\ \mathrm{lb/ft^2}$
Floor	Superimposed dead load $= 10\ \mathrm{lb/ft^2}$ Live load $= 65\ \mathrm{lb/ft^2}$ (includes $15\ \mathrm{lb/ft^2}$ for partitions)
Cladding	$8\ \mathrm{lb/ft^2}$
Structural Systems	
Floor/roof	Two-way slab systems [flat plate, flat plate with edge beams, two-way beam-supported slab, flat slab, and two-way joist system (waffle slab)]
Lateral	Moment-resisting frames in both directions

1.2.2 Building #2 – 12-Story Emergency Operations Center

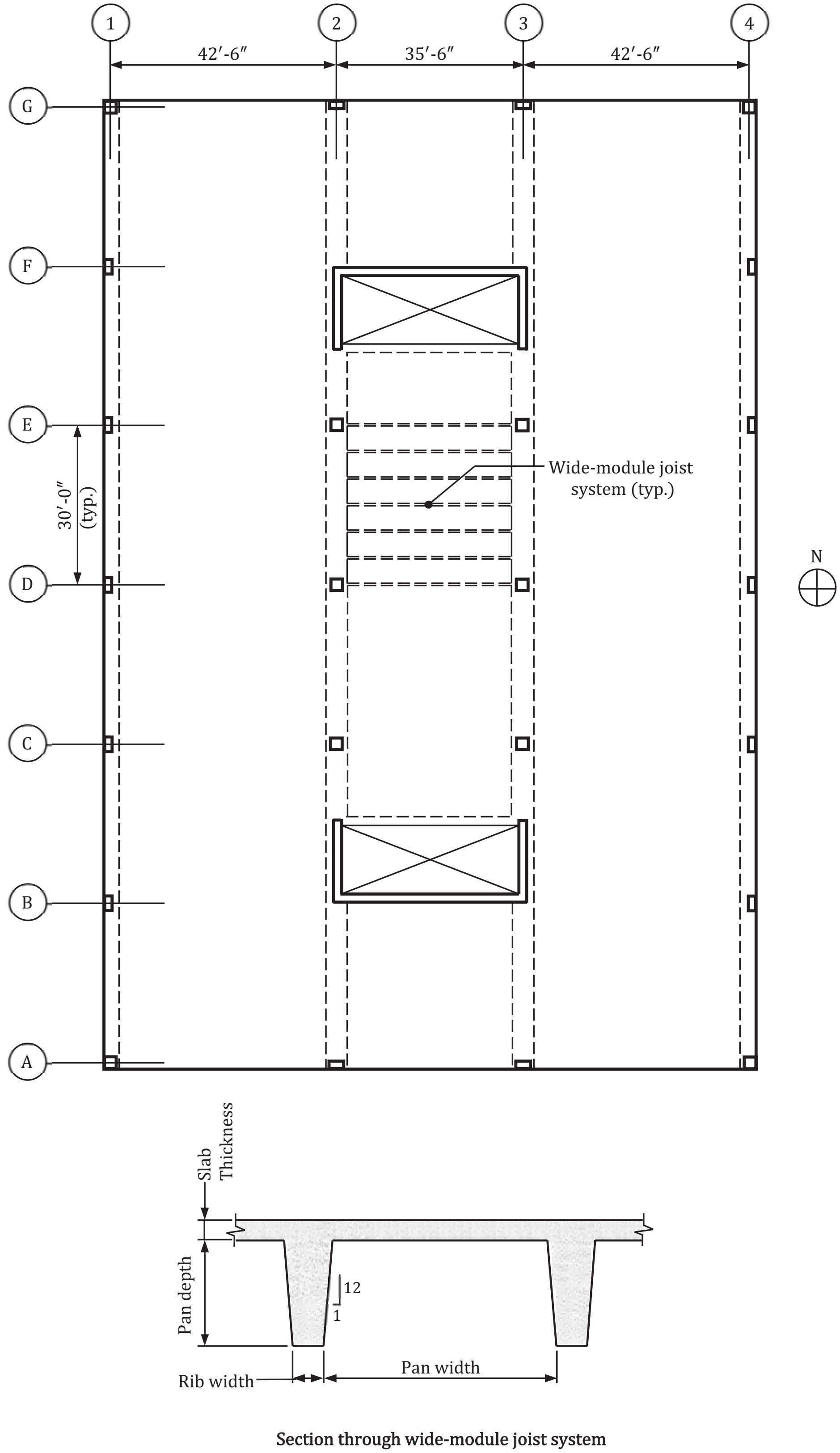


Figure 1.2 Building #2 – 12-story emergency operations center.

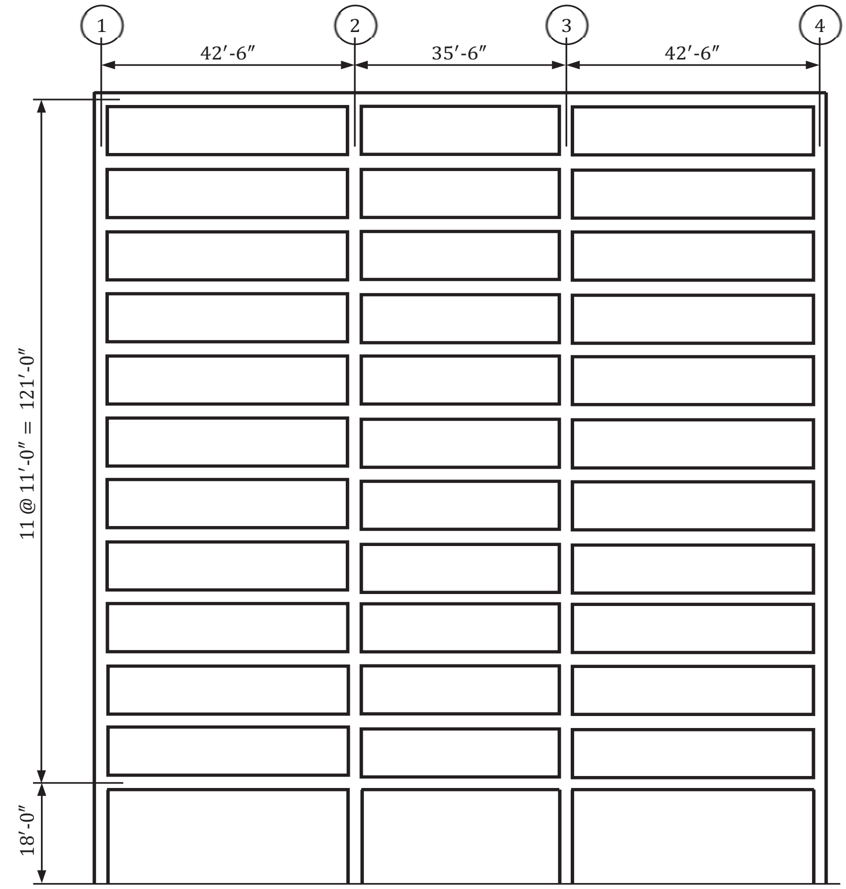


Figure 1.2 Building #2 – 12-story emergency operations center (cont.).

Table 1.2 Design Information for Building #2

Design Data	
Location	Latitude $= 40.74°$, Longitude $= -74.17°$
Occupancy	Essential facility
Topography	Not situated on a hill, ridge, or escarpment
Exposure category	B
Soil classification	Site Class D (stiff soil; determined)
Load Data	
Roof	Superimposed dead load $= 20$ lb/ft^2 Live load $= 20$ lb/ft^2
Floor	Superimposed dead load $= 20$ lb/ft^2 Live load $= 100$ lb/ft^2
Cladding	12 lb/ft^2
Structural Systems	
Floor/roof	Wide-module joist system
Lateral	Building frame system in both directions

1.2.3 Building #3 – 16-Story Residential Building

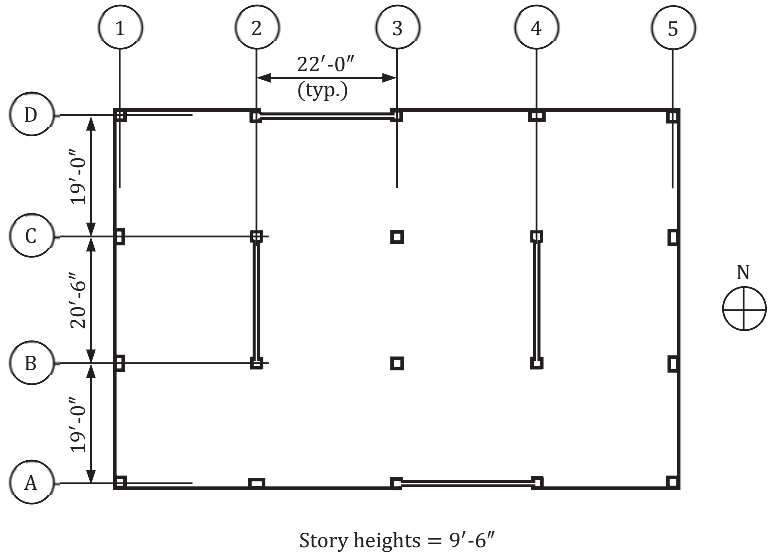


Figure 1.3 Building #3 – 16-story residential building.

Table 1.3 Design Information for Building #3

Design Data	
Location	Latitude = 47.61°, Longitude = −122.32°
Occupancy	Residential Group R
Topography	Not situated on a hill, ridge, or escarpment
Exposure category	B
Soil classification	Site Class D (stiff soil; determined)
Load Data	
Roof	Superimposed dead load = 10 lb/ft^2 Live load = 20 lb/ft^2
Floor	Superimposed dead load = 10 lb/ft^2 Live load = 40 lb/ft^2
Cladding	8 lb/ft^2
Structural Systems	
Floor/roof	Flat plate system
Lateral	Building frame system in both directions

1.2.4 Building #4 – 30-Story Office Building

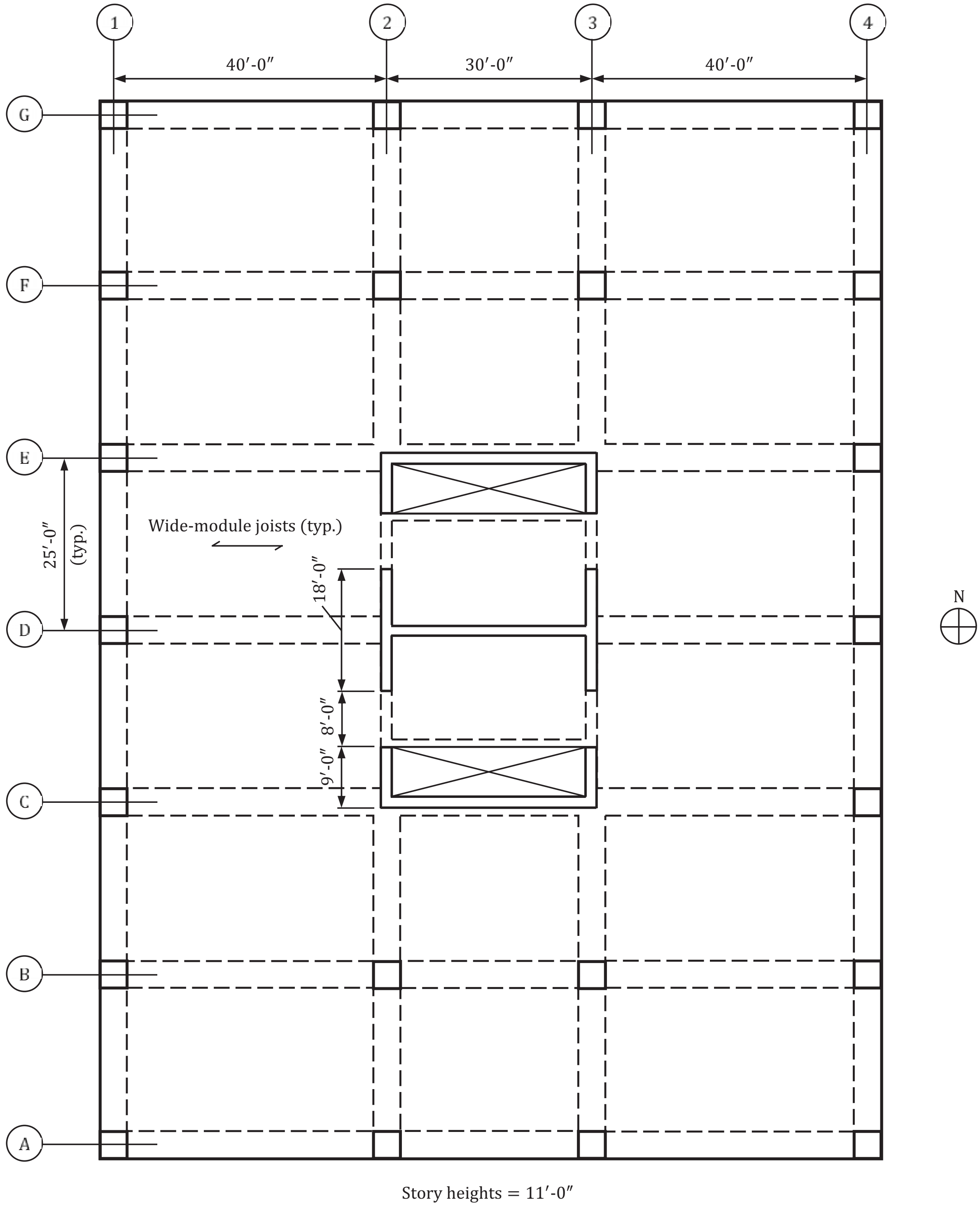


Figure 1.4 Building #4 – 30-story office building.

Table 1.4 Design Information for Building #4

Design Data	
Location	Latitude $= 36.17°$, Longitude $= -115.16°$
Occupancy	Business Group B
Topography	Not situated on a hill, ridge, or escarpment
Exposure category	B
Soil classification	Site Class D (default)
Load Data	
Roof	Superimposed dead load $= 25\ \mathrm{lb/ft^2}$ Live load $= 20\ \mathrm{lb/ft^2}$
Floor	Superimposed dead load $= 15\ \mathrm{lb/ft^2}$ Live load $= 65\ \mathrm{lb/ft^2}$ (includes $15\ \mathrm{lb/ft^2}$ for partitions)
Cladding	$10\ \mathrm{lb/ft^2}$
Structural Systems	
Floor/roof	Wide-module joist system
Lateral	Dual system in both directions

1.3 Organization of This Design Guide

Properties of concrete and nonprestressed steel reinforcing bars permitted to be used in the design of reinforced concrete members are covered in Chapter 2. ACI 318 strength reduction factors are also covered in this chapter.

Presented in Chapter 3 are typical loads and required load combinations applicable in the design of reinforced concrete buildings. Methods are given to determine the Seismic Design Category (SDC) of a building and live load reduction on structural members. Step-by-step procedures on how to determine wind forces on the main wind force resisting system (MWFRS), seismic forces on the seismic-force-resisting system (SFRS), and seismic design forces on diaphragms (including collectors) are also presented along with examples illustrating the load calculations for each of the example buildings.

Design and detailing requirements for one-way slabs are covered in Chapter 4. Included are methods to determine minimum slab thickness and the required flexural reinforcement. Typical reinforcement details are also given and examples are provided for a one-way slab that is part of a wide-module joist system.

Chapter 5 contains the design and detailing requirements for two-way slabs in buildings assigned to SDC A and B. Design aids are provided on how to determine the slab thickness based on serviceability and two-way shear strength requirements for flat plates, flat slabs, two-way beam-supported slabs, and two-way joists (waffle slabs). A general method to determine the section properties of any critical section for two-way shear is also given, including properties for polygon-shaped sections, which are required where shear reinforcement is provided. Examples include analysis and design for two-way slabs systems subjected to both gravity and lateral load effects, two-way shear design for slabs with headed shear studs and stirrups, and analysis for two-way shear with a slab opening adjacent to a column.

The design and detailing requirements for beams are covered in Chapter 6 and are applicable to buildings assigned to SDC A and B. In addition to design aids for sizing the section and selecting the reinforcement for flexure, shear, and torsion, methods to calculate both short- and long-term deflections are provided. Included are guidelines for economical detailing and design examples for wide-module joist systems and beams subjected to combined gravity and lateral load effects.

Chapter 7 covers the design and detailing requirements for columns in buildings assigned to SDC A and B. Included are methods to size the cross-section and to determine the required longitudinal and transverse reinforcement (ties and spirals) for both nonslender and slender columns. Examples include the determination of nominal and design strength interaction diagrams for rectangular and circular columns, columns subjected to combined gravity and lateral load effects, and the design and detailing of a corner column subjected to biaxial flexure and shear.

Structural walls in buildings assigned to SDC A, B, or C are covered in Chapter 8. Information is given on the moment magnification method, the alternative method for out-of-plane slender wall analysis, and the simplified design method. Typical reinforcement details are provided, including details for walls with openings, and examples are given for nonslender and slender walls subjected to axial forces and in-plane and out-of-plane lateral forces.

Design and detailing requirements for diaphragms are given in Chapter 9 for buildings assigned to SDC A, B, or C. Diaphragm modeling and analysis, in-plane stiffness modeling, and analysis methods for diaphragms with and without openings are the primary topics covered. Methods are presented on how to determine the center of rigidity, how to distribute the in-plane forces to the vertical elements of the lateral force-resisting system, and how to determine the required diaphragm and collector reinforcement.

Chapter 10 contains the design and detailing requirements for isolated spread footings, wall footings, combined spread footings, and drilled piers (caissons) supporting buildings assigned to SDC A and B. Procedures are presented on how to size the foundation member and how to determine and detail the required reinforcement for members subjected to axial forces and combined bending and axial forces.

Design and detailing requirements for beam-column and slab-column joints in buildings assigned to SDC A and B are covered in Chapter 11. Step-by-step procedures are given on how to determine joint forces in moment frames subjected to gravity loads only and to gravity and lateral loads. Examples are provided illustrating application of the requirements.

An overview of the design and detailing requirements for structures assigned to SDC B through F is given in Chapter 12. The requirements based on SDC are provided along with the applicable ACI 318 section numbers that must be satisfied. Included is an overview of seismic-force-resisting systems for reinforced concrete buildings, the corresponding design coefficients and factors, and the structural system limitations for these systems.

The design and detailing requirements for frame members and foundations in buildings assigned to SDC B and C are given in Chapter 13. A compilation of useful details is provided that clearly summarize the provisions for beams, columns, and beam-column joints in intermediate moment frames and for drilled piers (caissons). Included are examples for the design of the structural members in an intermediate moment frame and a foundation seismic tie.

Chapter 14 contains design and detailing requirements for the following structural members in buildings assigned to SDC D, E, or F:

- beams, columns, and beam-column joints in special moment frames;
- special structural walls;
- coupling beams;
- wall piers;
- diaphragms;
- shallow and deep foundations (including foundation seismic ties);
- and members not designated as part of the SFRS.

Numerous design aids and a catalogue of useful details are provided that summarize the ACI 318 requirements along with examples illustrating the proper application of the provisions.

References in this design guide are given in Appendix A. Appendix B contains properties of currently available steel reinforcing bars. The section index described in Section 1.1 above is in Appendix C.

Chapter 2
MATERIAL REQUIREMENTS AND STRENGTH REDUCTION FACTORS

2.1 Overview

Properties of concrete and nonprestressed reinforcement permitted to be used in the design of reinforced concrete members are covered in this chapter. According to ACI 4.2.1, design properties of concrete must be selected in accordance with the requirements in ACI Chapter 19. Similarly, the design properties of reinforcement must be selected in accordance with the requirements in ACI Chapter 20 (ACI 4.2.2).

Strength reduction factors, which are commonly referred to as resistance factors or ϕ-factors, play a key role in the determination of the design strength of a reinforced concrete member. ACI strength reduction factors are given in ACI Chapter 21 and are also covered in this chapter.

2.2 Material Requirements

2.2.1 Concrete Design Properties

Specified Compressive Strength

Stress-strain curves for compression tests on concrete mixtures of varying compressive strengths are given in Figure 2.1.

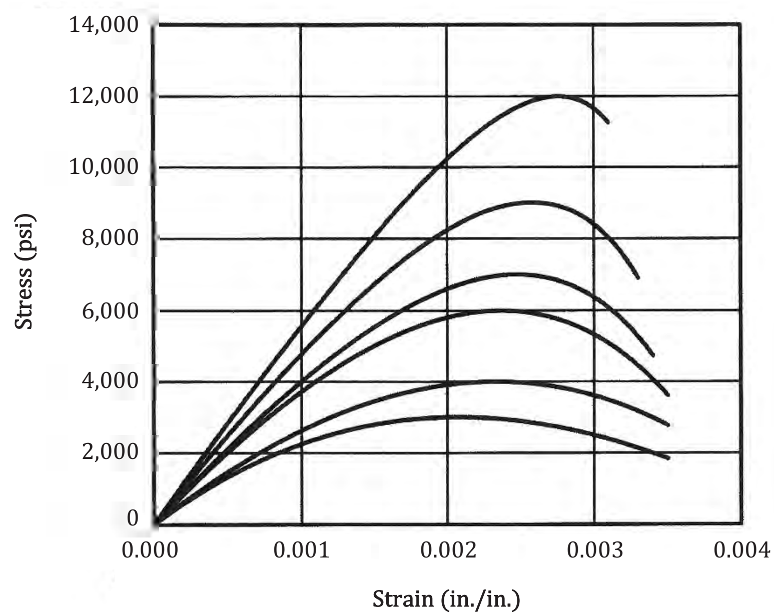


Figure 2.1 Stress-strain curves for compression tests on concrete mixtures of varying compressive strengths.

Limits on the specified compressive strength of concrete, f_c', are given in ACI Table 19.2.1.1, which apply to both normalweight and lightweight concrete [ACI 19.2.1.1(a)]. The limits on f_c' for cast-in-place structural members based on Seismic Design Category (SDC), including foundations supporting concrete structures with other than residential and utility use, are given in Table 2.1.

Table 2.1 Limits for Concrete Compressive Strength

Application	Minimum f'_c (psi)
General	2,500
Foundations for structures assigned to SDC A, B, or C	2,500
Foundations for structures assigned to SDC D, E, or F	3,000
Special moment frames Special structural walls with Grade 60 or 80 reinforcement	3,000
Special structural walls with Grade 100 reinforcement	5,000

A minimum f'_c of 5,000 psi for special structural walls with Grade 100 reinforcement is specified because test data are not available for lower concrete strengths. In general, minimum concrete strengths are increased for special seismic systems (special moment frames and special structural walls) with Grade 80 and Grade 100 reinforcement to enhance reinforcing bar anchorage and to improve structural performance by reducing the neutral axis depth.

The durability requirements in ACI 19.3.2.1 may have an impact on the minimum concrete compressive strength that must be specified [ACI 19.2.1.1(b)]. Exposure categories and classes are defined in ACI Table 19.3.1.1, and structural members must be assigned an exposure class for the applicable exposure category. Based on the severity of the anticipated exposure class, f'_c based on durability requirements may be higher than that required for structural purposes.

Structural strength requirements may also have an effect on the minimum f'_c [ACI 19.2.1.1(c)]. The only way to satisfy certain strength requirements for a structural member is to increase the compressive strength of the concrete.

No upper limits on f'_c are specified except for lightweight concrete in special moment frames, special structural walls, and their foundations; in such cases, f'_c must be less than or equal to 5,000 psi [ACI 19.2.1.1(d)]. This limit is imposed because of the very limited amount of experimental and field data available on the behavior and performance of members made with lightweight concrete. However, values of f'_c greater than 5,000 psi are permitted if it can be shown that members made with lightweight concrete provide strength and toughness levels equal to or exceeding those of comparable members made with normalweight concrete of the same compressive strength.

According to ACI 19.2.1.2, the specified compressive strength must be used for proportioning concrete mixtures in ACI 26.4.3 and for testing and acceptance of concrete in ACI 26.12.3.

Concrete compressive strength is typically determined for a concrete mixture on the basis of compression tests of cylinders molded and cured for a specified number of days in accordance with ASTM specifications. Standard cylinders are usually tested at 28 days. If f'_c is determined other than at 28 days, the test age must be indicated in the construction documents (ACI 19.2.1.3).

Modulus of Elasticity

The modulus of elasticity of concrete, E_c, which is used in the design of concrete members (including deflections and slenderness effects in columns) is permitted to be determined by ACI Equation (19.2.2.1.a) or (19.2.2.1.b) [ACI 19.2.2.1]:

For concrete with w_c between 90 and 160 lb/ft^3:

$$E_c = w_c^{1.5} 33\sqrt{f'_c} \quad \text{(psi)} \tag{2.1}$$

For normalweight concrete $(w_c \cong 145 \text{ lb/ft}^3)$:

$$E_c = 57{,}000\sqrt{f'_c} \quad \text{(psi)} \tag{2.2}$$

In Equation (2.1), w_c is the density (unit weight) of normalweight concrete or the equilibrium density of lightweight concrete. In both equations, f_c' has the units of pounds per square inch (psi).

It is permitted to specify E_c based on tests of concrete mixtures (ACI 19.2.2.2). Testing may be warranted where design conditions are sensitive to the value of E_c (for example, when calculating the deflection of a beam supporting a nonstructural member sensitive to deflections or vibration or when calculating the lateral stiffness of a tall building; see ACI R19.2.2.1).

Modulus of Rupture

The modulus of rupture for concrete, f_r, is determined by ACI Equation (19.2.3.1) [ACI 19.2.3.1]:

$$f_r = 7.5\lambda\sqrt{f_c'} \quad \text{(psi)} \tag{2.3}$$

In this equation, f_c' has the units of pounds per square inch (psi) and λ is a modification factor reflecting the reduced mechanical properties of lightweight concrete, which is determined in accordance with ACI 19.2.4 (see the following section).

The modulus of rupture is used in deflection calculations of beams and slabs.

Lightweight Concrete Modification Factor

The term λ is the modification factor reflecting the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength, and is determined based on either (1) the equilibrium density, w_c, of the concrete mixture or (2) the composition of the aggregate in the concrete mixture (see ACI 19.2.4). Values of λ based on w_c are given in Table 2.2 [ACI Table 19.2.4.1(a)] and values of λ based on composition of aggregates are given in Table 2.3 [ACI Table 19.2.4.1(b)]. Note that λ is permitted to be taken as 0.75 for lightweight concrete (ACI 19.2.4.2) and is equal to 1.0 for normalweight concrete (ACI 19.2.4.3).

Table 2.2 Values of λ Based on Equilibrium Density, w_c

Equilibrium Density, w_c	λ
$w_c \le 100\ \text{lb/ft}^3$	0.75
$100\ \text{lb/ft}^3 < w_c \le 135\ \text{lb/ft}^3$	$0.0075w_c \le 1.0$
$w_c > 135\ \text{lb/ft}^3$	1.0

Table 2.3 Values of λ Based on Composition of Aggregates

Concrete	Composition of Aggregates	λ
All-lightweight	Fine: ASTM C330 Coarse: ASTM C330	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330 and ASTM C33 Coarse: ASTM C330	0.75 to 0.85[(1)]
Sand-lightweight	Fine: ASTM C33 Coarse: ASTM C330	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33 Coarse: Combination of ASTM C330 and ASTM C33	0.85 to 1.0[(2)]

(1) Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.
(2) Linear interpolation from 0.85 to 1.0 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of aggregate.

2.2.2 Nonprestressed Steel Reinforcement

Material Properties

Yield Strength

The tensile stress-strain curve with a well-defined yield strength, f_y, for a steel reinforcing bar is given in Figure 2.2. This generic stress-strain curve has three distinct segments, which is characteristic of reinforcing bars with a yield strength of 60,000 psi or less:

- An initial linear elastic segment up to a well-defined yield strength
- A relatively flat yield plateau up to the onset of strain hardening, the strain at which is designated ε_{sh}
- A rounded strain-hardening segment

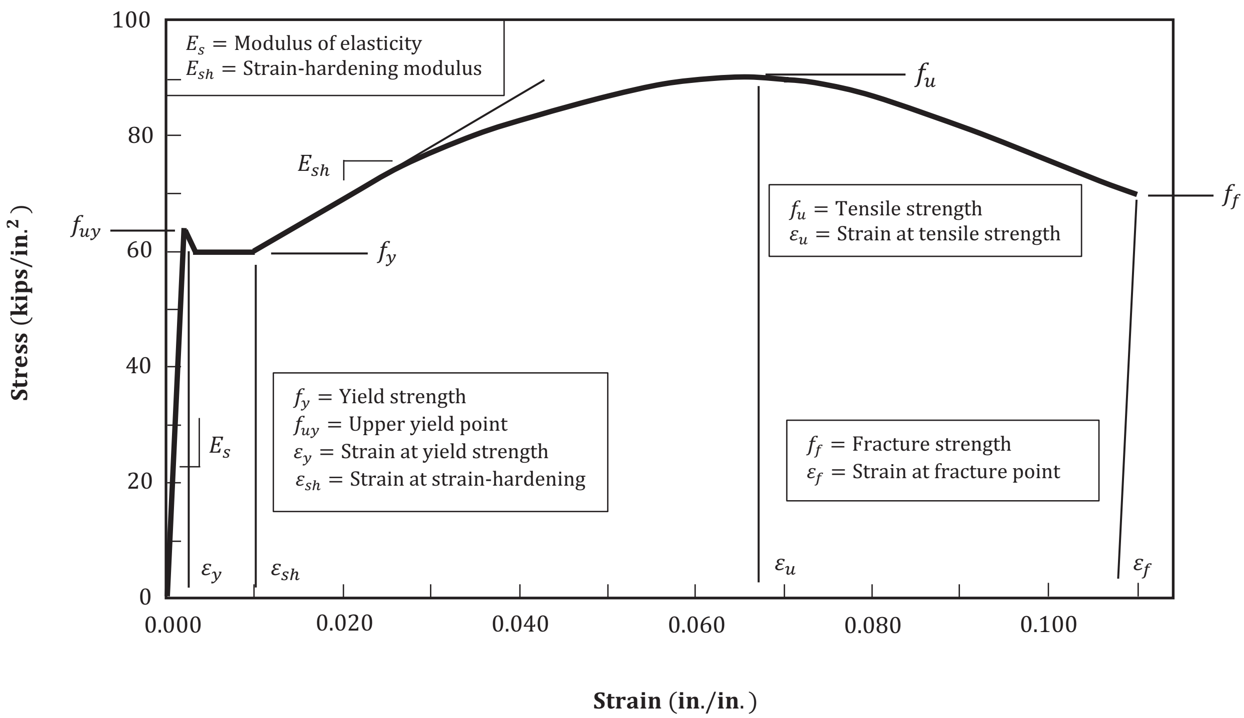


Figure 2.2 Stress-strain curve of a steel reinforcing bar with a well-defined yield strength.

Where the stress-strain diagram is characterized by a sharp-kneed or well-defined yield point, such as the one illustrated in Figure 2.2, the halt-of-force method can be used to determine the yield strength [ACI 20.2.1.2(b)]. In this method, an increasing force is applied to a tensile test specimen at a specified uniform rate. The load at which the force hesitates corresponds to the yield strength of the reinforcing bar.

The slope of the initial linear segment of the stress-strain curve is the modulus of elasticity, E_s. Test values of E_s may be of the order of 26,000,000 to 28,000,000 psi because the reinforcing bar area is not constant due to transverse deformations on the bars. However, for design purposes, E_s is permitted to be taken as 29,000,000 psi (see ACI 20.2.2.2).

The strain-hardening modulus, E_{sh}, is the slope tangent to the initial portion of the strain-hardening segment of the stress-strain curve. It is variable and not specified in any of the ASTM specifications nor described in ACI 318.

The stress-strain curves for some types of higher strength reinforcing bars can be more rounded in shape than the one shown in Figure 2.2; these are commonly referred to as round house or continuously yielding curves (see Figure 2.3 for a generic stress-strain curve for Grade 100 bars). After an initial linear-elastic segment, a gradual reduction in stiffness occurs; behavior becomes nonlinear before reaching a yield strength defined by the 0.2 percent offset method, which must be used to determine f_y where a stress-strain curve does not have a well-defined yield point [ACI 20.2.1.2(a)]. This is followed by gradual softening until the tensile strength, f_u, is reached.

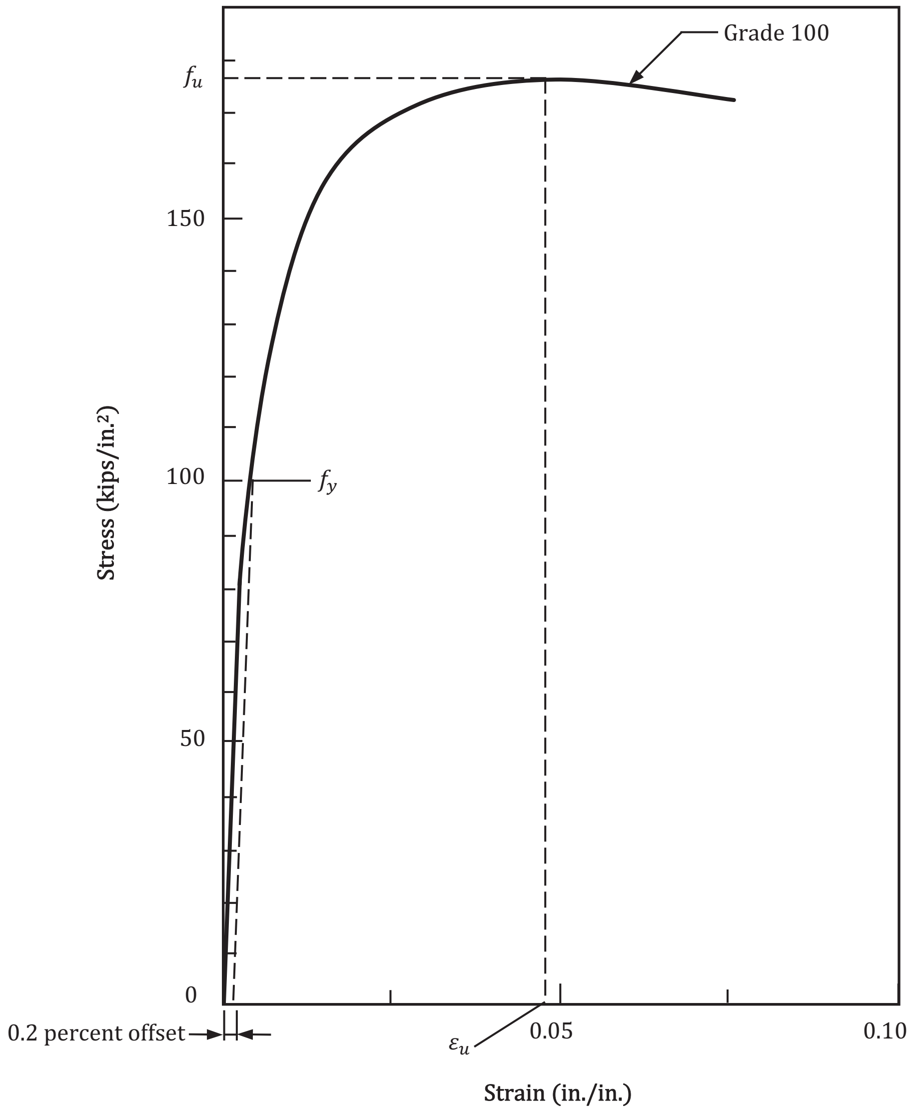


Figure 2.3 Rounded stress-strain curve for Grade 100 steel reinforcing bars.

In the 0.2 percent offset method, a strain is located on the strain axis a distance of 0.002 in./in. from the origin (see Figure 2.3). A line emanating from this point is then drawn parallel to the initial linear portion of the stress-strain curve. The point where this line intersects the stress-strain curve is defined as f_y.

Tensile Property and Uniform Elongation Requirements

Properties of deformed reinforcing bars must conform to ACI 20.2.1.3. The applicable ASTM specifications and additional requirements that must be satisfied for the available bar types are given in Table 2.4.

Table 2.4 ASTM Specifications and Additional Requirements for Deformed Reinforcing Bars

Steel Type	ASTM Specification/Additional Requirements
Carbon steel	ASTM A615/ACI Table 20.2.1.3(a)
Low-alloy steel	ASTM A706/ACI 20.2.1.3(b)(i)-(iii)
Axle steel, rail steel, and bars from rail steel	ASTM A996/Bars from rail steel must be Type R
Stainless steel	ASTM A955
Low-carbon chromium steel	ASTM A1035

The additional requirements in ACI Table 20.2.1.3(a) for ASTM A615 reinforcement are given in Table 2.5.

Table 2.5 Modified Tensile Strength and Additional Tensile Strength Property Requirements for ASTM A615 Reinforcement

	Grade 40	Grade 60	Grade 80	Grade 100
Minimum tensile strength (psi)	60,000	80,000	100,000	115,000
Minimum ratio of actual tensile strength to actual yield strength	1.10	1.10	1.10	1.10

The additional requirements that must be satisfied for ASTM A706 reinforcement are the following:

i. Tensile property requirements for ASTM A706 Grade 100 reinforcement must be as specified in ACI Table 20.2.1.3(b) [see Table 2.6]. Bend test requirements for ASTM A706 Grade 100 reinforcement must be the same as those for ASTM A706 Grade 80 reinforcement.

ii. Uniform elongation requirements for all grades of ASTM A706 reinforcement must be as specified in ACI Table 20.2.1.3(c) [see Table 2.7] where the uniform elongation is determined as the elongation at the maximum force sustained by the reinforcing bar test piece.

iii. For all grades of ASTM A706 reinforcement, the radius at the base of each bar deformation must be at least 1.5 times the height of the deformation. This requirement applies to all deformations, including transverse lugs, longitudinal ribs, grade ribs, grade marks, and intersections between deformations. Conformance must be assessed by measurements taken on newly-machined rolls used to manufacture reinforcing bars instead of measurements taken on reinforcing bar samples.

Table 2.6 Tensile Property Requirements for ASTM A706 Grade 100 Reinforcement

Minimum tensile strength, psi	117,000
Minimum ratio of actual tensile strength to actual yield strength	1.17
Minimum yield strength, psi	100,000
Maximum yield strength, psi	118,000
Minimum fracture elongation in 8 in., percent	10

Table 2.7 Uniform Elongation Requirements for ASTM A706 Reinforcement

Bar Size	Minimum Uniform Elongation, Percent		
	Grade 60	Grade 80	Grade 100
#3 through #10	9	7	6
#11, #14, #18	6	6	6

Plain bars, which are only permitted for spiral reinforcement, must conform to ASTM A615, A706, A955, or A1035 (ACI 20.2.1.4).

Headed deformed bars must conform to ASTM A970, including the requirements in Annex A1 for Class HA head dimensions (ACI 20.2.1.6). The limitation to Class HA head dimensions is due to the lack of test data for headed deformed bars that do not meet Class HA dimensional requirements.

Design Properties

Stress-Strain Relationships

For deformed reinforcing bars, it is permitted to assume the stress in the reinforcing bars, f_s, is proportional to strain in cases where the strain is below f_y, that is, $f_s = E_s \varepsilon_s$ (ACI 20.2.2.1). For strains greater than that corresponding to f_y, $f_s = f_y$; the increase in strength due to strain hardening is neglected for nominal strength calculations.

Modulus of Elasticity

As noted above, the modulus of elasticity of the reinforcing steel, E_s, is permitted to be taken as 29,000,000 psi regardless of reinforcement grade (ACI 20.2.2.2).

Permitted Values of Specified Yield Strength and Reinforcing Bar Types

In accordance with ACI 20.2.2.3 and 20.2.2.4, maximum values of f_y and permissible bar types for deformed reinforcing bars based on usage and application are given in ACI Table 20.2.2.4(a) [see Table 2.8]. Similar information is given in ACI Table 20.2.2.4(b) for plain reinforcement (see Table 2.9).

Table 2.8 Nonprestressed Deformed Reinforcing Bars

<table>
<tr><th>Usage</th><th colspan="2">Application</th><th>Maximum Value of f_y or f_{yt} Permitted for Design Calculations (psi)</th><th>ASTM Specification</th></tr>
<tr><td rowspan="3">• Flexure
• Axial force
• Shrinkage and temperature</td><td rowspan="2">Special seismic systems</td><td>Special moment frames</td><td>80,000</td><td rowspan="2">A706(2)</td></tr>
<tr><td>Special structural walls(1)</td><td>100,000</td></tr>
<tr><td colspan="2">Other</td><td>100,000(3),(4)</td><td>A615, A706, A955, A996, A1035</td></tr>
<tr><td rowspan="3">• Lateral support of longitudinal bars
• Concrete confinement</td><td colspan="2">Special seismic systems</td><td>100,000</td><td>A615, A706, A955, A996, A1035</td></tr>
<tr><td colspan="2">Spirals</td><td>100,000</td><td>A615, A706, A955, A996, A1035</td></tr>
<tr><td colspan="2">Other</td><td>80,000</td><td>A615, A706, A955, A996</td></tr>
<tr><td rowspan="5">Shear</td><td rowspan="2">Special seismic systems(5)</td><td>Special moment frames(6)</td><td>80,000</td><td rowspan="2">A615, A706, A955, A996</td></tr>
<tr><td>Special structural walls(7)</td><td>100,000</td></tr>
<tr><td colspan="2">Spirals</td><td>60,000</td><td>A615, A706, A955, A996</td></tr>
<tr><td colspan="2">Shear friction</td><td>60,000</td><td>A615, A706, A955, A996</td></tr>
<tr><td colspan="2">Stirrups, ties, hoops</td><td>60,000</td><td>A615, A706, A955, A996, A1035</td></tr>
<tr><td>Torsion</td><td colspan="2">Longitudinal and transverse</td><td>60,000</td><td>A615, A706, A955, A996</td></tr>
<tr><td rowspan="2">Anchor reinforcement</td><td colspan="2">Special seismic systems</td><td>80,000</td><td>A706(2)</td></tr>
<tr><td colspan="2">Other</td><td>80,000</td><td>A615, A706, A955, A996</td></tr>
</table>

(table continued on next page)

Table 2.8 Nonprestressed Deformed Reinforcing Bars (cont.)

Usage	Application	Maximum Value of f_y or f_{yt} Permitted for Design Calculations (psi)	ASTM Specification
Regions designed using the strut-and-tie method	Longitudinal ties	80,000	A615, A706, A955, A996
	Other	60,000	

(1) *All components of special structural walls, including coupling beams and wall piers.*
(2) *ASTM A615 Grade 60 bars are permitted provided the requirements of ACI 20.2.2.5(b) are satisfied.*
(3) *In slabs and beams that are not part of a special seismic system, reinforcing bars passing through or extending from special structural walls must satisfy ACI 20.2.2.5.*
(4) *Longitudinal reinforcement with* f_y*>80,000 psi is not permitted for intermediate moment frames and ordinary moment frames resisting earthquake demands, E.*
(5) *The maximum design value of* f_y *or* f_{yt} *is 80,000 psi for use in design calculations when used for shear reinforcement in diaphragms and foundations for load combinations including earthquake forces where the diaphragms or foundations are part of a building with a special seismic system.*
(6) *Shear reinforcement in this application includes stirrups, ties, hoops, and spirals in special moment frames.*
(7) *Shear reinforcement in this application includes all transverse reinforcement in special structural walls, coupling beams, and wall piers. Diagonal bars in coupling beams must comply with ASTM A706 or Footnote (2).*

Table 2.9 Nonprestressed Plain Spiral Reinforcing Bars

Usage	Application	Maximum Value of f_y or f_{yt} Permitted for Design Calculations (psi)	ASTM Specification
• Lateral support of longitudinal bars • Concrete confinement	Spirals in special seismic systems	100,000	A615, A706, A955, A1035
	Spirals	100,000	A615, A706, A955, A1035
Shear	Spirals	60,000	A615, A706, A955, A1035
Torsion in nonprestressed beams	Spirals	60,000	A615, A706, A955, A1035

Special Seismic Systems and Anchor Reinforcement

According to ACI 20.2.2.5, deformed nonprestressed longitudinal reinforcement resisting earthquake-induced bending moments, axial forces, or both, in special seismic systems and anchor reinforcement in SDCs C, D, E, and F must be in accordance with (a) or (b):

a. ASTM A706, Grade 60, 80, or 100 for special structural walls and Grade 60 and 80 for special moment frames.
b. ASTM A615 Grade 60 if (i) through (iv) are satisfied (ASTM A615 Grade 80 and Grade 100 are not permitted in special seismic systems because of concern associated with low-cycle fatigue behavior).
   i. Actual yield strength based on mill tests does not exceed f_y by more than 18,000 psi.
   ii. Ratio of the actual tensile strength to the actual yield strength is at least 1.25.
   iii. Minimum fracture elongation in 8 in. must be at least 14 percent for bar sizes #3 through #6, at least 12 percent for bar sizes #7 through #11, and at least 10 percent for bar sizes #14 and #18.
   iv. Minimum uniform elongation must be at least 9 percent for bar sizes #3 through #10 and at least 6 percent for bar sizes #11, #14, and #18.

The strain ε_u occurs at the tensile strength f_u is commonly referred to as uniform elongation, which is the largest elongation in the bar for which the tensile strains are uniform throughout the length of the bar (see Figure 2.2). This generally occurs right before the onset of necking. This is the maximum strain relied upon at a location where yielding of the reinforcing bar may occur (that is, in an anticipated plastic hinge region in the member).

The strain at the fracture point ε_f is the total elongation over a prescribed gauge length extending across the fracture of a reinforcing bar. According to ASTM A370, a reinforcing bar is marked with an initial 8-in. gauge length and is pulled to fracture. The ends of the fractured reinforcing bar are fit together and the distance between the gauge marks is re-measured. The total elongation is calculated as the percent increase in length relative to the original gauge length:

$$\text{Total elongation (\%)} = \frac{\text{Distance between gauge marks after fracture} - \text{Original gauge length}}{\text{Original gauge length}} \times 100 \tag{2.4}$$

2.2.3 Headed Shear Stud Reinforcement

Headed shear stud reinforcement and stud assemblies, which are typically used as shear reinforcement around columns in two-way reinforced concrete slabs, must conform to ASTM A1044 (ACI 20.4.1). Required stud head configurations are given in ACI Figure R20.4.1.

2.2.4 Durability of Steel Reinforcement

Specified Concrete Cover

Overview

Concrete cover, which is provided to reinforcing bars to protect them from weather and other effects, is measured from the concrete surface to the outermost surface of the reinforcement. Minimum specified concrete cover is given in ACI 20.5.1.2 through 20.5.1.4 (ACI 20.5.1.1). A greater cover than that prescribed in these sections may be required by the general building code for fire protection. Concrete floor finishes are permitted to be part of the required cover for nonstructural purposes (ACI 20.5.1.2).

Nonprestressed Cast-in-place Members

Minimum concrete cover for nonprestressed cast-in-place concrete members is given in ACI Table 20.5.1.3.1 (ACI 20.5.1.3.1; see Table 2.10).

Table 2.10 Minimum Concrete Cover for Cast-in-place Nonprestressed Concrete Members

Concrete Exposure	Member	Reinforcement	Specified Cover (in.)
Cast against and permanently in contact with the ground	All	All	3.0
Exposed to weather or in contact with the ground	All	#6 through #18 bars	2.0
		#5 bars and smaller	1.5
Not exposed to weather or in contact with the ground	Slabs, joists, and walls	#14 and #18 bars	1.5
		#11 bars and smaller	0.75
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	1.5

Deep Foundation Members

Specified concrete cover for cast-in-place deep foundation members (for example, piers and caissons) is given in ACI Table 20.5.1.3.4 (ACI 20.5.1.3.4; see Table 2.11).

Table 2.11 Minimum Concrete Cover for Cast-in-place Deep Foundation Members

Concrete Exposure	Reinforcement	Specified Cover (in.)
Cast against and permanently in contact with the ground and not enclosed by steel pipe, tube, permanent casing, or stable rock socket	All	3.0
Enclosed by steel pipe, tube, permanent casing, or stable rock socket	All	1.5

Bundled Reinforcing Bars

For bundled reinforcing bars, the specified concrete cover is equal to the smaller of the equivalent diameter of the bundle or 2.0 in. (ACI 20.5.1.3.5).

For bundled bars in members where the concrete is cast against and is permanently in contact with the ground, the specified cover is 3.0 in.

Headed Shear Stud Reinforcement

The specified concrete cover for heads and base rails in headed shear stud assemblies must be at least equal to that required for the reinforcement in the member (ACI 20.5.1.3.6; see ACI Figure R20.5.1.3.6) For example, the minimum concrete cover to the heads and rails is 0.75 in. for a headed shear stud assembly used as shear reinforcement in a two-way reinforced concrete slab not exposed to the elements and reinforced with longitudinal reinforcement smaller than #11 bars (see Table 2.10).

Corrosive Environments

For members in corrosive environments where it is expected the concrete will be exposed to external sources of chlorides (such as deicing salts, brackish water, seawater, or spray from these sources) or other severe exposure conditions, specified concrete cover should be increased above the minimum values specified in ACI 20.5.1.3 as deemed necessary (ACI 20.5.1.4.1). In such cases, the concrete should be proportioned to satisfy the requirements for the applicable exposure class specified in ACI Chapter 19.

A specified concrete cover of not less than 2.0 in. is recommended in ACI R20.5.1.4.1 for reinforcement in walls and slabs in corrosive environments. For other members, a 2.5-in. concrete cover is recommended.

Nonprestressed Coated Reinforcement

Coated reinforcing bars are used in applications where corrosion resistance is required (such as parking structures or other buildings and structures where the members are exposed to the elements). Nonprestressed coated reinforcing bars must conform to ACI Table 20.5.2.1 (ACI 20.5.2.1):

Zinc-coated (hot-dipped galvanized): ASTM A767

Epoxy-coated: ASTM A775 or A934

Zinc and epoxy dual-coated: ASTM A1055

The aforementioned coated bar types must conform to the requirements in ACI 20.2.1.3(a), (b), or (c).

2.3 Strength Reduction Factors

2.3.1 Overview

Strength reduction factors, ϕ, used in the determination of the design strength of a reinforced concrete member are given in ACI Chapter 21. The purposes of strength reduction factors are the following (ACI R21.1.1):

1. To account for the probability of under-strength members due to variations in material strengths and dimensions.

   The strength of concrete can vary because concrete is a composite material made of constituent materials whose properties vary. The strength of reinforcing steel can also vary but usually to a much lesser degree than concrete.

2. To account for inaccuracies in design equations.

   A number of assumptions and simplifications are made in the equations to determine nominal strengths. As such, inaccuracies are introduced that must be accounted for when determining the design strength of a reinforced concrete member.

3. To reflect the available ductility and required reliability of the reinforced concrete member under the load effects being considered.

   Reinforced concrete members that are more ductile, such as beams or slabs, are less sensitive to variations in concrete strength compared to members that are less ductile, such as columns. Spiral reinforcement confines the concrete in a column better than tied reinforcement. Thus, spiral columns are more ductile and have greater toughness than tied columns.

4. To reflect the importance of the reinforced concrete member in the structure.

   The failure of a vertical member, such as a column or wall, in a structure is usually considered more detrimental than failure of a horizontal member, such as a beam.

2.3.2 Strength Reduction Factors Based On Action or Structural Element

Strength reduction factors based on different types of actions and nonprestressed structural elements are given in ACI Table 21.2.1 (see ACI 21.2.1 and Table 2.12 for the strength reduction factors applicable in this publication). The strength reduction factor for shear in this table is not applicable to members in special moment frames (including beam-column joints), special structural walls (including diagonally reinforced coupling beams), and the other members noted in ACI 21.2.4 (see Section 2.3.4 of this publication).

Table 2.12 Strength Reduction Factors, ϕ

Action	Strength Reduction Factor, ϕ
Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with ACI 21.2.2 (see Section 2.3.3 of this publication)
Shear	0.75
Torsion	0.75
Bearing	0.65

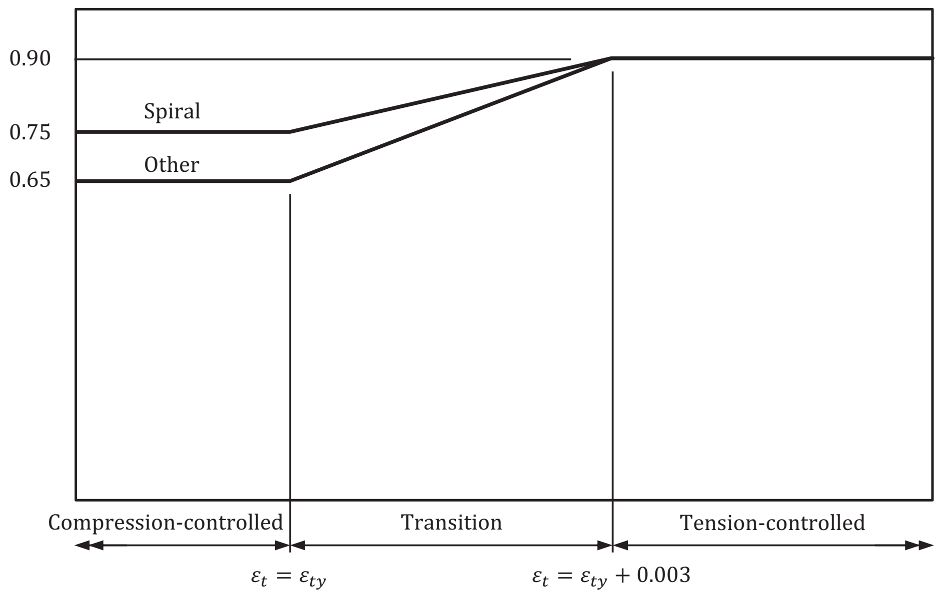
2.3.3 Strength Reduction Factors For Moment, Axial Force, Or Combined Moment and Axial Force

Strength reduction factors for reinforced concrete members subjected to bending moments, axial forces, or a combination of both are given in ACI Table 21.2.2 (see ACI 21.2.2, Table 2.13, and Figure 2.4).

Table 2.13 Strength Reduction Factors, ϕ, For Moment, Axal Force, Or Combined Moment and Axial Force

Net Tensile Strain, ε_t	Classification	Strength Reduction Factor, ϕ — Type of Transverse Reinforcement: Spirals Conforming to ACI 25.7.3	Other
$\varepsilon_t \le \varepsilon_{ty}$	Compression-controlled	0.75	0.65
$\varepsilon_{ty} < \varepsilon_t < \varepsilon_{ty} + 0.003$	Transition*	$0.75 + \dfrac{0.15(\varepsilon_t - \varepsilon_{ty})}{0.003}$	$0.65 + \dfrac{0.25(\varepsilon_t - \varepsilon_{ty})}{0.003}$
$\varepsilon_t \ge \varepsilon_{ty} + 0.003$	Tension-controlled	0.90	0.90

**For sections classified as transition, it is permitted to use ϕ corresponding to compression-controlled sections.*



$$\text{Spiral: } \phi = \begin{cases} 0.75 \text{ for } \varepsilon_t \leq \varepsilon_{ty} \\ 0.75 + \dfrac{0.15(\varepsilon_t - \varepsilon_{ty})}{0.003} \text{ for } \varepsilon_{ty} < \varepsilon_t < \varepsilon_{ty} + 0.003 \\ 0.90 \text{ for } \varepsilon_t \geq \varepsilon_{ty} + 0.003 \end{cases}$$

$$\text{Other: } \phi = \begin{cases} 0.65 \text{ for } \varepsilon_t \leq \varepsilon_{ty} \\ 0.65 + \dfrac{0.25(\varepsilon_t - \varepsilon_{ty})}{0.003} \text{ for } \varepsilon_{ty} < \varepsilon_t < \varepsilon_{ty} + 0.003 \\ 0.90 \text{ for } \varepsilon_t \geq \varepsilon_{ty} + 0.003 \end{cases}$$

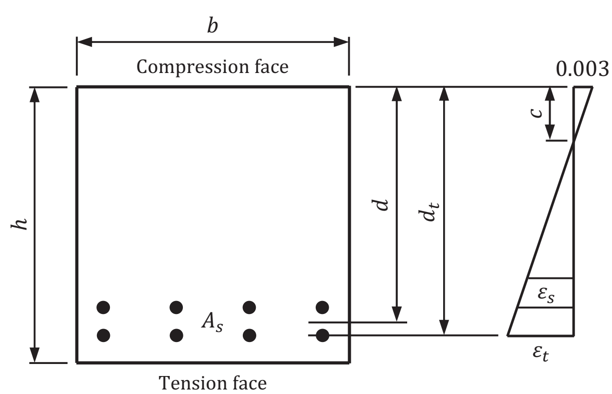


Figure 2.4 Variation of ϕ with net tensile strain ε_t.

The nominal strength of a member subjected to bending moments or combined bending moments and axial forces is determined assuming the strain in the extreme concrete compression fiber of the section is equal to 0.003 (see ACI 22.2.2.1). The net tensile strain, ε_t, is the tensile strain calculated in the extreme tension reinforcement at nominal strength (see Figure 2.4 for the case of a reinforced concrete beam with two layers of tension reinforcement at the section under consideration).

Compression-controlled sections are defined as sections where $\varepsilon_t \leq \varepsilon_{ty}$. The term ε_{ty} is the value of the net tensile strain in the extreme layer of longitudinal tension reinforcement used to define a compression-controlled section (ACI 21.2.2.1):

$$\varepsilon_{ty} = \frac{f_y}{E_s} \tag{2.5}$$

For Grade 60 reinforcement, ε_{ty} is permitted to be taken equal to 0.002 (ACI 21.2.2.1). Members subjected to only axial compression, like columns, are compression-controlled. In such cases, a brittle compression failure is expected with little or no warning prior to failure. As such, lower strength reduction factors are assigned to compression-controlled sections. Additionally, columns typically support areas much greater than beams, which are tension-controlled, and as noted in Section 2.3.1 of this publication, the consequences of column failure are generally more severe than those attributed to beam failure.

A section is defined as tension-controlled where $\varepsilon_t \geq \varepsilon_{ty} + 0.003$. This limit provides sufficient ductility for most applications and essentially provides a warning to failure (that is, excessive cracking and deflection are expected prior to failure). Beams and slabs must be tension-controlled.

A linear transition in the strength reduction factor is permitted between the limits for compression-controlled and tension-controlled sections (see Figure 2.4). It is permitted to use strength reduction factors corresponding to compression-controlled sections for sections classified as transition.

According to ACI 25.4.1.3, development lengths do not require a strength reduction factor. Because lap splice lengths are based on development lengths, a strength reduction factor is not required for lap splices as well. An allowance for strength reduction is included in the expressions in ACI Chapter 25 for determining development and splice lengths.

2.3.4 Strength Reduction Factors For Shear In Structures Relying On Special Moment Frames and Special Structural Walls

For structures that utilize special moment frames and special structural walls to resist earthquake effects E, the value of the strength reduction factor, ϕ, for shear is to be modified in accordance with ACI 21.2.4.1 through 21.2.4.4 (ACI 21.2.4).

For any member designed to resist E where the nominal shear strength of the member is less than the shear corresponding to the development of the nominal moment strength of the member, ϕ for shear must be taken as 0.60 (ACI 21.2.4.1). The nominal moment strength in such cases is to be calculated using the factored axial forces from load combinations in ACI Table 5.3.1 that include E. This provision is applicable to shear-controlled members, such as low-rise structural walls, portions of walls between openings, or diaphragms where nominal shear strength is less than the shear corresponding to the development of nominal moment strength. In most cases, it would be impractical to design the member otherwise.

For diaphragms, ϕ for shear must not exceed the value of ϕ for shear required for the vertical components of the primary seismic-force-resisting system (SFRS) [ACI 21.2.4.2]. This provision is intended to increase the strength of the diaphragm and its connections in buildings where $\phi = 0.60$ for the walls assigned to resist the earthquake forces. This requirement is also applicable to foundation elements supporting the primary vertical elements of the SFRS (ACI 21.2.4.3).

For beam-column joints of special moment frames and diagonally reinforced coupling beams, ϕ for shear is equal to 0.85 (ACI 21.2.4.4).

Chapter 3
DESIGN LOADS AND LOAD COMBINATIONS

3.1 Overview

According to ACI 4.3, the loads and load combinations considered in design must be in accordance with ACI Chapter 5. Presented in this chapter are the typical loads and required load combinations applicable in the design of reinforced concrete buildings. Methods are given to determine the Seismic Design Category (SDC) of a building and live load reduction on structural members. Step-by-step procedures on how to determine wind forces on the main wind force resisting system (MWFRS), seismic forces on the seismic-force-resisting system (SFRS), and seismic design forces on diaphragms (including collectors) are also presented.

3.2 Design Loads

In the design of reinforced concrete members, the loads that must be included are self-weight, applied loads, and the effects from wind, earthquakes, restraint of volume change, and differential settlement (ACI 5.2.1). Loads are to be determined in accordance with the general building code or by the building official (ACI 5.2.2).

The load effects included in the 2018 IBC and ASCE/SEI 7-16 are given in Table 3.1.

Table 3.1 Summary of Load Effects

Notation	Load Effect	Section/Chapter No.	
		IBC	ASCE/SEI 7
D	Dead loads	1606	3.1
D_i	Weight of ice	1614	10
E	Seismic load effects	1613	12.4.2
E_m	Seismic load effects including overstrength	1613	12.4.3
F	Loads due to fluids with well-defined pressures and maximum heights	—	—
F_a	Flood loads	1612	5
H	Loads due to lateral earth pressures, ground water pressure, or pressure of bulk materials	1610	3.2
L	Roof live loads greater than 20 lb/ft^2 and floor live load	1607	4
L_r	Roof live loads of 20 lb/ft^2 or less	1607	4
R	Rain loads	1611	8
S	Snow loads	1608	7
T	Cumulative effects of self-straining forces and effects	—	1.3.4
W	Loads due to wind pressure	1609	26 – 31
W_i	Wind-on-ice loads	1614	10

3.3 Seismic Design Category

The SDC of a building is determined in accordance with the general building code or determined by the building official (ACI 5.2.2). All buildings and structures must be assigned to an SDC in accordance with IBC 1613.2.5 or ASCE/SEI 11.6 (ACI 4.4.6.1). Figure 3.1 can be used to determine the SDC based on the requirements in these sections. The section, figure, and table numbers in the figure are from ASCE/SEI 7. The terms S_{DS} and S_{D1} are the design, 5 percent damped, spectral response acceleration parameters at short periods and at a period of 1 second, respectively, which are determined in accordance with ASCE/SEI 11.4.5.

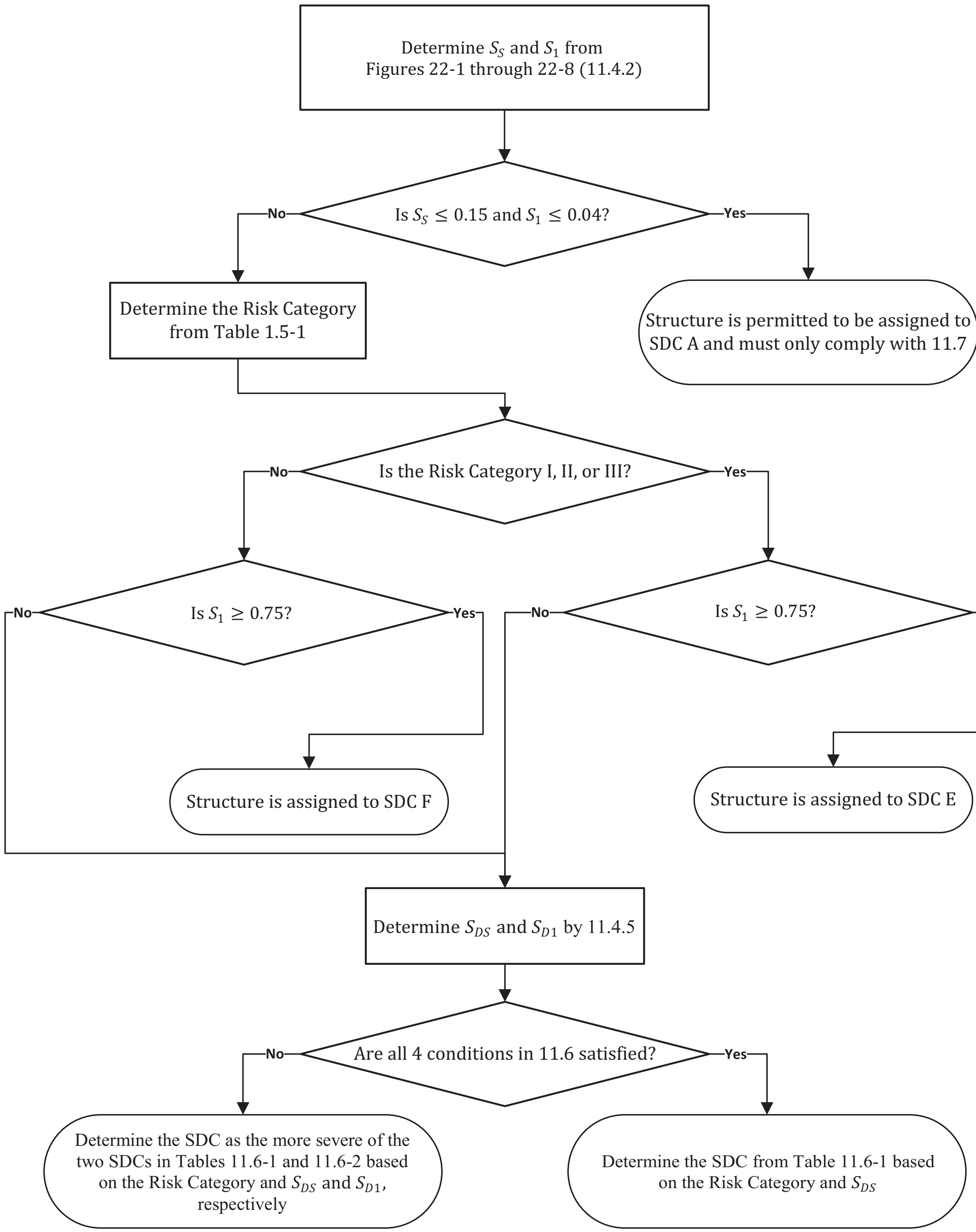


Figure 3.1 Determination of Seismic Design Category (SDC).

Structural members in buildings assigned to SDC A must satisfy all the applicable requirements in ACI 318 except for the requirements in ACI Chapter 18, which are not applicable (ACI 4.4.6.3). For buildings assigned to SDC B through F, the provisions of Chapter 18 must be satisfied in addition to the applicable requirements of other chapters (ACI 4.4.6.4). The requirements of ACI 4.4.6.5 pertain to structural members designated not to be part of the SFRS in buildings assigned to SDC B through F.

3.4 Live Load Reduction

According to ACI 5.2.3, live load reduction is permitted in accordance with the general building code. In the absence of a general building code, the live load reduction provisions in ASCE/SEI 4.7 and 4.8 for uniform live loads and uniform roof live loads, respectively, may be used.

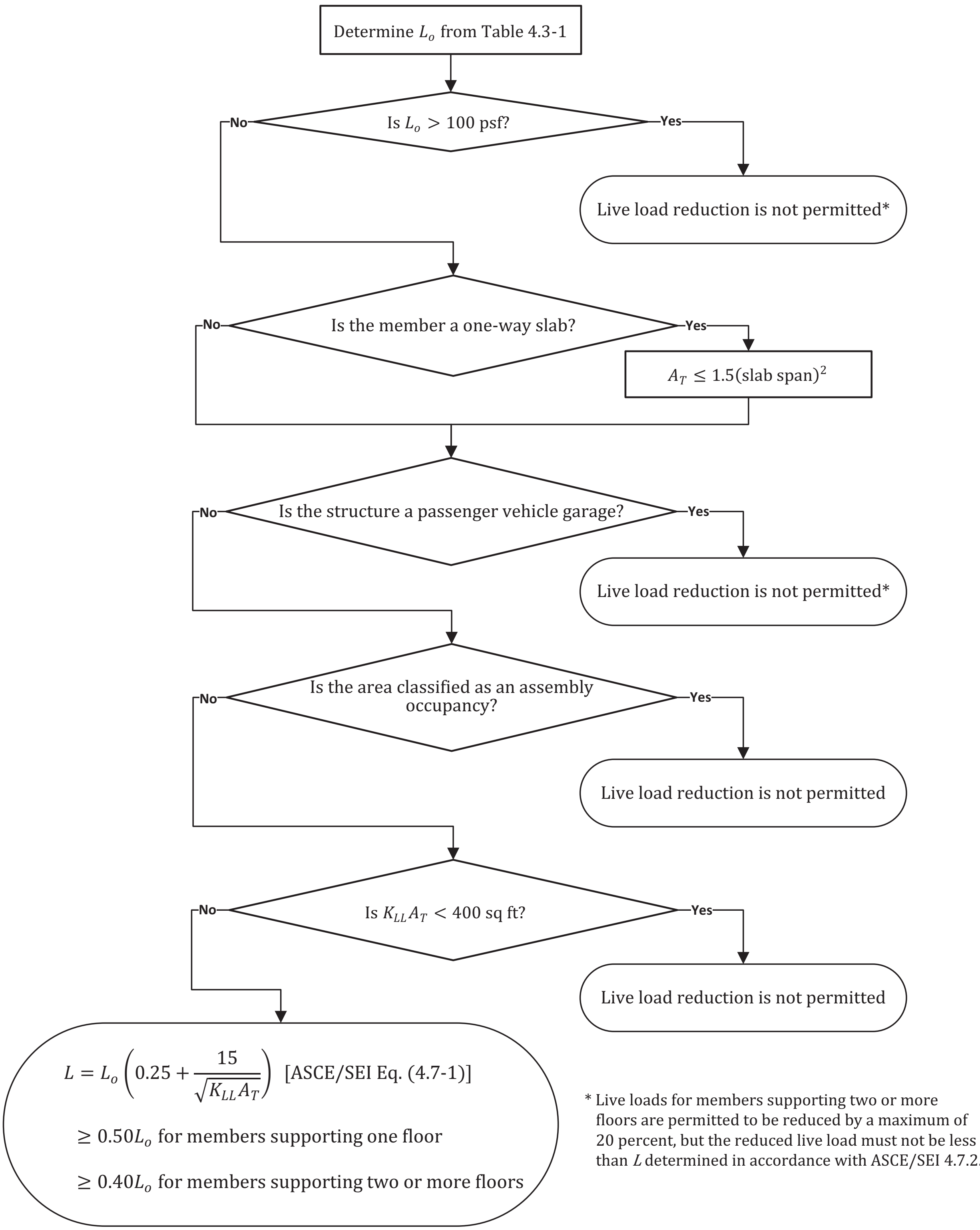


Figure 3.2 Determination of live load reduction in accordance with ASCE/SEI 4.7.

Reduced uniform live load, L, in accordance with ASCE/SEI 4.7 can be determined using the flowchart in Figure 3.2. In this figure, L_o is the uniformly distributed live load in ASCE/SEI Table 4.3-1, K_{LL} is the live load element factor in ASCE/SEI Table 4.7-1 (see ASCE/SEI Figure C4.7-1 and Table 3.2), and A_T is the tributary area in square feet. Similarly, reduced roof live load, L_r, can be determined by the flowchart in Figure 3.3 where R_1 and R_2 are reduction factors based on the tributary area, A_T, and the factor F, respectively. For pitched roofs, F is equal to the number of inches of rise per foot, and for arches or domes, it is the rise-to-span ratio multiplied by 32.

Table 3.2 Live Load Element Factor, K_{LL}

Element	K_{LL}
Interior columns	4
Exterior columns without cantilever slabs	4
Edge columns with cantilever slabs	3
Corner columns with cantilever slabs	2
Edge beams without cantilever slabs	2
Interior beams	2
All other members not identified, including • Edge beams with cantilever slabs • Cantilever beams • One-way slabs • Two-way slabs • Members without provisions for continuous shear transfer normal to their span	1

3.5 Load Factors and Combinations

The load combinations needed to determine the required strength, U, of a structural member are given in ACI Table 5.3.1 (see Table 3.3). The load combinations in IBC 1605.2 are the same as those in Table 3.3 with the following exceptions:

1. The variable f_1 in IBC Equations (16-3), (16-4), and (16-5) are not in ACI Equations (5.3.1c), (5.3.1d), and (5.3.1e). Instead, the load factor on the live load, L, in the ACI load combinations is equal to 1.0 with the exception that the load factor on L is permitted to equal 0.5 for all occupancies except for parking garages, areas occupied as places of public assembly, and areas where L is greater than 100 lb/ft^2 (ACI 5.3.3). This exception makes these load combinations the same in the IBC and ACI 318.
2. The variable f_2 in IBC Equation (16-5) is not in ACI Equation (5.3.1e). Instead, a load factor of 0.2 is applied to S. According to the second exception in ASCE/SEI 2.3.6, ASCE/SEI load combination 6 [which is the same as ACI Equation (5.3.1e)], S must be taken as either the flat roof snow load, p_f, or the sloped roof snow load, p_s. This means the balanced snow load defined in ASCE/SEI 7.3 for flat roofs and in ASCE/SEI 7.4 for sloped roofs can be used in ASCE/SEI load combination 6. Note that S in ASCE/SEI load combinations 2 and 4 is defined in the same way (see exception 2 in ASCE/SEI 2.3.1). Drift loads and unbalanced snow loads are covered by ASCE/SEI load combination 3.

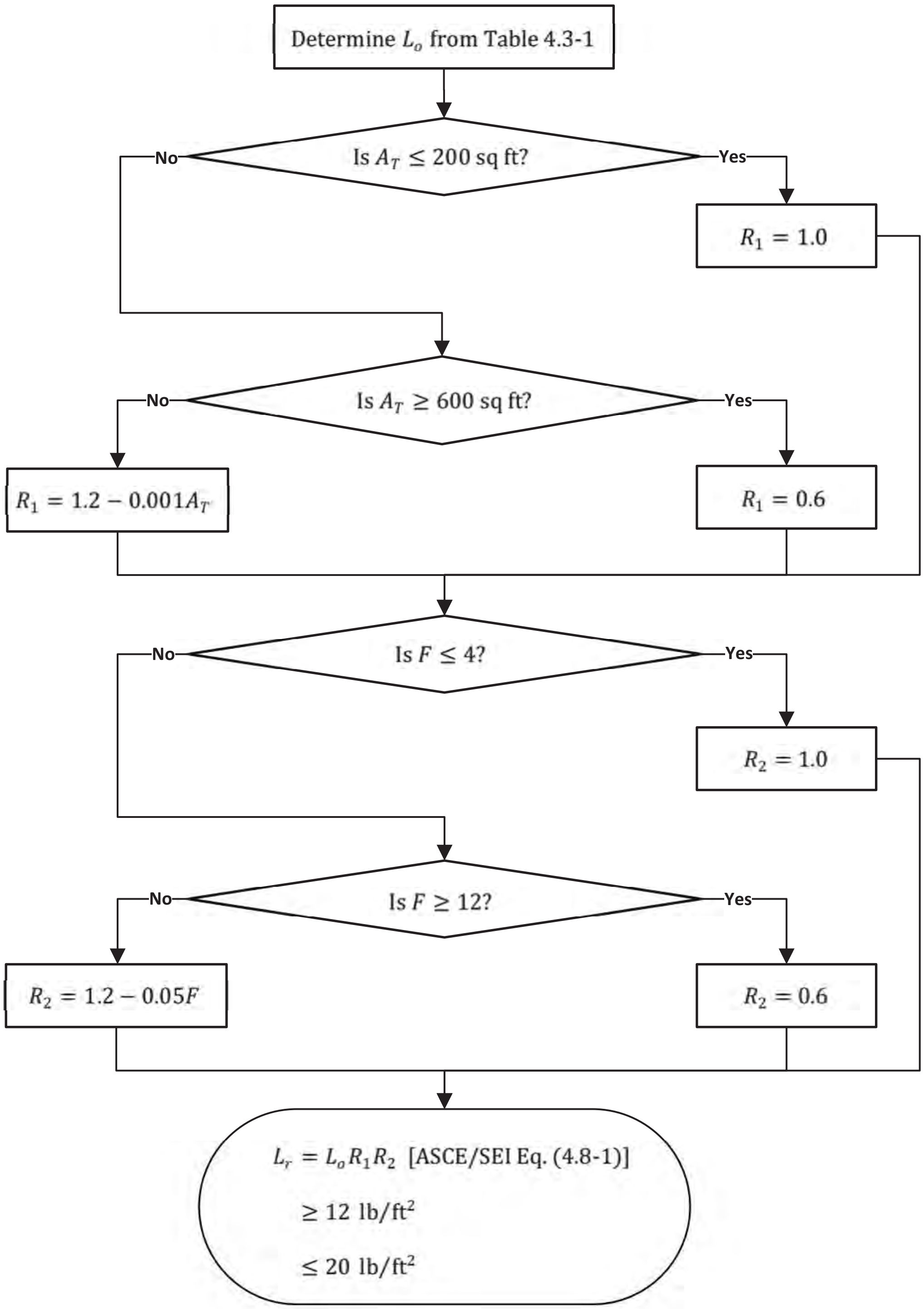


Figure 3.3 Determination of roof live load reduction in accordance with ASCE/SEI 4.8.

Table 3.3 ACI and ASCE/SEI Strength Design Load Combinations

ACI Equation Number	ASCE/SEI 7 Load Combination	Load Combination
5.3.1a	1	$U = 1.4D$
5.3.1b	2	$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
5.3.1c	3	$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$
5.3.1d	4	$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
5.3.1e	6	$U = 1.2D + 1.0E + 1.0L + 0.2S$
5.3.1f	5	$U = 0.9D + 1.0W$
5.3.1g	7	$U = 0.9D + 1.0E$

According to ACI 5.3.4, L must include the applicable effects from the following:

- Concentrated live loads (ASCE/SEI Table 4.3-1)
- Vehicular loads (ASCE/SEI 4.10)
- Crane loads (ASCE/SEI 4.9)
- Loads on handrails, guardrails, and vehicular barrier systems (ASCE/SEI 4.5)
- Impact effects (ASCE/SEI 4.6)
- Vibration effects

Where wind load, W, is provided at service-level loads, $1.6W$ must be used in place of $1.0W$ in ACI Equations (5.3.1d) and (5.3.1f), and $0.8W$ must be used in place of $0.5W$ in ACI Equation (5.3.1c) [ACI 5.3.5].

Seismic load effect, E, is defined in ASCE/SEI 12.4.2 as follows:

- For use in ACI Equation (5.3.1e) or load combination 6 in ASCE/SEI 2.3.6:

$$E = E_h + E_v = \rho Q_E + 0.2S_{DS}D \tag{3.1}$$

- For use in ACI Equation (5.3.1g) or load combination 7 in ASCE/SEI 2.3.6:

$$E = E_h - E_v = \rho Q_E - 0.2S_{DS}D \tag{3.2}$$

where E_h = effect of horizontal seismic forces $= \rho Q_E$

E_v = vertical seismic effect applied in the vertical downward direction $= 0.2S_{DS}D$

ρ = redundancy factor defined in ASCE/SEI 12.3.4 (ρ is permitted to be taken as 1.0 for buildings assigned to SDC B or C)

Q_E = effects from horizontal seismic forces (both sidesway right and left must be considered)

Seismic load combinations based on SDC using the provisions of ASCE/SEI 12.3.4.1 and 12.4.2 are given in Table 3.4.

Table 3.4 Seismic Load Combinations

SDC	ACI Equation Number	ASCE/SEI 7 Load Combination	Load Combination
B	5.3.1e	6	$U = 1.2D + Q_E + 1.0L + 0.2S$
	5.3.1g	7	$U = 0.9D + Q_E$
C	5.3.1e	6	$U = (1.2 + 0.2S_{DS})D + Q_E + 1.0L + 0.2S$
	5.3.1g	7	$U = (0.9 - 0.2S_{DS})D + Q_E$
D, E, and F	5.3.1e	6	$U = (1.2 + 0.2S_{DS})D + \rho Q_E + 1.0L + 0.2S$
	5.3.1g	7	$U = (0.9 - 0.2S_{DS})D + \rho Q_E$

Seismic load effect, E_m, is defined in ASCE/SEI 12.4.3 as follows:

- For use in ACI Equation (5.3.1e) or load combination 6 in ASCE/SEI 2.3.6:

$$E_m = E_{mh} + E_v = \Omega_o Q_E + 0.2S_{DS}D \tag{3.3}$$

- For use in ACI Equation (5.3.1g) or load combination 7 in ASCE/SEI 2.3.6:

$$E_m = E_{mh} - E_v = \Omega_o Q_E - 0.2S_{DS}D \tag{3.4}$$

where E_{mh} = effect of horizontal seismic forces, including overstrength $= \Omega_o Q_E$

Ω_o = overstrength factor given in ASCE/SEI Table 12.2-1 based on SFRS

Seismic load combinations based on SDC where seismic load effects including overstrength are required are given in Table 3.5.

Table 3.5 Seismic Load Combinations Where Seismic Load Effects Including Overstrength are Required

SDC	ACI Equation Number	ASCE/SEI 7 Load Combination	Load Combination
B	5.3.1e	6	$U = 1.2D + \Omega_o Q_E + 1.0L + 0.2S$
	5.3.1g	7	$U = 0.9D + \Omega_o Q_E$
C, D, E, and F	5.3.1e	6	$U = (1.2 + 0.2S_{DS})D + \Omega_o Q_E + 1.0L + 0.2S$
	5.3.1g	7	$U = (0.9 - 0.2S_{DS})D + \Omega_o Q_E$

The following load effects must be considered, where applicable, in combination with other load effects:

- Cumulative service effects of temperature, creep, shrinkage, and differential settlement, T, in accordance with ACI 5.3.6
- Effects of service lateral loads due to fluids, F, in accordance with ACI 5.3.7
- Effects of service load due to lateral earth pressure, H, in accordance with ACI 5.3.8
- Effects due to flood loads in accordance with ACI 5.3.9
- Effects due to atmospheric ice loads in accordance with ACI 5.3.10

Load combinations where flood loads, F_a, and atmospheric ice loads must be considered are given in Table 3.6.

Table 3.6 Strength Design Load Combination for Flood Loads and Atmospheric Ice Loads

Load Effect		ASCE/SEI load combination	Load Combination
Flood load, F_a *	V Zones or Coastal A Zones	4	$U = 1.2D + 1.0W + 2.0F_a + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
		5	$U = 0.9D + 1.0W + 2.0F_a$
	Noncoastal A Zones	4	$U = 1.2D + 0.5W + 1.0F_a + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
		5	$U = 0.9D + 0.5W + 1.0F_a$
Atmospheric ice load		2	$U = 1.2D + 1.6L + 0.2D_i + 0.5S$
		4	$U = 1.2D + 1.0L + D_i + W_i + 0.5S$
		4	$U = 1.2D + D_i$
		5	$U = 0.9D + D_i + W_i$

** Definitions of V Zones and Coastal A Zones are given in ASCE/SEI 5.2.*

According to ACI 5.3.2, load combinations must be investigated with one or more of the variable loads set equal to zero. It is possible that the most critical load effects on a member occur when one or more variable loads are not present.

Load combinations for extraordinary events and for general structural integrity loads are given in ASCE/SEI 2.5 and 2.6, respectively.

3.6 Determination of Wind Forces

The flowchart in Figure 3.4 can be used to determine the total design wind pressures at a height z above ground level on the MWFRS of an enclosed or partially enclosed, rigid or flexible reinforced concrete building without a parapet and with an essentially flat roof using Part 1 in Chapter 27 of ASCE/SEI 7. The section, figure, table, and equation numbers in Figure 3.4 are from ASCE/SEI 7. Horizontal internal pressures cancel out when determining total wind pressures on the MWFRS of a building.

For building structures, it is customary to calculate wind pressures at the roof and floor levels and to assume that the pressures are uniformly distributed over the tributary area at that level. Thus, the total wind force at a roof or floor level is determined by multiplying the total wind pressure at that level (windward plus leeward pressure) by the tributary area, which is equal to the tributary story height times the width of the windward face of the building (see Figure 3.5).

In lieu of using the figures in ASCE/SEI Chapter 26, the basic wind speed, V, can be obtained from Reference 4 or 5.

3.7 Determination of Seismic Forces

3.7.1 Seismic Forces on the SFRS

The flowchart in Figure 3.6 can be used to determine seismic forces over the height of a reinforced concrete SFRS in accordance with the Equivalent Lateral Force (ELF) Procedure of ASCE/SEI 12.8; it is applicable to sites where a site response analysis in accordance with ASCE/SEI 11.4.8 need not be performed and to structures without seismic isolation or damping systems. The section, figure, table, and equation numbers in Figure 3.6 are from ASCE/SEI 7. The type of SFRS permitted to be used is designated by the general building code or by the building official in areas without a legally adopted building code (ACI 4.4.6.2).

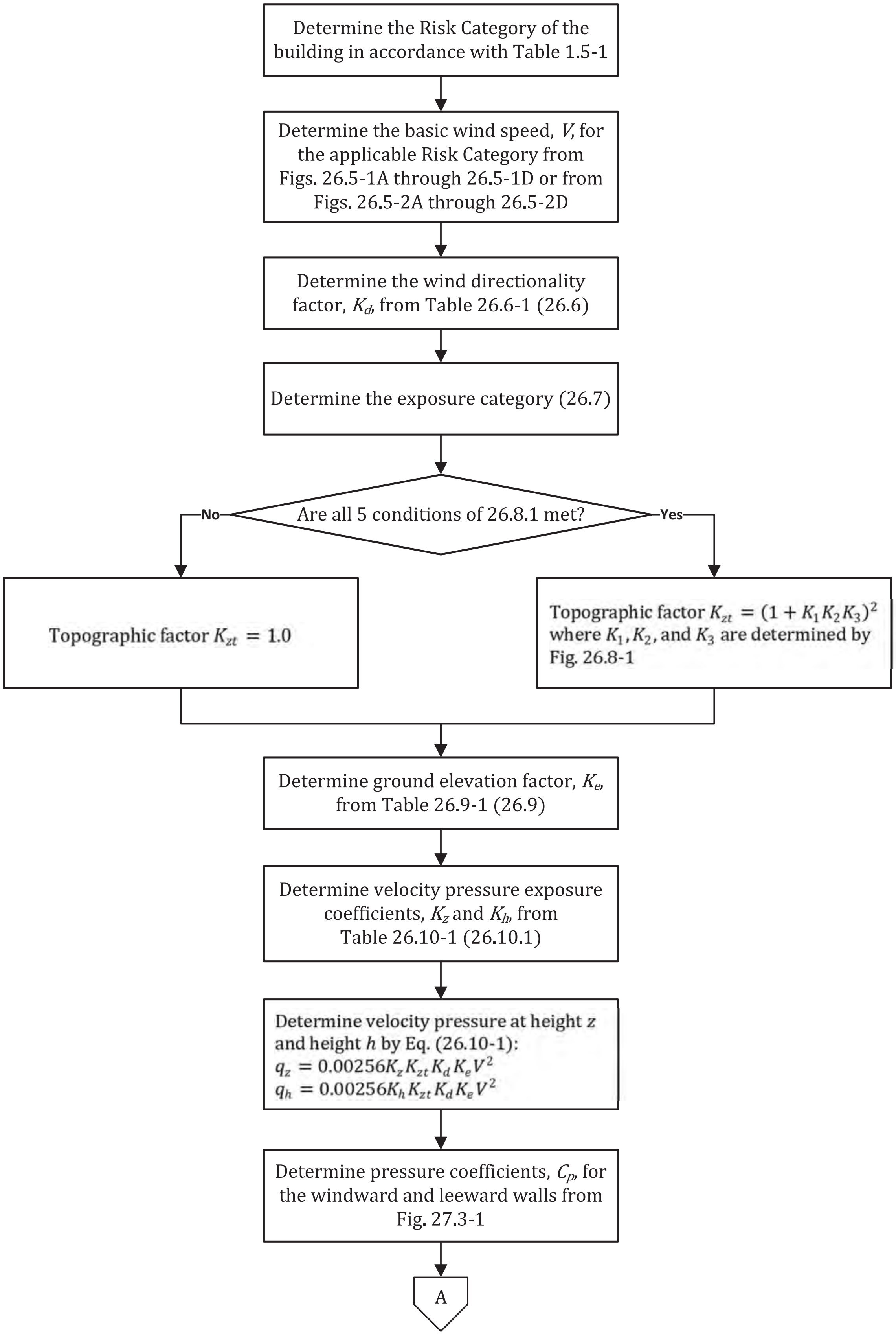


Figure 3.4 Determination of total wind pressure on the MWFRS of a building in accordance with Part 1 in Chapter 27 of ASCE/SEI 7.

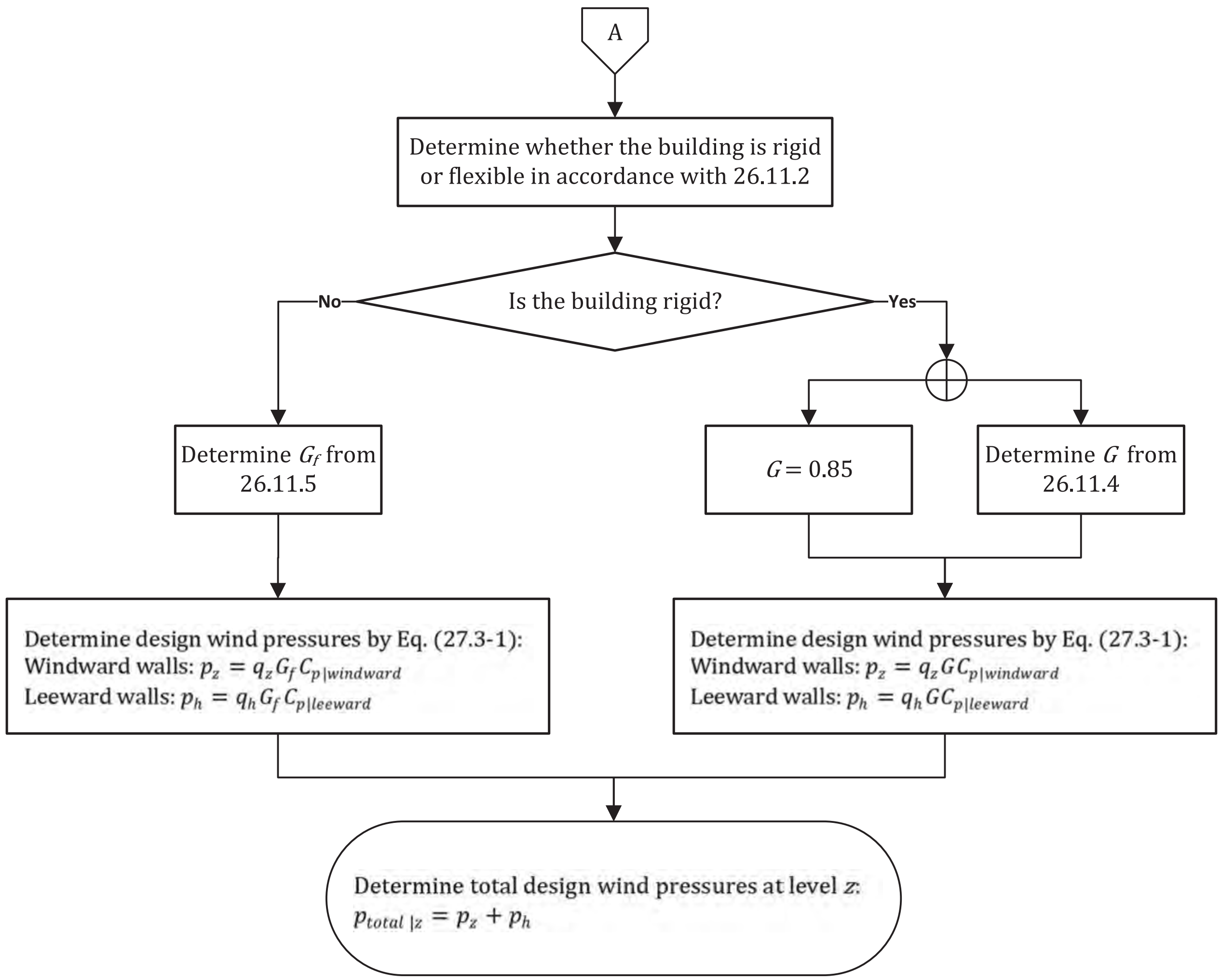
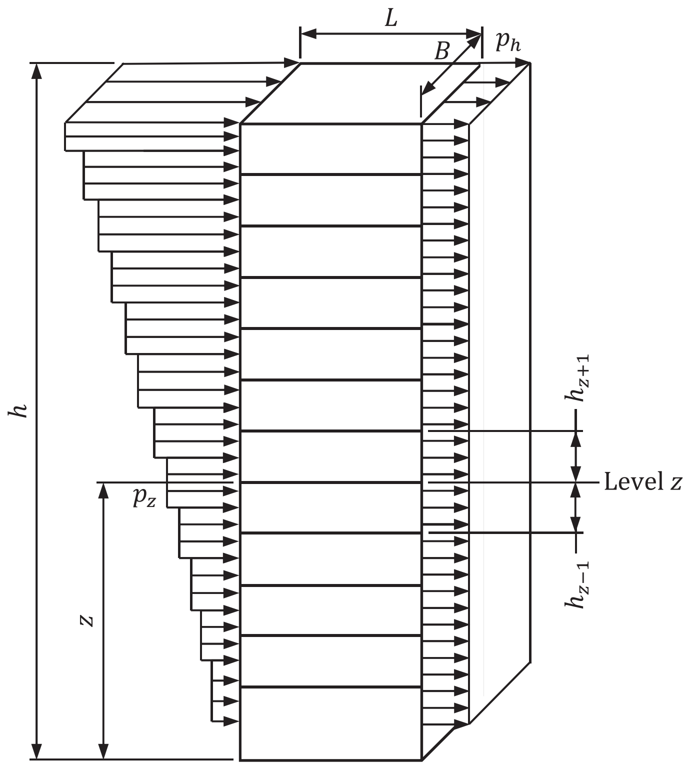


Figure 3.4 (cont.) Determination of total wind pressure on the MWFRS of a building in accordance with Part 1 in Chapter 27 of ASCE/SEI 7.



$$\text{Total wind force at level } z = (p_z + p_h)\left(\frac{h_{z+1} + h_{z-1}}{2}\right)B$$

Figure 3.5 Determination of the total wind force at a level of the MWFRS.

3.7.2 Seismic Forces on Diaphragms, Chords, and Collectors

Floor and roof diaphragms must be designed to resist the effects due to out-of-plane gravity loads and in-plane lateral forces using the load combinations in ACI 4.3 (ACI 4.4.7.1).

For buildings assigned to SDC B through F, diaphragms are to be designed for the larger of the forces in (1) and (2) below (ASCE/SEI 12.10.1.1):

1. Design seismic force, F_x, determined from the structural analysis (see Figure 3.6).
2. Diaphragm design force, F_{px}, determined by ASCE/SEI Equation (12.10-1):

$$F_{px} = \left(\sum_{i=x}^{n} F_i \Big/ \sum_{i=x}^{n} w_i\right) w_{px}$$

where w_{px} = weight tributary to the diaphragm at level x

F_i = portion of the seismic base shear, V, induced at level i (see Figure 3.6)

w_i = weight tributary to level i

Minimum and maximum values of F_{px} are determined by ASCE/SEI Equations (12.10-2) and (12.10-3), respectively, which are independent of the design base shear, V:

Minimum $F_{px} = 0.2 S_{DS} I_e w_{px}$

Maximum $F_{px} = 0.4 S_{DS} I_e w_{px}$

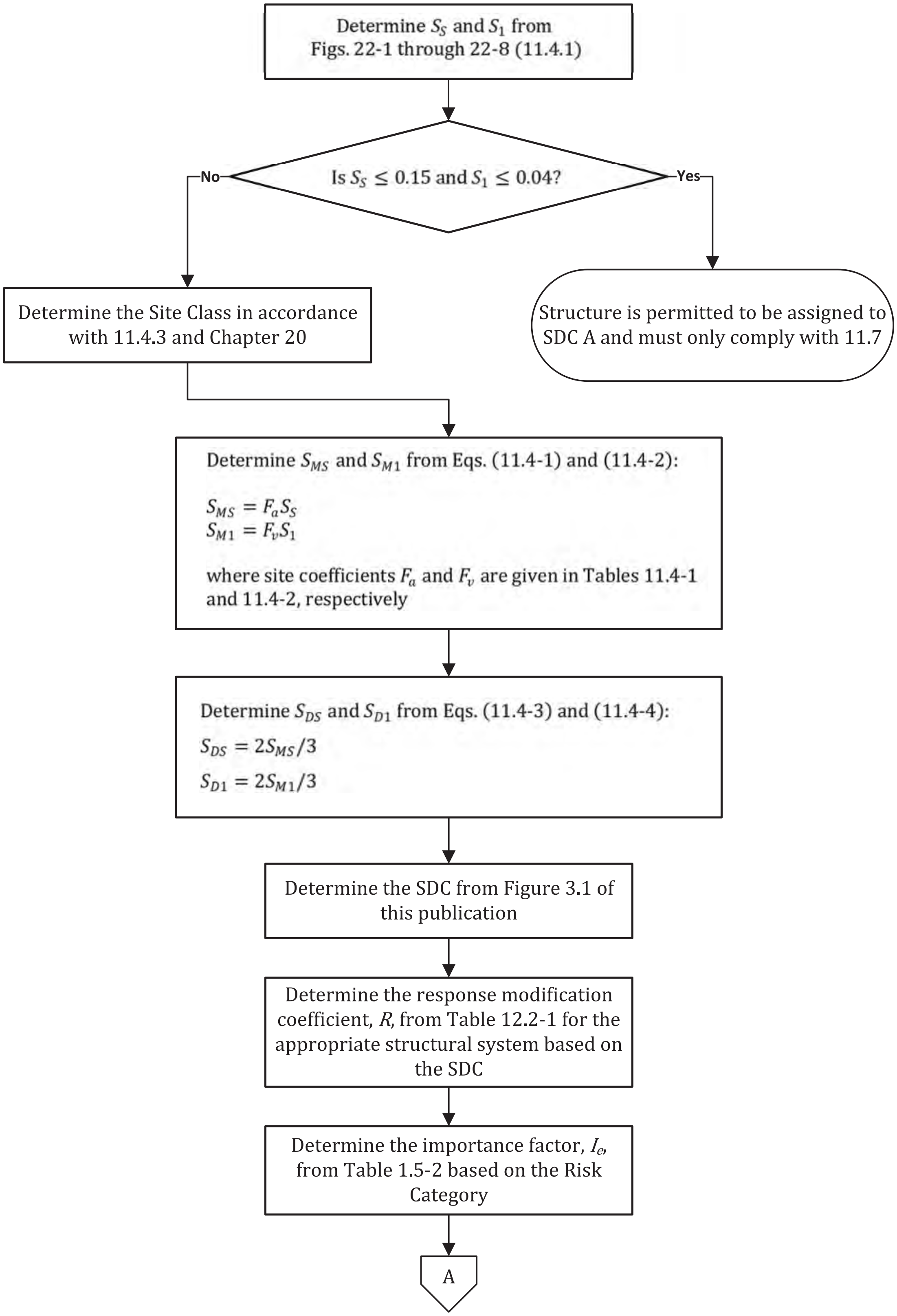
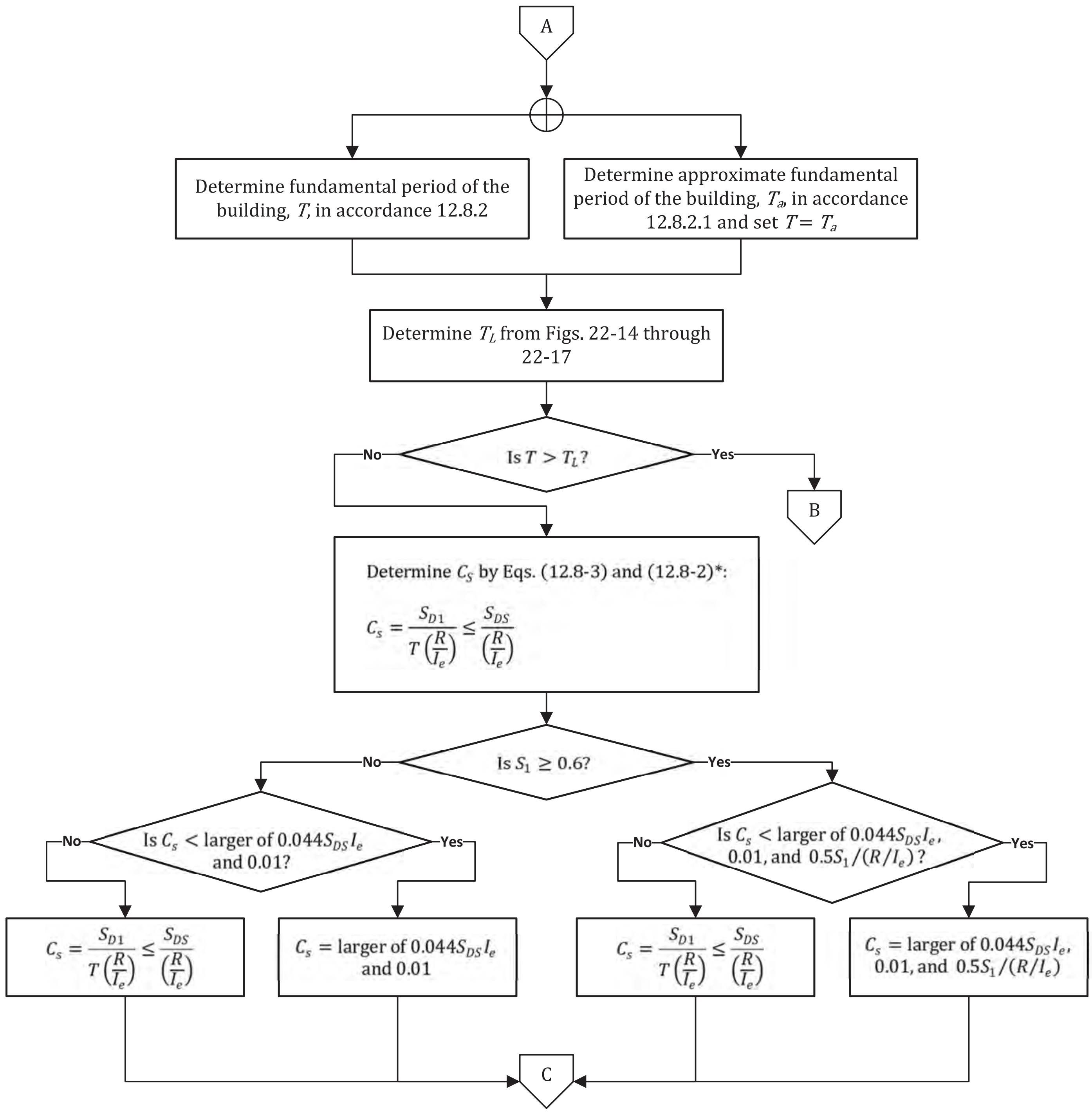


Figure 3.6 Determination of seismic forces on a SFRS in accordance with the ELF Procedure in ASCE/SEI 12.8.



* Value of C_S is permitted to be calculated using a value of 1.0 for S_{DS}, but not less than $0.7S_{DS}$, provided all criteria of 12.8.1.3 are met

Figure 3.6 (cont.) Determination of seismic forces on a SFRS in accordance with the ELF Procedure in ASCE/SEI 12.8.

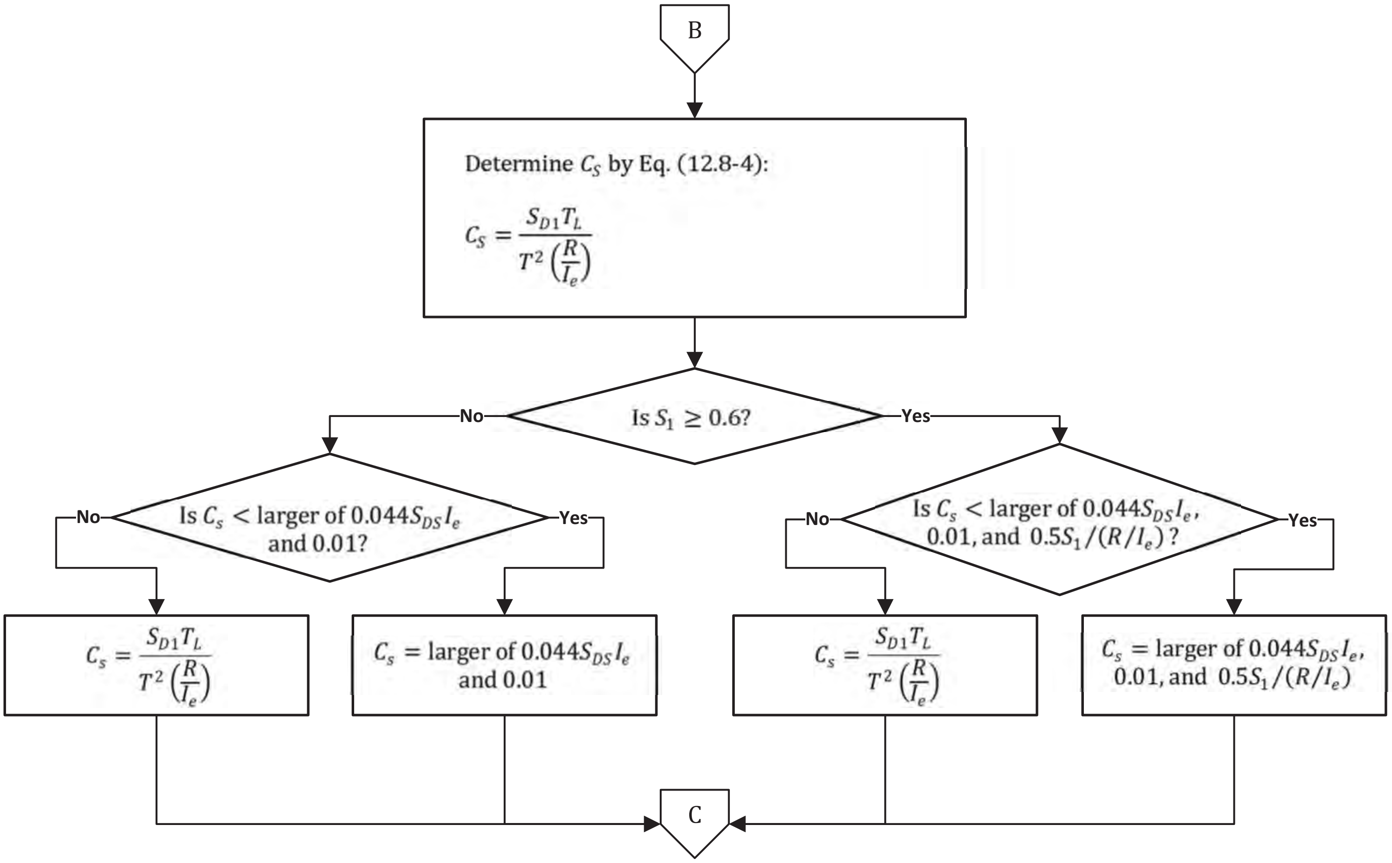


Figure 3.6 (cont.) Determination of seismic forces on a SFRS in accordance with the ELF Procedure in ASCE/SEI 12.8.

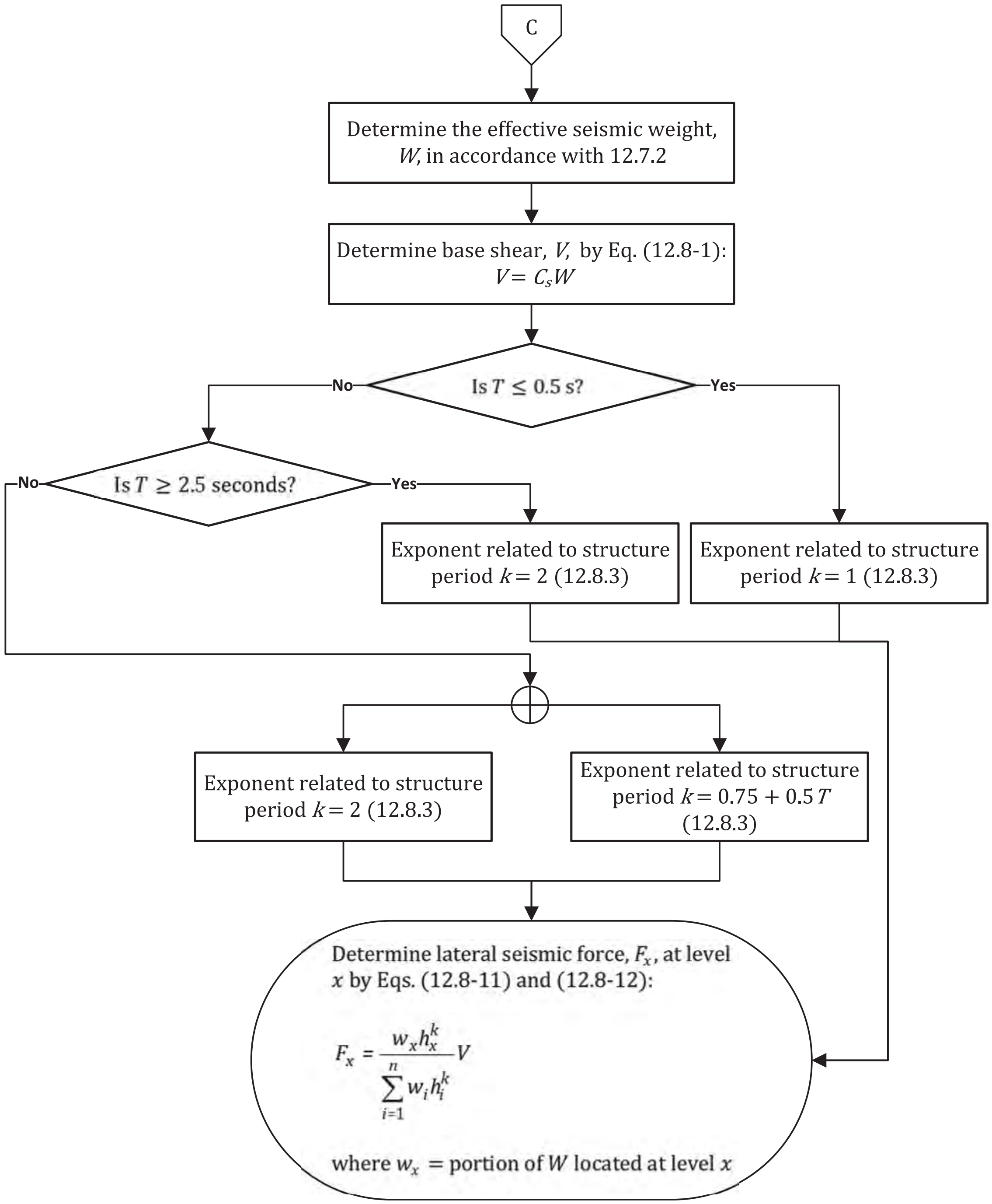


Figure 3.6 (cont.) Determination of seismic forces on a SFRS in accordance with the ELF Procedure in ASCE/SEI 12.8.

In addition to F_{px}, diaphragms must be designed for applicable transfer forces (see ASCE/SEI 12.10.1.1).

Collectors must be provided where required to transmit forces between diaphragms and the vertical elements of the LFRS (ACI 4.4.7.5). For buildings assigned to SDC C through F, collectors and their connections to the vertical elements of the SFRS must be designed to resist the effects from the maximum of the following forces (ASCE/SEI 12.10.2.1):

1. Forces calculated using the seismic load effects including overstrength of ASCE/SEI 12.4.3 with seismic forces determined by the ELF Procedure of ASCE/SEI 12.8 or the modal response spectrum analysis procedure of ASCE/SEI 12.9.1.

   In ACI Equations (5.3.1e) and (5.3.1g) or ASCE/SEI load combinations 6 and 7, use $E_{mh} = \Omega_o Q_E$ where Q_E is determined using F_x.
2. Forces calculated using the seismic load effects including overstrength of ASCE/SEI 12.4.3 with seismic forces determined by ASCE/SEI Equation (12.10-1) for diaphragms.

   In ACI Equations (5.3.1e) and (5.3.1g) or ASCE/SEI load combinations 6 and 7, use $E_{mh} = \Omega_o Q_E$ where Q_E is determined using F_{px}.
3. Forces calculated using the load combinations of ASCE/SEI 2.3.6 with seismic forces determined by ASCE/SEI Equation (12.10-2), which is the lower-limit diaphragm force.

   In ACI Equations (5.3.1e) and (5.3.1g) or ASCE/SEI load combinations 6 and 7, use $E_h = \rho Q_E$ where Q_E is determined using $F_{px} = 0.2 S_{DS} I_e w_{px}$.

Diaphragms and collectors in buildings assigned to SDC D, E, and F are designed in accordance with ACI Chapter 18 (ACI 4.4.7.6).

3.8 Examples

3.8.1 Example 3.1 – Determination of Wind Forces: Building #1

Determine the wind forces on the MWFRS in the north-south and east-west directions (see Figure 1.1). Assume the overall building dimensions are 95.67 ft in the north-south direction and 151.67 ft in the east-west direction.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the Risk Category ASCE/SEI Table 1.5-1

For the Business Group B occupancy, the Risk Category is II.

Step 2 – Determine the basic wind speed ASCE/SEI Figure 26.5-1B

For the given location of this Risk Category II building, $V = 107$ mph (which can be obtained from Ref. 4 or 5).

Step 3 – Determine the wind directionality factor ASCE/SEI Table 26.6-1

For the MWFRS of a building, $K_d = 0.85$.

Step 4 – Determine the exposure category ASCE/SEI 26.7

The exposure category is given as C in the design data.

Step 5 – Determine the topographic factor ASCE/SEI 26.8

The building is not located on a hill or escarpment, so $K_{zt} = 1.0$.

Step 6 – Determine the ground elevation factor ASCE/SEI Table 26.9-1

The ground elevation factor, K_e, is permitted to be 1.0 in all cases (see Note 1 in ASCE/SEI Table 26.9-1).

Step 7 – Determine the velocity pressure exposure coefficients ASCE/SEI Table 26.10-1

The velocity pressure exposure coefficients are determined from the following equations, which are given in Note 1 in ASCE/SEI Table 26.10-1:

$$K_z = \begin{cases} 2.01(15 / z_g)^{2/\alpha} \text{ for } z < 15 \text{ ft} \\ \\ 2.01(z / z_g)^{2/\alpha} \text{ for } 15 \text{ ft} \leq z \leq z_g \end{cases}$$

For Exposure C, $\alpha = 9.5$ and $z_g = 900$ ft from ASCE/SEI Table 26.11-1.

Values of K_z and K_h are given in Table 3.7.

Table 3.7 Velocity Pressure Exposure Coefficients, K_z, Building #1

Height Above Ground Level, z (ft)	K_z
60.0	1.14
48.5	1.09
37.0	1.03
25.5	0.95
14.0	0.85

Step 8 – Determine the velocity pressures ASCE/SEI Eq. (26.10-1)

$$q_z = 0.00256 K_z K_{zt} K_d K_e V^2 = 0.00256 \times K_z \times 1.0 \times 0.85 \times 1.0 \times 107^2 = 24.91 K_z$$

Values of q_z are given in Table 3.8.

Table 3.8 Velocity Pressures, q_z, Building #1

Height Above Ground Level, z (ft)	K_z	q_z (lb/ft^2)
60.0	1.14	28.4
48.5	1.09	27.2
37.0	1.03	25.7
25.5	0.95	23.7
14.0	0.85	21.2

Step 9 – Determine the external pressure coefficients ASCE/SEI Figure 27.3-1

For wind in the north-south direction:

- Windward wall: $C_p = 0.80$ for all L / B
- Leeward wall: $L / B = 95.67 / 151.67 = 0.63$; $C_p = -0.50$

For wind in the east-west direction:

- Windward wall: $C_p = 0.80$ for all L / B
- Leeward wall: $L / B = 151.67 / 95.67 = 1.59$; $C_p = -0.38$ from linear interpolation

Step 10 – Determine whether the building is rigid or not ASCE/SEI 26.11.2

Check if the natural frequency can be determined using the provisions of ASCE/SEI 26.11.3:

1. Building height $= 60.0\ \text{ft} < 300.0\ \text{ft}$
2. Building height $= 60.0\ \text{ft} < 4L_{eff} = \begin{cases} 4 \times 95.67 = 382.7\ \text{ft} \\ \\ 4 \times 151.67 = 606.7\ \text{ft} \end{cases}$ (building length in each direction is constant over the height, which means L_{eff} is equal to the building length parallel to the wind direction)

Therefore, the provisions of ASCE/SEI 26.11.3 may be used to determine the approximate natural frequency, n_a.

For concrete moment-resisting frame buildings [ASCE/SEI Eq. (26.11-3)]:

$$n_a = 43.5 / h^{0.9} = 43.5 / 60.0^{0.9} = 1.09\ \text{Hz} > 1.0\ \text{Hz}$$

Therefore, the building is rigid in both directions.

Step 11 – Determine the gust-effect factor ASCE/SEI 26.11.1

For rigid buildings, G is permitted to be taken as 0.85.

Step 12 – Determine the design wind pressures ASCE/SEI Eq. (27.3-1)

$$p_z = q_z G C_p = 0.85 q_z C_p$$

For wind in the north-south direction:

- Windward wall: $p_z = 0.85 q_z C_{p|windward} = 0.85 \times 0.80 \times q_z = 0.68 q_z$
- Leeward wall: $p_h = 0.85 q_h C_{p|leeward} = 0.85 \times 28.4 \times (-0.50) = -12.1\ \text{lb/ft}^2$
- Total pressure on MWFRS $= 0.68 q_z + 12.1\ \text{lb/ft}^2$

For wind in the east-west direction:

- Windward wall: $p_z = 0.85 q_z C_{p|windward} = 0.85 \times 0.80 \times q_z = 0.68 q_z$
- Leeward wall: $p_h = 0.85 q_h C_{p|leeward} = 0.85 \times 28.4 \times (-0.38) = -9.2\ \text{lb/ft}^2$
- Total pressure on MWFRS $= 0.68 q_z + 9.2\ \text{lb/ft}^2$

Total design wind pressures in the north-south and east-west directions are given in Table 3.9.

Table 3.9 Total Design Wind Pressures, p (lb/ft^2), Building #1

Height Above Ground Level, (ft)	North-South	East-West
60.0	31.4	28.5
48.5	30.6	27.7
37.0	29.6	26.7
25.5	28.2	25.3
14.0	26.5	23.6

The total design wind pressures are greater than the minimum wind pressure prescribed in ASCE/SEI 27.1.5, which is equal to 16.0 lb/ft^2 for walls.

Step 13 – Determine the total design wind forces

Total design wind forces are obtained at each level by multiplying the total design wind pressures in Table 3.9 by the tributary area for each level (see Table 3.10).

Table 3.10 Total Design Wind Forces, Building 1

Height Above Ground Level, z (ft)	North-South		East-West	
	Tributary Area (sq ft)	Wind Force (kips)	Tributary Area (sq ft)	Wind Force (kips)
60.0	872.1	27.4	550.1	15.7
48.5	1,744.2	53.4	1,100.2	30.5
37.0	1,744.2	51.6	1,100.2	29.4
25.5	1,744.2	49.2	1,100.2	27.8
14.0	1,933.8	51.3	1,219.8	28.8
	Σ	232.9		132.2

3.8.2 Example 3.2 – Determination of Wind Forces: Building #2

Determine the wind forces on the MWFRS in the north-south and east-west directions (see Figure 1.2). Assume the overall building dimensions are 182.0 ft in the north-south direction and 122.5 ft in the east-west direction.

Design data are given in Sect. 1.2.2.

Step 1 – Determine the Risk Category — ASCE/SEI Table 1.5-1

For an essential facility, the Risk Category is IV.

Step 2 – Determine the basic wind speed — ASCE/SEI Figure 26.5-1D

For the given location of this Risk Category IV building, $V = 129 \text{ mph}$ (which can be obtained from Ref. 4 or 5).

Step 3 – Determine the wind directionality factor — ASCE/SEI Table 26.6-1

For the MWFRS of a building, $K_d = 0.85$.

Step 4 – Determine the exposure category — ASCE/SEI 26.7

The exposure category is given as B in the design data.

Step 5 – Determine the topographic factor — ASCE/SEI 26.8

The building is not located on a hill or escarpment, so $K_{zt} = 1.0$.

Step 6 – Determine the ground elevation factor — ASCE/SEI Table 26.9-1

The ground elevation factor, K_e, is permitted to be 1.0 in all cases (see Note 1 in ASCE/SEI Table 26.9-1).

Step 7 – Determine the velocity pressure exposure coefficients — ASCE/SEI Table 26.10-1

The velocity pressure exposure coefficients are determined from the following equations, which are given in Note 1 in ASCE/SEI Table 26.10-1:

$$K_z = \begin{cases} 2.01(15 / z_g)^{2/\alpha} \text{ for } z < 15 \text{ ft} \\ \\ 2.01(z / z_g)^{2/\alpha} \text{ for } 15 \text{ ft} \le z \le z_g \end{cases}$$

For Exposure B, $\alpha = 7.0$ and $z_g = 1{,}200$ ft from ASCE/SEI Table 26.11-1.

Values of K_z and K_h are given in Table 3.11.

Table 3.11 Velocity Pressure Exposure Coefficients, K_z, Building #2

Height Above Ground Level, z (ft)	K_z
139.0	1.09
128.0	1.06
117.0	1.03
106.0	1.01
95.0	0.97
84.0	0.94
73.0	0.90
62.0	0.86
51.0	0.82
40.0	0.76
29.0	0.69
18.0	0.61

Step 8 – Determine the velocity pressures — ASCE/SEI Eq. (26.10-1)

$$q_z = 0.00256 K_z K_{zt} K_d K_e V^2 = 0.00256 \times K_z \times 1.0 \times 0.85 \times 1.0 \times 129^2 = 36.21 K_z$$

Values of q_z are given in Table 3.12.

Table 3.12 Velocity Pressures, q_z, Building #2

Height Above Ground Level, z (ft)	K_z	q_z (lb/ft^2)
139.0	1.09	39.5
128.0	1.06	38.4
117.0	1.03	37.3
106.0	1.01	36.6
95.0	0.97	35.1
84.0	0.94	34.0
73.0	0.90	32.6
62.0	0.86	31.1
51.0	0.82	29.7
40.0	0.76	27.5
29.0	0.69	25.0
18.0	0.61	22.1

Step 9 – Determine the external pressure coefficients — ASCE/SEI Figure 27.3-1

For wind in the north-south direction:

Windward wall: $C_p = 0.80$ for all L / B

Leeward wall: $L / B = 182.0 / 122.5 = 1.49$; $C_p = -0.40$ from linear interpolation

For wind in the east-west direction:

Windward wall: $C_p = 0.80$ for all L / B

Leeward wall: $L / B = 122.5 / 182.0 = 0.67$; $C_p = -0.50$

Step 10 – Determine whether the building is rigid or not — ASCE/SEI 26.11.2

Check if the natural frequency can be determined using the provisions of ASCE/SEI 26.11.3:

1. Building height $= 139.0$ ft < 300.0 ft
2. Building height $= 139.0 \text{ ft} < 4L_{eff} = \begin{cases} 4 \times 182.0 = 728.0 \text{ ft} \\ 4 \times 122.5 = 490.0 \text{ ft} \end{cases}$ (building length in each direction is constant over the height, which means L_{eff} is equal to the building length parallel to the wind direction)

Therefore, the provisions of ASCE/SEI 26.11.3 may be used to determine the approximate natural frequency, n_a.

For a concrete building with a MWFRS other than moment-resisting frames [ASCE/SEI Eq. (26.11-4)]:

$n_a = 75 / h = 75 / 139.0 = 0.54 \text{ Hz} < 1.0 \text{ Hz}$

Therefore, the building is flexible in both directions.

It can be determined the building is also flexible using ASCE/SEI Eq. (26.11-5), which is applicable for concrete shear wall buildings.

For comparison purposes, the fundamental frequencies in the north-south and east-west directions are equal to 0.72 Hz and 1.24 Hz, respectively, from a dynamic analysis of the structure, which signifies the building is flexible in the north-south direction and rigid in the east-west direction. The approximate natural frequency, n_a, is used in subsequent calculations.

Step 11 – Determine the gust-effect factor — ASCE/SEI 26.11.5

For flexible buildings, G_f is determined by ASCE/SEI Eq. (26.11-10).

Calculations for G_f in both the north-south and east-west directions are given in Table 3.13.

Table 3.13 Determination of Gust-Effect Factor, G_f, Building #2

Item	ASCE/SEI Reference
$g_Q = g_v = 3.4$	26.11.5
$g_R = \sqrt{2\ln(3{,}600 n_1)} + [0.577 / \sqrt{2\ln(3{,}600 n_1)}] = 4.0$	Eq. (26.11-11)
$z_{\min} = 30.0$ ft	Table 26.11-1
$\bar{z} = \text{greater of} \begin{cases} 0.6h = 83.4 \text{ ft} \quad \text{(governs)} \\ z_{\min} = 30.0 \text{ ft} \end{cases}$	26.11.4
$\ell = 320.0$ ft	Table 26.11-1
$c = 0.30$	Table 26.11-1
$\bar{\varepsilon} = 1 / 3.0$	Table 26.11-1
$I_{\bar{z}} = c(33 / \bar{z})^{1/6} = 0.26$	Eq. (26.11-7)
$L_{\bar{z}} = \ell(\bar{z} / 33)^{\bar{\varepsilon}} = 435.9$ ft	Eq. (26.11-9)
$Q = \sqrt{\dfrac{1}{1 + 0.63\left(\dfrac{B+h}{L_{\bar{z}}}\right)^{0.63}}} = \begin{cases} 0.83 \text{ in the N-S direction with } B = 122.5 \text{ ft} \\ 0.81 \text{ in the E-W direction with } B = 182.0 \text{ ft} \end{cases}$	Eq. (26.11-8)
Damping ratio $\beta = 0.015$ for concrete buildings	C26.11
$\bar{b} = 0.45$	Table 26.11-1
$\bar{\alpha} = 1 / 4.0$	Table 26.11-1
$\bar{V}_{\bar{z}} = \bar{b}\left(\dfrac{\bar{z}}{33}\right)^{\bar{\alpha}}\left(\dfrac{88}{60}\right)V = 107.4$ ft/sec	Eq. (26.11-16)
$N_1 = n_1 L_{\bar{z}} / \bar{V}_{\bar{z}} = 2.19$	Eq. (26.11-14)
$R_n = \dfrac{7.47 N_1}{(1 + 10.3 N_1)^{5/3}} = 0.09$	Eq. (26.11-13)
$\eta_h = 4.6 n_1 h / \bar{V}_{\bar{z}} = 3.2$	26.11.5

(table continued on next page)

Table 3.13 Determination of Gust-Effect Factor, G_f, Building #2 (cont.)

Item	ASCE/SEI Reference
$R_h = \frac{1}{\eta_h} - \frac{1}{2\eta_h^2}(1 - e^{-2\eta_h}) = 0.26$	Eq. (26.11-15a)
$\eta_B = 4.6n_1 B / \bar{V}_{\bar{z}} = \begin{cases} 2.8 \text{ in the N-S direction with } B = 122.5 \text{ ft} \\ 4.2 \text{ in the E-W direction with } B = 182.0 \text{ ft} \end{cases}$	26.11.5
$R_B = \frac{1}{\eta_B} - \frac{1}{2\eta_B^2}(1 - e^{-2\eta_B}) = \begin{cases} 0.29 \text{ in the N-S direction} \\ 0.21 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-15a)
$\eta_L = 15.4n_1 L / \bar{V}_{\bar{z}} = \begin{cases} 14.1 \text{ in the N-S direction with } L = 182.0 \text{ ft} \\ 9.5 \text{ in the E-W direction with } L = 122.5 \text{ ft} \end{cases}$	26.11.5
$R_L = \frac{1}{\eta_L} - \frac{1}{2\eta_L^2}(1 - e^{-2\eta_L}) = \begin{cases} 0.07 \text{ in the N-S direction} \\ 0.10 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-15a)
$R = \sqrt{R_n R_h R_B (0.53 + 0.47 R_L) / \beta} = \begin{cases} 0.51 \text{ in the N-S direction} \\ 0.44 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-12)
$G_f = 0.925\left(\frac{1 + 1.7 I_{\bar{z}} \sqrt{g_Q^2 Q^2 + g_R^2 R^2}}{1 + 1.7 g_v I_{\bar{z}}}\right) = \begin{cases} 0.94 \text{ in the N-S direction} \\ 0.90 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-10)

Step 12 – Determine the design wind pressures

ASCE/SEI Eq. (27.3-1)

$p_z = q_z G_f C_p$

For wind in the north-south direction:

- Windward wall: $p_z = q_z G_f C_{p|windward} = 0.94 \times 0.80 \times q_z = 0.75 q_z$
- Leeward wall: $p_h = q_h G_f C_{p|leeward} = 39.5 \times 0.94 \times (-0.40) = -14.9 \text{ lb/ft}^2$
- Total pressure on MWFRS $= 0.75 q_z + 14.9 \text{ lb/ft}^2$

For wind in the east-west direction:

- Windward wall: $p_z = q_z G_f C_{p|windward} = 0.90 \times 0.80 \times q_z = 0.72 q_z$
- Leeward wall: $p_h = q_h G_f C_{p|leeward} = 39.5 \times 0.90 \times (-0.50) = -17.8 \text{ lb/ft}^2$
- Total pressure on MWFRS $= 0.72 q_z + 17.8 \text{ lb/ft}^2$

Total design wind pressures in the north-south and east-west directions are given in Table 3.14.

Table 3.14 Total Design Wind Pressures, p (lb/ft^2), Building #2

Height Above Ground Level, z (ft)	North-South	East-West
139.0	44.5	46.2
128.0	43.7	45.5
117.0	42.9	44.7
106.0	42.4	44.2
95.0	41.2	43.1
84.0	40.4	42.3
73.0	39.4	41.3
62.0	38.2	40.2
51.0	37.2	39.2
40.0	35.5	37.6
29.0	33.7	35.8
18.0	31.5	33.7

The total design wind pressures are greater than the minimum wind pressure prescribed in ASCE/SEI 27.1.5, which is equal to 16.0 lb/ft^2 for walls.

Step 13 – Determine the total design wind forces

Total design wind forces are obtained at each level by multiplying the total design wind pressures in Table 3.14 by the tributary area for each level (see Table 3.15).

Table 3.15 Total Design Wind Forces, Building 2

Height Above Ground Level, z (ft)	North-South		East-West	
	Tributary Area (sq ft)	Wind Force (kips)	Tributary Area (sq ft)	Wind Force (kips)
139.0	673.8	30.0	1,001.0	46.3
128.0	1,347.5	58.9	2,002.0	91.1
117.0	1,347.5	57.8	2,002.0	89.5
106.0	1,347.5	57.1	2,002.0	88.3
95.0	1,347.5	55.5	2,002.0	86.3
84.0	1,347.5	54.5	2,002.0	84.7
73.0	1,347.5	53.1	2,002.0	82.7
62.0	1,347.5	51.5	2,002.0	80.5
51.0	1,347.5	50.1	2,002.0	78.5
40.0	1,347.5	47.8	2,002.0	75.3
29.0	1,347.5	45.4	2,002.0	71.7
18.0	1,776.3	56.0	2,639.0	88.9
	Σ	617.7		963.8

3.8.3 Example 3.3 – Determination of Wind Forces: Building #3

Determine the wind forces on the MWFRS in the north-south and east-west directions (see Figure 1.3). Assume the overall building dimensions are 61.0 ft in the north-south direction and 91.83 ft in the east-west direction.

Design data are given in Sect. 1.2.3.

Step 1 – Determine the Risk Category ASCE/SEI Table 1.5-1

For the Residential Group R occupancy, the Risk Category is II.

Step 2 – Determine the basic wind speed ASCE/SEI Figure 26.5-1B

For the given location of this Risk Category II building, $V = 98\ \text{mph}$ (which can be obtained from Ref. 4 or 5).

Step 3 – Determine the wind directionality factor ASCE/SEI Table 26.6-1

For the MWFRS of a building, $K_d = 0.85$.

Step 4 – Determine the exposure category ASCE/SEI 26.7

The exposure category is given as B in the design data.

Step 5 – Determine the topographic factor ASCE/SEI 26.8

The building is not located on a hill or escarpment, so $K_{zt} = 1.0$.

Step 6 – Determine the ground elevation factor ASCE/SEI Table 26.9-1

The ground elevation factor, K_e, is permitted to be 1.0 in all cases (see Note 1 in ASCE/SEI Table 26.9-1).

Step 7 – Determine the velocity pressure exposure coefficients ASCE/SEI Table 26.10-1

The velocity pressure exposure coefficients are determined from the following equations, which are given in Note 1 in ASCE/SEI Table 26.10-1:

$$K_z = \begin{cases} 2.01(15 / z_g)^{2/\alpha} & \text{for } z < 15\ \text{ft} \\ 2.01(z / z_g)^{2/\alpha} & \text{for } 15\ \text{ft} \le z \le z_g \end{cases}$$

For Exposure B, $\alpha = 7.0$ and $z_g = 1{,}200\ \text{ft}$ from ASCE/SEI Table 26.11-1.

Values of K_z and K_h are given in Table 3.16.

Table 3.16 Velocity Pressure Exposure Coefficients, K_z, Building #3

Height Above Ground Level, z (ft)	K_z
152.0	1.11
142.5	1.09
133.0	1.07
123.5	1.05

(table continued on next page)

Table 3.16 Velocity Pressure Exposure Coefficients, K_z, Building #3 (cont.)

Height Above Ground Level, z (ft)	K_z
114.0	1.03
104.5	1.00
95.0	0.97
85.5	0.95
76.0	0.91
66.5	0.88
57.0	0.84
47.5	0.80
38.0	0.75
28.5	0.69
19.0	0.62
9.5	0.57

Step 8 – Determine the velocity pressures ASCE/SEI Eq. (26.10-1)

$$q_z = 0.00256 K_z K_{zt} K_d K_e V^2 = 0.00256 \times K_z \times 1.0 \times 0.85 \times 1.0 \times 98^2 = 20.90 K_z$$

Values of q_z are given in Table 3.17.

Table 3.17 Velocity Pressures, q_z, Building #3

Height Above Ground Level, z (ft)	K_z	q_z (lb/ft^2)
152.0	1.11	23.2
142.5	1.09	22.8
133.0	1.07	22.4
123.5	1.05	22.0
114.0	1.03	21.5
104.5	1.00	20.9
95.0	0.97	20.3
85.5	0.95	19.9
76.0	0.91	19.0
66.5	0.88	18.4
57.0	0.84	17.6
47.5	0.80	16.7
38.0	0.75	15.7
28.5	0.69	14.4
19.0	0.62	13.0
9.5	0.57	11.9

Step 9 – Determine the external pressure coefficients ASCE/SEI Figure 27.3-1

For wind in the north-south direction:

Windward wall: $C_p = 0.80$ for all L / B

Leeward wall: $L / B = 61.0 / 91.83 = 0.66$; $C_p = -0.50$

For wind in the east-west direction:

Windward wall: $C_p = 0.80$ for all L / B

Leeward wall: $L / B = 91.83 / 61.0 = 1.51$; $C_p = -0.40$ from linear interpolation

Step 10 – Determine whether the building is rigid or not ASCE/SEI 26.11.2

Check if the natural frequency can be determined using the provisions of ASCE/SEI 26.11.3:

1. Building height $= 152.0 \text{ ft} < 300.0 \text{ ft}$
2. Building height $= 152.0 \text{ ft} < 4L_{eff} = \begin{cases} 4 \times 61.0 = 244.0 \text{ ft} \\ \\ 4 \times 91.83 = 367.3 \text{ ft} \end{cases}$ (building length in each direction is constant over the height, which means L_{eff} is equal to the building length parallel to the wind direction)

Therefore, the provisions of ASCE/SEI 26.11.3 may be used to determine the approximate natural frequency, n_a.

For a concrete building with a MWFRS other than moment-resisting frames [ASCE/SEI Eq. (26.11-4)]:

$n_a = 75 / h = 75 / 152.0 = 0.49 \text{ Hz} < 1.0 \text{ Hz}$

Therefore, the building is flexible in both directions.

It can be determined the building is also flexible using ASCE/SEI Eq. (26.11-5), which is applicable for concrete shear wall buildings.

For comparison purposes, the fundamental frequencies in the north-south and east-west directions are equal to 0.72 Hz and 0.76 Hz, respectively, from a dynamic analysis of the structure, which also signifies the building is flexible in both directions. The approximate natural frequency, n_a, is used in subsequent calculations.

Step 11 – Determine the gust-effect factor ASCE/SEI 26.11.5

For flexible buildings, G_f is determined by ASCE/SEI Eq. (26.11-10).

Calculations for G_f in both the north-south and east-west directions are given in Table 3.18.

Table 3.18 Determination of Gust-Effect Factor, G_f, Building #3

Item	ASCE/SEI Reference
$g_Q = g_v = 3.4$	26.11.5
$g_R = \sqrt{2\ln(3{,}600n_1)} + [0.577 / \sqrt{2\ln(3{,}600n_1)}] = 4.0$	Eq. (26.11-11)
$z_{\min} = 30.0 \text{ ft}$	Table 26.11-1

(table continued on next page)

Table 3.18 Determination of Gust-Effect Factor, G_f, Building #3 (cont.)

Item	ASCE/SEI Reference
$\bar{z} = \text{greater of} \begin{cases} 0.6h = 91.2 \text{ ft} \quad \text{(governs)} \\ z_{\min} = 30.0 \text{ ft} \end{cases}$	26.11.4
$\ell = 320.0$ ft	Table 26.11-1
$c = 0.30$	Table 26.11-1
$\bar{\varepsilon} = 1 / 3.0$	Table 26.11-1
$I_{\bar{z}} = c(33 / \bar{z})^{1/6} = 0.25$	Eq. (26.11-7)
$L_{\bar{z}} = \ell(\bar{z} / 33)^{\bar{\varepsilon}} = 449.1$ ft	Eq. (26.11-9)
$Q = \sqrt{\dfrac{1}{1 + 0.63\left(\dfrac{B+h}{L_{\bar{z}}}\right)^{0.63}}} = \begin{cases} 0.84 \text{ in the N-S direction with } B = 91.83 \text{ ft} \\ 0.85 \text{ in the E-W direction with } B = 61.0 \text{ ft} \end{cases}$	Eq. (26.11-8)
Damping ratio $\beta = 0.015$ for concrete buildings	C26.11
$\bar{b} = 0.45$	Table 26.11-1
$\bar{\alpha} = 1 / 4.0$	Table 26.11-1
$\bar{V}_{\bar{z}} = \bar{b}\left(\dfrac{\bar{z}}{33}\right)^{\bar{\alpha}}\left(\dfrac{88}{60}\right)V = 83.4$ ft/sec	Eq. (26.11-16)
$N_1 = n_1 L_{\bar{z}} / \bar{V}_{\bar{z}} = 2.64$	Eq. (26.11-14)
$R_n = \dfrac{7.47 N_1}{(1 + 10.3 N_1)^{5/3}} = 0.08$	Eq. (26.11-13)
$\eta_h = 4.6 n_1 h / \bar{V}_{\bar{z}} = 4.1$	26.11.5
$R_h = \dfrac{1}{\eta_h} - \dfrac{1}{2\eta_h^2}(1 - e^{-2\eta_h}) = 0.21$	Eq. (26.11-15a)
$\eta_B = 4.6 n_1 B / \bar{V}_{\bar{z}} = \begin{cases} 2.5 \text{ in the N-S direction with } B = 91.83 \text{ ft} \\ 1.7 \text{ in the E-W direction with } B = 61.0 \text{ ft} \end{cases}$	26.11.5
$R_B = \dfrac{1}{\eta_B} - \dfrac{1}{2\eta_B^2}(1 - e^{-2\eta_B}) = \begin{cases} 0.32 \text{ in the N-S direction} \\ 0.42 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-15a)
$\eta_L = 15.4 n_1 L / \bar{V}_{\bar{z}} = \begin{cases} 5.5 \text{ in the N-S direction with } L = 61.0 \text{ ft} \\ 8.3 \text{ in the E-W direction with } L = 91.83 \text{ ft} \end{cases}$	26.11.5

(table continued on next page)

Table 3.18 Determination of Gust-Effect Factor, G_f, Building #3 (cont.)

Item	ASCE/SEI Reference
$R_L = \frac{1}{\eta_L} - \frac{1}{2\eta_L^2}(1 - e^{-2\eta_L}) = \begin{cases} 0.17 \text{ in the N-S direction} \\ 0.11 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-15a)
$R = \sqrt{R_n R_h R_B (0.53 + 0.47 R_L) / \beta} = \begin{cases} 0.47 \text{ in the N-S direction} \\ 0.52 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-12)
$G_f = 0.925\left(\frac{1 + 1.7 I_{\bar{z}} \sqrt{g_Q^2 Q^2 + g_R^2 R^2}}{1 + 1.7 g_v I_{\bar{z}}}\right) = \begin{cases} 0.93 \text{ in the N-S direction} \\ 0.95 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-10)

Step 12 – Determine the design wind pressures, p_z ASCE/SEI Eq. (27.3-1)

$$p_z = q_z G_f C_p$$

For wind in the north-south direction:

- Windward wall: $p_z = q_z G_f C_{p|windward} = 0.93 \times 0.80 \times q_z = 0.74 q_z$
- Leeward wall: $p_h = q_h G_f C_{p|leeward} = 23.2 \times 0.93 \times (-0.50) = -10.8 \text{ lb/ft}^2$
- Total pressure on MWFRS $= 0.74 q_z + 10.8 \text{ lb/ft}^2$

For wind in the east-west direction:

- Windward wall: $p_z = q_z G_f C_{p|windward} = 0.95 \times 0.80 \times q_z = 0.76 q_z$
- Leeward wall: $p_h = q_h G_f C_{p|leeward} = 23.2 \times 0.95 \times (-0.40) = -8.8 \text{ lb/ft}^2$
- Total pressure on MWFRS $= 0.76 q_z + 8.8 \text{ lb/ft}^2$

Total design wind pressures in the north-south and east-west directions are given in Table 3.19.

Table 3.19 Total Design Wind Pressures, p (lb/ft^2), Building #3

Height Above Ground Level, z (ft)	North-South	East-West
152.0	28.0	26.4
142.5	27.7	26.1
133.0	27.4	25.8
123.5	27.1	25.5
114.0	26.7	25.1
104.5	26.3	24.7
95.0	25.8	24.2
85.5	25.5	23.9
76.0	24.9	23.2
66.5	24.4	22.8

(table continued on next page)

Table 3.19 Total Design Wind Pressures, p (lb/ft^2), Building #3 (cont.)

Height Above Ground Level, z (ft)	North-South	East-West
57.0	23.8	22.2
47.5	23.2	21.5
38.0	22.4	20.7
28.5	21.5	19.7
19.0	20.4	18.7
9.5	19.6	17.8

The total design wind pressures are greater than the minimum wind pressure prescribed in ASCE/SEI 27.1.5, which is equal to 16.0 lb/ft^2 for walls.

Step 13 – Determine the total design wind forces

Total design wind forces are obtained at each level by multiplying the total design wind pressures in Table 3.19 by the tributary area for each level (see Table 3.20).

Table 3.20 Total Design Wind Forces, Building #3

Height Above Ground Level, z (ft)	North-South		East-West	
	Tributary Area (sq ft)	Wind Force (kips)	Tributary Area (sq ft)	Wind Force (kips)
152.0	436.2	12.2	289.8	7.7
142.5	872.4	24.2	579.5	15.1
133.0	872.4	23.9	579.5	15.0
123.5	872.4	23.6	579.5	14.8
114.0	872.4	23.3	579.5	14.6
104.5	872.4	22.9	579.5	14.3
95.0	872.4	22.5	579.5	14.0
85.5	872.4	22.3	579.5	13.9
76.0	872.4	21.7	579.5	13.4
66.5	872.4	21.3	579.5	13.2
57.0	872.4	20.8	579.5	12.9
47.5	872.4	20.2	579.5	12.5
38.0	872.4	19.5	579.5	12.0
28.5	872.4	18.8	579.5	11.4
19.0	872.4	17.8	579.5	10.8
9.5	872.4	17.1	579.5	10.3
	Σ	332.1		205.9

3.8.4 Example 3.4 – Determination of Wind Forces: Building #4

Determine the wind forces on the MWFRS in the north-south and east-west directions (see Figure 1.4). Assume the overall building dimensions are 153.0 ft in the north-south direction and 113.0 ft in the east-west direction.

Design data are given in Sect. 1.2.4.

Step 1 – Determine the Risk Category ASCE/SEI Table 1.5-1

For the Business Group B occupancy, the Risk Category is II.

Step 2 – Determine the basic wind speed ASCE/SEI Figure 26.5-1B

For the given location of this Risk Category II building, $V = 98 \text{ mph}$ (which can be obtained from Ref. 4 or 5).

Step 3 – Determine the wind directionality factor ASCE/SEI Table 26.6-1

For the MWFRS of a building, $K_d = 0.85$.

Step 4 – Determine the exposure category ASCE/SEI 26.7

The exposure category is given as B in the design data.

Step 5 – Determine the topographic factor ASCE/SEI 26.8

The building is not located on a hill or escarpment, so $K_{zt} = 1.0$.

Step 6 – Determine the ground elevation factor ASCE/SEI Table 26.9-1

The ground elevation factor, K_e, is permitted to be 1.0 in all cases (see Note 1 in ASCE/SEI Table 26.9-1).

Step 7 – Determine the velocity pressure exposure coefficients ASCE/SEI Table 26.10-1

The velocity pressure exposure coefficients are determined from the following equations, which are given in Note 1 in ASCE/SEI Table 26.10-1:

$$K_z = \begin{cases} 2.01(15 / z_g)^{2/\alpha} \text{ for } z < 15 \text{ ft} \\ \\ 2.01(z / z_g)^{2/\alpha} \text{ for } 15 \text{ ft} \le z \le z_g \end{cases}$$

For Exposure B, $\alpha = 7.0$ and $z_g = 1{,}200 \text{ ft}$ from ASCE/SEI Table 26.11-1.

Values of K_z and K_h are given in Table 3.21.

Table 3.21 Velocity Pressure Exposure Coefficients, K_z, Building #4

Height Above Ground Level, z (ft)	K_z
330.0	1.39
319.0	1.38
308.0	1.36

(table continued on next page)

Table 3.21 Velocity Pressure Exposure Coefficients, K_z, Building #4 (cont.)

Height Above Ground Level, z (ft)	K_z
297.0	1.35
286.0	1.33
275.0	1.32
264.0	1.30
253.0	1.29
242.0	1.27
231.0	1.26
220.0	1.24
209.0	1.22
198.0	1.20
187.0	1.18
176.0	1.16
165.0	1.14
154.0	1.12
143.0	1.10
132.0	1.07
121.0	1.04
110.0	1.02
99.0	0.99
88.0	0.95
77.0	0.92
66.0	0.88
55.0	0.83
44.0	0.78
33.0	0.72
22.0	0.64
11.0	0.58

Step 8 – Determine the velocity pressures ASCE/SEI Eq. (26.10-1)

$$q_z = 0.00256 K_z K_{zt} K_d K_e V^2 = 0.00256 \times K_z \times 1.0 \times 0.85 \times 1.0 \times 98^2 = 20.90 K_z$$

Values of q_z are given in Table 3.22.

Table 3.22 Velocity Pressures, q_z, Building #4

Height Above Ground Level, z (ft)	K_z	q_z (lb/ft^2)
330.0	1.39	29.1
319.0	1.38	28.8
308.0	1.36	28.5
297.0	1.35	28.2
286.0	1.33	27.9
275.0	1.32	27.6
264.0	1.30	27.3
253.0	1.29	26.9
242.0	1.27	26.6
231.0	1.26	26.2
220.0	1.24	25.9
209.0	1.22	25.5
198.0	1.20	25.1
187.0	1.18	24.7
176.0	1.16	24.3
165.0	1.14	23.8
154.0	1.12	23.4
143.0	1.10	22.9
132.0	1.07	22.4
121.0	1.04	21.8
110.0	1.02	21.2
99.0	0.99	20.6
88.0	0.95	19.9
77.0	0.92	19.2
66.0	0.88	18.3
55.0	0.83	17.4
44.0	0.78	16.3
33.0	0.72	15.1
22.0	0.64	13.4
11.0	0.58	12.0

Step 9 – Determine the external pressure coefficients ASCE/SEI Figure 27.3-1

For wind in the north-south direction:

Windward wall: $C_p = 0.80$ for all L / B

Leeward wall: $L / B = 153.0 / 113.0 = 1.35$; $C_p = -0.43$ from linear interpolation

For wind in the east-west direction:

Windward wall: $C_p = 0.80$ for all L / B

Leeward wall: $L / B = 113.0 / 153.0 = 0.74$; $C_p = -0.50$

Step 10 – Determine whether the building is rigid or not — ASCE/SEI 26.11.2

Check if the natural frequency can be determined using the provisions of ASCE/SEI 26.11.3:

Building height $= 330.0 \text{ ft} > 300.0 \text{ ft}$

Therefore, the provisions of ASCE/SEI 26.11.3 may not be used to determine the approximate natural frequency, n_a.

From a dynamic analysis of the structure, the building is found to be flexible in both directions ($n_1 = 0.32 \text{ Hz}$ in the north-south direction and $n_1 = 0.40 \text{ Hz}$ in the east-west direction).

Step 11 – Determine the gust-effect factor — ASCE/SEI 26.11.5

For flexible buildings, G_f is determined by ASCE/SEI Eq. (26.11-10).

Calculations for G_f in both the north-south and east-west directions are given in Table 3.23.

Table 3.23 Determination of Gust-Effect Factor, G_f, Building #4

Item	ASCE/SEI Reference
$g_Q = g_v = 3.4$	26.11.5
$g_R = \sqrt{2\ln(3{,}600 n_1)} + \left[0.577 / \sqrt{2\ln(3{,}600 n_1)}\right] = \begin{cases} 3.9 \text{ in the N-S direction} \\ 4.0 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-11)
$z_{\min} = 30.0 \text{ ft}$	Table 26.11-1
$\bar{z} = \text{greater of} \begin{cases} 0.6h = 198.0 \text{ ft} \quad (\text{governs}) \\ z_{\min} = 30.0 \text{ ft} \end{cases}$	26.11.4
$\ell = 320.0 \text{ ft}$	Table 26.11-1
$c = 0.30$	Table 26.11-1
$\bar{\varepsilon} = 1 / 3.0$	Table 26.11-1
$I_{\bar{z}} = c(33 / \bar{z})^{1/6} = 0.22$	Eq. (26.11-7)
$L_{\bar{z}} = \ell(\bar{z} / 33)^{\bar{\varepsilon}} = 581.5 \text{ ft}$	Eq. (26.11-9)
$Q = \sqrt{\dfrac{1}{1 + 0.63\left(\dfrac{B+h}{L_{\bar{z}}}\right)^{0.63}}} = \begin{cases} 0.81 \text{ in the N-S direction with } B = 113.0 \text{ ft} \\ 0.80 \text{ in the E-W direction with } B = 153.0 \text{ ft} \end{cases}$	Eq. (26.11-8)
Damping ratio $\beta = 0.015$ for concrete buildings	C26.11
$\bar{b} = 0.45$	Table 26.11-1

(table continued on next page)

Table 3.23 Determination of Gust-Effect Factor, G_f, Building #4 (cont.)

Item	ASCE/SEI Reference
$\bar{\alpha} = 1 / 4.0$	Table 26.11-1
$\bar{V}_{\bar{z}} = \bar{b}\left(\frac{\bar{z}}{33}\right)^{\bar{\alpha}}\left(\frac{88}{60}\right)V = 101.2 \text{ ft/sec}$	Eq. (26.11-16)
$N_1 = n_1 L_{\bar{z}} / \bar{V}_{\bar{z}} = \begin{cases} 1.84 \text{ in the N-S direction} \\ 2.30 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-14)
$R_n = \frac{7.47N_1}{(1+10.3N_1)^{5/3}} = \begin{cases} 0.09 \text{ in the N-S direction} \\ 0.08 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-13)
$\eta_h = 4.6 n_1 h / \bar{V}_{\bar{z}} = \begin{cases} 4.8 \text{ in the N-S direction} \\ 6.0 \text{ in the E-W direction} \end{cases}$	26.11.5
$R_h = \frac{1}{\eta_h} - \frac{1}{2\eta_h^2}(1 - e^{-2\eta_h}) = \begin{cases} 0.19 \text{ in the N-S direction} \\ 0.15 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-15a)
$\eta_B = 4.6 n_1 B / \bar{V}_{\bar{z}} = \begin{cases} 1.6 \text{ in the N-S direction with } B = 113.0 \text{ ft} \\ 2.8 \text{ in the E-W direction with } B = 153.0 \text{ ft} \end{cases}$	26.11.5
$R_B = \frac{1}{\eta_B} - \frac{1}{2\eta_B^2}(1 - e^{-2\eta_B}) = \begin{cases} 0.44 \text{ in the N-S direction} \\ 0.29 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-15a)
$\eta_L = 15.4 n_1 L / \bar{V}_{\bar{z}} = \begin{cases} 7.5 \text{ in the N-S direction with } L = 153.0 \text{ ft} \\ 6.9 \text{ in the E-W direction with } L = 113.0 \text{ ft} \end{cases}$	26.11.5
$R_L = \frac{1}{\eta_L} - \frac{1}{2\eta_L^2}(1 - e^{-2\eta_L}) = \begin{cases} 0.12 \text{ in the N-S direction} \\ 0.13 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-15a)
$R = \sqrt{R_n R_h R_B (0.53 + 0.47R_L) / \beta} = \begin{cases} 0.54 \text{ in the N-S direction} \\ 0.37 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-12)
$G_f = 0.925\left(\frac{1 + 1.7I_{\bar{z}}\sqrt{g_Q^2 Q^2 + g_R^2 R^2}}{1 + 1.7 g_v I_{\bar{z}}}\right) = \begin{cases} 0.94 \text{ in the N-S direction} \\ 0.88 \text{ in the E-W direction} \end{cases}$	Eq. (26.11-10)

Step 12 – Determine the design wind pressures, p_z ASCE/SEI Eq. (27.3-1)

$p_z = q_z G_f C_p$

For wind in the north-south direction:

- Windward wall: $p_z = q_z G_f C_{p|windward} = 0.94 \times 0.80 \times q_z = 0.75 q_z$
- Leeward wall: $p_h = q_h G_f C_{p|leeward} = 29.1 \times 0.94 \times (-0.43) = -11.8\ \mathrm{lb/ft^2}$
- Total pressure on MWFRS $= 0.75 q_z + 11.8\ \mathrm{lb/ft^2}$

For wind in the east-west direction:

- Windward wall: $p_z = q_z G_f C_{p|windward} = 0.88 \times 0.80 \times q_z = 0.70 q_z$
- Leeward wall: $p_h = q_h G_f C_{p|leeward} = 29.1 \times 0.88 \times (-0.50) = -12.8\ \mathrm{lb/ft^2}$
- Total pressure on MWFRS $= 0.70 q_z + 12.8\ \mathrm{lb/ft^2}$

Total design wind pressures in the north-south and east-west directions are given in Table 3.24.

Table 3.24 Total Design Wind Pressures, p (lb/ft^2), Building #4

Height Above Ground Level, z (ft)	North-South	East-West
330.0	33.6	33.2
319.0	33.4	33.0
308.0	33.2	32.8
297.0	33.0	32.5
286.0	32.7	32.3
275.0	32.5	32.1
264.0	32.3	31.9
253.0	32.0	31.6
242.0	31.8	31.4
231.0	31.5	31.1
220.0	31.2	30.9
209.0	30.9	30.7
198.0	30.6	30.4
187.0	30.3	30.1
176.0	30.0	29.8
165.0	29.7	29.5
154.0	29.4	29.2
143.0	29.0	28.8
132.0	28.6	28.5
121.0	28.2	28.1
110.0	27.7	27.6
99.0	27.3	27.2

(table continued on next page)

Table 3.24 Total Design Wind Pressures, p (lb/ft^2), Building #4 (cont.)

Height Above Ground Level, z (ft)	North-South	East-West
88.0	26.7	26.7
77.0	26.2	26.2
66.0	25.5	25.6
55.0	24.9	25.0
44.0	24.0	24.2
33.0	23.1	23.4
22.0	21.9	22.2
11.0	20.8	21.2

The total design wind pressures are greater than the minimum wind pressure prescribed in ASCE/SEI 27.1.5, which is equal to 16.0 lb/ft^2 for walls.

Step 13 – Determine the total design wind forces

Total design wind forces are obtained at each level by multiplying the total design wind pressures in Table 3.24 by the tributary area for each level (see Table 3.25).

Table 3.25 Total Design Wind Forces, Building #4

Height Above Ground Level, z (ft)	North-South		East-West	
	Tributary Area (sq ft)	Wind Force (kips)	Tributary Area (sq ft)	Wind Force (kips)
330.0	621.5	20.9	841.5	27.9
319.0	1,243.0	41.5	1,683.0	55.5
308.0	1,243.0	41.3	1,683.0	55.2
297.0	1,243.0	41.0	1,683.0	54.7
286.0	1,243.0	40.7	1,683.0	54.4
275.0	1,243.0	40.4	1,683.0	54.0
264.0	1,243.0	40.2	1,683.0	53.7
253.0	1,243.0	39.8	1,683.0	53.2
242.0	1,243.0	39.5	1,683.0	52.9
231.0	1,243.0	39.2	1,683.0	52.3
220.0	1,243.0	38.8	1,683.0	52.0
209.0	1,243.0	38.4	1,683.0	51.7
198.0	1,243.0	38.0	1,683.0	51.2
187.0	1,243.0	37.7	1,683.0	50.7
176.0	1,243.0	37.3	1,683.0	50.2
165.0	1,243.0	36.9	1,683.0	49.7
154.0	1,243.0	36.5	1,683.0	49.1

(table continued on next page)

Table 3.25 Total Design Wind Forces, Building #4 (cont.)

Height Above Ground Level, z (ft)	North-South		East-West	
	Tributary Area (sq ft)	Wind Force (kips)	Tributary Area (sq ft)	Wind Force (kips)
143.0	1,243.0	36.1	1,683.0	48.5
132.0	1,243.0	35.6	1,683.0	48.0
121.0	1,243.0	35.1	1,683.0	47.3
110.0	1,243.0	34.4	1,683.0	46.5
99.0	1,243.0	33.9	1,683.0	45.8
88.0	1,243.0	33.2	1,683.0	44.9
77.0	1,243.0	32.6	1,683.0	44.1
66.0	1,243.0	31.7	1,683.0	43.1
55.0	1,243.0	31.0	1,683.0	42.1
44.0	1,243.0	29.8	1,683.0	40.7
33.0	1,243.0	28.7	1,683.0	39.4
22.0	1,243.0	27.2	1,683.0	37.4
11.0	1,243.0	25.9	1,683.0	35.7
	Σ	1,063.6		1,431.9

3.8.5 Example 3.5 – Determination of the Seismic Design Category: Building #1

Determine the SDC for (a) Site Class C, (b) Site Class D (default), and (c) Site Class E (see Figure 1.1).

Design data are given in Sect. 1.2.1.

Part (a): Site Class C

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

In lieu of determining the mapped acceleration parameters S_S and S_1 from ASCE/SEI Figures 22-1 and 22-2, respectively, use Ref. 4, 5, or 6: $S_S = 0.120$ and $S_1 = 0.062$ for the latitude and longitude of the site.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

Because $S_S = 0.120 < 0.150$ and $S_1 = 0.062 > 0.040$, the building is not permitted to be automatically assigned to SDC A.

Step 3 – Determine the Risk Category ASCE/SEI Table 1.5-1

For the Business Group B occupancy, the Risk Category is II.

Step 4 – Determine if the SDC is E or F ASCE/SEI 11.6

Because the Risk Category is II and $S_1 = 0.062 < 0.750$, the SDC is not E or F.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.4 and 11.4.5

For Site Class C and $S_S = 0.120 < 0.250$, $F_a = 1.30$ from ASCE/SEI Table 11.4-1.

For Site Class C and $S_1 = 0.062 < 0.100,\ F_v = 1.50$ from ASCE/SEI Table 11.4-2.

$S_{MS} = F_a S_S = 1.30 \times 0.120 = 0.156$

$S_{M1} = F_v S_1 = 1.50 \times 0.062 = 0.093$

$S_{DS} = 2S_{MS} / 3 = 0.104$

$S_{D1} = 2S_{M1} / 3 = 0.062$

Step 6 – Check if the SDC can be determined by ASCE/SEI Table 11.6-1 alone ASCE/SEI 11.6

Check if all four conditions in ASCE/SEI 11.6 are satisfied.

Determine the approximate fundamental period, T_a (ASCE/SEI 12.8.2.1).

For concrete moment-resisting frames in both directions:

$T_a = C_t h_n^x = 0.016 \times 60.0^{0.9} = 0.64 \text{ s}$
ASCE/SEI Eq. (12.8-7)

$0.8T_S = 0.8S_{D1} / S_{DS} = 0.48 \text{ s} < T_a = 0.64 \text{ s}$

Because $T_a > 0.8T_S$, the SDC cannot be determined by ASCE/SEI Table 11.6-1 alone (that is, condition 1 is not satisfied).

Step 7 – Determine the SDC ASCE/SEI Tables 11.6-1 and 11.6-2

From ASCE/SEI Table 11.6-1 with $S_{DS} = 0.104 < 0.167$ and Risk Category II, the SDC is A.

From ASCE/SEI Table 11.6-2 with $S_{D1} = 0.062 < 0.067$ and Risk Category II, the SDC is A.

Therefore, the SDC is A for this building.

Part (b): Site Class D (default)

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

From Step 1 in Part (a): $S_S = 0.120$ and $S_1 = 0.062$.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

Because $S_S = 0.120 < 0.150$ and $S_1 = 0.062 > 0.040$, the building is not permitted to be automatically assigned to SDC A.

Step 3 – Determine the Risk Category ASCE/SEI Table 1.5-1

For the Business Group B occupancy, the Risk Category is II.

Step 4 – Determine if the SDC is E or F ASCE/SEI 11.6

Because the Risk Category is II and $S_1 = 0.062 < 0.750$, the SDC is not E or F.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.4 and 11.4.5

For Site Class D (default) and $S_S = 0.120 < 0.250,\ F_a = 1.60$ from ASCE/SEI Table 11.4-1.

For Site Class D (default) and $S_1 = 0.062 < 0.100,\ F_v = 2.40$ from ASCE/SEI Table 11.4-2.

$$S_{MS} = F_a S_S = 1.60 \times 0.120 = 0.192$$

$$S_{M1} = F_v S_1 = 2.40 \times 0.062 = 0.149$$

$$S_{DS} = 2S_{MS} / 3 = 0.128$$

$$S_{D1} = 2S_{M1} / 3 = 0.099$$

Step 6 – Check if the SDC can be determined by ASCE/SEI Table 11.6-1 alone ASCE/SEI 11.6

Check if all four conditions in ASCE/SEI 11.6 are satisfied.

Determine the approximate fundamental period, T_a (ASCE/SEI 12.8.2.1).

From Step 6 in Part (a), $T_a = C_t h_n^x = 0.64$ s

$$0.8T_S = 0.8S_{D1} / S_{DS} = 0.62 \text{ s} < T_a = 0.64 \text{ s}$$

Because $T_a > 0.8T_S$, the SDC cannot be determined by ASCE/SEI Table 11.6-1 alone (that is, condition 1 is not satisfied).

Step 7 – Determine the SDC ASCE/SEI Tables 11.6-1 and 11.6-2

From ASCE/SEI Table 11.6-1 with $S_{DS} = 0.128 < 0.167$ and Risk Category II, the SDC is A.

From ASCE/SEI Table 11.6-2 with $0.067 < S_{D1} = 0.099 < 0.133$ and Risk Category II, the SDC is B.

Therefore, the SDC is B for this building.

Part (c): Site Class E

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

From Step 1 in Part (a): $S_S = 0.120$ and $S_1 = 0.062$.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

Because $S_S = 0.120 < 0.150$ and $S_1 = 0.062 > 0.040$, the building is not permitted to be automatically assigned to SDC A.

Step 3 – Determine the Risk Category ASCE/SEI Table 1.5-1

For the Business Group B occupancy, the Risk Category is II.

Step 4 – Determine if the SDC is E or F ASCE/SEI 11.6

Because the Risk Category is II and $S_1 = 0.062 < 0.750$, the SDC is not E or F.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.4 and 11.4.5

For Site Class E and $S_S = 0.120 < 0.250,\ F_a = 2.40$ from ASCE/SEI Table 11.4-1.

For Site Class E and $S_1 = 0.062 < 0.100,\ F_v = 4.20$ from ASCE/SEI Table 11.4-2.

$$S_{MS} = F_a S_S = 2.40 \times 0.120 = 0.288$$

$S_{M1} = F_v S_1 = 4.20 \times 0.062 = 0.260$

$S_{DS} = 2S_{MS} / 3 = 0.192$

$S_{D1} = 2S_{M1} / 3 = 0.173$

Step 6 – Check if the SDC can be determined by ASCE/SEI Table 11.6-1 alone ASCE/SEI 11.6

Check if all four conditions in ASCE/SEI 11.6 are satisfied.

Determine the approximate fundamental period, T_a (ASCE/SEI 12.8.2.1).

From Step 6 in Part (a), $T_a = C_t h_n^x = 0.64$ s

$0.8T_S = 0.8S_{D1} / S_{DS} = 0.62 \text{ s} < T_a = 0.64 \text{ s}$

Because $T_a > 0.8T_S$, the SDC cannot be determined by ASCE/SEI Table 11.6-1 alone (that is, condition 1 is not satisfied).

Step 7 – Determine the SDC ASCE/SEI Tables 11.6-1 and 11.6-2

From ASCE/SEI Table 11.6-1 with $0.167 < S_{DS} = 0.192 < 0.330$ and Risk Category II, the SDC is B.

From ASCE/SEI Table 11.6-2 with $0.133 < S_{D1} = 0.173 < 0.200$ and Risk Category II, the SDC is C.

Therefore, the SDC is C for this building.

3.8.6 Example 3.6 – Determination of the Seismic Design Category: Building #2

Determine the SDC for Site Class D (stiff soil; determined) [see Figure 1.2].

Design data are given in Sect. 1.2.2.

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

In lieu of determining the mapped acceleration parameters S_S and S_1 from ASCE/SEI Figures 22-1 and 22-2, respectively, use Ref. 4, 5, or 6: $S_S = 0.287$ and $S_1 = 0.060$ for the latitude and longitude of the site.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

Because $S_S = 0.287 > 0.150$ and $S_1 = 0.060 > 0.040$, the building is not permitted to be automatically assigned to SDC A.

Step 3 – Determine the Risk Category ASCE/SEI Table 1.5-1

For this essential facility, the Risk Category is IV.

Step 4 – Determine if the SDC is E or F ASCE/SEI 11.6

Because the Risk Category is IV and $S_1 = 0.060 < 0.750$, the SDC is not E or F.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.4 and 11.4.5

For Site Class D (stiff soil) and $0.250 < S_S = 0.287 < 0.500$, $F_a = 1.57$ from linear interpolation in ASCE/SEI Table 11.4-1.

For Site Class D (stiff soil) and $S_1 = 0.060 < 0.100$, $F_v = 2.40$ from ASCE/SEI Table 11.4-2.

$S_{MS} = F_a S_S = 1.57 \times 0.287 = 0.451$

$S_{M1} = F_v S_1 = 2.40 \times 0.060 = 0.144$

$S_{DS} = 2S_{MS} / 3 = 0.301$

$S_{D1} = 2S_{M1} / 3 = 0.096$

Step 6 – Check if the SDC can be determined by ASCE/SEI Table 11.6-1 alone ASCE/SEI 11.6

Check if all four conditions in ASCE/SEI 11.6 are satisfied.

Determine the approximate fundamental period, T_a (ASCE/SEI 12.8.2.1).

For all other structural systems in both directions:

$T_a = C_t h_n^x = 0.02 \times 139.0^{0.75} = 0.81 \text{ s}$ ASCE/SEI Eq. (12.8-7)

$0.8T_S = 0.8S_{D1} / S_{DS} = 0.26 \text{ s} < T_a = 0.81 \text{ s}$

Because $T_a > 0.8T_S$, the SDC cannot be determined by ASCE/SEI Table 11.6-1 alone (that is, condition 1 is not satisfied).

Step 7 – Determine the SDC ASCE/SEI Tables 11.6-1 and 11.6-2

From ASCE/SEI Table 11.6-1 with $0.167 < S_{DS} = 0.301 < 0.330$ and Risk Category IV, the SDC is C.

From ASCE/SEI Table 11.6-2 with $0.067 < S_{D1} = 0.096 < 0.133$ and Risk Category IV, the SDC is C.

Therefore, the SDC is C for this building.

3.8.7 Example 3.7 – Determination of the Seismic Design Category: Building #3

Determine the SDC for Site Class D (stiff soil; determined) [see Figure 1.3].

Design data are given in Sect. 1.2.3.

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

In lieu of determining the mapped acceleration parameters S_S and S_1 from ASCE/SEI Figures 22-1 and 22-2, respectively, use Ref. 4, 5, or 6: $S_S = 1.386$ and $S_1 = 0.483$ for the latitude and longitude of the site.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

Because $S_S = 1.386 > 0.150$ and $S_1 = 0.483 > 0.040$, the building is not permitted to be automatically be assigned to SDC A.

Step 3 – Determine the Risk Category ASCE/SEI Table 1.5-1

For the Residential Group R occupancy, the Risk Category is II.

Step 4 – Determine if the SDC is E or F ASCE/SEI 11.6

Because the Risk Category is II and $S_1 = 0.483 < 0.750$, the SDC is not E or F.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.4, 11.4.5, and 11.4.8

Because the Site Class is D and $S_1 = 0.483 > 0.200,$ a ground motion hazard analysis in accordance with ASCE/SEI 21.2 may be required. Assume the second exception in ASCE/SEI 11.4.8 is applicable; thus, such an analysis need not be performed.

For Site Class D (stiff soil) and $1.250 < S_S = 1.386 < 1.500,$ $F_a = 1.00$ from ASCE/SEI Table 11.4-1.

For Site Class D (stiff soil) and $0.400 < S_1 = 0.483 < 0.500,$ $F_v = 1.82$ from linear interpolation in ASCE/SEI Table 11.4-2.

$S_{MS} = F_a S_S = 1.00 \times 1.386 = 1.386$

$S_{M1} = F_v S_1 = 1.82 \times 0.483 = 0.879$

$S_{DS} = 2S_{MS} / 3 = 0.924$

$S_{D1} = 2S_{M1} / 3 = 0.586$

Step 6 – Check if the SDC can be determined by ASCE/SEI Table 11.6-1 alone ASCE/SEI 11.6

Check if all four conditions in ASCE/SEI 11.6 are satisfied.

Determine the approximate fundamental period, T_a (ASCE/SEI 12.8.2.1).

For all other structural systems in both directions:

$T_a = C_t h_n^x = 0.02 \times 152.0^{0.75} = 0.87 \text{ s}$ ASCE/SEI Eq. (12.8-7)

$0.8T_S = 0.8S_{D1} / S_{DS} = 0.51 \text{ s} < T_a = 0.87 \text{ s}$

Because $T_a > 0.8T_S$, the SDC cannot be determined by ASCE/SEI Table 11.6-1 alone (that is, condition 1 is not satisfied).

Step 7 – Determine the SDC ASCE/SEI Tables 11.6-1 and 11.6-2

From ASCE/SEI Table 11.6-1 with $S_{DS} = 0.924 > 0.500$ and Risk Category II, the SDC is D.

From ASCE/SEI Table 11.6-2 with $S_{D1} = 0.586 > 0.200$ and Risk Category II, the SDC is D.

Therefore, the SDC is D for this building.

3.8.8 Example 3.8 – Determination of the Seismic Design Category: Building #4

Determine the SDC for Site Class D (default) [see Figure 1.4].

Design data are given in Sect. 1.2.4.

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

In lieu of determining the mapped acceleration parameters S_S and S_1 from ASCE/SEI Figures 22-1 and 22-2, respectively, use Ref. 4, 5, or 6: $S_S = 0.583$ and $S_1 = 0.192$ for the latitude and longitude of the site.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

Because $S_S = 0.583 > 0.150$ and $S_1 = 0.192 > 0.040,$ the building is not permitted to be automatically be assigned to SDC A.

Step 3 – Determine the Risk Category ASCE/SEI Table 1.5-1

For the Business Group B occupancy, the Risk Category is II.

Step 4 – Determine if the SDC is E or F ASCE/SEI 11.6

Because the Risk Category is II and $S_1 = 0.192 < 0.750$, the SDC is not E or F.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.4, 11.4.5, and 11.4.8

Because the Site Class is D and $S_1 = 0.192 < 0.200$, a ground motion hazard analysis in accordance with ASCE/SEI 21.2 need not be performed.

For Site Class D (default) and $S_S = 0.583$, $F_a = 1.33$ from Table ASCE/SEI 11.4-1 by linear interpolation.

For Site Class D (default) and $S_1 = 0.192$, $F_v = 2.22$ from ASCE/SEI Table 11.4-2 by linear interpolation.

$S_{MS} = F_a S_S = 1.33 \times 0.583 = 0.775$

$S_{M1} = F_v S_1 = 2.22 \times 0.192 = 0.426$

$S_{DS} = 2S_{MS} / 3 = 0.517$

$S_{D1} = 2S_{M1} / 3 = 0.284$

Step 6 – Check if the SDC can be determined by ASCE/SEI Table 11.6-1 alone ASCE/SEI 11.6

Check if all four conditions in ASCE/SEI 11.6 are satisfied.

Determine the approximate fundamental period, T_a (ASCE/SEI 12.8.2.1).

For all other structural systems in both directions:

$T_a = C_t h_n^x = 0.02 \times 330.0^{0.75} = 1.55 \text{ s}$ ASCE/SEI Eq. (12.8-7)

$0.8T_S = 0.8S_{D1} / S_{DS} = 0.44 \text{ s} < T_a = 1.55 \text{ s}$

Because $T_a > 0.8T_S$, the SDC cannot be determined by ASCE/SEI Table 11.6-1 alone (that is, condition 1 is not satisfied).

Step 7 – Determine the SDC ASCE/SEI Tables 11.6-1 and 11.6-2

From ASCE/SEI Table 11.6-1 with $S_{DS} = 0.517 > 0.500$ and Risk Category II, the SDC is D.

From ASCE/SEI Table 11.6-2 with $S_{D1} = 0.284 > 0.200$ and Risk Category II, the SDC is D.

Therefore, the SDC is D for this building.

3.8.9 Example 3.9 – Determination of Seismic Forces: SFRS of Building #1 (Framing Option A)

Determine the seismic forces on the SFRS for Site Class D (default) [see Figure 1.1].

Design data are given in Sect. 1.2.1.

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

From Step 1 in Part (b) of Example 3.5, $S_S = 0.120$ and $S_1 = 0.062$.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

From Step 2 in Part (b) of Example 3.5, the building is not permitted to be automatically assigned to SDC A.

Step 3 – Determine the Site Class ASCE/SEI 11.4.3

From the design data, the Site Class is given as D (default).

Step 4 – Determine the acceleration parameters adjusted for site class effects ASCE/SEI 11.4.4

From Step 5 of Example 3.5, $S_{MS} = 0.192$ and $S_{M1} = 0.149$.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.5

From Step 5 of Example 3.5, $S_{DS} = 0.128$ and $S_{D1} = 0.099$.

Step 6 – Determine the SDC ASCE/SEI 11.6

From Step 7 of Example 3.5, the SDC is B.

Step 7 – Determine the response modification coefficient ASCE/SEI Table 12.2-1

Because the building is assigned to SDC B, ordinary reinforced concrete moment frames may be used in both directions with no limitations (SFRS C7).

For this SFRS, $R = 3.0$.

Step 8 – Determine the importance factor ASCE/SEI Table 1.5-2

For Risk Category II buildings, $I_e = 1.00$.

Step 9 – Determine the approximate period ASCE/SEI 12.8.2.1

From Step 6 of Example 3.5, $T_a = C_t h_n^x = 0.016 \times 60.0^{0.9} = 0.64$ s.

Step 10 – Determine the long-period transition period ASCE/SEI Figures 22-14 through 22-17

From ASCE/SEI Figure 22-14, $T_L = 12.0$ s (this period can also be determined from Ref. 4, 5, or 6).

Step 11 – Determine the seismic response coefficient ASCE/SEI 12.8.1.1

Because $T_a = 0.64\text{ s} < T_L = 12.0\text{ s}$, C_s is determined as follows:

$$C_s = \frac{S_{D1}}{T\left(\frac{R}{I_e}\right)} = \frac{0.099}{0.64 \times \left(\frac{3.0}{1.00}\right)} = 0.052$$ ASCE/SEI Eq. (12.8-3)

C_s need not exceed the following:

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.128}{\left(\frac{3.0}{1.00}\right)} = 0.043$$ ASCE/SEI Eq. (12.8-2)

Minimum C_s:

$$C_s = \text{ greater of } \begin{cases} 0.044 S_{DS} I_e = 0.006 \\ \\ 0.010 \end{cases}$$ ASCE/SEI Eq. (12.8-5)

Therefore, $C_s = 0.043$.

Step 12 – Determine the effective seismic weight ASCE/SEI 12.7.2

The effective seismic weight for this building includes the dead load of the structure (assuming a 9.5-in.-thick slab and 24.0 in. by 24.0 in. columns), the superimposed dead loads on the roof and floors, the 15 lb/ft^2 partition weight on the floors, and the weight of the cladding.

The effective seismic weights per floor and the total effective seismic weight, W, are given in Table 3.26.

Table 3.26 Seismic Forces and Story Shears, Building #1

Level	Story Weight, w_x (kips)	Height, h_x (ft)	$w_x h_x^k$	Lateral Force, F_x (kips)	Story Shear, V_x (kips)
R	2,043	60.0	163,264	149.3	149.3
5	2,368	48.5	150,704	137.8	287.1
4	2,368	37.0	112,813	103.2	390.3
3	2,368	25.5	75,750	69.2	459.5
2	2,399	14.0	40,401	37.0	496.5
Σ	***W*** = 11,546		542,932	496.5	

Step 13 – Determine the seismic base shear ASCE/SEI Eq. (12.8-1)

$$V = C_s W = 0.043 \times 11{,}546 = 496.5 \text{ kips}$$

Step 14 – Determine the exponent related to the structure period ASCE/SEI 12.8.3

$$k = 0.75 + 0.5T = 0.75 + (0.5 \times 0.64) = 1.07$$

Step 15 – Determine the seismic forces at each level ASCE/SEI Eqs. (12.8-11) and (12.8-12)

$$F_x = C_{vx} V = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k} V$$

Values of F_x are given in Table 3.26.

For example, at level 2:

$$F_x = \frac{2{,}399 \times 14.0^{1.07}}{542{,}932} \times 496.5 = 37.0 \text{ kips}$$

3.8.10 Example 3.10 – Determination of Seismic Forces: SFRS of Building #2

Determine the seismic forces on the SFRS for Site Class D (stiff soil; determined) [see Figure 1.2].

Design data are given in Sect. 1.2.2.

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

From Step 1 in Example 3.6, $S_S = 0.287$ and $S_1 = 0.060$.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

From Step 2 in Example 3.6, the building is not permitted to be automatically assigned to SDC A.

Step 3 – Determine the Site Class ASCE/SEI 11.4.3

From the design data, the Site Class is given as D (stiff soil).

Step 4 – Determine the acceleration parameters adjusted for site class effects ASCE/SEI 11.4.4

From Step 5 in Example 3.6, $S_{MS} = 0.451$ and $S_{M1} = 0.144$.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.5

From Step 5 in Example 3.6, $S_{DS} = 0.301$ and $S_{D1} = 0.096$.

Step 6 – Determine the SDC ASCE/SEI 11.6

From Step 7 in Example 3.6, the SDC is C.

Step 7 – Determine the response modification coefficient ASCE/SEI Table 12.2-1

Because the building is assigned to SDC C, a building frame system with ordinary reinforced concrete shear (structural) walls may be used in both directions with no limitations (SFRS B5).

For this SFRS, $R = 5.0$.

Step 8 – Determine the importance factor ASCE/SEI Table 1.5-2

For Risk Category IV buildings, $I_e = 1.50$.

Step 9 – Determine the approximate period ASCE/SEI 12.8.2.1

From Step 6 in Example 3.6, $T_a = C_t h_n^x = 0.02 \times 139.0^{0.75} = 0.81$ s.

Step 10 – Determine the long-period transition period ASCE/SEI Figures 22-14 through 22-17

From ASCE/SEI Figure 22-14, $T_L = 6.0$ s (this period can also be determined from Ref. 4, 5 or 6).

Step 11 – Determine the seismic response coefficient ASCE/SEI 12.8.1.1

Because $T_a = 0.81\text{ s} < T_L = 6.0\text{ s}$, C_s is determined as follows:

$$C_s = \frac{S_{D1}}{T\left(\frac{R}{I_e}\right)} = \frac{0.096}{0.81 \times \left(\frac{5.0}{1.50}\right)} = 0.036$$ ASCE/SEI Eq. (12.8-3)

C_s need not exceed the following:

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.301}{\left(\frac{5.0}{1.50}\right)} = 0.090$$ ASCE/SEI Eq. (12.8-2)

Minimum C_s:

$$C_s = \text{ greater of } \begin{cases} 0.044 S_{DS} I_e = 0.020 \\ \\ 0.010 \end{cases}$$ ASCE/SEI Eq. (12.8-5)

Therefore, $C_s = 0.036$.

Step 12 – Determine the effective seismic weight ASCE/SEI 12.7.2

The effective seismic weight for this building includes the dead load of the structure (assuming a $24 + 4.5 \times 7 + 53$ wide-module joist system; 32.0 in. by 28.5 in. interior beams; 30.0 in. by 28.5 in. edge beams; 28.0 in. by 28.0 in. and 18.0 in. by 32.0 in. columns in stories 1 through 6; 24.0 in. by 24.0 in. and 18.0 in. by 28.0 in. columns in stories 7 through 12; 12.0-in.-thick walls in stories 1 through 6; and 10.0-in.-thick walls in stories 7 through 12), the superimposed dead loads on the roof and floors, the 15 lb/ft^2 partition weight on the floors, and the weight of the cladding.

The effective seismic weights per floor and the total effective seismic weight, W, are given in Table 3.27.

Table 3.27 Seismic Forces and Story Shears, Building #2

Level	Story Weight, w_x (kips)	Height, h_x (ft)	$w_x h_x^k$	Lateral Force, F_x (kips)	Story Shear, V_x (kips)
R	3,418	139.0	1,046,331	236.0	236.0
12	3,621	128.0	1,007,377	227.2	463.2
11	3,621	117.0	907,662	204.7	667.9
10	3,621	106.0	809,438	182.5	850.4
9	3,621	95.0	712,833	160.8	1,011.2
8	3,621	84.0	618,006	139.4	1,150.6
7	3,652	73.0	529,645	119.4	1,270.0
6	3,687	62.0	442,433	99.8	1,369.8
5	3,687	51.0	352,740	79.6	1,449.4
4	3,687	40.0	266,111	60.0	1,509.4
3	3,687	29.0	183,255	41.3	1.550.7
2	3,840	18.0	109,761	24.8	1,575.5
Σ	W = 43,763		6,985,592	1,575.5	

Step 13 – Determine the seismic base shear ASCE/SEI Eq. (12.8-1)

$$V = C_s W = 0.036 \times 43{,}763 = 1{,}575.5 \text{ kips}$$

Step 14 – Determine the exponent related to the structure period ASCE/SEI 12.8.3

$$k = 0.75 + 0.5T = 0.75 + (0.5 \times 0.81) = 1.16$$

Step 15 – Determine the seismic forces at each level ASCE/SEI Eqs. (12.8-11) and (12.8-12)

$$F_x = C_{vx} V = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k} V$$

Values of F_x are given in Table 3.27.

For example, at level 5:

$$F_x = \frac{3{,}687 \times 51.0^{1.16}}{6{,}985{,}592} \times 1{,}575.5 = 79.6 \text{ kips}$$

3.8.11 Example 3.11 – Determination of Seismic Forces: SFRS of Building #3

Determine the seismic forces on the SFRS for Site Class D (stiff soil; determined) [see Figure 1.3].

Design data are given in Sect. 1.2.3.

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

From Step 1 in Example 3.7, $S_S = 1.386$ and $S_1 = 0.483$.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

From Step 2 in Example 3.7, the building is not permitted to be automatically assigned to SDC A.

Step 3 – Determine the Site Class ASCE/SEI 11.4.3

From the design data, the Site Class is given as D (stiff soil).

Step 4 – Determine the acceleration parameters adjusted for site class effects ASCE/SEI 11.4.4

From Step 5 in Example 3.7, $S_{MS} = 1.386$ and $S_{M1} = 0.879$.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.5

From Step 5 in Example 3.7, $S_{DS} = 0.924$ and $S_{D1} = 0.586$.

Step 6 – Determine the SDC ASCE/SEI 11.6

From Step 7 in Example 3.7, the SDC is D.

Step 7 – Determine the response modification coefficient ASCE/SEI Table 12.2-1

The height of this building is equal to 152.0 ft. Because the building is assigned to SDC D, a building frame system with special reinforced concrete shear (structural) walls may be used in both directions with a height limit of 160.0 ft (SFRS B4).

For this SFRS, $R = 6.0$.

Step 8 – Determine the importance factor ASCE/SEI Table 1.5-2

For Risk Category II buildings, $I_e = 1.00$.

Step 9 – Determine the approximate period ASCE/SEI 12.8.2.1

From Step 6 in Example 3.7, $T_a = C_t h_n^x = 0.02 \times 152.0^{0.75} = 0.87$ s.

From a dynamic analysis of the structure, the periods in the north-south and east-west directions are determined to be 1.4 s and 1.3 s, respectively. According to ASCE/SEI 12.8.2, the fundamental period, T, must not exceed the product of the coefficient for upper limit on calculated period, C_u, from ASCE/SEI Table 12.8-1 and the approximate fundamental period, T_a:

In the north-south direction: $T = 1.4 \text{ s} > C_u T_a = 1.4 \times 0.87 = 1.2 \text{ s}$

In the east-west direction: $T = 1.3 \text{ s} > C_u T_a = 1.4 \times 0.87 = 1.2 \text{ s}$

where $C_u = 1.4$ from ASCE/SEI Table 12.8-1 for $S_{D1} = 0.586 > 0.400$.

Therefore, $T = 1.2 \text{ s}$ is permitted to be used to determine the base shear, V, in both directions.

Step 10 – Determine the long-period transition period ASCE/SEI Figures 22-14 through 22-17

From ASCE/SEI Figure 22-14, $T_L = 6.0 \text{ s}$ (this period can also be determined from Ref. 4, 5, or 6).

Step 11 – Determine the seismic response coefficient ASCE/SEI 12.8.1.1

Because $T = 1.2 \text{ s} < T_L = 6.0 \text{ s}$, C_s is determined as follows:

$$C_s = \frac{S_{D1}}{T\left(\dfrac{R}{I_e}\right)} = \frac{0.586}{1.2 \times \left(\dfrac{6.0}{1.00}\right)} = 0.081 \qquad \text{ASCE/SEI Eq. (12.8-3)}$$

C_s need not exceed the following:

$$C_s = \frac{S_{DS}}{\left(\dfrac{R}{I_e}\right)} = \frac{0.924}{\left(\dfrac{6.0}{1.00}\right)} = 0.154 \qquad \text{ASCE/SEI Eq. (12.8-2)}$$

Minimum C_s:

$$C_s = \text{ greater of } \begin{cases} 0.044 S_{DS} I_e = 0.041 \\ \\ 0.010 \end{cases} \qquad \text{ASCE/SEI Eq. (12.8-5)}$$

It was assumed in Step 5 of Example 3.7 that a ground motion hazard analysis in accordance with ASCE/SEI 21.2 was not required because the second exception in ASCE/SEI 11.4.8 is applicable. Therefore, determine C_s based on that exception:

$$1.5T_S = 1.5S_{D1}/S_{DS} = 0.95 \text{ s} < T = 1.2 \text{ s} < T_L = 6.0 \text{ s}$$

Thus, C_s must be taken as 1.5 times the value computed in accordance with ASCE/SEI Eq. (12.8-3):

$$C_s = 1.5 \times 0.081 = 0.122$$

Step 12 – Determine the effective seismic weight ASCE/SEI 12.7.2

The effective seismic weight for this building includes the dead load of the structure (assuming an 8.0-in.-thick slab; 36.0 in. by 36.0 in. and 16.0 in. by 28.0 in. columns in stories 1 through 8; 30.0 in. by 30.0 in. and 12.0 in. by 24.0 in. columns in stories 9 through 16; and 24.0-in.-thick walls in stories 1 through 8 and 18.0-in.-thick walls in stories 9 through 16), the superimposed dead loads on the roof and floors, and the weight of the cladding.

The effective seismic weights per floor and the total effective seismic weight are given in Table 3.28.

Table 3.28 Seismic Forces and Story Shears, Building #3

Level	Story Weight, w_x (kips)	Height, h_x (ft)	$w_x h_x^k$	Lateral Force, F_x (kips)	Story Shear, V_x (kips)
R	704	152.0	620,957	194.1	194.1
16	808	142.5	653,223	204.2	398.3
15	808	133.0	595,129	186.0	584.3
14	808	123.5	538,470	168.3	752.6
13	808	114.0	483,318	151.1	903.7
12	808	104.5	429,752	134.3	1,038.0
11	808	95.0	377,866	118.1	1,156.1
10	808	85.5	327,767	102.4	1,258.5
9	841	76.0	291,001	90.9	1,349.4
8	874	66.5	252,535	78.9	1,428.3
7	874	57.0	205,089	64.1	1,492.4
6	874	47.5	160,342	50.1	1,542.5
5	874	38.0	118,637	37.1	1,579.6
4	874	28.5	80,455	25.1	1,604.7
3	874	19.0	46,540	14.5	1,619.2
2	874	9.5	18,257	5.7	1,624.9
Σ	W = 13,319		5,199,338	1,624.9	

Step 13 – Determine the seismic base shear ASCE/SEI Eq. (12.8-1)

$$V = C_s W = 0.122 \times 13{,}319 = 1{,}624.9 \text{ kips}$$

Step 14 – Determine the exponent related to the structure period ASCE/SEI 12.8.3

$$k = 0.75 + 0.5T = 0.75 + (0.5 \times 1.2) = 1.35$$

Step 15 – Determine the seismic forces at each level ASCE/SEI Eqs. (12.8-11) and (12.8-12)

$$F_x = C_{vx} V = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k} V$$

Values of F_x are given in Table 3.28.

For example, at level 2:

$$F_x = \frac{874 \times 9.5^{1.35}}{5{,}199{,}338} \times 1{,}624.9 = 5.7 \text{ kips}$$

3.8.12 Example 3.12 – Determination of Seismic Forces: SFRS of Building #4

Determine the seismic forces on the SFRS for Site Class D (default) [see Figure 1.4].

Design data are given in Sect. 1.2.4.

Step 1 – Determine the mapped acceleration parameters ASCE/SEI 11.4.2

From Step 1 in Example 3.8, $S_S = 0.583$ and $S_1 = 0.192$.

Step 2 – Determine if the building is permitted to be automatically assigned to SDC A ASCE/SEI 11.4.2

From Step 2 in Example 3.8, the building is not permitted to be automatically assigned to SDC A.

Step 3 – Determine the Site Class ASCE/SEI 11.4.3

From the design data, the Site Class is given as D (default).

Step 4 – Determine the acceleration parameters adjusted for site class effects ASCE/SEI 11.4.4

From Step 5 in Example 3.8, $S_{MS} = 0.775$ and $S_{M1} = 0.426$.

Step 5 – Determine the design acceleration parameters ASCE/SEI 11.4.5

From Step 5 in Example 3.8, $S_{DS} = 0.517$ and $S_{D1} = 0.284$.

Step 6 – Determine the SDC ASCE/SEI 11.6

From Step 7 in Example 3.8, the SDC is D.

Step 7 – Determine the response modification coefficient ASCE/SEI Table 12.2-1

Because the building is assigned to SDC D, a dual system with special reinforced concrete shear (structural) walls and special moment frames of reinforced concrete capable of resisting at least 25 percent of the prescribed seismic forces may be used with no limits in both directions (SFRS D3).

For this SFRS, $R = 7.0$.

A building frame system with special reinforced concrete shear (structural) walls is not permitted because the building height, which is equal to 330.0 ft, exceeds the height limit of 160.0 ft for this system in SDC D.

Step 8 – Determine the importance factor ASCE/SEI Table 1.5-2

For Risk Category II buildings, $I_e = 1.00$.

Step 9 – Determine the approximate period ASCE/SEI 12.8.2.1

From Step 6 in Example 3.8, $T_a = C_t h_n^x = 0.02 \times 330.0^{0.75} = 1.55$ s.

From a dynamic analysis of the structure, the periods in the north-south and east-west directions are determined to be 3.11 s and 2.52 s, respectively. According to ASCE/SEI 12.8.2, the fundamental period, T, must not exceed the product of the coefficient for upper limit on calculated period, C_u, from ASCE/SEI Table 12.8-1 and the approximate fundamental period, T_a:

In the north-south direction: $T = 3.28 \text{ s} > C_u T_a = 1.42 \times 1.55 = 2.20 \text{ s}$

In the east-west direction: $T = 3.19 \text{ s} > C_u T_a = 1.42 \times 1.55 = 2.20 \text{ s}$

where $C_u = 1.42$ from ASCE/SEI Table 12.8-1 by linear interpolation for $S_{D1} = 0.284$.

Therefore, $T = 2.20 \text{ s}$ is permitted to be used to determine the base shear, V, in both the north-south and east-west directions.

In order to use the ELF Procedure, structures exceeding 160.0 ft in height with no structural irregularities must satisfy the following (ASCE/SEI Table 12.6-1):

$$T < 3.5T_S = 3.5 S_{D1}/S_{DS} = 1.92 \text{ s}$$

Therefore, use $T = 1.90 \text{ s}$ in both directions.

Step 10 – Determine the long-period transition period ASCE/SEI Figures 22-14 through 22-17

From ASCE/SEI Figure 22-14, $T_L = 6.0 \text{ s}$ (this period can also be determined from Ref. 4, 5, or 6).

Step 11 – Determine the seismic response coefficient ASCE/SEI 12.8.1.1

Because $T < T_L = 6.0 \text{ s}$, C_s is determined as follows:

$$C_s = \frac{S_{D1}}{T\left(\frac{R}{I_e}\right)} = \frac{0.284}{1.90 \times \left(\frac{7.0}{1.00}\right)} = 0.021$$ ASCE/SEI Eq. (12.8-3)

C_s need not exceed the following:

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.517}{\left(\frac{7.0}{1.00}\right)} = 0.074$$ ASCE/SEI Eq. (12.8-2)

Minimum C_s:

$$C_s = \text{ greater of } \begin{cases} 0.044 S_{DS} I_e = 0.023 \\ \\ 0.010 \end{cases}$$ ASCE/SEI Eq. (12.8-5)

Thus, $C_s = 0.023$.

Step 12 – Determine the effective seismic weight ASCE/SEI 12.7.2

The effective seismic weight for this building includes the dead load of the structure (assuming wide-module roof and floor systems with $24 + 4.5 \times 7 + 53$; 36.0 in. by 36.0 in. columns in stories 1 through 10; 30.0 in. by 30.0 in. columns in stories 11 through 20; 24.0 in. by 24.0 in. columns in stories 21 through 30; beams (including collector beams) are as wide as the columns with a depth of 28.5 in.; coupling beams are wide as the walls with a depth of 42.0 in.; 24.0-in.-thick walls in stories 1 through 10; 18.0-in.-thick walls in stories 11 through 20; and 12.0-in.-thick walls in stories 21 through 30), the superimposed dead loads on the roof and floors, the weight of the partitions, and the weight of the cladding.

The effective seismic weights per floor and the total effective seismic weight are given in Table 3.29.

Table 3.29 Seismic Forces and Story Shears, Building #4

Level	Story Weight, w_x (kips)	Height, h_x (ft)	$w_x h_x^k$	Lateral Force, F_x (kips)	Story Shear, V_x (kips)
R	3,184	330.0	60,876,091	203.1	203.1
30	3,256	319.0	58,766,329	196.1	399.2
29	3,261	308.0	55,448,144	185.0	584.2
28	3,261	297.0	52,123,883	173.9	758.1
27	3,261	286.0	48,884,711	163.1	921.2
26	3,261	275.0	45,731,598	152.6	1,073.8
25	3,261	264.0	42,665,562	142.4	1,216.2
24	3,261	253.0	39,687,678	132.4	1,348.6
23	3,261	242.0	36,799,077	122.8	1,471.4
22	3,261	231.0	34,000,959	113.5	1,584.9
21	3,579	220.0	34,346,323	114.6	1,699.5
20	3,671	209.0	32,287,400	107.7	1,807.2
19	3,671	198.0	29,452,028	98.3	1,905.5
18	3,671	187.0	26,724,840	89.2	1,994.7
17	3,671	176.0	24,107,708	80.5	2,075.2
16	3,671	165.0	21,602,653	72.1	2,147.3
15	3,671	154.0	19,211,869	64.1	2,211.4
14	3,671	143.0	16,937,747	56.5	2,267.9
13	3,671	132.0	14,782,915	49.3	2,317.2
12	3,671	121.0	12,750,274	42.6	2,359.8
11	3,980	110.0	11,755,758	39.2	2,399.0
10	4,092	99.0	10,104,515	33.7	2,432.7
9	4,092	88.0	8,270,965	27.6	2,460.3
8	4,092	77.0	6,591,282	22.0	2,482.3
7	4,092	66.0	5,071,779	16.9	2,499.2
6	4,092	55.0	3,720,079	12.4	2,511.6

(table continued on next page)

Table 3.29 Seismic Forces and Story Shears, Building #4 (cont.)

Level	Story Weight, w_x (kips)	Height, h_x (ft)	$w_x h_x^k$	Lateral Force, F_x (kips)	Story Shear, V_x (kips)
5	4,092	44.0	2,545,688	8.5	2,520.1
4	4,092	33.0	1,561,023	5.2	2,525.3
3	4,092	22.0	783,527	2.6	2,527.9
2	4,092	11.0	241,159	1.0	2,528.9
Σ	W = 109,954		757,833,564	2,528.9	

Step 13 – Determine the seismic base shear ASCE/SEI Eq. (12.8-1)

$$V = C_s W = 0.023 \times 109,954 = 2,528.9 \text{ kips}$$

Step 14 – Determine the exponent related to the structure period ASCE/SEI 12.8.3

$$k = 0.75 + 0.5T = 0.75 + (0.5 \times 1.90) = 1.70$$

Step 15 – Determine the seismic forces at each level ASCE/SEI Eqs. (12.8-11) and (12.8-12)

$$F_x = C_{vx} V = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k} V$$

Values of F_x are given in Table 3.29.

For example, at the roof level:

$$F_x = \frac{3,184 \times 330.0^{1.70}}{757,833,564} \times 2,528.9 = 203.1 \text{ kips}$$

3.8.13 Example 3.13 – Determination of Seismic Forces: Diaphragms of Building #1 (Framing Option A)

Determine the seismic diaphragm design forces over the height of the building in accordance with ASCE/SEI 12.10.1.1 for Site Class D (default) [see Figure 1.1].

Design data are given in Sect. 1.2.1.

Step 1 – Determine the seismic forces on the SFRS ASCE/SEI 12.8.3

Seismic forces F_i over the height of the building are determined in Step 15 of Example 3.9 (see Table 3.26) and are given in Table 3.30.

Table 3.30 Seismic Diaphragm Design Forces, Building #1

Level	Story Weight, w_i (kips)	Lateral Force, F_i (kips)	Σw_i (kips)	ΣF_i (kips)	w_{px} (kips)	$\Sigma F_i / \Sigma w_i$	F_{px} (kips)	Design Force (kips)**
R	2,043	149.3	2,043	149.3	2,043	0.051*	104.2	149.3
5	2,368	137.8	4,411	287.1	2,368	0.051*	120.8	137.8

(table continued on next page)

Table 3.30 Seismic Diaphragm Design Forces, Building #1 (cont.)

Level	Story Weight, w_i (kips)	Lateral Force, F_i (kips)	Σw_i (kips)	ΣF_i (kips)	w_{px} (kips)	$\Sigma F_i / \Sigma w_i$	F_{px} (kips)	Design Force (kips)**
4	2,368	103.2	6,779	390.3	2,368	0.051*	120.8	120.8
3	2,368	69.2	9,147	459.5	2,368	0.050	118.4	118.4
2	2,399	37.0	11,546	496.5	2,399	0.043	103.2	103.2

* *Max.* $\Sigma F_i / \Sigma w_i = 0.051$

** *Max. of* F_i and F_{px}

Step 2 – Determine the seismic diaphragm forces ASCE/SEI Eqs. (12.10-1), (12.10-2), and (12.10-3)

$$0.2S_{DS}I_e w_{px} \le F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px} \le 0.4S_{DS}I_e w_{px}$$

Diaphragm forces F_{px} over the height of the building are given in Table 3.30. Values of the story weight, w_i, and the seismic lateral force, F_i, are given in Table 3.26 of Example 3.9. The weight tributary to the diaphragm at level x, w_{px}, is set equal to w_i.

It is evident from Table 3.30 that the maximum $F_{px} = 0.4S_{DS}I_e w_{px} = 0.4 \times 0.128 \times 1.00 \times w_{px} = 0.051 w_{px}$ governs at the three top levels of the building. For example, at the roof level:

$$F_{px} = \frac{\sum_{i=\mathrm{R}}^{\mathrm{R}} F_i}{\sum_{i=\mathrm{R}}^{\mathrm{R}} w_i} w_{px} = \frac{149.3}{2,043} w_{px} = 0.073 w_{px} > 0.4S_{DS}I_e w_{px} = 0.051 w_{px}, \text{ use } 0.051 w_{px}$$

Also, minimum $F_{px} = 0.2S_{DS}I_e w_{px} = 0.026 w_{px}$.

Step 3 – Determine the seismic diaphragm design forces ASCE/SEI 12.10.1.1

Seismic diaphragm design forces are the larger of F_i and F_{px}. The design forces over the height of the building are given in Table 3.30.

3.8.14 Example 3.14 – Determination of Seismic Forces: Diaphragms of Building #2

Determine the seismic diaphragm design forces over the height of the building in accordance with ASCE/SEI 12.10.1.1 (see Figure 1.2).

Design data are given in Sect. 1.2.2.

Step 1 – Determine the seismic forces on the SFRS ASCE/SEI 12.8.3

Seismic forces F_i over the height of the building are determined in Step 15 of Example 3.10 (see Table 3.27) and are given in Table 3.31.

Table 3.31 Seismic Diaphragm Design Forces, Building #2

Level	Story Weight, w_i (kips)	Lateral Force, F_i (kips)	Σw_i (kips)	ΣF_i (kips)	w_{px} (kips)	$\Sigma F_i/\Sigma w_i$	F_{px} (kips)	Design Force (kips)**
R	3,418	236.0	3,418	236.0	3,418	0.090*	307.6	307.6
12	3,621	227.2	7,039	463.2	3,621	0.090*	325.9	325.9
11	3,621	204.7	10,660	667.9	3,621	0.090*	325.9	325.9
10	3,621	182.5	14,281	850.4	3,621	0.090*	325.9	325.9
9	3,621	160.8	17,902	1,011.2	3,621	0.090*	325.9	325.9
8	3,621	139.4	21,523	1,150.6	3,621	0.090*	325.9	325.9
7	3,652	119.4	25,175	1,270.0	3,652	0.090*	328.7	328.7
6	3,687	99.8	28,862	1,369.8	3,687	0.090*	331.8	331.8
5	3,687	79.6	32,549	1,449.4	3,687	0.090*	331.8	331.8
4	3,687	60.0	36,236	1,509.4	3,687	0.090*	331.8	331.8
3	3,687	41.3	39,923	1.550.7	3,687	0.090*	331.8	331.8
2	3,840	24.8	43,763	1,575.5	3,840	0.090*	345.6	345.6

* *Min.* $\Sigma F_i / \Sigma w_i = 0.090$

** *Max. of* F_i and F_{px}

Step 2 – Determine the seismic diaphragm forces ASCE/SEI Eqs. (12.10-1), (12.10-2), and (12.10-3)

$$0.2S_{DS}I_e w_{px} \leq F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px} \leq 0.4S_{DS}I_e w_{px}$$

Diaphragm forces F_{px} over the height of the building are given in Table 3.31. Values of the story weight, w_i, and the seismic lateral force, F_i, are given in Table 3.27 of Example 3.10. The weight tributary to the diaphragm at level x, w_{px}, is set equal to w_i.

It is evident from Table 3.31 that the minimum $F_{px} = 0.2S_{DS}I_e w_{px} = 0.2 \times 0.301 \times 1.50 \times w_{px} = 0.090 w_{px}$ governs at all levels of the building. For example, at the second-floor level:

$$F_{px} = \frac{\sum_{i=2}^{R} F_i}{\sum_{i=2}^{R} w_i} w_{px} = \frac{1{,}575.5}{43{,}763} w_{px} = 0.036 w_{px} < 0.2S_{DS}I_e w_{px} = 0.090 w_{px}, \text{ use } 0.090 w_{px}$$

Also, maximum $F_{px} = 0.4S_{DS}I_e w_{px} = 0.180 w_{px}$.

Step 3 – Determine the seismic diaphragm design forces ASCE/SEI 12.10.1.1

Seismic diaphragm design forces are the larger of F_i and F_{px}. The design forces over the height of the building are given in Table 3.31.

3.8.15 Example 3.15 – Determination of Seismic Forces: Diaphragms of Building #3

Determine the seismic diaphragm design forces over the height of the building in accordance with ASCE/SEI 12.10.1.1 (see Figure 1.3).

Design data are given in Sect. 1.2.3.

Step 1 – Determine the seismic forces on the SFRS ASCE/SEI 12.8.3

Seismic forces F_i over the height of the building are determined in Step 15 of Example 3.11 (see Table 3.28) and are given in Table 3.32.

Table 3.32 Seismic Diaphragm Design Forces, Building #3

Level	Story Weight, w_i (kips)	Lateral Force, F_i (kips)	Σw_i (kips)	ΣF_i (kips)	w_{px} (kips)	$\Sigma F_i / \Sigma w_i$	F_{px} (kips)	Design Force (kips)**
R	704	194.1	704	194.1	704	0.276	194.1	194.1
16	808	204.2	1,512	398.3	808	0.263	212.9	212.9
15	808	186.0	2,320	584.3	808	0.252	203.5	203.5
14	808	168.3	3,128	752.6	808	0.241	194.4	194.4
13	808	151.1	3,936	903.7	808	0.230	185.8	185.8
12	808	134.3	4,744	1,038.0	808	0.219	177.0	177.0
11	808	118.1	5,552	1,156.1	808	0.208	168.3	168.3
10	808	102.4	6,360	1,258.5	808	0.198	160.0	160.0
9	841	90.9	7,201	1,349.4	841	0.187	157.3	157.3
8	874	78.9	8,075	1,428.3	874	0.185*	161.7	161.7
7	874	64.1	8,949	1,492.4	874	0.185*	161.7	161.7
6	874	50.1	9,823	1,542.5	874	0.185*	161.7	161.7
5	874	37.1	10,697	1,579.6	874	0.185*	161.7	161.7
4	874	25.1	11,571	1,604.7	874	0.185*	161.7	161.7
3	874	14.5	12,445	1,619.2	874	0.185*	161.7	161.7
2	874	5.7	13,319	1,624.9	874	0.185*	161.7	161.7

* *Min.* $\Sigma F_i / \Sigma w_i = 0.185$

** *Max. of* F_i and F_{px}

Step 2 – Determine the seismic diaphragm forces ASCE/SEI Eqs. (12.10-1), (12.10-2), and (12.10-3)

$$0.2 S_{DS} I_e w_{px} \le F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px} \le 0.4 S_{DS} I_e w_{px}$$

Diaphragm forces F_{px} over the height of the building are given in Table 3.32. Values of the story weight, w_i, and the seismic lateral force, F_i, are given in Table 3.28 of Example 3.11. The weight tributary to the diaphragm at level x, w_{px}, is set equal to w_i.

It is evident from Table 3.32 that the minimum $F_{px} = 0.2S_{DS}I_e w_{px} = 0.2 \times 0.924 \times 1.00 \times w_{px} = 0.185w_{px}$ governs at levels 2 through 8 of the building. For example, at the second-floor level:

$$F_{px} = \frac{\sum_{i=2}^{R} F_i}{\sum_{i=2}^{R} w_i} w_{px} = \frac{1,624.9}{13,319} w_{px} = 0.122w_{px} < 0.2S_{DS}I_e w_{px} = 0.185w_{px}, \text{ use } 0.185w_{px}$$

Also, maximum $F_{px} = 0.4S_{DS}I_e w_{px} = 0.370w_{px}$.

Step 3 – Determine the seismic diaphragm design forces ASCE/SEI 12.10.1.1

Seismic diaphragm design forces are the larger of F_i and F_{px}. The design forces over the height of the building are given in Table 3.32.

3.8.16 Example 3.16 – Determination of Seismic Forces: Diaphragms of Building #4

Determine the seismic diaphragm design forces over the height of the building in accordance with ASCE/SEI 12.10.1.1 (see Figure 1.4).

Design data are given in Sect. 1.2.4.

Step 1 – Determine the seismic forces on the SFRS ASCE/SEI 12.8.3

Seismic forces F_i over the height of the building are determined in Step 15 of Example 3.12 in the north-south and east-west directions (see Table 3.29) and are given in Table 3.33.

Table 3.33 Seismic Diaphragm Design Forces, Building #4

Level	Story Weight, w_i (kips)	Lateral Force, F_i (kips)	Σw_i (kips)	ΣF_i (kips)	w_{px} (kips)	$\Sigma F_i / \Sigma w_i$	F_{px} (kips)	Design Force (kips)**
R	3,184	203.1	3,184	203.1	3,184	0.103*	328.0	328.0
30	3,256	196.1	6,440	399.2	3,256	0.103*	335.4	335.4
29	3,261	185.0	9,701	584.2	3,261	0.103*	335.9	335.9
28	3,261	173.9	12,962	758.1	3,261	0.103*	335.9	335.9
27	3,261	163.1	16,223	921.2	3,261	0.103*	335.9	335.9
26	3,261	152.6	19,484	1,073.8	3,261	0.103*	335.9	335.9
25	3,261	142.4	22,745	1,216.2	3,261	0.103*	335.9	335.9
24	3,261	132.4	26,006	1,348.6	3,261	0.103*	335.9	335.9
23	3,261	122.8	29,267	1,471.4	3,261	0.103*	335.9	335.9
22	3,261	113.5	32,528	1,584.9	3,261	0.103*	335.9	335.9
21	3,579	114.6	36,107	1,699.5	3,579	0.103*	368.6	368.6

(table continued on next page)

Table 3.33 Seismic Diaphragm Design Forces, Building #4 (cont.)

Level	Story Weight, w_i (kips)	Lateral Force, F_i (kips)	Σw_i (kips)	ΣF_i (kips)	w_{px} (kips)	$\Sigma F_i / \Sigma w_i$	F_{px} (kips)	Design Force (kips)**
20	3,671	107.7	39,778	1,807.2	3,671	0.103*	378.1	378.1
19	3,671	98.3	43,449	1,905.5	3,671	0.103*	378.1	378.1
18	3,671	89.2	47,120	1,994.7	3,671	0.103*	378.1	378.1
17	3,671	80.5	50,791	2,075.2	3,671	0.103*	378.1	378.1
16	3,671	72.1	54,462	2,147.3	3,671	0.103*	378.1	378.1
15	3,671	64.1	58,133	2,211.4	3,671	0.103*	378.1	378.1
14	3,671	56.5	61,804	2,267.9	3,671	0.103*	378.1	378.1
13	3,671	49.3	65,475	2,317.2	3,671	0.103*	378.1	378.1
12	3,671	42.6	69,146	2,359.8	3,671	0.103*	378.1	378.1
11	3,980	39.2	73,126	2,399.0	3,980	0.103*	409.9	409.9
10	4,092	33.7	77,218	2,432.7	4,092	0.103*	421.5	421.5
9	4,092	27.6	81,310	2,460.3	4,092	0.103*	421.5	421.5
8	4,092	22.0	85,402	2,482.3	4,092	0.103*	421.5	421.5
7	4,092	16.9	89,494	2,499.2	4,092	0.103*	421.5	421.5
6	4,092	12.4	93,586	2,511.6	4,092	0.103*	421.5	421.5
5	4,092	8.5	97,678	2,520.1	4,092	0.103*	421.5	421.5
4	4,092	5.2	101,770	2,525.3	4,092	0.103*	421.5	421.5
3	4,092	2.6	105,862	2,527.9	4,092	0.103*	421.5	421.5
2	4,092	1.0	109,954	2,528.9	4,092	0.103*	421.5	421.5

* *Min.* $\Sigma F_i / \Sigma w_i = 0.103$

** *Max. of* F_i and F_{px}

Step 2 – Determine the seismic diaphragm forces ASCE/SEI Eqs. (12.10-1), (12.10-2), and (12.10-3)

$$0.2S_{DS}I_e w_{px} \le F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px} \le 0.4S_{DS}I_e w_{px}$$

Diaphragm forces F_{px} over the height of the building are given in Table 3.33. Values of the story weight, w_i, and the seismic lateral force, F_i, are given in Table 3.29 of Example 3.12. The weight tributary to the diaphragm at level x, w_{px}, is set equal to w_i.

It is evident from Table 3.33 that the minimum $F_{px} = 0.2S_{DS}I_e w_{px} = 0.2 \times 0.517 \times 1.00 \times w_{px} = 0.103 w_{px}$ governs at all levels of the building. For example, at the second-floor level:

$$F_{px} = \frac{\sum_{i=2}^{R} F_i}{\sum_{i=2}^{R} w_i} w_{px} = \frac{2,528.9}{109,954} w_{px} = 0.023 w_{px} < 0.2 S_{DS} I_e w_{px} = 0.103 w_{px}, \text{ use } 0.103 w_{px}$$

Also, maximum $F_{px} = 0.4 S_{DS} I_e w_{px} = 0.206 w_{px}$.

Step 3 – Determine the seismic diaphragm design forces ASCE/SEI 12.10.1.1

Seismic diaphragm design forces are the larger of F_i and F_{px}. The design forces over the height of the building are given in Table 3.33.

Chapter 4
ONE-WAY SLABS

4.1 Overview

One-way slabs are elements in one-way construction where the members are designed to support loads through bending in a single direction. Where the ratio of the long to the short side of a slab panel is greater than 2, load transfer is predominately by bending in the short direction, and the panel is defined as a one-way slab (see Figure 4.1). The main flexural reinforcement in a one-way slab system runs parallel to the direction of load transfer.

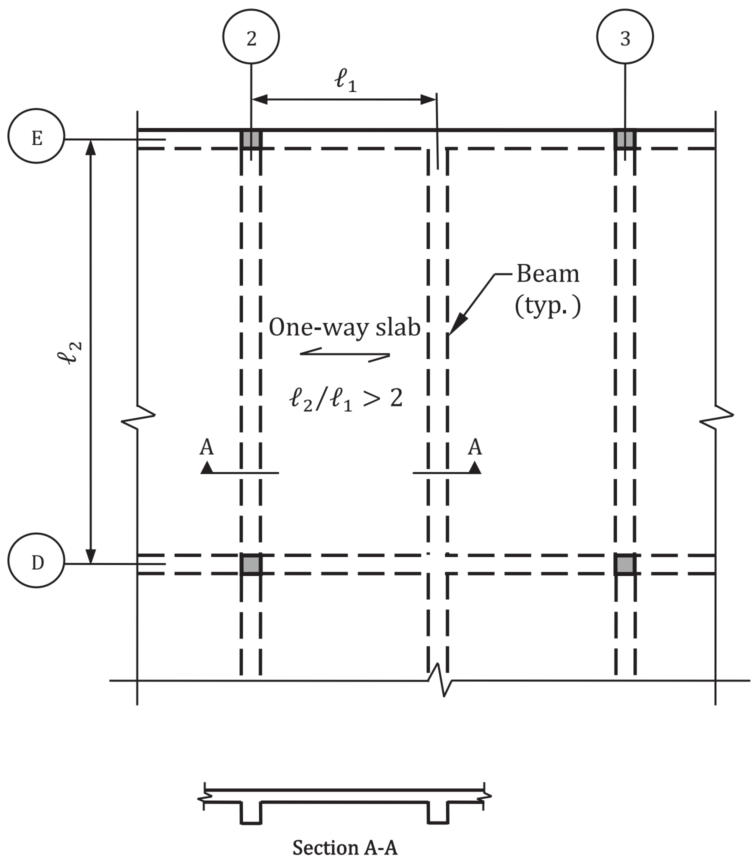


Figure 4.1 A one-way slab system.

The design and detailing of solid, cast-in-place one-way slabs with nonprestressed reinforcement are covered in this chapter. Provisions for one-way slabs are given in ACI Chapter 7.

4.2 Minimum Slab Thickness

One-way slabs must have sufficient thickness so that all applicable strength and serviceability requirements are satisfied. In lieu of calculating deflections in accordance with ACI 24.2 and subsequently checking that the deflections do not exceed the limits in ACI 24.2.2 (ACI 7.3.2), the minimum overall slab thickness, h, for solid, nonprestressed slabs not supporting or attached to partitions or other types of construction likely to be damaged by large deflections can be determined using the limits in ACI Table 7.3.1.1; these limits are applicable to one-way slabs with reinforcement that has a specified yield strength, f_y, equal to 60,000 psi and normalweight concrete (ACI 7.3.1.1). Modification factors for slabs with reinforcement other than 60,000 psi and lightweight concrete are given in ACI 7.3.1.1.1 and 7.3.1.1.2, respectively.

Minimum slab thicknesses based on various support conditions are given in Table 4.1. In the expressions for minimum h, ℓ is the span length of the one-way slab in inches; for cantilevers, ℓ is the clear projection of the cantilever in inches. Also, f_1 and f_2 are the modification factors for reinforcement grade and lightweight concrete, respectively.

Table 4.1 Minimum Thickness of Solid, Nonprestressed One-way Slabs

Support Condition	Normalweight Concrete		Lightweight Concrete(1)	
	f_y = 60 ksi	f_y other than 60 ksi(2)	f_y = 60 ksi(3)	f_y other than 60 ksi(2),(3)
Simply supported	$\ell / 20$	$(\ell / 20)f_1$	$(\ell / 20)f_2$	$(\ell / 20)f_1f_2$
One end continuous	$\ell / 24$	$(\ell / 24)f_1$	$(\ell / 24)f_2$	$(\ell / 24)f_1f_2$
Both ends continuous	$\ell / 28$	$(\ell / 28)f_1$	$(\ell / 28)f_2$	$(\ell / 28)f_1f_2$
Cantilever	$\ell / 10$	$(\ell / 10)f_1$	$(\ell / 10)f_2$	$(\ell / 10)f_1f_2$

(1) Applicable where equilibrium density, w_c, is in the range of 90 to 115 lb/ft³
(2) $f_1 = 0.4 + (f_y / 100{,}000)$ [f_y in psi]
(3) f_2 = greater of $(1.65 - 0.005w_c)$ and 1.09 [w_c in lb/ft^3]

For the usual case of continuous construction, the thickness of the slab, h, should be the same for all spans and it should be determined on the basis of the span yielding the largest minimum depth; this results in economical formwork (see Reference 7). For the one-way slab system in Figure 4.2 with normalweight concrete and Grade 60 reinforcement, the minimum slab thickness to be used for all the spans is equal to 7.7 in., which is rounded up to 8.0 in. Slab thickness is typically specified in whole-inch increments although half-inch increments are commonly used in wide-module joist construction.

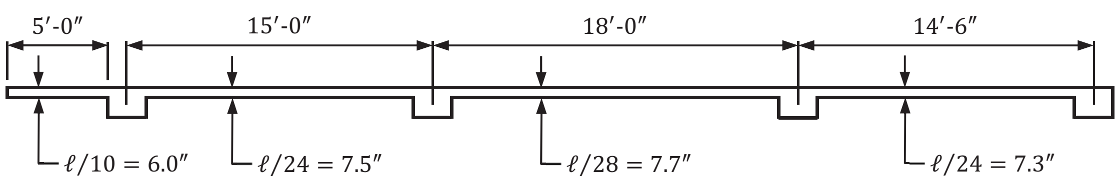


Figure 4.2 Calculation of minimum slab thickness for a one-way slab system.

Fire-resistance requirements of the general building code must also be considered when specifying a slab thickness (ACI 4.11.1). This is especially important for one-way slabs that are part of a joist system where the joists are spaced relatively closely together, which means the required slab thickness for serviceability is relatively small. In certain situations, the required slab thickness based on fire-resistance requirements found in IBC Table 721.1(2) is greater than that required by ACI 7.3.1 for serviceability (ACI 4.11.2).

4.3 Required Strength

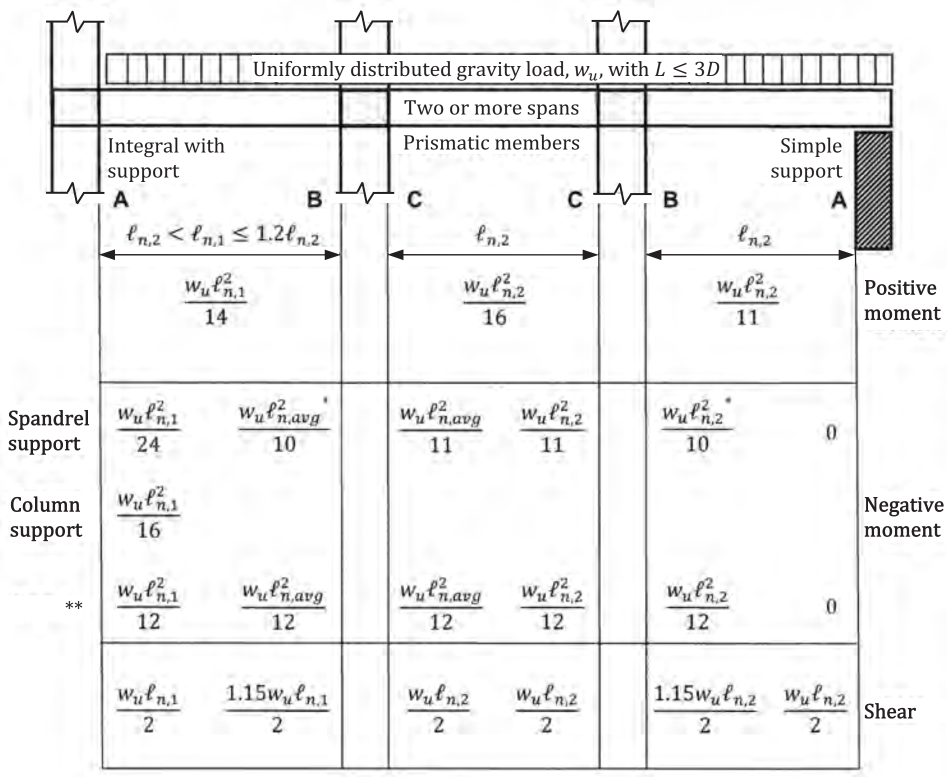
4.3.1 Analysis Methods

The analysis methods in ACI Chapter 6 in conjunction with the factored load combinations in ACI Chapter 5 are to be used to calculate required strength (see ACI 7.4.1.2 and 7.4.1.1, respectively). For the case of gravity loads, the simplified method for analysis of continuous beams and one-way slabs in ACI 6.5 is permitted to be used provided the conditions in ACI 6.5.1 are satisfied [ACI 6.2.3(a)]:

1. Members are prismatic
2. Gravity loads are uniformly distributed

3. The service live load, L, is less than or equal to 3 times the service dead load, D
4. There are at least two spans
5. The longer of two adjacent spans does not exceed the shorter span by more than 20 percent

The approximate factored bending moments, M_u, and factored shear forces, V_u, in ACI 6.5.2 and 6.5.4, respectively, are given in Figure 4.3 where w_u is the factored uniformly distributed gravity load on the one-way slab.



* For two-span condition, first interior negative moment $= w_u \ell_n^2/9$

$\ell_{n,avg} = (\ell_{n,1} + \ell_{n,2})/2$

** Applicable to slabs with spans equal to or less than 10 ft and beams where the ratio of the sum of column stiffness to beam stiffness is greater than 8 at each end of the span.

A – Interior face of exterior support
B – Exterior face of first interior support
C – Other faces of interior supports

Figure 4.3 Simplified analysis method for continuous beams and one-way slabs.

4.3.2 Critical Sections for Flexure and Shear

In accordance with ACI 7.4.2.1 and 7.4.3.1, M_u and V_u are permitted to be calculated at the faces of the supports for one-way slabs built integrally with the supports (see Figure 4.3). For flexural design, the critical sections occur at the faces of the supports where negative moments are maximum and in the span where positive moments are maximum. One-way slabs are permitted to be designed for the shear force at a critical section located a distance d from the face of the support where the conditions in ACI 7.4.3.2 are satisfied (see Figure 4.4):

- Support reaction, in direction of applied shear, introduces compression into the end region of the slab
- Loads are applied at or near the top surface of the slab
- No concentrated load occurs between the face of the support and the critical section

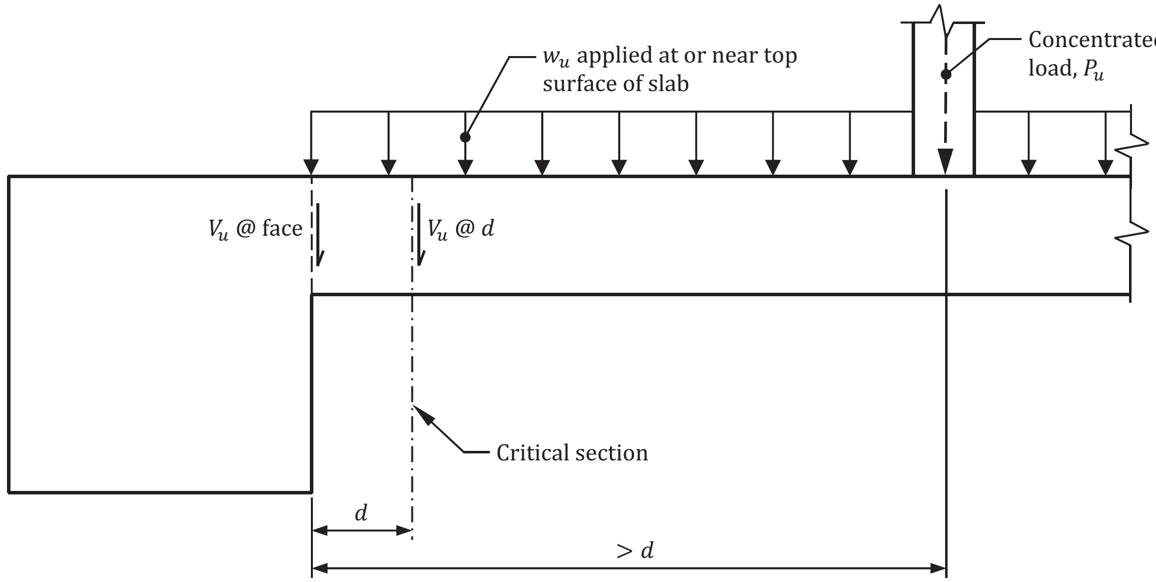


Figure 4.4 Critical section for shear in a one-way slab satisfying the conditions of ACI 7.4.3.2.

The critical section for shear must be taken at the face of the support where one or more of the three conditions in ACI 7.4.3.2 are not met. For example, the critical section must be at the face of the support for the framing configuration in Figure 4.5 because the supporting member is in tension.

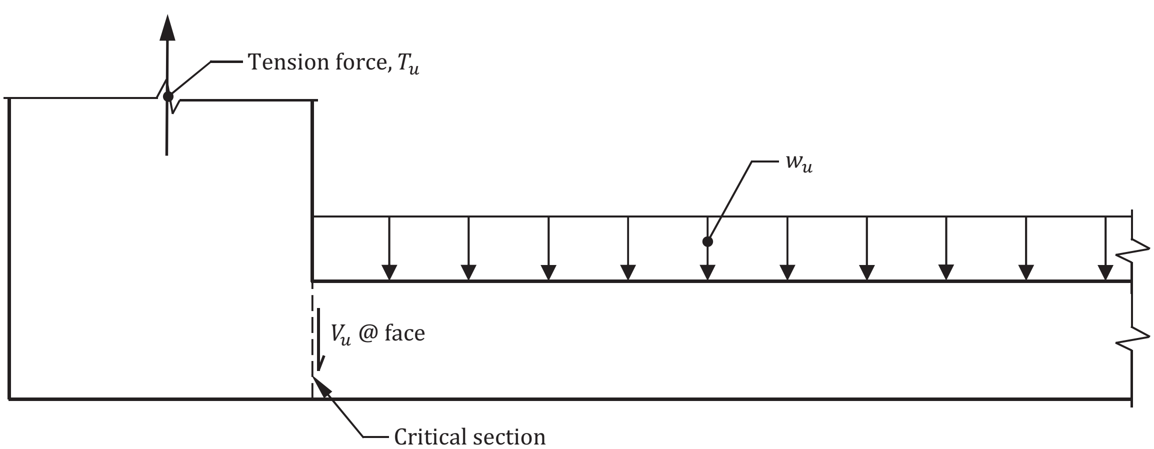


Figure 4.5 Critical section for shear in a one-way slab supported by a member in tension.

Moment redistribution is not permitted when bending moments are calculated using the simplified method (ACI 6.5.3). Where one or more of the conditions in ACI 6.5.1 is not satisfied, one of the other four methods of analysis given in ACI 6.2.3 must be used to determine M_u and V_u at the critical sections.

4.4 Design Strength

4.4.1 General

The following equations must be satisfied at any section in a one-way slab for each applicable factored load combination in ACI Table 5.3.1 (ACI 7.5.1.1):

$$\phi M_n \geq M_u \tag{4.1}$$

$$\phi V_n \geq V_u \tag{4.2}$$

Strength reduction factors, ϕ, are determined in accordance with ACI 21.2. Nonprestressed one-way slabs must be designed as tension-controlled in accordance with ACI Table 21.2.2 (ACI 7.3.3.1); thus, the net tensile strain in the extreme layer of the longitudinal tension reinforcement at nominal strength, ε_t, must be greater than or equal to $\varepsilon_{ty} + 0.003$ and $\phi = 0.90$ (ACI Table 21.2.2). The net tensile strain in the extreme layer of longitudinal tension reinforcement corresponding to compression-controlled sections, ε_{ty}, is equal to f_y / E_s (ACI 21.2.2.1). The modulus of elasticity of the reinforcement, E_s, is permitted to be taken as 29,000,000 psi regardless of the grade of the reinforcement (ACI 20.2.2.2). For shear, $\phi = 0.75$ (ACI Table 21.2.1).

Methods to determine the nominal strengths M_n and V_n are given in Sections 4.4.2 and 4.4.3, respectively.

4.4.2 Nominal Flexural Strength

The nominal flexural strength, M_n, of a rectangular section with one layer of tension reinforcement is determined in accordance with ACI 22.3, and is based on moment equilibrium of a rectangular section (see ACI 7.5.2.1 and Figure 4.6):

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \tag{4.3}$$

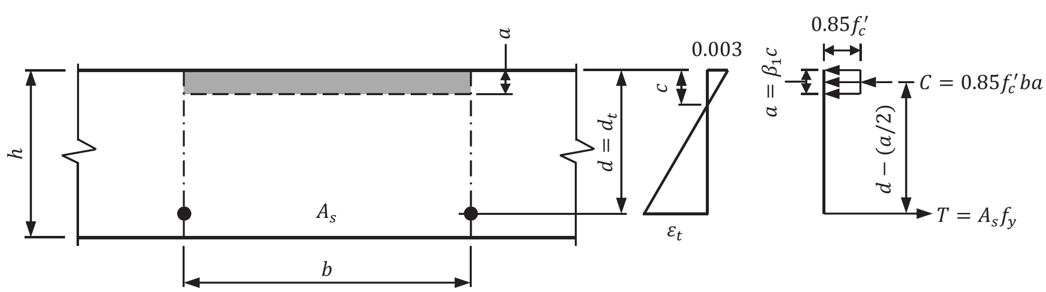


Figure 4.6 Strain and stress distributions at a positive moment section in a one-way slab.

In this equation, A_s is the total area of the flexural reinforcement. The strain and stress distributions in Figure 4.6 are at a positive moment section and are equally applicable at negative moment sections.

For one-way slabs, the average distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement, d, can be approximated as $h - 1.25$ in. Using an approximate d provides sufficient accuracy when determining the required amount of flexural reinforcement in typical one-way slab systems.

The depth of the equivalent stress block, a, is determined from force equilibrium, that is, it is determined by setting the resultant compressive force in the concrete, $C = 0.85 f_c' ba$, equal to the tension force in the reinforcement, $T = A_s f_y$, and solving for a (see Figure 4.6):

$$a = \frac{A_s f_y}{0.85 f_c' b} \tag{4.4}$$

It is assumed that a uniform stress equal to 85 percent of the concrete compressive strength, f_c', is distributed over the depth $a = \beta_1 c$, where c is the distance from the extreme compression fiber to the neutral axis (ACI 22.2.2.4.1).

The maximum strain in the concrete is assumed to be 0.003 (ACI 22.2.2.1), and the term b is the width of the compression face of the member, which is typically taken as 12 in. for a one-way slab.

The term β_1 is the factor relating the depth of the equivalent rectangular compression block, a, to the depth of the neutral axis, c; β_1 is defined in ACI Table 22.2.2.4.3 as follows (ACI 22.2.2.4.3):

- For $2{,}500 \text{ psi} \le f'_c \le 4{,}000 \text{ psi}$: $\beta_1 = 0.85$
- For $4{,}000 \text{ psi} < f'_c < 8{,}000 \text{ psi}$: $\beta_1 = 0.85 - [0.05(f'_c - 4{,}000) / 1{,}000]$ $[f'_c \text{ in psi}]$
- For $f'_c \ge 8{,}000 \text{ psi}$: $\beta_1 = 0.65$

4.4.3 Nominal Shear Strength

The nominal one-way shear strength, V_n, at a section is determined in accordance with ACI 22.5 (ACI 7.5.3.1):

$$V_n = V_c + V_s \tag{4.5}$$

In this equation, V_c is the nominal shear strength provided by concrete and V_s is the nominal shear strength provided by shear reinforcement. Shear reinforcement is typically not used in one-way slabs because of the issues associated with cost, fabrication, and placement of such reinforcement in a relatively thin slab; where additional shear strength is required, the slab thickness is often increased.

For nonprestressed members, V_c is determined by ACI Table 22.5.5.1 (ACI 22.5.5.1). Equation (c) in ACI Table 22.5.5.1 is applicable where no shear reinforcement is used in a section. This equation reduces to the following for the case where the axial force, N_u, is equal to zero:

$$V_c = 8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c}b_w d \tag{4.6}$$

In this equation, b_w is the width of the web of the section; for one-way slabs, $b_w = b$. According to ACI 22.5.5.1.1, V_c must not be taken greater than $5\lambda\sqrt{f'_c}b_w d$.

The size effect modification factor, λ_s, accounts for the phenomenon indicated in test results that the shear strength attributed to concrete in members without shear reinforcement does not increase in direct proportion with member depth. This factor is determined by ACI Equation (22.5.5.1.3) [ACI 22.5.5.1.3]:

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} \le 1.0 \tag{4.7}$$

It is evident from Equation (4.7) that λ_s is less than 1.0 for members with $d > 10.0$ in. For one-way slabs, this means that $\lambda_s = 1.0$ for slabs with an overall thickness less than about 11.0 in.

The term λ is the modification factor reflecting the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength, and is determined based on either (1) the equilibrium density, w_c, of the concrete mix or (2) the composition of the aggregate in the concrete mix (see ACI 22.5.1.5 and ACI 19.2.4.1). Values of λ based on w_c are given in Table 4.2 [ACI Table 19.2.4.1(a)] and values of λ based on composition of aggregates are given in Table 4.3 [ACI Table 19.2.4.1(b)]. Note that λ is permitted to be taken as 0.75 for lightweight concrete (ACI 19.2.4.2) and is equal to 1.0 for normalweight concrete (ACI 19.2.4.3).

Table 4.2 Values of λ Based on Equilibrium Density, w_c

Equilibrium Density, w_c	λ
$w_c \le 100 \text{ lb/ft}^3$	0.75
$100 \text{ lb/ft}^3 < w_c \le 135 \text{ lb/ft}^3$	$0.0075w_c \le 1.0$
$w_c > 135 \text{ lb/ft}^3$	1.0

Table 4.3 Values of λ Based on Composition of Aggregates

Concrete	Composition of Aggregates	λ
All-lightweight	Fine: ASTM C330 Coarse: ASTM C330	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330 and ASTM C33 Coarse: ASTM C330	0.75 to 0.85(1)
Sand-lightweight	Fine: ASTM C33 Coarse: ASTM C330	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33 Coarse: Combination of ASTM C330 and ASTM C33	0.85 to 1.0(2)

1. *Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.*
2. *Linear interpolation from 0.85 to 1.0 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of aggregate.*

The term ρ_w in Equation (4.6) is equal to the area of flexural reinforcement, A_s, at the section divided by $b_w d$. According to ACI R22.5.5.1, A_s may be taken as the sum of the areas of the longitudinal flexural reinforcement located more than two-thirds of the overall member depth away from the extreme compression fiber. For members with one layer of tension reinforcement, like one-way slabs, A_s is the total area of flexural reinforcement at that section.

Values of $\sqrt{f'_c}$ used to calculate V_c are limited to 100 psi, except as allowed in ACI 22.5.3.2, which is not applicable to one-way slabs (ACI 22.5.3.1). This limitation on f'_c is primarily due to the fact that there is a lack of test data and practical experience with concrete having compressive strengths greater than 10,000 psi. For economy, f'_c for one-way slabs usually does not exceed 5,000 psi (Reference 7).

In situations where $V_u > \phi V_n = \phi V_c$, the thickness of the slab and/or the concrete compressive strength should be increased, although increasing f'_c is less effective than increasing h (or, equivalently, d).

To minimize the likelihood of diagonal compression failure in the concrete and to limit the extent of cracking, the cross-sectional dimensions of a section must be selected to satisfy ACI Equation (22.5.1.2) [ACI 22.5.1.2]:

$$V_u \le \phi(V_c + 8\sqrt{f'_c}b_w d) \tag{4.8}$$

4.5 Determination of Required Reinforcement

4.5.1 Required Flexural Reinforcement

The required area of flexural reinforcement, A_s, at a critical section of a one-way slab is determined based on the required and design flexural strengths, M_u and ϕM_n, for tension-controlled sections. Required A_s can be determined by the following equation, which is based on Equations (4.1) and (4.3):

$$A_s = \frac{0.85 f'_c bd}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] \tag{4.9}$$

The nominal strength coefficient of resistance, R_n, is defined as follows:

$$R_n = \frac{M_u}{\phi b d^2} \tag{4.10}$$

where $\phi = 0.90$ for tension-controlled sections.

It is important to check that A_s calculated by Equation (4.9) is at least equal to the required minimum area of flexural reinforcement, $A_{s,min}$, which is equal to $0.0018A_g$ where the gross area of the slab, A_g, is equal to bh (ACI 7.6.1.1).

It is also important to verify the section is tension-controlled. The following relationship is obtained from the linear strain distribution where c_t is the depth of the neutral axis when $\varepsilon_t = \varepsilon_{ty} + 0.003$ for tension-controlled sections (see ACI 22.2.1.2 and Figure 4.6):

$$\frac{c_t}{d} = \frac{0.003}{\varepsilon_t + 0.003} = \frac{0.003}{\varepsilon_{ty} + 0.006} \tag{4.11}$$

Substituting $a_t = \beta_1 c_t$ into Equation (4.4) with c_t equal to that from Equation (4.11), the following equation can be used to calculate the area of flexural reinforcement, $A_{s,t}$, corresponding to tension-controlled sections for one-way slabs with $\varepsilon_t = \varepsilon_{ty} + 0.003$:

$$A_{s,t} = \frac{0.85\beta_1 f'_c bd}{f_y}\left(\frac{0.003}{\varepsilon_{ty} + 0.006}\right) \tag{4.12}$$

For Grade 60 reinforcement, $\varepsilon_{ty} = 60 / 29{,}000 = 0.00207$, and for $f'_c = 4{,}000$ psi, $\beta_1 = 0.85$. Equation (4.12) reduces to the following for these material properties:

$$A_{s,t} = 0.018bd \tag{4.13}$$

If A_s calculated by Equation (4.9) is greater than $A_{s,t}$, the latter of which is essentially the maximum amount of flexural reinforcement permitted at any section, the section is not tension-controlled $(\phi < 0.9)$, and h must be increased accordingly to attain a tension-controlled section.

4.5.2 Minimum Shrinkage and Temperature Reinforcement

A minimum amount of shrinkage and temperature reinforcement must be provided in a one-way slab perpendicular to the flexural reinforcement in accordance with ACI 24.4 (ACI 7.6.4.1). The minimum area of such reinforcement is equal to $A_{s,min} = 0.0018A_g$ (ACI 24.4.3.2).

The maximum center-to-center spacing of the shrinkage and temperature reinforcement is equal to the lesser of $5h$ and 18 in. (ACI 24.4.3.3), and these reinforcing bars must be designed to develop f_y in tension at all sections where required (ACI 24.4.3.4).

4.6 Reinforcement Detailing

4.6.1 Concrete Cover

Reinforcing bars are placed in one-way slabs with a minimum concrete cover to protect them from weather, fire, and other effects. Minimum cover requirements are given in ACI 20.5.1 (ACI 7.7.1.1). When general building code minimum cover requirements for fire protection are greater than ACI 20.5.1, ACI 4.11.2 requires the greater concrete cover thickness to govern. IBC Table 721.1(1) provides minimum concrete cover requirements for fire ratings ranging from 1-hr to 4-hr. For reinforcing bars in one-way slabs not exposed to weather or in contact with the ground and a maximum 2-hr. fire rating, the minimum cover is equal to 0.75 in. for #11 and smaller bars (ACI Table 20.5.1.3.1). For one-way slabs without transverse reinforcement, concrete cover is measured from the surface of the concrete to the outmost layer of reinforcement (see Figure 4.7).

4.6.2 Minimum Spacing of Flexural Reinforcing Bars

Minimum clear spacing of parallel reinforcing bars in a single horizontal layer is given in ACI 25.2 (ACI 7.7.2.1). These limits have been established primarily so that concrete can flow readily into the spaces between adjoining bars.

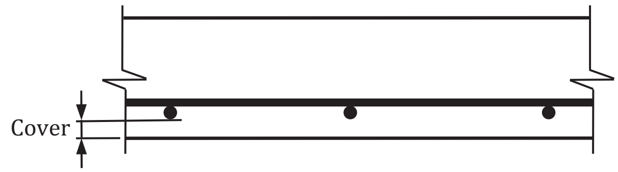


Figure 4.7 Concrete cover for one-way slabs.

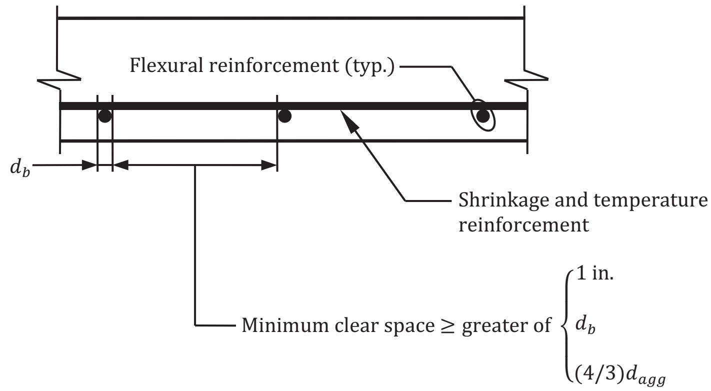


Figure 4.8 Minimum clear spacing requirements for reinforcing bars.

The spacing requirements are given in Figure 4.8 for a one-way slab where d_{agg} is the nominal maximum aggregate size in the concrete mix.

4.6.3 Maximum Spacing of Flexural Reinforcing Bars

Maximum center-to-center spacing of reinforcing bars is given in ACI 24.3 (ACI 7.7.2.2). The intent of these requirements is to control flexural cracking. In general, a larger number of finer cracks is preferable to a few wide cracks mainly for reasons of durability and appearance.

The maximum center-to-center bar spacing, $s,$ is determined by the equations in ACI Table 24.3.2 (ACI 24.3.2). The following equation is applicable to deformed reinforcing bars:

$$s \leq \text{lesser of} \begin{cases} 15\left(\dfrac{40{,}000}{f_s}\right) - 2.5c_c \\ \\ 12\left(\dfrac{40{,}000}{f_s}\right) \end{cases} \tag{4.14}$$

In this equation, c_c is the least distance from the surface of the flexural reinforcement to the tension face of the section and f_s is the calculated stress in the flexural reinforcement closet to the tension face at service loads (in psi). In lieu of calculating f_s using the unfactored bending moment at that section, f_s is permitted to be taken as $2f_y / 3$ (ACI 24.3.2.1). For Grade 60 reinforcement, $c_c = 0.75 \text{ in.},$ and $f_s = 2f_y / 3 = 40{,}000 \text{ psi},$ maximum center-to-center bar spacing $s = 12 \text{ in.}$ from Equation (4.14).

According to ACI 7.7.2.3, the maximum spacing of deformed flexural reinforcement in a one-way slab is equal to the lesser of $3h$ and 18 in. For typical one-way slab thicknesses, the maximum spacing based on crack control requirements usually governs.

4.6.4 Selection of Flexural Reinforcement

The size and spacing of the reinforcing bars at a critical section for flexure must be determined based on the required area of reinforcement determined by Equation (4.9) and the minimum and maximum spacing requirements given in Sections 4.6.2 and 4.6.3 of this publication, respectively.

In one-way slabs, A_s is typically calculated per foot width of slab. A bar size and spacing should be selected so that the provided A_s is equal to or slightly greater than the required A_s considering minimum reinforcement requirements and minimum and maximum bar spacing requirements. The information in Table 4.4 can be used to facilitate selection of bar size and spacing at a critical section for flexure in a one-way slab.

Table 4.4 Area of Reinforcement (in.^{2}) Based on Center-to-Center Spacing of the Bars

Bar Size	Spacing (in.)								
	4	5	6	7	8	9	10	11	12
#3	0.33	0.26	0.22	0.19	0.17	0.15	0.13	0.12	0.11
#4	0.60	0.48	0.40	0.34	0.30	0.27	0.24	0.22	0.20
#5	0.93	0.74	0.62	0.53	0.47	0.41	0.37	0.34	0.31
#6	1.32	1.06	0.88	0.75	0.66	0.59	0.53	0.48	0.44

4.6.5 Development of Flexural Reinforcement

Development of Deformed Bars in Tension

Development lengths of deformed reinforcement are given in ACI 25.4 (ACI 7.7.1.2). Provisions for the development of deformed reinforcing bars in tension are given in ACI 25.4.2. The tension development length, ℓ_d, is determined using the provisions of ACI 25.4.2.3 or 25.4.2.4 along with the modification factors in ACI 25.4.2.5. The requirements of ACI 25.4.2.3 are based on those in ACI 25.4.2.4, so the latter requirements are covered first.

Method 1 – ACI 25.4.2.4

The development length in tension of a deformed reinforcing bar, ℓ_d, is determined by ACI Equation (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \tag{4.15}$$

The terms in Equation (4.15) are as follows (ACI Table 25.4.2.5):

Modification factor for lightweight concrete, λ. This factor reflects the lower tensile strength of lightweight concrete:

$$\lambda = \begin{cases} 0.75 \text{ for lightweight concrete} \\ 1.0 \text{ for normalweight concrete} \end{cases} \tag{4.16}$$

Reinforcement grade factor, ψ_g. This factor accounts for the effect of reinforcement yield strength on the required development length:

$$\psi_g = \begin{cases} 1.0 \text{ for Grade 40 or Grade 60} \\ 1.15 \text{ for Grade 80} \\ 1.3 \text{ for Grade 100} \end{cases} \tag{4.17}$$

Grade 60 flexural reinforcement is typically used in one-way slabs, so $\psi_g = 1.0$.

Reinforcement coating factor, ψ_e. This factor accounts for the reduced bond strength between the concrete and epoxy-coated or zinc and epoxy dual-coated reinforcing bars:

$$\psi_e = \begin{cases} 1.5 \text{ for epoxy-coated or zinc and epoxy dual-coated bars with clear} \\ \text{cover} < 3d_b \text{ or clearing spacing} < 6d_b \\ \\ 1.2 \text{ for epoxy-coated or zinc and epoxy dual-coated bars for all other conditions} \\ \\ 1.0 \text{ for uncoated or zinc-coated (galvanized) bars} \end{cases} \tag{4.18}$$

Reinforcement size factor, ψ_s. This factor reflects the more favorable performance of smaller diameter reinforcing bars:

$$\psi_s = \begin{cases} 1.0 \text{ for \#7 and larger bars} \\ \\ 0.8 \text{ for \#6 and smaller bars} \end{cases} \tag{4.19}$$

Casting position factor, ψ_t. This factor reflects the adverse effects that can occur to the top reinforcement in a member due to vertical migration of water and mortar, which collect on the underside of the bars during placement of the concrete:

$$\psi_t = \begin{cases} 1.3 \text{ where more than 12 in. of fresh concrete is placed below the horizontal reinforcement} \\ \\ 1.0 \text{ in all other cases} \end{cases} \tag{4.20}$$

This factor is usually 1.0 for one-way slabs. Also, according to the footnote in ACI Table 25.4.2.5, the product $\psi_t \psi_e$ need not exceed 1.7.

Spacing or cover dimension, c_b. This term is defined as follows (see Figure 4.9):

$$c_b = \text{lesser of} \begin{cases} \text{distance from the center of a bar to the nearest concrete surface (lesser of } c_1 \text{ and } c_2) \\ \\ \text{one-half the center-to-center spacing of the bars being developed } (s / 2) \end{cases} \tag{4.21}$$

For one-way slabs, the distance from the center of the bar to the nearest concrete surface typically governs.

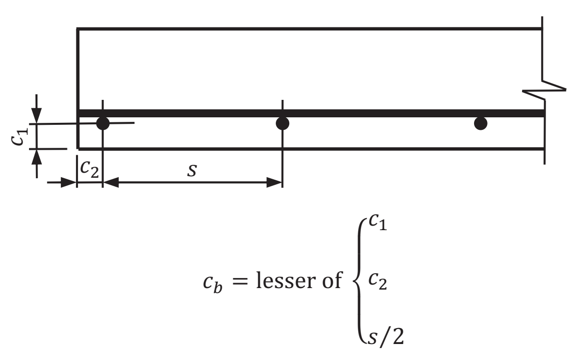


Figure 4.9 Spacing or cover dimension, c_b.

Transverse reinforcement index, K_{tr}. The transverse reinforcement index represents the effect of confining reinforcement across the potential splitting planes: larger amounts of confining reinforcement reduce the potential for splitting failure, thereby reducing the overall required development length. This index is determined by ACI Equation (25.4.2.4b):

$$K_{tr} = \frac{40A_{tr}}{sn} \tag{4.22}$$

where A_{tr} = total cross-sectional area of all transverse reinforcement within a spacing s that crosses the potential plane of splitting through the reinforcement being developed

s = center-to-center spacing of the transverse reinforcement

n = number of bars being developed across the plane of splitting

It is permitted to conservatively use $K_{tr} = 0$ if transverse reinforcement is present or required (ACI 25.4.2.4). For one-way slabs, where it is typical that transverse reinforcement is not used, $K_{tr} = 0$. Also, for reinforcing bars with $f_y \geq 80{,}000$ psi spaced closer than 6.0 in. on center, transverse reinforcement must be provided such that $K_{tr} \geq 0.5d_b$ (ACI 25.4.2.2).

The confining term $(c_b + K_{tr}) / d_b$ must be taken less than or equal to 2.5 in Equation (4.15) [ACI 25.4.2.4]. Tests have shown that when this term is less than 2.5, splitting failures are likely to occur. A pullout failure of the reinforcement is more likely when this term is greater than 2.5, so an increase in the anchorage capacity due to an increase in cover or amount of confining reinforcement is not likely.

Values of $\sqrt{f_c'}$ used in calculating ℓ_d must be less than or equal to 100 psi (ACI 25.4.1.4). This requirement is applicable to all the reinforcement development provisions in ACI 25.4.

The tension development length ℓ_d is permitted to be reduced in cases where the flexural reinforcement is greater than that required from analysis, except for the six cases in ACI 25.4.10.2 (ACI 25.4.10.1). Where permitted, the reduction factor applied to ℓ_d is equal to the required area of flexural reinforcement divided by the provided area of reinforcement.

Values of ℓ_d for deformed bars in one-way slabs based on normalweight concrete with $f_c' = 4{,}000$ psi, uncoated Grade 60 reinforcement, $\psi_t = 1.0$, $K_{tr} = 0$, and $c_b = (d_b / 2) + 0.75$ in. are given in Table 4.5. Calculated values of ℓ_d are rounded up to the next whole number. Values of ℓ_d for one-way slabs with coated bars can be obtained by multiplying the tabulated values by the appropriate value of ψ_e in Equation (4.18). Similarly, values of ℓ_d for one-way slabs made of lightweight concrete can be obtained by dividing the tabulated values by 0.75 in accordance with Equation (4.16).

Table 4.5 Tension Development Length, ℓ_d, for Deformed Bars in One-way Slabs in Accordance with ACI 25.4.2.4*

Bar Size	ℓ_d (in.)
#3	12
#4	15
#5	21
#6	29

* *Normalweight concrete with* $f_c' = 4{,}000$ psi
Uncoated Grade 60 reinforcement
$\psi_t = 1.0$
$K_{tr} = 0$
$c_b = (d_b / 2) + 0.75$ in.

Method 2 – ACI 25.4.2.3

The method given in ACI 25.4.2.3 to determine ℓ_d is based on the requirements given in ACI 25.4.2.4 and preselected values of the confining term $(c_b + K_{tr}) / d_b$. Two sets of spacing and cover cases are given in ACI Table 25.4.2.3. For one-way slabs, it is common for the clear spacing of the bars being developed to be at least $2d_b$ and the clear cover to be at least d_b. Therefore, the following equation from ACI Table 25.4.2.3 for the case of #6 and smaller longitudinal bars can be used to determine ℓ_d for reinforcing bars in one-way slabs:

$$\ell_d = \left(\frac{f_y \psi_t \psi_e \psi_g}{25\lambda\sqrt{f'_c}} \right) d_b \geq 12 \text{ in.} \tag{4.23}$$

Values of ℓ_d for deformed bars in one-way slabs based on normalweight concrete with $f'_c = 4{,}000$ psi, uncoated Grade 60 reinforcement, and $\psi_t = 1.0$ are given in Table 4.6. Calculated values of ℓ_d are rounded up to the next whole number. Values of ℓ_d for one-way slabs with coated bars can be obtained by multiplying the tabulated values by the appropriate value of ψ_e in Equation (4.18). Similarly, values of ℓ_d for one-way slabs made of lightweight aggregate concrete can be obtained by dividing the tabulated values by 0.75 in accordance with Equation (4.16).

Table 4.6 Tension Development Length, ℓ_d, for Deformed Bars in One-way Slabs in Accordance with ACI 25.4.2.3*

Bar Size	ℓ_d (in.)
#3	15
#4	19
#5	24
#6	29

* *Normalweight concrete with* $f'_c = 4{,}000$ psi
Uncoated Grade 60 reinforcement
$\psi_t = 1.0$

Development of Standard Hooks in Tension

Hooks are provided at the ends of reinforcing bars to provide additional anchorage where development length cannot be attained with straight bars. Standard hooks are defined in ACI 25.3.1 (see Figure 4.10).

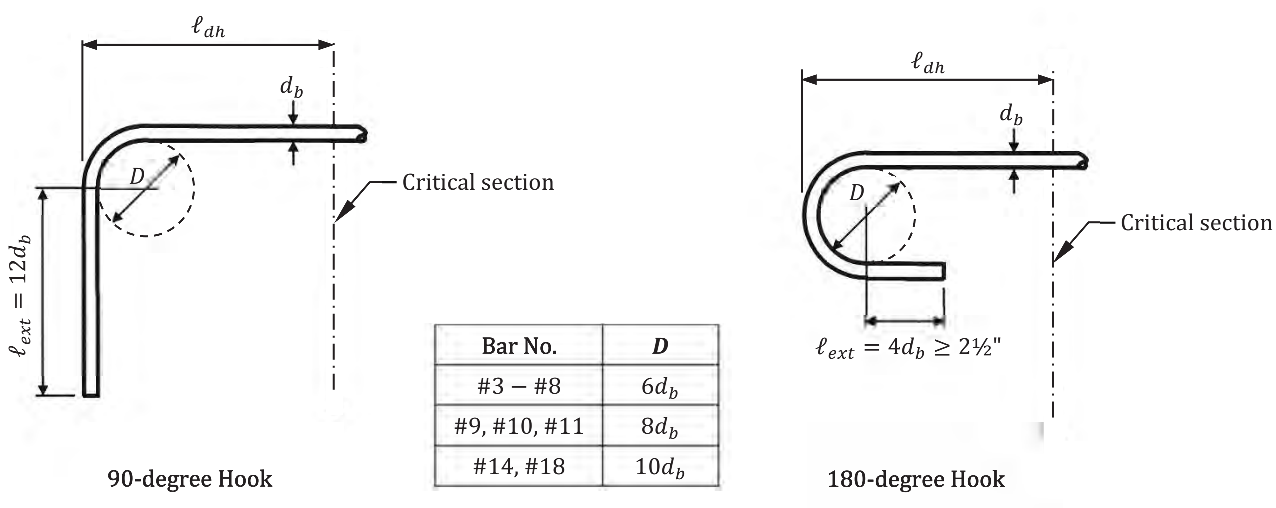


Figure 4.10 Standard hooks in accordance with ACI 25.3.1.

The development length of a deformed reinforcing bar in tension with a standard hook, ℓ_{dh}, is given in ACI 25.4.3.1:

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f'_c}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{4.24}$$

This development length is measured from the critical section to the outside face of the hook (see Figure 4.10). The modification factors in Equation (4.24) are given in Table 4.7 (see ACI Table 25.4.3.2).

Table 4.7 Modification Factors for Development of Hooked Bars in Tension

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Confining reinforcement, ψ_r	For #11 and smaller bars with $A_{th} \geq 0.4A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For #11 and smaller diameter hooked bars (1) terminating inside a column core with side cover normal to the plane of the hook ≥ 2.5 in. or (2) with side cover normal to the plane of the hook $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c / 15{,}000) + 0.6$ [f'_c in psi]
	$f'_c \geq 6{,}000$ psi	1.0

The term A_{th} is the total cross-sectional area of ties or stirrups confining hooked bars and A_{hs} is the total cross-sectional area of hooked bars being developed at the same critical section. For one-way slabs, it is typical for $A_{th} = 0$ (that is, ties or stirrups are not present). The term s is the center-to-center spacing of the hooked bars, which have a nominal diameter d_b.

The requirements in ACI 25.4.3.4 apply to hooks at discontinuous ends of members (that is, at ends of simply-supported members, at the free ends of cantilevers, and at exterior joints where members do not extend beyond the joint) where both side cover and top (or bottom) cover to the hook is less than 2.5 in. These provisions are typically applicable at beam-column joints, and thus, do not apply to one-way slabs in continuous construction where confinement is provided by the slab on both sides perpendicular to the plane of the hook.

Values of ℓ_{dh} for hooked deformed bars in one-way slabs based on normalweight concrete with $f'_c = 4{,}000$ psi, uncoated Grade 60 reinforcement, center-to-center hooked bar spacing $s \geq 6d_b$, and side cover normal to the plane of the hook $\geq 6d_b$ are given in Table 4.8. Calculated values of ℓ_{dh} are rounded up to the next whole number.

Table 4.8 Tension Development Length, ℓ_{dh}, for Hooked Deformed Bars in One-way Slabs*

Bar Size	ℓ_{dh} (in.)
#3	6
#4	6
#5	8
#6	10

* *Normalweight concrete with* $f'_c = 4{,}000$ psi
Uncoated Grade 60 reinforcement
$\psi_t = 1.0$
$s \geq 6d_b$
Side cover normal to the plane of the hook $\geq 6d_b$

Development of Headed Deformed Bars in Tension

Provisions for the development of headed deformed bars in tension are given in ACI 25.4.4. Examples of headed reinforcing bars are given in Figure 4.11.

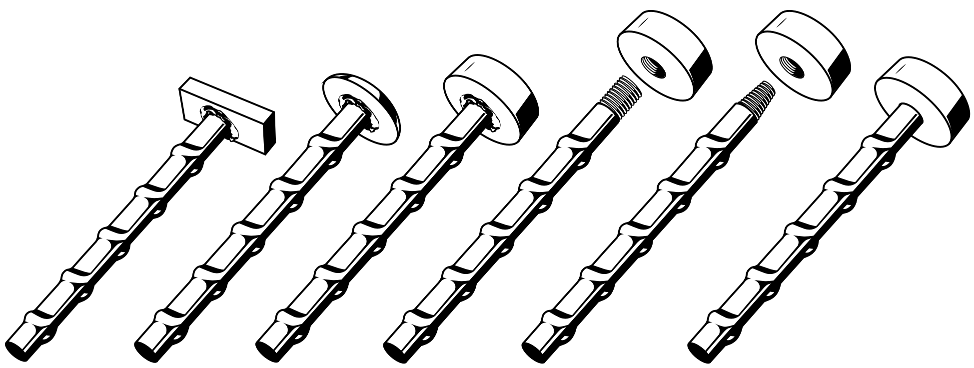


Figure 4.11 Headed reinforcing bars.

Headed deformed bars are permitted to be used only when the conditions of ACI 25.4.4.1 are satisfied:

(a) Bar must conform to ACI 20.2.1.6
(b) Bar size must be #11 or smaller
(c) Net bearing area of head, A_{brg}, must be at least $4A_b$ where A_b is the area of the bar
(d) Concrete must be normalweight
(e) Clear cover to the bar must be greater than or equal to $2d_b$ where d_b is the nominal diameter of the bar
(f) Center-to-center spacing between bars must be greater than or equal to $3d_b$

The development length of a headed deformed reinforcing bar in tension, ℓ_{dt}, is given in ACI 25.4.4.2:

$$\ell_{dt} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_p \psi_o \psi_c}{75\sqrt{f'_c}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{4.25}$$

This development length is measured from the critical section to the bearing face of the head (see ACI Figure R25.4.4.2a). The modification factors in Equation (4.25) are given in Table 4.9 (see ACI Table 25.4.4.3).

Table 4.9 Modification Factors for Development of Headed Bars in Tension

Modification Factor	Condition	Value of Factor
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Parallel tie reinforcement, ψ_p	For #11 and smaller bars with $A_{tt} \geq 0.3A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For headed bars (1) terminating inside a column core with side cover to bar ≥ 2.5 in. or (2) with side cover to bar $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c / 15{,}000) + 0.6$ [f'_c in psi]
	$f'_c \geq 6{,}000$ psi	1.0

The term A_{tt} is the total cross-sectional area of ties or stirrups acting as parallel tie reinforcement for headed bars and A_{ts} is the area of nonprestressed reinforcement in a tie or stirrup. For anchorages other than in beam-column joints (which is applicable to one-way slabs), A_{tt} must not be considered (ACI 25.4.4.5). The term s is the center-to-center spacing of the headed bars, which have a nominal diameter d_b.

The requirements of ACI 25.4.4.4 and 25.4.4.6 are not applicable to one-way slabs.

Values of ℓ_{dt} for headed deformed bars in one-way slabs based on normalweight concrete with $f'_c = 4{,}000$ psi, uncoated Grade 60 reinforcement, center-to-center headed bar spacing $s \geq 6d_b$, and side cover to the bar $\geq 6d_b$ are given in Table 4.10. Calculated values of ℓ_{dt} are rounded up to the next whole number.

Table 4.10 Tension Development Length, ℓ_{dt}, for Headed Deformed Bars in One-way Slabs*

Bar Size	ℓ_{dt} (in.)
#3	6
#4	6
#5	6
#6	8

* *Normalweight concrete with* $f'_c = 4{,}000$ psi
Uncoated Grade 60 reinforcement
$\psi_t = 1.0$
Side cover normal to the bar $\geq 6d_b$

Development of Mechanically Anchored Deformed Bars in Tension

It is permitted to use any mechanical attachment or device capable of developing f_y of the deformed bars provided it is approved by the building official in accordance with ACI 1.10 (ACI 25.4.5). The use of mechanical devices that do not meet the requirements in ACI 20.2.1.6 or are not developed in accordance with ACI 25.4.4 may be used provided test results are available that demonstrate the ability of the head and bar system to develop or anchor the required force in the bar.

Development of Positive and Negative Flexural Reinforcement

Flexural reinforcement must be properly developed or anchored in a reinforced concrete one-way slab for the slab to perform as intended in accordance with the strength design method. Critical sections for development of flexural reinforcement occur at the following (ACI 7.7.3.2):

1. Points of maximum stress (that is, sections of maximum bending moment)
2. Points along the span where adjacent reinforcement is terminated because it is no longer required to resist flexure

In continuous one-way slabs subjected to uniform gravity loads, the maximum positive and negative bending moments typically occur near midspan and at the faces of the supports, respectively. Positive and negative flexural reinforcing bars must be developed or anchored on both sides of these critical sections (ACI 7.7.3.1).

The required area of reinforcement at a critical section can be determined using the methods given in Section 4.5 of this publication.

For the one-way slab system in Figure 4.12, the total required area of negative reinforcement for the maximum negative factored bending moment, $(M_u^-)_A$, at critical section A is equal to A_s^-, and the total number of reinforcing bars at this location is equal to n where all the bars are the same size. Similarly, the total required area of positive reinforcement for the maximum positive factored bending moment, $(M_u^+)_C$, at critical section C is equal to A_s^+, and the total number of reinforcing bars at this location is equal to p. Requirements for the development of these two sets of reinforcing bars are discussed next.

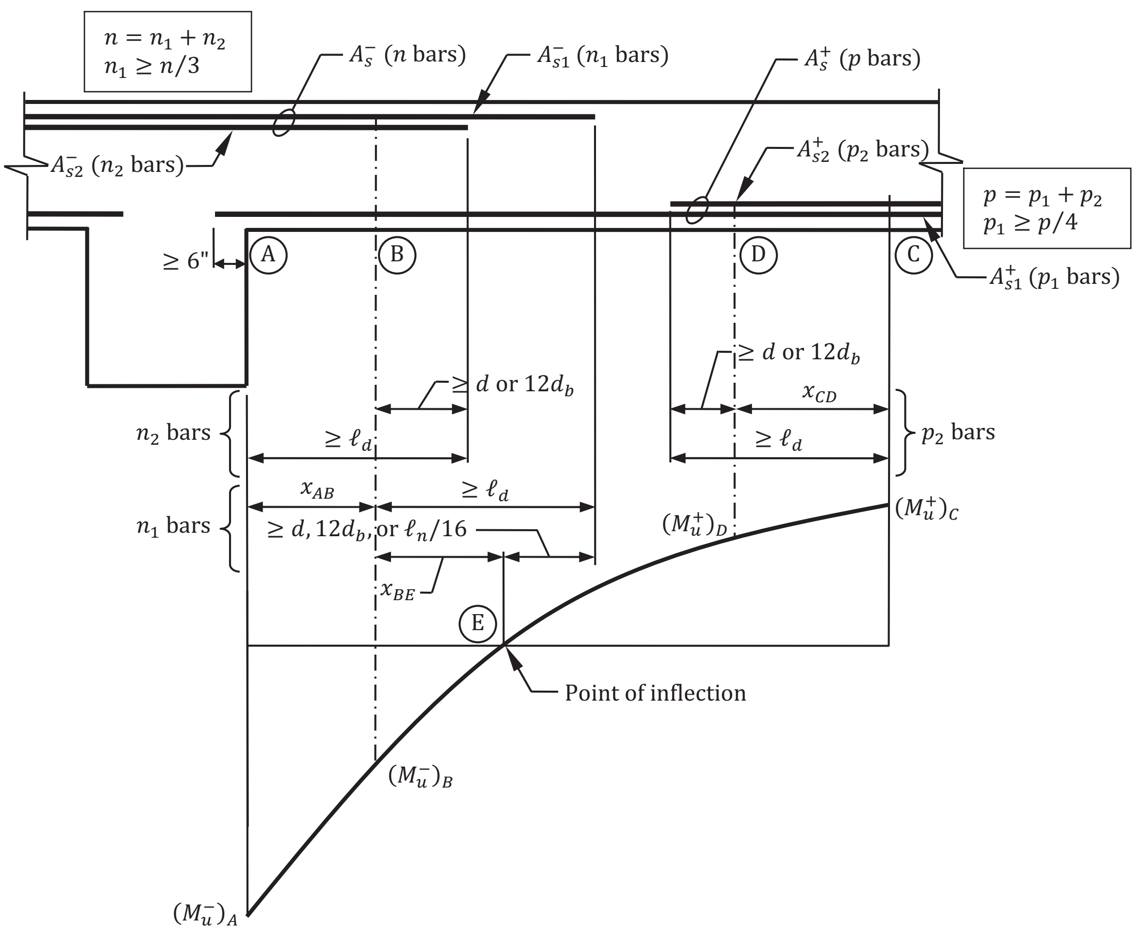


Figure 4.12 Development of flexural reinforcement in a one-way slab.

Negative Flexural Reinforcement

Assume a portion of A_s^- is cut off at section B where it is no longer required for flexural strength. This makes section B a critical section. At this location, the cutoff reinforcement has an area equal to A_{s2}^- and the number of reinforcing bars is equal to n_2. The area of the remaining portion of reinforcing bars is equal to $A_{s1}^- = A_s^- - A_{s2}^-$ and the corresponding number of reinforcing bars is equal to $n_1 = n - n_2$. This reinforcement must be able to resist the negative factored bending moment $(M_u^-)_B$ at section B.

Because section B is a critical section, n_1 bars must be adequately developed to the right of this section. This is achieved by extending the bars a minimum distance of ℓ_d past section B as shown in Figure 4.12 where ℓ_d is the development length in tension of a deformed bar, which is determined in accordance with ACI 25.4.2 (ACI 7.7.3.4).

According to ACI 7.7.3.8.4, at least one-third of the total negative reinforcement provided at a support must have an embedment length equal to the larger of d, $12d_b$, and $\ell_n / 16$ past the point of inflection where ℓ_n is the clear span length measured face-to-face of supports (which in this case are beams). Therefore, to satisfy this requirement, which accounts for possible shifting of the bending moment diagram at the point of inflection due to the approximate bending moment diagram customarily used in design, $n_1 \geq n / 3$. The minimum length of n_1 bars to the right of critical section A is equal to the following (ACI 7.7.3.4 and 7.7.3.8.4):

$$\text{Minimum length of } n_1 \text{ bars} = \text{greater of} \begin{cases} x_{AB} + \ell_d \\ \text{greater of} \begin{cases} x_{AB} + x_{BE} + d \\ x_{AB} + x_{BE} + 12d_b \\ x_{AB} + x_{BE} + (\ell_n / 16) \end{cases} \end{cases} \tag{4.26}$$

In these equations, x_{AB} is the distance from section A to the theoretical cutoff point at section B and x_{BE} is the distance from section B to the point of inflection at section E.

Because section A is a critical section, the bars cutoff at section B must be developed a distance equal to at least ℓ_d beyond that section. Additionally, ACI 7.7.3.3 stipulates that these bars must extend beyond the point where they are no longer required a distance equal to at least the greater of d and $12d_b$. The minimum length of n_2 bars to the right of critical section A is equal to the following:

$$\text{Minimum length of } n_2 \text{ bars} = \text{greater of} \begin{cases} \ell_d \\ \text{greater of} \begin{cases} x_{AB} + d \\ x_{AB} + 12d_b \end{cases} \end{cases} \tag{4.27}$$

Reinforcing bars are not permitted to be terminated in a tension zone unless one of the following conditions are satisfied (ACI 7.7.3.5):

1. At the cutoff point, $V_u \leq 2\phi V_n / 3$ where ϕV_n is the design shear strength of the section at that point (see Section 4.4.3 of this publication)
2. For #11 and smaller bars, continuing reinforcement provides at least double the area required for flexure at the cutoff point and $V_u \leq 3\phi V_n / 4$

The third condition in ACI 7.7.3.5, which is related to stirrup area in the section, is typically not applicable to one-way slabs.

The development of the negative reinforcing bars to the left of section A depends on its location within the framing system. At interior joints, like the one depicted in Figure 4.12, development is achieved by continuing the negative reinforcing bars into the span to the left of the beam. In an end span where the slab terminates at edge beams, a standard hook is provided at the ends of the negative reinforcing bars.

Positive Flexural Reinforcement

Assume a portion of A_s^+ is cut off at section D. This makes section D a critical section. At this location, the reinforcement has an area equal to A_{s2}^+ and the number of reinforcing bars is equal to p_2. The area of the remaining portion of reinforcing bars is equal to $A_{s1}^+ = A_s^+ - A_{s2}^+$ and the corresponding number of reinforcing bars is equal to $p_1 = p - p_2$. This reinforcement must be able to resist the negative factored bending moment $(M_u^+)_D$ at section D.

The bars cut off at section D must be developed to a distance greater than or equal to ℓ_d beyond the critical section C. Like in the case of the negative reinforcement, these bars must extend beyond the point they are no longer required by a distance equal to the greater of d and $12d_b$ (ACI 7.7.3.3). The minimum length of p_2 bars to the left of critical section C is equal to the following:

$$\text{Minimum length of } p_2 \text{ bars} = \text{greater of} \begin{cases} \ell_d \\ \text{greater of} \begin{cases} x_{CD} + d \\ x_{CD} + 12d_b \end{cases} \end{cases} \tag{4.28}$$

In this equation, x_{CD} is the distance between critical section C and the theoretical cutoff point located at section D.

The requirements of ACI 7.7.3.5 pertaining to reinforcement terminated in a tension zone are also applicable in this case.

According to ACI 7.7.3.8.2, at least one-fourth of the positive reinforcement must extend at least 6 in. into the support. Thus, $p_1 \geq p / 4$. Although not common in typical reinforced concrete construction, at least one-third of the maximum positive reinforcement must extend along the slab bottom into simple supports.

The diameter of the positive reinforcement is limited at points of inflection in accordance with ACI 7.7.3.8.3; this requirement helps to ensure the bars are developed in a length short enough such that the moment capacity is greater than the applied moment over the entire length of the slab:

$$\ell_d \leq \begin{cases} \dfrac{1.3M_n}{V_u} + \ell_a \text{ if the end of the reinforcement is confined by a compressive reaction} \\ \dfrac{M_n}{V_u} + \ell_a \text{ if the end of the reinforcement is not confined by a compressive reaction} \end{cases} \tag{4.29}$$

In this equation, M_n is the nominal flexural strength of the slab section assuming all the reinforcement at that section is stressed to f_y, V_u is the factored shear force at the section, and ℓ_a is the embedment length of the reinforcement beyond the inflection point, which is limited to the greater of d and $12d_b$. Additional information on this topic can be found in ACI R9.7.3.8.3 for beams.

The structural integrity requirements in ACI 7.7.7 also have an impact on development lengths (see Section 4.6.7 of this publication).

4.6.6 Splices of Reinforcement

Overview

Splices of deformed reinforcement in one-way slabs must be in accordance with ACI 25.5 (ACI 7.7.1.3). The primary reasons for splicing reinforcement are based on (1) restrictions related to transporting the reinforcing bars to the construction site and (2) limitations related to handling and placing the reinforcing bars in the field.

Lap splices, mechanical splices, and welded splices are common types of splices for flexural reinforcement and each type of splice is examined next.

Lap Splices

Lap splices are frequently specified and are usually the most economical type of splice. There are basically two types of lap splices: contact and noncontact. In a contact lap splice, the bars are generally in contact over a specified length and are tied together [see Figure 4.13(a)]. In a noncontact lap splice, the bars are not in contact, and the center-to-center spacing of the bars being spliced must not exceed the limits indicated in Figure 4.13(b) (see ACI 25.5.1.3). Contact lap splices are usually preferred because the bars are tied together and are less likely to displace during construction. The minimum clear spacing between a contact lap splice and adjacent splices or bars is the same as that for individual bars (ACI 25.5.1.2; see Figure 4.8).

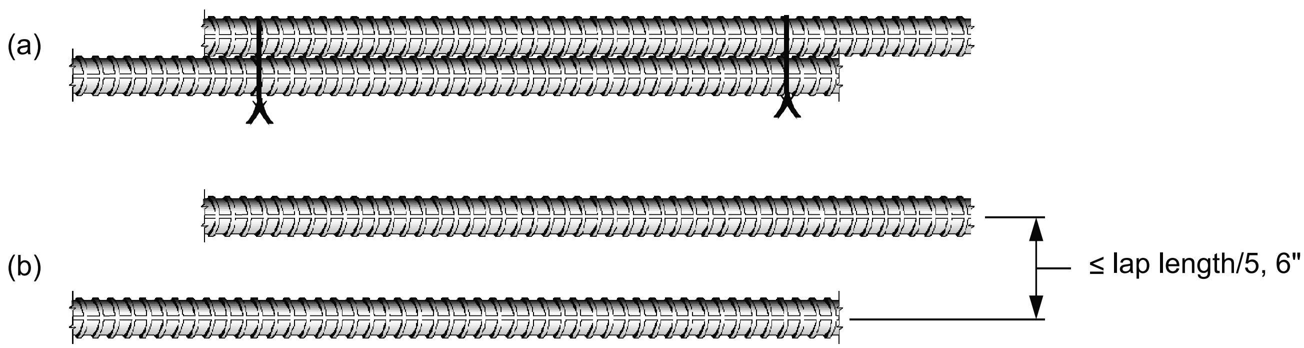


Figure 4.13 Lap splices (a) Contact lap splice (b) Noncontact lap splice.

Lap splices should be provided at locations away from maximum stress (that is, away from sections of maximum bending moment) and should be staggered wherever possible. Reinforcing bars provided for negative bending moments (top bars) should be spliced near the midspan of a member, and reinforcing bars provided for positive bending moments (bottom bars) should be spliced over the supports.

The required tension lap splice length, ℓ_{st}, depends on the tension development length of the bars, ℓ_d, the area of reinforcement provided over the length of the splice, and the percentage of reinforcement spliced at any one location. Lap splices are classified as Class A or Class B. Because experimental data on lap splices with #14 and #18 bars are sparse, the use of tension lap splices for these bars are prohibited except as permitted in ACI 25.5.5.3 for compression lap splices (ACI 25.5.1.1).

The length of a tension lap splice is given as a multiple of ℓ_d (ACI 25.5.2.1):

$$\text{Class A lap splice length: } \ell_{st} = 1.0\ell_d \geq 12 \text{ in.} \tag{4.30}$$

$$\text{Class B lap splice length: } \ell_{st} = 1.3\ell_d \geq 12 \text{ in.} \tag{4.31}$$

When determining ℓ_d to be used in determining ℓ_{st}, the 12-in. minimum length specified in ACI 25.4.2.1(b) and the excess reinforcement modification factor of ACI 25.4.10.1 are not applicable (ACI 25.5.1.4). Also, for reinforcing bars with $f_y \geq 80{,}000$ psi spaced closer than 6 in. on center, transverse reinforcement must be provided such that $K_{tr} \geq 0.5d_b$ (ACI 25.5.1.5).

The default splice classification for a tension lap splice is Class B. A Class A splice is permitted where both of the following conditions are satisfied (ACI Table 25.5.2.1):

- $A_{s,provided} / A_{s,required} \geq 2.0$ over the entire splice length where $A_{s,provided}$ and $A_{s,required}$ are the area of reinforcement provided at the splice location and the area of reinforcement required by analysis at the splice location, respectively
- Less than or equal to 50 percent of the reinforcement is spliced within the required lap splice length

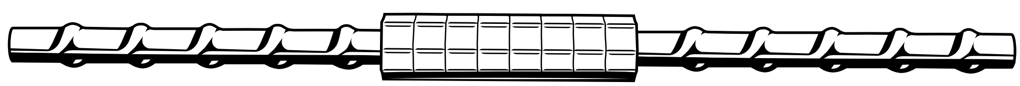
Where bars of different size are lap spliced in tension, ℓ_{st} is equal to the greater of ℓ_d of the larger bar and ℓ_{st} of the smaller bar (ACI 25.5.2.2).

Mechanical Splices

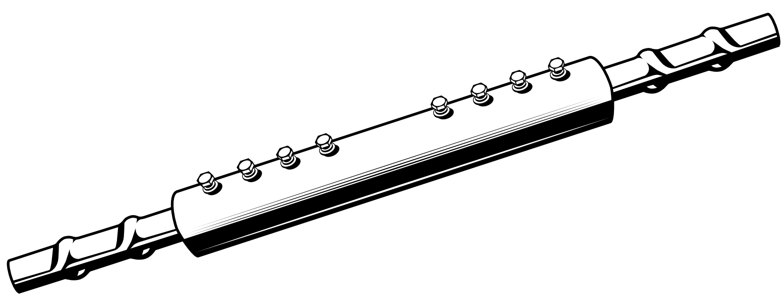
Mechanical splices are, in general, a complete assembly of a coupler, a coupling sleeve, or an end-bearing sleeve, including any additional material or components, required to accomplish the splicing of the reinforcing bars. Such splices must develop in tension or compression at least $1.25f_y$ of the bar (ACI 25.5.7.1). This is to ensure that yielding occurs in the reinforcing bar adjacent to the mechanical splice prior to the failure of the splice.

Illustrated in Figure 4.14 are three of the more popular types of mechanical splices. The cold-swaged coupling sleeve in Figure 4.14(a) uses a hydraulic swaging press with special dies to deform the sleeve around the ends of the spliced bars to produce positive mechanical interlock with the reinforcing bars. The shear screw coupling sleeve in Figure 4.14(b) consists of a coupling sleeve with shearhead screws, which are designed to shear off at a specified torque. The reinforcing bars are inserted to meet at the center of the coupling sleeve and then the screws are tightened. The tightening process embeds the pointed screws into the bars. The non-upset straight thread coupler in Figure 4.14(c) consists of a coupler with internal straight threads at each end that joins the two reinforcing bars with matching external threads. Additional information on these and other mechanical splices can be found in Reference 9.

(a)



(b)



(c)

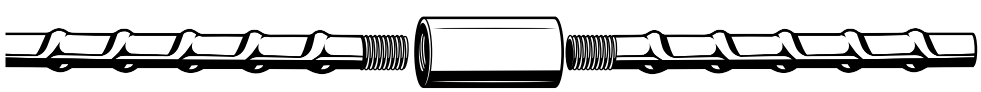


Figure 4.14 Mechanical splices (a) Cold-swaged coupling sleeve (b) Shear screw coupling sleeve (c) Non-upset straight thread coupler.

Mechanical splices can be advantageous in a number of situations, including the following:

- Where long lap splices are needed
- Where lap splices cause reinforcement congestion
- Where spacing of the flexural reinforcement is insufficient to permit lap splices

Welded Splices

ACI 25.5.7 permits the use of welded splices, which must conform to the requirements of ACI 26.6.4. Like mechanical splices, a full welded splice, which is generally intended for #6 and larger bars, must be able to develop at least $1.25f_y$ of the bar.

4.6.7 Structural Integrity Reinforcement

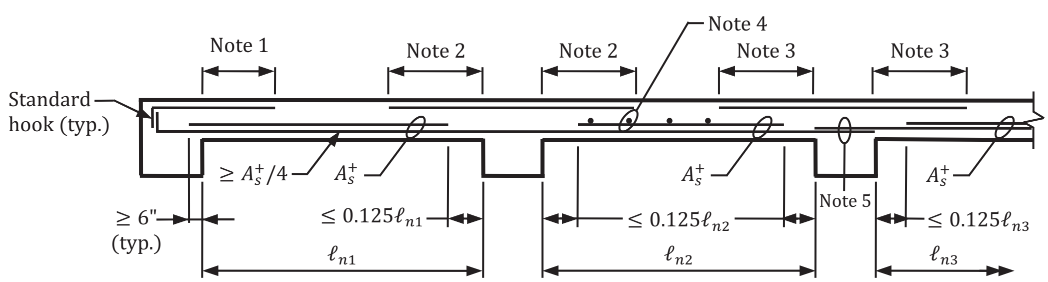
Requirements for structural integrity reinforcement in cast-in-place one-way slabs are given in ACI 7.7.7. The purpose of this reinforcement is to improve the redundancy and ductility in the structure so that in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be localized and the structure will have a higher probability of maintaining overall stability.

Three requirements are given in ACI 7.7.7:

1. Longitudinal structural integrity reinforcement must consist of at least one-quarter of the maximum positive moment reinforcement and it must be continuous.
2. Longitudinal structural integrity reinforcement at noncontinuous supports must be anchored to develop f_y at the face of the support.
3. Where splices are needed in the continuous structural integrity reinforcement, the reinforcement must be spliced near the supports. Mechanical or welded splices in accordance with ACI 25.5.7 or Class B tension lap splices in accordance with ACI 25.5.2 must be provided.

4.6.8 Recommended Flexural Reinforcement Details

Recommended flexural reinforcement details for one-way slabs based on the requirements covered above, including the structural integrity requirements of ACI 7.7.7, are given in Figure 4.15. The bar lengths given in the figure are based on a one-way slab subjected to uniformly distributed gravity loads; these lengths can be used for one-way slabs that have been designed using the simplified method of analysis in ACI 6.5 (see Section 4.3.1 of this publication). For one-way slabs subjected to the effects from other types of loads, required bar lengths must be determined by calculations.



Notes:
(1) $\geq \ell_{n1}/4$
(2) $\geq$ greater of $(\ell_{n1}/3, \ell_{n2}/3)$
(3) $\geq$ greater of $(\ell_{n2}/3, \ell_{n3}/3)$
(4) Shrinkage and temperature reinforcement (typ.)
(5) Class B lap splice or equivalent

Figure 4.15 Recommended flexural reinforcement details for one-way slabs.

4.7 Design Procedure

The design procedure in Figure 4.16 can be used in the design and detailing of one-way slabs. Included in the figure are the section numbers, table numbers, and figure numbers where specific information on that topic in this chapter can be found.

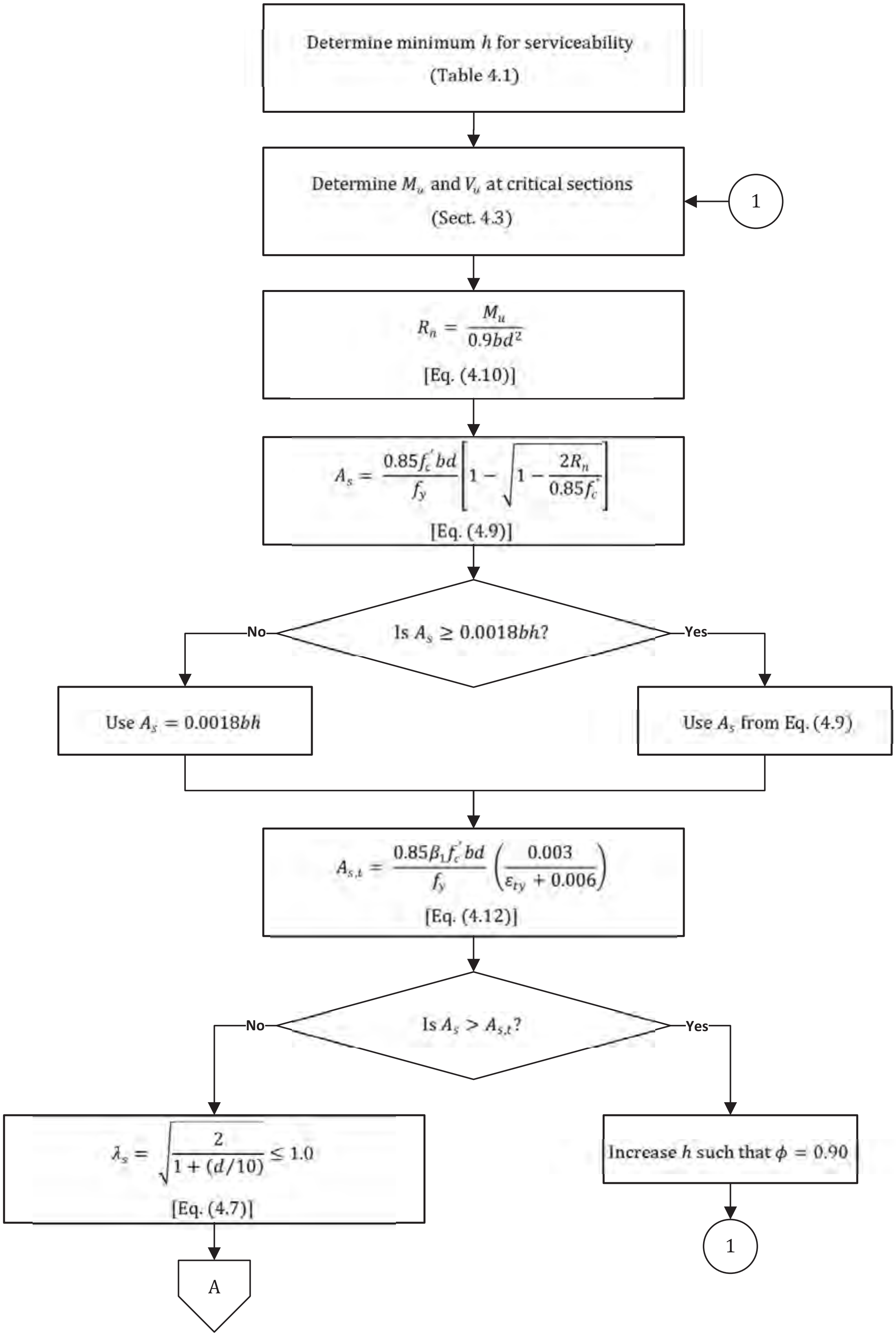


Figure 4.16 Design procedure for one-way slabs.

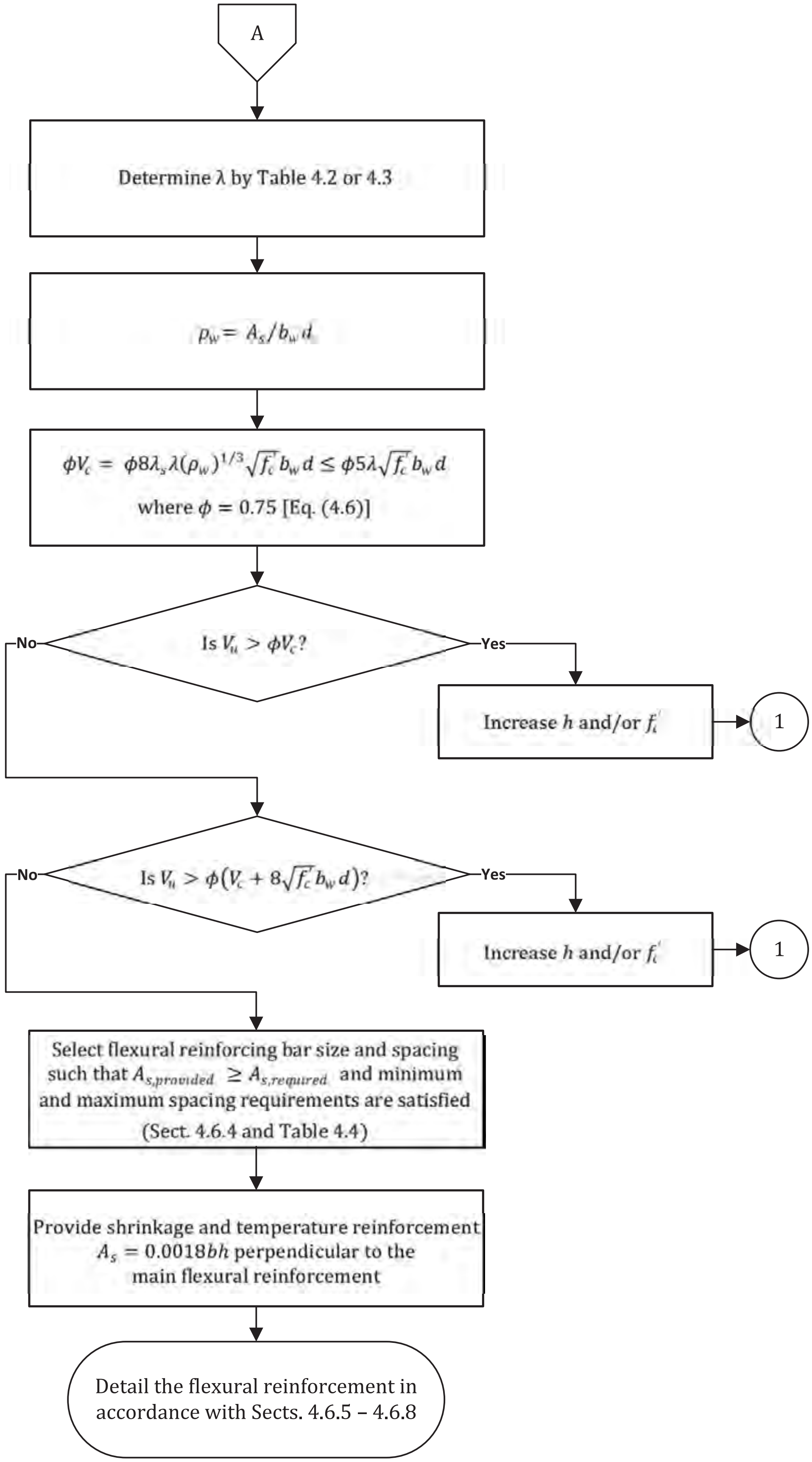


Figure 4.16 (cont.) Design procedure for one-way slabs.

4.8 Examples

4.8.1 Example 4.1 – Determination of Minimum Slab Thickness: One-way Slab System, Building #2, Normalweight Concrete

Determine the thickness of the one-way slab that is part of the wide-module joist system in Building #2 at a typical floor assuming the joists are spaced 5 ft on center and the rib width is equal to 7.0 in. (see Figure 1.2). Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2.

Step 1 – Determine the minimum slab thickness based on serviceability requirements ACI 7.3.1.1

Because all span lengths are equal, the governing minimum slab thickness is based on a support condition of one end continuous:

$$h = \frac{\ell}{24} = \frac{5.0 \times 12}{24} = 2.5 \text{ in.}$$ Table 4.1

Try a 4.5-in.-thick slab.

Step 2 – Check shear strength requirements ACI 7.5.1.1

- **Step 2a – Determine the maximum factored distributed load**

  For a one-foot design strip:

  $$\text{Dead load of slab} = (4.5 / 12) \times 150 = 56.3 \text{ lb/ft}$$
  $$\text{Superimposed dead load} = 20.0 \times 1.0 = 20.0 \text{ lb/ft}$$
  $$\text{Floor live load} = 100.0 \times 1.0 = 100.0 \text{ lb/ft}$$

  Maximum w_u is determined by ACI Eq. (5.3.1b): Table 3.3

  $$w_u = 1.2w_D + 1.6w_L = [1.2 \times (56.3 + 20.0)] + (1.6 \times 100.0) = 251.6 \text{ lb/ft}$$

- **Step 2b – Determine if the simplified analysis method can be used to determine the shear force** ACI 6.5.1

  (1) Members are prismatic

  (2) Gravity loads are uniformly distributed

  (3) $L = 100.0 \text{ lb/ft} < 3D = 3 \times (56.3 + 20.0) = 228.9 \text{ lb/ft}$

  (4) All spans are equal in length

  Therefore, the simplified analysis method can be used.

- **Step 2c – Determine the maximum factored shear force** ACI 6.5.4

  Maximum V_u occurs at the face of the first interior beam: Figure 4.3

  $$V_u = \frac{1.15 w_u \ell_n}{2} = \frac{1.15 \times (251.6 / 1{,}000) \times [(60.0 - 7.0) / 12]}{2} = 0.6 \text{ kips}$$

  where ℓ_n is conservatively taken as the clear distance between the bottom instead of the top of the tapered ribs (see Figure 1.2).

  Although the critical section for shear is permitted to be taken a distance d from the face of the support in this case, it is conservatively taken at the face of the support (ACI 7.4.3.2).

- **Step 2d – Check the shear strength requirements** ACI 7.5.1.1

$$V_c = 8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'}b_w d \le 5\lambda\sqrt{f_c'}b_w d \quad \text{Eq. (4.6)}$$

Assuming $d = 4.5 - 1.25 = 3.25$ in.:

$$\lambda_s = \sqrt{\frac{2}{1+(d/10)}} = \sqrt{\frac{2}{1+(3.25/10)}} = 1.2 > 1.0, \text{ use } 1.0 \quad \text{Eq. (4.7)}$$

$\lambda = 1.0$ for normalweight concrete ACI 19.2.4.3

Assuming minimum flexural reinforcement $(A_{s,min} = 0.0018A_g)$:

$$\rho_w = \frac{0.0018b_w h}{b_w d} = \frac{0.0018 \times 4.5}{3.25} = 0.0025$$

$$V_c = 8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'}b_w d = 8 \times 1.0 \times 1.0 \times 0.0025^{1/3} \times \sqrt{4{,}000} \times 12.0 \times 3.25/1{,}000 = 2.7 \text{ kips}$$

$$< 5\lambda\sqrt{f_c'}b_w d = 5 \times 1.0 \times \sqrt{4{,}000} \times 12.0 \times 3.25/1{,}000 = 12.3 \text{ kips}$$

For shear, $\phi = 0.75.$ ACI Table 21.2.1

$$\phi V_c = 0.75 \times 2.7 = 2.0 \text{ kips} > V_u = 0.6 \text{ kips}$$

Also, it is evident that $V_u < \phi(V_c + 8\sqrt{f_c'}b_w d)$. Eq. (4.8)

Therefore, a 4.5-in.-thick slab satisfies serviceability and shear strength requirements.

Note: The 4.5-in.-thick slab must also be checked for fire-resistance requirements based on the required fire-resistance rating prescribed in the governing building code.

4.8.2 Example 4.2 – Determination of Minimum Slab Thickness: One-way Slab System, Building #2, Lightweight Concrete

Determine if a 4.5-in.-thick slab that is part of the wide-module joist system in Building #2 is adequate at the roof level assuming lightweight concrete with an equilibrium density, w_c, equal to 110 pcf, $f_c' = 4{,}000$ psi, and Grade 60 reinforcement (see Figure 1.2).

Design data are given in Sect. 1.2.2. See Example 4.1.

Step 1 – Determine the minimum slab thickness based on serviceability requirements ACI 7.3.1.1

Because all span lengths are equal, minimum thickness based on a support condition of one end continuous governs:

$$f_2 = \text{greater of} \begin{cases} 1.65 - 0.005w_c = 1.65 - (0.005 \times 110) = 1.10 \\ \\ 1.09 \end{cases} \quad \text{Table 4.1}$$

$$h = \left(\frac{\ell}{24}\right)f_2 = \frac{60.0}{24} \times 1.10 = 2.8 \text{ in.} < 4.5 \text{ in.}$$

Therefore, a 4.5-in.-thick slab satisfies serviceability requirements.

Step 2 – Check shear strength requirements ACI 7.5.1.1

- **Step 2a – Determine the maximum factored distributed load**

  For a one-foot design strip:

  $$\text{Dead load of slab} = (4.5 / 12) \times 110 = 41.3 \text{ lb/ft}$$

  $$\text{Superimposed dead load} = 20.0 \times 1.0 = 20.0 \text{ lb/ft}$$

  $$\text{Roof live load} = 20.0 \times 1.0 = 20.0 \text{ lb/ft}$$

  Maximum w_u is determined by ACI Eq. (5.3.1c): Table 3.3

  $$w_u = 1.2w_D + 1.6w_{L_r} = [1.2 \times (41.3 + 20.0)] + (1.6 \times 20.0) = 105.6 \text{ lb/ft}$$

- **Step 2b – Determine if the simplified analysis method can be used to determine the shear force** ACI 6.5.1

  (1) Members are prismatic

  (2) Gravity loads are uniformly distributed

  (3) $L = 20.0 \text{ plf} < 3D = 3 \times (41.3 + 20.0) = 183.9 \text{ lb/ft}$

  (4) All spans are equal in length

  Therefore, the simplified analysis method can be used.

- **Step 2c – Determine the maximum factored shear force** ACI 6.5.4

  Maximum V_u occurs at the face of the first interior beam: Figure 4.3

  $$V_u = \frac{1.15 w_u \ell_n}{2} = \frac{1.15 \times (105.6 / 1{,}000) \times (53 / 12)}{2} = 0.3 \text{ kips}$$

  where ℓ_n is conservatively taken as the clear distance between the bottom instead of the top of the tapered ribs (see Figure 1.2).

  Although the critical section for shear is permitted to be taken a distance d from the face of the support in this case, it is conservatively taken at the face of the support (ACI 7.4.3.2).

- **Step 2d – Check the shear strength requirements** ACI 7.5.1.1

  $$V_c = 8\lambda_s \lambda (\rho_w)^{1/3} \sqrt{f'_c} b_w d \le 5\lambda \sqrt{f'_c} b_w d \qquad \text{Eq. (4.6)}$$

  Assuming $d = 4.5 - 1.25 = 3.25$ in.:

  $$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (3.25 / 10)}} = 1.2 > 1.0, \text{ use } 1.0 \qquad \text{Eq. (4.7)}$$

  $$\text{For } w_c = 110 \text{ pcf, } \lambda = 0.0075 w_c = 0.0075 \times 110 = 0.83 \qquad \text{Table 4.2}$$

  Assuming minimum flexural reinforcement $(A_{s,min} = 0.0018 A_g)$:

  $$\rho_w = \frac{0.0018 b_w h}{b_w d} = \frac{0.0018 \times 4.5}{3.25} = 0.0025$$

  $$V_c = 8\lambda_s \lambda (\rho_w)^{1/3} \sqrt{f'_c} b_w d = 8 \times 1.0 \times 0.83 \times 0.0025^{1/3} \times \sqrt{4{,}000} \times 12.0 \times 3.25 / 1{,}000 = 2.2 \text{ kips}$$

  $$< 5\lambda \sqrt{f'_c} b_w d = 5 \times 0.83 \times \sqrt{4{,}000} \times 12.0 \times 3.25 / 1{,}000 = 10.2 \text{ kips}$$

For shear, $\phi = 0.75$. ACI Table 21.2.1

$\phi V_c = 0.75 \times 2.2 = 1.7 \text{ kips} > V_u = 0.3 \text{ kips}$

Also, it is evident that $V_u < \phi(V_c + 8\sqrt{f_c'}b_w d)$. Eq. (4.8)

Therefore, the 4.5-in.-thick slab satisfies shear strength requirements.

Note: The 4.5-in.-thick slab must also be checked for fire-resistance requirements based on the required fire-resistance rating prescribed in the governing building code.

4.8.3 Example 4.3 – Determination of Required Reinforcement: One-way Slab System, Building #2

Determine the required reinforcement for the 4.5-in.-thick slab that is part of the wide-module joist system in Building #2 at a typical floor assuming normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement (see Figure 1.2).

Design data are given in Sect. 1.2.2. See Example 4.1.

Step 1 – Determine the factored bending moments along the span ACI 6.5

It was determined in Step 2b of Example 4.1 that the simplified analysis method can be used to determine factored bending moments and shear forces.

Because the slabs have spans less than 10 ft, the negative bending moments at the critical sections (face of supports) are equal to the following where $w_u = 251.6$ lb/ft is calculated in Step 2a of Example 4.1:

$$M_u^- = \frac{w_u \ell_n^2}{12} = \frac{(251.6 / 1{,}000) \times (53 / 12)^2}{12} = 0.41 \text{ ft-kips/ft}$$ Figure 4.3

Similarly, the maximum positive bending moment occurs near midspan in an end span; this moment is used for all spans:

$$M_u^+ = \frac{w_u \ell_n^2}{14} = \frac{(251.6 / 1{,}000) \times (53 / 12)^2}{14} = 0.35 \text{ ft-kips/ft}$$

Step 2 – Determine the required flexural reinforcement ACI 7.4.1.2

$$R_n = \frac{M_u}{\phi b d^2}$$ Eq. (4.10)

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right]$$ Eq. (4.9)

For tension-controlled sections, $\phi = 0.90$. ACI Table 21.2.2

A summary of the required A_s is given in Table 4.11 where $d = 4.5 - 1.25 = 3.25$ in.

Table 4.11 Summary of Required Flexural Reinforcement for the One-way Slab in Example 4.3

Location	M_u (ft-kips/ft)	R_n (psi)	A_s (in.^{2})
Face of support	0.41	43	0.03
Midspan	0.35	37	0.02

Minimum flexural reinforcement: ACI 7.6.1.1

$A_{s,min} = 0.0018A_g = 0.0018 \times 12.0 \times 4.5 = 0.10 \text{ in.}^2$

Because $A_{s,min} > A_s$ at all critical sections, use $A_s = 0.10 \text{ in.}^2$

Check if the sections are tension-controlled.

For Grade 60 reinforcement and $f_c' = 4{,}000$ psi:

$A_{s,t} = 0.018bd = 0.018 \times 12.0 \times 3.25 = 0.70 \text{ in.}^2$ Eq. (4.13)

Because $A_s = 0.10 \text{ in.}^2 < A_{s,t} = 0.70 \text{ in.}^2$, the sections are tension-controlled.

Step 3 – Select the reinforcing bars Sect. 4.6.4

Try #3 bars spaced at 12 in. on center for the negative and positive flexural reinforcement.

Provided $A_s = 0.11 \text{ in.}^2 >$ required $A_s = 0.10 \text{ in.}^2$ Table 4.4

Check spacing requirements.

The minimum center-to-center spacing of the bars, s_{min}, is equal to the greater of the following assuming a 0.75-in. maximum aggregate size, d_{agg}:

$$s_{min} = \text{greater of} \begin{cases} 1.0 \text{ in.} + d_b = 1.0 + 0.375 = 1.4 \text{ in.} \\ d_b + d_b = 2 \times 0.375 = 0.75 \text{ in.} \\ (4/3)d_{agg} + d_b = (4 \times 0.75/3) + 0.375 = 1.4 \text{ in.} \end{cases}$$ Figure 4.8

The maximum center-to-center spacing of the bars, s_{max}, is equal to the minimum of $3h = 3 \times 4.5 = 13.5$ in., 18.0 in., and the minimum of the following assuming a 0.75-in. concrete cover and $f_s = 2f_y/3$:

$$s_{max} = \text{lesser of} \begin{cases} 15\left(\dfrac{40{,}000}{f_s}\right) - 2c_c = 15 \times \left(\dfrac{40{,}000}{2 \times 60{,}000/3}\right) - (2 \times 0.75) = 13.5 \text{ in.} \\ 12\left(\dfrac{40{,}000}{f_s}\right) = 12 \times \left(\dfrac{40{,}000}{2 \times 60{,}000/3}\right) = 12.0 \text{ in.} \end{cases}$$ Eq. (4.14)

The 12.0-in. spacing of the #3 bars is greater than the minimum spacing of 1.4 in. and is equal to the maximum spacing of 12.0 in.

Use #3 bars spaced at 12 in. on center for the negative and positive flexural reinforcement.

Step 4 – Determine the shrinkage and temperature reinforcement ACI 7.6.4.1

Minimum shrinkage and temperature reinforcement is equal to the following:

$$A_{s,min} = 0.0018A_g = 0.0018 \times 12.0 \times 4.5 = 0.10 \text{ in.}^2 \qquad \text{ACI 24.4.3.2}$$

The maximum center-to-center spacing of the bars, s_{max}, is equal to the following:

$$s_{max} = \text{lesser of} \begin{cases} 5h = 5 \times 4.5 = 22.5 \text{ in.} \\ \\ 18.0 \text{ in.} \end{cases} \qquad \text{ACI 24.4.3.3}$$

Use #3 bars spaced at 12 in. on center for the shrinkage and temperature reinforcement (provided $A_s = 0.11 \text{ in.}^2 >$ required $A_{s,min} = 0.10 \text{ in.}^2$). This reinforcement is placed perpendicular to the flexural reinforcement (see Figure 4.15).

4.8.4 Example 4.4 – Determination of Lap Splice Lengths: One-way Slab System, Building #2

Determine the required lap splice length for the positive reinforcement in the 4.5-in.-thick slab that is part of the wide-module joist system in Building #2 at a typical floor assuming normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement (see Figure 1.2).

Design data are given in Sect. 1.2.2. See Examples 4.1 and 4.3.

Step 1 – Determine the tension development length ACI 25.4.2.4

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \qquad \text{Eq. (4.15)}$$

For normalweight concrete, $\lambda = 1.0$. Eq. (4.16)

For Grade 60 reinforcement, $\psi_g = 1.0$. Eq. (4.17)

For uncoated reinforcing bars, $\psi_e = 1.0$. Eq. (4.18)

For #3 reinforcing bars, $\psi_s = 0.8$. Eq. (4.19)

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$. Eq. (4.20)

$$c_b = \text{lesser of} \begin{cases} \text{cover} + (d_b / 2) = 0.75 + (0.375 / 2) = 0.9 \text{ in.} \\ \\ s / 2 = 12.0 / 2 = 6.0 \text{ in.} \end{cases} \qquad \text{Eq. (4.21)}$$

There is no transverse reinforcement in this one-way slab, so $K_{tr} = 0$. Eq. (4.22)

$$(c_b + K_{tr}) / d_b = (0.9 + 0) / 0.375 = 2.4 < 2.5$$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b = \left(\frac{3}{40} \frac{60,000}{1.0\sqrt{4,000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{2.4} \right) \times 0.375 = 8.9 \text{ in.} < 12.0 \text{ in., use } 12.0 \text{ in.}$$

The calculated value of ℓ_d matches the value in Table 4.5.

Step 2 – Determine the required lap splice length ACI 25.5.2.1

According to the structural integrity requirements in ACI 7.7.7, at least one-quarter of the maximum positive moment reinforcement must be used as structural integrity reinforcement and it must be continuous. Also, where splices are needed, Class B tension lap splices must be provided for this reinforcement near the supports (joists).

Class B lap splice, $\ell_{st} = 1.3\ell_d = 1.3 \times 8.9 = 11.6$ in. Eq. (4.31)

Use a 1 ft-0 in. lap splice length for the positive reinforcement in this one-way slab.

4.8.5 Example 4.5 – Determination of Reinforcement Details: One-way Slab System, Building #2

Determine the reinforcement details for the 4.5-in.-thick slab that is part of the wide-module joist system in Building #2 at a typical floor assuming normalweight concrete.

Design data are given in Sect. 1.2.2. See Examples 4.1, 4.3, and 4.4.

Reinforcement details for this one-way slab system are given in Figure 4.17, which are based on the information given in Figure 4.15; the details in Figure 4.15 can be used because the one-way slab in this example is subjected to uniformly distributed gravity loads.

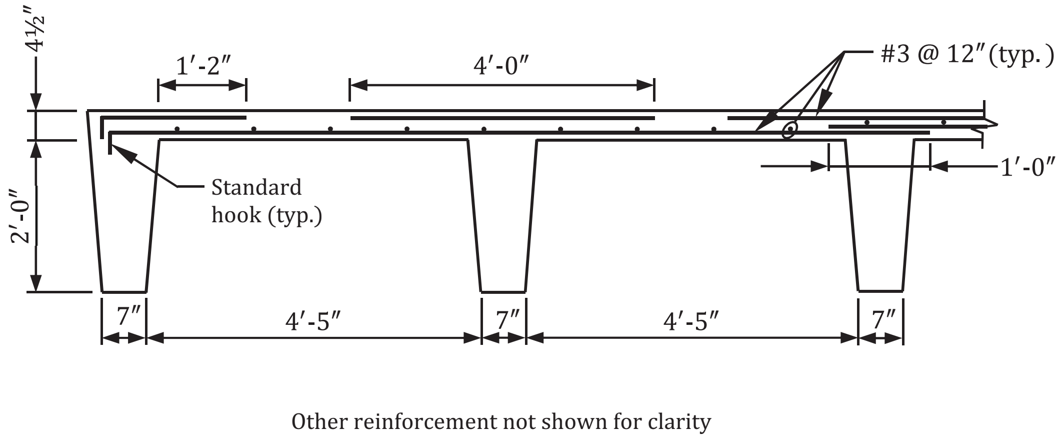


Figure 4.17 Reinforcement details for the one-way slab in Example 4.5.

Continuous positive reinforcement is provided because the span lengths are relatively small; this automatically satisfies one of the structural integrity reinforcement requirements of ACI 7.7.7 (see Figure 4.15). Also, 90-degree hooks are provided at the ends of the positive reinforcement in the end span at the noncontinuous supports (joists), which also satisfies one of the structural integrity reinforcement requirements. All other reinforcement in the system is not shown for clarity.

Chapter 5
TWO-WAY SLABS

5.1 Overview

Two-way slabs are elements in two-way construction where the slabs are designed to support loads through bending in two directions. Where the ratio of the long to the short side of a slab panel is 2 or less, load transfer is by bending predominately in two directions, and the panel is defined as a two-way slab. The main flexural reinforcement in a two-way slab system is parallel to the two orthogonal directions of load transfer.

Descriptions of common two-way slab systems are given in Table 5.1 (see also Figure 5.1).

Table 5.1 Two-way Slab Systems

System	Description
Flat plate	A two-way concrete slab supported directly on columns.
Flat slab	A two-way concrete slab supported directly on columns where the slab is thickened around the columns; the thickened portions of the slab are called drop panels.
Two-way beam-supported slab	A two-way concrete slab with column-line beams on all four sides of a panel.
Two-way joist (waffle slab)	Evenly spaced concrete joists spanning in both directions and a reinforced concrete slab cast integrally with the joists; solid slab sections are provided around the columns mainly to provide two-way shear resistance.
Flat plate voided concrete slab	A two-way concrete slab of uniform thickness containing regularly-spaced, hollow, plastic balls made of high-density, recycled polyethylene (HDPE) inside the concrete.

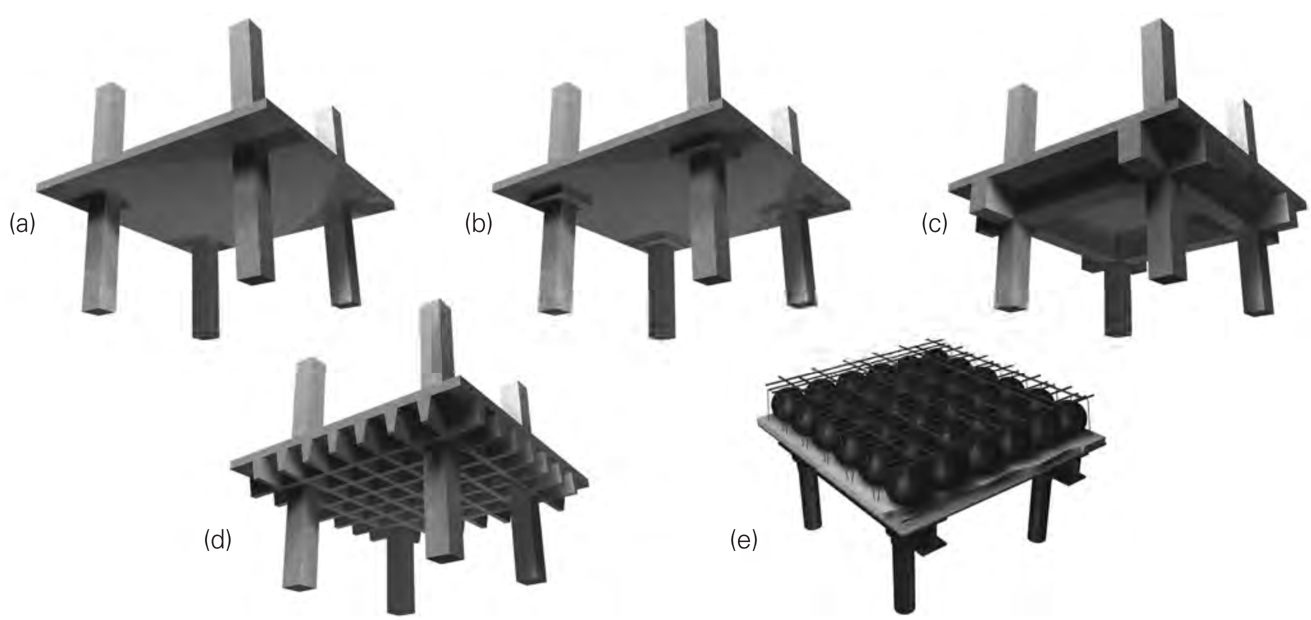


Figure 5.1 Two-way slab systems. (a) Flat plate. (b) Flat slab. (c) Two-way beam-supported slab. (d) Two-way joist. (e) Flat plate voided concrete slab.

The design and detailing of solid, cast-in-place two-way slabs with nonprestressed reinforcement in buildings assigned to Seismic Design Category (SDC) A and B are covered in this chapter. Two-way slabs that are part of the lateral force-resisting system (LFRS) in buildings assigned to SDC C are covered in Chapter 13 of this publication. Provisions for two-way slabs are given in ACI Chapter 8.

5.2 Minimum Slab Thickness

5.2.1 Overview

Two-way slabs must have sufficient thickness so all applicable strength and serviceability requirements are satisfied. Deflections are permitted to be calculated in accordance with ACI 24.2 and subsequently checked against the limits in ACI 24.2.2 (ACI 8.3.2.1). For slabs without interior beams spanning between supports on all sides where the flexural reinforcement has a specified yield strength, f_y, greater than 80,000 psi, the modulus of rupture, f_r, in calculating deflections is $5\sqrt{f_c'}$ instead of $7.5\lambda\sqrt{f_c'}$ (see ACI 19.2.3.1 and 8.3.1.1). For economy, Grade 60 reinforcement is commonly used in two-way slabs (Reference 7).

In lieu of performing deflection computations, minimum slab thickness, h, can be obtained using the provisions in ACI 8.3.1.1 for nonprestressed slabs without interior beams or in ACI 8.3.1.2 for nonprestressed slabs with beams spanning between supports on all sides. Methods to determine minimum slab thicknesses for the two-way systems illustrated in Figure 5.1 are given below.

The thickness of a concrete floor finish is permitted to be included in h where it is placed monolithically with the floor slab or where the floor finish is designed to be composite with the floor slab in accordance with ACI 16.4 (ACI 8.3.1.3).

Where single- or multiple-leg stirrups are used as part of the two-way shear resistance at critical sections in a two-way slab, the minimum slab thickness requirements in ACI 8.3.1.4 and 22.6.7.1 must be satisfied. In particular, the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement, d, must be at least equal to the greater of the following:

$$d \geq \text{greater of} \begin{cases} 6 \text{ in.} \\ \\ 16d_b \end{cases} \tag{5.1}$$

In this equation, d_b is the nominal diameter of the stirrups. For two-way slabs, an average d can be approximated as $h - 1.25$ in., which means the minimum h to satisfy the requirements of ACI 22.6.7.1 must be equal to the greater of 7.25 in. and $16d_b + 1.25$ in. This requirement essentially ensures the slab is thick enough so the stirrups can develop the specified yield strength, f_{yt}, considering the minimum inside bend diameters at the corners of the stirrups (see ACI 25.3.2).

5.2.2 Flat Plates

Minimum slab thickness for flat plates is permitted to be determined using the provisions in ACI 8.3.1.1 for exterior and interior panels without drop panels (see ACI Table 8.3.1.1 and Table 5.2). As noted previously, Grade 60 reinforcement is often used in flat plate systems.

Table 5.2 Minimum Thickness of Flat Plates

f_y (psi)	Exterior Panels		Interior Panels
	Without Edge Beams	With Edge Beams*	
40,000	$\ell_n / 33$	$\ell_n / 36$	$\ell_n / 36$
60,000	$\ell_n / 30$	$\ell_n / 33$	$\ell_n / 33$
80,000	$\ell_n / 27$	$\ell_n / 30$	$\ell_n / 30$

** Slabs with beams between columns along exterior edges. Exterior panels are considered to be without edge beams where* $\alpha_f < 0.8$.

In the expressions for minimum h, ℓ_n is the clear span length in the long direction of the panel measured face-to-face of supports in inches. Minimum slab thicknesses determined by Table 5.2 must be greater than or equal to 5.0 in. [ACI 8.3.1.1(a)].

The term α_f in the footnote of Table 5.2 is defined as the ratio of the flexural stiffness of the beam section to the flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels, if any, on each side of the beam:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \tag{5.2}$$

In this equation, E_{cb} and E_{cs} are the moduli of elasticity of the beam and slab concrete, respectively, which can be calculated by ACI Equation (19.2.2.1.a) where the density (unit weight) of normalweight concrete or the equilibrium density of lightweight concrete, w_c, is between 90 and 160 lb/ft^3, inclusive [ACI 19.2.2.1(a)]:

$$E_c = w_c^{1.5} 33 \sqrt{f_c'} \tag{5.3}$$

For normalweight concrete $(w_c = 135 \text{ to } 160 \text{ lb/ft}^3)$, E_c can be determined by ACI Equation (19.2.2.1.b) [ACI 19.2.2.1(b)]:

$$E_c = 57{,}000 \sqrt{f_c'} \tag{5.4}$$

In Equations (5.3) and (5.4), f_c' and E_c have the units of pounds per square inch. For the typical case of monolithic construction, $E_{cb} = E_{cs}$.

In lieu of using Equations (5.3) or (5.4), it is permitted to specify E_c based upon testing of concrete mixtures in accordance with ACI 19.2.2.2.

The terms I_b and I_s in Equation (5.2) are the moments of inertia of the gross sections of the beam and slab about the centroidal axes, respectively.

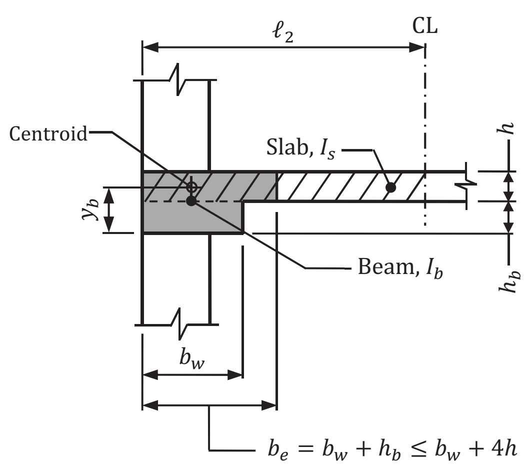


Figure 5.2 Effective beam and slab sections for computation of stiffness ratio, α_f, for edge beams.

A portion of the slab on the sides of a beam is permitted to be included when determining the section properties of the section (ACI 8.4.1.8). The effective slab width, b_e, permitted to be included in the determination of I_b is the lesser of the following for an edge beam (see Figure 5.2):

$$b_e = b_w + \text{lesser of} \begin{cases} h_b \\ 4h \end{cases} \tag{5.5}$$

The terms in Equation (5.5) are defined in Figure 5.2 for the case where the beam projects below the slab. In general, h_b is the greater of the beam projection above or below the slab (see ACI Figure R8.4.1.8).

The equations in Table 5.3 can be used to determine I_b and I_s for an edge beam [Note: These equations are also valid for interior beams where b_e is determined by Equation (5.6)]. The term y_b is the distance from the bottom of the combined beam section [which is the beam plus the slab section with an effective width, b_e, equal to that determined by Equation (5.5) or (5.6)] to the centroid of the section.

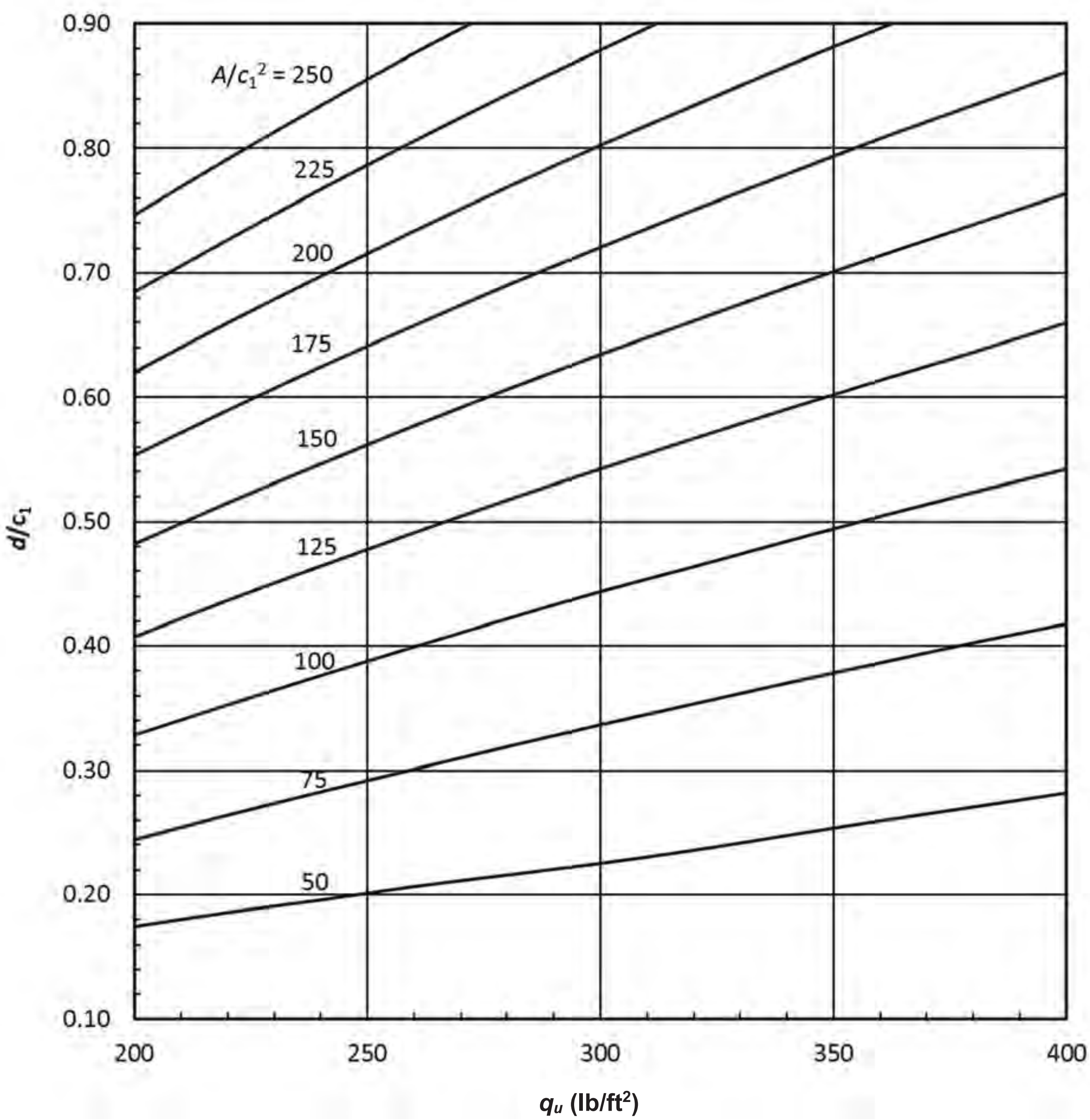


Figure 5.3 Preliminary slab thickness for flat plate systems.

Table 5.3 Equations for I_b and I_s for Edge and Interior Beams

y_b	I_b	I_s
$\frac{b_e h[h_b + (h/2)] + (b_w h_b^2 / 2)}{b_e h + b_w h_b}$	$\frac{b_w h_b^3}{12} + b_w h_b \left(y_b - \frac{h_b}{2}\right)^2 + \frac{b_e h^3}{12} + b_e h \left(h_b + \frac{h}{2} - y_b\right)^2$	$\frac{\ell_2 h^3}{12}$

For flat plate systems with edge beams, a preliminary slab thickness is determined using the expressions in Table 5.2 assuming $\alpha_f \geq 0.8$ because the slab thickness, h, is not known at this stage; this assumption is checked after α_f has been calculated based on that slab thickness.

The thickness of a flat plate is usually controlled by the serviceability requirements in Table 5.2 for span lengths in the range of 15 to 25 ft and live loads less than or equal to 40 lb/ft^2. Otherwise, two-way shear requirements typically govern the thickness. Shear stresses at edge and corner columns are particularly critical because of relatively large bending moments transferred from the slab to the columns at these locations. Because two-way shear requirements are related to flexural requirements, a closed-form solution for slab thickness that satisfies both sets of requirements cannot be obtained unless some simplifying assumptions are made. Figure 5.3 can be used to determine a preliminary slab thickness for flat plates considering two-way shear stresses. The information in the figure is based on the following assumptions:

- Square edge columns of size c_1 bending perpendicular to the edge with a three-sided critical section
- Column supports a tributary area A
- Concrete is normalweight with $f'_c = 4{,}000$ psi

In the figure, q_u is the total factored uniformly distributed load on the slab, which must include the factored self-weight of the slab; this weight can be estimated using a slab thickness based on the expressions in Table 5.2. A preliminary slab thickness, h, is obtained by adding 1.25 in. to the value of d acquired from Figure 5.3 for a given q_u and area ratio A / c_1^2.

For the usual case of continuous construction, it is typical for h to be the same for all spans and for it to be determined on the basis of the span yielding the largest minimum depth; this results in economical formwork (see Reference 7). For flat plates with edge beams, minimum depth requirements for the beams are given in Section 6.2 of this publication.

Fire-resistance requirements of the general building code must also be considered when specifying a slab thickness (ACI 4.11.1). Fire-resistance requirements are usually satisfied by providing a slab thickness that satisfies serviceability and/or shear strength requirements.

Where it is not viable to increase the slab thickness and/or the column size to satisfy two-way shear requirements, shear reinforcement or shear caps may be used. Information on common types of shear reinforcement is given in Section 5.4.4 of this publication.

5.2.3 Flat Slabs

Minimum slab thickness for flat slabs is permitted to be determined using ACI 8.3.1.1 for exterior and interior panels with drop panels (see ACI Table 8.3.1.1 and Table 5.4). The expressions in Table 5.4 cannot be used unless the dimensions of the drop panels conform to the requirements in ACI 8.2.4, which are illustrated in Figure 5.4. In the figure, ℓ_A and ℓ_B are the adjoining center-to-center span lengths (Note: The dimensional limitations of the drop panel must also be satisfied in the direction perpendicular to that shown in the figure). For economy, Grade 60 reinforcement is often used in flat slab systems.

Table 5.4 Minimum Thickness of Flat Slabs

f_y (psi)	Exterior Panels		Interior Panels
	Without Edge Beams	With Edge Beams*	
40,000	$\ell_n / 36$	$\ell_n / 40$	$\ell_n / 40$
60,000	$\ell_n / 33$	$\ell_n / 36$	$\ell_n / 36$
80,000	$\ell_n / 30$	$\ell_n / 33$	$\ell_n / 33$

** Slabs with beams between columns along exterior edges. Exterior panels are considered to be without edge beams where* $\alpha_f < 0.8$.

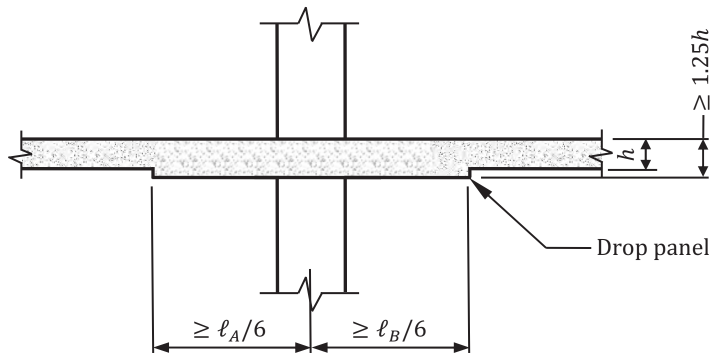
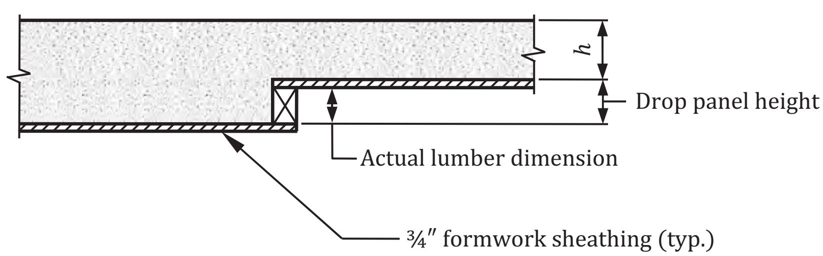


Figure 5.4 Dimensional requirements for drop panels.

Like in the case for flat plates, ℓ_n is the clear span length in the long direction of the panel measured face-to-face of supports in inches. Minimum slab thicknesses determined by the expressions in Table 5.4 must be greater than or equal to 4.0 in. [ACI 8.3.1.1(b)].

To achieve economical formwork, standard lumber dimensions should be used to form drop panels (References 7 and 10). Drop panel heights (that is, the depths of the projections below the slab) based on actual lumber dimensions and 0.75-in.-thick formwork sheathing are given in Table 5.5. Formwork costs unnecessarily increase if drop panel heights other than those in Table 5.5 are specified.

Table 5.5 Drop Panel Height for Formwork Economy



Lumber Size		Drop Panel Height
Nominal	Actual	
2x	1-1/2"	2-1/4"
4x	3-1/2"	4-1/4"
6x	5-1/2"	6-1/4"
8x	7-1/4"	8"

For the usual case of continuous construction, it is typical for h to be the same for all spans and for it to be determined on the basis of the span yielding the largest minimum depth; this results in economical formwork (see Reference 7). For flat slabs with edge beams, minimum depth requirements for the beams are given in Section 6.2 of this publication.

Fire-resistance requirements of the general building code must also be considered when specifying a slab thickness (ACI 4.11.1). Fire-resistance requirements are usually satisfied by providing a slab thickness that satisfies serviceability and/or shear requirements.

5.2.4 Two-way Beam-Supported Slabs

Minimum thickness requirements for slabs in two-way beam-supported slab systems are given in ACI 8.3.1.2 and are summarized in Table 5.6 (see ACI Table 8.3.1.2). The term α_{fm} is the average value of α_f for all beams on the edges of a panel. The equations in Table 5.3 can be used to determine the section properties of the combined beam section and the slab for edge and interior beams. For edge beams, the effective width, b_e, of the slab is determined by Equation (5.5) and for interior beams, b_e is determined by the following equation (see ACI 8.4.1.8 and Figure 5.5 for the case of an interior beam projecting below the slab):

$$b_e = b_w + \text{lesser of} \begin{cases} 2h_b \\ 8h \end{cases} \tag{5.6}$$

In general, h_b is the greater of the beam projection above or below the slab (see ACI Figure R8.4.1.8).

Table 5.6 Minimum Thickness of Two-way Slabs with Beams Spanning Between Supports on All Sides

α_{fm}	Minimum Thickness, h
$\alpha_{fm} \le 0.2$	ACI 8.3.1.1 applies (see Tables 5.2 and 5.4)
$0.2 < \alpha_{fm} \le 2.0$	$\dfrac{\ell_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \ge 5.0$ in.
$\alpha_{fm} > 2.0$	$\dfrac{\ell_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 9\beta} \ge 3.5$ in.

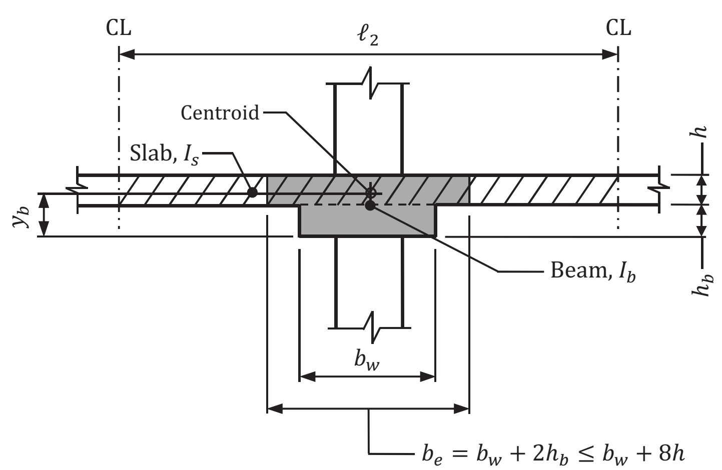


Figure 5.5 Effective beam and slab sections for computation of stiffness ratio, α_f, for interior beams.

In the expressions in Table 5.6, ℓ_n is the clear span in the long direction measured face-to-face of beams in inches and β is the ratio of the long-to-short dimensions of the clear spans in a panel. Also, f_y has the units of pounds per square inch.

In edge panels with edge beams where the stiffness ratio $\alpha_f < 0.80$, the minimum h determined by the equations in Table 5.6 must be increased by at least 10 percent (ACI 8.3.1.2.1).

For the usual case of continuous construction, it is typical for h to be the same for all spans and for it to be determined on the basis of the span yielding the largest minimum depth; this results in economical formwork (see Reference 7). For two-way beam-supported slabs, minimum depth requirements for the beams are given in Section 6.2 of this publication.

Fire-resistance requirements of the general building code must also be considered when specifying a slab thickness (ACI 4.11.1). Fire-resistance requirements are usually satisfied by providing a slab thickness that satisfies serviceability requirements.

5.2.5 Two-way Joists

General requirements for nonprestressed two-way joist systems are given in ACI 8.8. This type of system is formed by 30-, 41-, and 52-in. wide domes, resulting in 3-, 4-, and 5-ft modules, respectively (see Figure 5.6). Slab thickness, h, is typically controlled by fire-resistance requirements and the joist (rib) width, b_r, is set by the dome width, b_d, that is specified. The only depth to be determined to satisfy serviceability requirements is the dome depth, h_r. Standard form dimensions for two-way joist construction are given in Table 5.7.

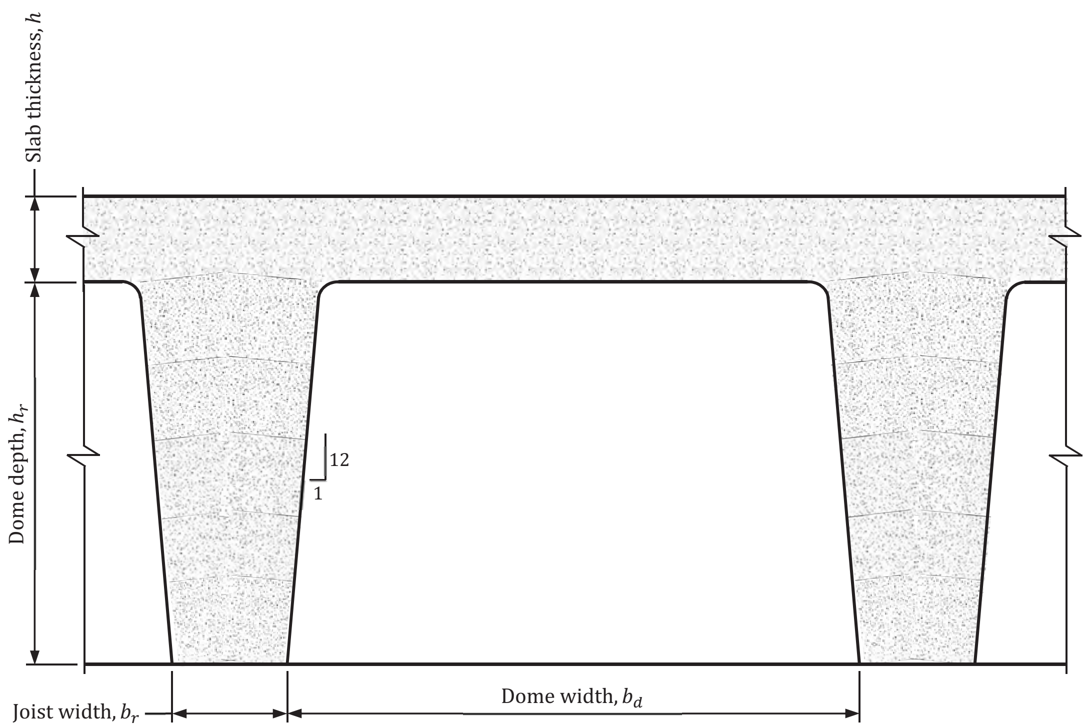


Figure 5.6 Cross-section of a two-way joist system.

Table 5.7 Standard Form Dimensions for Two-way Joist Construction

Dome Width, b_d (in.)	Dome Depth, h_r (in.)	Joist Width, b_r (in.)
30	8, 10, 12, 14, 16, 20, 24	6
41	14, 16, 20, 24	7
52	14, 16, 20, 24	8

For design purposes, a two-way joist system is considered to be a flat slab with the solid heads at the columns acting as drop panels. The cross-section of the two-way joist system is transformed into an equivalent section of uniform thickness, t_e, with the same width as the actual section, which is equal to $b_r + b_d$ (see Figure 5.6). The equivalent uniform slab thickness, t_e, is determined by setting the moment of inertia of the equivalent section equal to the gross moment of inertia of the actual two-way joist section, I_g:

$$\frac{(b_r + b_d)t_e^3}{12} = I_g \tag{5.7}$$

Solving for t_e:

$$t_e = \left(\frac{12I_g}{b_r + b_d}\right)^{1/3} \tag{5.8}$$

Equations to determine I_g for a two-way joist system are given in Table 5.8 for the case where the side faces of the domes are sloped 12:1 (see Figure 5.7).

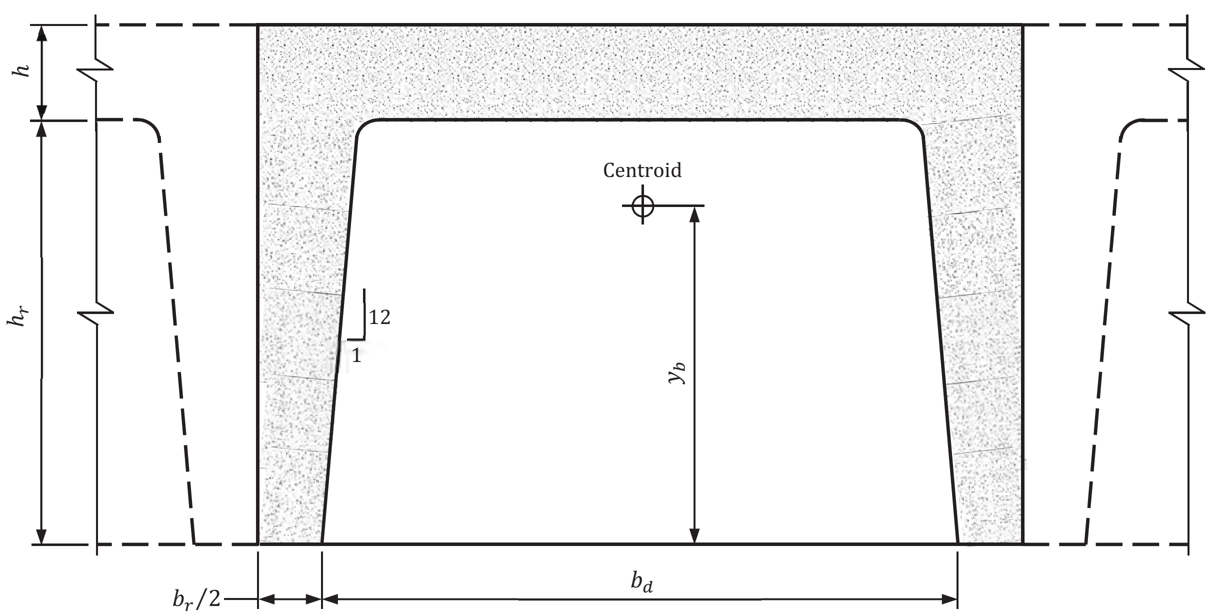


Figure 5.7 Determination of the gross moment of inertia, I_g for a two-way joist system.

Table 5.8 Equations to Determine I_g for Two-way Joist Systems

y_b	I_g
$\dfrac{\dfrac{b_r(h_r+h)^2}{2}+\dfrac{h_r^3}{18}+b_d h\left(h_r+\dfrac{h}{2}\right)}{b_r(h_r+h)+\dfrac{h_r^2}{12}+b_d h}$	$\dfrac{b_d h^3}{12}+b_d h\left(h_r+\dfrac{h}{2}-y_b\right)^2+\dfrac{b_r(h_r+h)^3}{12}+b_r(h_r+h)\left(y_b-\dfrac{h_r+h}{2}\right)^2 +\dfrac{h_r^4}{216}+\dfrac{h_r^2}{12}\left(\dfrac{2h_r}{3}-y_b\right)^2$

Equivalent slab thicknesses for a two-way joist system with a 3-ft module, a 6-in.-wide joist, and a 4.5-in.-thick slab are given in Table 5.9.

Table 5.9 Equivalent Slab Thickness, t_e, for a Two-way Joist System with a 3-ft Module

Dome Depth, h_d (in.)	Equivalent Slab Thickness, t_e (in.)
8	8.8
10	10.3
12	11.7
14	13.1
16	14.6
20	17.4
24	20.2

To satisfy serviceability requirements, an overall depth (h_r+h) must be provided such that the corresponding equivalent thickness, t_e, which is determined by Equation (5.8) [or, where applicable, by Table 5.9], is greater than or equal to the minimum required overall depth determined by the applicable expression in Table 5.4 using the longest clear span length, ℓ_n. For economy in formwork, the largest required overall depth from all the spans should be used wherever possible (Reference 7).

Two-way joist systems not satisfying the limitations of ACI 8.8.1.1 through 8.8.1.4 must be designed as slabs and beams (ACI 8.8.1.8).

5.2.6 Flat Plate Voided Concrete Slabs

Empirical serviceability requirements for flat plate voided concrete slabs systems are not specifically provided in ACI 318. Because flat plate voided concrete slab systems are very similar to two-way slab and two-way joist systems, the overall thickness can be initially estimated by $\ell_n/36$ for slabs with Grade 60 reinforcement (see Table 5.4).

In lieu of using an estimated overall slab thickness, it is recommended deflection calculations be performed in accordance with ACI 24.2. In addition to factors accounting for concrete cracking, the analysis must include the stiffness reduction factor, which accounts for the presence of the voids in the slab; such factors can be found in the manufacturers' literature (on average, this factor is equal to about 0.9). Calculated deflections must be less than or equal to the maximum permissible deflections in ACI Table 24.2.2 (ACI 8.3.2).

The largest required slab thickness from all the panels should be used wherever possible for overall economy. Additionally, the same size void formers should be used as often as practical throughout the project to facilitate placement in the field.

Design and detailing requirements for flat plate voided concrete slab systems can be found in Reference 11.

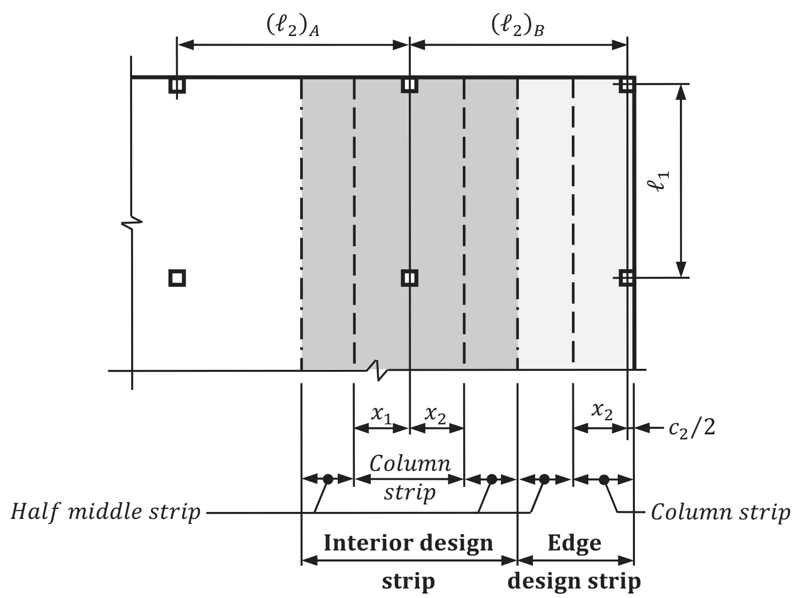
5.3 Required Strength

5.3.1 Analysis Methods

The analysis methods in ACI Chapter 6 in conjunction with the factored load combinations in ACI Chapter 5 must be used to calculate required strength (see ACI 8.4.1.2 and 8.4.1.1, respectively). The following analysis methods are relevant to two-way slabs (ACI 6.2.3 and 6.2.4):

(1) Linear elastic first-order analysis (ACI 6.6)
(2) Linear elastic second-order analysis (ACI 6.7)
(3) Inelastic analysis (ACI 6.8)
(4) Finite element analysis (ACI 6.9)
(5) Direct design method (DDM) [ACI 6.2.4.1(a)]
(6) Equivalent frame method (EFM) [ACI 6.2.4.1(b)]

For purposes of analysis, two-way slab panels, which are bounded by columns, beams, or wall centerlines on all sides (ACI 8.4.1.7), are permitted to be divided into columns strips and middle strips, the definitions of which are given in ACI 8.4.1.5 and 8.4.1.6, respectively. The widths of these strips depend on the span lengths ℓ_1 and ℓ_2, where ℓ_1 is the length of the span in the direction moments are being determined, measured center-to-center of supports, and ℓ_2 is the length of the span in the direction perpendicular to ℓ_1, measured center-to-center of supports (see Figure 5.8). Any column-line beams present are to be included in the column strip (ACI 8.4.1.5).



x_1 = lesser of $\ell_1/4$ or $(\ell_2)_A/4$

x_2 = lesser of $\ell_1/4$ or $(\ell_2)_B/4$

Figure 5.8 Design strips, column strips, and middle strips in a two-way slab system.

For slabs that are part of the LFRS, the results from a gravity load analysis are permitted to be combined with those from a lateral load analysis using the applicable load combinations in ACI Table 5.3.1 (ACI 8.4.1.9).

The DDM and the EFM are not included in ACI 318-19. Provisions for these methods are given in the 1971 through 2014 editions of ACI 318. The DDM is covered in Section 5.3.4 of this publication; the information in that section is primarily from ACI 8.10 in ACI 318-14.

5.3.2 Critical Sections for Flexure

Factored bending moments, M_u, are permitted to be calculated at the faces of the supports for two-way slabs built integrally with the supports (ACI 8.4.2.1). For flexural design, the critical sections in the design strips occur at the faces of the supports where negative moments are maximum and in the span where positive moments are maximum.

Studies of moment transfer between slabs and columns have shown that the factored slab moment resisted by the column at a joint, M_{sc}, due to gravity, lateral, and/or other load effects is transferred by a combination of flexure and eccentricity of shear. Requirements for transfer by flexure and transfer by eccentricity of shear are given in ACI 8.4.2.2 and 8.4.4.2, respectively.

The fraction of the factored slab moment resisted by the column transferred by flexure is equal to $\gamma_f M_{sc}$ where the factor γ_f is calculated by ACI Equation (8.4.2.2.2) [ACI 8.4.2.2.2]:

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} \tag{5.9}$$

In this equation, b_1 and b_2 are the dimensions of the critical section for two-way shear measured parallel and perpendicular to the direction of analysis, respectively. The critical section for two-way shear is located so its perimeter, b_o, is a minimum where the perimeter need not approach closer than $d/2$ to (1) edges or corners of columns, concentrated loads, or reaction areas, or (2) changes in slab thickness such as edges of column capitals, drop panels, or shear caps (ACI 22.6.4.1). For square or rectangular columns, concentrated loads, or reaction areas, the critical section is permitted to be defined using straight sides (ACI 22.6.4.1.1). For circular or polygon-shaped columns, the critical section is permitted to be defined by a square column that has the same area as the actual column (ACI 22.6.4.1.2). Properties of critical sections are given in Section 5.3.3 of this publication.

According to ACI 8.4.2.2.3, the effective slab width, b_{slab}, to be used to resist $\gamma_f M_{sc}$ is equal to the following for two-way slab systems without drop panels or shear caps:

$$b_{slab} = c_2 + \text{distance on each side equal to the lesser of} \begin{cases} 1.5h \\ \\ \text{distance to edge of slab} \end{cases} \tag{5.10}$$

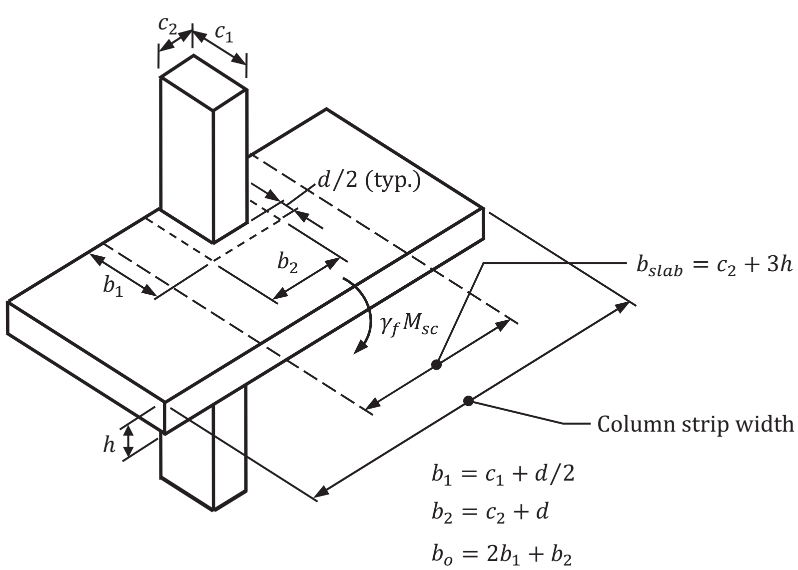


Figure 5.9 Effective slab width for transfer of moment at an edge column in a flat plate system.

For two-way systems with drop panels or shear caps, b_{slab} is equal to the following:

$$b_{slab} = c_2 + \text{distance on each side equal to the lesser of} \begin{cases} 1.5(h + h_1) \\ \\ \text{distance to the edge of the drop or cap} + 1.5h \end{cases} \tag{5.11}$$

In these equations, c_2 is the dimension of the column in the direction perpendicular to the direction of analysis, h is the thickness of the slab, and h_1 is the thickness of the drop panel or shear cap projection below the slab. For columns with capitals, the width of the capital is to be used in these equations instead of c_2.

The effective slab widths for an edge column in a flat plate system and for an interior column in a flat plate system with a shear cap are illustrated in Figures 5.9 and 5.10, respectively.

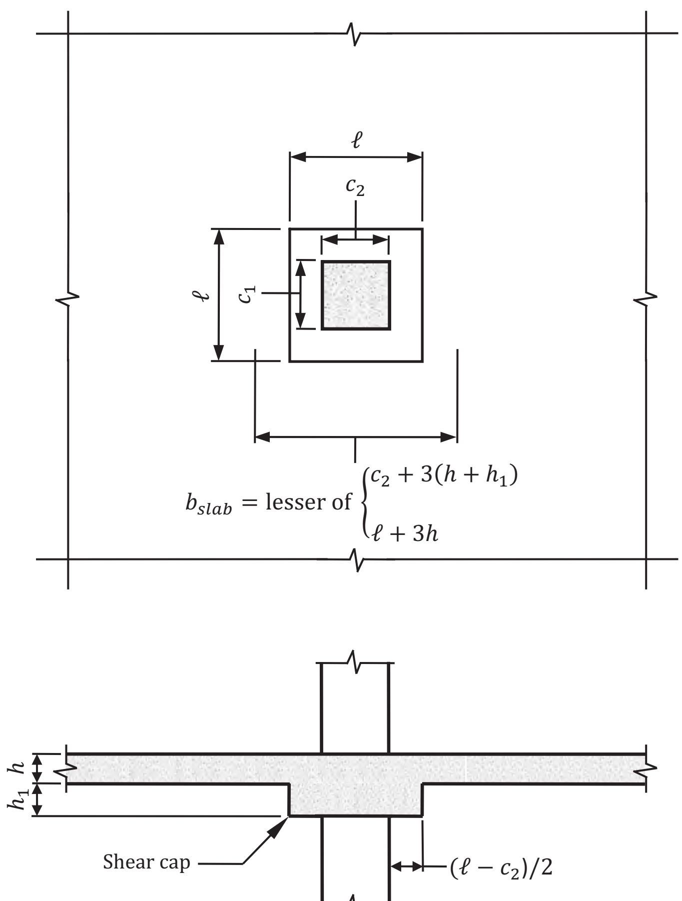


Figure 5.10 Effective slab width for transfer of moment at an interior column in a flat plate system with a shear cap.

The factor γ_f is permitted to be increased to the values in ACI Table 8.4.2.2.4 based on the limitations given in that table for different column locations (ACI 8.4.2.2.4; see Table 5.10). The terms v_c and v_{uv} in the table are the nominal shear strength provided by the concrete (ACI 22.6.5) and the factored shear stress on the slab critical section for two-way action, from the controlling load combination, without moment transfer, respectively. Information on how to calculate these stresses is given in Section 5.4.4 of this publication.

Table 5.10 Maximum Modified Values of γ_f for Nonprestressed Two-way Slabs

Column Location	Span Direction	v_{uv}	ε_t within b_{slab}	Maximum Modified γ_f
Corner column	Either direction	$\leq 0.50\phi v_c$	$\geq \varepsilon_{ty} + 0.003$	1.0
Edge column	Perpendicular to the edge	$\leq 0.75\phi v_c$	$\geq \varepsilon_{ty} + 0.003$	1.0
	Parallel to the edge	$\leq 0.40\phi v_c$	$\geq \varepsilon_{ty} + 0.008$	$\dfrac{1.25}{1+(2/3)\sqrt{b_1/b_2}} \leq 1.0$
Interior column	Either direction	$\leq 0.40\phi v_c$	$\geq \varepsilon_{ty} + 0.008$	$\dfrac{1.25}{1+(2/3)\sqrt{b_1/b_2}} \leq 1.0$

In order to be able to use the maximum modified values of γ_f in Table 5.10, the net tensile strain in the extreme layer of longitudinal tension reinforcement at nominal strength, ε_t, for the flexural reinforcement in b_{slab} must be greater than or equal to the strains indicated in the table. The strain $\varepsilon_{ty} + 0.003$ is the minimum strain corresponding to a tension-controlled section where $\varepsilon_{ty} = f_y / E_s$ is the value of the net tensile strain in the extreme layer of longitudinal tension reinforcement used to define a compression-controlled section (see ACI Table 21.2.2).

Increasing γ_f results in a decrease in γ_v, which is the factor used to determine the fraction of the factored slab moment resisted by the column transferred by eccentricity of shear: $\gamma_v = 1 - \gamma_f$ [see ACI Equation (8.4.4.2.2)]. It is evident that a decrease in γ_v results in a decrease in the moment $\gamma_v M_{sc}$ and a corresponding decrease in factored shear stresses, which may be advantageous in certain situations.

Once $\gamma_f M_{sc}$ has been determined, the required area of flexural reinforcement within b_{slab} can be determined using Equations (5.35) and (5.36) in Section 5.5.1 of this publication. Flexural reinforcement may need to be concentrated within b_{slab} to resist $\gamma_f M_{sc}$ (ACI 8.4.2.2.5). This can be achieved by providing a portion of the negative column strip reinforcement at a closer spacing within b_{slab} or by adding more reinforcement within b_{slab}.

A reversal of moment can occur at slab-column joints that are part of the LFRS. In such cases, both top and bottom flexural reinforcement must be concentrated within b_{slab}. ACI R8.4.2.2.4 recommends a ratio of top-to-bottom reinforcement of approximately 2 in such cases.

5.3.3 Critical Sections for Shear

One-way Shear

Factored shear forces, V_u, are permitted to be calculated at the faces of the supports for two-way slabs built integrally with the supports (ACI 8.4.3.1).

Two-way slabs are permitted to be designed for the factored shear force at a critical section located a distance d from the face of the support where the conditions in ACI 8.4.3.2 are satisfied (see Figure 5.11):

(1) Support reaction, in direction of applied shear, introduces compression into the end region of the slab
(2) Loads are applied at or near the top surface of the slab
(3) No concentrated load occurs between the face of the support and the critical section

The critical section for one-way shear must be taken at the face of the support where one or more of the three conditions in ACI 8.4.3.2 are not met.

Two-way Shear

The critical section for two-way shear is located so its perimeter, b_o, is a minimum where the perimeter need not approach closer than $d/2$ to (1) edges or corners of columns, concentrated loads, or reaction areas, or (2) changes in slab thickness such as edges of column capitals, drop panels, or shear caps (ACI 8.4.4.1.1 and 22.6.4.1).

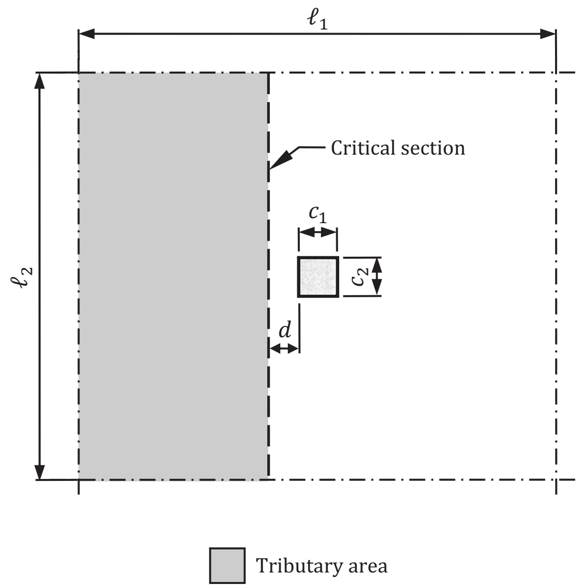


Figure 5.11 Critical section for one-way shear in a two-way slab that satisfies the conditions of ACI 8.4.3.2.

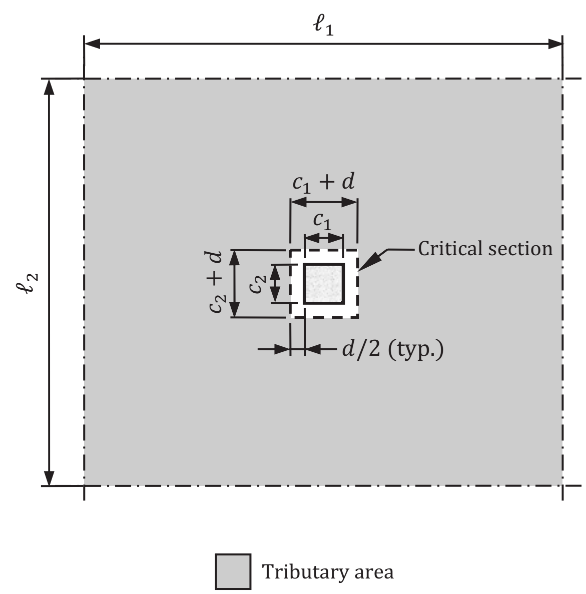


Figure 5.12 Critical section for two-way shear in a flat plate.

According to ACI 22.6.4.1.1, a critical section with straight sides is permitted for square or rectangular columns, concentrated loads, or reaction areas. The locations of the critical sections for two-way shear at an interior column in flat plate and flat slab systems are given in Figures 5.12 and 5.13, respectively. For flat slabs, shear requirements need to be checked at a distance $d_2 / 2$ from the face of the column and $d_1 / 2$ from the face of the drop panel because shear failure can occur at either location where d_2 and d_1 are the average distances from the extreme compression fiber to the centroid of the longitudinal tension reinforcement defined in Figure 5.13.

A shear cap is a projection below the slab similar to a drop panel; unlike a drop panel, it is used exclusively to increase the slab shear strength of the slab. According to ACI 8.2.5, shear caps must extend horizontally from the face of the column a distance equal to at least the thickness of the projection below the slab (see Figure 5.14 for the case of an interior column). Like slabs with drop panels, shear requirements must be checked at two locations.

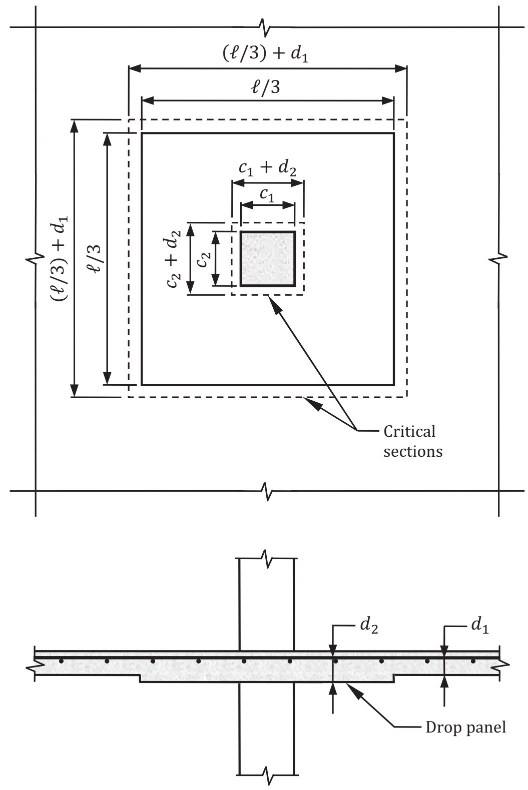


Figure 5.13 Critical sections for two-way shear in a flat slab.

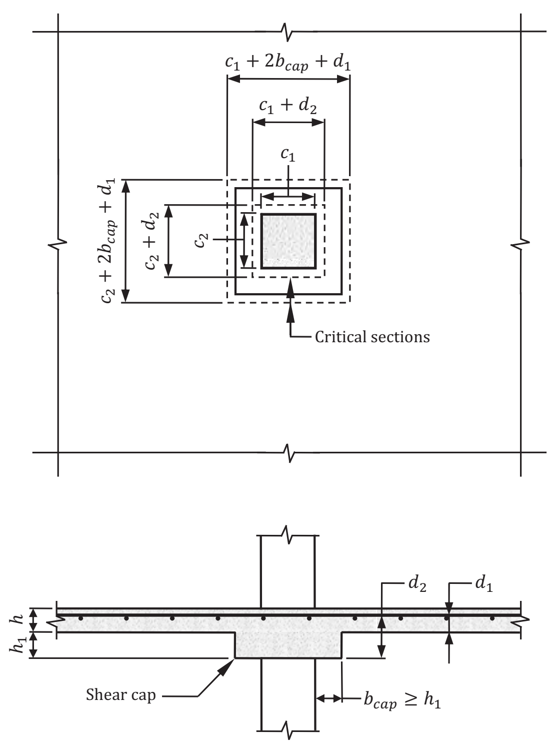


Figure 5.14 Critical sections for two-way shear at a shear cap.

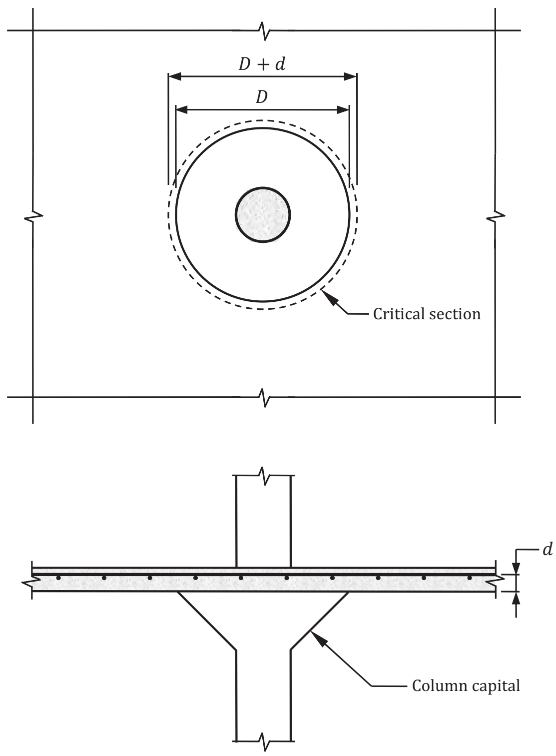


Figure 5.15 Critical section for two-way shear at a column capital.

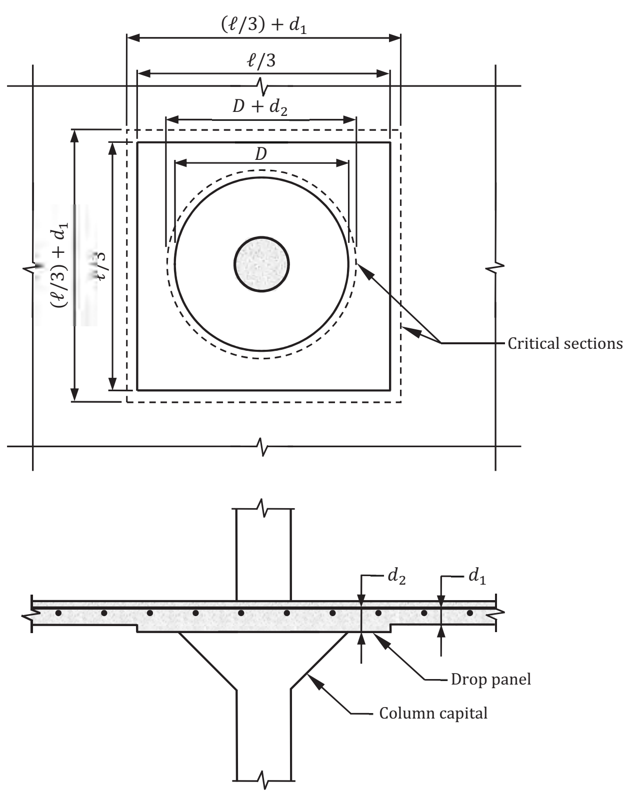


Figure 5.16 Critical sections for two-way shear at a column capital and drop panel.

The critical section for an interior column with a circular capital is illustrated in Figure 5.15. Because the capital is part of the column and not part of the slab, shear requirements need only be checked at the critical section located a distance $d/2$ from the face of the capital. In cases where a column capital and drop panel are both utilized, the shear strength must be checked at the critical section located a distance $d_2/2$ from the face of the capital and at the critical section located a distance $d_1/2$ from the face of the drop panel (see Figure 5.16).

Critical shear perimeters are not as clearly defined in cases where the slab edges cantilever beyond the face(s) of an edge column. The following general guidelines can be used based on the provisions of ACI 22.6.4.1, which requires the perimeter of the critical section, b_o, be a minimum.

Consider the edge column in Figure 5.17. The critical shear perimeter for this column is either three-sided or four-sided depending on the length of the cantilever (overhang), x. The following equation can be used to obtain minimum b_o:

$$b_o = \begin{cases} 2\{x + [c_1 + (d/2)]\} + (c_2 + d) \text{ where } x \le (c_2/2) + d & \text{[3-sided critical section]} \\ \\ 2[(c_1 + d) + (c_2 + d)] \text{ where } x > (c_2/2) + d & \text{[4-sided critical section]} \end{cases} \tag{5.12}$$

Similar equations can be derived for corner columns.

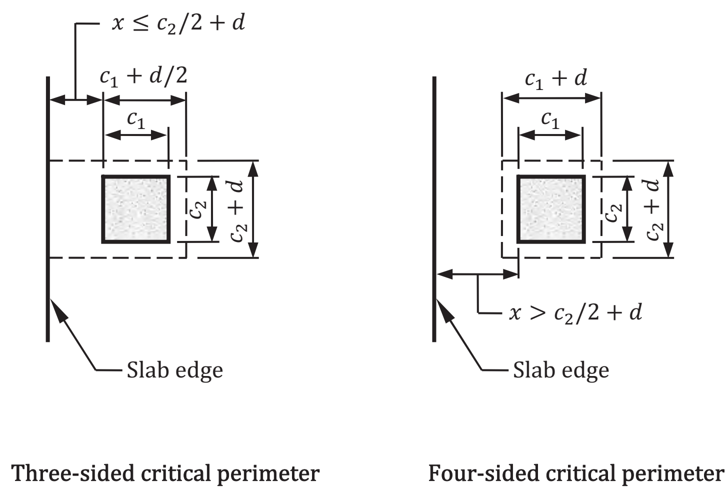


Figure 5.17 Critical section perimeters for edge columns with slab cantilevers.

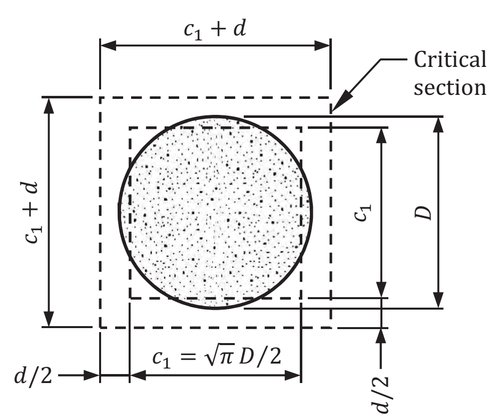


Figure 5.18 Critical section for two-way shear in slabs with circular columns.

For circular or regular polygon-shaped columns, the critical sections for two-way shear are permitted to be defined assuming a square column that has the same area as the circular or polygon-shaped column (ACI 22.6.4.1.2). For a circular column of diameter D, the size of the square column, c_1, is equal to the following:

$$c_1 = \frac{\sqrt{\pi}D}{2} \tag{5.13}$$

The dimensions of the critical section in this case are $b_1 = b_2 = c_1 + d$ (see Figure 5.18 for the case of an interior column). Note that this provision specifically refers to circular columns and not circular column capitals; the critical section for the latter is circular, as shown in Figures 5.15 and 5.16.

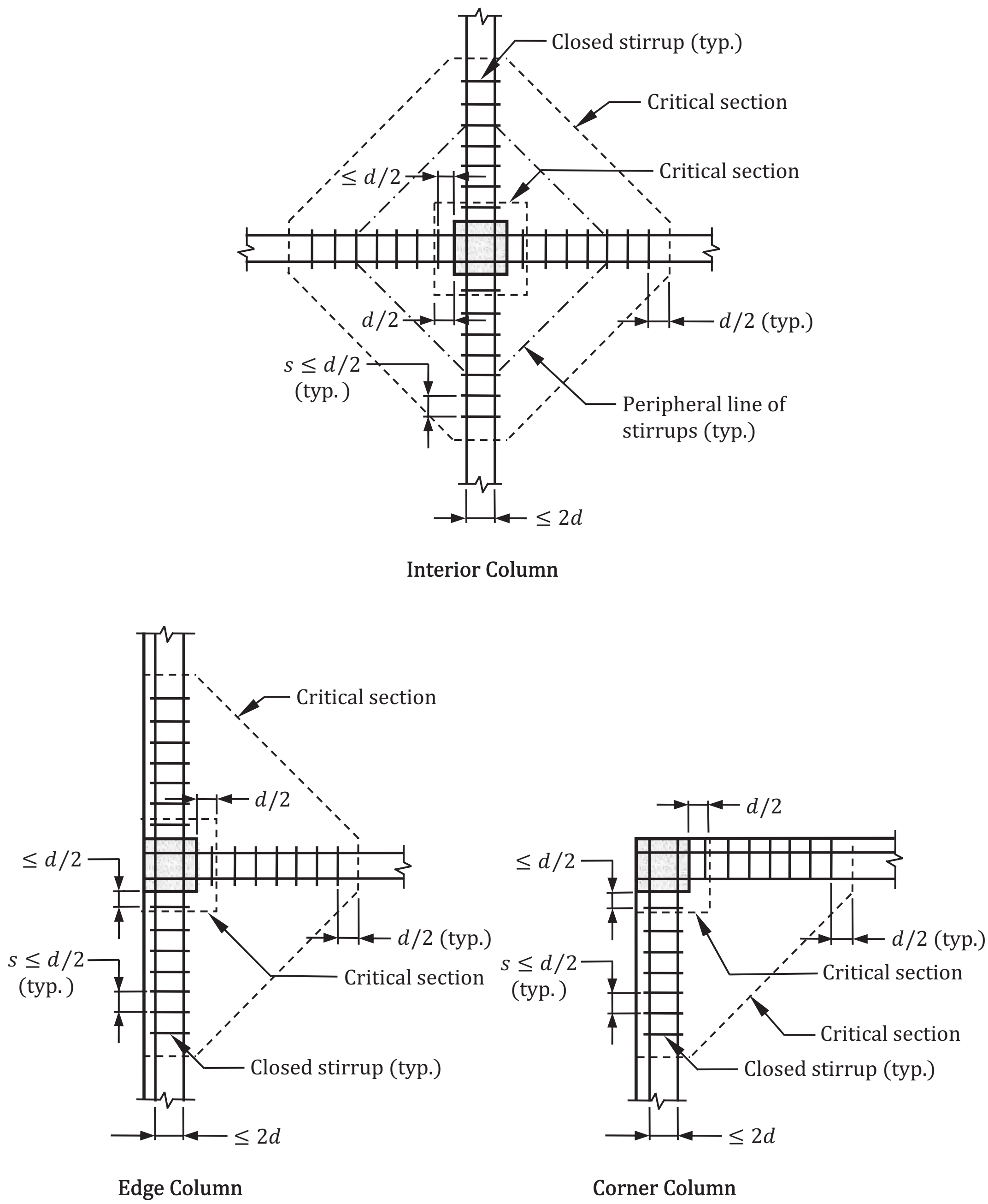


Figure 5.19 Critical sections for two-way shear in slabs reinforced with stirrups.

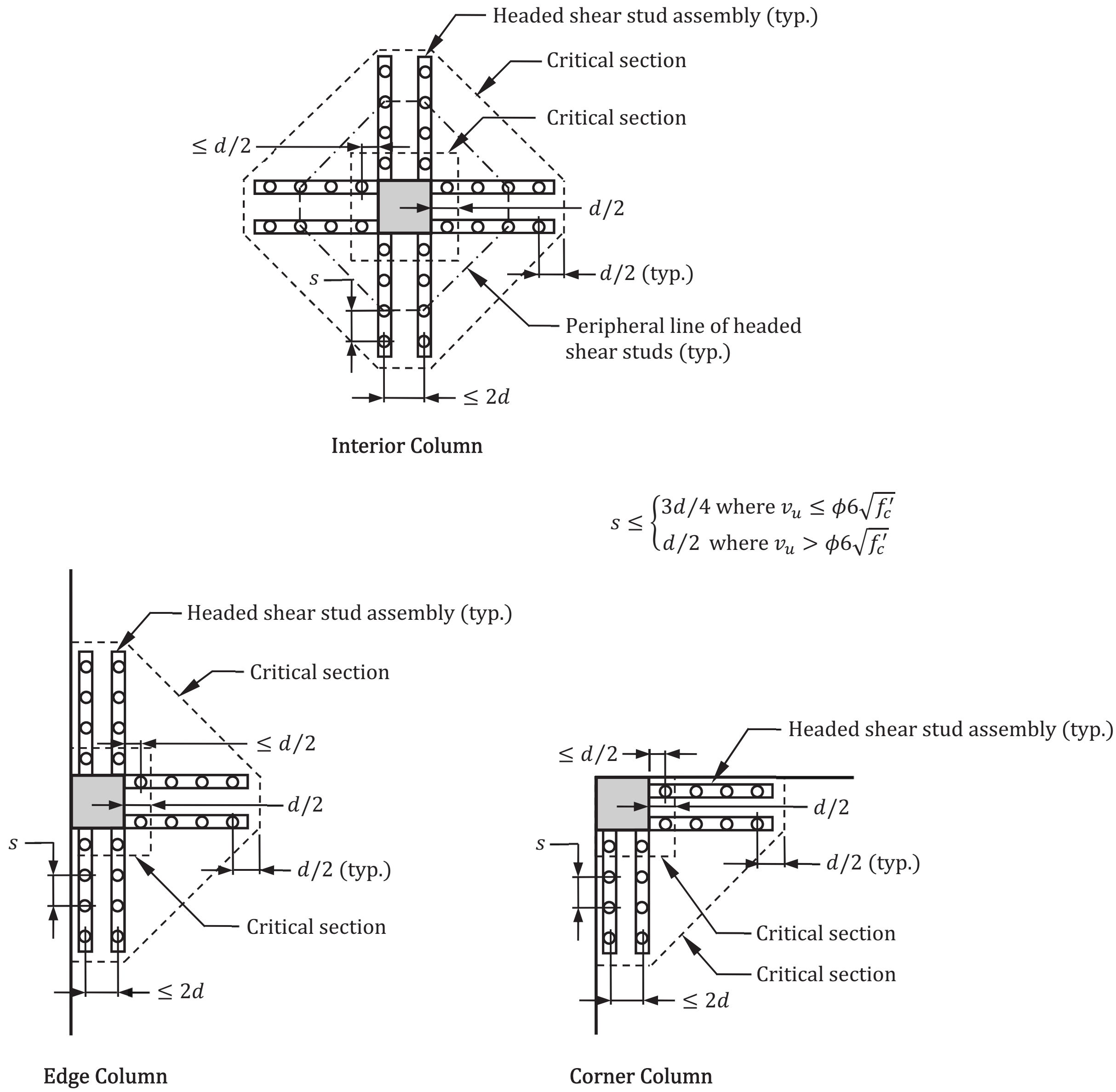


Figure 5.20 Critical sections for two-way shear in slabs reinforced with headed shear stud reinforcement.

Critical sections for two-way shear in slabs reinforced with stirrups or headed shear stud reinforcement are defined in ACI 22.6.4.2 (ACI 8.4.4.1.2). In addition to the critical section located a distance $d/2$ from the edges or corners of the columns, concentrated loads, or reaction areas, a critical section with a perimeter b_o located $d/2$ beyond the outermost peripheral line of shear reinforcement must also be considered. In order to minimize b_o, the shape of this critical section must be a polygon. The locations of the critical sections at interior, edge, and corner columns for slabs reinforced with stirrups are given in Figure 5.19 [also see ACI Figures R22.6.4.2(a)-(c)]. Similarly, the locations of the critical sections for slabs reinforced with headed shear stud reinforcement are given in Figure 5.20 (also see ACI Figure R8.7.7).

As discussed in Section 5.3.2 of this publication, $\gamma_v M_{sc}$ is the fraction of the factored slab moment resisted by the column transferred by eccentricity of shear where $\gamma_v = 1 - \gamma_f$ [ACI Equation (8.4.4.2.2)] and the factor γ_f is calculated by Equation (5.9), which depends on the dimensions of the critical section for shear. The moment $\gamma_v M_{sc}$ is applied to the centroid of the critical section (ACI 8.4.4.2.2) and produces factored shear stresses assumed to vary linearly about the centroid of the critical section in accordance with ACI 8.4.4.2.1 (ACI 8.4.4.2.3).

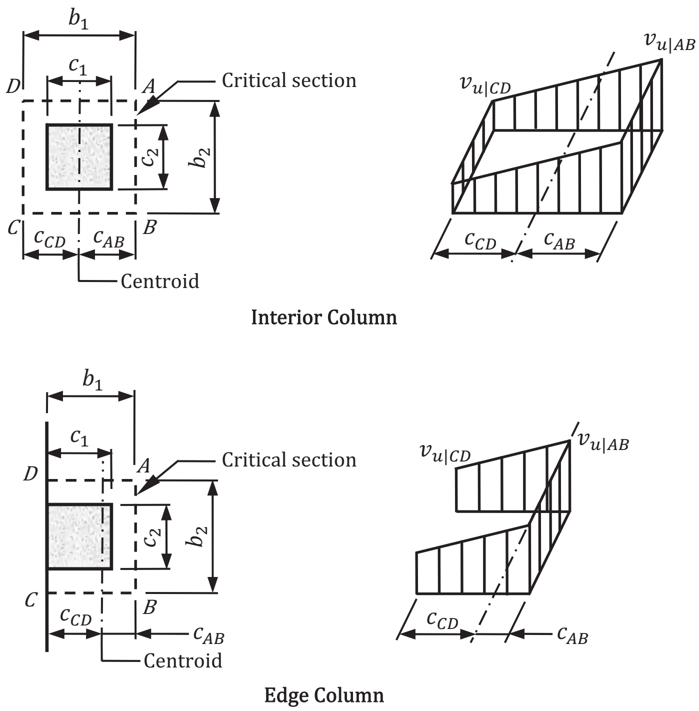


Figure 5.21 Assumed distribution of shear stresses due to direct factored shear and eccentricity of shear.

The assumed distributions of shear stresses due to the combination of direct factored shear, V_u, and eccentricity of shear, $\gamma_v M_{sc}$, on the faces of the critical section for an interior column and an edge column bending perpendicular to the edge are illustrated in Figure 5.21.

The total factored shear stresses on faces *AB* and *CD* of the critical section can be determined by the following equations:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c}$$

$$v_{u|CD} = \frac{V_u}{A_c} - \frac{\gamma_v M_{sc} c_{CD}}{J_c} \tag{5.14}$$

where A_c = area of concrete section resisting shear transfer $= b_o d$

b_o = perimeter of critical section for two-way shear

c_{AB} = distance from centroid of critical section to face *AB*

c_{CD} = distance from centroid of critical section to face *CD*

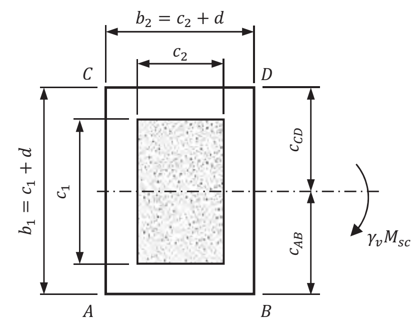
J_c = property of critical section analogous to the polar moment of inertia

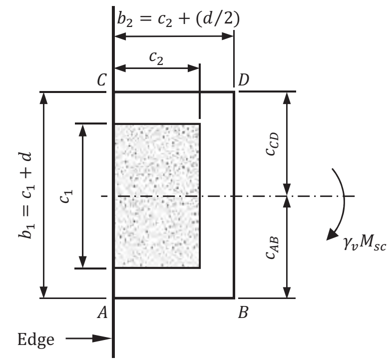
Section Properties of Critical Sections

Section properties of the critical section for two-way shear for rectangular columns, which are applicable to critical sections located a distance $d/2$ from the faces of the column, are given in Table 5.11. The critical shear perimeters for the edge and corner columns are based on the columns being flush with the slab edges (that is, no slab cantilevers).

Appendix D in Reference 10 contains tabulated values of the critical section properties for common slab thicknesses and square column sizes for each of the four cases in Table 5.11.

Table 5.11 Section Properties of the Critical Section for Rectangular Columns

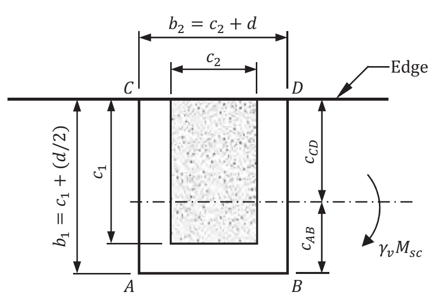


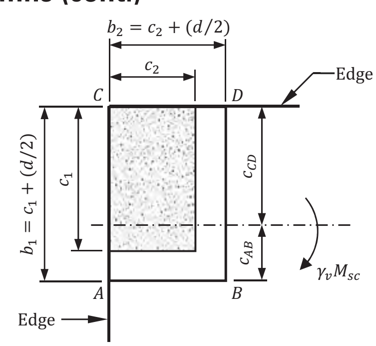


	Case	
Section Property	**1**	**2**
A_c	$2(b_1 + b_2)d$	$(b_1 + 2b_2)d$
c_{AB}	$b_1 / 2$	$b_1 / 2$
c_{CD}	$b_1 / 2$	$b_1 / 2$
J_c / c_{AB}	$\dfrac{b_1 d(b_1 + 3b_2) + d^3}{3}$	$\dfrac{b_1 d(b_1 + 6b_2) + d^3}{6}$
J_c / c_{CD}	$\dfrac{b_1 d(b_1 + 3b_2) + d^3}{3}$	$\dfrac{b_1 d(b_1 + 6b_2) + d^3}{6}$

Case 1: Interior rectangular column
Case 2: Edge rectangular column bending parallel to the edge

Table 5.11 Section Properties of the Critical Section for Rectangular Columns (cont.)



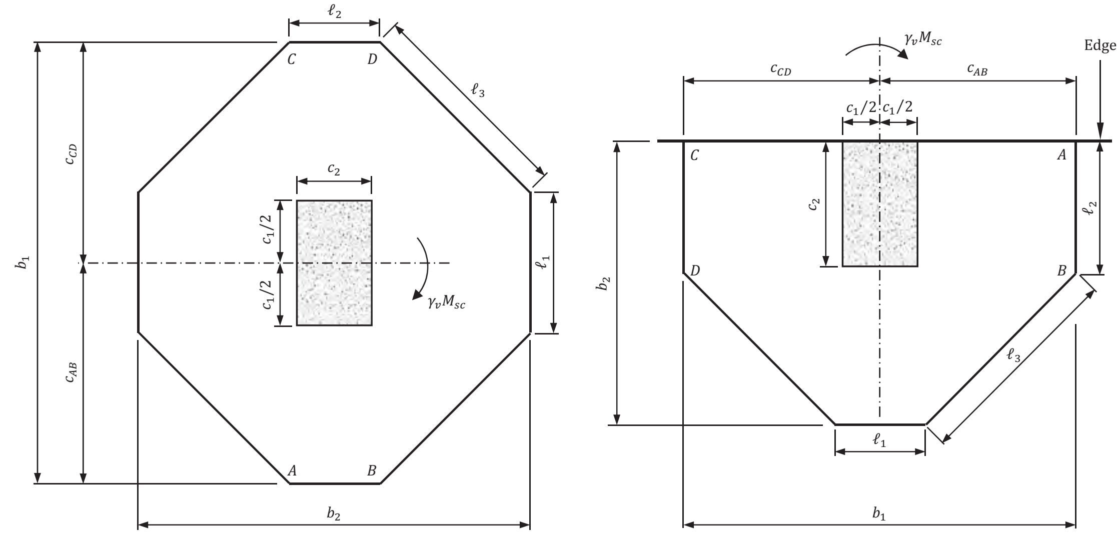


	Case	
Section Property	**3**	**4**
A_c	$(2b_1 + b_2)d$	$(b_1 + b_2)d$
c_{AB}	$\dfrac{b_1^2}{2b_1 + b_2}$	$\dfrac{b_1^2}{2(b_1 + b_2)}$
c_{CD}	$\dfrac{b_1(b_1 + b_2)}{2b_1 + b_2}$	$\dfrac{b_1(b_1 + 2b_2)}{2(b_1 + b_2)}$
J_c / c_{AB}	$\dfrac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1}$	$\dfrac{b_1^2 d(b_1 + 4b_2) + d^3(b_1 + b_2)}{6b_1}$
J_c / c_{CD}	$\dfrac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6(b_1 + b_2)}$	$\dfrac{b_1^2 d(b_1 + 4b_2) + d^3(b_1 + b_2)}{6(b_1 + 2b_2)}$

Case 3: Edge rectangular column bending perpendicular to the edge
Case 4: Corner rectangular column bending perpendicular to the edge

Section properties of polygon-shaped critical sections, like those illustrated in Figures 5.19 and 5.20 for two-way slabs with closed stirrups and headed shear stud reinforcement, respectively, are given in Table 5.12.

Table 5.12 Section Properties of Polygon-Shaped Critical Sections



	Case	
Section Property	**1**	**2**
A_c	$2d(\ell_1 + \ell_2 + 2\ell_3)$	$2d\left(\frac{\ell_1}{2} + \ell_2 + \ell_3\right)$
c_{AB}	$b_1 / 2$	$b_1 / 2$
c_{CD}	$b_1 / 2$	$b_1 / 2$
J_c / c_{AB}	$2d\left[\frac{\ell_1^3}{6b_1} + \frac{\ell_2 b_1}{2} + \frac{\ell_3}{3}\left(\frac{\ell_1^2}{b_1} + \ell_1 + b_1\right)\right]$	$2d\left[\frac{\ell_1^3}{12b_1} + \frac{\ell_2 b_1}{2} + \frac{\ell_3}{6}\left(\frac{\ell_1^2}{b_1} + \ell_1 + b_1\right)\right]$
J_c / c_{CD}	$2d\left[\frac{\ell_1^3}{6b_1} + \frac{\ell_2 b_1}{2} + \frac{\ell_3}{3}\left(\frac{\ell_1^2}{b_1} + \ell_1 + b_1\right)\right]$	$2d\left[\frac{\ell_1^3}{12b_1} + \frac{\ell_2 b_1}{2} + \frac{\ell_3}{6}\left(\frac{\ell_1^2}{b_1} + \ell_1 + b_1\right)\right]$

Case 1: Interior critical section with a rectangular column
Case 2: Edge critical section with a rectangular column bending parallel to the edge

(table continued on next page)

The equations in Table 5.12 are determined using the following general equations, which are suitable for critical sections with irregular geometries, such as polygon-shaped critical sections or critical sections affected by slab openings (Reference 12):

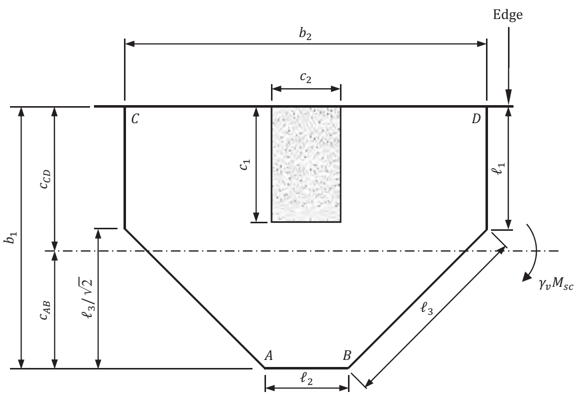
$$A_c = d\sum \ell \tag{5.15}$$

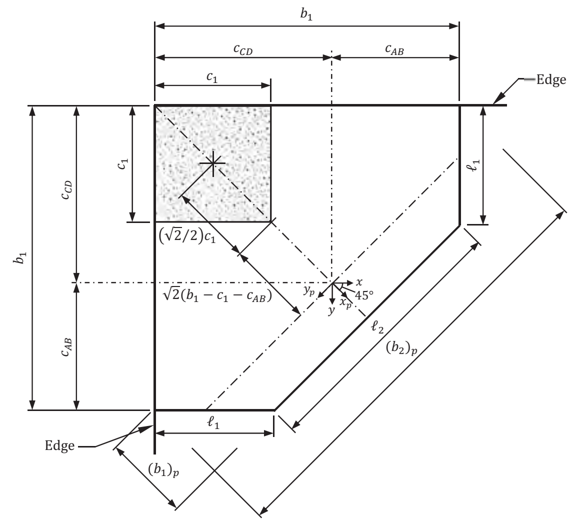
$$(J_c)_x = d\sum \frac{\ell}{3}(y_i^2 + y_i y_j + y_j^2)$$

(5.16)

$$(J_c)_y = d\sum \frac{\ell}{3}(x_i^2 + x_i x_j + x_j^2)$$

Table 5.12 Section Properties of Polygon-Shaped Critical Sections (cont.)





	Case	
Section Property	**3**	**4**
A_c	$2d\left(\ell_1 + \frac{\ell_2}{2} + \ell_3\right)$	$d(2\ell_1 + \ell_2)$
c_{AB}	$\dfrac{\sqrt{2}\ell_1^2 + 2\ell_1\ell_3 + \ell_3^2}{2\sqrt{2}\left(\ell_1 + \frac{\ell_2}{2} + \ell_3\right)}$	$\dfrac{\sqrt{2}\ell_1^2 + 2\ell_1\ell_2 + \ell_2^2}{2\sqrt{2}(2\ell_1 + \ell_2)}$
c_{CD}	$\dfrac{\ell_3}{\sqrt{2}} + \ell_1 - c_{AB}$	$\dfrac{\ell_2}{\sqrt{2}} + \ell_1 - c_{AB}$
J_c	$2d\left\{\ell_1\left[c_{CD}^2\left(1 - \frac{\ell_1}{c_{CD}}\right) + \frac{\ell_1^2}{3}\right] + \frac{\ell_2 c_{AB}^2}{2} + \frac{\ell_3}{3}\left[(c_{CD} - \ell_1)^2\left(1 - \frac{c_{AB}}{c_{CD} - \ell_1}\right) + c_{AB}^2\right]\right\}$	$\frac{d}{3}\left\{\ell_1\left[\left(\ell_1 + \frac{\ell_2}{\sqrt{2}} - c_{AB}\right)^2 + \left(\ell_1 + \frac{\ell_2}{\sqrt{2}} - c_{AB}\right)\left(\frac{\ell_2}{\sqrt{2}} - c_{AB}\right) + \left(\frac{\ell_2}{\sqrt{2}} - c_{AB}\right)^2\right] + \ell_2\left[3c_{AB}^2\left(1 - \frac{\ell_2}{\sqrt{2}c_{AB}}\right) + \frac{\ell_2^2}{2}\right] + 3\ell_1 c_{AB}^2\right\}$

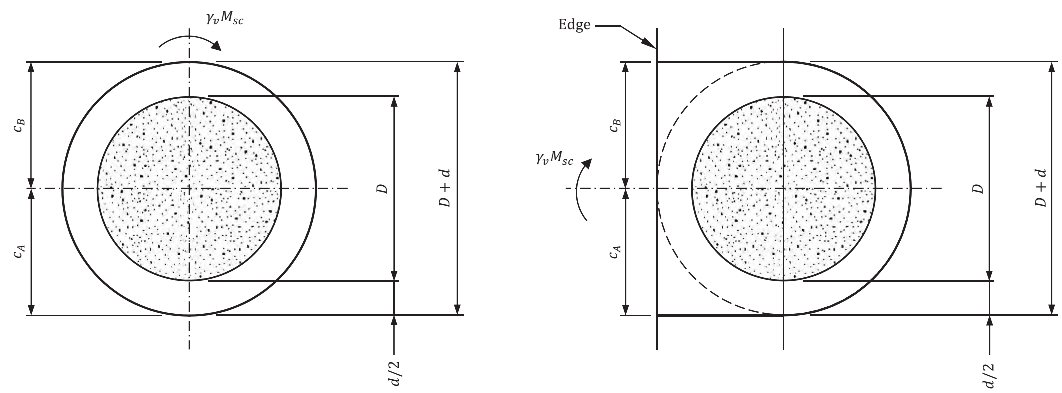
Case 3: Edge critical section with a rectangular column bending perpendicular to the edge
Case 4: Corner critical section with a square column bending perpendicular to the edge

The summation occurs over the number of straight segments in the critical section where each segment has a length equal to ℓ. The terms $y_i,\ y_j,\ x_i,$ and x_j are the coordinates of the ends of each straight segment along the y and x axes, respectively, with respect to the centroid of the critical section. The section properties J_c in Table 5.12 are conceptually different than those in Table 5.11; determining J_c for irregular critical sections using the methodology to determine J_c for regularly-shaped critical sections is very difficult. The section properties determined by Equations (5.15) and (5.16) for any shaped critical section are conservative (that is, they are smaller than those determined by the methodology used for the regularly-shaped critical sections in Table 5.11). Calculations for polygon-shaped critical sections for two-way slabs with closed stirrups and headed shear stud reinforcement are given in Examples 5.17 and 5.18, respectively.

Information on how to determine the critical section properties for rectangular corner columns is given in Reference 12. Also included is a method to calculate the critical section properties about the principal axes of the section. The principal axes for the square column in Table 5.12 are denoted by a subscript "*p*". For square columns, the principal axes occur along the diagonal of the column and at an angle oriented 90-degree from the diagonal.

In lieu of using an equivalent square column in accordance with ACI 22.6.4.1.2, section properties of the critical section for two-way shear for circular columns can be determined using the equations in Table 5.13. For edge and corner circular columns, the critical section consists of a partial circular section plus two straight tangential segments perpendicular to and extending to the edge(s) of the slab. It is assumed the column is set back a distance equal to $d\,/\,2$ from the edge(s), which provides conservative section properties for columns with larger setbacks. Similar section properties can be derived for circular columns at the edge(s) of the slab.

Table 5.13 Section Properties of the Critical Section for Circular Columns

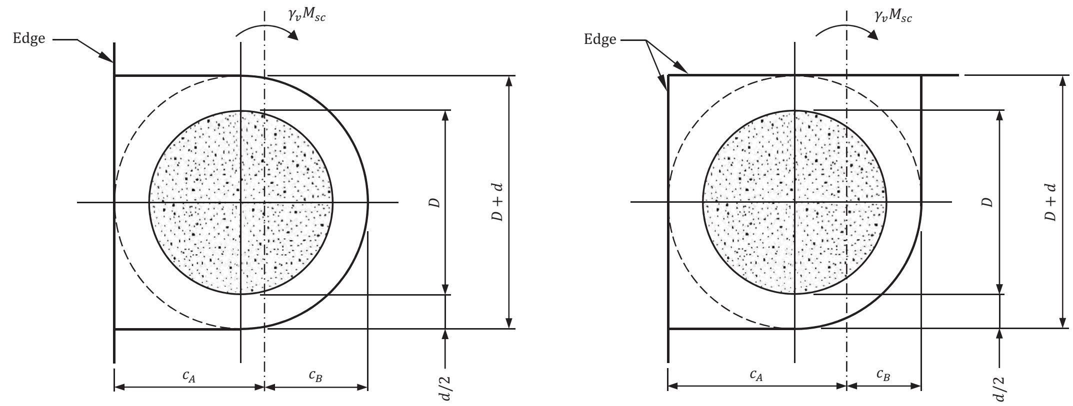


	Case	
Section Property	**1**	**2**
A_c	$\pi(D+d)d$	$(2+\pi)\left(\frac{D+d}{2}\right)d$
c_A	$\frac{D+d}{2}$	$\frac{D+d}{2}$
c_B	$\frac{D+d}{2}$	$\frac{D+d}{2}$
$J_c\,/\,c_A$	$\pi d\left(\frac{D+d}{2}\right)^2+\frac{d^3}{3}$	$\left(2+\frac{\pi}{2}\right)d\left(\frac{D+d}{2}\right)^2+\frac{d^3}{6}$
$J_c\,/\,c_B$	$\pi d\left(\frac{D+d}{2}\right)^2+\frac{d^3}{3}$	$\left(2+\frac{\pi}{2}\right)d\left(\frac{D+d}{2}\right)^2+\frac{d^3}{6}$

Case 1: Interior circular column
Case 2: Edge circular column bending parallel to the edge

(table continued on next page)

Table 5.13 Section Properties of the Critical Section for Circular Columns (cont.)



	Case	
Section Property	**3**	**4**
A_c	$(2+\pi)\left(\dfrac{D+d}{2}\right)d$	$\left(2+\dfrac{\pi}{2}\right)\left(\dfrac{D+d}{2}\right)d$
c_A	$\left(1+\dfrac{1}{2+\pi}\right)\dfrac{D+d}{2}$	$\left(1+\dfrac{1.5}{2+\dfrac{\pi}{2}}\right)\dfrac{D+d}{2}$
c_B	$\left(1-\dfrac{1}{2+\pi}\right)\dfrac{D+d}{2}$	$\left(1-\dfrac{1.5}{2+\dfrac{\pi}{2}}\right)\dfrac{D+d}{2}$
J_c/c_A	$\left(\dfrac{d}{1+\dfrac{1}{2+\pi}}\right)\left[2.043\left(\dfrac{D+d}{2}\right)^2+\dfrac{d^2}{3}\right]$	$\left(\dfrac{d}{1+\dfrac{1.5}{2+\dfrac{\pi}{2}}}\right)\left[1.472\left(\dfrac{D+d}{2}\right)^2+\dfrac{d^2}{6}\right]$
J_c / c_B	$\left(\dfrac{d}{1-\dfrac{1}{2+\pi}}\right)\left[2.043\left(\dfrac{D+d}{2}\right)^2+\dfrac{d^2}{3}\right]$	$\left(\dfrac{d}{1-\dfrac{1.5}{2+\dfrac{\pi}{2}}}\right)\left[1.472\left(\dfrac{D+d}{2}\right)^2+\dfrac{d^2}{6}\right]$

Case 3: Edge circular column bending perpendicular to the edge
Case 4: Corner circular column bending perpendicular to the edge

5.3.4 Direct Design Method

Overview

The Direct Design Method (DDM) is an approximate analysis method included in the 1971 through 2014 editions of ACI 318. The information given below is primarily from Section 8.10 of ACI 318-14. According to ACI 6.2.4.1(a), this method is permitted to be used to analyze two-way slab systems for gravity loads.

The DDM method gives reasonably conservative values of bending moments at the critical sections for two-way slabs subjected to gravity loads meeting the limitations of the method, which are summarized in Figure 5.22.

The fundamental concept of the DDM is that the slab is analyzed by dividing it into design strips (see Figure 5.8). Analysis of the slab system is performed in each direction independently, and 100 percent of the total gravity load must be carried in each direction.

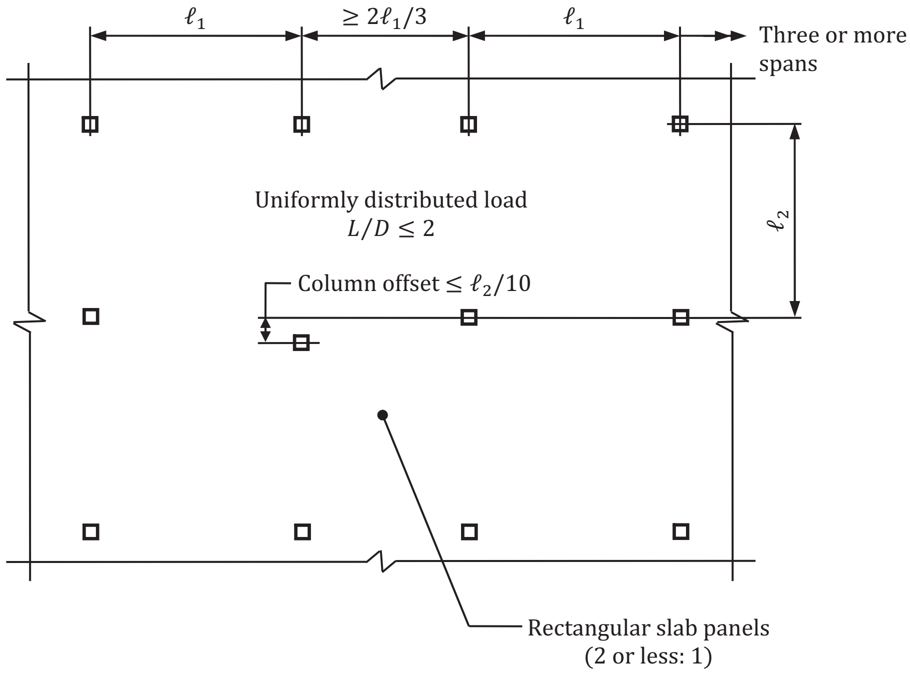


Figure 5.22 Limitations of the Direct Design Method.

Determination of Factored Bending Moments in a Design Strip

The three fundamental steps in the DDM used to determine factored bending moments in a design strip are as follows:

- Step 1: Determine the total factored static moment, M_o, in each span.

  The total factored static moment, M_o, in a span is determined by the following equation:

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} \tag{5.17}$$

where q_u = total factored uniformly distributed gravity load on the slab

ℓ_n = clear span in the direction of analysis measured from face to face of columns, capitals, brackets, or walls, which must be taken greater than or equal to $0.65\ell_1$ (see Figure 5.23 for the definition of clear span for rectangular and nonrectangular columns; for the purpose of determining ℓ_n, nonrectangular columns are converted to square columns of the same cross-sectional area)

ℓ_2 = length of the span in the direction perpendicular to ℓ_1, measured center-to-center of supports (see Figure 5.8)

Where ℓ_2 is different on each side of the design strip, an average of these transverse span lengths must be used in Equation (5.17). In the case of spans adjacent and parallel to a slab edge, ℓ_2 to be used in Equation (5.17) is equal to the distance from the edge of the slab to the centerline of the panel.

For varying span lengths in the direction of analysis, M_o is determined for each clear span length; however, M_o can be calculated based on the longest clear span and that value can conservatively be used for all spans in that design strip.

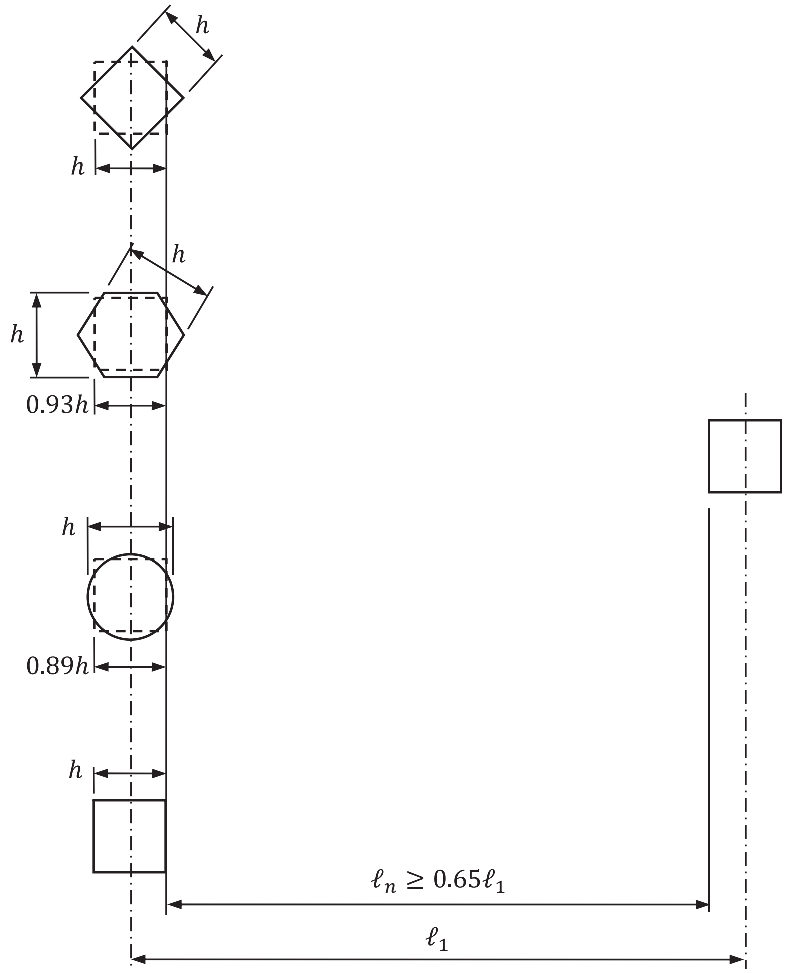


Figure 5.23 Definition of clear span length, ℓ_n

- Step 2: Distribute M_o into negative and positive bending moments in each span.

  Coefficients are used to distribute M_o into positive and negative bending moments at the critical sections within each span (that is, near or at midspan and the faces of the supports, respectively).

  A summary of the bending moment coefficients for an end span are given in Table 5.14 for the typical case of monolithic construction. A fully restrained exterior edge corresponds to a slab integrally constructed with a reinforced concrete wall that has a flexural stiffness much greater than that of the slab (that is, a relatively small amount of rotation occurs at the slab-wall connection).

Table 5.14 Bending Moment Coefficients for an End Span

Location	Two-way beam-supported slab	Flat Plate and Flat Slab		Exterior Edge Fully Restrained
		Without Edge Beams	With Edge Beams	
Exterior negative	0.16	0.26	0.30	0.65
Positive	0.57	0.52	0.50	0.35
Interior negative	0.70	0.70	0.70	0.65

For interior spans, the coefficients are equal to 0.65 and 0.35 at the negative and positive locations in the span, respectively.

The total factored bending moments in the design strips at the critical locations are obtained by multiplying M_o by the coefficients in Table 5.14 for an end span and by the applicable coefficients noted above for interior spans. For example, the total factored design strip bending moment at the exterior support of a flat plate or flat slab without edge beams is equal to $0.26M_o$.

- Step 3: Distribute the total negative and positive bending moments in the design strip to the column strips and middle strips.

  Once the total factored design positive and negative bending moments have been determined in Step 2, these moments are distributed to the column strips and middle strips using percentages based on studies of linearly elastic slabs with different beam stiffnesses. The percentages depend on the aspect ratio of the panel (ℓ_2 / ℓ_1) and the presence of column-line beams in one or both directions.

Distribution to Column Strips

Equations to determine the percentages at the critical sections in column strips are given in Table 5.15.

Table 5.15 Percentages of Factored Bending Moments in Column Strips

Location	Percentage*
Exterior support	$100 - 10\beta_t + 12\beta_t\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right)$
Positive	$60 + 30\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1.5 - \frac{\ell_2}{\ell_1}\right)$
Interior support	$75 + 30\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right)$

*Where $\beta_t > 2.5,$ use $\beta_t = 2.5;$ where $\alpha_{f1}\ell_2 / \ell_1 > 1.0,$ use $\alpha_{f1}\ell_2 / \ell_1 = 1.0$

In these equations, α_{f1} is the ratio of the flexural stiffness of the beam section to the flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels for the beams in the direction of analysis, and is calculated by Equation (5.2).

The term β_t is the ratio of the torsional stiffness of an edge beam section to the flexural stiffness of a slab width equal to the span length of the beam, measured center-to center of supports:

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \tag{5.18}$$

In this equation, E_{cb} and E_{cs} are the moduli of elasticity of the beam and slab concrete, respectively, which are typically equal for monolithic construction; I_s is the moment of inertia of the slab section using the full width of the slab tributary to the beam, that is, $I_s = \ell_2 h^3 / 12$; and C is the cross-sectional constant determined by dividing the beam section into its component rectangles, each having a smaller dimension x and a larger dimension y, and by summing the contribution of each rectangle:

$$C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3} \tag{5.19}$$

The subdivision is to be done in such a way as to maximize C. Equations for the calculation of C for an edge (spandrel) beam are given in Figure 5.24. The larger of C_A and C_B is to be used in Equation (5.18). Note that the effective width of slab permitted to be included in the edge beam is given in Figure 5.2.

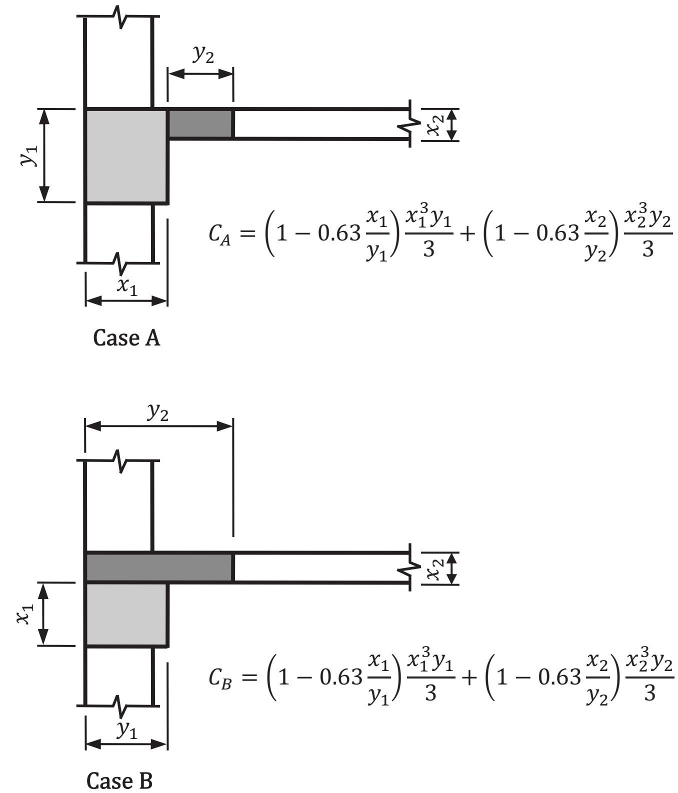


Figure 5.24 Calculation of cross-sectional constant C.

Values of C for common slab thicknesses and edge beam sizes are given in Appendix C of Reference 10.

In cases where β_t calculated by Equation (5.18) is greater than 2.5, β_t must be taken as 2.5 when determining the percentage of the factored bending moment at the exterior support by the equation in Table 5.15.

For slabs without beams between supports $(\alpha_{f1} = 0)$ and without edge beams $(\beta_t = 0)$, the percentages to use for distribution of total negative moments to column strips are 100 and 75 for exterior and interior supports, respectively. It is evident that all the exterior negative factored moment is assigned to the column strip unless an edge beam is provided with a relatively large torsional stiffness compared to the flexural stiffness of the slab. Similarly, the distribution of total positive moment to column strips is 60 percent.

Walls along column lines in the direction of analysis can be regarded as stiff beams with $\alpha_{f1}\ell_2 / \ell_1 > 1.0$, which means $\alpha_{f1}\ell_2 / \ell_1 = 1.0$ must be used in the equations in Table 5.15. If an exterior support is a reinforced concrete wall perpendicular to the direction of analysis and monolithic with the slab, it can be assumed the wall provides significant torsional resistance, that is, β_t can be taken as 2.5.

An exception to the percentages in Table 5.15 occurs where columns or walls have a relatively large width perpendicular to the direction of analysis. In particular, where the transverse width of a column or wall extends a distance greater than or equal to 75 percent the width of the design strip, ℓ_2, the negative factored bending moments are to be uniformly distributed across ℓ_2.

Distribution to Column-line Beams

For relatively stiff column-line beams $(\alpha_{f1}\ell_2 / \ell_1 \geq 1.0)$, it is assumed the beams attract a significant portion of the bending moments at the critical sections in the column strip. In such cases, the beams must be designed to resist 85 percent of the factored column strip moments based on the applicable percentages determined by the equations in Table 5.15.

Where $\alpha_{f1}\ell_2 / \ell_1 < 1.0$, the percentage resisted by the beam can be determined by linear interpolation between 85 and zero percent, which correspond to $\alpha_{f1}\ell_2 / \ell_1 = 1.0$ and $\alpha_{f1}\ell_2 / \ell_1 = 0$, respectively.

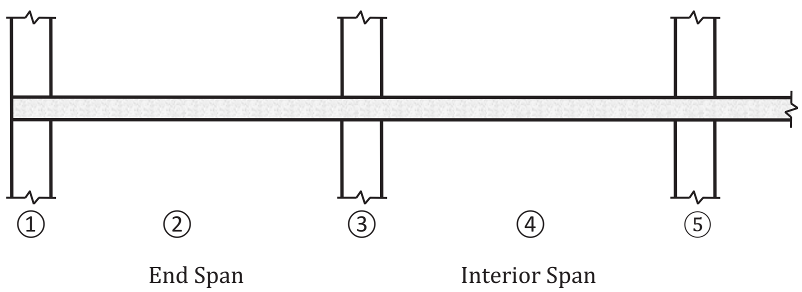
In addition to the factored bending moments from the column strip, column-line beams must also be designed to resist the effects from any loads applied directly to the beam. Loads located on the slab outside of the beam width must be distributed accordingly between the slab and beam.

Distribution to Middle Strips

The portions of the total factored design negative and positive bending moments that are not assigned to the column strip are assigned to the half middle strips in the design strip. A middle strip adjacent and parallel to a panel edge and supported by a wall must be designed for two times the factored bending moment assigned to the half middle strip corresponding to the first row of interior supports.

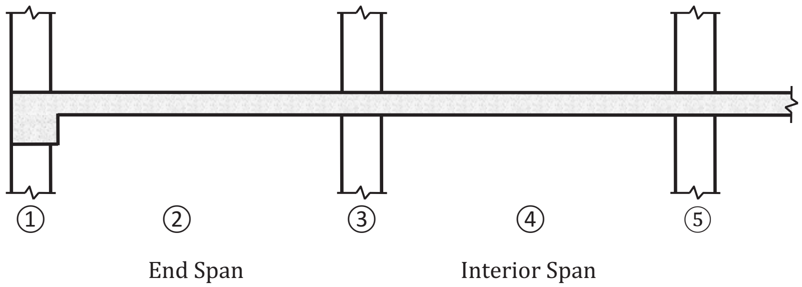
Design moment coefficients for total moments, column strip moments, and middle strip moments at the critical sections in (1) flat plates or flat slabs without edge beams, (2) flat plates or flat slabs with edge beams, (3) flat plates or flat slabs with an end span integral with a reinforced concrete wall, and (4) two-way beam-supported slabs are given in Tables 5.16, 5.17, 5.18, and 5.19, respectively. These coefficients are determined using the applicable percentages discussed previously.

Table 5.16 Design Moment Coefficients for Flat Plates or Flat Slabs Without Edge Beams



Slab Moments	End Span			Interior Span	
	①	②	③	④	⑤
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Total Moment	$0.26M_o$	$0.52M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.26M_o$	$0.31M_o$	$0.53M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	0	$0.21M_o$	$0.17M_o$	$0.14M_o$	$0.16M_o$

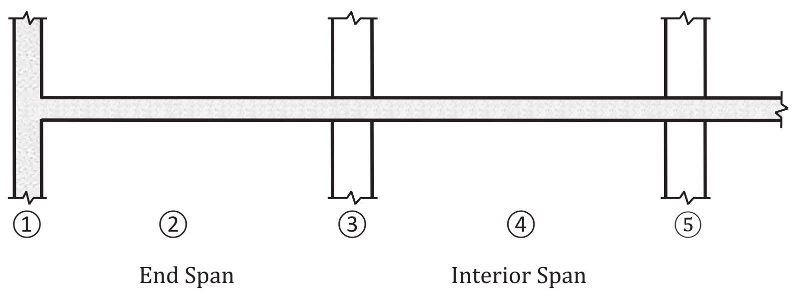
Table 5.17 Design Moment Coefficients for Flat Plates or Flat Slabs With Edge Beams*



Slab Moments	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Total Moment	$0.30M_o$	$0.50M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.23M_o$	$0.30M_o$	$0.53M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	$0.07M_o$	$0.20M_o$	$0.17M_o$	$0.14M_o$	$0.16M_o$

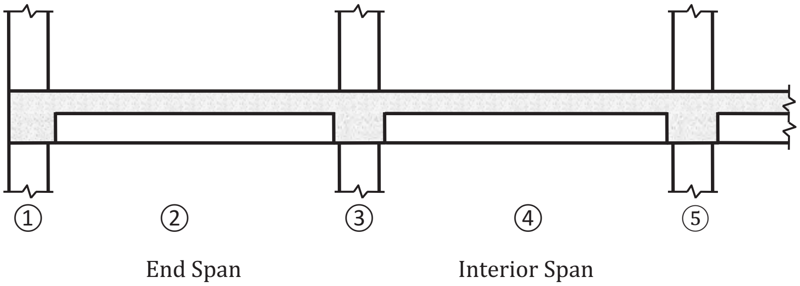
**Applicable to edge beams with* $\beta_t \geq 2.5$. *For edge beams with* $\beta_t < 2.5$, *exterior negative column strip moment is equal to* $(0.30 - 0.03\beta_t)M_o$.

Table 5.18 Design Moment Coefficients for Flat Plates or Flat Slabs With End Span Integral with a Reinforced Concrete Wall



Slab Moments	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Total Moment	$0.65M_o$	$0.35M_o$	$0.65M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.49M_o$	$0.21M_o$	$0.49M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	$0.16M_o$	$0.14M_o$	$0.16M_o$	$0.14M_o$	$0.16M_o$

Table 5.19 Design Moment Coefficients for Two-Way Beam-Supported Slabs



Span Ratio, ℓ_2 / ℓ_1	Slab and Beam Moments		End Span ① Exterior Negative	End Span ② Positive	End Span ③ First Interior Negative	Interior Span ④ Positive	Interior Span ⑤ Interior Negative
	Total Moment		$0.16M_o$	$0.57M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
0.5	Column Strip	Beam*	$0.12M_o$	$0.43M_o$	$0.54M_o$	$0.27M_o$	$0.50M_o$
		Slab	$0.02M_o$	$0.08M_o$	$0.09M_o$	$0.05M_o$	$0.09M_o$
	Middle Strip		$0.02M_o$	$0.06M_o$	$0.07M_o$	$0.03M_o$	$0.06M_o$
1.0	Column Strip	Beam*	$0.10M_o$	$0.37M_o$	$0.45M_o$	$0.22M_o$	$0.42M_o$
		Slab	$0.02M_o$	$0.06M_o$	$0.08M_o$	$0.04M_o$	$0.07M_o$
	Middle Strip		$0.04M_o$	$0.14M_o$	$0.17M_o$	$0.09M_o$	$0.16M_o$
2.0	Column Strip	Beam*	$0.06M_o$	$0.22M_o$	$0.27M_o$	$0.14M_o$	$0.25M_o$
		Slab	$0.01M_o$	$0.04M_o$	$0.05M_o$	$0.02M_o$	$0.04M_o$
	Middle Strip		$0.09M_o$	$0.31M_o$	$0.38M_o$	$0.19M_o$	$0.36M_o$

**Applicable to (1) edge beams with $\beta_t \geq 2.5$ and (2) column-line beams with $\alpha_{f1}\ell_2 / \ell_1 \geq 1.0$.*

Determination of Factored Moments in Columns and Walls

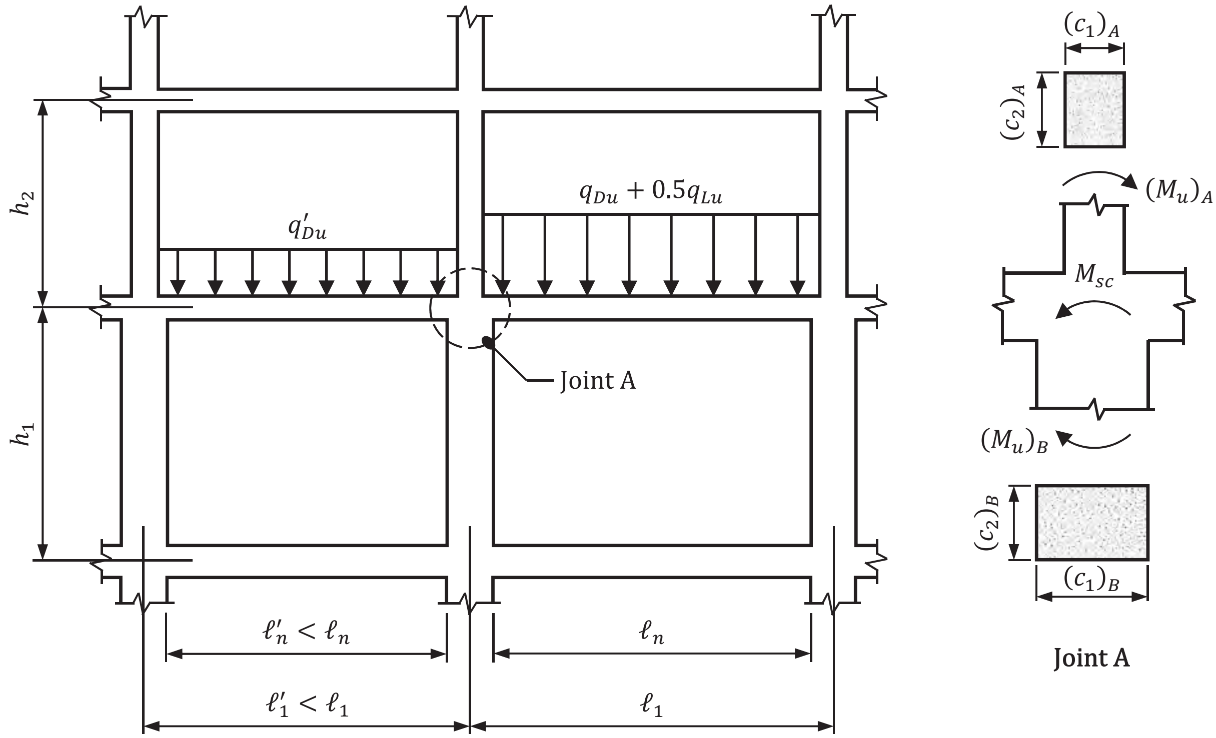
Columns and walls supporting a two-way slab system must be designed to resist the appropriate negative factored bending moments transferred from the slab. In lieu of a more exact analysis, the following equation can be used to determine the moment, M_{sc}, transferred at an interior support due to factored gravity loads for the case of two adjoining spans with one span longer than the other (see Figure 5.25):

$$M_{sc} = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q'_{Du}\ell'_2(\ell'_n)^2] \tag{5.20}$$

In this equation, q_{Du} and q_{Lu} are the uniformly distributed factored dead and live loads, respectively, on the longer span; q'_{Du} is the uniformly distributed factored dead load on the shorter span; ℓ_n and ℓ'_n are the longer and shorter clear span lengths, respectively; and, ℓ_2 and ℓ'_2 are the span lengths perpendicular to ℓ_1 and ℓ'_1, respectively.

Where $q_{Du} = q'_{Du,}$ $\ell_2 = \ell'_2$, and $\ell_n = \ell'_n$, Equation (5.20) reduces to the following:

$$M_{sc} = 0.035q_{Lu}\ell_2\ell_n^2 \tag{5.21}$$



$$M_{sc} = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q'_{Du}\ell'_2(\ell'_n)^2]$$

$$(k)_A = \frac{(c_2)_A(c_1)_A^3}{12h_2}$$

$$(k)_B = \frac{(c_2)_B(c_1)_B^3}{12h_1}$$

$$(M_u)_A = \frac{(k)_A}{(k)_A + (k)_B} M_{sc}$$

$$(M_u)_B = \frac{(k)_B}{(k)_A + (k)_B} M_{sc}$$

Figure 5.25 Determination of factored moments at interior supporting members.

The moment M_{sc} determined by either Equation (5.20) or (5.21) is distributed to the interior supporting members above and below the slab in direct proportion to the members' stiffnesses, which, in general, can be determined by dividing the cross-sectional moment of inertia of the member in the direction of analysis by the length of the member (this is based on the assumption the predominant contribution to the total displacement is due to flexure; see Figure 5.25). Where the cross-sectional dimensions of the supporting members above and below the slab are the same, M_{sc} is transferred on the basis of the inverse of the member lengths (that is, the longer member resists a lesser amount of M_{sc} than the shorter member).

At an exterior support, the total negative bending moment from the slab is transferred directly to the support. Like interior supports, this moment is transferred to the members above and below the slab in direct proportion to their stiffnesses. For two-way shear design at edge columns, the factored gravity bending moment to be transferred between the slab and edge column bending perpendicular to the edge, M_{sc}, must be $0.30M_o$ when the DDM is used to determine design strip bending moments.

Determination of Factored Shear in Slabs with Beams

In addition to the applicable bending moments in Table 5.19, beams must be designed to resist applicable tributary factored shear forces due to the factored loads. The tributary areas for beams in a two-way beam-supported system are given in Figure 5.26; the gray tributary area applies to the interior beam that spans between columns A and B. Where $\alpha_{f1}\ell_2 / \ell_1 \geq 1.0$, beams must be designed to resist 100 percent of the factored shear forces caused by the factored loads on the tributary area; this means the shear forces in the slab around the column are equal to zero.

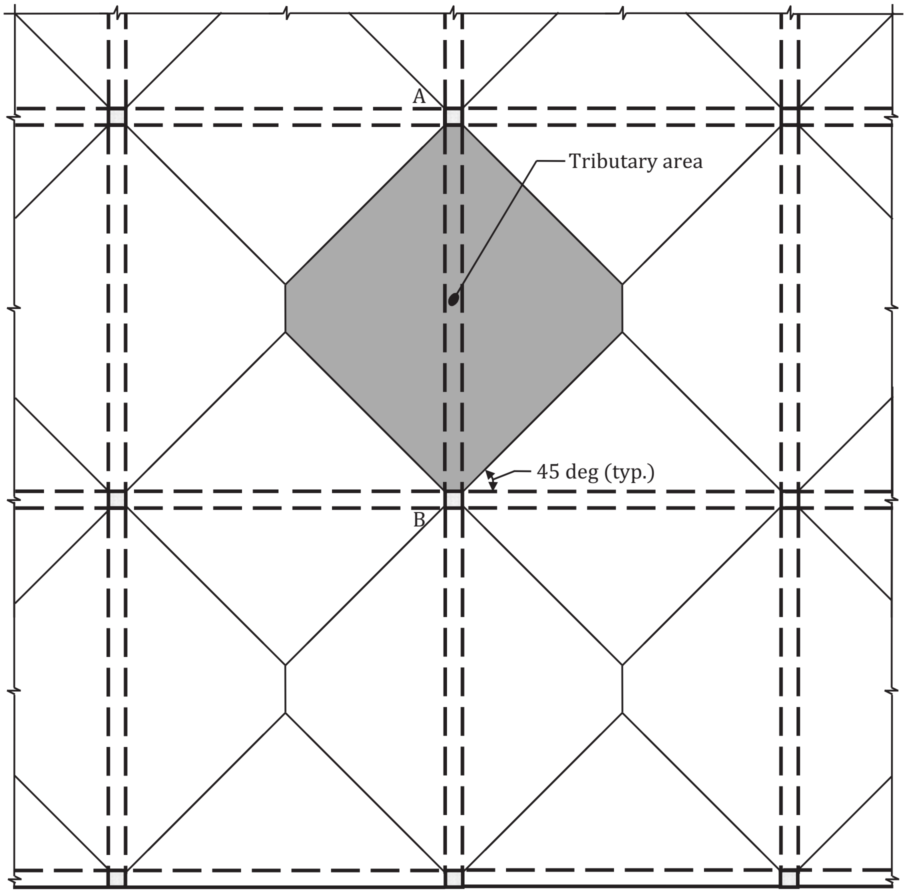


Figure 5.26 Tributary area for calculation of factored shear forces on an interior beam in a two-way beam-supported slab.

Where $\alpha_{f1}\ell_2 / \ell_1 < 1.0$, the shear forces resisted by the beam can be obtained by linear interpolation assuming the beam carries 100 percent of the load where $\alpha_{f1}\ell_2 / \ell_1 \geq 1.0$ (as noted above) and carries no load at $\alpha_{f1} = 0$. In cases where the beam carries less than 100 percent of the load, the shear forces not resisted by the beams must be resisted by the slab around the column. These shear forces and the applicable moments determined by the expressions in Table 5.19 cause shear stresses in the slab, which are calculated by Equation (5.14).

In addition to the shear forces noted above, column-line beams must also be designed to resist the factored shear forces due to any factored loads applied directly to the beam.

5.3.5 Lateral Loads

Effects from lateral loads on two-way systems can be determined using the analysis methods in ACI Chapter 6. Regardless of the analysis method, the effects of axial loads, cracking, and effects of load duration must be taken into consideration so drift caused by lateral loads is not underestimated. Where a linear elastic first-order analysis in accordance with ACI 6.6 is performed using factored loads, the section properties in ACI 6.6.3.1.1 and 6.6.3.1.2 are permitted to be used in lieu of performing a more rigorous analysis. In cases where service-level loads are applied to the structure, the moments of inertia given in ACI 6.6.3.2.2 are applicable. Similarly, where a linear elastic second-order analysis is performed in accordance with ACI 6.7 using factored loads, it is permitted to use the section properties calculated in accordance with ACI 6.6.3.1 (ACI 6.7.2.1.1); where service-level loads are used, the provisions of ACI 6.7.2.2.2 are applicable.

In two-way slab systems with column-line beams, the beams usually resist most or all of the lateral load effects because they are typically much stiffer than the slab.

For flat plate systems that are part of the seismic-force-resisting system (SFRS), the moment of inertia for the slab members must be determined by a model in substantial agreement with results of comprehensive tests and analysis; the moments of inertia of the other frame members must be in accordance with ACI 6.6.3.1.1 and 6.6.3.1.2 (ACI 6.6.3.1.3). Several models satisfying this requirement are referenced in ACI R6.6.3.1.3. To account for cracking, the slab moment of inertia is typically reduced between one-half and one-quarter of the uncracked moment of inertia. When determining drifts or second-order effects in columns, a lower-bound slab stiffness should be used in the analysis. In structures where slab-column frames interact with shear walls, a range of slab stiffnesses should be investigated in order to assess the importance of the interaction. Note that flat plates are not permitted to be part of the SFRS in buildings assigned to Seismic Design Category (SDC) D, E, or F (see ACI Chapter 18).

5.4 Design Strength

5.4.1 General

The following equations must be satisfied in a two-way slab system for each applicable factored load combination in ACI Table 5.3.1 (ACI 8.5.1.1):

$$\phi M_n \geq M_u \text{ at all sections along the span in each direction} \tag{5.22}$$

$$\phi M_n \geq \gamma_f M_{sc} \text{ within } b_{slab} \text{ (see Section 5.3.2 of this publication)} \tag{5.23}$$

$$\phi V_n \geq V_u \text{ at all sections along the span in each direction for one-way shear} \tag{5.24}$$

$$\phi v_n \geq v_u \text{ at the critical sections defined in ACI 8.4.4.1 for two-way shear} \tag{5.25}$$

Strength reduction factors, ϕ, are determined in accordance with ACI 21.2. Nonprestressed two-way slabs must be designed as tension-controlled in accordance with ACI Table 21.2.2 (ACI 8.3.3.1); thus, the net tensile strain in the extreme layer of the longitudinal tension reinforcement at nominal strength, ε_t, must be greater than or equal to $\varepsilon_{ty} + 0.003$ and $\phi = 0.90$ (ACI Table 21.2.2). The net tensile strain in the extreme layer of deformed longitudinal tension reinforcement corresponding to compression-controlled sections, ε_{ty}, is equal to f_y / E_s (ACI 21.2.2.1) where the modulus of elasticity of the reinforcement, E_s, is permitted to be taken as 29,000,000 psi regardless of the grade of the reinforcement (ACI 20.2.2.2). For shear, $\phi = 0.75$ (ACI Table 21.2.1).

Determination of nominal flexural and shear strengths are given in Sections 5.4.2 and 5.4.3, respectively.

5.4.2 Nominal Flexural Strength

The nominal flexural strength, M_n, of a rectangular section with one layer of tension reinforcement is determined in accordance with ACI 22.3, and is based on moment equilibrium of a rectangular section (see ACI 8.5.2.1 and Figure 5.27):

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \tag{5.26}$$

In this equation, A_s is the total area of the flexural reinforcement. The strain and stress distributions in Figure 5.27 are at a positive moment section and are equally applicable at negative moment sections.

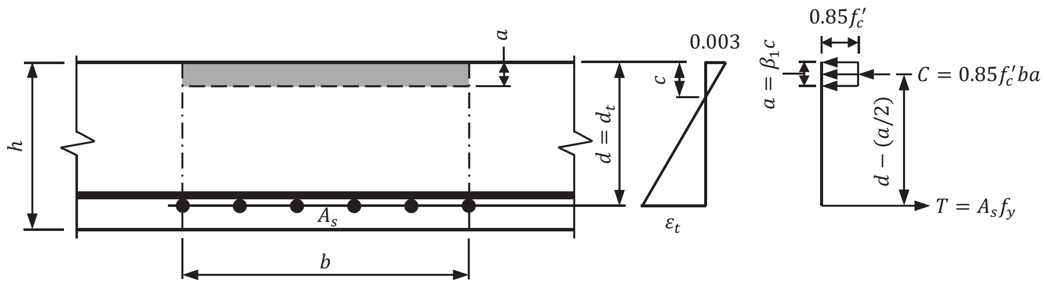
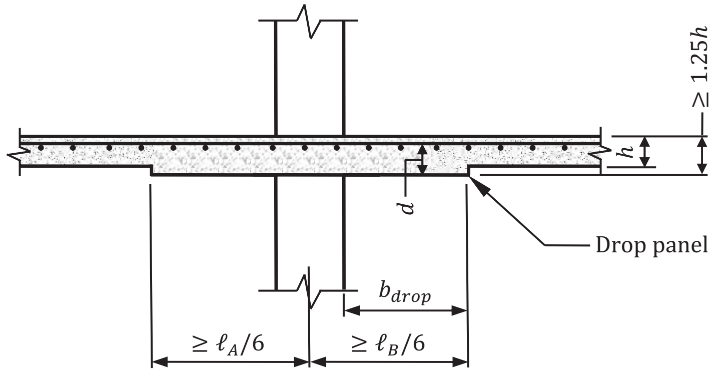


Figure 5.27 Strain and stress distributions at a positive moment section in a two-way slab.

For two-way slabs, the average distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement, d, can be approximated as $d = h - 1.25$ in. For flat slabs satisfying the dimensional requirements in ACI 8.2.4 for the drop panels, the d used in determining the negative flexural strength at the critical section, M_n^-, is calculated based on the limitation in ACI 8.5.2.2: the thickness of the drop panel below the slab, h_d, must be less than or equal to one-fourth the distance from the edge of the drop panel to the face of the column or column capital. This requirement essentially limits the value of d used in determining M_n (see Figure 5.28 for the case of an interior column with no capital).



$$d = 1.25h - d_b - \text{cover}$$

$$\leq h + (b_{drop}/4) - d_b - \text{cover}$$

Figure 5.28 Determination of $\boldsymbol{d}$ for slabs with drop panels.

The depth of the equivalent stress block, a, is determined from force equilibrium, that is, it is determined by setting the resultant compressive force in the concrete, $C = 0.85 f_c' ba$, equal to the tension force in the reinforcement, $T = A_s f_y$, and solving for a (see Figure 5.27):

$$a = \frac{A_s f_y}{0.85 f_c' b} \tag{5.27}$$

It is assumed a uniform stress equal to 85 percent of the concrete compressive strength, f_c', is distributed over the depth $a = \beta_1 c$, where c is the distance from the extreme compression fiber to the neutral axis (ACI 22.2.2.4.1). The maximum strain in the concrete is assumed to be 0.003 (ACI 22.2.2.1), and the term b is the width of the compression face of the member, which is typically the width of the column strip or middle strip.

The term β_1 is the factor relating the depth of the equivalent rectangular compression block, a, to the depth of the neutral axis, c; β_1 is defined in ACI Table 22.2.2.4.3 as follows (ACI 22.2.2.4.3):

- For $2{,}500 \text{ psi} \le f'_c \le 4{,}000 \text{ psi}$: $\beta_1 = 0.85$
- For $4{,}000 \text{ psi} < f'_c < 8{,}000 \text{ psi}$: $\beta_1 = 0.85 - [0.05(f'_c - 4{,}000) / 1{,}000]$
- For $f'_c \ge 8{,}000 \text{ psi}$: $\beta_1 = 0.65$

5.4.3 Nominal One-way Shear Strength

The nominal one-way shear strength, V_n, at a section is determined in accordance with ACI 22.5 (ACI 8.5.3.1.1):

$$V_n = V_c + V_s \tag{5.28}$$

In this equation, V_c is the nominal shear strength provided by concrete and V_s is the nominal shear strength provided by shear reinforcement. For nonprestressed members, V_c is determined by ACI Table 22.5.5.1 (ACI 22.5.5.1). Assuming shear reinforcement is not used in two-way slabs for one-way shear, Equation (c) in ACI Table 22.5.5.1 is applicable. This equation reduces to the following where the axial force, N_u, is equal to zero:

$$V_c = 8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c}b_w d \tag{5.29}$$

In this equation, b_w is the width of the plane extending across the entire slab width (ACI 8.5.3.1.1; see Figure 5.11 where $b_w = \ell_2$). According to ACI 22.5.5.1.1, V_c must not be taken greater than $5\lambda\sqrt{f'_c}b_w d$.

The size effect modification factor, λ_s, accounts for the phenomenon indicated in test results that the shear strength attributed to concrete in members without shear reinforcement does not increase in direct proportion with member depth; this factor is determined by ACI Equation (22.5.5.1.3) [ACI 22.5.5.1.3]:

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} \le 1.0 \tag{5.30}$$

It is evident from Equation (5.30) that λ_s is less than 1.0 for members with $d > 10$ in. This means $\lambda_s = 1.0$ for two-way slabs with an overall depth less than about 11.0 in.

The term λ is the modification factor reflecting the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength; it is determined based on either (1) the equilibrium density, w_c, of the concrete mix or (2) the composition of the aggregate in the concrete mix (see ACI 22.5.1.5 and ACI 19.2.4). Values of λ based on w_c are given in Table 5.20 [see ACI Table 19.2.4.1(a)] and values of λ based on composition of aggregates are given in Table 5.21 [see ACI Table 19.2.4.1(b)]. Note that λ is permitted to be taken as 0.75 for lightweight concrete (ACI 19.2.4.2) and is equal to 1.0 for normalweight concrete (ACI 19.2.4.3).

Table 5.20 Values of λ Based on Equilibrium Density, w_c

Equilibrium Density, w_c	λ
$w_c \le 100 \text{ lb/ft}^3$	0.75
$100 \text{ lb/ft}^3 < w_c \le 135 \text{ lb/ft}^3$	$0.0075 w_c \le 1.0$
$w_c > 135 \text{ lb/ft}^3$	1.0

Table 5.21 Values of λ Based on Composition of Aggregates

Concrete	Composition of Aggregates	λ
All-lightweight	Fine: ASTM C330 Coarse: ASTM C330	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330 and C33 Coarse: ASTM C330	0.75 to 0.85(1)
Sand-lightweight	Fine: ASTM C33 Coarse: ASTM C330	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33 Coarse: Combination of ASTM C330 and ASTM C33	0.85 to 1.0(2)

(1) *Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.*
(2) *Linear interpolation from 0.85 to 1.0 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of aggregate.*

The term ρ_w in Equation (5.29) is equal to the area of flexural reinforcement, A_s, at the section divided by $b_w d$. According to ACI R22.5.5.1, A_s may be taken as the sum of the areas of the longitudinal flexural reinforcement located more than two-thirds of the overall member depth away from the extreme compression fiber. For members with one layer of tension reinforcement, like two-way slabs, A_s is the total area of flexural reinforcement at that section, which has a width equal to b_w.

Values of $\sqrt{f_c'}$ used to calculate V_c are limited to 100 psi, except as allowed in ACI 22.5.3.2 (ACI 22.5.3.1). This limitation on f_c' is primarily due to a lack of test data and practical experience with concrete having compressive strengths greater than 10,000 psi. For economy, it is typical for f_c' in two-way slabs to be less than or equal to 5,000 psi (Reference 7).

One-way shear rarely governs in a two-way slab system, so shear reinforcement for one-way shear is not required; thus, $V_n = V_c$.

To minimize the likelihood of diagonal compression failure in the concrete and to limit the extent of cracking, the cross-sectional dimensions of a section must be selected to satisfy ACI Equation (22.5.1.2) [ACI 22.5.1.2]:

$$V_u \leq \phi(V_c + 8\sqrt{f_c'}b_w d) \qquad (5.31)$$

5.4.4 Nominal Two-way Shear Strength

Overview

For two-way shear, v_n is determined in accordance with ACI 22.6 (ACI 8.5.3.1.2). Provisions to determine nominal two-way shear strengths for slabs without and with shear reinforcement are given below.

Two-way Shear Strength Provided by Concrete in Slabs without Shear Reinforcement

For two-way slab systems without shear reinforcement, Equation (5.25) must be satisfied at the critical sections defined in Section 5.3.3 of this publication where $v_n = v_c$. The stress corresponding to the nominal two-way shear strength provided by the concrete, v_c, is the least of the values calculated by the equations in ACI Table 22.6.5.2 (ACI 22.6.5):

$$v_c = \text{ least of } \begin{cases} 4\lambda_s\lambda\sqrt{f_c'} \\ \left(2 + \dfrac{4}{\beta}\right)\lambda_s\lambda\sqrt{f_c'} \\ \left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f_c'} \end{cases} \qquad (5.32)$$

The size effect modification factor, λ_s, is determined by Equation (5.30) [ACI 22.5.5.1.3] and λ is the modification factor reflecting the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength (see Tables 5.20 and 5.21).

The term β is the ratio of the long to short dimensions of a rectangular column, concentrated load, or reaction area. The constant α_s is equal to 40 for interior columns, 30 for edge columns, and 20 for corner columns. Reference to these types of columns does not suggest the actual location of a column in a building; instead, α_s references the number of critical sections available to resist shear around a column. For example, α_s is equal to 30 for the interior column in Figure 5.29 adjacent to a relatively large opening; in this case, only 3 sides of the critical section are available to resist the shear stress. Additional information on the effects of openings in two-way slab systems is given in Section 5.4.5 of this publication.

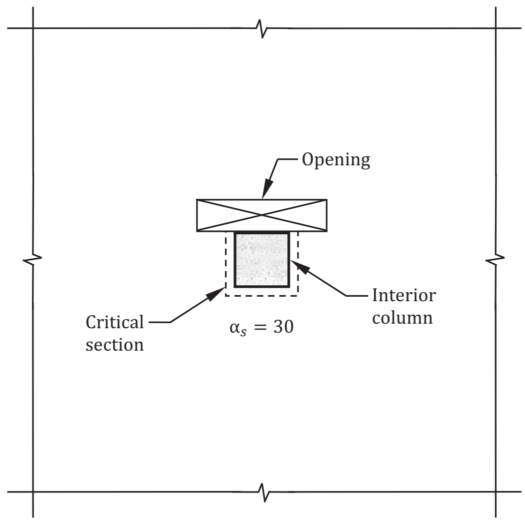


Figure 5.29 Effect of an opening on the critical section of an interior column.

Two-way Shear Strength Provided by Concrete in Slabs with Shear Reinforcement

For two-way slab systems with shear reinforcement (stirrups or headed shear stud reinforcement), v_c is determined in accordance with ACI 22.6.6. Equations to determine v_c at the various critical section locations are given in Table 5.22 (see ACI Table 22.6.6.1).

Table 5.22 Nominal Two-way Shear Strength Provided by Concrete in Slabs with Shear Reinforcement

Shear Reinforcement	Critical Sections	v_c
Stirrups	ACI 22.6.4.1 and 22.6.4.2 (Figure 5.19)	$2\lambda_s\lambda\sqrt{f'_c}$
Headed shear stud reinforcement	ACI 22.6.4.1 (Figure 5.20)	least of $\begin{cases} 3\lambda_s\lambda\sqrt{f'_c} \\ \left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f'_c} \\ \left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c} \end{cases}$
	ACI 22.6.4.2 (Figure 5.20)	$2\lambda_s\lambda\sqrt{f'_c}$

The size effect modification factor, λ_s, determined by Equation (5.30) is permitted to be taken as 1.0 where one of the following two conditions is satisfied (ACI 22.6.6.2):

1. Stirrups designed and detailed in accordance with ACI 8.7.6 are provided where $A_v / s \geq 2\sqrt{f'_c}b_o / f_{yt}$.
2. Smooth headed shear stud reinforcement with a stud shaft length less than or equal to 10 in. designed and detiled in accordance with ACI 8.7.7 are provided where $A_v / s \geq 2\sqrt{f'_c}b_o / f_{yt}$.

In these equations, A_v is the total area of stirrup legs or headed shear stud reinforcement on one peripheral line geometrically similar to the perimeter of the column section within the spacing s. Information on shear reinforcement in two-way slabs is given below.

Two-way Shear Strength Provided by Single- or Multiple-leg Stirrups

Two-way shear strength of two-way slabs can be increased by shear reinforcement consisting of properly anchored single- or multiple-leg stirrups (see ACI 8.7.6 and Figure 5.30). Stirrups are permitted to be used provided d of the two-way slab is greater than or equal to the larger of 6 in. or $16d_b$ where d_b is the diameter of the stirrups (ACI 22.6.7.1; see Figure 5.31).

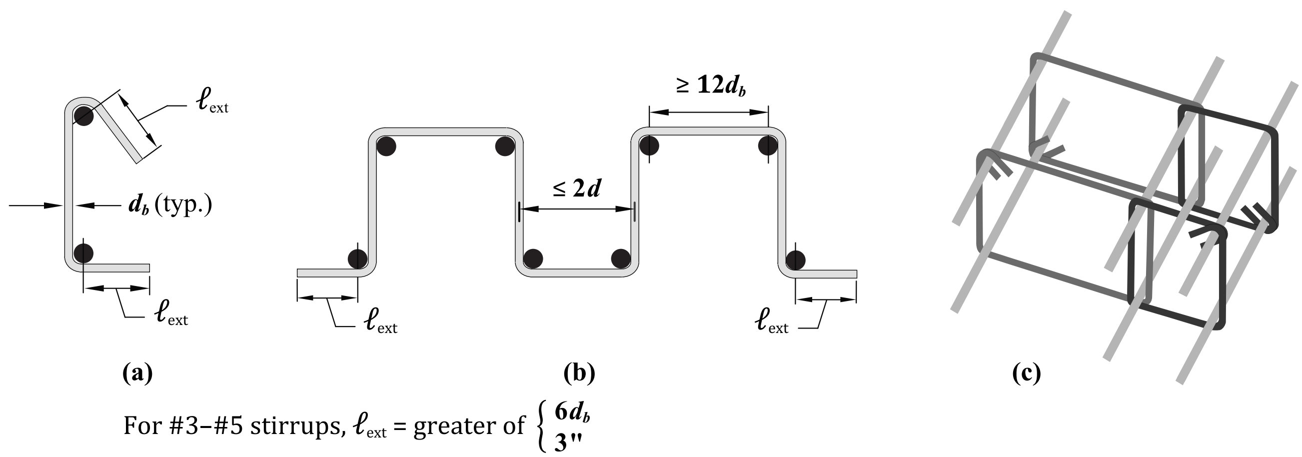


Figure 5.30 Stirrup shear reinforcement for use in two-way slabs. (a) Single-leg stirrup. (b) Multiple-leg stirrup. (c) Closed stirrups.

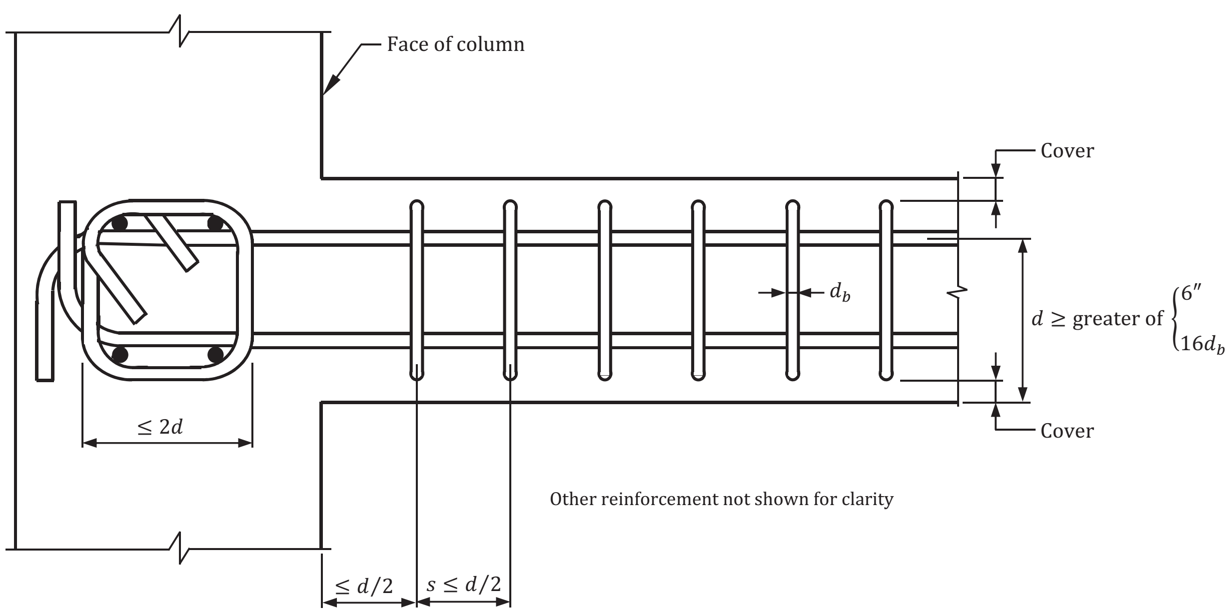


Figure 5.31 Cover, spacing, and depth requirements for closed stirrups.

The nominal shear strength, v_s, is determined in accordance with ACI 22.6.7.2:

$$v_s = \frac{A_v f_{yt}}{b_o s} \tag{5.33}$$

In this equation, A_v is the area of all stirrup legs on one peripheral line geometrically similar to the perimeter of the column section. For example, at the interior column depicted in Figure 5.19, stirrups are provided parallel to each column face. Along one peripheral line of stirrups, there are two vertical stirrup legs on each face, so for four faces, $A_v = 4 \times 2A_b = 8A_b$ where A_b is equal to the area of one stirrup leg.

The term s is the spacing of the peripheral lines of stirrups in the direction perpendicular to the column face. The location and spacing of the stirrups must conform with ACI Table 8.7.6.3 (see ACI 8.7.6.3 and Figure 5.19). Methods to determine s are given in Section 5.5.2 of this publication.

Two-way Shear Strength Provided by Headed Shear Stud Reinforcement

Headed shear stud reinforcement conforming to ACI 20.5 can be used alone or in conjunction with stirrups, drop panels, shear caps, or column capitals to increase the nominal two-way shear strength of two-way slabs (ACI 8.7.7).

A typical headed shear stud arrangement is shown in Figure 5.32. Large headed studs are welded to flat rails, and the assembly is positioned on chairs around the columns and subsequently nailed to the formwork. The size and spacing of the studs and the length of the base rail depend on the shear requirements.

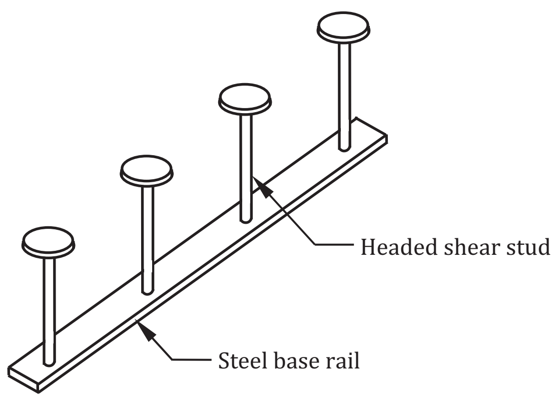


Figure 5.32 Headed shear stud reinforcement.

Like reinforcing bars, sufficient cover must be provided to protect the assembly from weather, fire, and other effects (see Figure 5.33). Minimum cover requirements for headed shear stud reinforcement are given in ACI 20.5.1.3.6: the minimum cover to the heads and base rails is the same as that required for the flexural reinforcement in a two-way slab, which is given in ACI Table 20.5.1.3.1 for cast-in-place nonprestressed members. For two-way slabs not exposed to weather or in contact with the ground, the minimum cover is 0.75 in. for #11 and smaller bars. The overall height of the headed shear stud assembly must conform to the requirements in ACI 8.7.7.1.1.

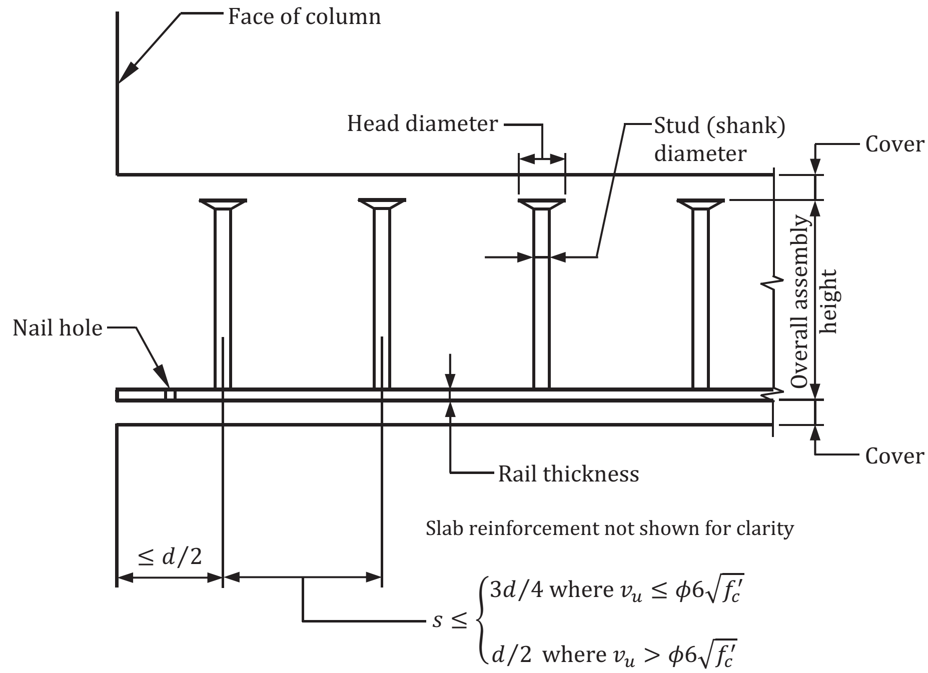


Figure 5.33 Cover and spacing requirements for headed shear stud reinforcement.

The nominal shear strength, v_s, is determined in accordance with ACI 22.6.8.2. ACI Equation (22.6.8.2) is used to determine v_s; this equation is the same as ACI Equation (22.6.7.2) for stirrups [see Equation (5.33)]. The main difference is that for the case of headed shear studs, A_v is the cross-sectional area of all headed shear studs on one peripheral line geometrically similar to the perimeter of the column section. For the interior column in Figure 5.20, $A_v = 8A_b$ where A_b is the area of one headed shear stud.

The term s is the spacing of the peripheral lines of headed shear studs in the direction perpendicular to the column face. The location and spacing of the headed shear studs must conform with ACI Table 8.7.7.1.2 (see ACI 8.7.7.1.2 and Figures 5.20 and 5.33). Methods to determine s are given in Section 5.5.2 of this publication.

ACI Equation (22.6.8.3) must also be satisfied where headed shear stud reinforcement is provided (ACI 22.6.8.3):

$$\frac{A_v}{s} \ge \frac{2\sqrt{f'_c}b_o}{f_{yt}} \tag{5.34}$$

Summary of Nominal Two-way Shear Strength Requirements

The information in Table 5.23 can be used to determine the nominal two-way shear strength, v_n, for two-way slab systems without and with shear reinforcement. Included in this table are the maximum permitted values of v_n for two-way slabs with shear reinforcement, which are equal to the values of v_u in ACI Table 22.6.6.3 divided by the strength reduction factor, ϕ (ACI 22.6.6.3).

Table 5.23 Nominal Two-way Shear Strength Requirements for Two-way Slabs

Shear Reinforcement	Critical Sections	Nominal Strengths	
None	ACI 22.6.4.1 (Figures 5.12 – 5.16)	$v_n = v_c$	$\text{least of} \begin{cases} 4\lambda_s\lambda\sqrt{f'_c} \\ \left(2+\dfrac{4}{\beta}\right)\lambda_s\lambda\sqrt{f'_c} \\ \left(2+\dfrac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c} \end{cases}$
		v_s	0
Stirrups	ACI 22.6.4.1 (Figure 5.19)	v_c	$2\lambda_s\lambda\sqrt{f'_c}$
		v_s	$\dfrac{A_v f_{yt}}{b_o s}$
		v_n	$2\lambda_s\lambda\sqrt{f'_c} + \dfrac{A_v f_{yt}}{b_o s} \le 6\sqrt{f'_c}$
	ACI 22.6.4.2 (Figure 5.19)	$v_n = v_c$	$2\lambda_s\lambda\sqrt{f'_c}$
		v_s	0
Headed Shear Stud Reinforcement	ACI 22.6.4.1 (Figure 5.20)	v_c	$\text{least of} \begin{cases} 3\lambda_s\lambda\sqrt{f'_c} \\ \left(2+\dfrac{4}{\beta}\right)\lambda_s\lambda\sqrt{f'_c} \\ \left(2+\dfrac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c} \end{cases}$
		v_s	$\dfrac{A_v f_{yt}}{b_o s} \ge 2\sqrt{f'_c}$
		v_n	$\text{least of} \left\{ \begin{matrix} 3\lambda_s\lambda\sqrt{f'_c} \\ \left(2+\dfrac{4}{\beta}\right)\lambda_s\lambda\sqrt{f'_c} \\ \left(2+\dfrac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c} \end{matrix} \right\} + \dfrac{A_v f_{yt}}{b_o s} \le 8\sqrt{f'_c}$
	ACI 22.6.4.2 (Figure 5.20)	$v_n = v_c$	$2\lambda_s\lambda\sqrt{f'_c}$
		v_s	0

5.4.5 Openings in Two-way Slab Systems

Openings for mechanical ductwork, electrical and plumbing risers, stairs, elevators, and other elements are commonly required in two-way slab systems. The effects of such openings must be considered in design (ACI 8.2.2).

According to ACI 8.5.4.1, openings of any size are permitted in a two-way slab system provided an analysis of the system with the openings is performed that shows all strength and serviceability requirements are satisfied. For two-way slab systems without beams, such an analysis need not be performed when the provisions of ACI 8.5.4.2 are satisfied; the first three of these provisions are given in Table 5.24 (see Figure 5.34).

Table 5.24 Requirements for Openings in Two-way Slab Systems in Accordance with ACI 8.5.4.2

Location	Maximum Opening Size	Additional Requirements
Area common to intersecting middle strips	Openings of any size are permitted	Total quantity of reinforcement in the panel must be at least that required for the panel without the opening
Area common to intersecting columns strips	One-eighth the width of the column strip in either span	A quantity of reinforcement at least equal to that interrupted by an opening must be added to the sides of the opening
Area common to one column strip and one middle strip	Not more than one-fourth of the reinforcement in either strip must be interrupted by openings	A quantity of reinforcement at least equal to that interrupted by an opening must be added to the sides of the opening

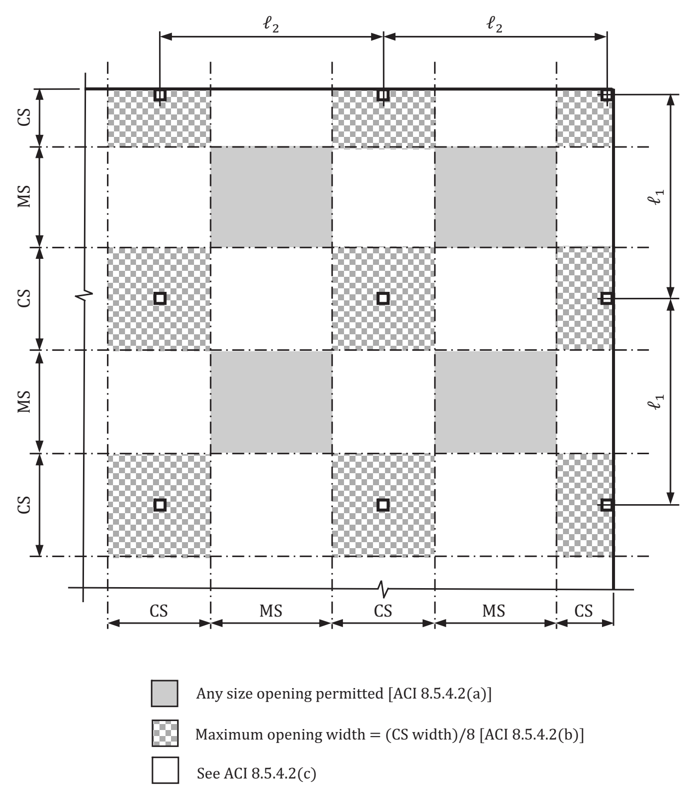


Figure 5.34 Permitted openings in two-way slab systems without beams in accordance with ACI 8.5.4.2.

The closer an opening is to a column, the greater effect it has on the critical shear perimeter available to resist shear forces. In particular, where an opening is located closer than $4h$ from the periphery of a column, concentrated load, or reaction area, the provisions of ACI 22.6.4.3 must be satisfied [ACI 8.5.4.2(d)]: The perimeter of the critical section must be reduced by a length equal to the projection of the opening formed by two straight lines that extend from the centroid of the column, concentrated load, or reaction area and are tangent to the boundaries of the opening. The ineffective portions of b_o are illustrated in Figure 5.35 for a column in a flat plate system with openings for systems without and with shear reinforcement.

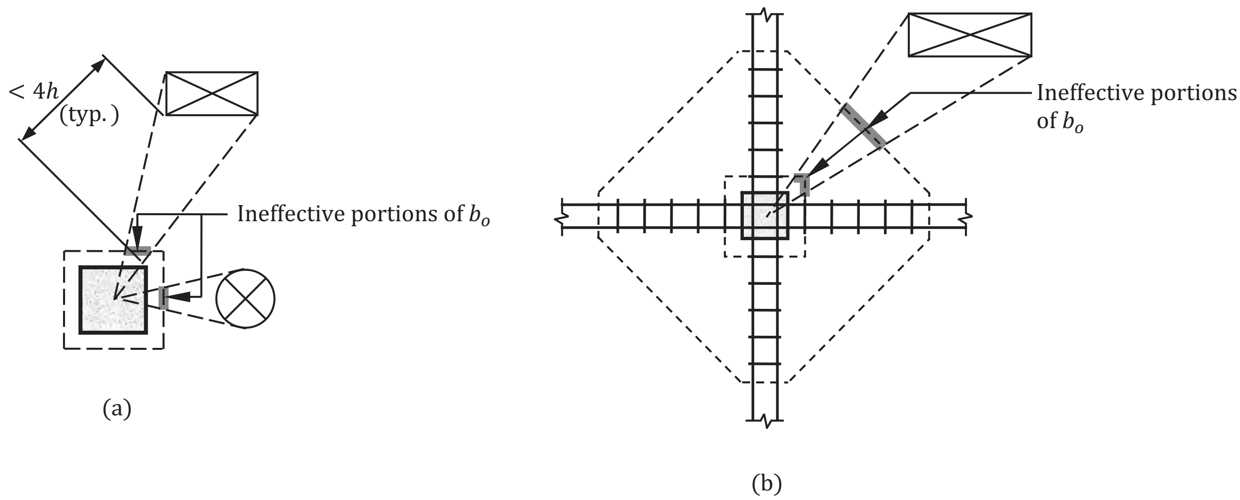


Figure 5.35 Effect of openings on critical shear perimeter in two-way slabs.
(a) Without shear reinforcement. (b) With shear reinforcement.

The section properties of the remaining effective segments of the critical section can be determined using Equations (5.15) and (5.16) in Section 5.3.3 of this publication. The shear stresses due to the direct shear force and the portion of the moment transferred to the column by eccentricity of shear are calculated by Equation (5.14) using the reduced section properties of the critical section. The maximum shear stresses must be less than or equal to applicable design two-way shear strengths, ϕv_n, in Table 5.23.

5.5 Determination of Required Reinforcement

5.5.1 Required Flexural Reinforcement

The required area of flexural reinforcement, A_s, at a critical section in a column strip or middle strip in a two-way slab system is determined based on the required and design flexural strengths M_u and ϕM_n for tension-controlled sections. Using Equations (5.22) and (5.26), the required A_s can be determined by the following equation:

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] \tag{5.35}$$

The nominal strength coefficient of resistance, R_n, is defined as follows:

$$R_n = \frac{M_u}{\phi b d^2} \tag{5.36}$$

and $\phi = 0.90$ for tension-controlled sections. Factored bending moments, M_u, in Equation (5.36) can be obtained at the critical sections in the column strips and middle strips using the DDM where applicable (see Tables 5.16 through 5.19).

The area of flexural reinforcement at any critical section in a two-way slab must not be less than $A_{s,min} = 0.0018A_g$ where the gross area A_g is equal to the thickness of the slab, h, times the width of the column strip or middle strip (ACI 8.6.1.1).

The required area of flexural reinforcement in the slab width b_{slab} can also be determined by Equation (5.35) where R_n in Equation (5.36) is calculated using $M_u = \gamma_f M_{sc}$ [see Equation (5.23)]. In this case, $A_{s,min}$ is equal to the following (see ACI 8.6.1.2 and Section 5.3.2 of this publication):

$$A_{s,min} = \begin{cases} 0.0018hb_{slab} \text{ where } v_{uv} \leq \phi 2\lambda_s\lambda\sqrt{f'_c} \\ \\ \text{greater of } \begin{cases} 0.0018hb_{slab} \\ \\ 5v_{uv}b_{slab}b_o / \phi\alpha_s f_y \end{cases} \text{where } v_{uv} > \phi 2\lambda_s\lambda\sqrt{f'_c} \end{cases} \tag{5.37}$$

In this equation, $\phi = 0.75$ for shear and v_{uv} is the factored shear stress on the slab critical section for two-way action from the controlling load combination without moment transfer:

$$v_{uv} = \frac{V_u}{A_c} \tag{5.38}$$

It is also important to verify the section is tension-controlled. The following relationship is obtained from the linear strain distribution where c_t is the depth of the neutral axis when $\varepsilon_t = \varepsilon_{ty} + 0.003$ for tension-controlled sections (see Figure 5.27):

$$\frac{c_t}{d} = \frac{0.003}{\varepsilon_t + 0.003} = \frac{0.003}{\varepsilon_{ty} + 0.006} \tag{5.39}$$

Substituting $a_t = \beta_1 c_t$ into Equation (5.27) with c_t from Equation (5.39), the following equation can be used to calculate the area of flexural reinforcement, $A_{s,t}$, corresponding to tension-controlled sections for two-way slabs with $\varepsilon_t = \varepsilon_{ty} + 0.003$:

$$A_{s,t} = \frac{0.85\beta_1 f'_c bd}{f_y}\left(\frac{0.003}{\varepsilon_{ty} + 0.006}\right) \tag{5.40}$$

For Grade 60 reinforcement, $\varepsilon_{ty} = 60 / 29{,}000 = 0.00207$ and for $f'_c = 4{,}000$ psi, $\beta_1 = 0.85$. For these material properties, Equation (5.40) reduces to the following:

$$A_{s,t} = 0.018bd \tag{5.41}$$

In these equations, b is the width of the column strip or middle strip. If A_s calculated by Equation (5.35) is greater than $A_{s,t}$, the section is not tension-controlled $(\phi < 0.9)$, and h must be increased accordingly to attain a tension-controlled section. A similar check must be performed for the flexural reinforcement within b_{slab}.

5.5.2 Required Shear Reinforcement

Where required, the size and spacing of shear reinforcement for two-way slabs can be determined using the information provided in Table 5.23.

Stirrups

Prior to determining the required size and spacing of the stirrups, it should be verified that the maximum shear stress, v_u, at the critical section defined by ACI 22.6.4.1 is less than or equal to $\phi 6\sqrt{f_c'}$ in accordance with ACI Table 22.6.6.3. Where $v_u > \phi 6\sqrt{f_c'}$, h and/or f_c' must be increased accordingly to satisfy this requirement.

The total number of stirrup legs, n, provided on one peripheral line geometrically similar to the perimeter of the column depend on the number of sides of the critical section (see Figure 5.19). Assuming a stirrup bar size with an area equal to A_b, the following equation can be used to determine the required spacing, s (see Table 5.23):

$$s \le \text{least of} \begin{cases} \dfrac{\phi n A_b f_{yt}}{(v_u - \phi 2\lambda_s \lambda\sqrt{f_c'})b_o} \\ \\ d/2 \end{cases} \tag{5.42}$$

Stirrups can be terminated where the design strength of the concrete can resist the factored shear stress without the contribution of the stirrups; this occurs where $v_u \le \phi 2\lambda_s\lambda\sqrt{f_c'}$ (see Figure 5.19 and Table 5.23).

Headed Shear Stud Reinforcement

Prior to determining the required size and spacing of the headed shear stud reinforcement, it should be verified that the maximum shear stress, v_u, at the critical section defined by ACI 22.6.4.1 is less than or equal to $\phi 8\sqrt{f_c'}$ in accordance with ACI Table 22.6.6.3. Where $v_u > \phi 8\sqrt{f_c'}$, h and/or f_c' must be increased accordingly to satisfy this requirement.

The number of headed shear studs, n, provided on one peripheral line geometrically similar to the perimeter of the column depend on (1) the number of sides of the critical section and (2) the spacing requirement parallel to the column face in ACI Table 8.7.7.1.2 (see Figure 5.20). Assuming a headed shear stud with an area equal to A_b, the following equation can be used to determine the required spacing, s (see Table 5.23):

$$s \le \text{least of} \begin{cases} \dfrac{\phi n A_b f_{yt} / b_o}{v_u - \text{least of} \begin{cases} \phi 3\lambda_s\lambda\sqrt{f_c'} \\ \\ \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s\lambda\sqrt{f_c'} \\ \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f_c'} \end{cases}} \\ \\ n A_b f_{yt} / 2\sqrt{f_c'} b_o \\ \\ 3d/4 \text{ where } v_u \le \phi 6\sqrt{f_c'} \\ \\ d/2 \text{ where } v_u > \phi 6\sqrt{f_c'} \end{cases} \tag{5.43}$$

Headed shear studs can be terminated where the design strength of the concrete can resist the factored shear stress without the contribution of the headed shear studs; this occurs where $v_u \le \phi 2\lambda_s\lambda\sqrt{f_c'}$ (see Figure 5.20 and Table 5.23).

As noted previously, ACI 20.4 requires headed shear stud reinforcement and stud assemblies conform to ASTM A1044 (Reference 13). According to that document, the minimum yield strength of the stud material, f_{yt}, is equal to 51,000 psi. Also, the area of a headed shear stud should be obtained from manufacturers' literature.

5.6 Reinforcement Detailing

5.6.1 Concrete Cover

Reinforcing bars, stirrups, and headed shear stud assemblies are placed in two-way slabs with a minimum concrete cover to protect them from weather, fire, and other effects. Minimum cover requirements are given in ACI 20.5.1 (ACI 8.7.1.1). For reinforcing bars in two-way slabs not exposed to weather or in contact with the ground, the minimum cover is equal to 0.75 in. for #11 and smaller bars (ACI Table 20.5.1.3.1). Concrete cover is measured from the surface of the concrete to the following:

- Outermost layer of flexural reinforcement for two-way slabs without shear reinforcement (Figure 5.36)
- Outermost surface of the stirrup bars for two-way slabs with shear reinforcement consisting of stirrups (Figure 5.31)
- Underside of the base rail and outermost surface of the head for two-way slabs with shear reinforcement consisting of headed shear studs (Figure 5.33)

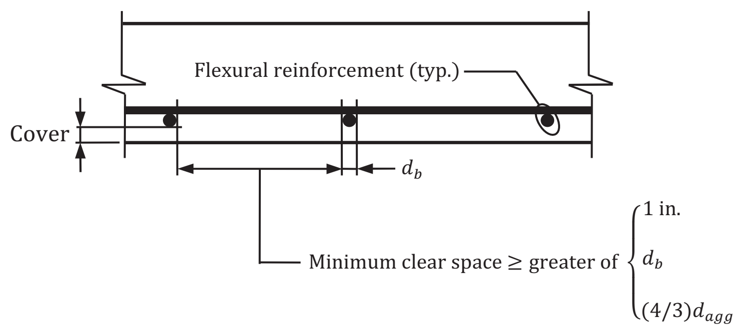


Figure 5.36 Minimum clear cover and spacing requirements for reinforcing bars in two-way slabs.

5.6.2 Minimum Spacing of Flexural Reinforcing Bars

Minimum clear spacing of parallel reinforcing bars in a single horizontal layer is given in ACI 25.2 (ACI 8.7.2.1). These limits have been established primarily so concrete can flow readily into the spaces between adjoining bars.

The spacing requirements are summarized in Figure 5.36 for a two-way slab system where d_{agg} is the nominal maximum aggregate size in the concrete mix.

5.6.3 Maximum Spacing of Flexural Reinforcing Bars

Except for two-way joist systems, maximum center-to-center spacing of flexural reinforcing bars is equal to the lesser of $2h$ and 18 in. at critical sections and $3h$ and 18 in. at other sections (ACI 8.7.2.2). In addition to controlling crack widths, this requirement takes into consideration the effect that could be caused by loads concentrated on small areas of the slab.

5.6.4 Selection of Flexural Reinforcement

The size and spacing of the reinforcing bars at a critical section for flexure must be determined based on the required area of reinforcement determined by Equation (5.35) and the minimum and maximum spacing requirements given in Sections 5.6.2 and 5.6.3 of this publication, respectively.

In two-way slabs, A_s is calculated at the critical sections in the column strips and middle strips. A bar size and spacing should be selected so that the provided A_s is equal to or slightly greater than the required A_s considering minimum reinforcement requirements and minimum and maximum bar spacing requirements. The flexural reinforcing bars are uniformly distributed in the design strips. Economy is generally achieved by using the largest size reinforcing bars satisfying strength and maximum spacing requirements and to repeat bar size and spacing as often as possible.

In order to satisfy Equation (5.23), the provided area of reinforcement in the column strip within b_{slab} may need to be increased; this is typically achieved by adding reinforcing bars of the same size as those in the column strip considering the requirements for bar spacing. Where additional reinforcing bars are required, the bars should be uniformly spaced within b_{slab} and a special detail must be provided that clearly illustrates the extent and location of the reinforcing bars.

5.6.5 Corner Restraint in Slabs

At exterior corners of slabs supported by stiff elements such as walls and edge beams with $\alpha_f > 1.0$, corner reinforcement must be provided that satisfies the requirements of ACI 8.7.3. This reinforcement is required because the stiff elements restrain the slab and cause additional bending moments at these locations.

Corner reinforcement must be provided in both the top and bottom of the slab, and the reinforcement in each layer in each direction must be designed for a factored moment, M_u, equal to the largest positive bending moment per unit width in the slab panel. The top and bottom reinforcement must be placed parallel and perpendicular to the diagonal, respectively, for a distance of at least one-fifth of the longer of the two span lengths in the corner panel [see Figure 5.37(a)]. In lieu of the diagonal reinforcement layout, reinforcement placed parallel to the edges is permitted (see ACI 8.7.3.1.3 and Figure 5.37(b)]; this layout is preferred compared to the diagonal layout because the reinforcing bars are simpler to place in the field. In either option, the maximum center-to-center spacing of the corner reinforcement is equal to $2h$.

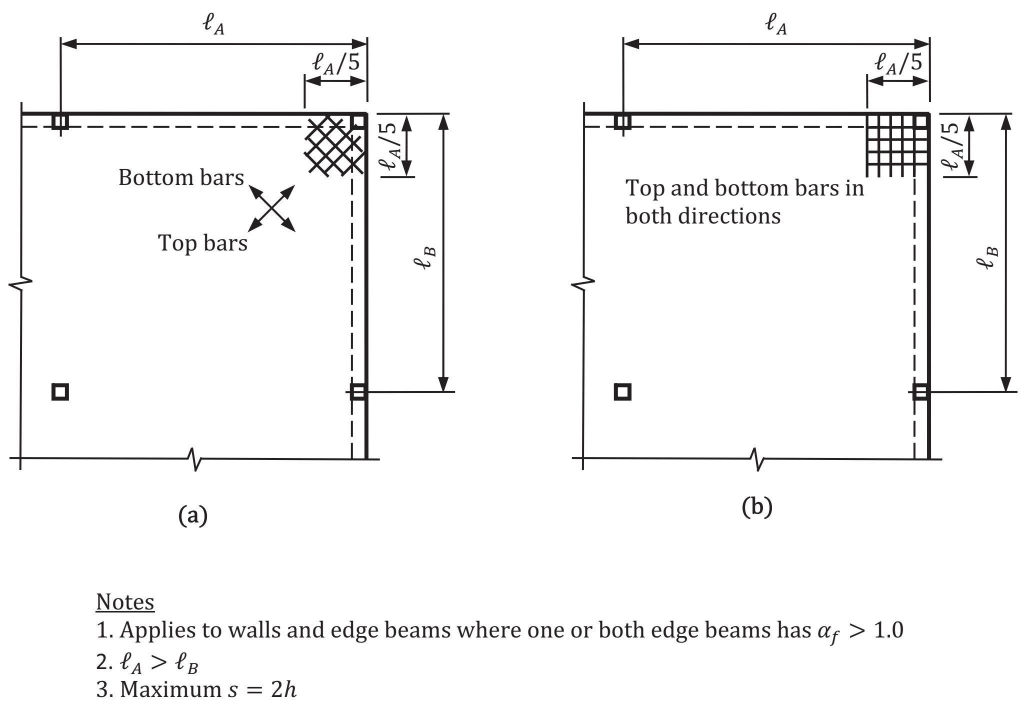


Figure 5.37 Required reinforcement at corners of slabs supported by stiff edge members.
(a) Reinforcement parallel and perpendicular to the diagonal. (b) Reinforcement parallel to the slab edges.

5.6.6 Termination of Flexural Reinforcement

Requirements for termination locations of positive and negative flexural reinforcement perpendicular to a discontinuous edge in column and middle strips are given in ACI 8.7.4.1. For slabs supported directly on edge (spandrel) beams, columns, and walls, the positive flexural reinforcement must extend into the supporting member at least 6 in. from its face (see ACI 8.7.4.1.1 and Figure 5.38). Straight, hooked, or headed bars may be used in such cases. Hooked or headed reinforcing bars must be provided at the ends of negative reinforcement and the reinforcement must be developed in tension at the face of the supporting member with development lengths determined in accordance with ACI 25.4 (ACI 8.7.1.2). Information on how to determine tension development lengths for hooked bars and for headed bars can be found in Section 4.6.5 of this publication; the information in that section for one-way slabs is also applicable to two-way slabs.

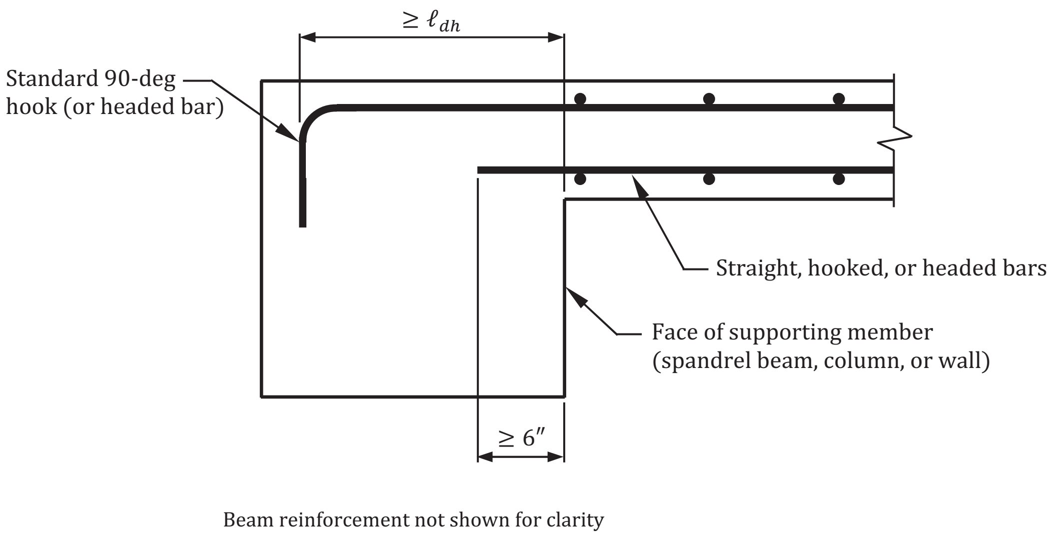
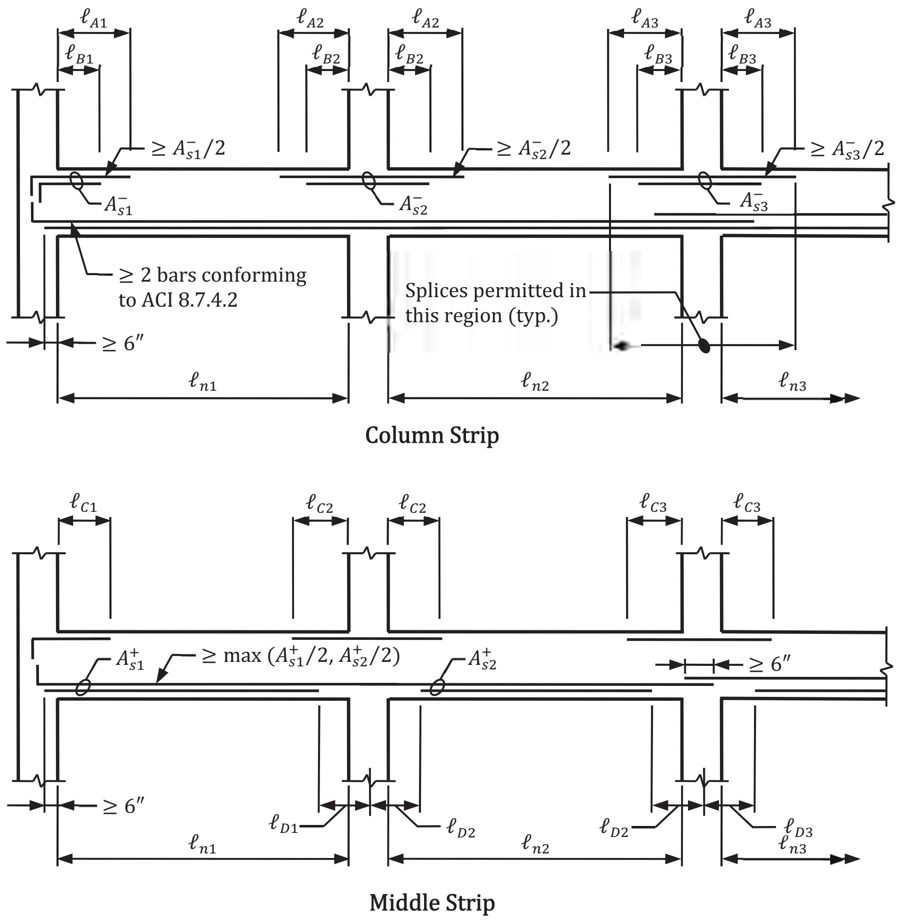


Figure 5.38 Termination of flexural reinforcement in a two-way slab supported by an edge (spandrel) beam, column, or wall.

Anchorage of reinforcement is permitted within a slab where the slab is not supported by a spandrel beam or a wall at a discontinuous edge or where a slab cantilevers beyond the support (ACI 8.7.4.1.2).

For two-way slab systems without beams, the flexural reinforcement must have the minimum extensions given in ACI Figure 8.7.4.1.3 (ACI 8.7.4.1.3; see Figure 5.39); these extensions are applicable to systems supporting only gravity loads. The purpose of the minimum extension length of $5d$ for the negative reinforcing bars, which governs where ℓ_n / h is less than about 15, is to help intercept potential punching shear cracks that can form in relatively thick two-way slabs (for example, in transfer slabs and podium slabs; see ACI Figure R8.7.4.1.3). In other words, the prescribed minimum extension length of $0.30\ell_n$ may not be sufficient to intercept potential punching shear cracks in such cases.

Where a two-way slab resists the effects from combined gravity and lateral forces, the bar lengths must be based on analysis, but must not be less than those given in ACI Figure 8.7.4.1.3. Precise locations of inflection points cannot be explicitly found using approximate methods, such as the DDM Method, because they depend on the ratio of the panel dimensions, the ratio of live to dead load, and the continuity conditions at the edges of the panels. Also, other than performing a more refined analysis, there is no explicit way of determining the distribution of applied combined loads to the column and middle strips. A conservative approach in such cases is to make at least 25 percent of the negative reinforcing bars in the column strip continuous; this is consistent with the requirement pertaining to two-way slabs in intermediate moment frames (see ACI 18.4.5.4).



Notes:

1. $\ell_{A1} \geq \begin{cases} \text{greater of } 0.30\ell_{n1} \text{ and } 5d \text{ for two-way slabs without drop panels} \\ \text{greater of } 0.33\ell_{n1} \text{ and } 5d \text{ for two-way slabs with drop panels} \end{cases}$

2. $\ell_{A2} \geq \begin{cases} \text{greater of } 0.30\ell_{n1}, 0.30\ell_{n2}, \text{ and } 5d \text{ for two-way slabs without drop panels} \\ \text{greater of } 0.33\ell_{n1}, 0.33\ell_{n2}, \text{ and } 5d \text{ for two-way slabs with drop panels} \end{cases}$

3. $\ell_{A3} \geq \begin{cases} \text{greater of } 0.30\ell_{n2}, 0.30\ell_{n3}, \text{ and } 5d \text{ for two-way slabs without drop panels} \\ \text{greater of } 0.33\ell_{n2}, 0.33\ell_{n3}, \text{ and } 5d \text{ for two-way slabs with drop panels} \end{cases}$

4. $\ell_{B1} \geq 0.20\ell_{n1}$

5. $\ell_{B2} \geq$ greater of $0.20\ell_{n1}$ and $0.20\ell_{n2}$

6. $\ell_{B3} \geq$ greater of $0.20\ell_{n2}$ and $0.20\ell_{n3}$

7. $\ell_{C1} \geq 0.22\ell_{n1}$

8. $\ell_{C2} \geq$ greater of $0.22\ell_{n1}$ and $0.22\ell_{n2}$

9. $\ell_{C3} \geq$ greater of $0.22\ell_{n2}$ and $0.22\ell_{n3}$

10. $\ell_{D1} \leq 0.15\ell_{n1}$

11. $\ell_{D2} \leq 0.15\ell_{n2}$

12. $\ell_{D3} \leq 0.15\ell_{n3}$

Figure 5.39 Minimum bar lengths for two-way slabs without beams.

5.6.7 Splices of Reinforcement

Splices of deformed reinforcement in two-way slabs must be in accordance with ACI 25.5 (ACI 8.7.1.3). The primary reasons for splicing reinforcement are based on (1) restrictions related to transporting the reinforcing bars to the construction site and (2) limitations related to handling and placing the reinforcing bars in the field.

Lap splices, mechanical splices, and welded splices are common types of splices for flexural reinforcement. The information provided in Section 4.6.6 of this publication for splices of reinforcing bars in one-way slabs is also applicable to two-way slabs.

5.6.8 Structural Integrity Reinforcement

Structural integrity reinforcement requirements for cast-in-place two-way slabs are given in ACI 8.7.4.2. The purpose of this reinforcement is to improve the redundancy and ductility in the structure so in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be localized and the structure will have a higher probability of maintaining overall stability.

Two requirements are given in ACI 8.7.4.2 (see Figure 5.39):

(1) All the bottom flexural reinforcement in the column strips in both directions must be continuous or spliced using mechanical or welded splices in accordance with ACI 25.5.7 or Class B tension lap splices in accordance with ACI 25.5.2. Splices must be located in accordance with ACI Figure 8.7.4.1.3.

(2) At least two of the bottom reinforcing bars in the column strips in both directions must pass within the region bounded by the longitudinal reinforcement of the column and must be anchored at exterior supports using hooks or headed bars.

5.6.9 Shear Reinforcement Details

Reinforcement details for stirrups and headed shear studs are given in ACI 8.7.6 and 8.7.7, respectively (see Section 5.5.2 of this publication). These details are given in Figures 5.19, 5.30, and 5.31 for stirrups and in Figures 5.20 and 5.33 for headed shear studs.

5.7 Design Procedure

The general design procedure in Figure 5.40 can be used in the design and detailing of two-way slabs assuming the DDM is used to determine the factored bending moments at the critical sections in the column and middle strips. Included in the figure are the section and equation numbers where specific information on that topic can be found in this chapter.

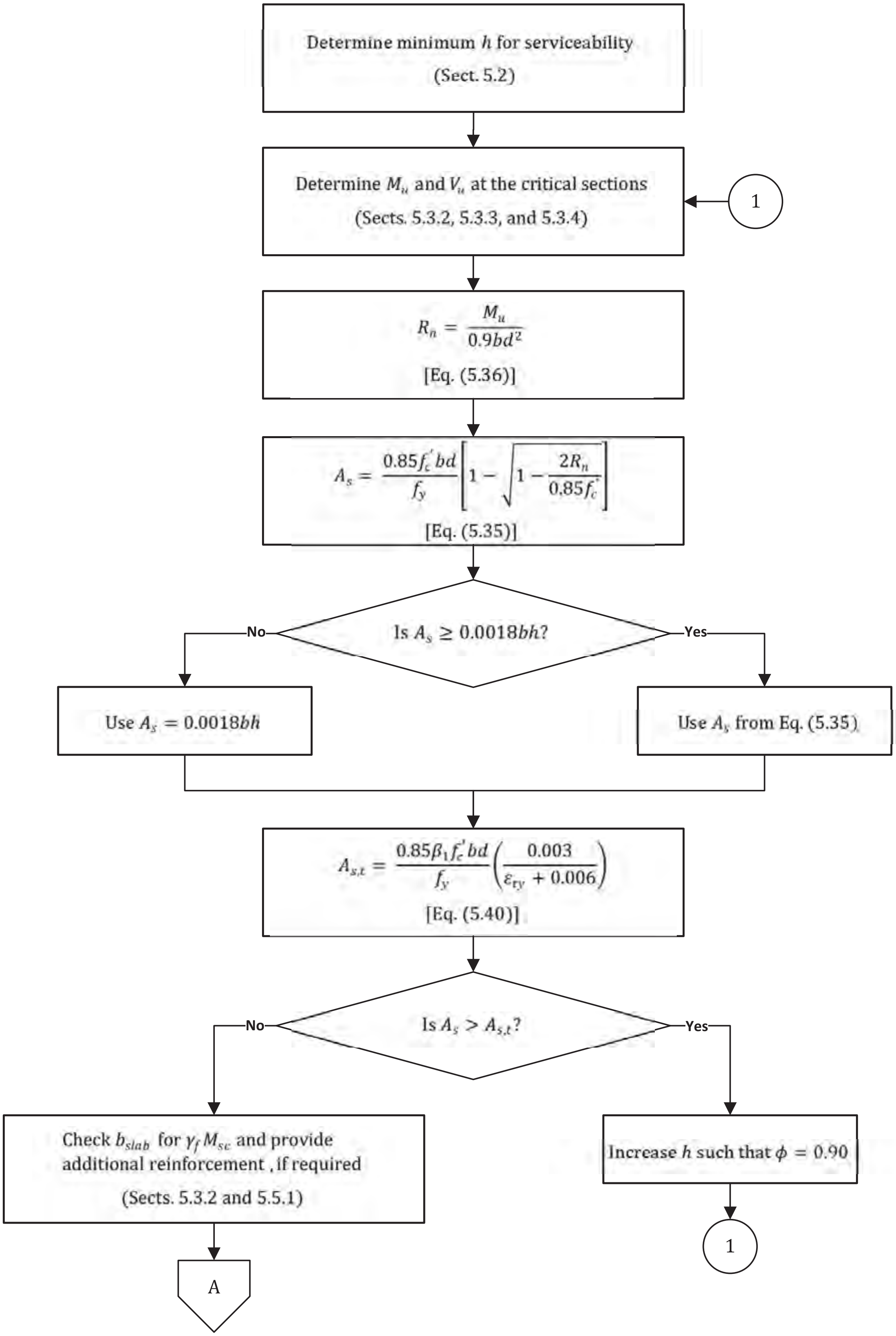


Figure 5.40 Design procedure for two-way slabs.

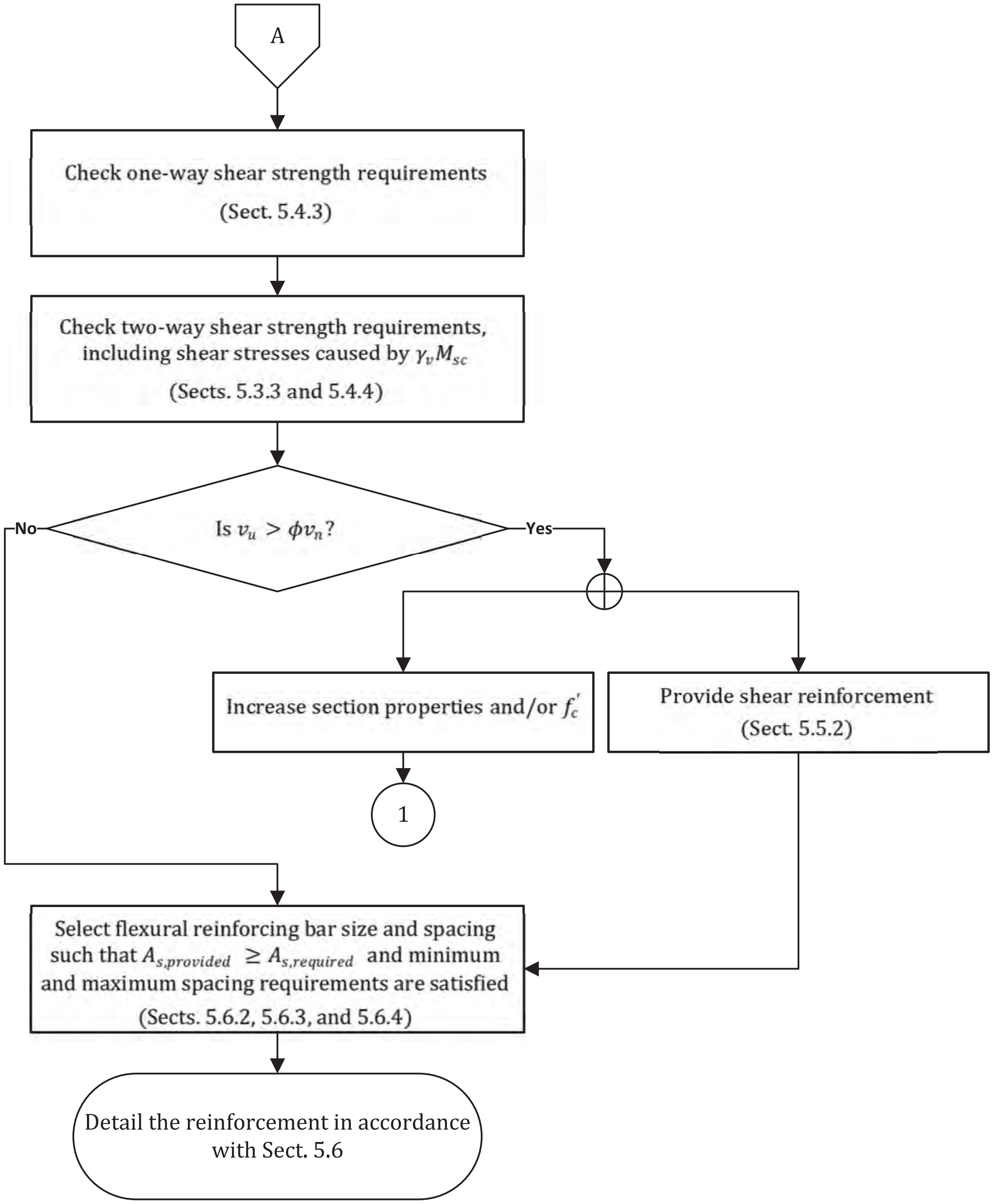


Figure 5.40 Design procedure for two-way slabs (cont.).

5.8 Examples

5.8.1 Example 5.1 – Determination of Minimum Slab Thickness: Flat Plate System, Building #1 (Framing Option A)

Determine the minimum slab thickness for the flat plate system in Figure 1.1 (Framing Option A) at a typical floor with 24 in. by 24 in. columns. Assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the minimum slab thickness based on serviceability requirements ACI 8.3.1.1

The longest clear span length, ℓ_n, measured face-to-face of columns is equal to the following:

$$\ell_n = (25.0 \times 12) - 24.0 = 276.0 \text{ in.}$$ Figure 5.23

For exterior panels without edge beams:

$$h = \frac{\ell_n}{30} = \frac{276.0}{30} = 9.2 \text{ in.}$$ Table 5.2

For interior panels:

$$h = \frac{\ell_n}{33} = \frac{276.0}{33} = 8.4 \text{ in.}$$

Try a 9.5-in. slab thickness for all panels. This thickness is greater than the minimum thickness of 5.0 in. prescribed in ACI 8.3.1.1(a).

Step 2 – Determine a preliminary slab thickness based on two-way shear strength requirements Sect. 5.2.2

Use Figure 5.3 to determine a preliminary h based on gravity load effects.

$$\text{Dead load of slab} = (9.5 / 12) \times 150.0 = 118.8 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$$

$$\text{Live load} = 65.0 \text{ lb/ft}^2$$

Maximum $q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$ ACI Eq. (5.3.1b)

For the edge columns along column lines A or E, tributary area, A, is equal to the following:

$$A = 25.0 \times \left(\frac{23.5}{2} + \frac{24}{2 \times 12} \right) = 318.8 \text{ sq ft}$$

For the edge columns along column lines 1 or 7, tributary area, A, is equal to the following:

$$A = 23.5 \times \left(\frac{25.0}{2} + \frac{24}{2 \times 12} \right) = 317.3 \text{ sq ft}$$

Determine the area ratio based on the largest tributary area:

$$A / c_1^2 = (318.8 \times 144) / 24^2 = 79.7$$

$$d / c_1 \cong 0.32$$ Figure 5.3

Therefore, $h = (0.32 \times 24.0) + 1.25 = 8.9 \text{ in.}$

Preliminary calculations indicate a 9.5-in. slab thickness is adequate for serviceability and shear strength.

A final slab thickness is established once two-way shear requirements are checked at all columns (see Sect. 5.4.4). Because the columns and portions of the slab form the LFRS, the total two-way shear stress is due to the combination of gravity and lateral load effects.

Comments. To illustrate the impact column size has on the required slab thickness, determine a preliminary slab thickness assuming 20-in.-square edge columns instead of the 24-in.-square columns used in this example:

$$\ell_n = (25.0 \times 12) - 20.0 = 280.0 \text{ in.}$$

For exterior panels without edge beams:

$$h = \frac{\ell_n}{30} = \frac{280.0}{30} = 9.3 \text{ in.}$$

$$A = 25.0 \times \left(\frac{23.5}{2} + \frac{20}{2 \times 12} \right) = 314.6 \text{ sq ft}$$

$$A / c_1^2 = (314.6 \times 144) / 20^2 = 113.3$$

$$d / c_1 \cong 0.45$$ Figure 5.3

Therefore, $h = (0.45 \times 20.0) + 1.25 = 10.3 \text{ in.}$

In this case, preliminary calculations indicate a 10.5-in. slab thickness is adequate for serviceability and shear strength.

5.8.2 Example 5.2 – Determination of Minimum Slab Thickness: Flat Plate System with Edge Beams, Building #1 (Framing Option B)

Determine the minimum slab thickness for the flat plate system in Figure 1.1 (Framing Option B) at a typical floor with 28 in. by 24 in. edge beams and 24 in. by 24 in. columns. Assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the minimum slab thickness based on serviceability requirements — ACI 8.3.1.1

The longest clear span length, ℓ_n, measured face-to-face of columns is equal to the following:

$$\ell_n = (25.0 \times 12) - 24.0 = 276.0 \text{ in.}$$ Figure 5.23

For exterior panels with edge beams:

$$h = \frac{\ell_n}{33} = \frac{276.0}{33} = 8.4 \text{ in.}$$ Table 5.2

According to the footnote in Table 5.2, this minimum slab thickness is based on the assumption $\alpha_f \geq 0.8$ for the edge beams. Check this assumption for the 28 in. by 24 in. edge beams assuming an 8.5-in.-thick slab.

Determine the effective slab width, b_e:

$$b_e = b_w + \text{lesser of} \begin{cases} h_b \\ 4h \end{cases} = 28.0 + \text{lesser of} \begin{cases} 24.0 - 8.5 = 15.5 \\ 4 \times 8.5 = 34.0 \end{cases} = 28.0 + 15.5 = 43.5 \text{ in.}$$ Eq. (5.5), Figure 5.2

Determine the section properties of the beam:

$$y_b = \frac{b_e h[h_b + (h/2)] + (b_w h_b^2 / 2)}{b_e h + b_w h_b} = 13.3 \text{ in.}$$ Table 5.3

$$I_b = \frac{b_w h_b^3}{12} + b_w h_b \left(y_b - \frac{h_b}{2} \right)^2 + \frac{b_e h^3}{12} + b_e h \left(h_b + \frac{h}{2} - y_b \right)^2 = 39{,}666 \text{ in.}^4$$

Determine the moment inertia of the slab using the largest ℓ_2 because this results in the maximum I_s, and hence, the smallest α_f:

$$I_s = \frac{\ell_2 h^3}{12} = \frac{[(25.0 \times 12/2) + (24.0/2)] \times 8.5^3}{12} = 8{,}291 \text{ in.}^4$$

Therefore, assuming the same concrete mixture is used in the slab and beams $(E_{cs} = E_{cb})$:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{39{,}666}{8{,}291} = 4.8 > 0.8$$ Eq. (5.2)

Thus, the assumption that $\alpha_f \geq 0.8$ is correct.

For interior panels:

$$h = \frac{\ell_n}{33} = \frac{276.0}{33} = 8.4 \text{ in.}$$ Table 5.2

Try an 8.5-in. slab thickness for all panels. This thickness is greater than the minimum thickness of 5.0 in. prescribed in ACI 8.3.1.1(a).

Step 2 – Determine a preliminary slab thickness based on two-way shear strength requirements Sects. 5.3.3, 5.4.4

Because of the edge beams, two-way shear is not critical at the edge and corner columns.

Check two-way shear requirements at an interior column assuming the lateral load effects are resisted by the exterior frames, that is, the interior columns are subjected to gravity load effects only. The two-way shear stress due to gravity load moment transfer at an interior column is not considered at this stage.

$$\text{Dead load of slab} = (8.5/12) \times 150.0 = 106.3 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$$

Live load $= 65.0$ lb/ft^2

Maximum $q_u = 1.2q_D + 1.6q_L = [1.2 \times (106.3 + 10.0)] + (1.6 \times 65.0) = 243.6$ lb/ft^2 ACI Eq. (5.3.1b)

$d = 8.5 - 1.25 = 7.25$ in.

Section properties of the critical section:

$b_1 = b_2 = 24.0 + 7.25 = 31.25$ in. Table 5.11, Case 1

$A_c = 2(b_1 + b_2)d = 2 \times (2 \times 31.25) \times 7.25 = 906.3$ in.^{2}

Factored shear force at the critical section:

$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) = 243.6 \times [(25.0 \times 23.5) - (31.25 / 12)^2] / 1{,}000 = 141.5$ kips Figure 5.12

Maximum shear stress, v_u:

$$v_u = \frac{V_u}{A_c} = \frac{141{,}500}{906.3} = 156.1 \text{ psi}$$ Eq. (5.14)

Assuming no shear reinforcement is used, the governing design two-way shear strength is equal to the following for a square, interior column:

$$\phi v_c = \phi 4 \lambda_s \lambda \sqrt{f'_c} = 0.75 \times 4 \times 1.0 \times 1.0 \sqrt{4{,}000} = 189.7 \text{ psi} > v_u = 156.1 \text{ psi}$$ Eq. (5.32)

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (7.25 / 10)}} = 1.1 > 1.0, \text{ use } 1.0$$ Eq. (5.30)

and

$\lambda = 1.0$ for normalweight concrete Table 5.20

Preliminary calculations indicate a slab thickness equal to 8.5 in. is adequate for serviceability and shear strength.

A final slab thickness is established once two-way shear requirements are checked at all columns, including the shear stress due to the bending moments transferred from the slab (see Sect. 5.4.4).

5.8.3 Example 5.3 – Determination of Minimum Slab Thickness: Two-way Beam-Supported Slab System, Building #1 (Framing Option C)

Determine the minimum slab thickness for the two-way beam-supported slab system in Figure 1.1 (Framing Option C) at a typical floor with 28 in. by 24 in. beams and 24 in. by 24 in. columns. Assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the minimum slab thickness based on serviceability requirements ACI 8.3.1.2

The longest clear span length, ℓ_n, measured face-to-face of beams is equal to the following:

$\ell_n = (25.0 \times 12) - 28.0 = 272.0$ in.

The minimum slab thickness is determined on the basis of the stiffness ratio α_{fm}. Because the slab thickness is not known at this stage, α_f and α_{fm} cannot be determined. In lieu of assuming a slab thickness, a minimum slab thickness is determined assuming $\alpha_{fm} > 2.0$; this assumption will be checked after the slab thickness has been calculated.

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{200,000}\right)}{36 + 9\beta} = \frac{272.0 \times \left(0.8 + \frac{60,000}{200,000}\right)}{36 + \{9 \times [(25.0 \times 12) - 28.0] / [(23.5 \times 12) - 28.0]\}} = 6.6 \text{ in.} > 3.5 \text{ in.} \qquad \text{Table 5.6}$$

Try a 7.0-in. slab thickness and calculate α_f for the beams and α_{fm} for the panels.

- Edge beams on column lines 1 and 7

  Determine the effective slab width, b_e:

$$b_e = b_w + \text{lesser of } \begin{cases} h_b \\ 4h \end{cases} = 28.0 + \text{lesser of } \begin{cases} 24.0 - 7.0 = 17.0 \\ 4 \times 7.0 = 28.0 \end{cases} = 28.0 + 17.0 = 45.0 \text{ in.} \qquad \text{Eq. (5.5), Figure 5.2}$$

  Determine the section properties of the beam:

$$y_b = \frac{b_e h[h_b + (h/2)] + (b_w h_b^2 / 2)}{b_e h + b_w h_b} = 13.3 \text{ in.} \qquad \text{Table 5.3}$$

$$I_b = \frac{b_w h_b^3}{12} + b_w h_b\left(y_b - \frac{h_b}{2}\right)^2 + \frac{b_e h^3}{12} + b_e h\left(h_b + \frac{h}{2} - y_b\right)^2 = 40,047 \text{ in.}^4$$

  Determine the moment inertia of the slab, I_s:

$$I_s = \frac{\ell_2 h^3}{12} = \frac{[(25.0 \times 12/2) + (24.0/2)] \times 7.0^3}{12} = 4,631 \text{ in.}^4$$

  Therefore,

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{40,047}{4,631} = 8.7 \qquad \text{Eq. (5.2)}$$

- Edge beams on column lines A and E

  Because the beam size on column lines 1 and 7 is the same as that on column lines A and E, $I_b = 40,047$ in.^{4}

$$I_s = \frac{\ell_2 h^3}{12} = \frac{[(23.5 \times 12/2) + (24.0/2)] \times 7.0^3}{12} = 4,373 \text{ in.}^4$$

  Therefore,

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{40,047}{4,373} = 9.2 \qquad \text{Eq. (5.2)}$$

- Interior beams on column lines 2 through 6

  Determine the effective slab width, b_e:

$$b_e = b_w + \text{lesser of}\begin{cases}2h_b\\ \\ 8h\end{cases} = 28.0 + \text{lesser of}\begin{cases}2\times(24.0-7.0)=34.0\\ \\ 8\times 7.0 = 56.0\end{cases} = 28.0 + 34.0 = 62.0 \text{ in.}$$ Eq. (5.6), Figure 5.5

  Determine the section properties of the beam:

$$y_b = \frac{b_e h[h_b + (h/2)] + (b_w h_b^2/2)}{b_e h + b_w h_b} = 14.2 \text{ in.}$$ Table 5.3

$$I_b = \frac{b_w h_b^3}{12} + b_w h_b\left(y_b - \frac{h_b}{2}\right)^2 + \frac{b_e h^3}{12} + b_e h\left(h_b + \frac{h}{2} - y_b\right)^2 = 45{,}927 \text{ in.}^4$$

  Determine the moment inertia of the slab, I_s:

$$I_s = \frac{\ell_2 h^3}{12} = \frac{(25.0\times 12)\times 7.0^3}{12} = 8{,}575 \text{ in.}^4$$

  Therefore,

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{45{,}927}{8{,}575} = 5.4$$ Eq. (5.2)

- Interior beams along column lines B through D

  Because the beam size on column lines 2 through 6 is the same as that on column lines B through D, $I_b = 45{,}927 \text{ in.}^4$

$$I_s = \frac{\ell_2 h^3}{12} = \frac{(23.5\times 12)\times 7.0^3}{12} = 8{,}061 \text{ in.}^4$$

  Therefore,

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{45{,}927}{8{,}061} = 5.7$$ Eq. (5.2)

Determine α_{fm} for each type of panel:

- Corner panel: $\alpha_{fm} = (9.2+5.4+5.7+8.7)/4 = 7.3$
- Edge panel on column line A or E: $\alpha_{fm} = (9.2+5.4+5.7+5.4)/4 = 6.4$
- Edge panel on column line 1 or 7: $\alpha_{fm} = (5.7+5.4+5.7+8.7)/4 = 6.4$
- Interior panel: $\alpha_{fm} = (5.7+5.4+5.7+5.4)/4 = 5.6$

Because $\alpha_{fm} > 2.0$ for all panels, the initial assumption is correct.

Try a 7.0-in. slab thickness for all panels. This thickness is greater than the minimum thickness of 3.5 in. prescribed in ACI 8.3.1.2.

Step 2 – Determine a preliminary slab thickness based on two-way shear strength requirements Sects. 5.3.3, 5.4.4

Because of the column-line beams, two-way shear in the slab is not critical at any of the columns.

Where the DDM is used, column-line beams must be designed to resist 100 percent of the factored shear force due to gravity loads where $\alpha_{f1}\ell_2 / \ell_1 \geq 1.0$. This criterion is satisfied for all the beams in this example, which means the factored shear forces in the slab around the columns are equal to zero. The beams must be designed for the combined effects due to gravity and lateral loads.

Preliminary calculations indicate a slab thickness equal to 7.0 in. is adequate for serviceability and shear strength.

5.8.4 Example 5.4 – Determination of Minimum Slab Thickness: Flat Slab System With Edge Beams, Building #1 (Framing Option D)

Determine the minimum slab thickness for the flat slab system in Figure 1.1 (Framing Option D) at a typical floor with standard drop panels, 28 in. by 24 in. edge beams, and 24 in. by 24 in. columns. Assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the minimum slab thickness based on serviceability requirements ACI 8.3.1.1

Check the provided drop panel dimensions:

- At interior columns:

  $\text{Minimum drop panel length for the longest span length} = \ell_1 / 3 = 25.0 / 3 = 8.3 \text{ ft} < 8.5 \text{ ft}$ Figure 5.4

- At edge and corner columns:

  $\text{Minimum drop panel length for the longest span length} = \ell_1 / 3 = 25.0 / 6 = 4.17 \text{ ft} < 5.25 - [24.0 / (2 \times 12)] = 4.25 \text{ ft}$

Thus, the requirements of ACI 8.2.4(b) are satisfied for the plan dimensions of the drop panels. Once a slab thickness is determined, the projection of the drop panel below the slab must be equal to at least one-fourth of the slab thickness in order to satisfy the requirement in ACI 8.2.4(a).

The longest clear span length, ℓ_n, measured face-to-face of columns is equal to the following:

$$\ell_n = (25.0 \times 12) - 24.0 = 276.0 \text{ in.}$$ Figure 5.23

For exterior panels with edge beams:

$$h = \frac{\ell_n}{36} = \frac{276.0}{36} = 7.7 \text{ in.}$$ Table 5.4

According to the footnote in Table 5.4, this minimum slab thickness is based on the assumption $\alpha_f \geq 0.8$ for the edge beams. Check this assumption for the 28 in. by 24 in. edge beams assuming an 8.0-in.-thick slab.

Determine the effective slab width, b_e:

$$b_e = b_w + \text{lesser of} \begin{cases} h_b \\ 4h \end{cases} = 28.0 + \text{lesser of} \begin{cases} 24.0 - 8.0 = 16.0 \\ 4 \times 8.0 = 32.0 \end{cases} = 28.0 + 16.0 = 44.0 \text{ in.}$$ Eq. (5.5), Figure 5.2

Determine the section properties of the beam:

$$y_b = \frac{b_e h[h_b + (h/2)] + (b_w h_b^2/2)}{b_e h + b_w h_b} = 13.3 \text{ in.}$$ Table 5.3

$$I_b = \frac{b_w h_b^3}{12} + b_w h_b\left(y_b - \frac{h_b}{2}\right)^2 + \frac{b_e h^3}{12} + b_e h\left(h_b + \frac{h}{2} - y_b\right)^2 = 39{,}820 \text{ in.}^4$$

Determine the moment inertia of the slab using the largest ℓ_2 because this results in the maximum I_s, and hence, the smallest α_f:

$$I_s = \frac{\ell_2 h^3}{12} = \frac{[(25.0\times 12/2) + (24.0/2)]\times 8.0^3}{12} = 6{,}912 \text{ in.}^4$$

Therefore, assuming the same concrete mixture is used in the slab and beams $(E_{cs} = E_{cb})$:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{39{,}820}{6{,}912} = 5.8 > 0.8$$ Eq. (5.2)

Thus, the assumption that $\alpha_f \geq 0.8$ is correct.

For interior panels:

$$h = \frac{\ell_n}{36} = \frac{276.0}{36} = 7.7 \text{ in.}$$ Table 5.4

Try an 8.0-in. slab thickness for all panels. This thickness is greater than the minimum thickness of 4.0 in. prescribed in ACI 8.3.1.1(b).

Step 2 – Determine a preliminary slab thickness based on two-way shear strength requirements Sects. 5.3.3, 5.4.4

Because of the edge beams, two-way shear is not critical at the edge and corner columns.

Check two-way shear requirements at an interior column assuming the lateral load effects are resisted by the exterior frames, that is, the interior columns are subjected to gravity load effects only. The two-way shear stress due to gravity load moment transfer at an interior column is not considered at this stage.

- Check the shear stress at the critical section located a distance equal to $d/2$ from the face of the column.

  Minimum drop panel thickness $= h/4 = 8.0/4 = 2.0$ in. Figure 5.4

  For formwork economy, select a 2.25-in. drop panel height. Table 5.5

  Dead load of slab $= (8.0/12)\times 150.0 = 100.0 \text{ lb/ft}^2$

  Dead load of drop panel projection below the slab $= (2.25/12)\times 150.0\times 8.5\times 8.5/1{,}000 = 2.0$ kips

  Superimposed dead load $= 10.0 \text{ lb/ft}^2$

  Live load $= 65.0 \text{ lb/ft}^2$

Maximum $q_u = 1.2q_D + 1.6q_L = [1.2 \times (100.0 + 10.0)] + (1.6 \times 65.0) = 236.0 \text{ lb/ft}^2$ ACI Eq. (5.3.1b)

$d_2 = 10.25 - 1.25 = 9.0$ in. Figure 5.13

Section properties of the critical section:

$b_1 = b_2 = 24.0 + 9.0 = 33.0$ in. Table 5.11, Case 1

$A_c = 2(b_1 + b_2)d_2 = 2 \times (2 \times 33.0) \times 9.0 = 1{,}188.0 \text{ in.}^2$

Factored shear force at the critical section:

$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) + \text{factored weight of the drop panel projection below the slab}$

$= \{236.0 \times [(25.0 \times 23.5) - (33.0 / 12)^2] / 1{,}000\} + (1.2 \times 2.0) = 139.3 \text{ kips}$

Maximum shear stress, v_u:

$$v_u = \frac{V_u}{A_c} = \frac{139{,}300}{1{,}188.0} = 117.3 \text{ psi}$$ Eq. (5.14)

Assuming no shear reinforcement is used, the governing design two-way shear strength is equal to the following for a square, interior column:

$$\phi v_c = \phi 4 \lambda_s \lambda \sqrt{f'_c} = 0.75 \times 4 \times 1.0 \times 1.0 \sqrt{4{,}000} = 189.7 \text{ psi} > v_u = 117.3 \text{ psi}$$ Eq. (5.32)

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d_2 / 10)}} = \sqrt{\frac{2}{1 + (9.0 / 10)}} = 1.03 > 1.0, \text{ use } 1.0$$ Eq. (5.30)

and

$\lambda = 1.0$ for normalweight concrete Table 5.20

- Check the shear stress at the critical section located a distance equal to $d / 2$ from the face of the drop panel.

$d_1 = 8.0 - 1.25 = 6.75$ in. Figure 5.13

Section properties of the critical section:

$b_1 = b_2 = (8.5 \times 12) + 6.75 = 108.75$ in. Table 5.11, Case 1

$A_c = 2(b_1 + b_2)d_1 = 2 \times (2 \times 108.75) \times 6.75 = 2{,}936.3 \text{ in.}^2$

Factored shear force at the critical section:

$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) = 236.0 \times [(25.0 \times 23.5) - (108.75 / 12)^2] / 1{,}000 = 119.3 \text{ kips}$ Figure 5.13

Maximum shear stress, v_u:

$$v_u = \frac{V_u}{A_c} = \frac{119{,}300}{2{,}936.3} = 40.6 \text{ psi} < \phi v_c = 189.7 \text{ psi}$$ Eq. (5.14)

Preliminary calculations indicate a slab thickness equal to 8.0 in. is adequate for serviceability and shear strength.

A final slab thickness is established once two-way shear requirements are checked at all columns, including the shear stress from the bending moments transferred from the slab (see Sect. 5.4.4).

5.8.5 Example 5.5 – Determination of Minimum Thickness: Two-way Joist System, Building #1 (Framing Option E)

Determine the minimum overall thickness for the two-way joist system in Figure 1.1 (Framing Option E) at a typical floor assuming a 3-ft module, a 4.5-in.-thick slab, and 30 in. by 30 in. columns. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the minimum overall thickness based on serviceability requirements ACI 8.3.1.1

Two-way joist systems are considered as flat slabs for purposes of design. Sect. 5.2.5

The longest clear span length, ℓ_n, measured face-to-face of columns is equal to the following:

$$\ell_n = (50.0 \times 12) - 30.0 = 570.0 \text{ in.}$$ Figure 5.23

For exterior panels without edge beams:

$$h = \frac{\ell_n}{33} = \frac{570.0}{33} = 17.3 \text{ in.}$$ Table 5.4

A 20-in. dome depth plus a 4.5-in. slab provides an equivalent thickness $t_e = 17.4 \text{ in.} > 17.3 \text{ in.}$ Table 5.9

The equivalent thickness is determined as follows:

$$y_b = \frac{\dfrac{b_r(h_r + h)^2}{2} + \dfrac{h_r^3}{18} + b_d h\left(h_r + \dfrac{h}{2}\right)}{b_r(h_r + h) + \dfrac{h_r^2}{12} + b_d h} = 16.7 \text{ in.}$$ Table 5.8, Figure 5.7

where $b_r = 6.0$ in., $h_r = 20.0$ in., $h = 4.5$ in., and $b_d = 30.0$ in.

$$I_g = \frac{b_d h^3}{12} + b_d h\left(h_r + \frac{h}{2} - y_b\right)^2 + \frac{b_r(h_r + h)^3}{12} + b_r(h_r + h)\left(y_b - \frac{h_r + h}{2}\right)^2 + \frac{h_r^4}{216} + \frac{h_r^2}{12}\left(\frac{2h_r}{3} - y_b\right)^2 = 15{,}769 \text{ in.}^4$$

$$t_e = \left(\frac{12 I_g}{b_r + b_d}\right)^{1/3} = \left(\frac{12 \times 15{,}769}{6.0 + 30.0}\right)^{1/3} = 17.4 \text{ in.}$$ Eq. (5.8)

Try a 20-in. dome plus a 4.5-in. slab for all panels.

Step 2 – Determine a preliminary overall thickness based on two-way shear strength requirements Sects. 5.3.3, 5.4.4

Check the shear stress at the critical section located a distance equal to $d / 2$ from the face of an interior column.

$\text{Dead load of joists plus solid heads} = 173.3 \text{ lb/ft}^2$

$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$

$\text{Live load} = 65.0 \text{ lb/ft}^2$

Maximum $q_u = 1.2q_D + 1.6q_L = [1.2 \times (173.3 + 10.0)] + (1.6 \times 65.0) = 324.0 \text{ lb/ft}^2$ ACI Eq. (5.3.1b)

$d = 24.5 - 1.25 = 23.25 \text{ in.}$

Section properties of the critical section at an interior column:

$b_1 = b_2 = 30.0 + 23.25 = 53.25 \text{ in.}$ Table 5.11, Case 1

$A_c = 2(b_1 + b_2)d = 2 \times (2 \times 53.25) \times 23.25 = 4{,}952.3 \text{ in.}^2$

Factored shear force at the critical section:

$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) = 324.0 \times [(50.0 \times 31.33) - (53.25 / 12)^2] / 1{,}000 = 501.2 \text{ kips}$

Maximum shear stress, v_u:

$$v_u = \frac{V_u}{A_c} = \frac{501{,}200}{4{,}952.3} = 101.2 \text{ psi}$$ Eq. (5.14)

Assuming no shear reinforcement is used, the governing design two-way shear strength is equal to the following for a square, interior column:

$$\phi v_c = \phi 4 \lambda_s \lambda \sqrt{f'_c} = 0.75 \times 4 \times 0.78 \times 1.0 \sqrt{4{,}000} = 148.0 \text{ psi} > v_u = 101.2 \text{ psi}$$ Eq. (5.32)

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (23.25 / 10)}} = 0.78$$ Eq. (5.30)

and

$\lambda = 1.0 \text{ for normalweight concrete}$ Table 5.20

Preliminary calculations indicate an overall thickness of 24.5 in. is adequate for serviceability and shear strength.

A final overall thickness is established once two-way shear strength requirements are satisfied at all columns. Because the two-way joist system is also part of the LFRS and is subjected to lateral load effects, the maximum v_u is larger than that calculated above for gravity loads only.

5.8.6 Example 5.6 – Determination of Minimum Slab Thickness: Flat Plate System, Building #3

Determine the minimum slab thickness for the flat plate system in Figure 1.3 at the second-floor level with 24 in. by 24 in. interior and corner columns and 16 in. by 28 in. edge columns. Assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.3.

Step 1 – Determine the minimum slab thickness based on serviceability requirements ACI 8.3.1.1

The longest clear span length, ℓ_n, is equal to the following for an exterior panel:

$\ell_n = (22.0 \times 12) - (24.0 / 2) - (16.0 / 2) = 244.0$ in. Figure 5.23

For exterior panels without edge beams:

$$h = \frac{\ell_n}{30} = \frac{244.0}{30} = 8.1 \text{ in.}$$ Table 5.2

For interior panels:

$$h = \frac{\ell_n}{33} = \frac{(22.0 \times 12) - 24.0}{33} = 7.3 \text{ in.}$$

Try an 8.0-in. slab thickness for all panels. This thickness is slightly less than that required by ACI 8.3.1.1 and is greater than the minimum thickness of 5.0 in. prescribed in ACI 8.3.1.1(a).

Step 2 – Determine a preliminary slab thickness based on two-way shear strength requirements Sects. 5.3.3, 5.4.4

Check two-way shear requirements at a 16 in. by 28 in. edge column bending perpendicular to the edge for gravity loads only (the shear walls in the building resist lateral load effects).

$\text{Dead load of slab} = (8.0 / 12) \times 150.0 = 100.0 \text{ lb/ft}^2$

$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$

$\text{Live load} = 40.0 \text{ lb/ft}^2$

Maximum $q_u = 1.2q_D + 1.6q_L = \{1.2 \times (100.0 + 10.0)] + (1.6 \times 40.0) = 196.0 \text{ lb/ft}^2$ ACI Eq. (5.3.1b)

$d = 8.0 - 1.25 = 6.75$ in.

The maximum combined shear stress occurs at edge columns B1, C1, B5, and C5 bending perpendicular to the edge.

Section properties of the critical section:

$b_1 = c_1 + (d / 2) = 16.0 + (6.75 / 2) = 19.38$ in. Table 5.11, Case 3

$b_2 = c_2 + d = 28.0 + 6.75 = 34.75$ in.

$A_c = (2b_1 + b_2)d = [(2 \times 19.38) + 34.75] \times 6.75 = 496.2 \text{ in.}^2$

$$\frac{J_c}{c_{AB}} = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} = 4{,}070 \text{ in.}^3$$

Factored shear force at the critical section:

$$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) = 196.0 \times \left\{\left[\left(\frac{22.0}{2} + \frac{16.0}{2 \times 12}\right) \times \left(\frac{20.5 + 19.0}{2}\right)\right] - \frac{19.38 \times 34.75}{144}\right\} / 1{,}000 = 44.3 \text{ kips}$$

Assuming the DDM can be used, the required M_{sc} at this location is equal to $0.30M_o$. Sect. 5.3.4

$$\ell_2 = \frac{20.5 + 19.0}{2} = 19.8 \text{ ft}$$

$$\ell_n = 22.0 - \frac{16.0}{2 \times 12} - \frac{24.0}{2 \times 12} = 20.3 \text{ ft}$$

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} = \frac{196.0 \times 19.8 \times 20.3^2}{8 \times 1{,}000} = 199.9 \text{ ft-kips} \qquad \text{Eq. (5.17)}$$

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.33 \qquad \text{ACI Eq. (8.4.4.2.2)}$$

Maximum shear stress, $v_{u|AB}$:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{44{,}300}{496.2} + \frac{0.33 \times (0.30 \times 199.9) \times 12{,}000}{4{,}070} = 89.3 + 58.4 = 147.7 \text{ psi} \qquad \text{Eq. (5.14)}$$

$$\phi v_c = \text{ least of } \begin{cases} \phi 4 \lambda_s \lambda \sqrt{f_c'} = 189.7 \text{ psi} > v_{u|AB} = 147.7 \text{ psi} \\ \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s \lambda \sqrt{f_c'} = 200.3 \text{ psi} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s \lambda \sqrt{f_c'} = 225.6 \text{ psi} \end{cases} \qquad \text{Eq. (5.32)}$$

where $\lambda_s = 1.0,\ \lambda = 1.0,\ \beta = 28.0 / 16.0 = 1.8,\ \alpha_s = 30,$ and $b_o = (2 \times 19.38) + 34.75 = 73.5$ in.

Preliminary calculations indicate a slab thickness equal to 8.0 in. is adequate for serviceability and shear strength.

A final slab thickness is established once two-way shear requirements are checked at all columns (see Sect. 5.4.4).

5.8.7 Example 5.7 – Determination of Required Flexural Reinforcement: Flat Plate System, Building #1 (Framing Option A), SDC A

Determine the required flexural reinforcement in an interior design strip in the north-south direction for the flat plate system in Figure 1.1 (Framing Option A) at the second-floor level with a 9.5-in.-thick slab and 24 in. by 24 in. columns (see Figure 5.41). Assume the Site Class is C, which results in the building being assigned to SDC A [see Example 3.5, Part (a)]. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Step 1 – Determine the widths of the column strip and middle strip ACI 8.4.1.5, 8.4.1.6

$$\text{Width of column strip} = \text{lesser of} \begin{cases} \ell_1 / 2 = 23.5 / 2 = 11.75 \text{ ft} \\ \ell_2 / 2 = 25.0 / 2 = 12.50 \text{ ft} \end{cases} \qquad \text{Figure 5.8}$$

$$\text{Width of middle strip} = 25.0 - 11.75 = 13.25 \text{ ft}$$

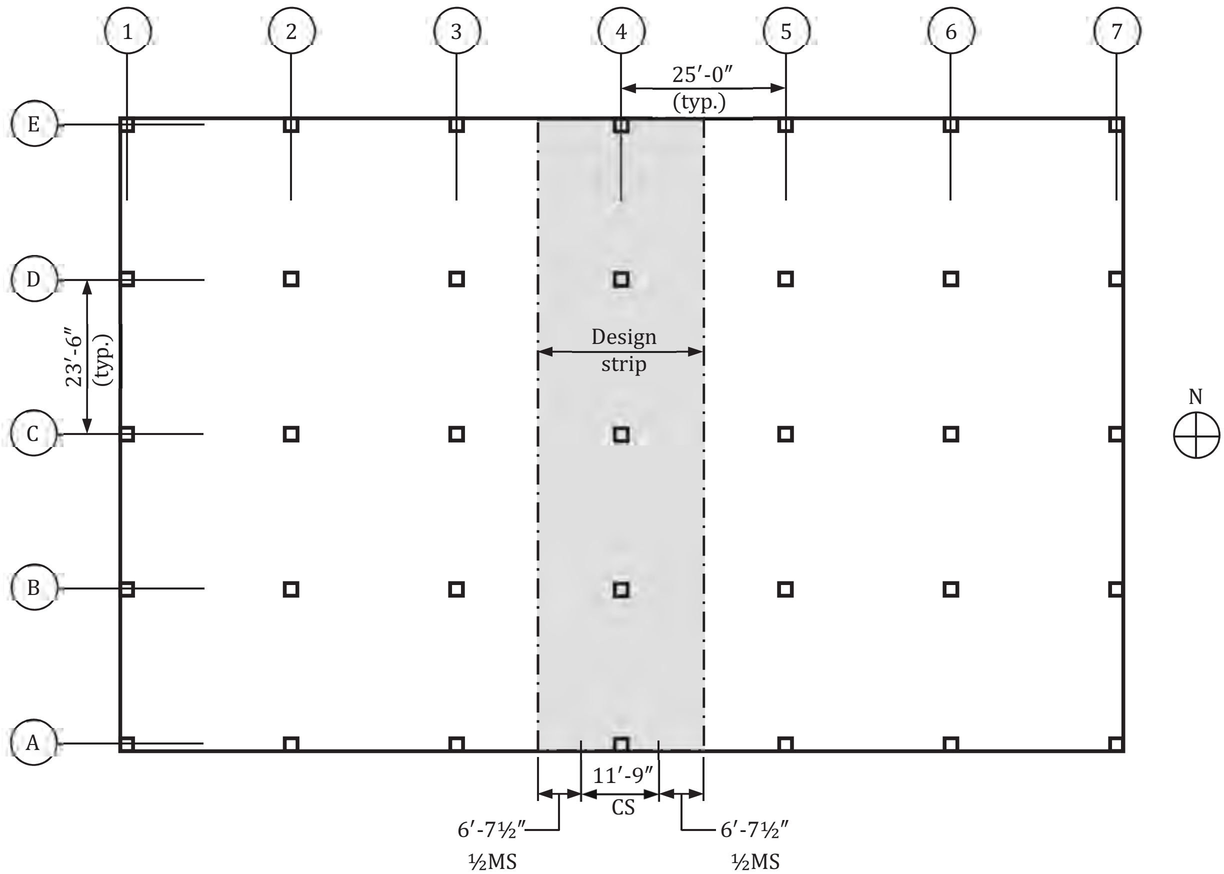


Figure 5.41 Design strips for the flat plate system in Example 5.7.

Step 2 – Check if the Direct Design Method can be used to determine gravity load bending moments Sect. 5.3.4

Check the following limitations: Figure 5.22

- There must be 3 or more continuous spans in each direction…There are 4 and 6 continuous spans in the north-south and east-west directions, respectively.
- Successive panel lengths measured center-to-center of supports in each direction must not differ by more than one-third the longer span…All the panel lengths are equal in each direction.
- Panels must be rectangular with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2… $\text{Largest ratio of panel lengths} = 25.0 / 23.5 = 1.1 < 2.0.$
- Column offset must not exceed 10 percent of the span in the direction of the offset from either axis between centerlines of successive columns…None of the columns are offset.
- All loads must be due to gravity only and uniformly distributed over an entire panel…This method will be used only for gravity loads, which are uniformly distributed over the entire floor system.
- The unfactored live load must not exceed 2 times the unfactored dead load… $\text{Unfactored live load} = 65\ \text{lb/ft}^2$; assuming an 9.5-in.-thick slab, $\text{unfactored dead load} = (9.5 \times 150.0 / 12) + 10 = 129\ \text{lb/ft}^2$; $L / D = 0.5 < 2.$
- For a panel with beams between supports on all sides, the following equation must be satisfied for the beams in the two perpendicular directions: $0.2 \le \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 \le 5.0$…There are no beams in this floor system, so this limitation is not applicable.

Because all the limitations are satisfied, the DDM can be used to determine the bending moments in the column and middle strips.

Step 3 – Determine the total factored static moment in each span

Dead load of slab $= (9.5/12) \times 150.0 = 118.8\ \text{lb/ft}^2$

Superimposed dead load $= 10.0\ \text{lb/ft}^2$

Live load $= 65.0\ \text{lb/ft}^2$

$\ell_2 = 25.0\ \text{ft}$ Figure 5.8

$\ell_n = 23.5 - (24.0/12) = 21.5\ \text{ft}$ Figure 5.23

Because the flat plate is part of the LFRS, service dead and live load bending moments are determined, which will be combined with those due to the effects from lateral forces using the appropriate load combinations:

Dead load: $M_{oD} = \dfrac{q_D \ell_2 \ell_n^2}{8} = \dfrac{(118.8 + 10.0) \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 186.1\ \text{ft-kips}$ Eq. (5.17)

Live load: $M_{oL} = \dfrac{q_L \ell_2 \ell_n^2}{8} = \dfrac{65.0 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 93.9\ \text{ft-kips}$

These total factored static moments are the same for all spans of the interior design strip in the direction of analysis.

Step 4 – Distribute the total factored static moment to the column strips and middle strips Table 5.16

Design moment coefficients for a flat plate system without edge beams are given in Table 5.16. A summary of the service dead and live load bending moments in the column strip and middle strip in an end span and interior span is given in Table 5.25.

Table 5.25 Bending Moments (ft-kips) due to Service Dead and Live Loads at the Second-Floor Level for the Flat Plate System in Example 5.7

Design Strip	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Column strip	$0.26M_{oD} = -48.4$ $0.26M_{oL} = -24.4$	$0.31M_{oD} = 57.7$ $0.31M_{oL} = 29.1$	$0.53M_{oD} = -98.6$ $0.53M_{oL} = -49.8$	$0.21M_{oD} = 39.1$ $0.21M_{oL} = 19.7$	$0.49M_{oD} = -91.2$ $0.49M_{oL} = -46.0$
Middle strip	0	$0.21M_{oD} = 39.1$ $0.21M_{oL} = 19.7$	$0.17M_{oD} = -31.6$ $0.17M_{oL} = -16.0$	$0.14M_{oD} = 26.1$ $0.14M_{oL} = 13.2$	$0.16M_{oD} = -29.8$ $0.16M_{oL} = -15.0$

Step 5 – Determine the bending moments due to wind forces ACI 6.6

Design wind forces in the north-south direction on Building #1 are determined in Example 3.1 and are given in Table 3.10.

A three-dimensional model of the building was constructed using Reference 14 and a linear first-order analysis was performed using the north-south wind loads in Table 3.10 applied to the centroid of the building face at the roof and floor levels (see Figure 5.42). In the model, the columns are fixed at the base.

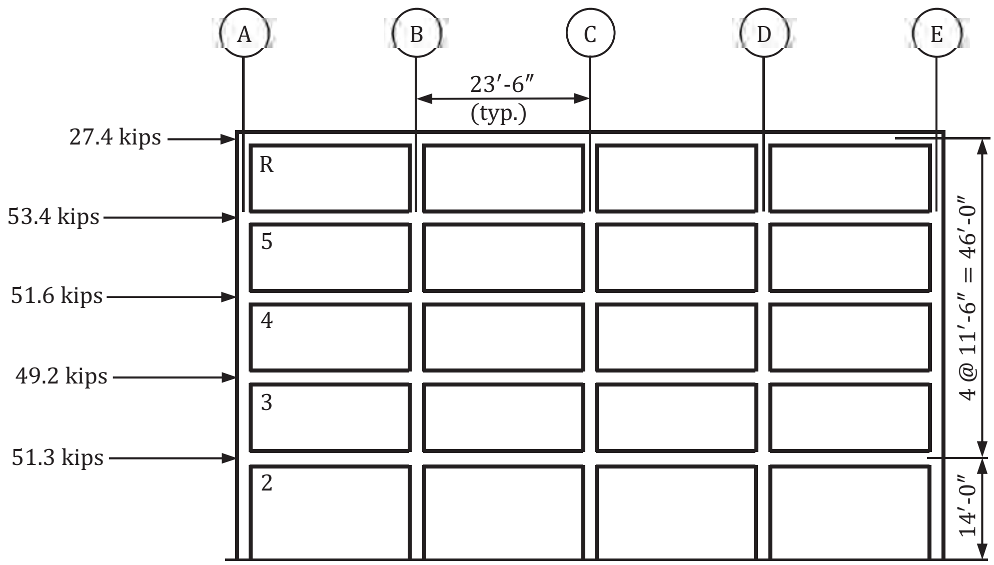


Figure 5.42 Total wind loads applied to Building #1.

The following reduced moments of inertia are used to account for cracked sections: ACI Table 6.6.3.1.1(a)

- Columns: $I = 0.70I_g$
- Slabs: $I = 0.25I_g$

A finite element analysis was performed to determine the extent over which the slab resists the effects from the wind loads assuming all the frames in the north-south direction are part of the LFRS. From the analysis, it was determined the majority of the wind load effects at an interior design strip are resisted by slab strips centered on the columns with widths approximately equal to the column strip width; thus, all the wind load effects are assigned to the column strips in the direction of analysis in this example.

The bending moments in the column strip due to wind loads are given in Table 5.26 for end and interior spans at the second-floor level. The "plus-minus" sign preceding the tabulated values signifies the wind loads can act in both the north direction and the south direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the east elevation of the building).

Table 5.26 Bending Moments (ft-kips) due to Wind Loads at the Second-Floor Level for the Flat Plate System in Example 5.7

Design Strip	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Column strip	±16.6	—	±13.8	—	±12.0

Step 6 – Determine the bending moments due to seismic forces

As noted in the example statement, it is assumed the Site Class is C for this building. From Example 3.5, Part (a), the building is assigned to SDC A for this case. Therefore, effects due to seismic forces need not be considered.

It can be determined the lateral forces based on the general structural integrity requirements of ASCE/SEI 1.4.2 are less than those due to wind forces, and, thus, need not be considered in this example.

Step 7 – Determine the combined factored bending moments due to gravity and wind forces ACI Table 5.3.1

The design bending moments from the governing load combinations are given in Table 5.27. The gravity load moments from the DDM are combined with the bending moment from the lateral load analysis (ACI 8.4.1.9). It is evident at all locations that the maximum moments are equal to those from the combination of the gravity load effects.

Table 5.27 Design Bending Moments (ft-kips) at the Second-Floor Level for the Flat Plate System in Example 5.7

Load Combination	Location		End Span			Interior Span	
			1	2	3	4	5
			Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
$1.2D + 1.6L$	Column strip		−97.1	115.8	−198.0	78.4	−183.0
	Middle strip		0	78.4	−63.5	52.4	−59.8
$1.2D + 1.0W + 0.5L$	Column strip	SSR	−53.7	83.8	−157.0	56.8	−144.7
		SSL	−86.9	83.8	−129.4	56.8	−120.1
	Middle strip		0	56.8	−46.0	37.9	−43.3
$0.9D + 1.0W$	Column strip	SSR	−27.0	51.9	−102.5	35.2	−94.4
		SSL	−60.2	51.9	−74.9	35.2	−69.8
	Middle strip		0	35.2	−28.4	23.5	−26.8

Step 8 – Determine the required flexural reinforcement

The required flexural reinforcement is determined at the critical sections in the column strip and middle strip using the following equations where b is the width of the column strip or middle strip and $d = 9.5 - 1.25 = 8.25$ in.:

$$A_s = \frac{0.85 f'_c b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] \qquad \text{Eq. (5.35)}$$

$$R_n = \frac{M_u}{\phi b d^2} \qquad \text{Eq. (5.36)}$$

A summary of the required flexural reinforcement is given in Table 5.28. The provided areas of reinforcement are greater than or equal to the minimum required in accordance with ACI 8.6.1.1 and the number of bars are selected so the provided spacing of the reinforcing bars is less than the maximum required in ACI 8.7.2.2. It is evident the provided areas of reinforcement at all critical sections in the column strip and middle strip are less than the area of flexural reinforcement, $A_{s,t}$, corresponding to tension-controlled sections, which is determined by Eq. (5.41); therefore, all the critical sections are tension-controlled, which satisfies ACI 8.3.3.1.

Table 5.28 Required Flexural Reinforcement at the Second-Floor Level for the Flat Plate System in Example 5.7

Location			M_u (ft-kips)	b (in.)	A_s (in.^{2})*	Reinforcement*
End Span	Column strip	Exterior Negative	−97.1	141.0	2.67	9-#5
		Positive	115.8	141.0	3.20	11-#5
		First Interior Negative	−198.0	141.0	5.57	18-#5
	Middle strip	Exterior Negative	0	159.0	2.72	9-#5
		Positive	78.4	159.0	2.72	9-#5
		First Interior Negative	−63.5	159.0	2.72	9-#5
Interior Span	Column strip	Positive	78.4	141.0	2.41	8-#5
		Negative	−183.0	141.0	5.13	17-#5
	Middle strip	Positive	52.4	159.0	2.72	9-#5
		Negative	−59.8	159.0	2.72	9-#5

* *Min. A_s for the column strip* $= 0.0018 \times 141.0 \times 9.5 = 2.41$ in.^{2}
Min. A_s for the middle strip $= 0.0018 \times 159.0 \times 9.5 = 2.72$ in.^{2}
Max. spacing = lesser of ($2h$, 18.0 in.) = 18.0 in.
For $b = 141.0$ in., $141.0 / 18.0 = 7.8$ spaces, say minimum of 8 bars
For $b = 159.0$ in., $159.0 / 18.0 = 8.8$ spaces, say minimum of 9 bars
For the column strip: $A_{s,t} = 0.018bd = 20.9$ in.^{2}
For the middle strip: $A_{s,t} = 0.018bd = 23.6$ in.^{2}

Step 9 – Check that the flexural reinforcement is adequate for moment transfer requirements — ACI 8.4.2.2

- Edge columns

Determine the required reinforcement based on $M_{sc} = 0.30M_o$. — Sect. 5.3.4

$$M_{sc} = 0.30(1.2M_{oD} + 1.6M_{oL}) = 0.30 \times [(1.2 \times 186.1) + (1.6 \times 93.9)] = 112.1 \text{ ft-kips}$$

This moment is greater than 86.9 ft-kips, which is the moment due to the load combination — Table 5.27

$1.2D + 1.0W + 0.5L$.

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.62 \qquad \text{Eq.(5.9)}$$

where

$b_1 = c_1 + (d/2) = 24.0 + (8.25/2) = 28.13$ in. — Table 5.11, Case 3

$b_2 = c_2 + d = 24.0 + 8.25 = 32.25$ in.

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

The moment $\gamma_f M_{sc} = 0.62 \times 112.1 = 69.5 \text{ ft-kips}$ must be transferred over the effective slab width b_{slab}:

$b_{slab} = c_2 + 3h = 24.0 + (3 \times 9.5) = 52.5 \text{ in., say } 52.0 \text{ in.}$ Eq. (5.10)

Determine the required A_s:

$$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{69.5 \times 12}{0.9 \times 52.0 \times 8.25^2} = 0.262 \text{ ksi}$$ Eq. (5.36)

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4 \times 52.0 \times 8.25}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.262}{0.85 \times 4}}\right] = 1.95 \text{ in.}^2$$ Eq. (5.35)

This required area of steel is equivalent to 7-#5 bars. Provide the 7-#5 bars by concentrating 7 of the 9-#5 column strip bars within the 52.0-in. width over the column (see Table 5.28). For symmetry, add 2-#5 bars and check the bar spacing:

For 7-#5 within the 52.0-in. width, $\text{bar spacing} = 52.0 / 7 = 7.4 \text{ in.} < 18.0 \text{ in.}$

For 4-#5 within the $141.0 - 52.0 = 89.0\text{-in. width, bar spacing} = 89.0 / 4 = 22.3 \text{ in.} > 18.0 \text{ in.}$

Therefore, provide 2 additional bars within the 89.0-in. width to satisfy maximum spacing requirements. A total of 13-#5 bars are required at the edge columns within the column strip, with 7 of the 13-#5 bars concentrated within a width of 52.0 in.

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2 q_D + 1.6 q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$$

$$d = 9.5 - 1.25 = 8.25 \text{ in.}$$

$$A_c = (2b_1 + b_2)d = [(2 \times 28.13) + 32.25)] \times 8.25 = 730.2 \text{ in.}^2$$ Table 5.11, Case 3

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 258.6 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 32.25}{144}\right)\right\} / 1{,}000 = 80.8 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_{u(D+L)}}{A_c} = \frac{80{,}800}{730.2} = 110.7 \text{ psi} > \phi 2 \lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0 \sqrt{4{,}000} = 94.9 \text{ psi}$$ Eq. (5.38)

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (8.25 / 10)}} = 1.1 > 1.0, \text{ use } 1.0$$ Eq. (5.30)

and

$\lambda = 1.0$ for normalweight concrete Table 5.20

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi\alpha_s f_y} = \dfrac{5 \times 110.7 \times 52.0 \times [(2 \times 28.13) + 32.25)]}{0.75 \times 30 \times 60{,}000} = 1.89 \text{ in.}^2 \end{cases} \qquad \text{Eq. (5.37)}$$

Provided A_s within $b_{slab} = 7 \times 0.31 = 2.17 \text{ in.}^2 > A_{s,min} = 1.89 \text{ in.}^2$

(2) Load combination: $1.2D + 1.0W + 0.5L$

$$q_u = 1.2q_D + 0.5q_L = [1.2 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 187.1 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 187.1 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 32.25}{144}\right)\right\} / 1{,}000 = 58.5 \text{ kips}$$

$$V_{u(W)} = (16.6 + 13.8) / 21.5 = 1.4 \text{ kips} \qquad \text{Table 5.26}$$

Therefore,

$$v_{uv} = \frac{V_{u(D+L)} + V_{u(W)}}{A_c} = \frac{58{,}500 + 1{,}400}{730.2} = 82.0 \text{ psi}$$

$$< \phi 2\lambda_s\lambda\sqrt{f'_c} = 0.75 \times 2 \times 1.0 \times 1.0\sqrt{4{,}000} = 94.9 \text{ psi} \qquad \text{Eq. (5.38)}$$

Provided A_s within $b_{slab} = 7 \times 0.31 = 2.17 \text{ in.}^2 > A_{s,min} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2$

Therefore, no additional reinforcement is required to satisfy the minimum reinforcement requirements of ACI 8.6.1.2.

Reinforcement details for the top bars at the edge columns are given in Figure 5.43.

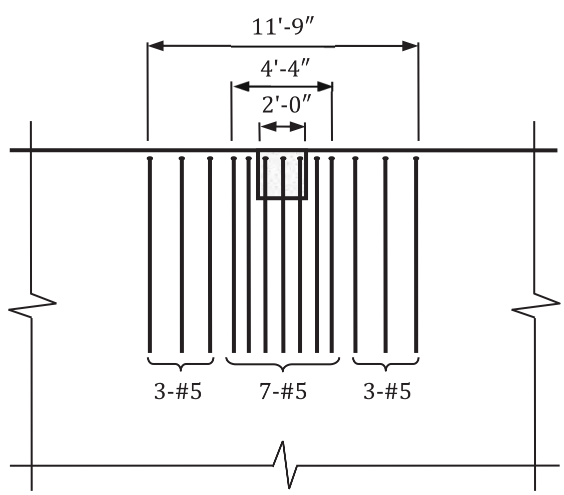


Figure 5.43 Reinforcement details for the top reinforcing bars at the edge columns in the flat plate system in Example 5.7.

- Interior columns

  Determine the required reinforcement based on the largest transfer moment at the interior columns.

  In lieu of a more exact analysis, the factored slab moment, M_{sc}, transferred to an interior column due to gravity load effects and combined gravity and wind load effects can be determined from the following equations where the spans in the direction of analysis and perpendicular to the direction of analysis are equal.

  For gravity loads only:

  $$M_{sc(L)} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (1.6 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 42.1 \text{ ft-kips} \quad \text{Eq. (5.21)}$$

  For gravity and wind loads at the first interior columns:

  $M_{sc(L)} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (0.5 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 13.2 \text{ ft-kips}$ due to gravity load effects

  $M_{sc(W)} = 13.8 + 12.0 = 25.8 \text{ ft-kips}$ due to wind load effects — Table 5.26

  Total $M_{sc} = 13.2 + 25.8 = 39.0 \text{ ft-kips}$

  For gravity and wind loads at the interior column:

  $M_{sc(L)} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (0.5 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 13.2 \text{ ft-kips}$ due to gravity load effects

  $M_{sc(W)} = 2 \times 12.0 = 24.0 \text{ ft-kips}$ due to wind load effects

  Total $M_{sc} = 13.2 + 24.0 = 37.2 \text{ ft-kips}$

  Use $M_{sc} = 42.1 \text{ ft-kips}$ at all interior columns.

  $$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.60 \quad \text{Eq. (5.9)}$$

  where

  $b_1 = c_1 + d = 24.0 + 8.25 = 32.25 \text{ in.}$ — Table 5.11, Case 1

  $b_2 = c_2 + d = 24.0 + 8.25 = 32.25 \text{ in.}$

  Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

  The moment $\gamma_f M_{sc} = 0.60 \times 42.1 = 25.3 \text{ ft-kips}$ must be transferred over the effective slab width b_{slab}:

  $$b_{slab} = c_2 + 3h = 24.0 + (3 \times 9.5) = 52.5 \text{ in., say } 52.0 \text{ in.} \quad \text{Eq. (5.10)}$$

  Determine the required A_s:

  $$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{25.3 \times 12}{0.9 \times 52.0 \times 8.25^2} = 0.095 \text{ ksi} \quad \text{Eq. (5.36)}$$

  $$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4 \times 52.0 \times 8.25}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.095}{0.85 \times 4}}\right] = 0.69 \text{ in.}^2 \quad \text{Eq. (5.35)}$$

This required area of steel is equivalent to 3-#5 bars. With a uniform bar spacing in the column strip, 6 of the 18-#5 bars are within the 52.0-in. effective slab width at the first interior columns, so no additional reinforcement is required to satisfy moment transfer requirements at these locations. Similarly, 7 of the 17-#5 bars are within the 52.0-in. effective slab width at the interior column, so no additional reinforcement is required to satisfy moment transfer requirements at this location.

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$$

$$A_c = 2(b_1 + b_2)d = 2 \times (2 \times 32.25) \times 8.25 = 1{,}064.3 \text{ in.}^2$$ Table 5.11, Case 1

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 258.6 \times \left[(23.5 \times 25.0) - \left(\frac{32.25}{12}\right)^2\right] / 1{,}000 = 150.1 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_{u(D+L)}}{A_c} = \frac{150{,}100}{1{,}064.3} = 141.0 \text{ psi} > \phi 2 \lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0 \sqrt{4{,}000} = 94.9 \text{ psi}$$ Eq. (5.38)

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d/10)}} = \sqrt{\frac{2}{1 + (8.25/10)}} = 1.1 > 1.0, \text{ use } 1.0$$ Eq. (5.30)

and

$\lambda = 1.0$ for normalweight concrete Table 5.20

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi \alpha_s f_y} = \dfrac{5 \times 141.0 \times 52.0 \times [2 \times (2 \times 32.25)]}{0.75 \times 40 \times 60{,}000} = 2.63 \text{ in.}^2 \end{cases}$$ Eq. (5.37)

At the first interior columns, provided A_s within $b_{slab} = 6 \times 0.31 = 1.86 \text{ in.}^2 < A_{s,min} = 2.63 \text{ in.}^2$

At the interior column, provided A_s within $b_{slab} = 7 \times 0.31 = 2.17 \text{ in.}^2 < A_{s,min} = 2.63 \text{ in.}^2$

Based on this load combination, additional reinforcement must be provided within b_{slab} to satisfy the minimum reinforcement requirements of ACI 8.6.1.2.

(2) Load combination: $1.2D + 1.0W + 0.5L$

$$q_u = 1.2q_D + 0.5q_L = [1.2 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 187.1 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 187.1 \times \left[(23.5 \times 25.0) - \left(\frac{32.25}{12}\right)^2\right] / 1{,}000 = 108.6 \text{ kips}$$

At the first interior columns, $V_{u(W)} = (16.6 + 13.8) / 21.5 = 1.4$ kips Table 5.26

At the interior column, $V_{u(W)} = (12.0 + 12.0) / 21.5 = 1.1$ kips

Therefore, maximum shear stress is equal to the following:

$$v_{uv} = \frac{V_{u(D+L)} + V_{u(W)}}{A_c} = \frac{108{,}600 + 1{,}400}{1{,}064.3} = 103.4 \text{ psi}$$

Eq. (5.38)

$$> \phi 2\lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0\sqrt{4{,}000} = 94.9 \text{ psi}$$

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2 \\ \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi\alpha_s f_y} = \dfrac{5 \times 103.5 \times 52.0 \times [2 \times (2 \times 32.25)]}{0.75 \times 40 \times 60{,}000} = 1.93 \text{ in.}^2 \end{cases}$$

Eq. (5.37)

The required $A_{s,min}$ from this load combination is less than the required $A_{s,min}$ from the $1.2D + 1.6L$ load combination, so required $A_{s,min} = 2.63 \text{ in.}^2$

To satisfy minimum reinforcement requirements within b_{slab}, 9-#5 bars must be placed within the 52.0-in. effective slab width ($A_{s,provided} = 9 \times 0.31 = 2.79 \text{ in.}^2 > A_{s,min} = 2.63 \text{ in.}^2$) at the first interior columns, with the remaining 9-#5 bars to be placed within the $141.0 - 52.0 = 89.0$-in. width of the column strip. For symmetry, add 1-#5 bar to the 89.0-in. width. Thus, 19-#5 bars must be provided within the column strip at the first interior columns with 9-#5 bars concentrated within the 52.0-in. effective slab width. The reinforcement details for this case are shown in Figure 5.44.

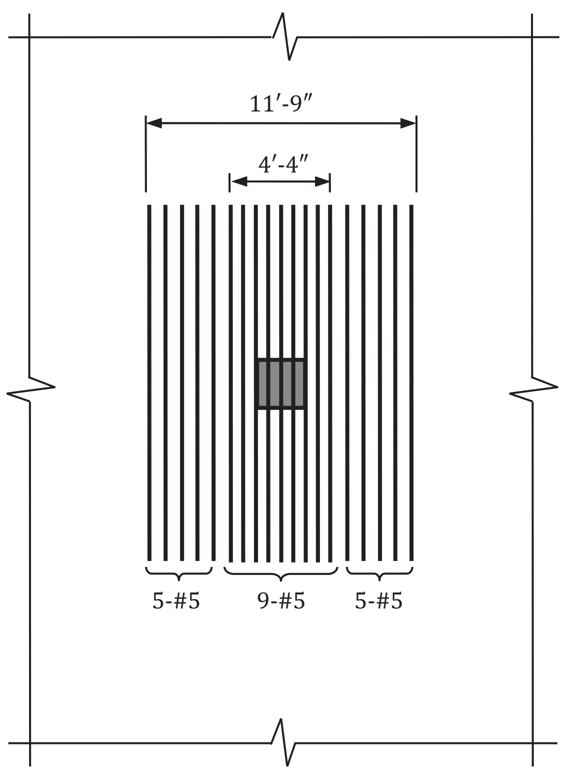


Figure 5.44 Reinforcement details for the top reinforcing bars at the first interior columns in the flat plate system in Example 5.7.

Similarly, 9-#5 bars must be placed within the 52.0-in. effective slab width at the interior column. The remaining 8-#5 bars are provided within the 89.0-in. width of the column strip.

Step 10 – Determine the reinforcement details ACI 8.7

The lengths of the top reinforcing bars in Figure 5.39 cannot be used in the column strip because of the effects due to wind loads. To account for the wind load effects, 25 percent of the top bars in the column strips are made continuous over the spans. In this case, the maximum amount of top reinforcement occurs at the first interior supports (19-#5 bars), so 5-#5 bars are made continuous.

The remaining bars in the column strip and the bars in the middle strip can be terminated at the locations identified in Figure 5.39; this figure also includes the structural integrity requirements of ACI 8.7.4.2.

A Class B tension lap splice is provided over the supports for the bottom bars in the column strip. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s\psi_g}{\left(\dfrac{c_b+K_{tr}}{d_b}\right)}\right)d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #5 reinforcing bars, $\psi_s = 0.8$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{lesser of} \begin{cases} \text{cover} + (d_b / 2) = 0.75 + (0.625 / 2) = 1.1 \text{ in.} \\ s / 2 = 12.8 / 2 = 6.4 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (1.1 + 0) / 0.625 = 1.8 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40}\frac{60{,}000}{1.0\sqrt{4{,}000}}\frac{1.0\times1.0\times0.8\times1.0}{1.8}\right)\times 0.625 = 19.8 \text{ in.} > 12.0 \text{ in.}$$

$\text{Class B lap splice length} = 1.3\ell_d = 1.3\times19.8 = 25.7 \text{ in.}$ ACI Table 25.5.2.1

Provide a 2 ft-2 in. lap splice length.

Reinforcement details for the columns strip and middle strip are given in Figure 5.45.

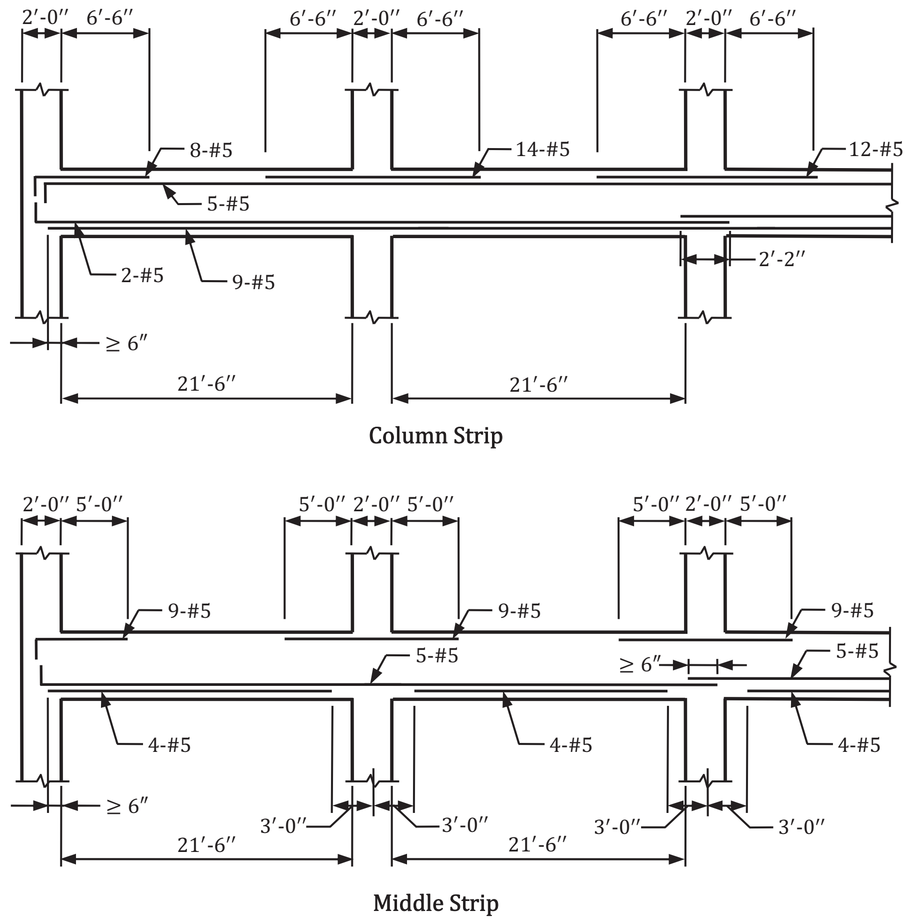


Figure 5.45 Reinforcement details for an interior design strip in the north-south direction. for the flat plate system in Example 5.7.

5.8.8 Example 5.8 – Determination of Required Flexural Reinforcement: Flat Plate System With Edge Beams, Building #1 (Framing Option B), SDC A

Determine the required flexural reinforcement in an interior design strip in the north-south direction for the flat plate system in Figure 1.1 (Framing Option B) at the second-floor level with an 8.5-in.-thick slab, 28 in. by 24 in. edge beams, and 24 in. by 24 in. columns (see Figure 5.46). Assume the Site Class is C. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the widths of the column strip and middle strip ACI 8.4.1.5, 8.4.1.6

$$\text{Width of column strip} = \text{lesser of} \begin{cases} \ell_1 / 2 = 23.5 / 2 = 11.75 \text{ ft} \\ \ell_2 / 2 = 25.0 / 2 = 12.50 \text{ ft} \end{cases}$$ Figure 5.8

$$\text{Width of middle strip} = 25.0 - 11.75 = 13.25 \text{ ft}$$

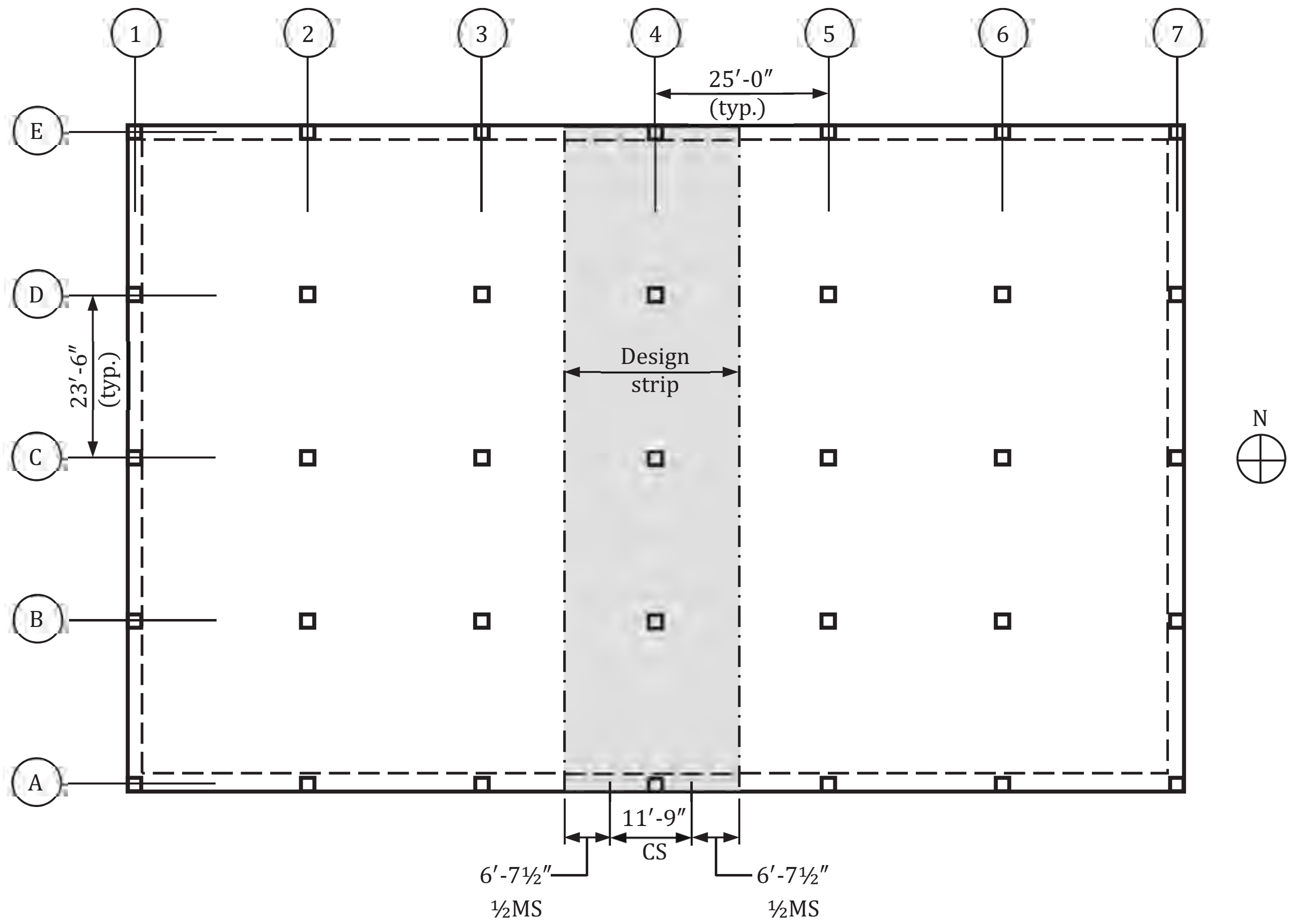


Figure 5.46 Design strips for the flat plate system in Example 5.8.

Step 2 – Check if the Direct Design Method can be used to determine gravity load bending moments

Sect. 5.3.4

Check the following limitations: Figure 5.22

- There must be 3 or more continuous spans in each direction…There are 4 and 6 continuous spans in the north-south and east-west directions, respectively.
- Successive panel lengths measured center-to-center of supports in each direction must not differ by more than one-third the longer span…All the panel lengths are equal in each direction.
- Panels must be rectangular with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2… Largest ratio of panel lengths $= 25.0 / 23.5 = 1.1 < 2.0$.
- Column offset must not exceed 10 percent of the span in the direction of the offset from either axis between centerlines of successive columns…None of the columns are offset.
- All loads must be due to gravity only and uniformly distributed over an entire panel…This method will be used only for gravity loads, which are uniformly distributed over the entire floor system.
- The unfactored live load must not exceed 2 times the unfactored dead load… Unfactored live load $= 65$ lb/ft^2; assuming an 8.5-in.-thick slab, unfactored dead load $= (8.5 \times 150.0 / 12) + 10 = 116$ lb/ft^2; $L / D = 0.6 < 2$.
- For a panel with beams between supports on all sides, the following equation must be satisfied for the beams in the two perpendicular directions: $0.2 \le \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 \le 5.0$…There are only perimeter beams in this floor system, so this limitation is not applicable.

Because all the limitations are satisfied, the DDM can be used to determine the bending moments in the column and middle strips.

Step 3 – Determine the total factored static moment in each span

$$\text{Dead load of slab} = (8.5 / 12) \times 150.0 = 106.3 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$$

$$\text{Live load} = 65.0 \text{ lb/ft}^2$$

$$\ell_2 = 25.0 \text{ ft}$$ Figure 5.8

$$\ell_n = 23.5 - (24.0 / 12) = 21.5 \text{ ft}$$ Figure 5.23

The flat plate system is not part of the LFRS (only the columns and edge beams form the LFRS); therefore, it must resist the effects from gravity loads only.

Maximum $q_u = 1.2q_D + 1.6q_L = [1.2 \times (106.3 + 10.0)] + (1.6 \times 65.0) = 243.6 \text{ lb/ft}^2$ ACI Eq. (5.3.1b)

Total factored static moment M_o:

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} = \frac{243.6 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 351.9 \text{ ft-kips}$$ Eq. (5.17)

The total factored static moment is the same for all spans of the interior design strip in the direction of analysis.

Step 4 – Distribute the total factored static moment to the column strips and middle strips Table 5.17

Design moment coefficients for a flat plate system with edge beams are given in Table 5.17. The tabulated exterior negative column strip moment coefficients are applicable in cases where the term $\beta_t \geq 2.5$ where β_t is determined by the following equation:

$$\beta_t = \frac{E_{cb} C}{2 E_{cs} I_s}$$ Eq. (5.18)

$$b_e = b_w + \text{lesser of} \begin{cases} h_b \\ \\ 4h \end{cases} = 28.0 + \text{lesser of} \begin{cases} 24.0 - 8.5 = 15.5 \\ \\ 4 \times 8.5 = 34.0 \end{cases} = 28.0 + 15.5 = 43.5 \text{ in.}$$ Eq. (5.5), Figure 5.2

Cross-sectional constant C is determined by the following equation:

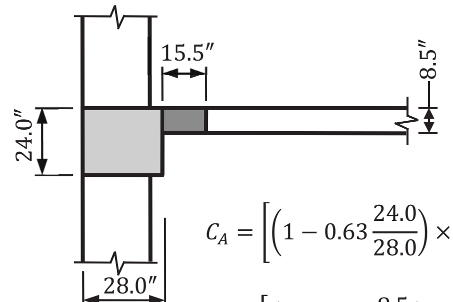
$$C = \sum \left(1 - 0.63 \frac{x}{y}\right) \frac{x^3 y}{3}$$ Eq. (5.19)

Calculations for C are given in Figure 5.47, where it is found that $C = 61{,}428 \text{ in.}^4$

$$I_s = \frac{\ell_2 h^3}{12} = \frac{(25.0 \times 12) \times 8.5^3}{12} = 15{,}353 \text{ in.}^4$$

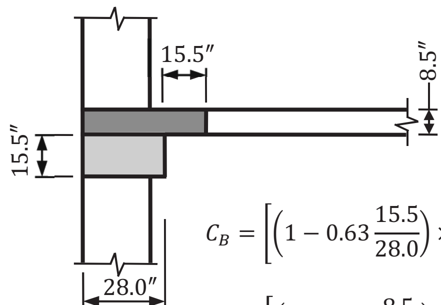
Therefore, with the slab and beam cast monolithically with the same concrete mixture (that is, $E_{cb} = E_{cs}$):

$$\beta_t = \frac{61{,}428}{2 \times 15{,}353} = 2.0$$



$$C_A = \left[\left(1 - 0.63\frac{24.0}{28.0}\right) \times \frac{24.0^3 \times 28.0}{3}\right] + \left[\left(1 - 0.63\frac{8.5}{15.5}\right) \times \frac{8.5^3 \times 15.5}{3}\right] = 61{,}428 \text{ in.}^4$$

Case A



$$C_B = \left[\left(1 - 0.63\frac{15.5}{28.0}\right) \times \frac{15.5^3 \times 28.0}{3}\right] + \left[\left(1 - 0.63\frac{8.5}{43.5}\right) \times \frac{8.5^3 \times 43.5}{3}\right] = 30{,}444 \text{ in.}^4$$

Case B

Figure 5.47 Calculations for cross-sectional constant ***C***.

Because $\beta_t < 2.5$, the exterior negative column strip bending moment is equal to the following:

$(0.30 - 0.03\beta_t)M_o = [0.30 - (0.03 \times 2.0)]M_o = 0.24M_o$ Table 5.17

A summary of the design bending moments in the column strip and middle strip in an end span and in an interior span is given in Table 5.29.

Table 5.29 Design Bending Moments (ft-kips) at the Second-Floor Level for the Flat Plate System in Example 5.8

Design Strip	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Column strip	$0.24M_o = -84.5$	$0.30M_o = 105.6$	$0.53M_o = -186.5$	$0.21M_o = 73.9$	$0.49M_o = -172.4$
Middle strip	$0.06M_o = -21.1$	$0.20M_o = 70.4$	$0.17M_o = -59.8$	$0.14M_o = 49.3$	$0.16M_o = -56.3$

Step 5 – Determine the bending moments due to wind forces

Design wind forces in the north-south direction on Building #1 are determined in Example 3.1 and are given in Table 3.10.

Because it has been assumed all the wind load effects are resisted by the perimeter moment-resisting frames, the slabs resist only gravity load effects, which are given in Table 5.29.

Step 6 – Determine the bending moments due to seismic forces

As noted in the example statement, it is assumed the Site Class is C for this building. From Example 3.5, Part (a), the building is assigned to SDC A for this case. Therefore, effects due to seismic forces need not be considered.

It is also assumed the lateral forces based on the general structural integrity requirements of ASCE/SEI 1.4.2 are resisted by the perimeter moment-resisting frames, and, thus, need not be applied to the slabs.

Step 7 – Determine the required flexural reinforcement

The required flexural reinforcement is determined at the critical sections in the column strip and middle strip using the following equations where b is the width of the column strip or middle strip and $d = 8.5 - 1.25 = 7.25$ in.:

$$A_s = \frac{0.85 f'_c bd}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] \quad \text{Eq. (5.35)}$$

$$R_n = \frac{M_u}{\phi b d^2} \quad \text{Eq. (5.36)}$$

A summary of the required flexural reinforcement is given in Table 5.30. The provided areas of reinforcement are greater than or equal to the minimum required in accordance with ACI 8.6.1.1 and the number of bars are selected so that the provided spacing of the reinforcing bars is less than the maximum required in ACI 8.7.2.2. It is evident the provided areas of reinforcement at all critical sections in the column strip and middle strip are less than the area of flexural reinforcement, $A_{s,t}$, corresponding to tension-controlled sections, which is determined by Eq. (5.41); therefore, all the critical sections are tension-controlled, which satisfies ACI 8.3.3.1.

Table 5.30 Required Flexural Reinforcement at the Second-Floor Level for the Flat Plate System in Example 5.8

Location			M_u (ft-kips)	b (in.)	A_s (in.^{2})*	Reinforcement*
End Span	Column strip	Exterior Negative	−84.5	141.0	2.65	9-#5
		Positive	105.6	141.0	3.33	11-#5
		First Interior Negative	−186.5	141.0	6.03	20-#5
	Middle strip	Exterior Negative	−21.1	159.0	2.43	10-#5
		Positive	70.4	159.0	2.43	10-#5
		First Interior Negative	−59.8	159.0	2.43	10-#5
Interior Span	Column strip	Positive	73.9	141.0	2.31	9-#5
		Negative	−172.4	141.0	5.54	18-#5
	Middle strip	Positive	49.3	159.0	2.43	10-#5
		Negative	−56.3	159.0	2.43	10-#5

* *Min. A_s for the column strip* $= 0.0018 \times 141.0 \times 8.5 = 2.16$ in.^{2}

Min. A_s for the middle strip $= 0.0018 \times 159.0 \times 8.5 = 2.43$ in.^{2}

Max. spacing $=$ lesser of $(2h,\ 18.0$ in.$) = 17.0$ in.

For $b = 141.0$ in., $141.0 / 17.0 = 8.3$ spaces, say minimum of 9 bars

For $b = 159.0$ in., $159.0 / 17.0 = 9.4$ spaces, say minimum of 10 bars

For the column strip: $A_{s,t} = 0.018bd = 18.4$ in.^{2}

For the middle strip: $A_{s,t} = 0.018bd = 20.8$ in.^{2}

Step 8 – Check that the flexural reinforcement is adequate for moment transfer requirements ACI 8.4.2.2

- Edge columns

  The factored slab moments resisted by the edge columns need not be checked in this example because of the beams at the perimeter of the slab. These edge beams must be designed for shear forces and torsional moments transferred from the slab.

- Interior columns

  Determine the required reinforcement based on the largest transfer moment at the interior columns.

  In lieu of a more exact analysis, the factored slab moment, M_{sc}, transferred to an interior column due to the gravity load effects can be determined from the following equation where the spans in the direction of analysis and perpendicular to the direction of analysis are equal:

$$M_{sc} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (1.6 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 42.1 \text{ ft-kips}$$ Eq. (5.21)

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.60$$ Eq. (5.9)

where

$$b_1 = c_1 + d = 24.0 + 7.25 = 31.25 \text{ in.}$$ Table 5.11, Case 1

$$b_2 = c_2 + d = 24.0 + 7.25 = 31.25 \text{ in.}$$

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

The moment $\gamma_f M_{sc} = 0.60 \times 42.1 = 25.3$ ft-kips must be transferred over the effective slab width b_{slab}:

$$b_{slab} = c_2 + 3h = 24.0 + (3 \times 8.5) = 49.5 \text{ in., say } 49.0 \text{ in.}$$ Eq. (5.10)

Determine the required A_s:

$$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{25.3 \times 12}{0.9 \times 49.0 \times 7.25^2} = 0.131 \text{ ksi}$$ Eq. (5.36)

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4 \times 49.0 \times 7.25}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.131}{0.85 \times 4}}\right] = 0.79 \text{ in.}^2$$ Eq. (5.35)

This required area of steel is equivalent to 3-#5 bars. With a uniform spacing of bars in the column strip, 6 of the 20-#5 bars are within the 49.0-in. effective slab width at the first interior columns, so no additional reinforcement is required to satisfy moment transfer requirements at these locations. Similar calculations show that no additional reinforcement is required at the interior column where 18-#5 bars are provided in the column strip.

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

$$A_c = 2(b_1 + b_2)d = 2 \times (2 \times 31.25) \times 7.25 = 906.3 \text{ in.}^2$$ Table 5.11, Case 1

Factored shear force at the critical section:

$$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) = 243.6 \times \left[(23.5 \times 25.0) - \left(\frac{31.25}{12}\right)^2\right] / 1{,}000 = 141.5 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_u}{A_c} = \frac{141,500}{906.3} = 156.1 \text{ psi} > \phi 2\lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0\sqrt{4,000} = 94.9 \text{ psi}$$ Eq. (5.38)

where

$$\lambda_s = \sqrt{\frac{2}{1+(d/10)}} = \sqrt{\frac{2}{1+(7.25/10)}} = 1.1 > 1.0, \text{ use } 1.0$$ Eq. (5.30)

and

$\lambda = 1.0$ for normalweight concrete Table 5.20

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 8.5 \times 49.0 = 0.75 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi \alpha_s f_y} = \dfrac{5 \times 156.1 \times 49.0 \times [2 \times (2 \times 31.25)]}{0.75 \times 40 \times 60,000} = 2.66 \text{ in.}^2 \end{cases}$$ Eq. (5.37)

At the first interior columns, provided A_s within $b_{slab} = 6 \times 0.31 = 1.86 \text{ in.}^2 < A_{s,min} = 2.66 \text{ in.}^2$

At the interior column, provided A_s within $b_{slab} = 6 \times 0.31 = 1.86 \text{ in.}^2 < A_{s,min} = 2.66 \text{ in.}^2$

Therefore, additional reinforcement must be provided within b_{slab} to satisfy the minimum reinforcement requirements of ACI 8.6.1.2.

To satisfy minimum reinforcement requirements within b_{slab}, 9-#5 bars must be placed within the 49.0-in. effective slab width $(A_{s,provided} = 9 \times 0.31 = 2.79 \text{ in.}^2 > A_{s,min} = 2.66 \text{ in.}^2)$ at the first interior columns, with the remaining 11-#5 bars to be placed within the $141.0 - 49.0 = 92.0$-in. width of the column strip. For symmetry, add 1-#5 bar to the 92.0-in. width. Thus, 21-#5 bars must be provided within the column strip at the first interior columns with 9-#5 bars concentrated within the 49.0-in. effective slab width. The reinforcement detail for this case is similar to that shown in Figure 5.44.

Similarly, 9-#5 bars must be placed within the 49.0-in. effective slab width at the interior column. The remaining 9-#5 bars are provided within the 92.0-in. width of the column strip. For symmetry, add 1-#5 bar to the 92.0-in. width. Thus, 19-#5 bars must be provided within the column strip at the interior column with 9-#5 bars concentrated within the 49.0-in. effective slab width.

Step 9 – Determine the reinforcement details ACI 8.7

The lengths of the reinforcing bars in Figure 5.39 can be used because the slab is subjected to the effects from uniformly distributed gravity loads only.

A Class B tension lap splice is provided over the supports for the bottom bars in the column strip. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #5 reinforcing bars, $\psi_s = 0.8$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{lesser of} \begin{cases} \text{cover} + (d_b / 2) = 0.75 + (0.625 / 2) = 1.1 \text{ in.} \\ \\ s / 2 = 12.8 / 2 = 6.4 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (1.1 + 0) / 0.625 = 1.8 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{1.8} \right) \times 0.625 = 19.8 \text{ in.} > 12.0 \text{ in.}$$

Class B lap splice length $= 1.3\ell_d = 1.3 \times 19.8 = 25.7$ in. ACI Table 25.5.2.1

Provide a 2 ft-2 in. lap splice length.

Reinforcement details for the columns strip and middle strip are given in Figure 5.48. For simpler detailing, the lengths of the negative reinforcement at all critical sections in the column strip are set equal to 30 percent of the clear span length, which is greater than $5d = 3.0$ ft (see Figure 5.39). The structural integrity requirements in ACI 8.7.4.2 are included.

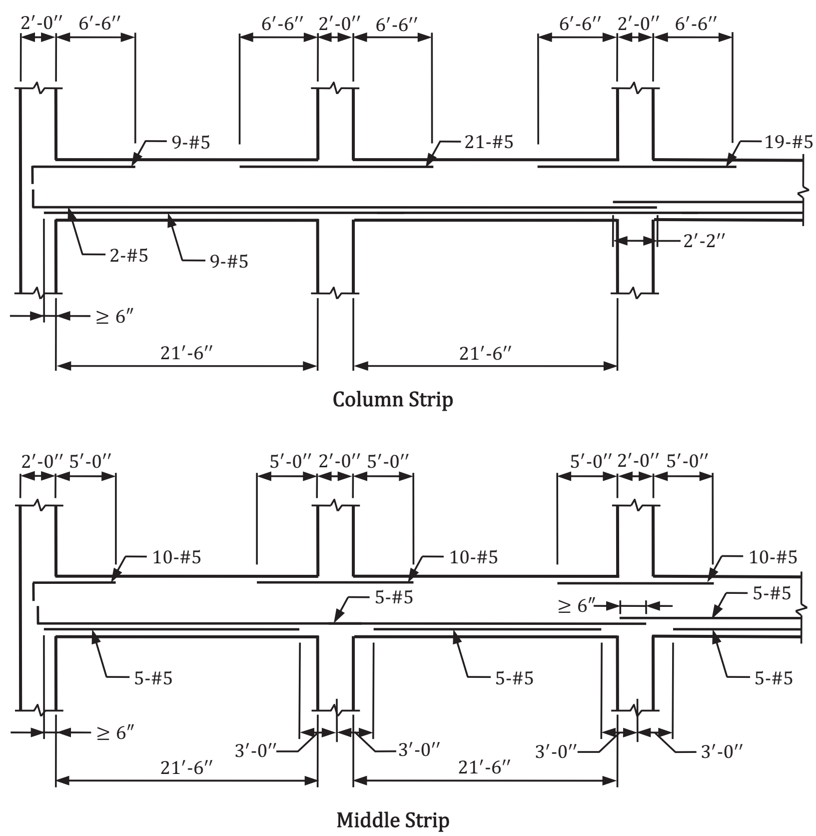


Figure 5.48 Reinforcement details for an interior design strip in the north-south direction for the flat plate system in Example 5.8.

5.8.9 Example 5.9 – Determination of Required Flexural Reinforcement: Two-way Beam-Supported Slab System, Building #1 (Framing Option C), SDC A

Determine the required flexural reinforcement in an interior design strip in the north-south direction for the two-way beam-supported slab system in Figure 1.1 (Framing Option C) at the second-floor level with a 7.0-in.-thick slab, 28 in. by 24 in. beams, and 24 in. by 24 in. columns (see Figure 5.49). Assume the Site Class is C. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

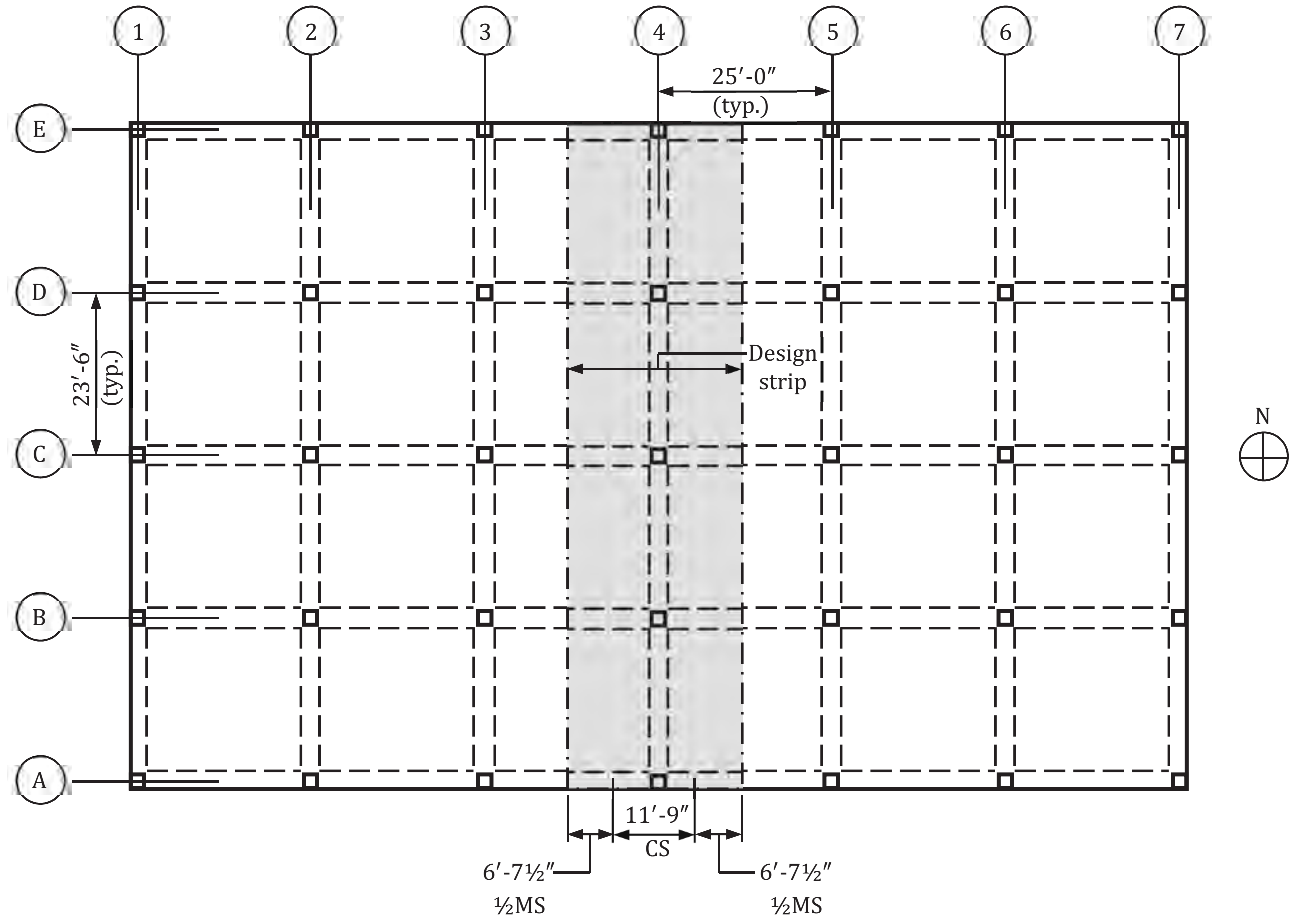


Figure 5.49 Design strips for the two-way beam-supported slab system in Example 5.9.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the widths of the column strip and middle strip

ACI 8.4.1.5, 8.4.1.6

$$\text{Width of column strip} = \text{lesser of} \begin{cases} \ell_1 / 2 = 23.5 / 2 = 11.75 \text{ ft} \\ \ell_2 / 2 = 25.0 / 2 = 12.50 \text{ ft} \end{cases}$$

Figure 5.8

$$\text{Width of middle strip} = 25.0 - 11.75 = 13.25 \text{ ft}$$

Step 2 – Check if the Direct Design Method can be used to determine gravity load bending moments Sect. 5.3.4

Check the following limitations: Figure 5.22

- There must be 3 or more continuous spans in each direction…There are 4 and 6 continuous spans in the north-south and east-west directions, respectively.
- Successive panel lengths measured center-to-center of supports in each direction must not differ by more than one-third the longer span…All the panel lengths are equal in each direction.
- Panels must be rectangular with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2… $\text{Largest ratio of panel lengths} = 25.0 / 23.5 = 1.1 < 2.0$.
- Column offset must not exceed 10 percent of the span in the direction of the offset from either axis between centerlines of successive columns…None of the columns are offset.
- All loads must be due to gravity only and uniformly distributed over an entire panel…This method will be used only for gravity loads, which are uniformly distributed over the entire floor system.
- The unfactored live load must not exceed 2 times the unfactored dead load… $\text{Unfactored live load} = 65\ \text{lb/ft}^2$; assuming a 7.0-in.-thick slab, $\text{unfactored dead load} = (7.0 \times 150.0 / 12) + 10 = 98\ \text{lb/ft}^2$; $L / D = 0.7 < 2$.
- For a panel with beams between supports on all sides, the following equation must be satisfied for the beams in the two perpendicular directions: $0.2 \le \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 \le 5.0$…Check the relative stiffnesses for interior, edge, and corner panels where α_f are determined in Example 5.3.

  Interior panel:

  North-south interior beam: $\alpha_{f1} = 5.4$

  East-west interior beam: $\alpha_{f2} = 5.7$

  $0.2 < \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 = (5.4 \times 25.0^2) / (5.7 \times 23.5^2) = 1.1 < 5.0$

  Edge panel (north or south face):

  North-south interior beam: $\alpha_{f1} = 5.4$

  East-west edge beam: $\alpha_{f2} = 9.2$

  $0.2 < \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 = (5.4 \times 25.0^2) / (9.2 \times 23.5^2) = 0.7 < 5.0$

  Edge panel (east or west face):

  North-south edge beam: $\alpha_{f1} = 8.7$

  East-west interior beam: $\alpha_{f2} = 5.7$

  $0.2 < \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 = (8.7 \times 25.0^2) / (5.7 \times 23.5^2) = 1.7 < 5.0$

  Corner panel:

  North-south edge beam: $\alpha_{f1} = 8.7$

  East-west edge beam: $\alpha_{f2} = 9.2$

  $0.2 < \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 = (8.7 \times 25.0^2) / (9.2 \times 23.5^2) = 1.1 < 5.0$

Because all the limitations are satisfied, the DDM can be used to determine the bending moments in the column and middle strips.

Step 3 – Determine the total factored static moment in each span

Dead load of slab $= (7.0 / 12) \times 150.0 = 87.5 \text{ lb/ft}^2$

Superimposed dead load $= 10.0 \text{ lb/ft}^2$

Live load $= 65.0 \text{ lb/ft}^2$

$\ell_2 = 25.0 \text{ ft}$ — Figure 5.8

$\ell_n = 23.5 - (24.0 / 12) = 21.5 \text{ ft}$ — Figure 5.23

The slab in this system is not part of the LFRS (only the columns and beams form the LFRS); therefore, the slab must resist the effects from gravity loads only.

Maximum $q_u = 1.2q_D + 1.6q_L = [1.2 \times (87.5 + 10.0)] + (1.6 \times 65.0) = 221.0 \text{ lb/ft}^2$ — ACI Eq. (5.3.1b)

Total factored static moment M_o:

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} = \frac{221.0 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 319.2 \text{ ft-kips} \qquad \text{Eq. (5.17)}$$

The total factored static moment is the same for all spans of an interior design strip in the direction of analysis.

Step 4 – Distribute the total factored static moment to the column strips and middle strips

Total design moments are determined using Table 5.14 and the information in Step 2 of Sect. 5.3.4. A summary of the total design strip moments is given in Table 5.31.

Table 5.31 Total Design Bending Moments (ft-kips) at the Second-Floor Level for the Two-way Beam-Supported Slab System in Example 5.9

End Span			Interior Span	
1	2	3	4	5
Exterior Negative	**Positive**	**First Interior Negative**	**Positive**	**Interior Negative**
$0.16M_o = -51.1$	$0.57M_o = 181.9$	$0.70M_o = -223.4$	$0.35M_o = 111.7$	$0.65M_o = -207.5$

Determine the percentages for factored bending moments in the column strip: — Table 5.15

- Exterior support: $100 - 10\beta_t + 12\beta_t \left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right)$

  Interior beams in north-south direction: $\alpha_{f1} = 5.4$ — Example 5.3

  $\alpha_{f1}\ell_2 / \ell_1 = 5.4 \times 25.0 / 23.5 = 5.8 > 1.0, \text{ use } 1.0$

  $$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \qquad \text{Eq. (5.18)}$$

  $$b_e = b_w + \text{lesser of} \begin{cases} h_b \\ 4h \end{cases} = 28.0 + \text{lesser of} \begin{cases} 24.0 - 7.0 = 17.0 \\ 4 \times 7.0 = 28.0 \end{cases} = 28.0 + 17.0 = 45.0 \text{ in.} \qquad \text{Eq. (5.5), Figure 5.2}$$

Cross-sectional constant C is determined by the following equation:

$$C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3} \qquad \text{Eq. (5.19)}$$

Calculations for C are given in Figure 5.50, where it is found that $C = 60{,}791$ in.^{4}

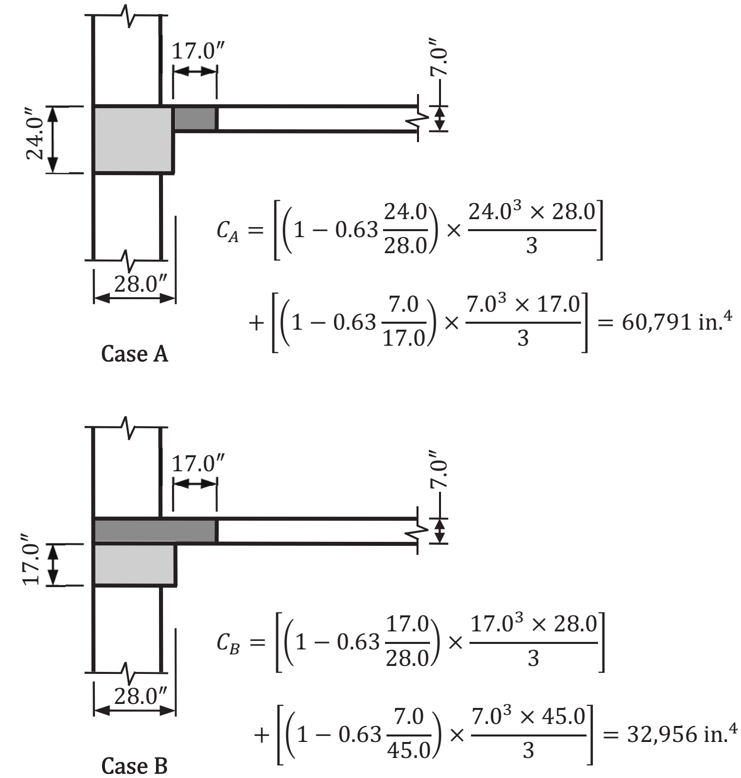


Figure 5.50 Calculations for cross-sectional constant ***C***.

$$I_s = \frac{\ell_2 h^3}{12} = \frac{(25.0 \times 12) \times 7.0^3}{12} = 8{,}575 \text{ in.}^4$$

With the slab and beam cast monolithically with the same concrete mixture (that is, $E_{cb} = E_{cs}$):

$$\beta_t = \frac{60{,}791}{2 \times 8{,}575} = 3.6 > 2.5, \text{ use } 2.5$$

Therefore,

$$100 - (10 \times 2.5) + \left[(12 \times 2.5) \times 1.0 \times \left(1 - \frac{25.0}{23.5}\right)\right] = 73.1\%$$

- Positive: $60 + 30\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1.5 - \frac{\ell_2}{\ell_1}\right) = 60 + (30 \times 1.0) \times \left(1.5 - \frac{25.0}{23.5}\right) = 73.1\%$
- Interior support: $75 + 30\left(\frac{\alpha_{f1}\ell_2}{\ell_1}\right)\left(1 - \frac{\ell_2}{\ell_1}\right) = 75 + (30 \times 1.0) \times \left(1 - \frac{25.0}{23.5}\right) = 73.1\%$

Determine percentages for factored bending moments in the middle strip:

- Exterior support: $100 - 73.1 = 26.9\%$
- Positive: $100 - 73.1 = 26.9\%$
- Interior support: $100 - 73.1 = 26.9\%$

Determine percentages for factored bending moments in the column-line beams:

Because $\alpha_{f1}\ell_2 / \ell_1 = 5.8 > 1.0$, the beams must resist 85% of the total column strip moments:

- Exterior support: $0.85 \times 73.1 = 62.1\%$
- Positive: $0.85 \times 73.1 = 62.1\%$
- Interior support: $0.85 \times 73.1 = 62.1\%$

A summary of the design bending moments in the column strip and middle strip in an end span and interior span is given in Table 5.32.

Table 5.32 Design Bending Moments (ft-kips) at the Second-Floor Level for the Two-way Beam-Supported Slab System in Example 5.9

Design Strip		End Span			Interior Span	
		1	2	3	4	5
		Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Total		$0.16M_o = -51.1$	$0.57M_o = 181.9$	$0.70M_o = -223.4$	$0.35M_o = 111.7$	$0.65M_o = -207.5$
Column strip	Total	$0.12M_o = -38.3$	$0.42M_o = 134.1$	$0.51M_o = -162.8$	$0.26M_o = 83.0$	$0.48M_o = -153.2$
	Beam	$0.10M_o = -31.9$	$0.36M_o = 114.9$	$0.43M_o = -137.3$	$0.22M_o = 70.2$	$0.41M_o = -130.9$
	Slab	$0.02M_o = -6.4$	$0.06M_o = 19.2$	$0.08M_o = -25.5$	$0.04M_o = 12.8$	$0.07M_o = -22.3$
Middle strip		$0.04M_o = -12.8$	$0.15M_o = 47.9$	$0.19M_o = -60.7$	$0.09M_o = 28.7$	$0.17M_o = -54.3$

The following illustrates the calculation of the design bending moments at the exterior negative support:

$$\text{Total moment in column strip} = -0.731 \times 0.16M_o = -0.12M_o = -0.12 \times 319.2 = -38.3 \text{ ft-kips}$$

$$\text{Moment in beam} = -0.85 \times (0.12M_o) = -0.10M_o = -0.10 \times 319.2 = -31.9 \text{ ft-kips}$$

$$\text{Moment in slab} = -(1 - 0.85) \times (0.12M_o) = -0.02M_o = -0.02 \times 319.2 = -6.4 \text{ ft-kips}$$

$$\text{Moment in middle strip} = -(0.16 - 0.12)M_o = -0.04M_o = -0.04 \times 319.2 = -12.8 \text{ ft-kips}$$

The bending moments at the other critical sections can be obtained in a similar fashion.

Step 5 – Determine the bending moments due to wind forces — ACI 6.6

Design wind forces in the north-south direction on Building #1 are determined in Example 3.1 and are given in Table 3.10.

Because it has been assumed all the wind load effects are resisted by the moment-resisting frames along all the column lines, the slabs resist only gravity load effects, which are given in Table 5.32.

Step 6 – Determine the bending moments due to seismic forces

As noted in the example statement, it is assumed the Site Class is C for this building. From Example 3.5, Part (a), the building is assigned to SDC A for this case. Therefore, effects due to seismic forces need not be considered.

It is also assumed the lateral forces based on the general structural integrity requirements of ASCE/SEI 1.4.2 are resisted by the moment-resisting frames, and, thus, need not be applied to the slabs.

Step 7 – Determine the required flexural reinforcement

The required flexural reinforcement is determined at the critical sections in the column strip and middle strip using the following equations:

$$A_s = \frac{0.85 f_c' bd}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] \qquad \text{Eq. (5.35)}$$

$$R_n = \frac{M_u}{\phi b d^2} \qquad \text{Eq. (5.36)}$$

where b is the width of the column strip or middle strip and $d = 7.0 - 1.25 = 5.75$ in.

A summary of the required flexural reinforcement is given in Table 5.33 for the slab. The provided areas of reinforcement are greater than or equal to the minimum required in accordance with ACI 8.6.1.1 and the number of bars are selected so that the provided spacing of the reinforcing bars is less than the maximum required in ACI 8.7.2.2. It is evident the provided areas of reinforcement at all critical sections in the column strip and middle strip are less than the area of flexural reinforcement, $A_{s,t}$, corresponding to tension-controlled sections, which is determined by Eq. (5.41); therefore, all the critical sections are tension-controlled, which satisfies ACI 8.3.3.1.

Table 5.33 Required Flexural Reinforcement at the Second-Floor Level for the Two-way Beam-Supported Slab System in Example 5.9

Location			M_u (ft-kips)	b (in.)	A_s (in.^{2})*	Reinforcement*
End Span	Column strip	Exterior Negative	−6.4	141.0	1.78	11-#4
		Positive	19.2	141.0	1.78	11-#4
		First Interior Negative	−25.5	141.0	1.78	11-#4
	Middle strip	Exterior Negative	−12.8	159.0	2.00	12-#4
		Positive	47.9	159.0	2.00	12-#4
		First Interior Negative	−60.7	159.0	2.40	12-#4
Interior Span	Column strip	Positive	12.8	141.0	1.78	11-#4
		Negative	−22.3	141.0	1.78	11-#4
	Middle strip	Positive	28.7	159.0	2.00	12-#4
		Negative	−54.3	159.0	2.14	12-#4

* *Min. A_s for the column strip* $= 0.0018 \times 141.0 \times 7.0 = 1.78$ in.^{2}

Min. A_s for the middle strip $= 0.0018 \times 159.0 \times 7.0 = 2.00$ in.^{2}

Max. spacing = lesser of $(2h,\ 18.0$ in.$) = 14.0$ in.

For $b = 141.0$ in., $141.0 / 14.0 = 10.1$ spaces, say minimum of 11 bars

For $b = 159.0$ in., $159.0 / 14.0 = 11.4$ spaces, say minimum of 12 bars

For the column strip: $A_{s,t} = 0.018bd = 14.6$ in.^{2}

For the middle strip: $A_{s,t} = 0.018bd = 16.5$ in.^{2}

Step 8 – Check that the flexural reinforcement is adequate for moment transfer requirements ACI 8.4.2.2

The factored slab moments resisted by the columns need not be checked in this example because of the column-line beams. These beams must be designed for shear forces and torsional moments transferred from the slab.

Step 9 – Determine the reinforcement details ACI 8.7

The lengths of the reinforcing bars in Figure 5.39 can be used because the slab is subjected to the effects from uniformly distributed gravity loads only.

A Class B tension lap splice is provided over the supports for the bottom bars in the column strip. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #4 reinforcing bars, $\psi_s = 0.8$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{lesser of} \begin{cases} \text{cover} + (d_b / 2) = 0.75 + (0.50 / 2) = 1.0 \text{ in.} \\ s / 2 = 12.8 / 2 = 6.4 \text{ in.} \end{cases}$$

Set $K_{tr} = 0.$ ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (1.0 + 0) / 0.500 = 2.0 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{2.0} \right) \times 0.500 = 14.2 \text{ in.} > 12.0 \text{ in.}$$

$\text{Class B lap splice length} = 1.3\ell_d = 1.3 \times 14.2 = 18.5 \text{ in.}$ ACI Table 25.5.2.1

Provide a 2 ft-0 in. lap splice length.

Reinforcement details for the columns strip and middle strip are given in Figure 5.51. For simpler detailing, the lengths of the negative reinforcement at all critical sections in the column strip are set equal to 30 percent of the clear span length, which is greater than $5d = 2.4$ ft. The structural integrity requirements in ACI 8.7.4.2 are included.

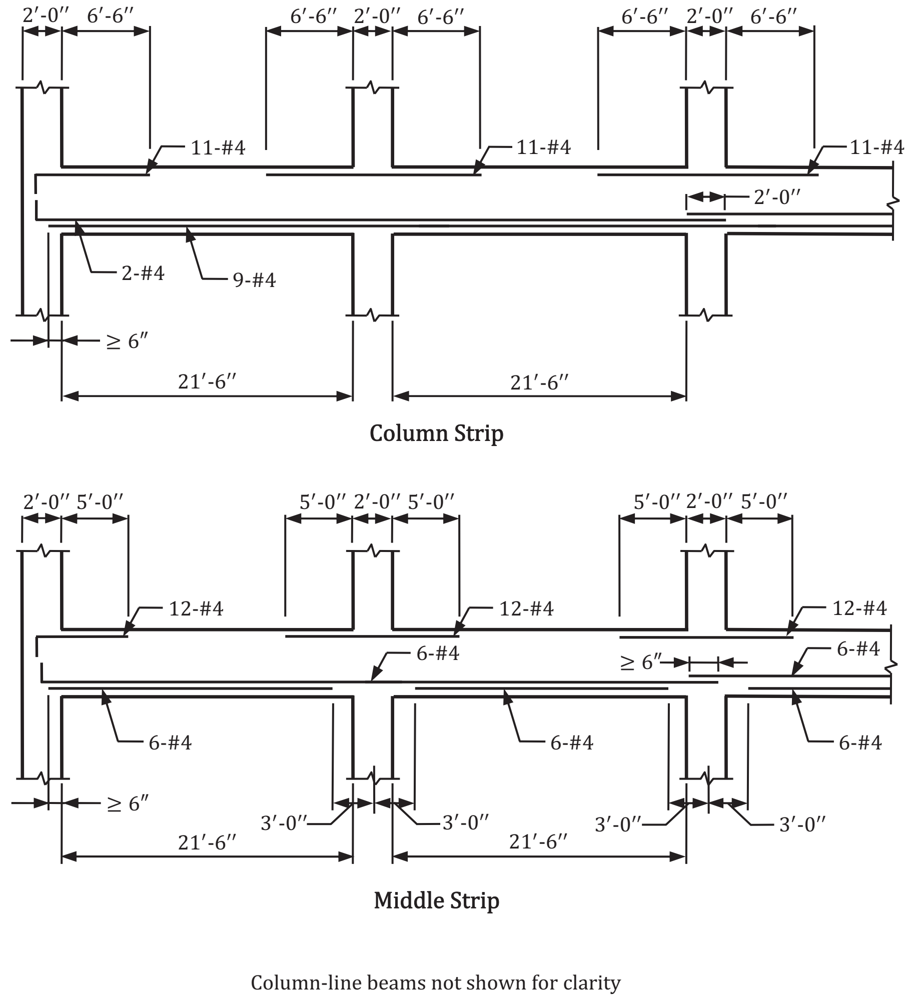


Figure 5.51 Reinforcement details for an interior design strip in the north-south direction for the two-way beam-supported slab system in Example 5.9.

5.8.10 Example 5.10 – Determination of Required Flexural Reinforcement: Flat Slab System With Edge Beams, Building #1 (Framing Option D), SDC A

Determine the required flexural reinforcement in an interior design strip in the north-south direction for the flat slab system in Figure 1.1 (Framing Option D) at the second-floor level with an 8.0-in.-thick slab, 28 in. by 24 in. edge beams, 24 in. by 24 in. columns, and a 2.25-in. drop panel projection below the slab (see Figure 5.52). Assume the Site Class is C. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the widths of the column strip and middle strip — ACI 8.4.1.5, 8.4.1.6

$$\text{Width of column strip} = \text{lesser of} \begin{cases} \ell_1 / 2 = 23.5 / 2 = 11.75 \text{ ft} \\ \ell_2 / 2 = 25.0 / 2 = 12.50 \text{ ft} \end{cases}$$

Figure 5.8

$$\text{Width of middle strip} = 25.0 - 11.75 = 13.25 \text{ ft}$$

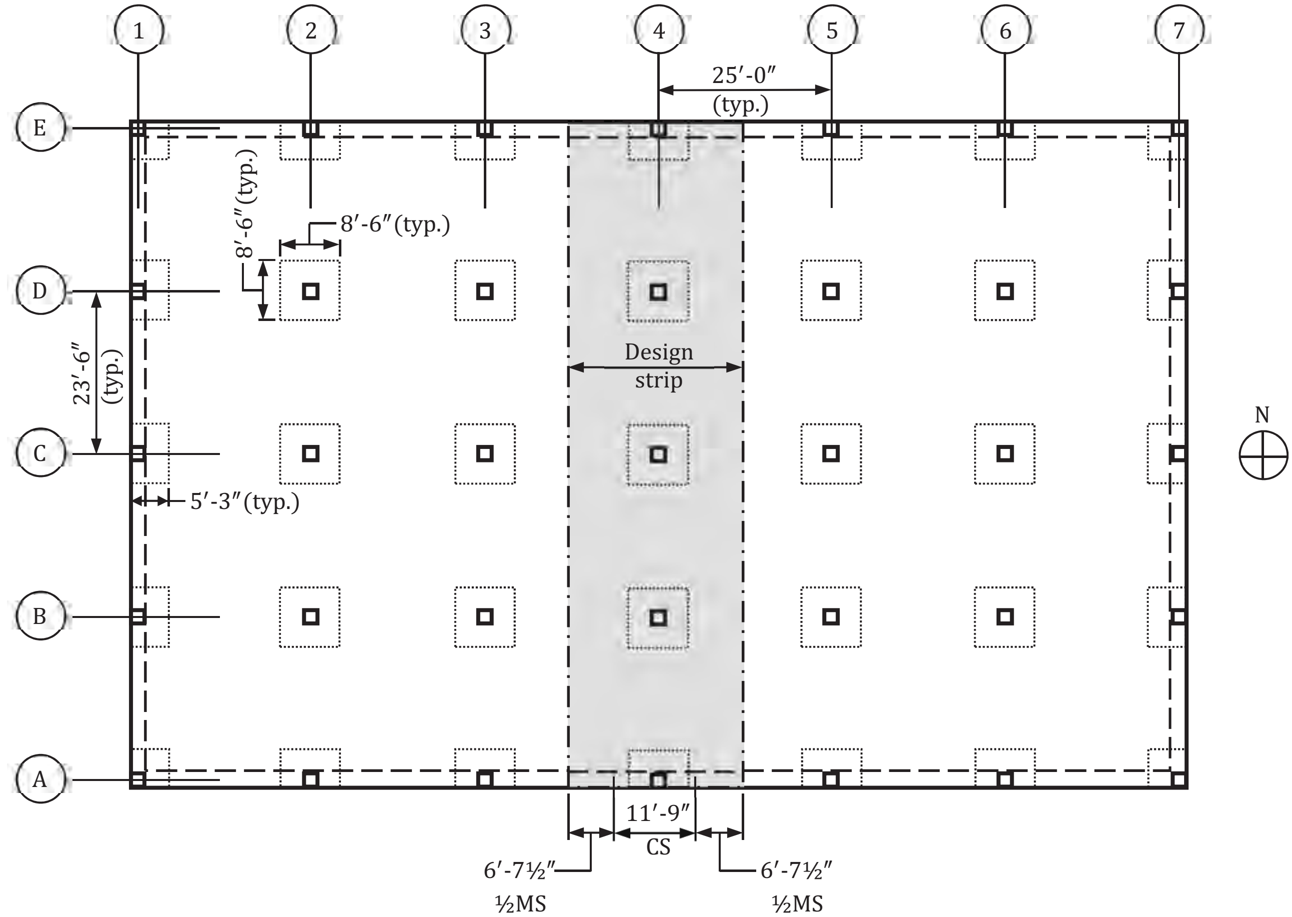


Figure 5.52 Design strips for the flat slab system in Example 5.10.

Step 2 – Check if the Direct Design Method can be used to determine gravity load bending moments Sect. 5.3.4

Check the following limitations: Figure 5.22

- There must be 3 or more continuous spans in each direction…There are 4 and 6 continuous spans in the north-south and east-west directions, respectively.
- Successive panel lengths measured center-to-center of supports in each direction must not differ by more than one-third the longer span…All the panel lengths are equal in each direction.
- Panels must be rectangular with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2… $\text{Largest ratio of panel lengths} = 25.0 / 23.5 = 1.1 < 2.0$.
- Column offset must not exceed 10 percent of the span in the direction of the offset from either axis between centerlines of successive columns…None of the columns are offset.
- All loads must be due to gravity only and uniformly distributed over an entire panel…This method will be used only for gravity loads, which are uniformly distributed over the entire floor system.
- The unfactored live load must not exceed 2 times the unfactored dead load… $\text{Unfactored live load} = 65\ \text{lb/ft}^2$; assuming an 8.0-in.-thick slab, $\text{unfactored dead load} = (8.0 \times 150.0 / 12) + 10 = 110\ \text{lb/ft}^2$; $L / D = 0.6 < 2$.
- For a panel with beams between supports on all sides, the following equation must be satisfied for the beams in the two perpendicular directions: $0.2 \le \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 \le 5.0$…There are only perimeter beams in this floor system, so this limitation is not applicable.

Because all the limitations are satisfied, the DDM can be used to determine the bending moments in the column and middle strips.

Step 3 – Determine the total factored static moment in each span

$$\text{Dead load of slab} = (8.0 / 12) \times 150.0 = 100.0 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$$

$$\text{Live load} = 65.0 \text{ lb/ft}^2$$

$\ell_2 = 25.0$ ft — Figure 5.8

$\ell_n = 23.5 - (24.0 / 12) = 21.5$ ft — Figure 5.23

The flat slab system is not part of the LFRS (only the columns and edge beams form the LFRS); therefore, the flat slab must resist the effects from gravity loads only.

Maximum $q_u = 1.2q_D + 1.6q_L = [1.2 \times (100.0 + 10.0)] + (1.6 \times 65.0) = 236.0 \text{ lb/ft}^2$ — ACI Eq. (5.3.1b)

Total factored static moment M_o:

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} = \frac{236.0 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 340.9 \text{ ft-kips} \qquad \text{Eq. (5.17)}$$

The total factored static moment is the same for all spans of the interior design strip in the direction of analysis.

Step 4 – Distribute the total factored static moment to the column strips and middle strips — Table 5.17

Design moment coefficients for a flat slab system with edge beams are given in Table 5.17. The tabulated exterior negative column strip moment coefficients are applicable in cases where the term $\beta_t \geq 2.5$ where β_t is determined by the following equation:

$$\beta_t = \frac{E_{cb} C}{2 E_{cs} I_s} \qquad \text{Eq. (5.18)}$$

$$b_e = b_w + \text{lesser of} \begin{cases} h_b \\ 4h \end{cases} = 28.0 + \text{lesser of} \begin{cases} 24.0 - 8.0 = 16.0 \\ 4 \times 8.0 = 32.0 \end{cases} = 28.0 + 16.0 = 44.0 \text{ in.} \qquad \text{Eq. (5.5), Figure 5.2}$$

Cross-sectional constant C is determined by the following equation:

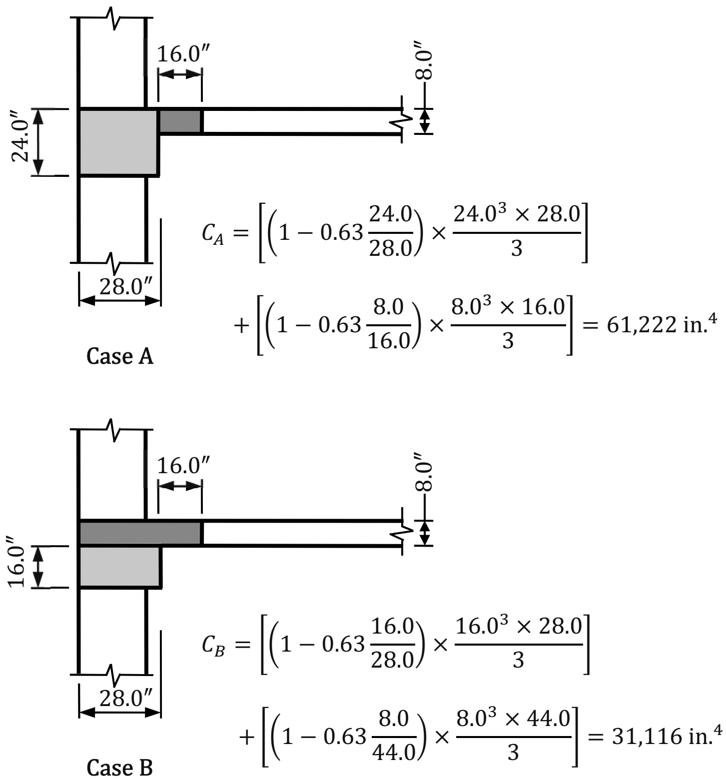
$$C = \sum \left(1 - 0.63 \frac{x}{y}\right) \frac{x^3 y}{3} \qquad \text{Eq. (5.19)}$$

Calculations for C are given in Figure 5.53, where it is found that $C = 61{,}222 \text{ in.}^4$

$$I_s = \frac{\ell_2 h^3}{12} = \frac{(25.0 \times 12) \times 8.0^3}{12} = 12{,}800 \text{ in.}^4$$

Therefore, with the slab and beam cast monolithically with the same concrete (that is, $E_{cb} = E_{cs}$):

$$\beta_t = \frac{61{,}222}{2 \times 12{,}800} = 2.4$$



$$C_A = \left[\left(1 - 0.63\frac{24.0}{28.0}\right) \times \frac{24.0^3 \times 28.0}{3}\right] + \left[\left(1 - 0.63\frac{8.0}{16.0}\right) \times \frac{8.0^3 \times 16.0}{3}\right] = 61{,}222 \text{ in.}^4$$

$$C_B = \left[\left(1 - 0.63\frac{16.0}{28.0}\right) \times \frac{16.0^3 \times 28.0}{3}\right] + \left[\left(1 - 0.63\frac{8.0}{44.0}\right) \times \frac{8.0^3 \times 44.0}{3}\right] = 31{,}116 \text{ in.}^4$$

Figure 5.53 Calculations for cross-sectional constant ***C*.**

Because $\beta_t < 2.5$, the exterior negative column strip bending moment is equal to the following:

$$(0.30 - 0.03\beta_t)M_o = [0.30 - (0.03 \times 2.4)]M_o = 0.23M_o$$ Table 5.17

A summary of the design bending moments in the column strip and middle strip in an end span and interior span is given in Table 5.34.

Table 5.34 Design Bending Moments (ft-kips) at the Second-Floor Level for the Flat Slab System in Example 5.10

Design Strip	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Column strip	$0.23M_o = -78.4$	$0.30M_o = 102.3$	$0.53M_o = -180.7$	$0.21M_o = 71.6$	$0.49M_o = -167.0$
Middle strip	$0.07M_o = -23.9$	$0.20M_o = 68.2$	$0.17M_o = -58.0$	$0.14M_o = 47.7$	$0.16M_o = -54.5$

Step 5 – Determine the bending moments due to wind forces ACI 6.6

Design wind forces in the north-south direction on Building #1 are determined in Example 3.1 and are given in Table 3.10.

Because it has been assumed all the wind load effects are resisted by the perimeter moment-resisting frames, the slabs resist only gravity load effects, which are given in Table 5.34.

Step 6 – Determine the bending moments due to seismic forces

As noted in the example statement, it is assumed the Site Class is C for this building. From Example 3.5, Part (a), the building is assigned to SDC A for this case. Therefore, effects due to seismic forces need not be considered.

It is also assumed the lateral forces based on the general structural integrity requirements of ASCE/SEI 1.4.2 are resisted by the perimeter moment-resisting frames, and, thus, need not be applied to the slabs.

Step 7 – Determine the required flexural reinforcement

The required flexural reinforcement is determined at the critical sections in the column strip and middle strip using the following equations:

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] \qquad \text{Eq. (5.35)}$$

$$R_n = \frac{M_u}{\phi b d^2} \qquad \text{Eq. (5.36)}$$

where b is the width of the column strip or middle.

It is shown in Example 5.4 that the dimensions of the drop panel satisfy the requirements of ACI 8.2.4. Therefore, at negative critical sections in the column strip:

$$d = \text{lesser of} \begin{cases} (8.0 + 2.25) - 1.25 = 9.0 \text{ in.} \\ 8.0 + (39.0\ /\ 4) - 1.25 = 16.5 \text{ in.} \end{cases} \qquad \text{Figure 5.28}$$

where $b_{drop} = (8.5 \times 12\ /\ 2) - (24.0\ /\ 2) = 39.0$ in.

At all other critical sections, $d = 8.0 - 1.25 = 6.75$ in.

A summary of the required flexural reinforcement is given in Table 5.35. The provided areas of reinforcement are greater than or equal to then the minimum required in accordance with ACI 8.6.1.1 and the number of bars are selected so that the provided spacing of the reinforcing bars is less than the maximum required in ACI 8.7.2.2. It is evident the provided areas of reinforcement at all critical sections in the column strip and middle strip are less than the area of flexural reinforcement, $A_{s,t}$, corresponding to tension-controlled sections, which is determined by Eq. (5.41); therefore, all the critical sections are tension-controlled, which satisfies ACI 8.3.3.1.

Table 5.35 Required Flexural Reinforcement at the Second-Floor Level for the Flat Slab System in Example 5.10

Location			M_u (ft-kips)	b (in.)	d (in.)	A_s (in.^{2})*	Reinforcement*
End Span	Column strip	Exterior Negative	−78.4	141.0	9.00	2.60	9-#5
		Positive	102.3	141.0	6.75	3.48	12-#5
		First Interior Negative	−180.7	141.0	9.00	4.61	15-#5
	Middle strip	Exterior Negative	−23.9	159.0	6.75	2.29	10-#5
		Positive	68.2	159.0	6.75	2.29	10-#5
		First Interior Negative	−58.0	159.0	6.75	2.29	10-#5
Interior Span	Column strip	Positive	71.6	141.0	6.75	2.41	9-#5
		Negative	−167.0	141.0	9.00	4.25	14-#5
	Middle strip	Positive	47.7	159.0	6.75	2.29	10-#5
		Negative	−54.5	159.0	6.75	2.29	10-#5

* *Min. A_s for the column strip at negative critical sections* $= 0.0018 \times 141.0 \times 10.25 = 2.60$ in.^{2}

Min. A_s for the column strip at positive critical sections $= 0.0018 \times 141.0 \times 8.0 = 2.03$ in.^{2}

Min. A_s for the middle strip $= 0.0018 \times 159.0 \times 8.0 = 2.29$ in.^{2}

Max. spacing = lesser of $(2h,\ 18.0\text{ in.}) = 16.0$ in.

For $b = 141.0$ in., $141.0 / 16.0 = 8.8$ spaces, say minimum of 9 bars

For $b = 159.0$ in., $159.0 / 16.0 = 9.9$ spaces, say minimum of 10 bars

For the column strip at negative critical sections: $A_{s,t} = 0.018bd = 26.0$ in.^{2}

For the column strip at positve critical sections: $A_{s,t} = 0.018bd = 17.1$ in.^{2}

For the middle strip: $A_{s,t} = 0.018bd = 19.3$ in.^{2}

Step 8 – Check that the flexural reinforcement is adequate for moment transfer requirements ACI 8.4.2.2

- Edge columns

  The factored slab moments resisted by the edge columns need not be checked in this example because of the beams at the perimeter of the slab. These edges beams must be designed for shear forces and torsional moments transferred from the slab.

- Interior columns

  Determine the required reinforcement based on the largest transfer moment at the interior columns.

  In lieu of a more exact analysis, the factored slab moment, M_{sc}, transferred to an interior column due to the gravity load effects can be determined from the following equation where the spans in the direction of analysis and perpendicular to the direction of analysis are equal:

$$M_{sc} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (1.6 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 42.1 \text{ ft-kips} \quad \text{Eq. (5.21)}$$

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.60 \quad \text{Eq. (5.9)}$$

where

$b_1 = c_1 + d = 24.0 + 9.0 = 33.0 \text{ in.}$ Table 5.11, Case 1

$b_2 = c_2 + d = 24.0 + 9.0 = 33.0 \text{ in.}$

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

The moment $\gamma_f M_{sc} = 0.60 \times 42.1 = 25.3 \text{ ft-kips}$ must be transferred over the effective slab width b_{slab}:

$$b_{slab} = \text{lesser of} \begin{cases} c_2 + 3(h + h_1) = 24.0 + (3 \times 10.25) = 54.8 \text{ in., say } 55.0 \text{ in.} \\ \\ c_2 + 2(b_{drop} + 1.5h) = 24.0 + \{2 \times [39.0 + (1.5 \times 8.0)]\} = 126.0 \text{ in.} \end{cases}$$ Eq. (5.11)

Determine the required A_s:

$$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{25.3 \times 12}{0.9 \times 55.0 \times 9.0^2} = 0.076 \text{ ksi}$$ Eq. (5.36)

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4 \times 55.0 \times 9.0}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.076}{0.85 \times 4}}\right] = 0.63 \text{ in.}^2$$ Eq. (5.35)

This required area of steel is equivalent to 3-#5 bars. With the bars uniformly spaced in the column strip, 5 of the 15-#5 bars are within the 55.0-in. effective slab width at the first interior column, so no additional reinforcement is required to satisfy moment transfer requirements at these locations. Similar calculations show that no additional reinforcement is required at the interior column.

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

Determine the governing factored two-way shear stress, v_{uv}, at the interior columns.

$A_c = 2(b_1 + b_2)d = 2 \times (2 \times 33.0) \times 9.0 = 1{,}188.0 \text{ in.}^2$ Table 5.11, Case 1

Factored shear force at the critical section:

$$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) = 236.0 \times \left[(23.5 \times 25.0) - \left(\frac{33.0}{12}\right)^2\right] / 1{,}000 = 136.9 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_u}{A_c} = \frac{136{,}900}{1{,}188.0} = 115.2 \text{ psi} > \phi 2 \lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0 \sqrt{4{,}000} = 94.9 \text{ psi}$$ Eq. (5.38)

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d/10)}} = \sqrt{\frac{2}{1 + (9.0/10)}} = 1.03 > 1.0, \text{ use } 1.0$$ Eq. (5.30)

and

$\lambda = 1.0 \text{ for normalweight concrete}$ Table 5.20

Thus,

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 10.25 \times 55.0 = 1.02 \text{ in.}^2 \\ \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi\alpha_s f_y} = \dfrac{5 \times 115.2 \times 55.0 \times [2 \times (2 \times 33.0)]}{0.75 \times 40 \times 60{,}000} = 2.32 \text{ in.}^2 \end{cases} \qquad \text{Eq. (5.37)}$$

At the first interior columns, provided A_s within $b_{slab} = 5 \times 0.31 = 1.55 \text{ in.}^2 < A_{s,min} = 2.32 \text{ in.}^2$

At the interior column, provided A_s within $b_{slab} = 6 \times 0.31 = 1.86 \text{ in.}^2 < A_{s,min} = 2.32 \text{ in.}^2$

Additional reinforcement must be provided in b_{slab} to satisfy the minimum reinforcement requirements of ACI 8.6.1.2.

To satisfy minimum reinforcement requirements within b_{slab}, 8-#5 bars must be placed within the 55.0-in. effective slab width $(A_{s,provided} = 8 \times 0.31 = 2.48 \text{ in.}^2 > A_{s,min} = 2.32 \text{ in.}^2)$ at the first interior columns, with the remaining 7-#5 bars to be placed within the $141.0 - 55.0 = 86.0\text{-in.}$ width of the column strip. For symmetry, add 1-#5 bar to the 86.0-in. width. Thus, 16-#5 bars must be provided within the column strip at the first interior columns with 8-#5 bars concentrated within the 55.0-in. effective slab width. The reinforcement detail for this case is similar to that shown in Figure 5.44.

Similarly, 8-#5 bars must be placed within the 55.0-in. effective slab width at the interior column. The remaining 6-#5 bars are provided within the 86.0-in. width of the column strip.

Step 9 – Determine the reinforcement details — ACI 8.7

The lengths of the reinforcing bars in Figure 5.39 can be used because the slab is subjected to the effects from uniformly distributed gravity loads only.

A Class B tension lap splice is provided over the supports for the bottom bars in the column strip. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t\psi_e\psi_s\psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)} \right) d_b \geq 12.0 \text{ in.} \qquad \text{ACI Eq. (25.4.2.4a)}$$

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #5 reinforcing bars, $\psi_s = 0.8$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{lesser of} \begin{cases} \text{cover} + (d_b / 2) = 0.75 + (0.625 / 2) = 1.1 \text{ in.} \\ \\ s / 2 = 11.8 / 2 = 5.9 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (1.1 + 0) / 0.625 = 1.8 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60,000}{1.0\sqrt{4,000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{1.8} \right) \times 0.625 = 19.8 \text{ in.} > 12.0 \text{ in.}$$

Class B lap splice length $= 1.3\ell_d = 1.3 \times 19.8 = 25.7$ in. ACI Table 25.5.2.1

Provide a 2 ft-2 in. lap splice length.

Reinforcement details for the columns strip and middle strip are given in Figure 5.54. For simpler detailing, the lengths of the negative reinforcement at all critical sections in the column strip are set equal to 30 percent of the clear span length, which is greater than $5d = 3.8$ ft (see Figure 5.39). The structural integrity requirements in ACI 8.7.4.2 are included.

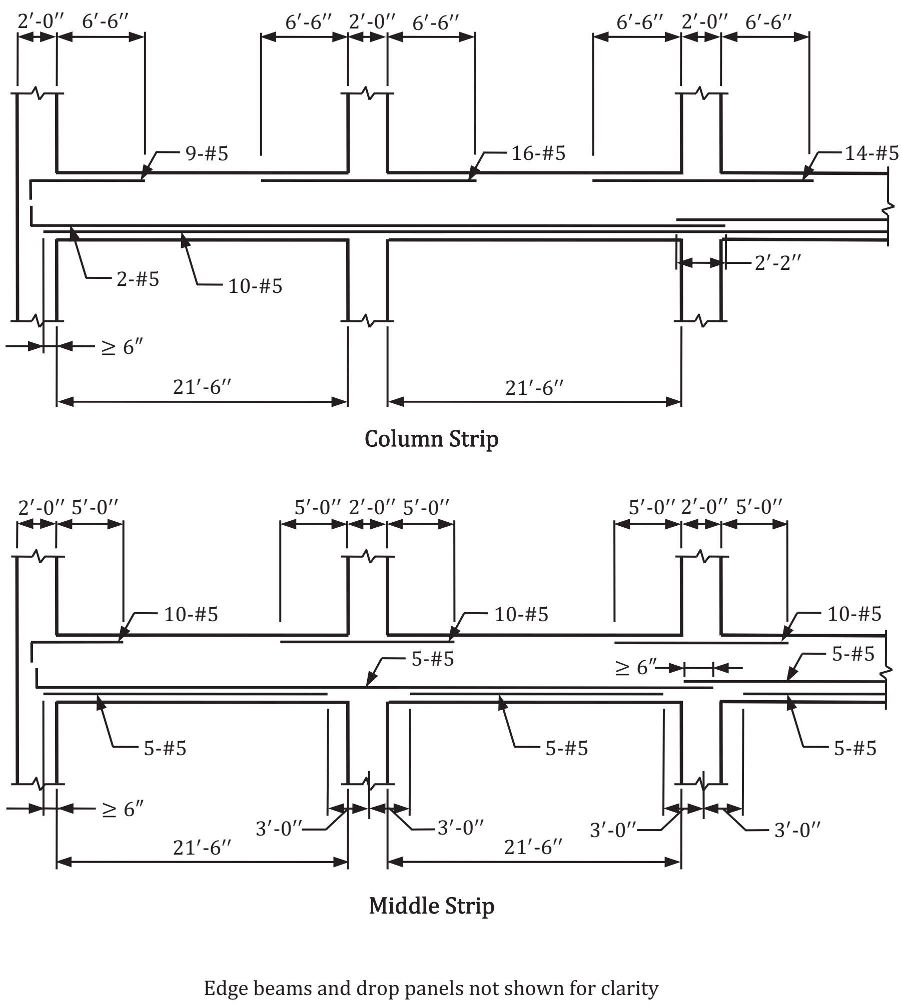


Figure 5.54 Reinforcement details for an interior design strip in the north-south direction for the flat slab system in Example 5.10.

5.8.11 Example 5.11 – Determination of Required Flexural Reinforcement: Two-way Joist System, Building #1 (Framing Option E), SDC A

Determine the required flexural reinforcement in an interior design strip in the east-west direction for the two-way joist system in Figure 1.1 (Framing Option E) at the second-floor level with a 3-ft module, a 4.5-in.-thick slab, a 20-in. dome, and 30 in. by 30 in. columns (see Figure 5.55). Assume the Site Class is C. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

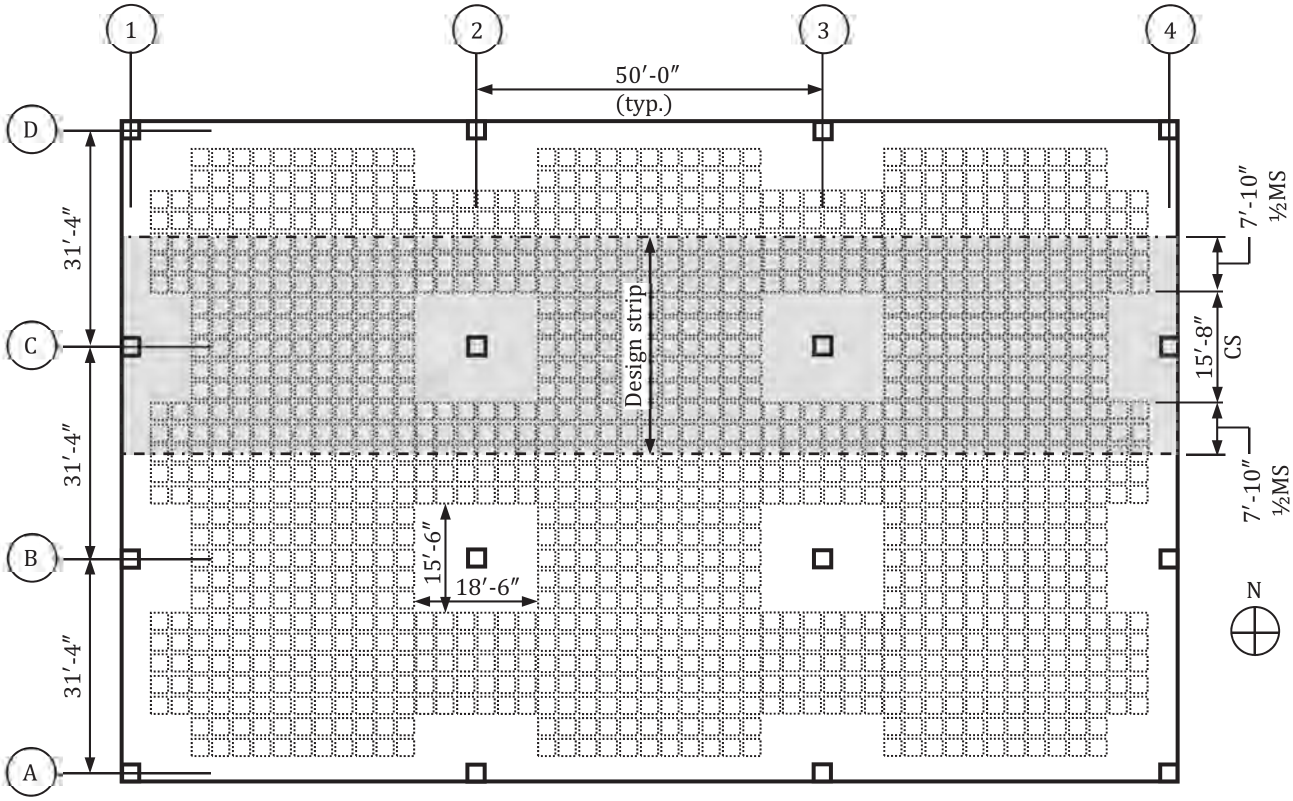


Figure 5.55 Design strips for the two-way joist system in Example 5.11.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the widths of the column strip and middle strip ACI 8.4.1.5, 8.4.1.6

$$\text{Width of column strip} = \text{lesser of} \begin{cases} \ell_1 / 2 = 50.0 / 2 = 25.0 \text{ ft} \\ \ell_2 / 2 = 31.33 / 2 = 15.67 \text{ ft} \end{cases}$$ Figure 5.8

$$\text{Width of middle strip} = 31.33 - 15.67 = 15.67 \text{ ft}$$

Step 2 – Check if the Direct Design Method can be used to determine gravity load bending moments Sect. 5.3.4

Check the following limitations: Figure 5.22

- There must be 3 or more continuous spans in each direction…There are 3 continuous spans in the north-south and east-west directions.

- Successive panel lengths measured center-to-center of supports in each direction must not differ by more than one-third the longer span…All the panel lengths are equal in each direction.
- Panels must be rectangular with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2… $\text{Largest ratio of panel lengths} = 50.0 / 31.33 = 1.6 < 2.0$.
- Column offset must not exceed 10 percent of the span in the direction of the offset from either axis between centerlines of successive columns…None of the columns are offset.
- All loads must be due to gravity only and uniformly distributed over an entire panel…This method will be used only for gravity loads, which are uniformly distributed over the entire floor system.
- The unfactored live load must not exceed 2 times the unfactored dead load… $\text{Unfactored live load} = 65\ \text{lb/ft}^2$; $\text{unfactored dead load} = 173.3 + 10 = 183.3\ \text{lb/ft}^2$; $L / D = 0.4 < 2$.
- For a panel with beams between supports on all sides, the following equation must be satisfied for the beams in the two perpendicular directions: $0.2 \le \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 \le 5.0$…There are no beams in this floor system, so this limitation is not applicable.

Because all the limitations are satisfied, the DDM can be used to determine the bending moments in the column and middle strips.

Step 3 – Determine the total factored static moment in each span

$\text{Dead load of two-way joist} = 173.3\ \text{lb/ft}^2$

$\text{Superimposed dead load} = 10.0\ \text{lb/ft}^2$

$\text{Live load} = 65.0\ \text{lb/ft}^2$

$\ell_2 = 31.33\ \text{ft}$ Figure 5.8

$\ell_n = 50.0 - (30.0 / 12) = 47.5\ \text{ft}$ Figure 5.23

Because the two-way joist system is part of the LFRS, service dead and live load bending moments are determined, which will be combined with those due to the effects from lateral forces:

Dead load: $$M_{oD} = \frac{q_D \ell_2 \ell_n^2}{8} = \frac{(173.3 + 10.0) \times 31.33 \times 47.5^2}{8 \times 1{,}000} = 1{,}619.7\ \text{ft-kips}$$ Eq. (5.17)

Live load: $$M_{oL} = \frac{q_L \ell_2 \ell_n^2}{8} = \frac{65.0 \times 31.33 \times 47.5^2}{8 \times 1{,}000} = 574.3\ \text{ft-kips}$$

These total factored static moments are the same for all spans of the interior design strip in the direction of analysis.

Step 4 – Distribute the total factored static moment to the column strips and middle strips Table 5.16

As noted in Sect. 5.2.5, a two-way joist system is considered to be a flat slab for purposes of design. Design moment coefficients for a flat slab system without edge beams are given in Table 5.16. A summary of the service dead and live load bending moments in the column strip and middle strip in an end span and interior span is given in Table 5.36.

Table 5.36 Bending Moments (ft-kips) due to Service Dead and Live Loads at the Second-Floor Level for the Two-way Joist System in Example 5.11

Design Strip	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Column strip	$0.26M_{oD} = -421.1$ $0.26M_{oL} = -149.3$	$0.31M_{oD} = 502.1$ $0.31M_{oL} = 178.0$	$0.53M_{oD} = -858.4$ $0.53M_{oL} = -304.4$	$0.21M_{oD} = 340.1$ $0.21M_{oL} = 120.6$	$0.49M_{oD} = -793.7$ $0.49M_{oL} = -281.4$
Middle strip	0	$0.21M_{oD} = 340.1$ $0.21M_{oL} = 120.6$	$0.17M_{oD} = -275.3$ $0.17M_{oL} = -97.6$	$0.14M_{oD} = 226.8$ $0.14M_{oL} = 80.4$	$0.16M_{oD} = -259.2$ $0.16M_{oL} = -91.9$

Step 5 – Determine the bending moments due to wind forces ACI 6.6

Design wind forces in the east-west direction on Building #1 are determined in Example 3.1 and are given in Table 3.10.

A three-dimensional model of the building was constructed using Reference 14 and a linear first-order analysis was performed using the east-west wind loads in Table 3.10 applied to the centroid of the building face at the roof and floor levels (see Figure 5.56). In the model, the columns are fixed at the base.

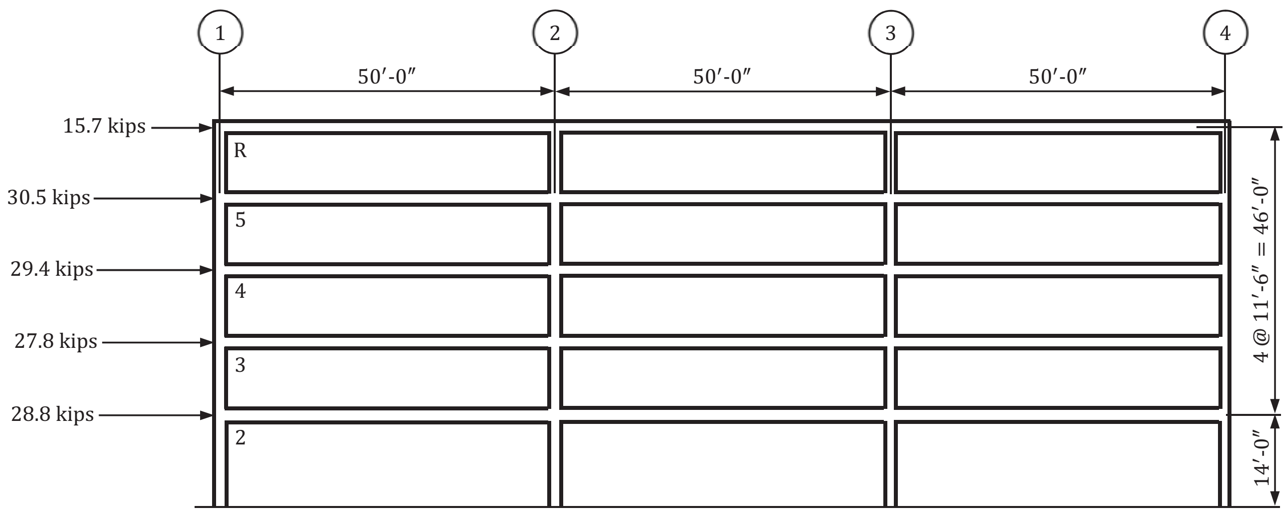


Figure 5.56 Total wind loads applied to Building #1.

The following reduced moments of inertia are used to account for cracked sections: ACI Table 6.6.3.1.1(a)

- Columns: $I = 0.70I_g$
- Two-way joists: $I = 0.25I_g$

A finite element analysis was performed to determine the extent over which the slab system resists the effects from the wind loads. From the analysis, it was determined the majority of the wind load effects at an interior design strip are resisted by slab strips centered on the columns with widths approximately equal to the column strip width; thus, all the wind load effects are assigned to the column strips in the direction of analysis in this example.

The bending moments in the column strip due to wind loads are given in Table 5.37 for the end and interior spans. The "plus-minus" sign preceding the tabulated values signifies the wind loads can act in both the east direction and the west direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the south elevation of the building).

Table 5.37 Bending Moments (ft-kips) due to Wind Loads at the Second-Floor Level for the Two-way Joist System in Example 5.11

Design Strip	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Column strip	±55.7	—	±51.9	—	±44.6

Step 6 – Determine the bending moments due to seismic forces

As noted in the example statement, it is assumed the Site Class is C for this building. From Example 3.5, Part (a), the building is assigned to SDC A for this case. Therefore, effects due to seismic forces need not be considered.

It can be determined the lateral forces based on the general structural integrity requirements of ASCE/SEI 1.4.2 are less than those due to wind forces, and, thus, need not be considered in this example.

Step 7 – Determine the combined factored bending moments due to gravity and wind forces ACI Table 5.3.1

The design bending moments from the governing load combinations are given in Table 5.38. The gravity load moments from the DDM are combined with the bending moment from the lateral load analysis (ACI 8.4.1.9). It is evident that at all locations, the maximum moments are equal to those from the combination of the gravity load effects.

Table 5.38 Design Bending Moments (ft-kips) at the Second-Floor Level for the Two-way Joist System in Example 5.11

Load Combination	Location		End Span			Interior Span	
			1	2	3	4	5
			Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
$1.2D + 1.6L$	Column strip		−744.2	887.3	−1,517.1	601.1	−1,402.7
	Middle strip		0	601.1	−486.6	400.8	−458.1
$1.2D + 1.0W + 0.5L$	Column strip	SSR	−524.3	691.5	−1,234.2	468.4	−1,048.5
		SSL	−635.7	691.5	−1,130.4	468.4	−1,137.7
	Middle strip		0	468.4	−379.2	312.4	−357.0
$0.9D + 1.0W$	Column strip	SSR	−323.3	451.9	−824.5	306.1	−669.7
		SSL	−434.7	451.9	−720.7	306.1	−758.9
	Middle strip		0	306.1	−247.9	204.1	−233.3

Step 8 – Determine the required flexural reinforcement

The required flexural reinforcement is determined at the critical sections in the column strip and middle strip using the following equations:

$$A_s = \frac{0.85 f'_c bd}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] \qquad \text{Eq. (5.35)}$$

$$R_n = \frac{M_u}{\phi b d^2} \qquad \text{Eq. (5.36)}$$

In this example, there are 6 ribs in the column strip and 4 ribs in the middle strip (2 ribs are in each of the half middle strips).

- Column strip flexural reinforcement

  At the negative critical sections, the total negative factored moment M_u^- is resisted by the solid area of concrete around the column, and A_s^- is determined using M_u^-, $b = 186.0$ in., and $d = 24.5 - 1.25 = 23.25$ in. (the width of the solid head perpendicular to the direction of analysis is 186.0 in., which is 2.0 in. less than the width of the columns strip; the 186.0-in. width is used to calculate A_s).

  At the positive critical sections, the total positive factored moment M_u^+ is shared equally between the 6 ribs, and A_s^+ is determined for each rib using $M_u^+ / 6$, $b = b_d + b_r = 30.0 + 6.0 = 36.0$ in., and $d = 24.5 - 1.25 = 23.25$ in.

- Middle strip flexural reinforcement

  At the negative critical sections, the total negative factored moment M_u^- is resisted by 4 ribs, and A_s^- is determined using M_u^-, $b = \text{number of ribs} \times b_r = 4 \times 6.0 = 24.0$ in., and $d = 24.5 - 1.25 = 23.25$ in.

  At the positive critical sections, the total positive factored moment M_u^+ is shared equally between the 4 ribs, and A_s^+ is determined per rib using $M_u^+ / 4$, $b = b_d + b_r = 30.0 + 6.0 = 36.0$ in., and $d = 24.5 - 1.25 = 23.25$ in.

  The minimum required flexural reinforcement at negative critical sections in the column and middle strips is equal to the following:

  $$A_{s,min} = 0.0018bh \qquad \text{ACI 8.6.1.1}$$

  At all other critical sections, the minimum required flexural reinforcement can be determined using the requirements for beams:

  $$A_{s,min} = \text{greater of} \begin{cases} 3\sqrt{f'_c}\, b_w d / f_y \\ 200 b_w d / f_y \end{cases} \qquad \text{ACI 9.6.1.2}$$

  A summary of the required flexural reinforcement is given in Table 5.39, which is applicable for both the end and interior spans. It can be determined all sections are tension-controlled.

Table 5.39 Required Flexural Reinforcement at the Second-Floor Level for the Two-way Joist System in Example 5.11

Location		M_u (ft-kips)	b (in.)	A_s (in.^{2})*	Reinforcement*
Column strip	Exterior Negative	−744.2	186.0	8.20	14-#7
	Positive	887.3 / 6 = 147.9	36.0	1.44	2-#8/rib
	First Interior Negative	−1,517.1	186.0	14.96	25-#7
Middle strip	Exterior Negative	0	24.0	1.52	21-#5
	Positive	601.1 / 4 = 150.3	36.0	1.46	2-#8/rib
	First Interior Negative	−486.6	24.0	5.06	21-#5

* *Min. A_s for negative moment in the column strip* $= 0.0018 \times 186.0 \times 24.5 = 8.20$ in.^{2}
Min. A_s for negative moment in the middle strip $= 0.0018 \times 188.0 \times 4.5 = 1.52$ in.^{2};
minimum number of bars $= b$ / max. spacing $= 188.0 / (2 \times 4.5) = 20.9$, say 21 bars
Min. A_s for all other critical sections = greater of $[3\sqrt{f_c'}b_w d / f_y,\ 200 b_w d / f_y]$

Step 9 – Check that the flexural reinforcement is adequate for moment transfer requirements — ACI 8.4.2.2

- Edge columns

Determine the required reinforcement based on $M_{sc} = 0.30M_o$. — Sect. 5.3.4

$$M_{sc} = 0.30(1.2M_{oD} + 1.6M_{oL}) = 0.30 \times [(1.2 \times 1{,}619.7) + (1.6 \times 574.3)] = 858.8 \text{ ft-kips}$$

This moment is greater than 635.7 ft-kips, which is the moment due to the load combination $1.2D + 1.0W + 0.5L$. — Table 5.38

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.63 \qquad \text{Eq. (5.9)}$$

where

$$b_1 = c_1 + (d/2) = 30.0 + (23.25/2) = 41.63 \text{ in.} \qquad \text{Table 5.11, Case 3}$$

$$b_2 = c_2 + d = 30.0 + 23.25 = 53.25 \text{ in.}$$

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

The moment $\gamma_f M_{sc} = 0.63 \times 858.8 = 541.0$ ft-kips must be transferred over the effective slab width b_{slab}:

$$b_{slab} = c_2 + 3h = 30.0 + (3 \times 24.5) = 103.5 \text{ in., say } 103.0 \text{ in.} \qquad \text{Eq. (5.10)}$$

Determine the required A_s:

$$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{541.0 \times 12}{0.9 \times 103.0 \times 23.25^2} = 0.130 \text{ ksi} \qquad \text{Eq. (5.36)}$$

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4 \times 103.0 \times 23.25}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.130}{0.85 \times 4}}\right] = 5.29 \text{ in.}^2 \qquad \text{Eq. (5.35)}$$

This required area of steel is equivalent to 9-#7 bars. Provide the 9-#7 bars by concentrating 9 of the 14-#7 column strip bars within the 103.0-in. width over the column (see Table 5.39). For symmetry, add 1-#7 bar and check bar spacing:

For 9-#7 within the 103.0-in. width, $\text{bar spacing} = 103.0 / 9 = 11.4 \text{ in.} < 18.0 \text{ in.}$

For 6-#7 within the $186.0 - 103.0 = 83.0\text{-in. width}$, $\text{bar spacing} = 83.0 / 6 = 13.8 \text{ in.} < 18.0 \text{ in.}$

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2q_D + 1.6q_L = [1.2 \times (173.3 + 10.0)] + (1.6 \times 65.0) = 324.0 \text{ lb/ft}^2$$

$$A_c = (2b_1 + b_2)d = [(2 \times 41.63) + 53.25)] \times 23.25 = 3{,}173.9 \text{ in.}^2$$ Table 5.11, Case 3

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 324.0 \times \left\{\left[31.33 \times \left(\frac{50.0}{2} + \frac{30.0}{2 \times 12}\right)\right] - \left(\frac{41.63 \times 53.25}{144}\right)\right\} / 1{,}000 = 261.5 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_{u(D+L)}}{A_c} = \frac{261{,}500}{3{,}173.9} = 82.4 \text{ psi} > \phi 2\lambda_s\lambda\sqrt{f_c'} = 0.75 \times 2 \times 0.8 \times 1.0\sqrt{4{,}000} = 75.9 \text{ psi}$$ Eq. (5.38)

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (23.25 / 10)}} = 0.8 < 1.0$$ Eq. (5.30)

$\lambda = 1.0$ for normalweight concrete Table 5.20

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 24.5 \times 103.0 = 4.54 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi\alpha_s f_y} = \dfrac{5 \times 82.4 \times 103.0 \times [(2 \times 41.63) + 53.25)]}{0.75 \times 30 \times 60{,}000} = 4.29 \text{ in.}^2 \end{cases}$$ Eq. (5.37)

Provided A_s within $b_{slab} = 9 \times 0.60 = 5.40 \text{ in.}^2 > A_{s,min} = 4.54 \text{ in.}^2$, so no additional reinforcement is required based on this load combination.

(2) Load combination: $1.2D + 1.0W + 0.5L$

$$q_u = 1.2q_D + 0.5q_L = [1.2 \times (173.3 + 10.0)] + (0.5 \times 65.0) = 252.5 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 252.5 \times \left\{\left[31.33 \times \left(\frac{50.0}{2} + \frac{30.0}{2 \times 12}\right)\right] - \left(\frac{41.63 \times 53.25}{144}\right)\right\} / 1{,}000 = 203.8 \text{ kips}$$

$$V_{u(W)} = (55.7 + 51.9) / 47.5 = 2.3 \text{ kips}$$ Table 5.37

Therefore,

$$v_{uv} = \frac{V_{u(D+L)} + V_{u(W)}}{A_c} = \frac{203,800 + 2,300}{3,173.9} = 64.9 \text{ psi}$$

Eq. (5.38)

$$< \phi 2\lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 0.8 \times 1.0\sqrt{4,000} = 75.9 \text{ psi}$$

Provided A_s within $b_{slab} = 9 \times 0.60 = 5.40 \text{ in.}^2 > A_{s,min} = 0.0018 \times 24.5 \times 103.0 = 4.54 \text{ in.}^2$

No additional reinforcement is required to satisfy moment transfer requirements at the edge columns.

- Interior columns

Determine the required reinforcement based on the largest transfer moment at the interior columns.

In lieu of a more exact analysis, the factored slab moment, M_{sc}, transferred to an interior column due to the gravity load effects and gravity and wind load effects can be determined from the following equations where the spans in the direction of analysis and perpendicular to the direction of analysis are equal.

For gravity loads only:

$$M_{sc(L)} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (1.6 \times 65.0) \times 31.33 \times 47.5^2 / 1,000 = 257.3 \text{ ft-kips}$$ Eq. (5.21)

For gravity and wind loads:

$M_{sc(L)} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (0.5 \times 65.0) \times 31.33 \times 47.5^2 / 1,000 = 80.4 \text{ ft-kips}$ due to gravity load effects

$M_{sc} = 51.9 + 44.6 = 96.5 \text{ ft-kips}$ due to wind load effects — Table 5.37

Total $M_{sc} = 80.4 + 96.5 = 176.9 \text{ ft-kips}$

Therefore, use $M_{sc} = 257.3 \text{ ft-kips}$ at the interior columns.

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.60$$ Eq. (5.9)

where

$b_1 = c_1 + d = 30.0 + 23.25 = 53.25 \text{ in.}$ — Table 5.11, Case 1

$b_2 = c_2 + d = 30.0 + 23.25 = 53.25 \text{ in.}$

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

The moment $\gamma_f M_{sc} = 0.60 \times 257.3 = 154.4 \text{ ft-kips}$ must be transferred over the effective slab width b_{slab}:

$b_{slab} = c_2 + 3h = 30.0 + (3 \times 24.5) = 103.5 \text{ in., say } 103.0 \text{ in.}$ — Eq. (5.10)

Determine the required A_s:

$$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{154.4 \times 12}{0.9 \times 103.0 \times 23.25^2} = 0.037 \text{ ksi}$$ Eq. (5.36)

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4 \times 103.0 \times 23.25}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.037}{0.85 \times 4}}\right] = 1.48 \text{ in.}^2$$ Eq. (5.35)

This required area of steel is equivalent to 3-#7 bars. With the bars uniformly spaced within the column strip, 13 of the 25-#7 bars are within the 103.0-in. effective slab width at the interior columns, so no additional reinforcement is required to satisfy moment transfer requirements at this location.

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

(1) Load combination: $1.2D + 1.6L$

$$A_c = 2(b_1 + b_2)d = 2 \times (2 \times 53.25) \times 23.25 = 4{,}952.3 \text{ in.}^2$$ Table 5.11, Case 1

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 324.0 \times \left[(31.33 \times 50.0) - \left(\frac{53.25}{12}\right)^2\right] / 1{,}000 = 501.2 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_{u(D+L)}}{A_c} = \frac{501{,}200}{4{,}952.3} = 101.2 \text{ psi} > \phi 2 \lambda_s \lambda \sqrt{f'_c} = 0.75 \times 2 \times 0.8 \times 1.0\sqrt{4{,}000} = 75.9 \text{ psi}$$ Eq. (5.38)

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (23.25 / 10)}} = 0.8 < 1.0$$ Eq. (5.30)

and

$\lambda = 1.0$ for normalweight concrete Table 5.20

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 24.5 \times 103.0 = 4.54 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi \alpha_s f_y} = \dfrac{5 \times 101.2 \times 103.0 \times [2 \times (2 \times 53.25)]}{0.75 \times 40 \times 60{,}000} = 6.17 \text{ in.}^2 \end{cases}$$ Eq. (5.37)

(2) Load combination: $1.2D + 1.0W + 0.5L$

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 252.5 \times \left[(31.33 \times 50.0) - \left(\frac{53.25}{12}\right)^2\right] / 1{,}000 = 390.6 \text{ kips}$$

$$V_{u(W)} = (51.9 + 44.6) / 47.5 = 2.0 \text{ kips}$$ Table 5.37

Therefore, the maximum shear stress is equal to the following:

$$v_{uv} = \frac{V_{u(D+L)} + V_{u(W)}}{A_c} = \frac{390{,}600 + 2{,}000}{4{,}952.3} = 79.3 \text{ psi}$$

$$> \phi 2 \lambda_s \lambda \sqrt{f'_c} = 0.75 \times 2 \times 0.8 \times 1.0\sqrt{4{,}000} = 75.9 \text{ psi}$$ Eq. (5.38)

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 24.5 \times 103.0 = 4.54 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi \alpha_s f_y} = \dfrac{5 \times 79.4 \times 103.0 \times [2 \times (2 \times 53.25)]}{0.75 \times 40 \times 60{,}000} = 4.84 \text{ in.}^2 \end{cases}$$ Eq. (5.37)

Thus, the $1.2D + 1.6L$ load combination governs with respect to minimum reinforcement requirements.

Provided A_s within $b_{slab} = 13 \times 0.60 = 7.80 \text{ in.}^2 > A_{s,min} = 6.17 \text{ in.}^2$

Thus, no additional reinforcement is required to satisfy moment transfer requirements at the interior columns.

Step 10 – Determine the reinforcement details ACI 8.7

The lengths of the top reinforcing bars in the 4.5-in.-thick slab shown in Figure 5.39 cannot be used in the column strip because of the effects due to wind loads. To account for the wind load effects, 25 percent of the top bars in the column strips are made continuous over the spans. In this case, the maximum amount of top reinforcement occurs at the first interior support (25-#7 bars), so 7-#7 bars are made continuous.

The remaining bars in the column strip and the top bars in the middle strip can be terminated at the locations identified in Figure 5.39.

According to ACI 8.8.1.6, at least one bottom bar in each rib must be continuous and must be anchored to develop f_y at the faces of the supports for structural integrity. The other bottom bars can be cutoff in the column strips and middle strips in accordance with the lengths shown in Figure 5.39.

A Class B tension lap splice is provided over the supports for the bottom bars in the column strip. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #8 reinforcing bars, $\psi_s = 1.0$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{lesser of} \begin{cases} \text{cover} + (d_b / 2) = 0.75 + (1.0 / 2) = 1.25 \text{ in.} \\ \\ s / 2 = [6.0 - (2 \times 0.75) - 1.0] / 2 = 1.75 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (1.25 + 0) / 1.0 = 1.25 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{1.25} \right) \times 1.0 = 56.9 \text{ in.} > 12.0 \text{ in.}$$

Class B lap splice length $= 1.3\ell_d = 1.3 \times 56.9 = 74.0$ in. ACI Table 25.5.2.1

Provide a 6 ft-2 in. lap splice length.

Reinforcement details for the columns strip and middle strip are given in Figure 5.57.

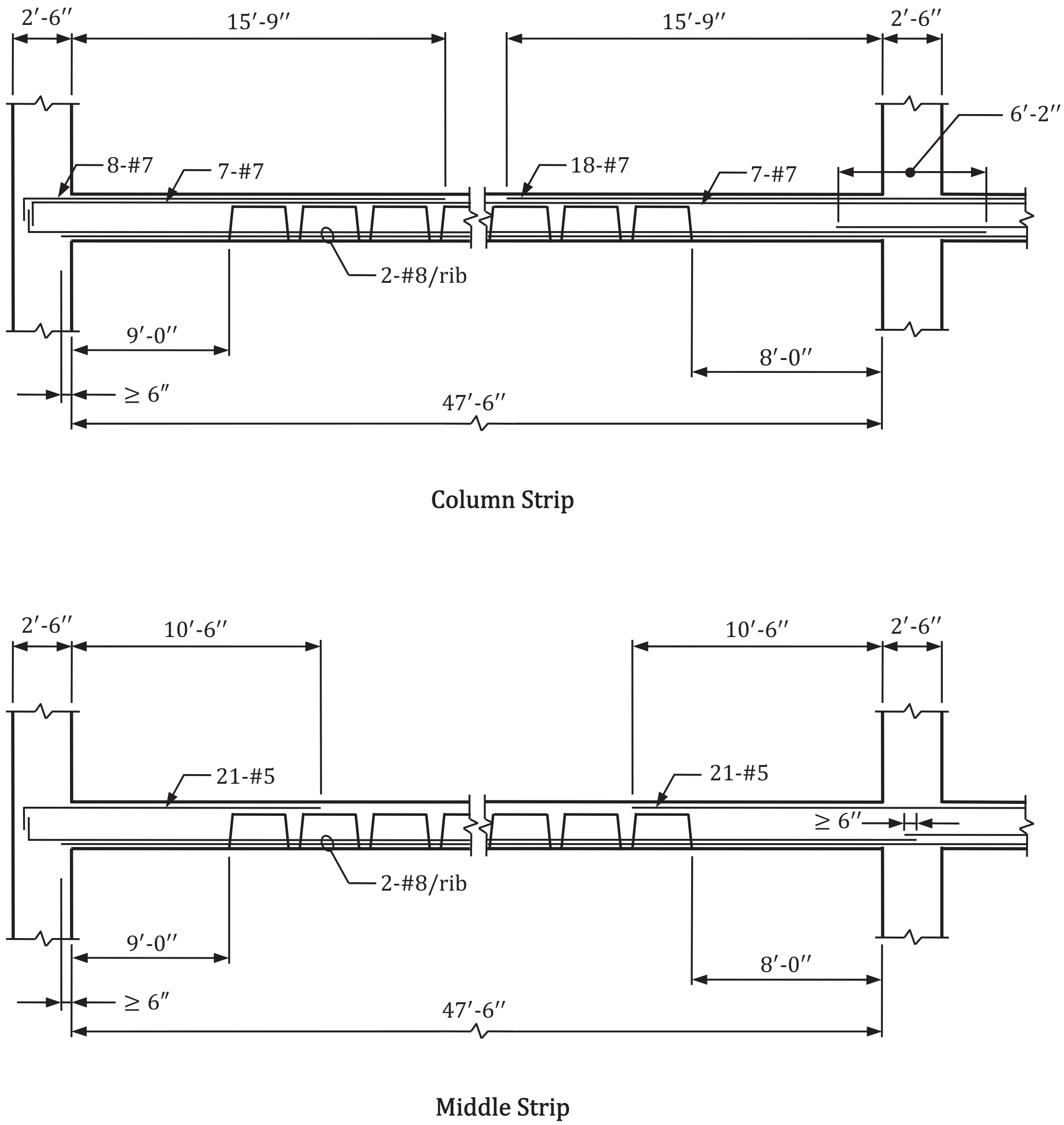


Figure 5.57 Reinforcement details for an interior design strip in the east-west direction for the two-way joist system in Example 5.11.

5.8.12 Example 5.12 – Determination of Required Flexural Reinforcement: Flat Plate System, Building #1 (Framing Option A), SDC B

Determine the required flexural reinforcement in an interior design strip in the north-south direction for the flat plate system in Figure 1.1 (Framing Option A) at the second-floor level with a 9.5-in.-thick slab and 24 in. by 24 in. columns (see Figure 5.41). Assume the Site Class is D. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the widths of the column strip and middle strip ACI 8.4.1.5, 8.4.1.6

$$\text{Width of column strip} = \text{lesser of} \begin{cases} \ell_1 / 2 = 23.5 / 2 = 11.75 \text{ ft} \\ \\ \ell_2 / 2 = 25.0 / 2 = 12.5 \text{ ft} \end{cases}$$ Figure 5.8

$$\text{Width of middle strip} = 25.0 - 11.75 = 13.25 \text{ ft}$$

Step 2 – Check if the Direct Design Method can be used to determine gravity load bending moments Sect. 5.3.4

Check the following limitations: Figure 5.22

- There must be 3 or more continuous spans in each direction…There are 4 and 6 continuous spans in the north-south and east-west directions, respectively.
- Successive panel lengths measured center-to-center of supports in each direction must not differ by more than one-third the longer span…All the panel lengths are equal in each direction.
- Panels must be rectangular with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2… $\text{Largest ratio of panel lengths} = 25.0 / 23.5 = 1.1 < 2.0$.
- Column offset must not exceed 10 percent of the span in the direction of the offset from either axis between centerlines of successive columns…None of the columns are offset.
- All loads must be due to gravity only and uniformly distributed over an entire panel…This method will be used only for gravity loads, which are uniformly distributed over the entire floor system.
- The unfactored live load must not exceed 2 times the unfactored dead load… $\text{Unfactored live load} = 65 \text{ lb/ft}^2$; assuming an 9.5-in.-thick slab, $\text{unfactored dead load} = (9.5 \times 150.0 / 12) + 10 = 129 \text{ lb/ft}^2$; $L / D = 0.5 < 2$.
- For a panel with beams between supports on all sides, the following equation must be satisfied for the beams in the two perpendicular directions: $0.2 \le \alpha_{f1}\ell_2^2 / \alpha_{f2}\ell_1^2 \le 5.0$…There are no beams in this floor system, so this limitation is not applicable.

Because all the limitations are satisfied, the DDM can be used to determine the bending moments in the column and middle strips.

Step 3 – Determine the total factored static moment in each span

$$\text{Dead load of slab} = (9.5 / 12) \times 150.0 = 118.8 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$$

$$\text{Live load} = 65.0 \text{ lb/ft}^2$$

$$\ell_2 = 25.0 \text{ ft}$$ Figure 5.8

$$\ell_n = 23.5 - (24.0 / 12) = 21.5 \text{ ft}$$ Figure 5.23

Because the flat plate is part of the LFRS, service dead and live load bending moments are determined, which will be combined with those due to the effects from lateral forces:

Dead load: $$M_{oD} = \frac{q_D \ell_2 \ell_n^2}{8} = \frac{(118.8 + 10.0) \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 186.1 \text{ ft-kips}$$ Eq. (5.17)

Live load: $$M_{oL} = \frac{q_L \ell_2 \ell_n^2}{8} = \frac{65.0 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 93.9 \text{ ft-kips}$$

These total factored static moments are the same for all spans of the interior design strip in the direction of analysis.

Step 4 – Distribute the total factored static moment to the column strips and middle strips Table 5.16

Design moment coefficients for a flat plate system without edge beams are given in Table 5.16. A summary of the service dead and live load bending moments in the column strip and middle strip in an end span and interior span is given in Table 5.40.

Table 5.40 Bending Moments (ft-kips) due to Service Dead and Live Loads at the Second-Floor Level for the Flat Plate System in Example 5.12

Design Strip	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Column strip	$0.26M_{oD} = -48.4$ $0.26M_{oL} = -24.4$	$0.31M_{oD} = 57.7$ $0.31M_{oL} = 29.1$	$0.53M_{oD} = -98.6$ $0.53M_{oL} = -49.8$	$0.21M_{oD} = 39.1$ $0.21M_{oL} = 19.7$	$0.49M_{oD} = -91.2$ $0.49M_{oL} = -46.0$
Middle strip	0	$0.21M_{oD} = 39.1$ $0.21M_{oL} = 19.7$	$0.17M_{oD} = -31.6$ $0.17M_{oL} = -16.0$	$0.14M_{oD} = 26.1$ $0.14M_{oL} = 13.2$	$0.16M_{oD} = -29.8$ $0.16M_{oL} = -15.0$

Step 5 – Determine the bending moments due to wind forces ACI 6.6

Design wind forces in the north-south direction on Building #1 are determined in Example 3.1 and are given in Table 3.10.

A three-dimensional model of the building was constructed using Reference 14 and a linear first-order analysis was performed using the north-south wind loads in Table 3.10 applied to the centroid of the building face at the roof and floor levels (see Figure 5.42). In the model, the columns are fixed at the base.

The following reduced moments of inertia are used to account for cracked sections: ACI Table 6.6.3.1.1(a)

- Columns: $I = 0.70I_g$
- Slabs: $I = 0.25I_g$

A finite element analysis was performed to determine the extent over which the slab resists the effects from the wind loads. From the analysis, it was determined the majority of the wind load effects at an interior design strip are resisted by slab strips centered on the columns with widths approximately equal to the column strip width; thus, all the wind load effects are assigned to the column strips in the direction of analysis in this example.

The bending moments in the column strip due to wind loads in an end span and interior span are given in Table 5.41. The "plus-minus" sign preceding the tabulated values signifies the wind loads can act in both the north direction and the south direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the east elevation of the building).

Table 5.41 Bending Moments (ft-kips) due to Wind Loads at the Second-Floor Level for the Flat Plate System in Example 5.12

Design Strip	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Column strip	±16.6	—	±13.8	—	±12.0

Step 6 – Determine the bending moments due to seismic forces

As noted in the example statement, it is assumed the Site Class is D for this building. From Example 3.5, Part (b), the building is assigned to SDC B for this case. Therefore, effects due to seismic forces must be considered.

Design seismic forces in the north-south direction on Building #1 are determined in Example 3.9 and are given in Table 3.26.

The three-dimensional model described in Step 5 was used to perform the linear first-order analysis using the north-south seismic loads in Table 3.26, which are applied to the center of mass at each level (rigid diaphragms are assigned at each level). Like in the case of wind loads, the majority of the seismic load effects at an interior design strip are resisted by slab strips centered on the columns with widths approximately equal to the column strip width; thus, all the seismic load effects are assigned to the column strips in the direction of analysis in this example.

The bending moments in the column strip due to seismic loads in an end span and interior span are given in Table 5.42. The “plus-minus” sign preceding the tabulated values signifies the seismic loads can act in both the north direction and the south direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the east elevation of the building).

Table 5.42 Bending Moments (ft-kips) due to Seismic Loads at the Second-Floor Level for the Flat Plate System in Example 5.12

Design Strip	End Span			Interior Span	
	1	2	3	4	5
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
Column strip	±42.4	—	±35.4	—	±31.6

It can be determined the lateral forces based on the general structural integrity requirements of ASCE/SEI 1.4.2 are less than those due to wind and seismic forces, and, thus, need not be considered in this example.

Comparing Tables 5.41 and 5.42, it is evident the bending moments due to seismic load effects are larger than those due to wind load effects. Because both types of effects are assigned a load factor equal to 1.0 in the design load combinations, only the bending moments due to the seismic load effects are considered in the remainder of this example.

Step 7 – Determine the combined factored bending moments due to gravity and seismic forces

ACI Table 5.3.1

The design bending moments from the governing load combinations are given in Table 5.43. The seismic load combinations are given in Table 3.4 for SDC B. The gravity load moments from the DDM are combined with the bending moment from the lateral load analysis (see ACI 8.4.1.9).

Table 5.43 Design Bending Moments (ft-kips) at the Second-Floor Level for the Flat Plate System in Example 5.12

Load Combination	Location		End Span			Interior Span	
			1	2	3	4	5
			Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
$1.2D + 1.6L$	Column strip		−97.1	115.8	−198.0	78.4	−183.0
	Middle strip		0	78.4	−63.5	52.4	−59.8
$1.2D + Q_E + 0.5L$	Column strip	SSR	−27.9	83.8	−178.6	56.8	−164.0
		SSL	−112.7	83.8	−107.8	56.8	−100.8
	Middle strip		0	56.8	−45.9	37.9	−43.3
$0.9D + Q_E$	Column strip	SSR	1.2	51.9	−124.1	35.2	−113.7
		SSL	−86.0	51.9	−53.3	35.2	−50.5
	Middle strip		0	35.2	−28.4	23.5	−26.8

Step 8 – Determine the required flexural reinforcement

The required flexural reinforcement is determined at the critical sections in the column strip and middle strip using the following equations where b is the width of the column strip or middle strip and $d = 9.5 - 1.25 = 8.25$ in.:

$$A_s = \frac{0.85 f'_c b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] \quad \text{Eq. (5.35)}$$

$$R_n = \frac{M_u}{\phi b d^2} \quad \text{Eq. (5.36)}$$

A summary of the required flexural reinforcement is given in Table 5.44. The provided areas of reinforcement are greater than or equal to the minimum required in accordance with ACI 8.6.1.1 and the number of bars are selected so that the provided spacing of the reinforcing bars is less than the maximum required in ACI 8.7.2.2. It is evident the provided areas of reinforcement at all critical sections in the column strip and middle strip are less than the area of flexural reinforcement, $A_{s,t}$, corresponding to tension-controlled sections, which is determined by Eq. (5.41); therefore, all the critical sections are tension-controlled, which satisfies ACI 8.3.3.1. Note that the 11-#5 bottom bars in the end span required for the factored positive moment are also adequate to resist the 1.2 ft-kip positive moment at the face of the exterior support due to moment reversal (see Table 5.43).

Table 5.44 Required Flexural Reinforcement at the Second-Floor Level for the Flat Plate System in Example 5.12

Location			M_u (ft-kips)	b (in.)	A_s (in.^{2})*	Reinforcement*
End Span	Column strip	Exterior Negative	−112.7	141.0	3.11	10-#5
		Positive	115.8	141.0	3.20	11-#5
		First Interior Negative	−198.0	141.0	5.57	18-#5
	Middle strip	Exterior Negative	0	159.0	2.72	9-#5
		Positive	78.4	159.0	2.72	9-#5
		First Interior Negative	−63.5	159.0	2.72	9-#5
Interior Span	Column strip	Positive	78.4	141.0	2.41	8-#5
		Negative	−183.0	141.0	5.13	17-#5
	Middle strip	Positive	52.4	159.0	2.72	9-#5
		Negative	−59.8	159.0	2.72	9-#5

* *Min. A_s for the column strip* $= 0.0018 \times 141.0 \times 9.5 = 2.41$ in.^{2}
Min. A_s for the middle strip $= 0.0018 \times 159.0 \times 9.5 = 2.72$ in.^{2}
Max. spacing $=$ lesser of $(2h,\ 18.0$ in.$) = 18.0$ in.
For $b = 141.0$ in., $141.0 / 18.0 = 7.8$ spaces, say minimum of 8 bars
For $b = 159.0$ in., $159.0 / 18.0 = 8.8$ spaces, say minimum of 9 bars
For the column strip: $A_{s,t} = 0.018bd = 20.9$ in.^{2}
For the middle strip: $A_{s,t} = 0.018bd = 23.6$ in.^{2}

Step 9 – Check that the flexural reinforcement is adequate for moment transfer requirements ACI 8.4.2.2

- Edge columns

Determine the reinforcement based on the largest transfer moment at the edge columns:

$$M_{sc} = 112.7 \text{ ft-kips} > 0.30M_o = 0.30(1.2M_{oD} + 1.6M_{oL}) = 0.30 \times [(1.2 \times 186.1) + (1.6 \times 93.9)]$$ Table 5.44

$$= 112.1 \text{ ft-kips}$$

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.62$$ Eq. (5.9)

where

$$b_1 = c_1 + (d/2) = 24.0 + (8.25/2) = 28.13 \text{ in.}$$ Table 5.11, Case 3

$$b_2 = c_2 + d = 24.0 + 8.25 = 32.25 \text{ in.}$$

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

The moment $\gamma_f M_{sc} = 0.62 \times 112.7 = 69.9$ ft-kips must be transferred over the effective slab width b_{slab}:

$b_{slab} = c_2 + 3h = 24.0 + (3 \times 9.5) = 52.5$ in., say 52.0 in. Eq. (5.10)

Determine the required A_s:

$$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{69.9 \times 12}{0.9 \times 52.0 \times 8.25^2} = 0.263 \text{ ksi}$$ Eq. (5.36)

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4 \times 52.0 \times 8.25}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.263}{0.85 \times 4}}\right] = 1.96 \text{ in.}^2$$ Eq. (5.35)

This required area of steel is equivalent to 7-#5 bars. Provide the 7-#5 bars by concentrating 7 of the 10-#5 column strip bars within the 52.0-in. width over the column (see Table 5.44). For symmetry, add 1-#5 bar and check the bar spacing:

For 7-#5 within the 52.0-in. width, bar spacing $= 52.0 / 7 = 7.4$ in. < 18.0 in.

For 4-#5 within the $141.0 - 52.0 = 89.0$-in. width, bar spacing $= 89.0 / 4 = 22.3$ in. > 18.0 in.

Therefore, provide 2 additional bars within the 89.0-in. width to satisfy maximum spacing requirements. A total of 13-#5 bars are required at the edge columns within the column strip, with 7 of the 13-#5 bars concentrated within a width of 52.0 in.

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

(1) Load combination: $1.2D + 1.6L$

$q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$

$d = 9.5 - 1.25 = 8.25$ in.

$A_c = (2b_1 + b_2)d = [(2 \times 28.13) + 32.25)] \times 8.25 = 730.2 \text{ in.}^2$ Table 5.11, Case 3

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 258.6 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 32.25}{144}\right)\right\} / 1{,}000 = 80.8 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_{u(D+L)}}{A_c} = \frac{80{,}800}{730.2} = 110.7 \text{ psi} > \phi 2 \lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0 \sqrt{4{,}000} = 94.9 \text{ psi}$$ Eq. (5.38)

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (8.25 / 10)}} = 1.1 > 1.0, \text{ use } 1.0$$ Eq. (5.30)

$\lambda = 1.0$ for normalweight concrete Table 5.20

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi\alpha_s f_y} = \dfrac{5 \times 110.7 \times 52.0 \times [(2 \times 28.13) + 32.25)]}{0.75 \times 30 \times 60{,}000} = 1.89 \text{ in.}^2 \end{cases} \quad \text{Eq. (5.37)}$$

Provided A_s within $b_{slab} = 7 \times 0.31 = 2.17 \text{ in.}^2 > A_{s,min} = 1.89 \text{ in.}^2$

Based on this load combination, no additional reinforcement must be provided to satisfy the minimum reinforcement requirements of ACI 8.6.1.2.

(2) Load combination: $1.2D + Q_E + 0.5L$

$$q_u = 1.2q_D + 0.5q_L = [1.2 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 187.1 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 187.1 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 32.25}{144}\right)\right\} / 1{,}000 = 58.5 \text{ kips}$$

$$V_{u(Q_E)} = (42.4 + 35.4) / 21.5 = 3.6 \text{ kips} \quad \text{Table 5.42}$$

Therefore,

$$v_{uv} = \frac{V_{u(D+L)} + V_{u(Q_E)}}{A_c} = \frac{58{,}500 + 3{,}600}{730.2} = 85.1 \text{ psi}$$

$$< \phi 2\lambda_s\lambda\sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0\sqrt{4{,}000} = 94.9 \text{ psi} \quad \text{Eq. (5.38)}$$

Provided A_s within $b_{slab} = 7 \times 0.31 = 2.17 \text{ in.}^2 > A_{s,min} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2$

Reinforcement details for the top bars at the edge columns are the same as those shown in Figure 5.43.

- Interior columns

Determine the required reinforcement based on the largest transfer moment at the interior columns.

In lieu of a more exact analysis, the factored slab moment, M_{sc}, transferred to an interior column due to the gravity load effects and gravity and seismic load effects can be determined from the following equations where the spans in the direction of analysis and perpendicular to the direction of analysis are equal.

For gravity loads only:

$$M_{sc(L)} = 0.035q_{Lu}\ell_2\ell_n^2 = 0.035 \times (1.6 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 42.1 \text{ ft-kips} \quad \text{Eq. (5.21)}$$

For gravity and seismic loads at the first interior columns:

$M_{sc(L)} = 0.035q_{Lu}\ell_2\ell_n^2 = 0.035 \times (0.5 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 13.2 \text{ ft-kips}$ due to gravity load effects

$M_{sc(Q_E)} = 35.4 + 31.6 = 67.0 \text{ ft-kips}$ due to seismic load effects — Table 5.42

Total $M_{sc} = 13.2 + 67.0 = 80.2 \text{ ft-kips}$

For gravity and seismic loads at the interior column:

$M_{sc(L)} = 0.035q_{Lu}\ell_2\ell_n^2 = 0.035 \times (0.5 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 13.2 \text{ ft-kips}$ due to gravity load effects

$M_{sc(Q_E)} = 2 \times 31.6 = 63.2$ ft-kips due to seismic load effects

Total $M_{sc} = 13.2 + 63.2 = 76.4$ ft-kips

Therefore, use $M_{sc} = 80.2$ ft-kips at all interior columns.

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.60$$ Eq. (5.9)

where

$b_1 = c_1 + d = 24.0 + 8.25 = 32.25$ in. Table 5.11, Case 1

$b_2 = c_2 + d = 24.0 + 8.25 = 32.25$ in.

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

The moment $\gamma_f M_{sc} = 0.60 \times 80.2 = 48.1$ ft-kips must be transferred over the effective slab width b_{slab}:

$b_{slab} = c_2 + 3h = 24.0 + (3 \times 9.5) = 52.5$ in., say 52.0 in. Eq. (5.10)

Determine the required A_s:

$$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{48.1 \times 12}{0.9 \times 52.0 \times 8.25^2} = 0.181 \text{ ksi}$$ Eq. (5.36)

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4 \times 52.0 \times 8.25}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.181}{0.85 \times 4}}\right] = 1.33 \text{ in.}^2$$ Eq. (5.35)

This required area of steel is equivalent to 5-#5 bars. With uniform bar spacing in the column strip, 6 of the 18-#5 bars are within the 52.0-in. effective slab width at the first interior columns, so no additional reinforcement is required to satisfy moment transfer requirements at these locations. Similarly, no additional reinforcement is required to satisfy moment transfer requirements at the interior column.

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

(1) Load combination: $1.2D + 1.6L$

$A_c = 2(b_1 + b_2)d = 2 \times (2 \times 32.25) \times 8.25 = 1{,}064.3$ in.^{2} Table 5.11, Case 1

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 258.6 \times \left[(23.5 \times 25.0) - \left(\frac{32.25}{12}\right)^2\right] / 1{,}000 = 150.1 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_{u(D+L)}}{A_c} = \frac{150{,}100}{1{,}064.3} = 141.0 \text{ psi} > \phi 2 \lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0\sqrt{4{,}000} = 94.9 \text{ psi}$$ Eq. (5.38)

where

$$\lambda_s = \sqrt{\frac{2}{1+(d/10)}} = \sqrt{\frac{2}{1+(8.25/10)}} = 1.1 > 1.0, \text{ use } 1.0 \qquad \text{Eq. (5.30)}$$

and

$\lambda = 1.0$ for normalweight concrete — Table 5.20

$$A_{s,min} = \text{greater of } \begin{cases} 0.0018hb_{slab} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi\alpha_s f_y} = \dfrac{5 \times 141.0 \times 52.0 \times [2 \times (2 \times 32.25)]}{0.75 \times 40 \times 60{,}000} = 2.63 \text{ in.}^2 \end{cases} \qquad \text{Eq. (5.37)}$$

At the first interior columns, provided A_s within $b_{slab} = 6 \times 0.31 = 1.86 \text{ in.}^2 < A_{s,min} = 2.63 \text{ in.}^2$

At the interior column, provided A_s within $b_{slab} = 7 \times 0.31 = 2.17 \text{ in.}^2 < A_{s,min} = 2.63 \text{ in.}^2$

Based on this load combination, additional reinforcement is required to satisfy the minimum reinforcement requirements of ACI 8.6.1.2

(2) Load combination: $1.2D + Q_E + 0.5L$

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 187.1 \times \left[(23.5 \times 25.0) - \left(\frac{32.25}{12}\right)^2\right] / 1{,}000 = 108.6 \text{ kips}$$

At the first interior columns, $V_{u(Q_E)} = (42.4 + 35.4)/21.5 = 3.6 \text{ kips}$ — Table 5.42

At the interior column, $V_{u(Q_E)} = (31.6 + 31.6)/21.5 = 2.9 \text{ kips}$

Therefore, the maximum shear stress is equal to the following:

$$v_{uv} = \frac{V_{u(D+L)} + V_{u(Q_E)}}{A_c} = \frac{108{,}600 + 3{,}600}{1{,}064.3} = 105.4 \text{ psi}$$

$$> \phi 2\lambda_s\lambda\sqrt{f'_c} = 0.75 \times 2 \times 1.0 \times 1.0\sqrt{4{,}000} = 94.9 \text{ psi} \qquad \text{Eq. (5.38)}$$

$$A_{s,min} = \text{greater of } \begin{cases} 0.0018hb_{slab} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi\alpha_s f_y} = \dfrac{5 \times 105.4 \times 52.0 \times [2 \times (2 \times 32.25)]}{0.75 \times 40 \times 60{,}000} = 1.96 \text{ in.}^2 \end{cases} \qquad \text{Eq. (5.37)}$$

The required $A_{s,min}$ from this load combination is less than the required $A_{s,min}$ from the $1.2D + 1.6L$ load combination, so required $A_{s,min} = 2.63 \text{ in.}^2$

To satisfy minimum reinforcement requirements within b_{slab}, 9-#5 bars must be placed within the 52.0-in. effective slab width ($A_{s,provided} = 9 \times 0.31 = 2.79 \text{ in.}^2 > A_{s,min} = 2.63 \text{ in.}^2$) at the first interior columns, with the remaining 9-#5 bars to be placed within the $141.0 - 52.0 = 89.0$-in. width of the column strip. For symmetry, add 1-5 bar to the 89.0-in. width. Thus, 19-#5 bars must be provided within the column strip at the first interior columns with 9-#5 bars concentrated within the 52.0-in. effective slab width. The reinforcement detail for this case is similar to that shown in Figure 5.44.

Similarly, 9-#5 bars must be placed within the 52.0-in. effective slab width at the interior column. The remaining 8-#5 bars are provided within the 89.0-in. width of the column strip.

Step 10 – Determine the reinforcement details ACI 8.7

The lengths of the top reinforcing bars in Figure 5.39 cannot be used in the column strip because of the effects due to seismic loads. To account for the seismic load effects, 25 percent of the top bars in the column strips are made continuous over the spans. In this case, the maximum amount of top reinforcement occurs at the first interior supports (19-#5 bars), so 5-#5 bars are made continuous.

The remaining bars in the column strip and the bars in the middle strip can be terminated at the locations identified in Figure 5.39; this figure also includes the structural integrity requirements of ACI 8.7.4.2.

A Class B tension lap splice is provided over the supports for the bottom bars in the column strip. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #5 reinforcing bars, $\psi_s = 0.8$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{lesser of} \begin{cases} \text{cover} + \left(d_b / 2\right) = 0.75 + (0.625 / 2) = 1.1 \text{ in.} \\ \\ s / 2 = 12.8 / 2 = 6.4 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (1.1 + 0) / 0.625 = 1.8 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{1.8} \right) \times 0.625 = 19.8 \text{ in.} > 12.0 \text{ in.}$$

Class B lap splice length $= 1.3\ell_d = 1.3 \times 19.8 = 25.7$ in. ACI Table 25.5.2.1

Provide a 2 ft-2 in. lap splice length.

Reinforcement details for the column strip and middle strip are given in Figure 5.58, which are the same as those in Figure 5.45 for SDC A.

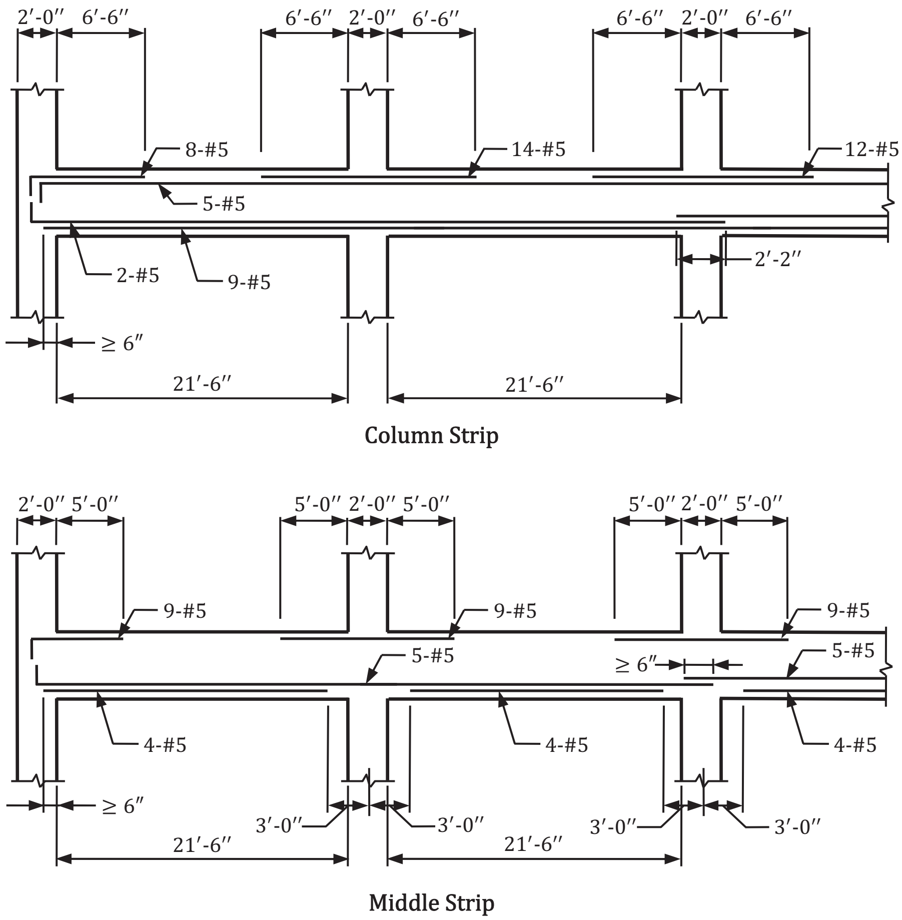


Figure 5.58 Reinforcement details for an interior design strip in the north-south direction for the flat plate system in Example 5.12.

Comments. Throughout this example, increasing γ_f to the values in ACI Table 8.4.2.2.4 was not considered. As an exercise, determine if γ_f can be increased to the maximum modified value of 1.0 in Table 5.10 for the edge columns bending perpendicular to the edge for a typical interior design strip in the north-south direction.

From Step 9, maximum $v_{uv} = 110.7 \text{ psi}$ for an edge column, which corresponds to the load combination $1.2D + 1.6L$.

Assuming there is no shear reinforcement in the slab, v_c is determined by Eq. (5.32). For a square edge column bending perpendicular to the edge, the first of the three equations governs:

$$v_c = 4\lambda_s \lambda \sqrt{f'_c} = 4 \times 1.0 \times 1.0\sqrt{4{,}000} = 253.0 \text{ psi}$$

Therefore,

$$0.75\phi v_c = 0.75 \times 0.75 \times 253.0 = 142.3 \text{ psi} > v_{uv} = 110.7 \text{ psi}$$

Determine ε_t within b_{slab} (7-#5 bars are provided within $b_{slab} = 52.0$ in.):

$$c = \frac{a}{\beta_1} = \frac{A_s f_y}{0.85 f'_c b \beta_1} = \frac{(7 \times 0.31) \times 60}{0.85 \times 4 \times 52.0 \times 0.85} = 0.9 \text{ in.}$$

$$\varepsilon_t = 0.003\left(\frac{d}{c} - 1\right) = 0.003\left(\frac{8.25}{0.9} - 1\right) = 0.025 > \varepsilon_{ty} + 0.003 = \left(\frac{60}{29{,}000}\right) + 0.003 = 0.005$$

Thus, γ_f can be taken as 1.0 at the edge columns in this example.

If $\gamma_f = 1.0$, $\gamma_f M_{sc} = 1.0 \times 112.7 = 112.7$ ft-kips, and 11-#5 bars are required within the 52.0-in. effective slab width (compared to 7-#5 bars with $\gamma_f = 0.62$). Providing this additional flexural reinforcement at the edge columns results in the shear stress caused by eccentricity of shear on these critical sections equal to zero because $\gamma_v = 1 - \gamma_f = 0$, and, thus, $\gamma_v M_{sc} = 0$; this shear stress is usually relatively large because M_{sc} at the edge columns in flat plate systems without edge beams is relatively large (see Example 5.13).

5.8.13 Example 5.13 – Check of Shear Strength Requirements: Flat Plate System, Building #1 (Framing Option A), SDC B

Check the shear strength requirements at the columns in an interior design strip in the north-south direction for the flat plate system in Figure 1.1 (Framing Option A) at the second-floor level with a 9.5-in.-thick slab and 24 in. by 24 in. columns. Assume the Site Class is D. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Edge Columns

Step 1 – Check one-way shear strength requirements — ACI 8.4.3

The critical section is located a distance d from the face of the column.

Because the panel is rectangular, one-way shear strength requirements must be checked at critical sections A and B (see Figure 5.59). Also, the gravity load combination $1.2D + 1.6L$ governs in this case.

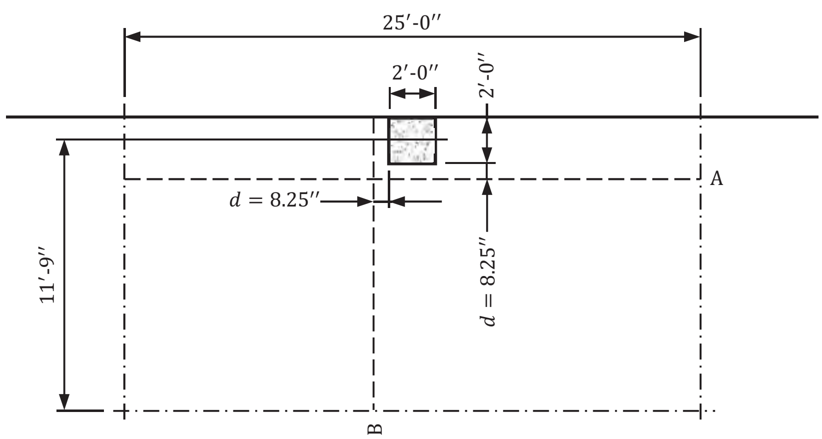


Figure 5.59 Critical sections for one-way shear at an edge column in Example 5.13.

$$\text{Dead load of slab} = (9.5/12) \times 150.0 = 118.8 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$$

$$\text{Live load} = 65.0 \text{ lb/ft}^2$$

Maximum $q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6$ lb/ft^2 — ACI Eq. (5.3.1b)

Maximum factored shear force on critical section A:

$$V_{u(A)} = 258.6 \times [11.75 - (2.0/2) - (8.25/12)] \times 25.0 / 1{,}000 = 65.1 \text{ kips}$$

Maximum factored shear force on critical section B:

$$V_{u(B)} = 258.6 \times [(25.0/2) - (2.0/2) - (8.25/12)] \times [11.75 + (2.0/2)] / 1{,}000 = 35.7 \text{ kips}$$

Design shear strength:

$$\phi V_c = \phi 8 \lambda_s \lambda (\rho_w)^{1/3} \sqrt{f_c'} b_w d \leq \phi 5 \lambda \sqrt{f_c'} b_w d \qquad \text{Eq. (5.29)}$$

$$\lambda_s = \sqrt{\frac{2}{1 + (d/10)}} = \sqrt{\frac{2}{1 + (8.25/10)}} = 1.1 > 1.0, \text{ use } 1.0 \qquad \text{Eq. (5.30)}$$

$\lambda = 1.0$ for normalweight concrete — Table 5.20

From Example 5.12, the tension reinforcement in the design strip at the location of the edge column consists of 22-#5 bars (13-#5 bars in the column strip and 9-#5 bars in the middle strip) in the north-south direction (see Figure 5.58).

Thus,

$$\rho_{w(A)} = \frac{22 \times 0.31}{(25.0 \times 12) \times 8.25} = 0.0028$$

$$\phi V_{c(A)} = 0.75 \times 8 \times 1.0 \times 1.0 \times (0.0028)^{1/3} \times \sqrt{4{,}000} \times (25.0 \times 12) \times 8.25 / 1{,}000 = 132.4 \text{ kips} > V_{u(A)} = 65.1 \text{ kips}$$

$$< 0.75 \times 5 \times 1.0 \times \sqrt{4{,}000} \times (25.0 \times 12) \times 8.25 / 1{,}000 = 587.0 \text{ kips}$$

One-way shear requirements are also satisfied at critical section B (calculations not shown here).

Step 2 – Check two-way shear strength requirements — ACI 8.4.4

The critical section is located a distance $d/2$ from the face of the column.

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2 q_D + 1.6 q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$$

$$b_1 = c_1 + (d/2) = 24.0 + (8.25/2) = 28.13 \text{ in.}$$

$$b_2 = c_2 + d = 24.0 + 8.25 = 32.25 \text{ in.}$$

Factored shear force at the critical section (see Figure 5.60):

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 258.6 \times \left\{ \left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12} \right) \right] - \left(\frac{28.13 \times 32.25}{144} \right) \right\} / 1{,}000 = 80.8 \text{ kips}$$

For the gravity load combination where the DDM is used to determine design bending moments, $M_{sc(D+L)} = 0.30 M_o$:

Dead load: $$M_{oD} = \frac{q_D \ell_2 \ell_n^2}{8} = \frac{(118.1 + 10.0) \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 186.1 \text{ ft-kips} \qquad \text{Eq. (5.17)}$$

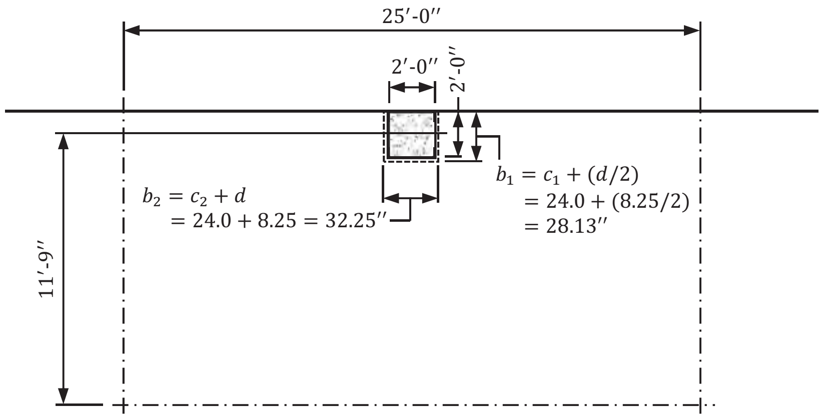


Figure 5.60 Critical section for two-way shear at an edge column in Example 5.13.

Live load: $M_{oL} = \dfrac{q_L \ell_2 \ell_n^2}{8} = \dfrac{65.0 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 93.9$ ft-kips

$M_o = 1.2M_{oD} + 1.6M_{oL} = (1.2 \times 186.1) + (1.6 \times 93.9) = 373.6$ ft-kips

$M_{sc(D+L)} = 0.30 \times 373.6 = 112.1$ ft-kips

$A_c = (2b_1 + b_2)d = [(2 \times 28.13) + 32.25)] \times 8.25 = 730.2$ in.^{2} Table 5.11, Case 3

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2\,/\,3)\sqrt{b_1/b_2}} = 1 - 0.62 = 0.38$$ ACI Eq. (8.4.4.2.2)

$$\frac{J_c}{c_{AB}} = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} = 7{,}460 \text{ in.}^3$$

Total factored shear stress:

$$v_{u|AB} = \frac{V_{u(D+L)}}{A_c} + \frac{\gamma_v M_{sc(D+L)} c_{AB}}{J_c} = \frac{80{,}800}{730.2} + \frac{0.38 \times 112.1 \times 12{,}000}{7{,}460} = 110.7 + 68.5 = 179.2 \text{ psi}$$ Eq. (5.14)

$$\phi v_c = \text{ least of } \begin{cases} \phi 4\lambda_s \lambda \sqrt{f_c'} = 189.7 \text{ psi} > v_{u|AB} = 179.2 \text{ psi} \\[2ex] \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s \lambda \sqrt{f_c'} = 284.6 \text{ psi} \\[2ex] \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s \lambda \sqrt{f_c'} = 227.5 \text{ psi} \end{cases}$$ Eq. (5.32)

where $\beta = 24.0\,/\,24.0 = 1.0,\ \alpha_s = 30,$ and $b_o = (2 \times 28.13) + 32.25 = 88.5$ in.

(2) Load combination: $1.2D + Q_E + 0.5L$

$$q_u = 1.2q_D + 0.5q_L = [1.2 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 187.1 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 187.1 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 32.25}{144}\right)\right\} / 1{,}000 = 58.5 \text{ kips}$$

$$V_{u(Q_E)} = (42.4 + 35.4) / 21.5 = 3.6 \text{ kips}$$ Table 5.42

Total factored shear force:

$$V_u = V_{u(D+L)} + V_{u(Q_E)} = 58.5 + 3.6 = 62.1 \text{ kips}$$

For other than gravity load combinations, M_{sc} is equal to that from the actual load combination:

$$M_{sc} = 112.7 \text{ ft-kips}$$ Table 5.44

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{62{,}100}{730.2} + \frac{0.38 \times 112.7 \times 12{,}000}{7{,}460} = 85.1 + 68.9 = 154.0 \text{ psi}$$ Eq. (5.14)

$$\phi v_c = \phi 4\lambda_s\lambda\sqrt{f_c'} = 189.7 \text{ psi} > v_{u|AB} = 154.0 \text{ psi}$$ Eq. (5.32)

Shear strength requirements are satisfied at the exterior columns.

First interior columns

Step 1 – Check one-way shear strength requirements — ACI 8.4.3

The critical section is located a distance d from the face of the column.

Because the panel is rectangular, one-way shear strength requirements must be checked at critical sections A and B (see Figure 5.61). Also, the gravity load combination $1.2D + 1.6L$ governs in this case.

Maximum $q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10)] + (1.6 \times 65) = 258.6 \text{ lb/ft}^2$ ACI Eq. (5.3.1b)

Maximum factored shear force on critical section A:

$$V_{u(A)} = 258.6 \times [(23.5 / 2) - (2.0 / 2) - (8.25 / 12)] \times 25.0 / 1{,}000 = 65.1 \text{ kips}$$

Maximum factored shear force on critical section B:

$$V_{u(B)} = 258.6 \times [(25.0 / 2) - (2.0 / 2) - (8.25 / 12)] \times 23.5 / 1{,}000 = 65.7 \text{ kips}$$

Design shear strength:

$$\phi V_c = \phi 8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'}b_w d \le \phi 5\lambda\sqrt{f_c'}b_w d$$ Eq. (5.29)

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (8.25 / 10)}} = 1.1 > 1.0, \text{ use } 1.0$$ Eq. (5.30)

$\lambda = 1.0$ for normalweight concrete Table 5.20

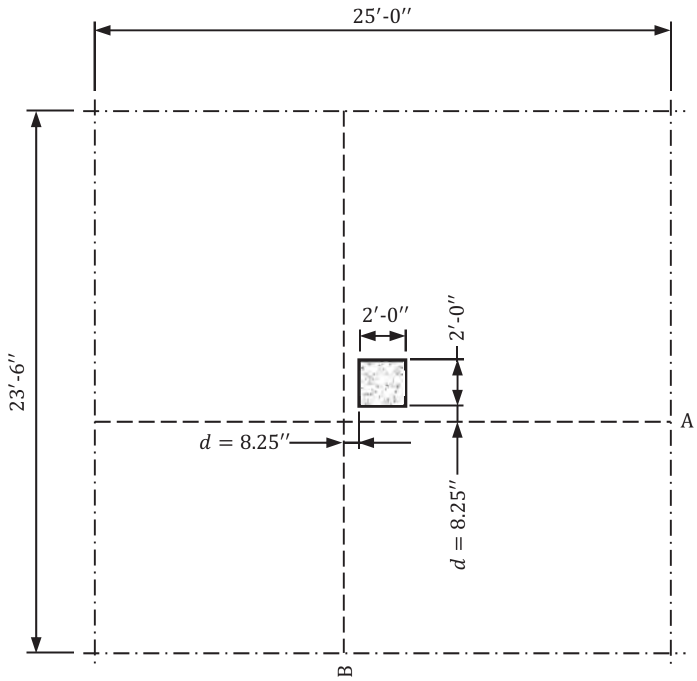


Figure 5.61 Critical sections for one-way shear at the first interior column in Example 5.13.

From Example 5.12, the tension reinforcement in the design strip at the location of the first interior column consists of 28-#5 bars (19-#5 bars in the column strip and 9-#5 bars in the middle strip) in the north-south direction (see Figure 5.58).

Thus,

$$\rho_{w(A)} = \frac{28 \times 0.31}{(25.0 \times 12) \times 8.25} = 0.0035$$

$$\phi V_{c(A)} = 0.75 \times 8 \times 1.0 \times 1.0 \times (0.0035)^{1/3} \times \sqrt{4{,}000} \times (25.0 \times 12) \times 8.25 \, / \, 1{,}000 = 142.6 \text{ kips} > V_{u(A)} = 65.1 \text{ kips}$$

$$< 0.75 \times 5 \times 1.0 \times \sqrt{4{,}000} \times (25.0 \times 12) \times 8.25 \, / \, 1{,}000 = 587.0 \text{ kips}$$

Assuming the tension reinforcement in the design strip at the location of the first interior column consists of 28-#5 bars in the east-west direction:

$$\rho_{w(B)} = \frac{28 \times 0.31}{(23.5 \times 12) \times 8.25} = 0.0037$$

$$\phi V_{c(B)} = 0.75 \times 8 \times 1.0 \times 1.0 \times (0.0037)^{1/3} \times \sqrt{4{,}000} \times (23.5 \times 12) \times 8.25 \, / \, 1{,}000 = 136.6 \text{ kips} > V_{u(A)} = 65.7 \text{ kips}$$

$$< 0.75 \times 5 \times 1.0 \times \sqrt{4{,}000} \times (23.5 \times 12) \times 8.25 \, / \, 1{,}000 = 551.8 \text{ kips}$$

Step 2 – Check two-way shear strength requirements ACI 8.4.4

The critical section is located a distance $d \, / \, 2$ from the face of the column.

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$$

$$b_1 = c_1 + d = 24.0 + 8.25 = 32.25 \text{ in.}$$

$$b_2 = c_2 + d = 24.0 + 8.25 = 32.25 \text{ in.}$$

Factored shear force at the critical section (see Figure 5.62):

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 258.6 \times \left[(25.0 \times 23.5) - \left(\frac{32.25^2}{144}\right)\right] / 1{,}000 = 150.1 \text{ kips}$$

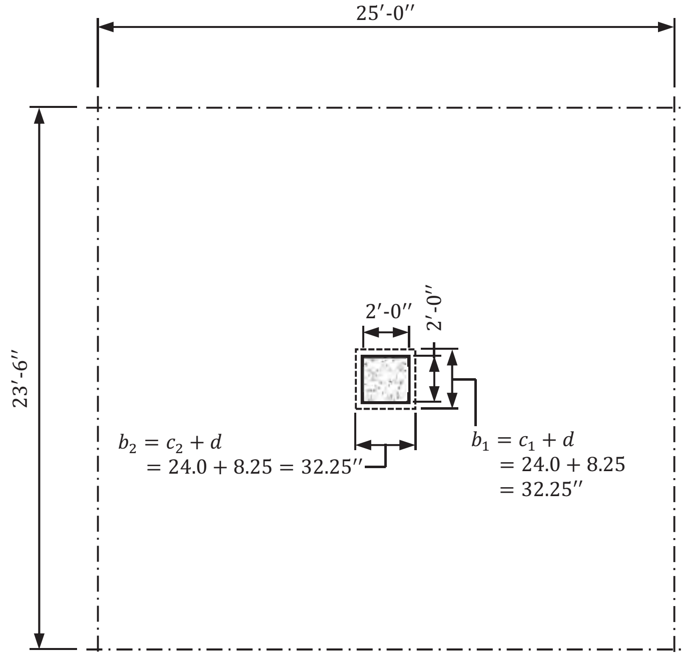


Figure 5.62 Critical section for two-way shear at the first interior column in Example 5.13.

In lieu of a more exact analysis, the factored slab moment, M_{sc}, transferred to an interior column due to the gravity load effects can be determined from the following equation where the spans in the direction of analysis and perpendicular to the direction of analysis are equal:

$$M_{sc} = M_{sc(L)} = 0.035q_{Lu}\ell_2\ell_n^2 = 0.035 \times (1.6 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 42.1 \text{ ft-kips} \qquad \text{Eq. (5.21)}$$

$$A_c = 2(b_1 + b_2)d = [2 \times (2 \times 32.25)] \times 8.25 = 1{,}064.3 \text{ in.}^2 \qquad \text{Table 5.11, Case 1}$$

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} = 1 - 0.60 = 0.40 \qquad \text{ACI Eq. (8.4.4.2.2)}$$

$$\frac{J_c}{c_{AB}} = \frac{b_1d(b_1 + 3b_2) + d^3}{3} = 11{,}628 \text{ in.}^3$$

Total factored shear stress:

$$v_{u|AB} = \frac{V_{u(D+L)}}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{150{,}100}{1{,}064.3} + \frac{0.40 \times 42.1 \times 12{,}000}{11{,}628} = 141.0 + 17.4 = 158.4 \text{ psi} \qquad \text{Eq. (5.14)}$$

$$\phi v_c = \text{ least of } \begin{cases} \phi 4 \lambda_s \lambda \sqrt{f'_c} = 189.7 \text{ psi} > v_{u|AB} = 158.4 \text{ psi} \\ \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s \lambda \sqrt{f'_c} = 284.6 \text{ psi} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s \lambda \sqrt{f'_c} = 216.2 \text{ psi} \end{cases} \qquad \text{Eq. (5.32)}$$

where $\beta = 24.0 / 24.0 = 1.0,\ \alpha_s = 40,$ and $b_o = 4 \times 32.25 = 129.0$ in.

(2) Load combination: $1.2D + Q_E + 0.5L$

$$q_u = 1.2q_D + 0.5q_L = [1.2 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 187.1 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 187.1 \times \left[(25.0 \times 23.5) - \left(\frac{32.25^2}{144}\right)\right] / 1{,}000 = 108.6 \text{ kips}$$

$$V_{u(Q_E)} = (42.4 + 35.4) / 21.5 = 3.6 \text{ kips} \qquad \text{Table 5.42}$$

Total factored shear force:

$$V_u = V_{u(D+L)} + V_{u(Q_E)} = 108.6 + 3.6 = 112.2 \text{ kips}$$

From gravity load effects:

$$M_{sc} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (0.5 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 13.2 \text{ ft-kips} \qquad \text{Eq. (5.21)}$$

From seismic load effects:

$$M_{sc} = 35.4 + 31.6 = 67.0 \text{ ft-kips}$$

Total $M_{sc} = 13.2 + 67.0 = 80.2$ ft-kips

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{112{,}200}{1{,}064.3} + \frac{0.40 \times 80.2 \times 12{,}000}{11{,}628} = 105.4 + 33.1 = 138.5 \text{ psi} \qquad \text{Eq. (5.14)}$$

$$\phi v_c = \phi 4 \lambda_s \lambda \sqrt{f'_c} = 189.7 \text{ psi} > v_{u|AB} = 138.5 \text{ psi} \qquad \text{Eq. (5.32)}$$

Shear strength requirements are satisfied at the first interior columns.

5.8.14 Example 5.14 – Check of Shear Strength Requirements: Flat Plate System, Building #1 (Framing Option A), SDC B, Shear Cap

Check the two-way shear strength requirements at an edge column in an interior design strip in the north-south direction for the flat plate system in Figure 1.1 (Framing Option A) at the second-floor level with an 9.5-in.-thick slab, a shear cap with a 4.25-in. projection below the slab, and 20 in. by 20 in. columns. Assume the Site Class is D. Also assume normalweight concrete with $f_c' = 4{,}000$ psi.

Step 1 – Check two-way shear stress at the critical section around the column ACI 8.4.4

$\text{Dead load of slab} = (9.5 / 12) \times 150.0 = 118.8 \text{ lb/ft}^2$

Given $h_1 = 4.25$ in., minimum shear cap projection $b_{cap} = 4.25$ in. Figure 5.14

Assume $b_{cap} = 5.0$ in. at the three faces of the column.

$\text{Weight of the shear cap below the slab} = (4.25 / 12) \times 0.150 \times [(20.0 + 5.0 + 5.0) \times (20.0 + 5.0)] / 144 = 0.3 \text{ kips}$

$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$

$\text{Live load} = 65.0 \text{ lb/ft}^2$

The critical section is located a distance $d_2 / 2 = (9.5 + 4.25 - 1.25) / 2 = 6.25$ in. from the face of the column (see Figure 5.63).

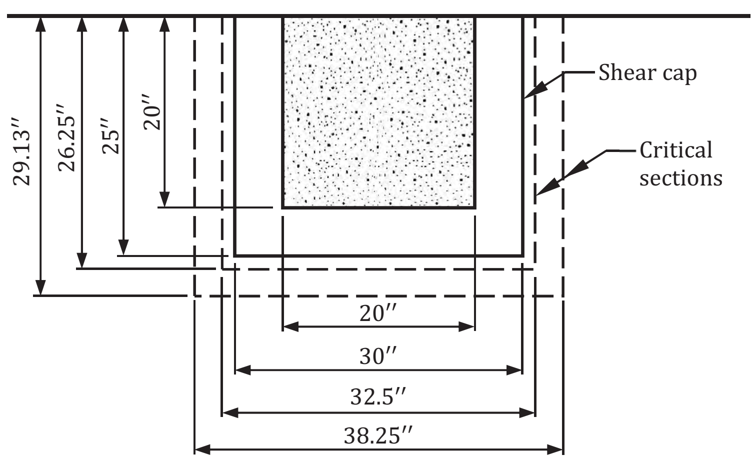


Figure 5.63 Critical sections for two-way shear at the edge column in Example 5.14.

(1) Load combination: $1.2D + 1.6L$

$q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$

$b_1 = c_1 + (d_2 / 2) = 20.0 + (12.5 / 2) = 26.25 \text{ in.}$

$b_2 = c_2 + d_2 = 20.0 + 12.5 = 32.5 \text{ in.}$

Factored shear force at the critical section:

$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) + \text{factored weight of the shear cap below the slab}$

$$= 258.6 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{20.0}{2 \times 12}\right)\right] - \left(\frac{26.25 \times 32.5}{144}\right)\right\} / 1{,}000 + (1.2 \times 0.3) = 80.2 \text{ kips}$$

For the gravity load combination where the DDM is used to determine design bending moments,

$M_{sc(D+L)} = 0.30M_o$:

$\ell_2 = 25.0$ ft Figure 5.8

$\ell_n = 23.5 - (20.0/12) = 21.83$ ft Figure 5.23

Dead load: $M_{oD} = \dfrac{q_D \ell_2 \ell_n^2}{8} = \dfrac{(118.8 + 10.0) \times 25.0 \times 21.83^2}{8 \times 1{,}000} = 191.8$ ft-kips Eq. (5.17)

Live load: $M_{oL} = \dfrac{q_L \ell_2 \ell_n^2}{8} = \dfrac{65.0 \times 25.0 \times 21.83^2}{8 \times 1{,}000} = 96.8$ ft-kips

$M_o = 1.2M_{oD} + 1.6M_{oL} = (1.2 \times 191.8) + (1.6 \times 96.8) = 385.0$ ft-kips

$M_{sc(D+L)} = 0.30 \times 385.0 = 115.5$ ft-kips

$A_c = (2b_1 + b_2)d_2 = [(2 \times 26.25) + 32.5)] \times 12.5 = 1{,}062.5$ in.^{2} Table 5.11, Case 3

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

$\gamma_v = 1 - \gamma_f = 1 - \dfrac{1}{1 + (2/3)\sqrt{b_1/b_2}} = 1 - 0.62 = 0.38$ ACI Eq. (8.4.4.2.2)

$$\frac{J_c}{c_{AB}} = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} = 11{,}035 \text{ in.}^3$$

Total factored shear stress:

$v_{u|AB} = \dfrac{V_{u(D+L)}}{A_c} + \dfrac{\gamma_v M_{sc(D+L)} c_{AB}}{J_c} = \dfrac{80{,}200}{1{,}062.5} + \dfrac{0.38 \times 115.5 \times 12{,}000}{11{,}035} = 75.5 + 47.7 = 123.2$ psi Eq. (5.14)

$$\phi v_c = \text{ least of } \begin{cases} \phi 4\lambda_s \lambda \sqrt{f_c'} = 189.7 \text{ psi} > v_{u|AB} = 123.2 \text{ psi} \\ \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s \lambda \sqrt{f_c'} = 284.6 \text{ psi} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s \lambda \sqrt{f_c'} = 304.1 \text{ psi} \end{cases}$$ Eq. (5.32)

where $\beta = 20.0/20.0 = 1.0$, $\alpha_s = 30$, and $b_o = (2 \times 26.25) + 32.5 = 85.0$ in.

(2) Load combination: $1.2D + Q_E + 0.5L$

$q_u = 1.2q_D + 0.5q_L = [1.2 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 187.1$ lb/ft^2

$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) + \text{factored weight of the shear cap below the slab}$

$$= \frac{187.1}{1{,}000} \times \left\{ \left[25.0 \times \left(\frac{23.5}{2} + \frac{20.0}{2 \times 12} \right) \right] - \left(\frac{26.25 \times 32.5}{144} \right) \right\} + (1.2 \times 0.3) = 58.2 \text{ kips}$$

Assume the bending moments due to seismic load effects are the same as those given in Table 5.42.

$V_{u(Q_E)} = (42.4 + 35.4) / 21.5 = 3.6 \text{ kips}$

Total factored shear force:

$V_u = V_{u(D+L)} + V_{u(Q_E)} = 58.2 + 3.6 = 61.8 \text{ kips}$

For other than gravity load combinations, M_{sc} is equal to that from the actual load combination.

For an edge column subjected to gravity loads effects:

$M_{sc(D+L)} = 0.26(1.2M_{oD} + 0.5M_{oL}) = 0.26 \times [(1.2 \times 191.8) + (0.5 \times 96.8)] = 72.4 \text{ ft-kips}$ Table 5.16

From seismic load effects, $M_{sc(Q_E)} = 42.4 \text{ ft-kips}$ Table 5.42

Total $M_{sc} = 72.4 + 42.4 = 114.8 \text{ ft-kips}$

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{61{,}800}{1{,}062.5} + \frac{0.38 \times 114.8 \times 12{,}000}{11{,}035} = 58.2 + 47.4 = 105.6 \text{ psi}$$ Eq. (5.14)

$\phi v_c = 189.7 \text{ psi} > v_{u|AB} = 105.6 \text{ psi}$ Eq. (5.32)

Step 2 – Check two-way shear stress at the critical section around the shear cap ACI 8.4.4

(1) Load combination: $1.2D + 1.6L$

$b_1 = c_1 + b_{cap} + (d_1 / 2) = 20.0 + 5.0 + (8.25 / 2) = 29.13 \text{ in.}$ Figure 5.63

$b_2 = c_2 + 2b_{cap} + d_1 = 20.0 + (2 \times 5.0) + 8.25 = 38.25 \text{ in.}$

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 258.6 \times \left\{ \left[25.0 \times \left(\frac{23.5}{2} + \frac{20.0}{2 \times 12} \right) \right] - \left(\frac{29.13 \times 38.25}{144} \right) \right\} / 1{,}000 = 79.4 \text{ kips}$$

$A_c = (2b_1 + b_2)d = [(2 \times 29.13) + 38.25)] \times 8.25 = 796.2 \text{ in.}^2$ Table 5.11, Case 3

$$\frac{J_c}{c_{AB}} = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} = 8{,}772 \text{ in.}^3$$

Total factored shear stress:

$$v_{u|AB} = \frac{V_{u(D+L)}}{A_c} + \frac{\gamma_v M_{sc(D+L)} c_{AB}}{J_c} = \frac{79{,}400}{796.2} + \frac{0.38 \times 115.5 \times 12{,}000}{8{,}772} = 99.7 + 60.0 = 159.7 \text{ psi} \qquad \text{Eq. (5.14)}$$

$$\phi v_c = \text{ least of } \begin{cases} \phi 4 \lambda_s \lambda \sqrt{f_c'} = 189.7 \text{ psi} > v_{u|AB} = 159.7 \text{ psi} \\ \phi \left(2 + \dfrac{4}{\beta}\right) \lambda_s \lambda \sqrt{f_c'} = 253.0 \text{ psi} \\ \phi \left(2 + \dfrac{\alpha_s d}{b_o}\right) \lambda_s \lambda \sqrt{f_c'} = 216.5 \text{ psi} \end{cases} \qquad \text{Eq. (5.32)}$$

where $\beta = 30.0 / 25.0 = 1.2,\ \alpha_s = 30,$ and $b_o = (2 \times 29.13) + 38.25 = 96.5$ in.

(2) Load combination: $1.2D + Q_E + 0.5L$

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 187.1 \times \left\{ \left[25.0 \times \left(\frac{23.5}{2} + \frac{20.0}{2 \times 12} \right) \right] - \left(\frac{29.13 \times 38.25}{144} \right) \right\} / 1{,}000 = 57.4 \text{ kips}$$

$$V_{u(Q_E)} = (42.4 + 35.4) / 21.5 = 3.6 \text{ kips} \qquad \text{Table 5.42}$$

Total factored shear force:

$$V_u = V_{u(D+L)} + V_{u(Q_E)} = 57.4 + 3.6 = 61.0 \text{ kips}$$

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{61{,}000}{796.2} + \frac{0.38 \times 114.8 \times 12{,}000}{8{,}772} = 76.6 + 59.7 = 136.3 \text{ psi} \qquad \text{Eq. (5.14)}$$

$$\phi v_c = 189.7 \text{ psi} > v_{u|AB} = 136.3 \text{ psi} \qquad \text{Eq. (5.32)}$$

Two-way shear requirements are satisfied at the edge columns.

5.8.15 Example 5.15 – Check of Shear Strength Requirements: Flat Plate System, Building #1 (Framing Option A), SDC B, Slab Opening

Check two-way shear strength requirements at an edge column in an interior design strip in the north-south direction for the flat plate system in Figure 1.1 (Framing Option A) at the second-floor level with a 9.5-in.-thick slab and 24 in. by 24 in. columns given the 8-in.-diameter slab opening in Figure 5.64. Assume the Site Class is D. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Step 1 – Check if the size of the opening is permitted ACI 8.5.4.2(b)

The 8-in.-diameter opening occurs within two intersecting column strips. Without performing an analysis of the system with the opening, the maximum opening size permitted in such cases is one-eighth the width of the governing column strip:

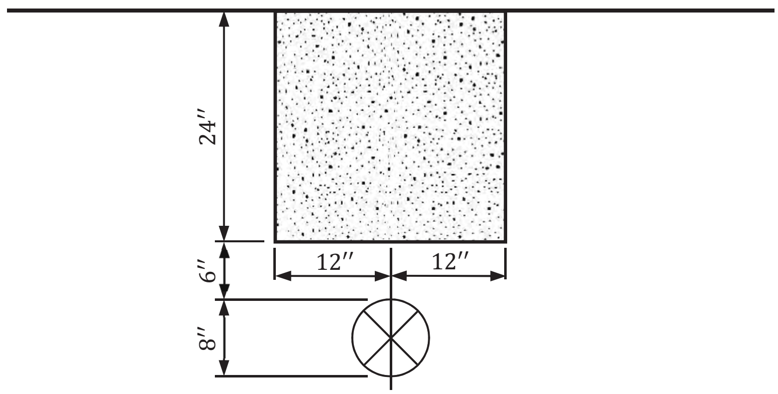


Figure 5.64 Slab opening in Example 5.15.

$$\text{Governing column strip width} = \text{lesser of} \begin{cases} \text{lesser of} \begin{cases} 23.5 / 2 = 11.75 \text{ ft} \\ \\ 25.0 / 2 = 12.50 \text{ ft} \end{cases} \quad \text{(north-south column strip)} \\ \\ \text{lesser of} \begin{cases} (23.5 / 4) + 1.0 = 6.88 \text{ ft} \\ \\ (25.0 / 4) + 1.0 = 7.25 \text{ ft} \end{cases} \quad \text{(east-west column strip)} \end{cases}$$

Figure 5.8

This opening is permitted at the indicated location because the 8-in. diameter is less than $(6.88 \times 12) / 8 = 10.3$ in.

Step 2 – Check if portions of the critical section are ineffective ACI 22.6.4.3

A portion of b_o is considered ineffective where an opening is located closer than $4h$ from the periphery of the column:

$4h = 4 \times 9.5 = 38.0$ in.

Because the opening is located 6.0 in. from the face of the opening, which is less than 38.0 in., a portion of b_o enclosed by straight lines projecting from the centroid of the column and tangent to the boundaries of the opening must be considered ineffective.

The length of the critical section adjacent to the opening is $b_2 = 24.0 + 8.25 = 32.25$ in. (see Figure 5.65). Based on the requirements of ACI 22.6.4.3, the ineffective length of the critical section can be obtained from similar triangles:

$$\text{Ineffective length} = \frac{2 \times [12 + (8.25 / 2)] \times 4}{12 + 6 + 4} = 5.9 \text{ in., say } 6.0 \text{ in.}$$

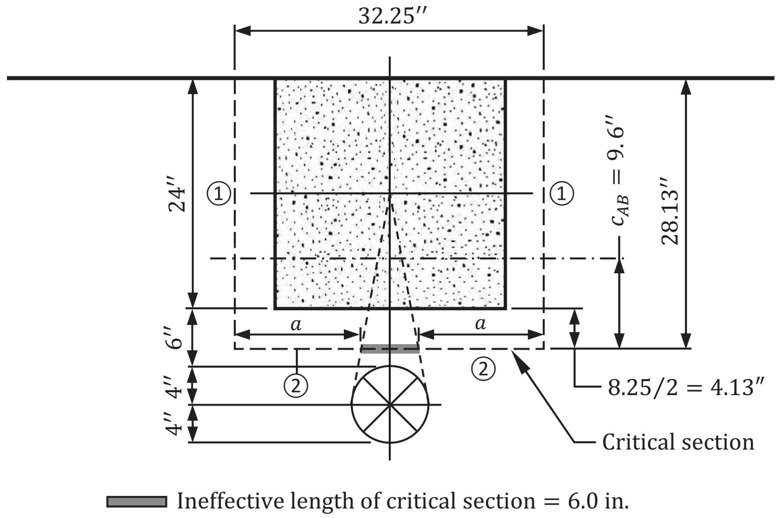


Figure 5.65 Ineffective portion of b_o for the edge column in Example 5.15.

Step 3 – Check two-way shear strength requirements — ACI 8.4.4

Dead load of slab $= (9.5/12) \times 150.0 = 118.8$ lb/ft^2

Superimposed dead load $= 10.0$ lb/ft^2

Live load $= 65.0$ lb/ft^2

(1) Load combination: $1.2D + 1.6L$

$q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10)] + (1.6 \times 65) = 258.6$ lb/ft^2 — ACI Eq. (5.3.1b)

$b_1 = c_1 + (d/2) = 24.0 + (8.25/2) = 28.13$ in.

$b_2 = c_2 + d -$ length of ineffective portion of critical section $= 24.0 + 8.25 - 6.0 = 26.25$ in.

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 258.6 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 26.25}{144}\right)\right\} / 1{,}000 = 81.1 \text{ kips}$$

For the gravity load combination where the DDM is used to determine design bending moments, $M_{sc(D+L)} = 0.30M_o$:

Dead load: $$M_{oD} = \frac{q_D\ell_2\ell_n^2}{8} = \frac{(118.8 + 10.0) \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 186.1 \text{ ft-kips}$$ Eq. (5.17)

Live load: $$M_{oL} = \frac{q_L\ell_2\ell_n^2}{8} = \frac{65.0 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 93.9 \text{ ft-kips}$$

$M_o = 1.2M_{oD} + 1.6M_{oL} = (1.2 \times 186.1) + (1.6 \times 93.9) = 373.6$ ft-kips

$M_{sc(D+L)} = 0.30 \times 373.6 = 112.1$ ft-kips

Section properties of the critical section are determined based on the straight segments of the effective portions of the section (see Figure 5.65):

$$c_{AB} = \frac{2\times 28.13\times(28.13/2)}{(2\times 28.13)+(32.25-6.0)} = 9.6 \text{ in.}$$

$$A_c = d\sum \ell = 8.25\times[(2\times 28.13)+(32.25-6.0)] = 680.7 \text{ in.}^2 \qquad \text{Eq. (5.15)}$$

Segment 1:

$$y_i = -9.6 \text{ in.},\ y_j = 28.13 - 9.6 = 18.5 \text{ in.}$$

$$\frac{\ell}{3}\sum(y_i^2 + y_iy_j + y_j^2) = \frac{28.13}{3}\times[(-9.6)^2 + (-9.6\times 18.5) + 18.5^2] = 2{,}408 \text{ in.}^3$$

Segment 2:

$$y_i = y_j = -9.6 \text{ in.}$$

$$\frac{\ell}{3}\sum(y_i^2 + y_iy_j + y_j^2) = \frac{(32.25-6.0)/2}{3}\times[3\times(-9.6)^2] = 1{,}210 \text{ in.}^3$$

$$J_c = d\sum\frac{\ell}{3}(y_i^2 + y_iy_j + y_j^2) = 8.25\times[(2\times 2{,}408)+(2\times 1{,}210)] = 59{,}697 \text{ in.}^4 \qquad \text{Eq. (5.16)}$$

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

$$\gamma_v = 1-\gamma_f = 1-\frac{1}{1+(2/3)\sqrt{b_1/b_2}} = 1-0.59 = 0.41 \qquad \text{ACI Eq. (8.4.4.2.2)}$$

Total factored shear stress:

$$v_{u|AB} = \frac{V_{u(D+L)}}{A_c} + \frac{\gamma_v M_{sc(D+L)}c_{AB}}{J_c} = \frac{81{,}100}{680.7} + \frac{0.41\times 112.1\times 12{,}000\times 9.6}{59{,}697} = 119.1 + 88.7 = 207.8 \text{ psi} \qquad \text{Eq. (5.14)}$$

$$\phi v_c = \text{ least of } \begin{cases} \phi 4\lambda_s\lambda\sqrt{f_c'} = 189.7 \text{ psi} < v_{u|AB} = 207.8 \text{ psi} \\ \phi\left(2+\dfrac{4}{\beta}\right)\lambda_s\lambda\sqrt{f_c'} = 284.6 \text{ psi} \\ \phi\left(2+\dfrac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f_c'} = 237.2 \text{ psi} \end{cases} \qquad \text{Eq. (5.32)}$$

where $\beta = 24.0/24.0 = 1.0,\ \alpha_s = 30,$ and $b_o = (2\times 28.13)+26.25 = 82.5$ in.

(2) Load combination: $1.2D + Q_E + 0.5L$

$$q_u = 1.2q_D + 0.5q_L = [1.2\times(118.8+10.0)] + (0.5\times 65.0) = 187.1 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 187.1\times\left\{\left[25.0\times\left(\frac{23.5}{2}+\frac{24.0}{2\times 12}\right)\right]-\left(\frac{28.13\times 26.25}{144}\right)\right\}/1{,}000 = 58.7 \text{ kips}$$

$$V_{u(Q_E)} = (42.4 + 35.4) / 21.5 = 3.6 \text{ kips}$$ Table 5.42

Total factored shear force:

$$V_u = V_{u(D+L)} + V_{u(Q_E)} = 58.7 + 3.6 = 62.3 \text{ kips}$$

For other than gravity load combinations, M_{sc} is equal to that from the actual load combination:

$$M_{sc} = 112.7 \text{ ft-kips}$$ Table 5.44

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{62,300}{680.7} + \frac{0.41 \times 112.7 \times 12,000 \times 9.6}{59,697} = 91.5 + 89.2 = 180.7 \text{ psi}$$ Eq. (5.14)

$$\phi v_c = \phi 4 \lambda_s \lambda \sqrt{f'_c} = 189.7 \text{ psi} > v_{u|AB} = 180.7 \text{ psi}$$ Eq. (5.32)

Shear strength requirements are not satisfied at the exterior columns because $v_{u|AB} > \phi v_c$ for the gravity load combination $1.2D + 1.6L$.

Comments. Comparing the results from this example to those in Example 5.13 where there is no opening adjacent to the edge column, it is evident the 8-in.-diameter opening has a significant impact on two-way shear strength.

The section properties of the critical section are determined in this example using the conservative method outlined in Sect. 5.3.3 of this publication. Determine the section properties based on the actual properties of the critical section with the 6.0-in. ineffective segment:

$$a = (32.25 - 6.0) / 2 = 13.13 \text{ in.}$$ Figure 5.65

$$b_1 = c_1 + (d / 2) = 24.0 + (8.25 / 2) = 28.13 \text{ in.}$$

$$b_o = 2(b_1 + a) = 2 \times (28.13 + 13.13) = 82.5 \text{ in.}$$

$$A_c = b_o d = 82.5 \times 8.25 = 680.6 \text{ in.}^2$$

$$c_{AB} = \frac{2b_1 d(b_1 / 2)}{A_c} = \frac{2 \times 28.13 \times 8.25 \times (28.13 / 2)}{680.6} = 9.6 \text{ in.}$$

For the two faces $b_1 = 28.13$ in.:

$$(J_c)_1 = I_{xx} + I_{zz} = \left\{ \frac{2db_1^3}{12} + 2db_1 \left[\frac{b_1}{2} - c_{AB} \right]^2 \right\} + \frac{2b_1 d^3}{12} = 42,492 \text{ in.}^4$$

For the two faces $a = 13.13$ in.:

$$(J_c)_2 = I_{xx} = 2adc_{AB}^2 = 19,966 \text{ in.}^4$$

Therefore, $J_c = (J_c)_1 + (J_c)_2 = 62,458 \text{ in.}^4$

Total factored shear stress due to gravity loads:

$$v_{u|AB} = \frac{V_{u(D+L)}}{A_c} + \frac{\gamma_v M_{sc(D+L)} c_{AB}}{J_c} = \frac{81,100}{680.6} + \frac{0.41 \times 112.1 \times 12,000 \times 9.6}{62,458} = 119.2 + 84.8 = 204.0 \text{ psi} > \phi v_c = 189.7 \text{ psi}$$

It is evident shear strength requirements are not satisfied using the actual properties of the critical section.

Note that shear strength requirements are satisfied if $f_c' = 5{,}000$ psi:

$$\phi v_c = \phi 4\lambda_s \lambda \sqrt{f_c'} = 212.1 \text{ psi} > v_{u|AB} = 204.0 \text{ psi}$$

If the concrete compressive strength cannot be increased, it may be possible to satisfy two-way shear strength requirements by investigating whether γ_f can be increased to 1.0 (thereby decreasing γ_v to zero) in accordance with ACI 8.4.2.2.4. It is determined in the Comments section at the end of Example 5.12 that γ_f can be taken as 1.0 provided the amount of negative flexural reinforcement at the critical section is increased accordingly. Therefore, assuming $f_c' = 4{,}000$ psi, the maximum two-way shear stress $v_{u|AB} = 119.1 \text{ psi} < \phi v_c = 189.7$ psi for the gravity load combination $1.2D + 1.6L$ with the opening, which means two-way shear requirements are satisfied.

It is unlikely an opening of this size will have a major impact on flexural or serviceability requirements. In regard to detailing requirements, a quantity of reinforcement at least equal to that interrupted by the opening must be added on the sides of the opening [ACI 8.5.4.2(b)].

5.8.16 Example 5.16 – Check of Shear Strength Requirements: Flat Slab System With Edge Beams, Building #1 (Framing Option D), SDC A, Circular Columns

Check two-way shear strength requirements at an interior column in an interior design strip in the north-south direction at the second-floor level for the flat slab system in Figure 1.1 (Framing Option D) with an 8.0-in.-thick slab, a drop panel with a 2.25-in. projection below the slab, and 20-in. circular columns. Assume the Site Class is C. Also assume normalweight concrete with $f_c' = 4{,}000$ psi. Interior design strips are not part of the LFRS.

Step 1 – Check two-way shear stress at the critical section around the column — ACI 8.4.4

Dead load of slab $= (8.0 / 12) \times 150.0 = 100.0 \text{ lb/ft}^2$

Dead load of drop panel below the slab $= (2.25 / 12) \times 150.0 \times 8.5 \times 8.5 / 1{,}000 = 2.0$ kips

Superimposed dead load $= 10.0 \text{ lb/ft}^2$

Live load $= 65.0 \text{ lb/ft}^2$

Maximum $q_u = 1.2 q_D + 1.6 q_L = [1.2 \times (100.0 + 10.0)] + (1.6 \times 65.0) = 236.0 \text{ lb/ft}^2$ — ACI Eq. (5.3.1b)

$d_2 = 10.25 - 1.25 = 9.0$ in. — Figure 5.13

Determine the section properties of the critical section assuming a square column of equivalent area (see Figure 5.66):

$c_1 = c_2 = \sqrt{\pi} D / 2 = \sqrt{\pi} \times 20.0 / 2 = 17.7$ in. — ACI 22.6.4.1.2

$b_1 = b_2 = c_1 + d_2 = 17.7 + 9.0 = 26.7$ in.

$A_c = 2(b_1 + b_2) d_2 = [2 \times (2 \times 26.7)] \times 9.0 = 961.2 \text{ in.}^2$ — Table 5.11, Case 1

Factored shear force at the critical section:

$$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) + \text{factored weight of the drop panel below the slab}$$

$$= 236.0 \times \left[(25.0 \times 23.5) - \left(\frac{26.7^2}{144}\right)\right] / 1{,}000 + (1.2 \times 2.0) = 139.9 \text{ kips}$$

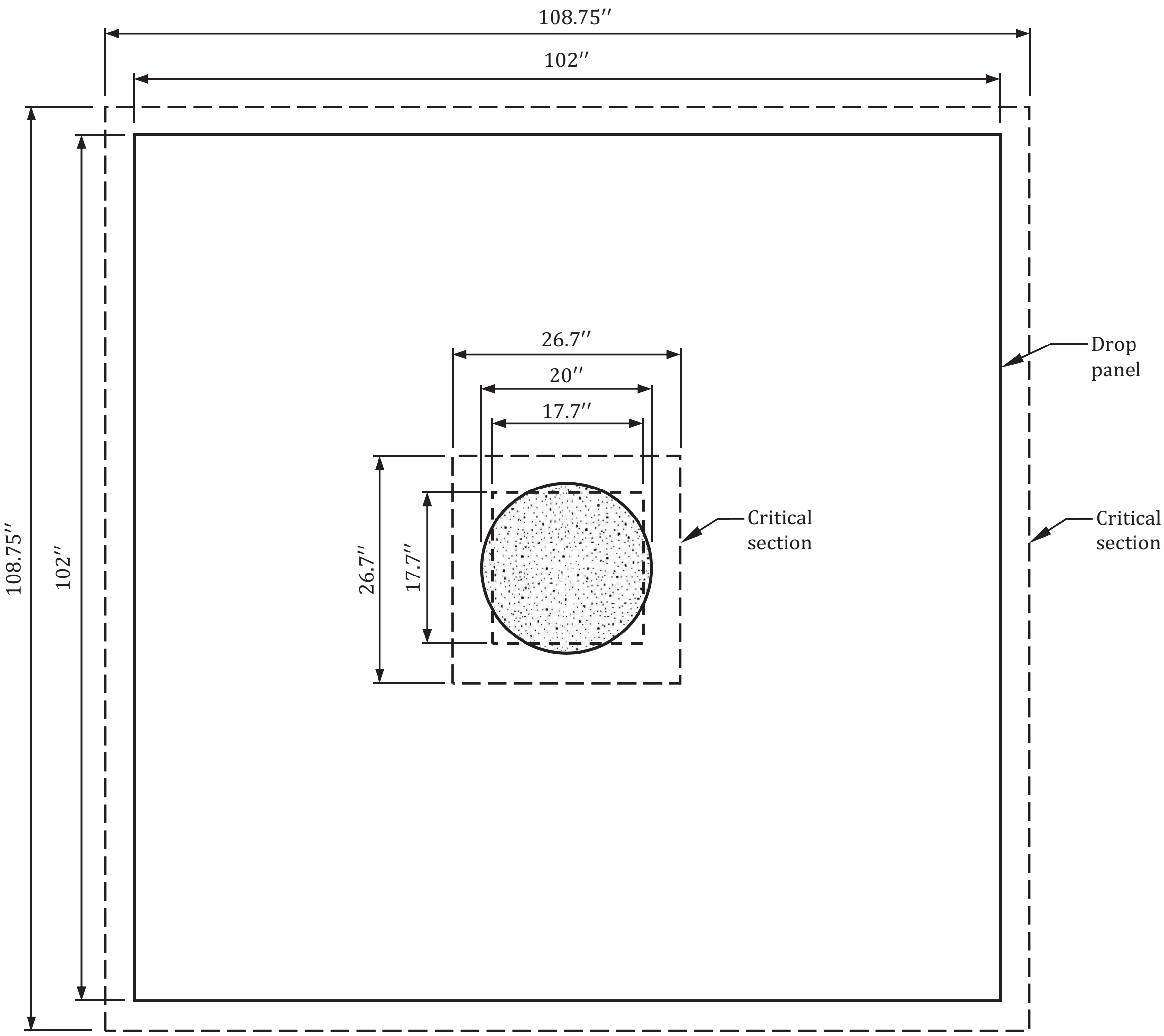


Figure 5.66 Critical sections for the interior column in Example 5.15.

In lieu of a more exact analysis, the factored slab moment, M_{sc}, transferred to an interior column due to the gravity load effects can be determined from the following equation where the spans in the direction of analysis and perpendicular to the direction of analysis are equal:

$$M_{sc} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (1.6 \times 65.0) \times 25.0 \times 22.0^2 / 1{,}000 = 44.0 \text{ ft-kips}$$ Eq. (5.21)

where $\ell_n = 23.5 - (17.7 / 12) = 22.0$ ft. Figure 5.23

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 1 - 0.60 = 0.40$$ ACI Eq. (8.4.4.2.2)

$$\frac{J_c}{c_{AB}} = \frac{b_1 d(b_1 + 3b_2) + d^3}{3} = 8{,}798 \text{ in.}^3$$ Table 5.11, Case 1

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{139,900}{961.2} + \frac{0.40 \times 44.0 \times 12,000}{8,798} = 145.6 + 24.0 = 169.6 \text{ psi} \quad \text{Eq. (5.14)}$$

$$\phi v_c = \text{ least of } \begin{cases} \phi 4\lambda_s \lambda \sqrt{f_c'} = 189.7 \text{ psi} > v_{u|AB} = 169.6 \text{ psi} \\ \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s \lambda \sqrt{f_c'} = 284.6 \text{ psi} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s \lambda \sqrt{f_c'} = 254.8 \text{ psi} \end{cases} \quad \text{Eq. (5.32)}$$

where $\beta = 17.7 / 17.7 = 1.0,\ \alpha_s = 40,\ b_o = 4 \times 26.7 = 106.8 \text{ in.},\ \lambda_s = 1.0, \text{ and } \lambda = 1.0.$

Step 2 – Check two-way shear stress at the critical section around the drop panel ACI 8.4.4

$d_1 = 8.0 - 1.25 = 6.75$ in. Figure 5.13

Section properties of the critical section:

$b_1 = b_2 = (8.5 \times 12) + 6.75 = 108.75$ in. Table 5.11, Case 1

$A_c = 2(b_1 + b_2)d_1 = 2 \times (2 \times 108.75) \times 6.75 = 2,936.3 \text{ in.}^2$

Factored shear force at the critical section:

$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) = 236.0 \times [(25.0 \times 23.5) - (108.75 / 12)^2] / 1,000 = 119.3 \text{ kips}$

The transfer moment M_{sc} is negligible at this location and is not included in the calculation of the maximum shear stress.

Maximum shear stress, v_u:

$$v_u = \frac{V_u}{A_c} = \frac{119,300}{2,936.3} = 40.6 \text{ psi} < \phi v_c = 189.7 \text{ psi} \quad \text{Eq. (5.14)}$$

Two-way shear requirements are satisfied at the interior columns.

Comments. Instead of calculating the section properties of the critical section using an equivalent square column, calculate the section properties based on the circular section and check two-way shear requirements.

$A_c = \pi(D + d_2)d_2 = \pi \times (20.0 + 9.0) \times 9.0 = 820.0 \text{ in.}^2$ Table 5.13, Case 1

$$\frac{J_c}{c_A} = \pi d_2 \left(\frac{D + d_2}{2}\right)^2 + \frac{d_2^3}{3} = \left[\pi \times 9.0 \times \left(\frac{20.0 + 9.0}{2}\right)^2\right] + \frac{9.0^3}{3} = 6,188 \text{ in.}^3$$

Factored shear force at the critical section:

$$V_u = q_u(\ell_1 \ell_2 - A) + \text{factored weight of the drop panel below the slab}$$

$$= 236.0 \times [(25.0 \times 23.5) - (\pi \times 14.5^2 / 144)] / 1,000 + (1.2 \times 2.0) = 140.0 \text{ kips}$$

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{140{,}000}{820.0} + \frac{0.40 \times 44.0 \times 12{,}000}{6{,}188} = 170.7 + 34.1 = 204.8 \text{ psi} > \phi v_c = 189.7 \text{ psi} \qquad \text{Eq. (5.14)}$$

Two-way shear strength requirements can be satisfied by increasing the drop panel projection below the slab to 4.25 in.

5.8.17 Example 5.17 – Determination of Shear Reinforcement: Flat Plate System, Building #1 (Framing Option A), SDC B, Stirrups

Check two-way shear strength requirements at the edge and first interior columns in an interior design strip in the north-south direction for the flat plate system in Figure 1.1 (Framing Option A) at the second-floor level with a 9.5-in.-thick slab and 20 in. by 20 in. columns. Assume the Site Class is D. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement. Provide shear reinforcement in the form of closed stirrups if needed.

Edge columns

Step 1 – Check two-way shear strength requirements ACI 8.4.4

Dead load of slab $= (9.5/12) \times 150.0 = 118.8$ lb/ft^2

Superimposed dead load $= 10.0$ lb/ft^2

Live load $= 65.0$ lb/ft^2

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$$

$$b_1 = c_1 + (d/2) = 20.0 + (8.25/2) = 24.13 \text{ in.}$$

$$b_2 = c_2 + d = 20.0 + 8.25 = 28.25 \text{ in.}$$

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 258.6 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{20.0}{2 \times 12}\right)\right] - \left(\frac{24.13 \times 28.25}{144}\right)\right\} / 1{,}000 = 80.1 \text{ kips}$$

For the gravity load combination where the DDM is used to determine design bending moments, $M_{sc(D+L)} = 0.30M_o$:

$\ell_2 = 25.0$ ft Figure 5.8

$\ell_n = 23.5 - (20.0/12) = 21.83$ ft Figure 5.23

$$\text{Dead load: } M_{oD} = \frac{q_D \ell_2 \ell_n^2}{8} = \frac{(118.8 + 10.0) \times 25.0 \times 21.83^2}{8 \times 1{,}000} = 191.8 \text{ ft-kips} \qquad \text{Eq. (5.17)}$$

$$\text{Live load: } M_{oL} = \frac{q_L \ell_2 \ell_n^2}{8} = \frac{65.0 \times 25.0 \times 21.83^2}{8 \times 1{,}000} = 96.8 \text{ ft-kips}$$

$$M_o = 1.2M_{oD} + 1.6M_{oL} = (1.2 \times 191.8) + (1.6 \times 96.8) = 385.0 \text{ ft-kips}$$

$M_{sc(D+L)} = 0.30 \times 385.0 = 115.5$ ft-kips

$A_c = (2b_1 + b_2)d = [(2 \times 24.13) + 28.25)] \times 8.25 = 631.2$ in.^{2} Table 5.11, Case 3

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} = 1 - 0.62 = 0.38$$ ACI Eq. (8.4.4.2.2)

$$\frac{J_c}{c_{AB}} = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} = 5{,}647 \text{ in.}^3$$

Total factored shear stress:

$$v_{u|AB} = \frac{V_{u(D+L)}}{A_c} + \frac{\gamma_v M_{sc(D+L)} c_{AB}}{J_c} = \frac{80{,}100}{631.2} + \frac{0.38 \times 115.5 \times 12{,}000}{5{,}647} = 126.9 + 93.3 = 220.2 \text{ psi}$$ Eq. (5.14)

$$\phi v_c = \text{ least of } \begin{cases} \phi 4 \lambda_s \lambda \sqrt{f_c'} = 189.7 \text{ psi} < v_{u|AB} = 220.2 \text{ psi} \\ \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s \lambda \sqrt{f_c'} = 284.6 \text{ psi} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s \lambda \sqrt{f_c'} = 248.3 \text{ psi} \end{cases}$$ Eq. (5.32)

where $\beta = 20.0 / 20.0 = 1.0$, $\alpha_s = 30$, and $b_o = (2 \times 24.13) + 28.25 = 76.5$ in.

(2) Load combination: $1.2D + Q_E + 0.5L$

$q_u = 1.2q_D + 0.5q_L = [1.2 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 187.1$ lb/ft^2

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 187.1 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{20.0}{2 \times 12}\right)\right] - \left(\frac{24.13 \times 28.25}{144}\right)\right\} / 1{,}000 = 58.0 \text{ kips}$$

Assume the bending moments due to seismic load effects are the same as those given in Table 5.42.

$V_{u(Q_E)} = (42.4 + 35.4) / 21.5 = 3.6$ kips

Total factored shear force:

$V_u = V_{u(D+L)} + V_{u(Q_E)} = 58.0 + 3.6 = 61.6$ kips

For other than gravity load combinations, M_{sc} is equal to that from the actual load combination:

$M_{sc} = [1.2 \times (0.26 \times 191.8)] + [0.5 \times (0.26 \times 96.8)] + 42.4 = 114.8$ ft-kips

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{61{,}600}{631.2} + \frac{0.38 \times 114.8 \times 12{,}000}{5{,}647} = 97.6 + 92.7 = 190.3 \text{ psi}$$ Eq. (5.14)

$\phi v_c = 189.7 \text{ psi} < v_{u|AB} = 190.3 \text{ psi}$ Eq. (5.32)

Two-way shear strength requirements are not satisfied at the edge columns based on both load combinations.

Step 2 – Determine the size and spacing of closed stirrups ACI 8.7.6

- **Step 2a – Check if stirrups can be utilized in the slab** ACI 22.6.7.1

  Assuming #4 stirrups:

$$\text{Average } d = 8.25 \text{ in.} > \text{greater of } \begin{cases} 6.0 \text{ in.} \\ \\ 16d_b = 16 \times 0.50 = 8.0 \text{ in.} \end{cases}$$

  Thus, stirrups are permitted to be used to increase the design two-way shear strength.

- **Step 2b – Check the maximum shear strength permitted with stirrups** ACI Table 22.6.6.3

  From Step 1, maximum $v_{u|AB} = 220.2 \text{ psi} < \phi 6\sqrt{f_c'} = 284.6 \text{ psi}$

- **Step 2c – Determine the design shear strength of the concrete with stirrups** ACI Table 22.6.6.1

$$\phi v_c = \phi 2\lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0 \times \sqrt{4{,}000} = 94.9 \text{ psi}$$ Table 5.23

  where $\lambda_s = \sqrt{\dfrac{2}{1+(d\,/\,10)}} = \sqrt{\dfrac{2}{1+(8.25\,/\,10)}} = 1.1 > 1.0, \text{ use } 1.0$ Eq. (5.30)

  and

  $\lambda = 1.0 \text{ for normalweight concrete}$ Table 5.20

  Note: λ_s is permitted to be taken as 1.0 for two-way slabs with stirrups that satisfy the provisions of ACI 22.6.6.2(a).

- **Step 2d – Determine the required area and spacing of the stirrups** ACI 22.6.7.2

  Maximum stirrup spacing $s \le d\,/\,2 = 8.25\,/\,2 = 4.1 \text{ in.}$ Figure 5.19

  Try $s = 4.0 \text{ in.}$

  Total required area of stirrups, A_v, for the three-sided, rectangular critical section located a distance $d\,/\,2$ from the face of the column (see Figure 5.67):

$$A_v = \frac{(v_{u|AB} - \phi v_c) b_o s}{\phi f_{yt}} = \frac{(220.2 - 94.9) \times [(2 \times 24.13) + 28.25] \times 4.0}{0.75 \times 60{,}000} = 0.85 \text{ in.}^2$$ Table 5.23

$$\text{Required area of stirrups per side} = \frac{0.85}{3} = 0.28 \text{ in.}^2$$

$$\text{Required area of one stirrup leg per side} = \frac{0.28}{2} = 0.14 \text{ in.}^2 < A_{v(provided)} = 0.20 \text{ in.}^2$$

  Use #4 stirrups spaced at 4.0 in. on center $[A_{v(provided)} \text{ per side} = 2 \times 0.20 = 0.40 \text{ in.}^2 > 0.28 \text{ in.}^2]$

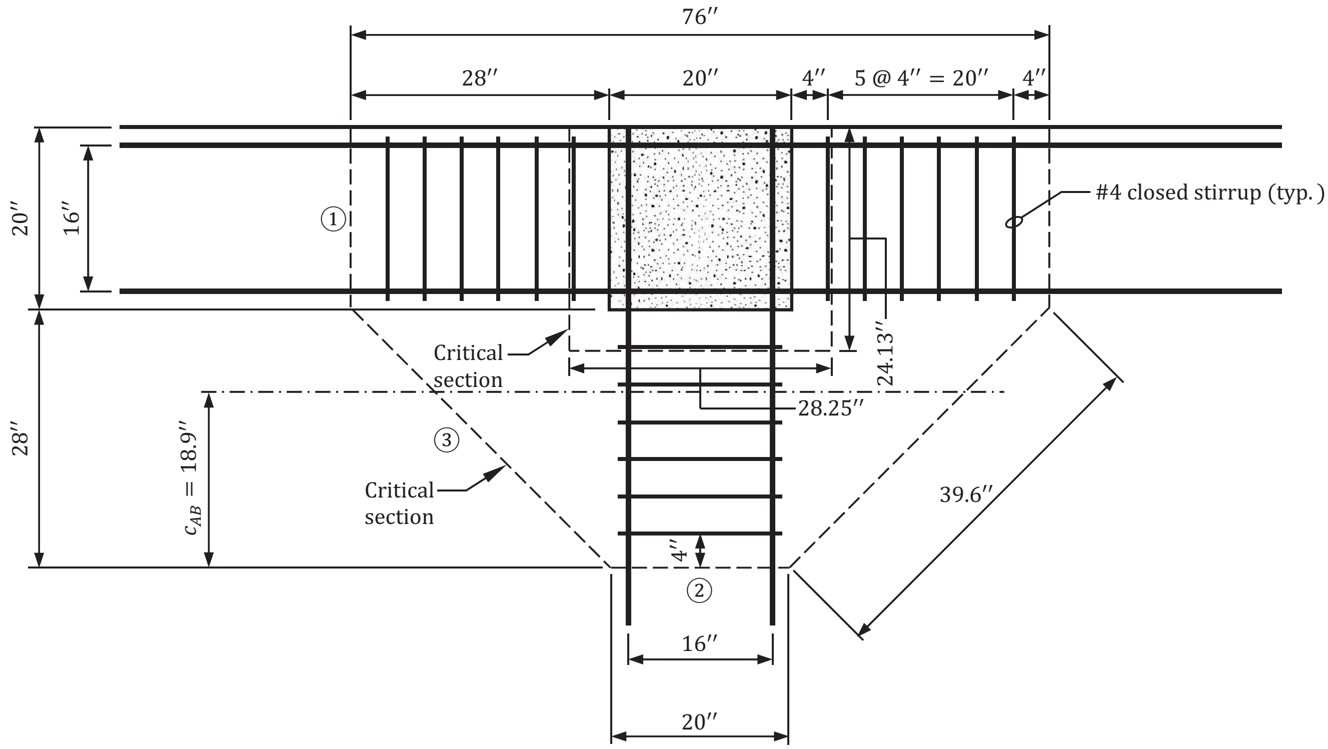


Figure 5.67 Closed stirrup shear reinforcement at the edge columns in Example 5.17.

- **Step 2e – Determine the distance from the faces of the column where the stirrups can be terminated**

Stirrups can be terminated where the design strength of the concrete can resist the factored shear stress without stirrups.

At the critical section located a distance $d/2$ from the outermost peripheral line of stirrups:

$$\phi v_n = \phi v_c = \phi 2\lambda_s \lambda \sqrt{f'_c} = 94.9 \text{ psi}$$ Table 5.23

Assume 6 stirrups are provided parallel to each face of the column and check if the maximum factored shear stress at the critical section located a distance of $d/2$ from the centerline of the outermost peripheral line of stirrups is less than or equal to ϕv_n.

Section properties of the polygonal critical section (see Figure 5.67):

$$c_{AB} = \frac{\left[2\times(20.0)\times\left(28.0+\dfrac{20.0}{2}\right)\right]+\left[2\times 39.6\times\left(\dfrac{28.0}{2}\right)\right]}{(3\times 20.0)+(2\times 39.6)} = 18.9 \text{ in.}$$ Table 5.12, Case 3

$$A_c = d\sum \ell = 8.25\times[(3\times 20.0)+(2\times 39.6)] = 1{,}148.4 \text{ in.}^2$$ Eq. (5.15)

<u>**Segment 1**</u>**:**

$y_i = 48.0 - 18.9 = 29.1$ in., $y_j = 28.0 - 18.9 = 9.1$ in.

$$\frac{\ell}{3}\sum(y_i^2 + y_i y_j + y_j^2) = \frac{20.0}{3}\times[29.1^2 + (29.1\times 9.1) + 9.1^2] = 7{,}963 \text{ in.}^3$$

Segment 2:

$$y_i = y_j = -18.9 \text{ in.}$$

$$\frac{\ell}{3}\sum(y_i^2 + y_i y_j + y_j^2) = \frac{20.0}{3}\times[3\times(-18.9)^2] = 7{,}144 \text{ in.}^3$$

Segment 3:

$$y_i = 9.1 \text{ in.},\ y_j = -18.9 \text{ in.}$$

$$\frac{\ell}{3}\sum(y_i^2 + y_i y_j + y_j^2) = \frac{39.6}{3}\times[9.1^2 - (9.1\times18.9) + 18.9^2] = 3{,}538 \text{ in.}^3$$

$$J_c = d\sum\frac{\ell}{3}(y_i^2 + y_i y_j + y_j^2) = 8.25\times[(2\times7{,}963) + 7{,}144 + (2\times3{,}538) = 248{,}705 \text{ in.}^4$$ Eq. (5.16)

$$V_u = q_u(\ell_1\ell_2 - A) = 258.6\times\left\{\left[25.0\times\left(\frac{23.5}{2}+\frac{20.0}{2\times12}\right)\right]-\left[\frac{(76.0\times20.0)+(28.0\times20.0)+28.0^2}{144}\right]\right\}/1{,}000 = 76.2 \text{ kips}$$

The transfer moment M_{sc} is essentially zero at this critical section, but for calculation purposes, is conservatively taken as $0.3M_o/2 = 115.5/2 = 57.8$ ft-kips.

$$\gamma_v = 1-\gamma_f = 1-\frac{1}{1+(2/3)\sqrt{b_1/b_2}} = 1-\frac{1}{1+(2/3)\sqrt{48.0/76.0}} = 0.35$$ ACI Eq. (8.4.4.2.2)

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{76{,}200}{1{,}148.4} + \frac{0.35\times57.8\times12{,}000\times18.9}{248{,}705}$$

Eq. (5.14)

$$= 66.4 + 18.57 = 84.9 \text{ psi} < \phi v_n = 94.9 \text{ psi}$$

Therefore, the 6-#4 closed stirrups provided on each face of the column are adequate.

First interior columns

Check two-way shear strength requirements ACI 8.4.4

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2q_D + 1.6q_L = [1.2\times(118.8+10.0)] + (1.6\times65.0) = 258.6 \text{ lb/ft}^2$$

$$b_1 = c_1 + d = 20.0 + 8.25 = 28.25 \text{ in.}$$

$$b_2 = c_2 + d = 20.0 + 8.25 = 28.25 \text{ in.}$$

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1 b_2) = 258.6\times\left[(25.0\times23.5)-\left(\frac{28.25^2}{144}\right)\right]/1{,}000 = 150.5 \text{ kips}$$

In lieu of a more exact analysis, the factored slab moment, M_{sc}, transferred to an interior column due to the gravity load effects can be determined from the following equation where the spans in the direction of analysis and perpendicular to the direction of analysis are equal:

$$M_{sc} = M_{sc(L)} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (1.6 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 42.1 \text{ ft-kips} \qquad \text{Eq. (5.21)}$$

$$A_c = 2(b_1 + b_2)d = [2 \times (2 \times 28.25)] \times 8.25 = 932.3 \text{ in.}^2 \qquad \text{Table 5.11, Case 1}$$

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 1 - 0.60 = 0.40 \qquad \text{ACI Eq. (8.4.4.2.2)}$$

$$\frac{J_c}{c_{AB}} = \frac{b_1 d(b_1 + 3b_2) + d^3}{3} = 8{,}966 \text{ in.}^3$$

Total factored shear stress:

$$v_{u|AB} = \frac{V_{u(D+L)}}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{150{,}500}{932.3} + \frac{0.40 \times 42.1 \times 12{,}000}{8{,}966} = 161.4 + 22.5 = 183.9 \text{ psi} \qquad \text{Eq. (5.14)}$$

$$\phi v_c = \text{ least of } \begin{cases} \phi 4 \lambda_s \lambda \sqrt{f_c'} = 189.7 \text{ psi} > v_{u|AB} = 183.9 \text{ psi} \\ \phi \left(2 + \dfrac{4}{\beta}\right) \lambda_s \lambda \sqrt{f_c'} = 284.6 \text{ psi} \\ \phi \left(2 + \dfrac{\alpha_s d}{b_o}\right) \lambda_s \lambda \sqrt{f_c'} = 233.4 \text{ psi} \end{cases} \qquad \text{Eq. (5.32)}$$

where $\beta = 20.0 / 20.0 = 1.0,\ \alpha_s = 40,\ \text{and } b_o = 4 \times 28.25 = 113.0 \text{ in.}$

(2) Load combination: $1.2D + Q_E + 0.5L$

$$q_u = 1.2 q_D + 0.5 q_L = [1.2 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 187.1 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 187.1 \times \left[(25.0 \times 23.5) - \left(\frac{28.25^2}{144}\right)\right] / 1{,}000 = 108.9 \text{ kips}$$

$$V_{u(Q_E)} = (42.4 + 35.4) / 21.5 = 3.6 \text{ kips} \qquad \text{Table 5.42}$$

Total factored shear force:

$$V_u = V_{u(D+L)} + V_{u(Q_E)} = 108.8 + 3.6 = 112.4 \text{ kips}$$

From gravity load effects:

$$M_{sc} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (0.5 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 13.2 \text{ ft-kips} \qquad \text{Eq. (5.21)}$$

From seismic load effects:

$$M_{sc} = 35.4 + 31.6 = 67.0 \text{ ft-kips}$$

Total $M_{sc} = 13.2 + 67.0 = 80.2$ ft-kips

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{112{,}500}{932.3} + \frac{0.40 \times 80.2 \times 12{,}000}{8{,}966} = 120.7 + 42.9 = 163.6 \text{ psi}$$ Eq. (5.14)

$$\phi v_c = 189.7 \text{ psi} > v_{u|AB} = 163.6 \text{ psi}$$ Eq. (5.32)

Shear strength requirements are satisfied at the first interior columns.

5.8.18 Example 5.18 – Determination of Shear Reinforcement: Flat Plate System, Building #1 (Framing Option A), SDC B, Headed Shear Studs

Check two-way shear strength requirements at the edge columns in an interior design strip in the north-south direction for the flat plate system in Figure 1.1 (Framing Option A) at the second-floor level with a 9.5-in.-thick slab and 20 in. by 20 in. columns. Assume the Site Class is D. Also assume normalweight concrete with $f'_c = 4{,}000$ psi. Provide shear reinforcement in the form of headed shear studs if needed with $f_{yt} = 51{,}000$ psi.

Step 1 – Check two-way shear strength requirements ACI 8.4.4

Dead load of slab $= (9.5 / 12) \times 150.0 = 118.8$ lb/ft^2

Superimposed dead load $= 10.0$ lb/ft^2

Live load $= 65.0$ lb/ft^2

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2 q_D + 1.6 q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$$

$$b_1 = c_1 + (d / 2) = 20.0 + (8.25 / 2) = 24.13 \text{ in.}$$ Table 5.11, Case 3

$$b_2 = c_2 + d = 20.0 + 8.25 = 28.25 \text{ in.}$$

Factored shear force at the critical section:

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 258.6 \times \left\{ \left[25.0 \times \left(\frac{23.5}{2} + \frac{20.0}{2 \times 12} \right) \right] - \left(\frac{24.13 \times 28.25}{144} \right) \right\} / 1{,}000 = 80.1 \text{ kips}$$

For the gravity load combination where the DDM is used to determine design bending moments, $M_{sc(D+L)} = 0.30 M_o$:

$\ell_2 = 25.0$ ft Figure 5.8

$\ell_n = 23.5 - (20.0 / 12) = 21.83$ ft Figure 5.23

Dead load: $$M_{oD} = \frac{q_D \ell_2 \ell_n^2}{8} = \frac{(118.8 + 10.0) \times 25.0 \times 21.83^2}{8 \times 1{,}000} = 191.8 \text{ ft-kips}$$ Eq. (5.17)

Live load: $$M_{oL} = \frac{q_L \ell_2 \ell_n^2}{8} = \frac{65.0 \times 25.0 \times 21.83^2}{8 \times 1{,}000} = 96.8 \text{ ft-kips}$$

$$M_o = 1.2 M_{oD} + 1.6 M_{oL} = (1.2 \times 191.8) + (1.6 \times 96.8) = 385.0 \text{ ft-kips}$$

$M_{sc(D+L)} = 0.30 \times 385.0 = 115.5$ ft-kips

$A_c = (2b_1 + b_2)d = [(2 \times 24.13) + 28.25)] \times 8.25 = 631.2$ in.^{2} Table 5.11, Case 3

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 1 - 0.62 = 0.38 \qquad \text{ACI Eq. (8.4.4.2.2)}$$

$$\frac{J_c}{c_{AB}} = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} = 5,647 \text{ in.}^3$$

Total factored shear stress:

$$v_{u|AB} = \frac{V_{u(D+L)}}{A_c} + \frac{\gamma_v M_{sc(D+L)} c_{AB}}{J_c} = \frac{80,100}{631.2} + \frac{0.38 \times 115.5 \times 12,000}{5,647} = 126.9 + 93.3 = 220.2 \text{ psi} \qquad \text{Eq. (5.14)}$$

$$\phi v_c = \text{ least of } \begin{cases} \phi 4 \lambda_s \lambda \sqrt{f_c'} = 189.7 \text{ psi} < v_{u|AB} = 220.2 \text{ psi} \\ \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s \lambda \sqrt{f_c'} = 284.6 \text{ psi} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s \lambda \sqrt{f_c'} = 248.3 \text{ psi} \end{cases} \qquad \text{Eq. (5.32)}$$

where $\beta = 20.0 / 20.0 = 1.0,\ \alpha_s = 30,$ and $b_o = (2 \times 24.13) + 28.25 = 76.5$ in.

(2) Load combination: $1.2D + Q_E + 0.5L$

$q_u = 1.2q_D + 0.5q_L = [1.2 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 187.1$ lb/ft^2

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 187.1 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{20.0}{2 \times 12}\right)\right] - \left(\frac{24.13 \times 28.25}{144}\right)\right\} / 1,000 = 58.0 \text{ kips}$$

Assume the bending moments due to seismic load effects are the same as those given in Table 5.42.

$V_{u(Q_E)} = (42.4 + 35.4) / 21.5 = 3.6$ kips

Total factored shear force:

$V_u = V_{u(D+L)} + V_{u(Q_E)} = 58.0 + 3.6 = 61.6$ kips

For other than gravity load combinations, M_{sc} is equal to that from the actual load combination:

$M_{sc} = [1.2 \times (0.26 \times 191.8)] + [0.5 \times (0.26 \times 96.8)] + 42.4 = 114.8$ ft-kips

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{61,600}{631.2} + \frac{0.38 \times 114.8 \times 12,000}{5,647} = 97.6 + 92.7 = 190.3 \text{ psi} \qquad \text{Eq. (5.14)}$$

$\phi v_c = 189.7 \text{ psi} < v_{u|AB} = 190.3 \text{ psi}$ Eq. (5.32)

Two-way shear strength requirements are not satisfied at the edge columns based on both load combinations.

Step 2 – Determine the size and spacing of headed shear stud reinforcement ACI 8.7.7

- **Step 2a – Check the maximum shear strength permitted with headed shear studs** ACI Table 22.6.6.3

From Step 1, maximum $v_{u|AB} = 220.2 \text{ psi} < \phi 8\sqrt{f'_c} = 379.5 \text{ psi}$

- **Step 2b – Determine the design shear strength of the concrete with headed shear studs** ACI Table 22.6.6.1

$$\phi v_c = \text{least of} \begin{cases} \phi 3\lambda_s \lambda \sqrt{f'_c} = 142.3 \text{ psi} \\ \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s \lambda \sqrt{f'_c} = 284.6 \text{ psi} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s \lambda \sqrt{f'_c} = 248.3 \text{ psi} \end{cases}$$

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d/10)}} = \sqrt{\frac{2}{1 + (8.25/10)}} = 1.1 > 1.0, \text{ use } 1.0, \quad \text{Eq. (5.30)}$$

$\lambda = 1.0$ for normalweight concrete, and $\beta = 20.0/20.0 = 1.0$, $\alpha_s = 30$, Table 5.20
and $b_o = (2 \times 24.13) + 28.25 = 76.5$ in.

Note: λ_s is permitted to be taken as 1.0 for two-way slabs with smooth headed shear stud reinforcement that satisfies the provisions of ACI 22.6.6.2(b); this provision is satisfied in this example (see Step 2c below).

- **Step 2c – Determine the required size and spacing of the headed shear studs** ACI 22.6.8

Maximum spacing between adjacent lines of shear studs $= 2d = 16.5$ in. Figure 5.20

Provide three lines of headed shear studs on each face of the column:

Maximum spacing $\cong 20.0/2 = 10.0$ in. < 16.5 in.

Assuming ½-in.-diameter studs ($A_b = 0.196 \text{ in.}^2$), the required spacing is the following:

$$s = \frac{\phi A_v f_{yt}}{(v_{u|AB} - \phi v_c) b_o} = \frac{0.75 \times (9 \times 0.196) \times 51{,}000}{(220.2 - 142.3) \times 76.5} = 11.3 \text{ in.} \quad \text{Table 5.23}$$

In this equation, A_v is the cross-sectional area of all the headed studs on one peripheral line parallel to the perimeter of the column section (see Figure 5.68).

Because $v_{u|AB} = 220.2 \text{ psi} < \phi 6\sqrt{f'_c} = 284.6$ psi, maximum spacing $= 0.75d = 6.2$ in. Figure 5.20

Assuming a 6.0-in. spacing, check the requirements of ACI 22.6.8.3:

$$\frac{A_v}{s} = \frac{9 \times 0.196}{6.0} = 0.29 \text{ in.}^2/\text{in.} > \frac{2\sqrt{f'_c} b_o}{f_{yt}} = 0.19 \text{ in.}^2/\text{in.} \quad \text{ACI Eq. (22.6.8.3)}$$

Use ½-in.-diameter headed shear studs spaced at 6.0 in. on center.

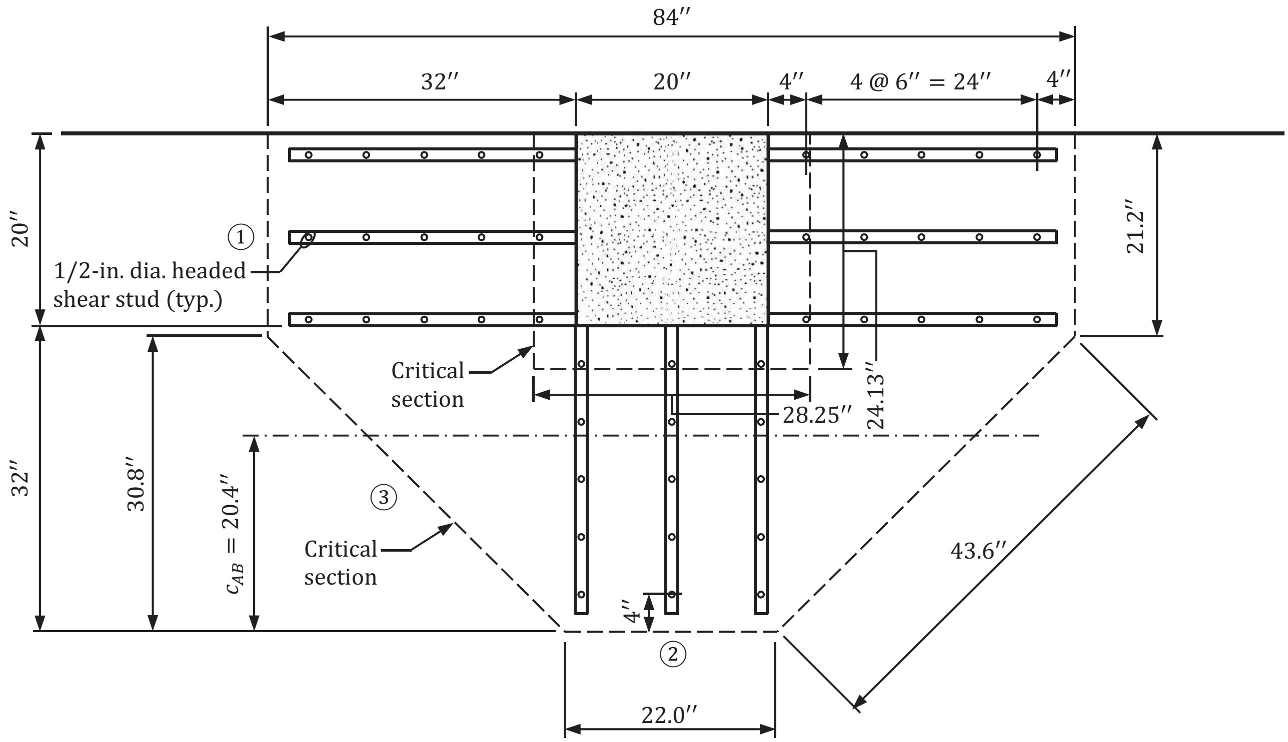


Figure 5.68 Headed shear stud reinforcement at the edge columns in Example 5.18.

- **Step 2d – Determine the distance from the faces of the column where the headed shear studs can be terminated**

Headed shear studs can be terminated where the design strength of the concrete can resist the factored shear stress without the headed shear studs.

At the critical section located a distance $d/2$ from the outermost peripheral line of headed shear studs:

$$\phi v_n = \phi v_c = \phi 2\lambda_s \lambda \sqrt{f'_c} = 94.9 \text{ psi} \qquad \text{Table 5.23}$$

Assume 5 headed shear studs are provided in each line parallel to each face of the column and check if the maximum factored shear stress at the critical section located a distance of $d/2$ from the centerline of the outermost peripheral line of headed studs is less than or equal to ϕv_n.

Section properties of the polygonal critical section (see Figure 5.68):

$$c_{AB} = \frac{\{(2\times 21.2)\times[30.8+(21.2/2)]\}+[2\times 43.6\times(30.8/2)]}{(2\times 21.2)+(2\times 43.6)+22.0} = 20.4 \text{ in.} \qquad \text{Table 5.12, Case 3}$$

$$A_c = d\sum \ell = 8.25\times[(2\times 21.2)+(2\times 43.6)+22.0] = 1{,}250.7 \text{ in.}^2 \qquad \text{Eq. (5.15)}$$

Segment 1:

$$y_i = 52.0 - 20.4 = 31.6 \text{ in.},\ y_j = 30.8 - 20.4 = 10.4 \text{ in.}$$

$$\frac{\ell}{3}\sum(y_i^2 + y_i y_j + y_j^2) = \frac{21.2}{3}\times[31.6^2 + (31.6\times 10.4) + 10.4^2] = 10{,}143 \text{ in.}^3$$

Segment 2:

$$y_i = y_j = -20.4 \text{ in.}$$

$$\frac{\ell}{3}\sum(y_i^2 + y_i y_j + y_j^2) = \frac{22.0}{3} \times [3 \times (-20.4)^2] = 9{,}156 \text{ in.}^3$$

Segment 3:

$$y_i = 10.4 \text{ in.}, \; y_j = -20.4 \text{ in.}$$

$$\frac{\ell}{3}\sum(y_i^2 + y_i y_j + y_j^2) = \frac{43.6}{3} \times [10.4^2 - (10.4 \times 20.4) + (-20.4)^2] = 4{,}537 \text{ in.}^3$$

$$J_c = d\sum\frac{\ell}{3}(y_i^2 + y_i y_j + y_j^2) = 8.25 \times [(2 \times 10{,}143) + (2 \times 4{,}537) + 9{,}156 = 317{,}757 \text{ in.}^4$$ Eq. (5.16)

$$V_u = q_u(\ell_1 \ell_2 - A) = 258.6 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{20.0}{2 \times 12}\right)\right] - \left[\frac{(84.0 \times 21.2) + (30.8 \times 22.0) + 30.8^2}{144}\right]\right\} / 1{,}000 = 75.2 \text{ kips}$$

The transfer moment M_{sc} is essentially zero at this critical section, but for calculation purposes, is conservatively taken as $0.3M_o / 2 = 115.5 / 2 = 57.8$ ft-kips.

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 1 - \frac{1}{1 + (2/3)\sqrt{52.0 / 84.0}} = 0.34$$ ACI Eq. (8.4.4.2.2)

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{75{,}200}{1{,}250.7} + \frac{0.34 \times 57.8 \times 12{,}000 \times 20.4}{317{,}757}$$ Eq. (5.14)

$$= 60.1 + 15.1 = 75.2 \text{ psi} < \phi v_n = 94.9 \text{ psi}$$

Therefore, the ½-in.-diameter headed shear studs spaced at 6.0 in. on center with 3 lines of headed studs on each face of the column are adequate.

Chapter 6
BEAMS

6.1 Overview

Beams are members subjected primarily to flexure and shear, with or without axial force or torsion. One of the primary functions of a beam is to transfer roof and floor loads to supporting members (columns, walls, or other beams, the latter of which are often referred to as girders). The main flexural reinforcement in a beam runs parallel to the direction of load transfer.

The design and detailing of cast-in-place beams with nonprestressed reinforcement are covered in this chapter for members subjected to a factored axial force, P_u, less than $0.10f'_cA_g$ where A_g is the gross cross-sectional area of the beam (see ACI 9.3.3.1). Provisions for beams are given in ACI Chapter 9, which are applicable to members in buildings assigned to Seismic Design Category (SDC) A and B.

Nonprestressed one-way joist systems consist of a monolithic combination of regularly spaced ribs with a clear spacing between the ribs of no more than 30 in. and a one-way slab meeting the dimensional requirements in ACI 9.8.1.2 through 9.8.1.4. The ribs and supporting beams are designed in accordance with ACI 9.8. This system was once popular but is generally not used anymore. Wide-module joist systems, which are also referred to as skip joist systems, are formed by 53-in.- or 66-in.-wide pan forms and have essentially taken the place of standard one-way joist systems. The evenly spaced joists (or, ribs) support the loads from a one-way slab cast integrally with the joists and beams (or, girders) [see Figure 6.1]. Because this system does not meet the clear spacing requirements between ribs in ACI 9.8.1.4 for standard one-way joist systems, the nominal shear strength of the concrete, V_c, is not permitted to be increased by 1.1 times the value prescribed in ACI 22.5 (ACI 9.8.1.5); thus, the joists and beams in the wide-module system must be designed in accordance with the provisions in ACI Chapter 9 excluding the requirements in ACI 9.8.

Figure 6.1 A wide-module joist system.

Deep beams, which are defined in ACI 9.9.1.1, are not covered in this chapter. These members are commonly designed using the strut-and-tie method given in ACI Chapter 23 (ACI 9.9.1.3). Numerous references are available that illustrate the design of deep beams by this method, including Reference 15.

6.2 Sizing the Cross-Section

6.2.1 Determining the Beam Depth

Beams must have sufficient depth so all applicable strength and serviceability requirements are satisfied. In lieu of calculating deflections in accordance with ACI 24.2 and subsequently checking the deflections do not exceed the limits in ACI 24.2.2 (ACI 9.3.2.1), the minimum overall depth, h, for nonprestressed beams not supporting or attached to partitions or other types of construction likely to be damaged by large deflections can be determined using the expressions in ACI Table 9.3.1.1, which are applicable for reinforcement with a specified yield strength, f_y, equal to 60,000 psi and normalweight concrete (ACI 9.3.1.1). Modification factors for beams with reinforcement other than 60,000 psi and made of lightweight concrete are given in ACI 9.3.1.1.1 and 9.3.1.1.2, respectively.

Minimum beam depths based on various support conditions are given in Table 6.1. In the expressions for minimum h, ℓ is the span length of the beam in inches; for cantilevers, ℓ is the clear projection of the cantilever in inches. Also, f_1 and f_2 are the modification factors for reinforcement grade and lightweight concrete, respectively.

Table 6.1 Minimum Depth of Nonprestressed Beams

Support Condition	Normalweight Concrete		Lightweight Concrete(1)	
	f_y = 60 ksi	f_y other than 60 ksi(2)	f_y = 60 ksi(3)	f_y other than 60 ksi(2),(3)
Simply supported	$\ell / 16$	$(\ell / 16)f_1$	$(\ell / 16)f_2$	$(\ell / 16)f_1f_2$
One end continuous	$\ell / 18.5$	$(\ell / 18.5)f_1$	$(\ell / 18.5)f_2$	$(\ell / 18.5)f_1f_2$
Both ends continuous	$\ell / 21$	$(\ell / 21)f_1$	$(\ell / 21)f_2$	$(\ell / 21)f_1f_2$
Cantilever	$\ell / 8$	$(\ell / 8)f_1$	$(\ell / 8)f_2$	$(\ell / 8)f_1f_2$

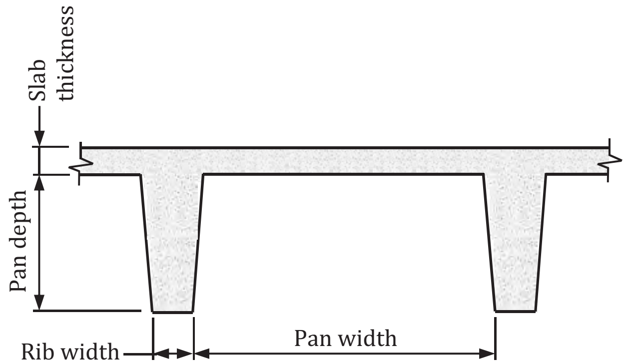
(1) *Applicable where equilibrium density,* w_c, *is in the range of 90 to 115 lb/ft*^{3}
(2) $f_1 = 0.4 + (f_y / 100{,}000)$ [f_y in psi]
(3) f_2 = greater of $(1.65 - 0.005w_c)$ and 1.09 [w_c in lb/ft^3]

For the usual case of continuous construction, the depth of the beams, h, must be the same for all spans and it should be determined on the basis of the span yielding the largest minimum depth; this results in economical formwork (see Reference 7). The minimum h for an end span condition governs in the case of equal end and interior spans. In cases where the interior span is greater than $21 / 18.5 = 1.14$ times the end span, the minimum h for the interior span condition governs.

Beam depth is typically specified in whole-inch increments although half-inch increments are commonly used in wide-module joist construction (see below).

As noted in Section 6.1, standard pan forms are used in constructing wide-module joist systems. The pan depths corresponding to the standard pan widths are given in Figure 6.2. A 4.5-in.-thick slab is typically specified in such systems because structural requirements are minimal. Therefore, the overall depth of the system (slab thickness plus pan depth) must be greater than or equal to the applicable minimum depth given in Table 6.1 to satisfy serviceability requirements. To achieve economical formwork, the depths of the beams must be the same depth as the joists: The formwork associated with beams deeper than the joists is more complex to build and additional shoring is required to support the beam formwork, which results in unnecessary increases in construction time and cost. For economy, the joists typically span in the long direction, which means their depth usually controls the overall depth of the wide-module system.

Fire-resistance requirements of the general building code must also be considered when specifying a beam depth and slab thickness (ACI 4.11.1). In most cases, the required beam depth based on fire-resistance requirements is smaller than that required by ACI 9.3.1 for serviceability.



Pan Width (in.)	Pan Depth (in.)
53	16, 20, 24
66	14, 16, 20, 24

Figure 6.2 Standard form dimensions for wide-module joist systems.

The depth of any beam can be determined based on the deflection requirements in ACI 24.2. For beams supporting construction likely to be damaged by large deflections (such as glass partition walls), deflection calculations should be performed in accordance with that section. Maximum permissible deflections are given in ACI Table 24.2.2 for members supporting or attached to nonstructural elements likely to be damaged by large deflections and for members that are not. Methods on how to calculate immediate and time-dependent deflections for beams are given in Section 6.7 of this publication. This information is also applicable to calculation of deflections for one-way slabs.

6.2.2 Determining the Beam Width

The width of a beam, b_w, can be determined based on flexural strength requirements, that is, by satisfying the following strength design equation: $\phi M_n = M_u$. At this stage of the design, the required area of flexural reinforcement, A_s, is typically not known. However, a range for A_s can be determined based on required minimum and maximum areas of reinforcement. The minimum area of flexural reinforcement is specified in ACI 9.6.1. Also, beams subjected to relatively small or no axial loads must be designed as tension-controlled sections (ACI 9.3.3.1), which essentially sets an upper limit on the area of flexural reinforcement provided in a section. The provided area of reinforcement must be within this range. For Grade 60 reinforcement and a specified concrete compressive strength, f'_c, equal to 4,000 psi, minimum and maximum A_s are equal to the following:

- $$\text{Minimum } A_s = \text{ larger of } \begin{cases} \dfrac{3\sqrt{f'_c}b_w d}{f_y} = \dfrac{3 \times \sqrt{4{,}000} \times b_w d}{60{,}000} = 0.0032 b_w d \\[2ex] \dfrac{200 b_w d}{f_y} = \dfrac{200 b_w d}{60{,}000} = 0.0033 b_w d \quad \text{(governs)} \end{cases}$$

- $$\text{Maximum } A_s = \frac{0.85\beta_1 c_t f'_c b_w}{f_y} = \frac{0.85\beta_1[0.003d \,/\, (\varepsilon_{ty} + 0.006)] f'_c b_w}{f_y}$$

$$= \frac{0.85 \times 0.85 \times \left(\dfrac{0.003d}{0.00207 + 0.006}\right) \times 4 \times b_w}{60} = 0.0179 b_w d$$

Using an initial estimate of $0.01b_w d$ for the area of the flexural reinforcement (which is approximately equal to the average of the minimum and maximum values), b_w can be obtained by setting $\phi M_n = M_u$ and solving for b_w :

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right) = \phi A_s f_y \left(d - \frac{0.5A_s f_y}{0.85f_c' b_w}\right) = \phi(0.01b_w d^2) f_y \left(1 - \frac{0.0059f_y}{f_c'}\right)$$

$$b_w = \frac{M_u}{0.009d^2 f_y \left(1 - \dfrac{0.0059f_y}{f_c'}\right)}$$

For beams with $f_y = 60{,}000$ psi and $f_c' = 4{,}000$ psi:

$$b_w = \frac{2.03M_u}{d^2}$$

In this equation, M_u has the units of in.-kips and b_w and d have the units of inches.

The above equation can be rewritten in the following form where M_u has the units of ft-kips and b_w and d have the units of inches:

$$b_w = \frac{24.4M_u}{d^2} \tag{6.1}$$

The largest factored bending moment M_u along the spans should be used in Equation (6.1) to determine b_w. For beams with one layer of longitudinal tension reinforcement, d can be taken as $h - 2.5$ in. where h is the depth of the beam determined for serviceability (see Section 6.2.1 of this publication) and 2.5 in. is the sum of the concrete cover in accordance with ACI Table 20.5.1.3.1 for beams not exposed to weather or in contact with the ground (1.5 in.), the diameter of a #4 stirrup (0.5 in.), and one-half the diameter of a #8 longitudinal bar $[(1.0 / 2) = 0.5 \text{ in.}]$. To achieve economical formwork, the same b_w should be used for all beams along the span. Also, the same beam size should be used as often as possible throughout the structure.

Providing a beam width greater than or equal to that determined by Equation (6.1) results in cross-sectional dimensions satsifying both strength and serviceability requirements.

The following equation can be used to determine b_w for a beam with any grade of reinforcing steel, concrete compressive strength, and reinforcement ratio $\rho = A_s / b_w d$:

$$b_w = \frac{M_u \times 12{,}000}{0.9\rho f_y \left(1 - \dfrac{0.5\rho f_y}{0.85f_c'}\right) d^2} \tag{6.2}$$

where M_u has the units of ft-kips; f_y and f_c' have the units of psi; and d and b_w have the units of inches. The reinforcement ratio ρ must be greater than or equal to the greater of $3\sqrt{f_c'} / f_y$ and $200 / f_y$ (ACI 9.6.1.2) and less than or equal to $0.319\beta_1 f_c' / f_y$ (ACI 9.3.3.1) where the term β_1 is defined in ACI Table 22.2.2.4.3 as follows (ACI 22.2.2.4.3):

- For $2{,}500 \text{ psi} \le f_c' \le 4{,}000$ psi: $\beta_1 = 0.85$
- For $4{,}000 \text{ psi} < f_c' < 8{,}000$ psi: $\beta_1 = 0.85 - [0.05(f_c' - 4{,}000) / 1{,}000]$
- For $f_c' \ge 8{,}000$ psi: $\beta_1 = 0.65$

Equations (6.1) or (6.2) can also be used to determine the required width of the joists in a wide-module joist system. Joist width can be tailored to satisfy virtually any requirement. In usual situations, the thinnest practical width is typically adequate for structural requirements. Generally, a 7-in.-wide joist is used with a 53-in. pan and a 6-in.-wide joist is used with a 66-in. pan resulting in 5-ft and 6-ft modules, respectively. Column-line joists can be made part of the lateral force-resisting system (LFRS) and any practical width (not necessarily the same as the adjoining joists) can be provided to resist combined gravity and lateral load effects.

6.2.3 General Guidelines for Sizing Beams for Economy

The following general guidelines for sizing beams should be followed for economy (see Reference 7):

- Use whole-inch increments for beam dimensions, except for the depth of beams in wide-module joist systems where half-inch beam depths are used for formwork economy where applicable.
- Use beam widths in multiples of 2 or 3 in.
- Use constant beam size from span to span, and vary the reinforcement as required.
- Use wide, shallower beams rather than narrow, deeper beams.
- Use beam widths at least 4 in. wider than the supporting column widths, wherever possible.
- Use uniform width and depth of beams throughout the building, wherever possible.

6.3 Required Strength

6.3.1 Analysis Methods

Overview

The analysis methods in ACI Chapter 6 in conjunction with the factored load combinations in ACI Chapter 5 are to be used to calculate required strength (see ACI 9.4.1.2 and 9.4.1.1, respectively).

Bending Moments and Shear Forces

Five methods of analysis are given in ACI 6.2.3 to determine the factored bending moments and shear forces at the critical sections in a beam (see Section 6.3.2 of this publication for the locations of the critical sections for flexure and shear).

For the case of gravity loads, the simplified method for analysis of continuous beams and one-way slabs in ACI 6.5 is permitted to be used to determine bending moments and shear forces provided the conditions in ACI 6.5.1 are satisfied [ACI 6.2.3(a)]:

(1) Members are prismatic
(2) Gravity loads are uniformly distributed
(3) The service live load, L, is less than or equal to 3 times the service dead load, D
(4) There are at least two spans
(5) The longer of two adjacent spans does not exceed the shorter span by 20 percent

The approximate factored bending moments, M_u, and factored shear forces, V_u, in ACI 6.5.2 and 6.5.4, respectively, are given in Figure 4.3 of this publication where w_u is the factored uniformly distributed gravity load on the continuous beams.

Moment redistribution is not permitted when bending moments are calculated using the simplified method (ACI 6.5.3).

Where one or more of the conditions in ACI 6.5.1 is not satisfied, one of the other four methods of analysis given in ACI 6.2.3 must be used to determine M_u and V_u at the critical sections.

Torsional Moments

Factored torsional moments, T_u, can be obtained using the analysis methods in ACI Chapter 6. It is permitted to take the torsional loading from a slab as uniformly distributed along the span of a beam (ACI 9.4.4.1).

Once T_u is determined at the critical section (see Section 6.3.2 of this publication for the location of the critical section for torsion), the next step is to determine if torsional effects on a beam need to be considered or not. Torsional effects on a reinforced concrete beam need not be considered if $T_u < \phi T_{th}$ where T_{th} is the threshold torsional moment (ACI 9.5.4.1). For nonprestressed members with solid cross-sections, T_{th} is given in ACI Table 22.7.4.1(a):

$$T_{th} = \begin{cases} \lambda\sqrt{f_c'}\left(\dfrac{A_{cp}^2}{p_{cp}}\right) \\ \\ \lambda\sqrt{f_c'}\left(\dfrac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\dfrac{N_u}{4A_g\lambda\sqrt{f_c'}}} \end{cases} \tag{6.3}$$

The first of these equations is applicable to members not subjected to an axial force. The second equation is applicable where the member is subjected to a factored axial force N_u (in pounds), which is taken as positive for compression forces acting on the gross area of the beam, A_g, and negative for tension forces.

The term λ is the modification factor reflecting the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength, and is determined based on either (1) the equilibrium density, w_c, of the concrete mix or (2) the composition of the aggregate in the concrete mix (see ACI 22.7.1.3 and ACI 19.2.4). Values of λ based on w_c are given in Table 6.2 [ACI Table 19.2.4.1(a)] and values of λ based on composition of aggregates are given in Table 6.3 [ACI Table 19.2.4.1(b)]. Note that λ is permitted to be taken as 0.75 for lightweight concrete (ACI 19.2.4.2) and is equal to 1.0 for normalweight concrete (ACI 19.2.4.3).

Table 6.2 Values of λ Based on Equilibrium Density, w_c

Equilibrium Density, w_c	λ
$w_c \leq 100$ lb/ft^3	0.75
100 lb/ft$^3 < w_c \leq 135$ lb/ft^3	$0.0075w_c \leq 1.0$
$w_c > 135$ lb/ft^3	1.0

Table 6.3 Values of λ Based on Composition of Aggregates

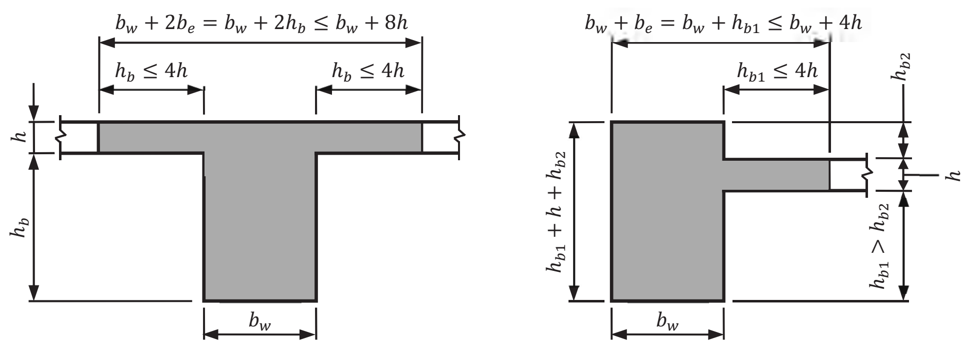
Concrete	Composition of Aggregates	λ
All-lightweight	Fine: ASTM C330 Coarse: ASTM C330	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330 and C33 Coarse: ASTM C330	0.75 to 0.85[(1)]
Sand-lightweight	Fine: ASTM C33 Coarse: ASTM C330	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33 Coarse: Combination of ASTM C330 and ASTM C33	0.85 to 1.0[(2)]

(1) Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.
(2) Linear interpolation from 0.85 to 1.0 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of aggregate.

The term A_{cp} is the area enclosed by the outside perimeter of the concrete cross-section, and p_{cp} is the outside perimeter of the concrete cross-section. For beams cast monolithically with a slab, a portion of the slab may be able to contribute to torsional resistance. The overhanging flange width, b_e, permitted to be used in the calculation of A_{cp} and p_{cp} is given in ACI 9.2.4.4:

$$b_e = \text{least of} \begin{cases} h_b \\ \\ 4h \end{cases} \tag{6.4}$$

where h_b is the greater of the projection of the beam above or below the slab (see Figure 6.3).



Interior Beam	Edge Beam
$b_e = h_b \leq 4h$	$b_e = h_{b1} \leq 4h$
$A_{cp} = b_w h_b + (b_w + 2b_e)h$	$A_{cp} = b_w(h_{b1} + h + h_{b2}) + hb_e$
$p_{cp} = 2(h + h_b + b_w + 2b_e)$	$p_{cp} = 2(h_{b1} + h + h_{b2} + b_w + b_e)$

Figure 6.3 Torsional section properties A_{cp} and p_{cp} for interior and edge beams cast monolithically with the slab.

Torsional section properties A_{cp} and p_{cp} for an interior beam and an upturned edge beam cast monolithically with the slab are also given in Figure 6.3. Note that the torsional section properties for the edge beam are applicable for any value of h_{b1} and h_{b2}, including a value of zero ($h_{b1} = 0$ corresponds to the case where the bottom of the slab is flush with the bottom of the beam and $h_{b2} = 0$ corresponds to the case where the top of the slab is flush with the top of the beam).

The contribution of the overhanging flange(s) must be neglected in cases where the parameter (A_{cp}^2 / p_{cp}) calculated for a solid beam with flanges is less than (A_{cp}^2 / p_{cp}) calculated for the same solid beam ignoring the flanges [ACI 9.2.4.4(b)]. For hollow sections, the overhanging flange(s) must be neglected where (A_g^2 / p_{cp}) for the beam with flanges is less than that for the same beam ignoring the flanges where A_g is the area of the concrete only and does not include the area of the void.

For nonprestressed members with hollow cross-sections, T_{th} is given in ACI Table 22.7.4.1(b):

$$T_{th} = \begin{cases} \lambda\sqrt{f'_c}\left(\dfrac{A_g^2}{p_{cp}}\right) \\ \\ \lambda\sqrt{f'_c}\left(\dfrac{A_g^2}{p_{cp}}\right)\sqrt{1 + \dfrac{N_u}{4A_g\lambda\sqrt{f'_c}}} \end{cases} \tag{6.5}$$

Once it has been established that torsional effects must be considered (that is, where $T_u \geq \phi T_{th}$), the next step is to determine whether T_u from analysis can be reduced in accordance with the provisions in ACI 22.7.3 (ACI 9.4.4.4).

The beam supporting the cantilever slab in ACI Figure R22.7.3a is unable to redistribute the effects due to the torsional moment (it is not part of an indeterminate structure where redistribution of the applied torsional moment can occur); thus, it must be designed for the torsional moment T_u from analysis. This type of torsion is referred to as equilibrium torsion because T_u is required to maintain equilibrium (ACI 22.7.3.1).

The spandrel beam in ACI Figure R22.7.3b is part of an indeterminate structure where redistribution of torsional moments can occur. This type of torsion is referred to as compatibility torsion. In this case, the beam can be designed for a torsional moment T_u equal to the design cracking torsional moment, ϕT_{cr}, which is less than that obtained from analysis (ACI 22.7.3.2). For both solid and hollow sections, T_{cr} is determined by the equations in ACI Table 22.7.5.1 (ACI 22.7.5.1):

$$T_{cr} = \begin{cases} 4\lambda\sqrt{f_c'}\left(\dfrac{A_{cp}^2}{p_{cp}}\right) \\[2ex] 4\lambda\sqrt{f_c'}\left(\dfrac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\dfrac{N_u}{4A_g\lambda\sqrt{f_c'}}} \end{cases} \tag{6.6}$$

The first of these equations is applicable to members not subjected to an axial force. The second equation is applicable where the member is subjected to a factored axial force N_u (in pounds), which is taken as positive for compression forces acting on the gross area of the beam, A_g, and negative for tension forces. Note that $T_{cr} = 4T_{th}$ for members with solid cross-sections.

Adjoining members must be designed for the redistributed bending moments and shear forces due to application of the compatibility torsional moment (ACI 22.7.3.3). The beams framing into the spandrel beam in ACI Figure R22.7.3b must be designed for a concentrated moment at their ends equal to the compatibility torsional moment $T_u = \phi T_{cr}$; that moment is redistributed to the positive and interior negative critical sections of the beam and are algebraically combined with the bending moments corresponding to gravity and any other applicable loads. Typically, the negative moment at the exterior support of the beam framing into the spandrel beam is reduced compared to that prior to redistribution and the positive and negative moments at the other critical sections are increased.

In cases where torsional effects must be considered and T_u from analysis is less than ϕT_{cr}, the section should be designed to resist the factored torsional moment from analysis.

6.3.2 Critical Sections for Flexure, Shear, and Torsion

In accordance with ACI 9.4.2.1, 9.4.3.1, and 9.4.4.2, M_u, V_u, and T_u are permitted to be calculated at the faces of the supports for beams built integrally with the supports. For flexural design, the critical sections occur at the faces of the supports where negative moments are maximum and in the span where positive moments are maximum.

Beams are permitted to be designed for the shear force at a critical section located a distance d from the face of the support where the conditions in ACI 9.4.3.2 are satisfied (see Figure 6.4):

(1) Support reaction, in direction of applied shear, introduces compression into the end region of the beam
(2) Loads are applied at or near the top surface of the beam
(3) No concentrated load occurs between the face of the support and the critical section

The critical section for shear must be taken at the face of the support where one or more of the three conditions in ACI 9.4.3.2 are not met. For example, the critical section must be at the face of the support for the framing configuration in Figure 6.5 because the supporting member is in tension.

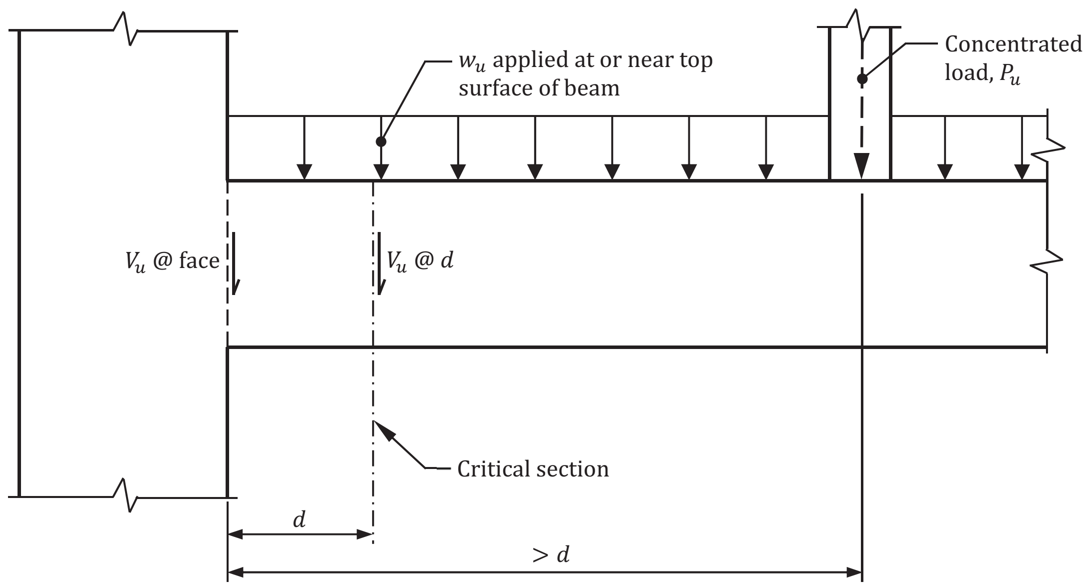


Figure 6.4 Critical section for shear in a beam satisfying the conditions of ACI 9.4.3.2.

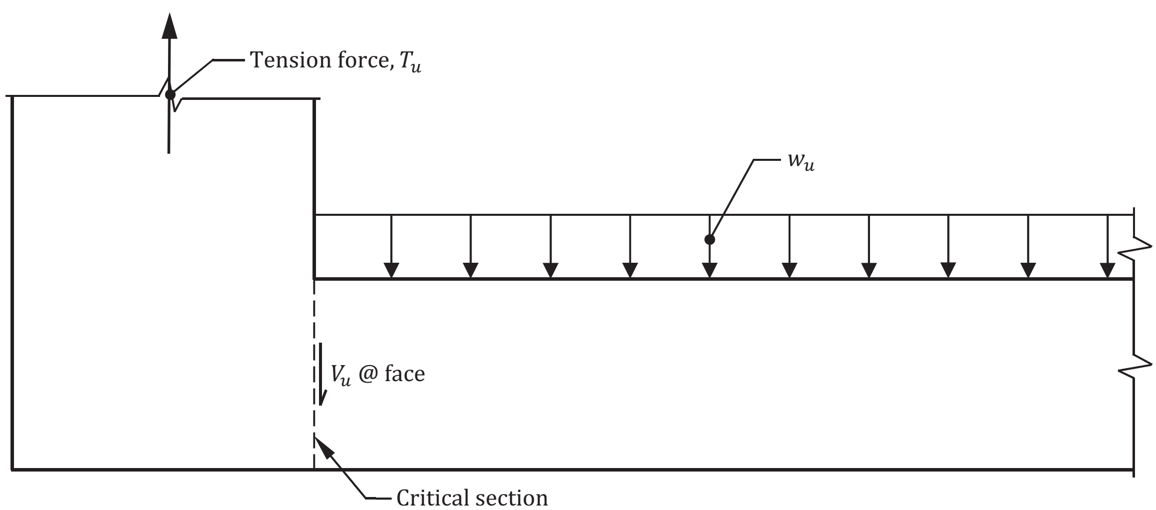


Figure 6.5 Critical section for shear in a beam supported by a member in tension.

Beams are permitted to be designed for the torsional moment at a critical section located a distance d from the face of the support provided no concentrated torsional moments occur within that distance (ACI 9.4.4.3); otherwise, the critical section must be taken at the face of the support. The edge beam in Figure 6.6 is subjected to a uniformly distributed torsional moment, t_u, from the slab and a concentrated torsional moment, $T_{u,beam}$, from the beam framing into it. The critical section for torsion can be taken a distance d from the face of the column in this case because $T_{u,beam}$ occurs at a distance greater than d from the face of the column. As such, the edge beam can be designed for the torsional moment at the location of the critical section (that is, at T_u @ d).

6.3.3 Redistribution of Moments in Continuous Flexural Members

Bending moments calculated in accordance with ACI 6.8 at supports of continuous flexural members are permitted to be increased or decreased in accordance with ACI 6.6.5, except where the moments have been computed using the approximate coefficients in ACI 6.5. It is customary to reduce the negative moments at the supports, which results in an increase of the positive moments in the span.

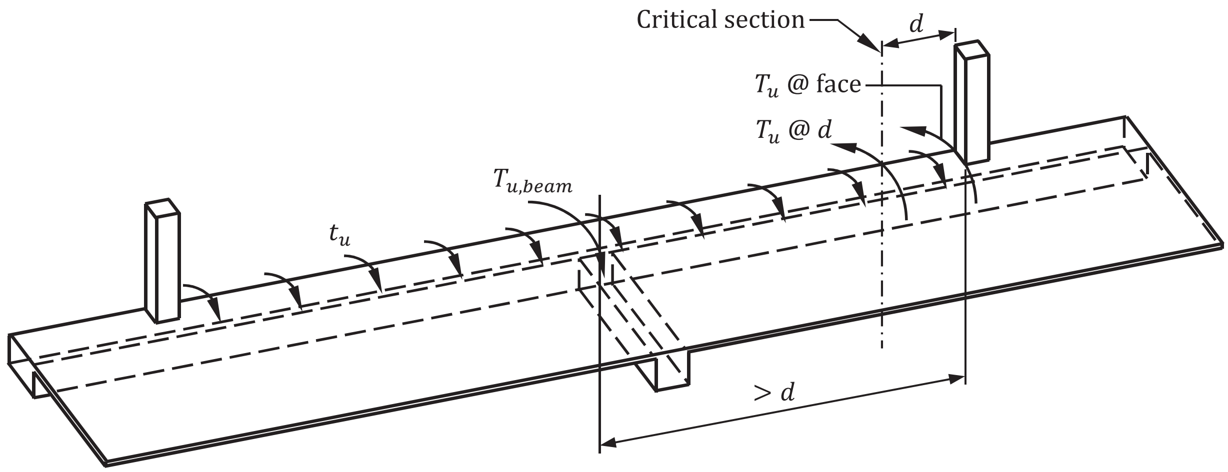


Figure 6.6 Critical section for torsion in a beam where a concentrated torsional moment occurs at a distance greater than ***d*** from the face of the support.

The maximum percentage of redistribution permitted is the lesser of 1,000 times the net tensile strain in the reinforcement, ε_t, or 20 percent.

Moment redistribution is dependent on adequate ductility in plastic hinge regions, which develop at points of maximum moment and which cause a shift in the elastic bending moment diagram. Thus, redistribution of negative moments is only permitted where ε_t is greater than or equal to 0.0075 at the section in which the moment is reduced. Adjustments of the negative moments at the supports are made for each loading configuration, considering pattern live loading. It is important to ensure that static equilibrium is maintained at all joints before and after moment redistribution. Thus, a decrease in the negative moments at the supports warrants an increase in the positive moment in the span under consideration.

Adjusted negative bending moments at the faces of the supports, $(M_u^-)_{adj}$, are equal to $[1-(A\%/100)]M_u^-$ where $A\%$ is the lesser of $1{,}000\varepsilon_t\%$ and 20% and M_u^- is the factored negative moment at the face of the support from analysis.

The adjusted positive moments are obtained based on equilibrium considering the adjusted negative bending moments at the faces of the supports.

6.4 Design Strength

6.4.1 General

For each applicable factored load combination in ACI Table 5.3.1, the following equations must be satisfied at any section in a beam (ACI 9.5.1.1):

$$\phi M_n \geq M_u \tag{6.7}$$

$$\phi V_n \geq V_u \tag{6.8}$$

$$\phi T_n \geq T_u \tag{6.9}$$

$$\phi P_n \geq P_u \tag{6.10}$$

Strength reduction factors, ϕ, are determined in accordance with ACI 21.2. Nonprestressed beams with $P_u < 0.10 f_c' A_g$ must be designed as tension-controlled sections in accordance with ACI Table 21.2.2 (ACI 9.3.3.1); thus, the net tensile strain in the extreme layer of the longitudinal tension reinforcement at nominal strength, ε_t, must be greater than or equal to $\varepsilon_{ty} + 0.003$ and $\phi = 0.90$ (ACI Table 21.2.2). The net tensile strain in the extreme layer of longitudinal tension reinforcement corresponding to compression-controlled sections, ε_{ty}, is equal to f_y / E_s (ACI 21.2.2.1). The modulus of elasticity of the reinforcement, E_s, is permitted to be taken as 29,000,000 psi regardless of the grade of the reinforcement (ACI 20.2.2.2). For shear and torsion, $\phi = 0.75$ (ACI Table 21.2.1).

Methods to determine the nominal strengths M_n, V_n, and T_n are given in Sections 6.4.2, 6.4.3, and 6.4.4, respectively.

6.4.2 Nominal Flexural Strength

Rectangular Sections with Tension Reinforcement Only

Single Layer of Tension Reinforcement. The nominal flexural strength, M_n, of a rectangular section with one layer of tension reinforcement and with $P_u < 0.10 f'_c A_g$ is determined in accordance with ACI 22.3, and is based on moment equilibrium of a rectangular section (see ACI 9.5.2.1 and Figure 6.7):

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \tag{6.11}$$

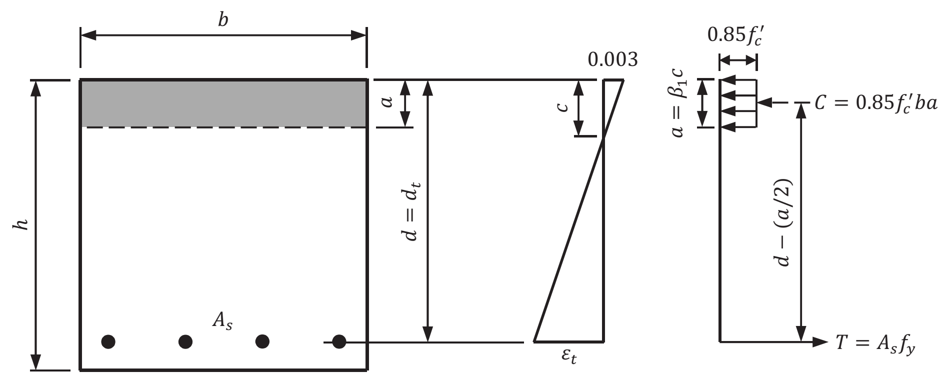


Figure 6.7 Strain and stress distributions at a positive moment section in a beam.

In this equation, A_s is the total area of flexural reinforcement. The strain and stress distributions in Figure 6.7 are at a positive moment section and are equally applicable at negative moment sections.

The depth of the equivalent stress block, a, is determined from force equilibrium, that is, it is determined by setting the resultant compressive force in the concrete, $C = 0.85 f'_c ba$, equal to the tension force in the reinforcement, $T = A_s f_y$, and solving for a (see Figure 6.7):

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{6.12}$$

It is assumed a uniform stress equal to 85 percent of the concrete compressive strength, f'_c, is distributed over the depth $a = \beta_1 c$, where c is the distance from the extreme compression fiber to the neutral axis (ACI 22.2.2.4.1). The ultimate strain in the concrete is assumed to be 0.003 (ACI 22.2.2.1), and the term b is the width of the compression face of the member.

Multiple Layers of Tension Reinforcement. Under certain conditions, the required tension reinforcement cannot fit within a single layer. For example, in beams with a width restriction due to architectural constraints, it may not be possible to select a practical bar size while satisfying the minimum clear spacing requirements between the bars in ACI 25.2.1. In such cases, the longitudinal bars are provided in more than one layer.

The nominal flexural strength, M_n, for sections with multiple layers of reinforcement is determined in the same way as that for sections with a single layer of reinforcement. The main difference is that for beams with two or more layers, the reinforcement in the inner layer(s) may not yield. Therefore, it is important to check whether all the reinforcement in the section yields or not; M_n is then determined accordingly based on this check.

Consider the rectangular beam in Figure 6.8 with two layers of reinforcement. Assume all the bars are the same size and are located below the neutral axis. Also assume the yield strain $\varepsilon_{ty} = f_y / E_s$ occurs at a distance d_{ty} from the extreme compression fiber. The following relationship is established between ε_{ty} and c:

$$\frac{c}{0.003} = \frac{d_{ty} - c}{\varepsilon_{ty}} \tag{6.13}$$

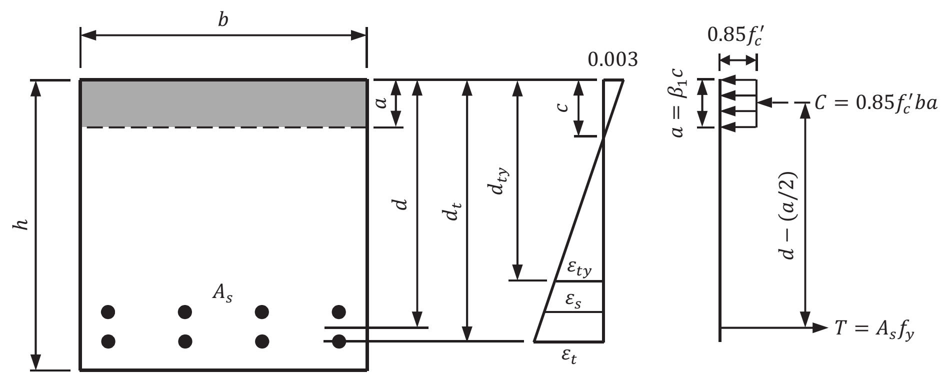


Figure 6.8 A reinforced concrete beam with multiple layers of tension reinforcement.

Solving for d_{ty} results in the following:

$$d_{ty} = c\left(1 + \frac{\varepsilon_{ty}}{0.003}\right) \tag{6.14}$$

For Grade 60 reinforcement, Equation (6.14) reduces to the following:

$$d_{ty} = c\left(1 + \frac{60 / 29{,}000}{0.003}\right) = 1.7c \tag{6.15}$$

Reinforcement located a distance equal to or greater than d_{ty} from the extreme compression fiber yields, and M_n is calculated by Equation (6.11) assuming all the reinforcement is concentrated at d, which is the distance from the extreme compression fiber to the centroid of all the reinforcement (see Figure 6.8). Where the reinforcement in one or more layers does not yield, it are separated from the reinforcement in the layer(s) that yield, and the actual stress in those bars $(f_s = \varepsilon_s E_s < f_y)$ is used to calculate M_n.

Rectangular Sections with Tension and Compression Reinforcement

Reinforcement is typically added to the compression zone of a beam section for the following reasons: (1) to increase the nominal flexural strength, M_n, where the dimensions of the beam are limited (due to architectural constraints, for example) and the area of tensile reinforcement required to resist M_u is greater than that corresponding to a tension-controlled section and (2) to help reduce long-term deflections of the beam (see Section 6.7 of this publication for a discussion on long-term deflections). Beams with both tension and compression reinforcement are commonly referred to as doubly reinforced beams.

Reinforcement in the compression zone contributes to M_n, though the contribution is usually relatively small. Additionally, ε_t is larger when compression reinforcement is used in a section, which results in more ductile behavior.

Transverse reinforcement in the form of stirrups are typically required in beams to resist factored shear forces. Stirrups must be anchored at both the top and bottom of the section to longitudinal reinforcement to properly develop them in tension (see Section 6.4.3 of this publication). Thus, reinforcing bars in the compression zone must be provided wherever stirrups are required. As noted above, the contribution of the compression reinforcement to M_n is usually relatively small, and, thus, is often not considered in design. However, if compression reinforcement is needed to make the section tension-controlled, the following information can be used to determine M_n.

Nominal Flexural Strength When the Compression Reinforcement Yields. Strain and stress distributions in a doubly reinforced section are given in Figure 6.9 where the total area of compression reinforcement, A_s', is located a distance d' from the extreme compression fiber. Assuming the yield strain, ε_{ty}, occurs at a distance d_y' from the extreme compression fiber, the following relationship is established between d_y' and c:

$$\frac{c}{0.003} = \frac{c - d_y'}{\varepsilon_{ty}} \tag{6.16}$$

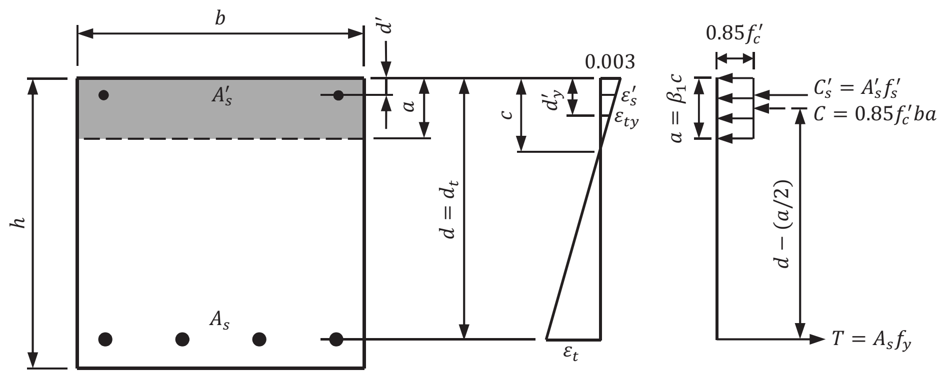


Figure 6.9 Strain and stress distributions in a doubly reinforced concrete beam.

Solving for d_y' results in the following:

$$d_y' = c\left(1 - \frac{\varepsilon_{ty}}{0.003}\right) \tag{6.17}$$

For Grade 60 reinforcement, Equation (6.17) reduces to the following:

$$d_y' = c\left(1 - \frac{60 / 29{,}000}{0.003}\right) = 0.31c \tag{6.18}$$

Compression reinforcement located a distance equal to or less than d_y' from the extreme compression fiber yields ($f_s' = f_y$), and the depth of the equivalent stress block, a, can be obtained by satisfying force equilibrium (that is, by setting $T = C + C_s'$ and solving for a):

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b} \tag{6.19}$$

The nominal flexural strength in this case can be determined by the following equation:

$$M_n = (A_s - A_s')f_y\left(d - \frac{a}{2}\right) + A_s'f_y(d - d') \tag{6.20}$$

Nominal Flexural Strength When the Compression Reinforcement Does Not Yield. When the compression reinforcement does not yield $(f_s' < f_y)$, the depth of the stress block, a, cannot be determined by Equation (6.19) because the magnitude of f_s' is unknown. A relationship between f_s' and c can be obtained from strain compatibility:

$$\frac{c}{0.003} = \frac{c - d'}{\varepsilon_s'} \tag{6.21}$$

Substituting $\varepsilon_s' = f_s' / E_s$ into Equation (6.21) and solving for f_s' results in the following:

$$f_s' = 0.003E_s\left(1 - \frac{d'}{c}\right) \tag{6.22}$$

From force equilibrium, c can be obtained from the following equation:

$$c = \frac{-b_1 \pm \sqrt{b_1^2 + 348a_1A_s'd'}}{2a_1} \tag{6.23}$$

where $a_1 = 0.85f_c'b\beta_1$ and $b_1 = 87A_s' - A_sf_y$. Note that f_c' and f_y have the units of kips per square inch in the preceding equations.

The nominal flexural strength can be determined by the following equation:

$$M_n = 0.85f_c'ab\left(d - \frac{a}{2}\right) + A_s'f_s'(d - d') \tag{6.24}$$

where $a = \beta_1 c$ and the terms c and f_s' are determined by Equations (6.23) and (6.22), respectively.

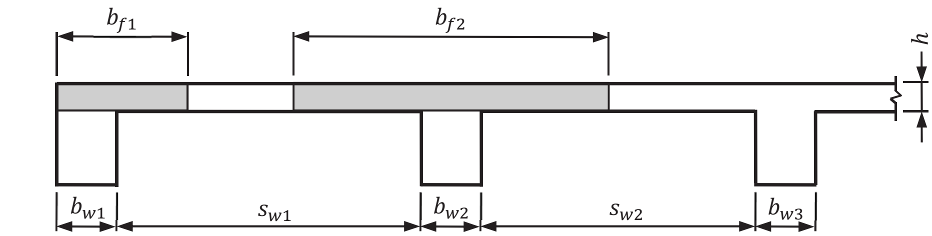
T-Beams and Inverted L-Beams with Tension Reinforcement

Effective Flange Width. In typical reinforced concrete construction, the beams and slabs are cast together with reinforcement extending between the members, which results in a monolithic structure. Therefore, the beams do not act alone but rather work with a portion of the slab to resist the effects from the applied loads.

In general, the effective flange width, b_f, of a beam (that is, the portion of the slab that works with the beam) depends on the clear distance between adjoining beam webs (s_w), slab thickness (h), and clear span length of the beam (ℓ_n). Dimensional limits for b_f for nonprestressed beams supporting monolithic slabs on both sides of the beam web (interior beams or T-beams) and on one side of the beam web (edge beams or inverted L-beams) are given in ACI 6.3.2.1 (see Figure 6.10).

Nominal Flexural Strength – Flange in Tension. The flange of a T-beam or an inverted L-beam is in tension at locations of negative moment in a continuous system (that is, at the faces of the supports). The strain and stress distributions for a T-beam in this case are illustrated in Figure 6.11. An inverted L-beam has similar distributions.

The nominal flexural strength in this case is determined by Equation (6.11) for a rectangular section with a single layer of reinforcement using $b = b_w$. If needed, the contribution of the positive reinforcement in the web (not shown in Figure 6.11) can be included and M_n can be determined using Equation (6.20) or (6.24).



$$b_{f1} = b_{w1} + \text{least of} \begin{cases} 6h \\ s_{w1}/2 \\ \ell_{n1}/12 \end{cases} \qquad b_{f2} = b_{w2} + \text{least of} \begin{cases} 16h \\ (s_{w1}/2) + (s_{w2}/2) \\ \ell_{n2}/4 \end{cases}$$

Figure 6.10 Effective flange widths for T-beams and inverted L-beams.

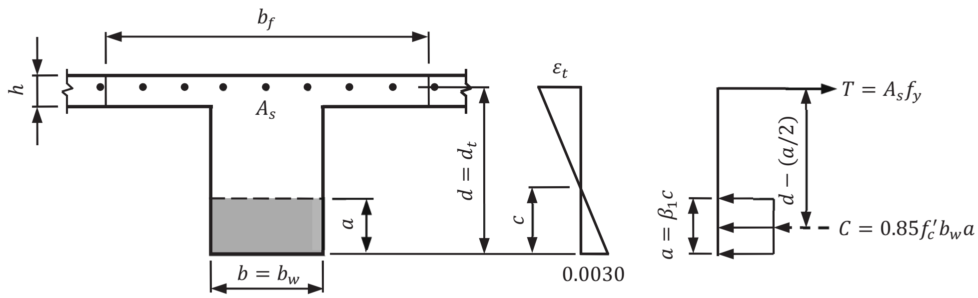


Figure 6.11 Strain and stress distributions in a T-beam with the flange in tension.

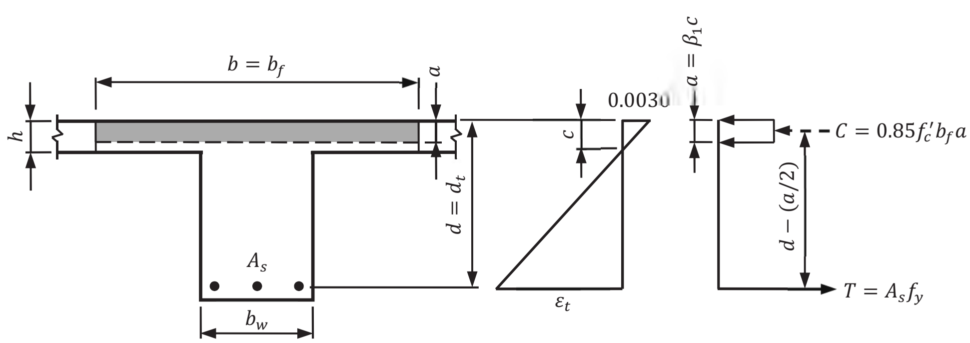


Figure 6.12 Strain and stress distributions in a T-beam with the flange in compression and $a \leq h$.

Nominal Flexural Strength – Flange in Compression. At locations of positive moment in a continuous system, a portion of the flange or the entire flange of a T-beam or inverted L-beam is in compression. The determination of M_n depends on whether the depth of the equivalent stress block, a, is less than or greater than the flange thickness, h.

Depth of Stress Block Less Than or Equal to the Flange Thickness $(a \leq h)$. The depth of the equivalent stress block, a, is initially determined by Equation (6.12) for sections with one layer of reinforcement or by Equation (6.19) for doubly reinforced sections. If it is found that $a \leq h$, the compressive zone is rectangular with a width equal to b_f (see Figure 6.12). Equation (6.11) can be used to determine M_n for sections with one layer of reinforcement. Similarly, Equation (6.20) or (6.24) can be used if compression reinforcement is included.

Depth of Stress Block Greater Than the Flange Thickness $(a > h)$. Where a determined by Equation (6.12) or (6.19) falls within the web, the compressive zone is T- or L-shaped as opposed to rectangular (see Figure 6.13 for a T-beam). The nominal flexural strength of the section in this case can be determined by the following equation:

$$M_n = A_{sf} f_y \left(d - \frac{h}{2} \right) + \left(A_s - A_{sf} \right) f_y \left(d - \frac{a}{2} \right) \tag{6.25}$$

where $A_{sf} = 0.85 f_c'(b_f - b_w)h / f_y$ and $a = (A_s - A_{sf}) f_y / 0.85 f_c' b_w$.

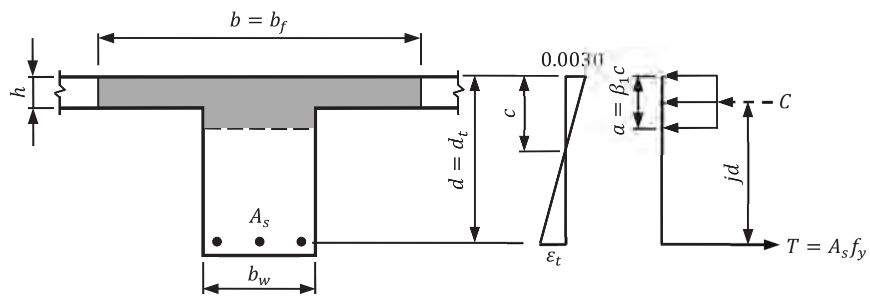


Figure 6.13 Strain and stress distribution in a T-beam with the flange in compression and $a>h$.

6.4.3 Nominal Shear Strength

Overview

The nominal one-way shear strength, V_n, at a section is determined in accordance with ACI 22.5 (ACI 9.5.3.1):

$$V_n = V_c + V_s \tag{6.26}$$

where V_c is the nominal shear strength provided by concrete and V_s is the nominal shear strength provided by shear reinforcement.

Nominal Shear Strength Provided by Concrete

For nonprestressed members, V_c is determined by the applicable equation in ACI Table 22.5.5.1 (ACI 22.5.5.1). The equations in that table are given in Table 6.4 along with the maximum permitted V_c specified in ACI 22.5.5.1.1.

Table 6.4 Nominal Shear Strength Provided by Concrete, V_c

Area of Shear Reinforcement, A_v*	V_c**
$\geq A_{v,min}$	Either of $\left\{ \begin{array}{l} \left(2\lambda\sqrt{f_c'} + \frac{N_u}{6A_g} \right) b_w d \\ \left[8\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g} \right] b_w d \end{array} \right. \leq 5\lambda\sqrt{f_c'} b_w d$
$< A_{v,min}$	$\left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g} \right] b_w d \leq 5\lambda\sqrt{f_c'} b_w d$

**Minimum area of shear reinforcement, $A_{v,min}$, is determined by ACI Table 9.6.3.4*
$^{**}N_u / 6A_g \leq 0.05 f_c'$

The minimum area of shear reinforcement in a beam, $A_{v,min}$, is determined in accordance with ACI 9.6.3.4 (see Section 6.5.2 of this publication). The required area of shear reinforcement, A_v, is often greater than or equal to $A_{v,min}$ at the critical section.

The modification factor, λ, that reflects the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength is given in Table 6.2 based on equilibrium density and in Table 6.3 based on composition of aggregates in the concrete mix.

The size effect modification factor, λ_s, accounts for the phenomenon indicated in test results that the shear strength attributed to concrete in members without shear reinforcement does not increase in direct proportion with member depth. This factor is determined by ACI Equation (22.5.5.1.3) [ACI 22.5.5.1.3]:

$$\lambda_s = \sqrt{\frac{2}{1+(d/10)}} \leq 1.0 \tag{6.27}$$

It is evident from Equation (6.27) that λ_s is less than 1.0 for members with $d > 10.0$ in. This means $\lambda_s = 1.0$ for beams with an overall depth less than about 12.5 in. As noted above, because A_v is often greater than or equal to $A_{v,min}$, this factor is usually not applicable when determining V_c for beams.

The term ρ_w is equal to the area of flexural reinforcement, A_s, at the section divided by $b_w d$ where b_w is the width of the web of the beam. According to ACI R22.5.5.1, A_s may be taken as the sum of the areas of the longitudinal flexural reinforcement located more than two-thirds of the overall member depth away from the extreme compression fiber. For members with one layer of tension reinforcement, A_s is the total area of flexural reinforcement at that section.

According to the first Note in ACI Table 22.5.5.1, the axial force, N_u, which has the units of pounds, is to be taken as positive for compression forces acting on the gross area of the beam, A_g, and is to be taken as negative for tension forces.

Values of $\sqrt{f_c'}$ used to calculate V_c are limited to 100 psi (ACI 22.5.3.1), except as allowed in ACI 22.5.3.2 (which permits values of $\sqrt{f_c'}$ to be greater than 100 psi provided the minimum shear reinforcement in ACI 9.6.3.4 or the minimum transverse torsional reinforcement in ACI 9.6.4.2, if applicable, is provided in the section). This limitation on f_c' is primarily due to the fact that there is a lack of test data and practical experience with concrete having compressive strengths greater than 10,000 psi. For economy, f_c' for beams should not exceed 5,000 psi (Reference 7).

To minimize the likelihood of diagonal compression failure in the concrete and to limit the extent of cracking, the cross-sectional dimensions of a section must be selected to satisfy ACI Equation (22.5.1.2) [ACI 22.5.1.2]:

$$V_u \leq \phi\left(V_c + 8\sqrt{f_c'}b_w d\right) \tag{6.28}$$

In typical situations, beams are subjected to shear forces acting along one axis only, so the requirements in ACI 22.5.1.10 and 22.5.1.11 for interaction of shear forces along orthogonal axes are usually not applicable.

Nominal Shear Strength Provided by Shear Reinforcement

The nominal shear strength provided by shear reinforcement, V_s, is to be determined by ACI 22.5.8 (ACI 22.5.1.6). The following types of shear reinforcement are permitted in nonprestressed members:

(1) Stirrups, ties, or hoops perpendicular to the longitudinal axis of the member [ACI 22.5.8.5.1(a)]

(2) Welded wire reinforcement with wires located perpendicular to the longitudinal axis of the member [ACI 22.5.8.5.1(b)]

(3) Spiral reinforcement [ACI 22.5.8.5.1(c)]

(4) Inclined stirrups making an angle of at least 45 degrees with the longitudinal axis of the member and crossing the plane of the potential shear crack (ACI 22.5.8.5.2)

(5) Longitudinal reinforcement bent an angle of 30 degrees or more with respect to the longitudinal axis of the member (ACI 22.5.8.6)

Stirrups oriented perpendicular to the axis of the member and anchored to the longitudinal flexural reinforcement are the most commonly used type of shear reinforcement for beams in buildings assigned to SDC A or B. In buildings assigned to SDC C and above, spirals or hoops in accordance with the provisions in Chapter 18 of ACI 318 must be provided (see Chapters 12 and 14 of this publication). Inclined stirrups and bent longitudinal bars are essentially not used any longer and, thus, are not covered here.

The nominal shear strength provided by stirrups is determined by ACI Equation (22.5.8.5.3) [ACI 22.5.8.5.3]:

$$V_s = \frac{A_v f_{yt} d}{s} \tag{6.29}$$

In this equation, A_v is the total area of shear reinforcement within the center-to-center spacing, s, of the stirrups. For the two-legged stirrups depicted in Figure 6.14 (which are commonly referred to U-stirrups because of their shape), $A_v = 2A_b$ where A_b is the area of the stirrup bar. In general, A_v is equal to A_b times the number of stirrup legs provided (ACI 22.5.8.5.5).

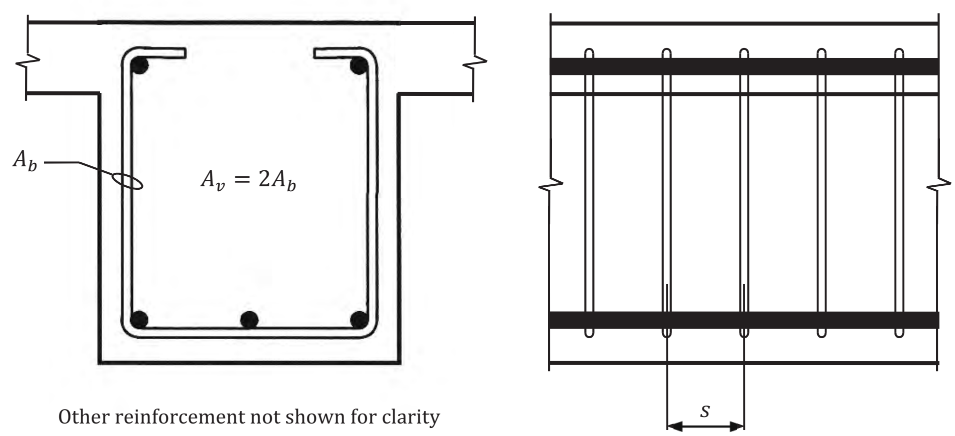


Figure 6.14 Two-legged U-stirrup configuration.

Locating stirrups solely around the perimeter of a wide beam is not fully effective. Thus, stirrup legs are required in the interior of a wide beam. The maximum stirrup spacing across the width of a member is given in ACI Table 9.7.6.2.2; this requirement essentially translates to a minimum number of stirrup legs needed across the width of a beam. In such cases, shear reinforcement usually consists of sets of U-stirrups. For the configuration illustrated in Figure 6.15, a single, open stirrup extends the full net width of the beam (the net width is equal to the full beam width minus the total side cover). A stirrup cap consisting of a horizontal bar with a 135-degree hook at one end and a 90-degree hook at the other end is provided at the top of the configuration, which also extends the full net width of the beam. Providing a full-width stirrup helps in maintaining the required concrete cover and facilitates installation of the longitudinal reinforcement in the beam. Two sets of identical U-stirrups with 135-degree hooks at each end are placed symmetrically within the interior of the beam. In this configuration, $A_v = 6A_b$.

In relatively narrow beams, a continuous bent bar, which forms a series of single-leg stirrups along the length of the member, can be used as shear reinforcement where required (ACI R25.7.1.3; see Figure 6.16). ACI 25.7.1.3(c) permits #3 and #4 bars to be used as shear reinforcement in standard joist construction (which is defined in ACI 9.8) where the bar is anchored by a standard hook, which is not necessarily engaging any of the longitudinal rein-

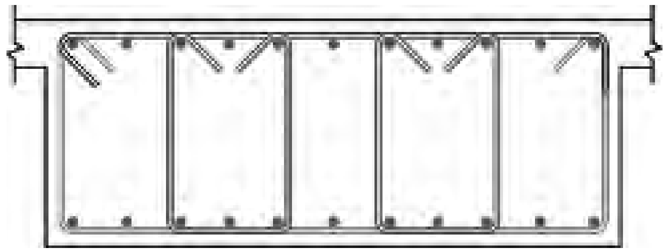


Figure 6.15 Multiple-leg stirrup configuration for wide beams.

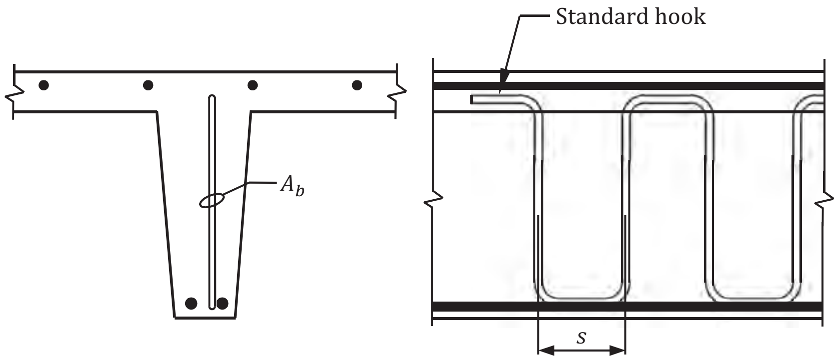


Figure 6.16 Single-leg stirrup configuration for joists and narrow beams.

forcement. Although this type of shear reinforcement is permitted specifically for standard joist construction, it is assumed that it can be used in joists that are part of a wide-module joist system or for any narrow beam where U-stirrups cannot be used. In this configuration, $A_v = A_b$.

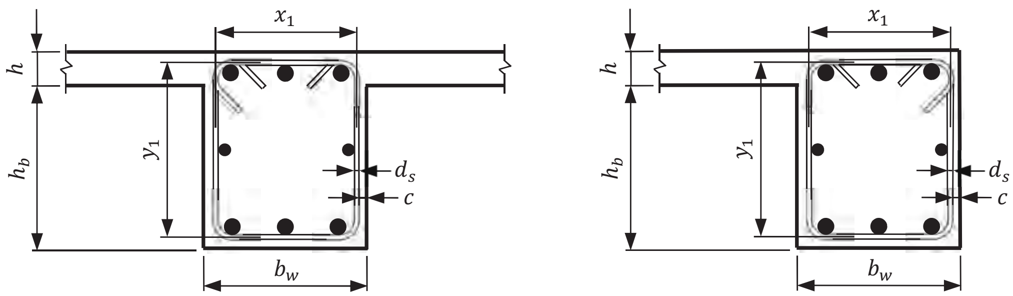
6.4.4 Nominal Torsional Strength

Where torsional effects must be considered $(T_u \geq \phi T_{th})$, nominal torsional strength T_n is determined in accordance with ACI 22.7 (ACI 9.5.4.2). For nonprestressed members, T_n is equal to the following (ACI 22.7.6.1):

$$T_n = \text{lesser of} \begin{cases} \dfrac{2A_o A_t f_{yt}}{s} \cot\theta \\[2ex] \dfrac{2A_o A_\ell f_y}{p_h} \tan\theta \end{cases} \tag{6.30}$$

The term A_o is the gross area of the beam cross-section enclosed by the torsional shear flow path after cracking. In lieu of determining it by analysis, A_o is permitted to be approximated as $0.85A_{oh}$ where A_{oh} is the area enclosed by the centerline of the outmost closed transverse torsional reinforcement in the section (see ACI 22.7.6.1.1 and ACI Figure R22.7.6.1.1). For the interior and edge beams in Figure 6.17, $A_{oh} = x_1 y_1$ where x_1 and y_1 are the distances between the centerlines of the outmost closed stirrups in the section. Closed stirrups must be used because cracking due to torsional moments can occur on all faces of a beam. The perimeter of the centerline of the outermost closed transverse torsional reinforcement, p_h, is also given in Figure 6.17 for solid sections. Note that the definitions of A_{oh} and p_{cp} in Figure 6.17 are the same for hollow sections.

The term A_t is the area of one leg of the outermost closed stirrups, which have a center-to-center spacing of s along the length of the member. Regardless of the total number of stirrup legs, only the outermost legs resist torsional effects; this is consistent with the thin-walled tube methodology that forms the basis of the torsional provisions in ACI 22.7.



d_s = diameter of closed stirrup

c = clear cover to closed stirrup

Interior Beam **Edge Beam**

$$x_1 = b_w - 2c - d_s$$

$$y_1 = h_b + h - 2c - d_s$$

$$A_{oh} = x_1 y_1$$

$$p_h = 2(x_1 + y_1)$$

Figure 6.17 Torsional section properties A_{oh} and p_h.

The total area of longitudinal reinforcement required to resist the effects of torsion, A_ℓ, must be uniformly distributed around the perimeter of the beam and is combined with the longitudinal reinforcement required for flexure (see Section 6.5.4 of this publication).

The angle θ is related to the orientation of the compression diagonals that are part of the space truss assumed to resist the effects from torsion after cracking (the transverse and longitudinal reinforcement are the other parts of the truss; see Figure 6.18). ACI 22.7.6.1.2(a) permits θ to be taken as 45 degrees for nonprestressed members instead of determining it from analysis.

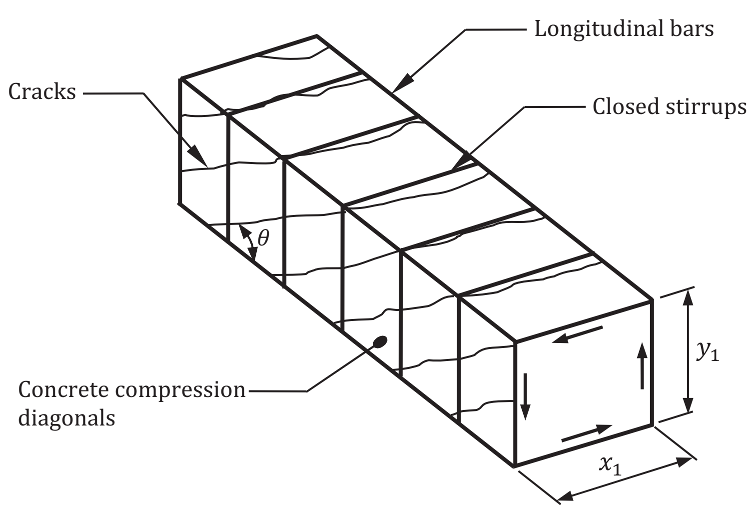


Figure 6.18 Space truss analogy for a reinforced concrete beam subjected to torsional loading.

In order to help reduce unsightly cracking and to prevent crushing of the inclined concrete compression struts due to shear and torsion, the cross-sectional dimensions of solid and hollow sections are limited by ACI Equations (22.7.7.1a) and (27.7.7.1b), respectively (ACI 22.7.7.1):

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \tag{6.31}$$

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \tag{6.32}$$

The terms on the left-hand side of these equations are the stresses due to shear and torsion, respectively.

In solid sections, shear stresses act over the full width of the section, while those due to torsion are resisted by a thin-walled tube [see ACI Figure R22.7.7.1(b)]. Thus, the stresses cannot be directly added together. Equation (6.31) represents an elliptical interaction between shear and torsional stresses.

For hollow sections, the shear and torsional stresses are directly additive on one side wall of the section [see ACI Figure R22.7.7.1(a)]. This linear interaction of stresses is evident in Equation (6.32). For hollow sections where the wall thickness varies around the perimeter of the section, Equation (6.32) must be evaluated where the summation of the shear and torsional stresses on the left-hand side of the equation is a maximum (ACI 22.7.7.1.2). Also, where the wall thickness of a hollow section is less than A_{oh} / p_h, the torsional stress $T_u p_h / 1.7A_{oh}^2$ in Equation (6.32) must be taken as $T_u / 1.7A_{oh}t$ where t is the thickness of the wall of the hollow section at the location where stresses are being evaluated (ACI 22.7.7.2).

The dimensions of the cross-section must be increased where Equation (6.31) or (6.32) is not satisfied. Increasing the cross-sectional dimensions of a section typically has a greater impact than increasing f_c'.

Solid sections with an aspect ratio $h / b_t \geq 3$ (where b_t is the width of that part of the cross-section containing the closed stirrups provided to resist torsion) are permitted to be designed by a procedure other than that given in ACI 318 provided the procedure has demonstrated that the results are in substantial agreement with results from comprehensive tests (ACI 9.5.4.6). In such cases, the minimum reinforcement requirements of ACI 9.6.4 and the detailing requirements of ACI 9.7.5 and 9.7.6.3 need not be satisfied. One such design procedure is referenced in ACI R9.5.4.7.

6.5 Determination of Required Reinforcement

6.5.1 Required Flexural Reinforcement

Rectangular Sections with Tension Reinforcement Only

Single Layer of Tension Reinforcement. The required area of flexural reinforcement, A_s, at a critical section of a beam with a single layer of tension reinforcement is determined based on the required and design flexural strengths M_u and ϕM_n for tension-controlled sections. Using Equations (6.7) and (6.11), the required A_s can be determined by the following equation:

$$A_s = \frac{0.85 f_c' bd}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] \tag{6.33}$$

The nominal strength coefficient of resistance, R_n, is defined as follows:

$$R_n = \frac{M_u}{\phi b d^2} \tag{6.34}$$

where $\phi = 0.90$ for tension-controlled sections.

It is important to check that A_s calculated by Equation (6.33) is at least equal to the following minimum area of flexural reinforcement, $A_{s,min}$ (ACI 9.6.1.2):

$$A_{s,min} = \text{larger of} \begin{cases} \dfrac{3\sqrt{f'_c}b_w d}{f_y} \\ \\ \dfrac{200 b_w d}{f_y} \end{cases} \tag{6.35}$$

For 4,000-psi concrete and Grade 60 reinforcement, $200b_w d / f_y$ governs. The value of f_y in Equation (6.35) is limited to a maximum of 80,000 psi.

Minimum flexural reinforcement determined by Equation (6.35) need not be satisfied at any section where at least an area of reinforcement equal to $1.3A_s$ is provided where A_s is calculated by Equation (6.33) [ACI 9.6.1.3].

It is also important to verify the section is tension-controlled. The following relationship is obtained from the linear strain distribution where c_t is the depth of the neutral axis when $\varepsilon_t = \varepsilon_{ty} + 0.003$ for tension-controlled sections (see ACI 22.2.1.2 and Figure 6.7):

$$\frac{c_t}{d} = \frac{0.003}{\varepsilon_t + 0.003} = \frac{0.003}{\varepsilon_{ty} + 0.006} \tag{6.36}$$

Substituting $a_t = \beta_1 c_t$ into Equation (6.12) with c_t from Equation (6.36), the following equation can be used to calculate the area of flexural reinforcement, $A_{s,t}$, corresponding to tension-controlled sections for beams with $\varepsilon_t = \varepsilon_{ty} + 0.003$:

$$A_{s,t} = \frac{0.85\beta_1 f'_c bd}{f_y}\left(\frac{0.003}{\varepsilon_{ty} + 0.006}\right) \tag{6.37}$$

For Grade 60 reinforcement, $\varepsilon_{ty} = 60 / 29{,}000 = 0.00207$, and for $f'_c = 4{,}000$ psi, $\beta_1 = 0.85$. For these material properties, Equation (6.37) reduces to the following:

$$A_{s,t} = 0.018bd \tag{6.38}$$

If A_s calculated by Equation (6.33) is greater than $A_{s,t}$, which is essentially the maximum amount of flexural reinforcement permitted at any section, the section is not tension-controlled $(\phi < 0.9)$, and the dimensions of the beam must be modified accordingly to attain a tension-controlled section.

Multiple Layers of Tension Reinforcement. The required A_s is determined in the same way as for sections with one layer of tension reinforcement using d and d_t (see Figure 6.8). For beams with two layers of tension reinforcement, it is conservative to determine A_s using $d_t = d$. However, it is permitted to use d_t instead of the smaller d; this may be advantageous in cases where the section is at or very close to the tension-controlled strain limit, ε_t.

The reinforcement ratio $\rho = A_s / bd$ based on d is equal to the following considering force equilibrium on the section ($T = A_s f_y = \rho bd f_y = C$; see Figure 6.8):

$$\rho = \frac{C}{bdf_y} \tag{6.39}$$

Similarly, the reinforcement ratio $\rho_t = A_{s,t} / bd$ corresponding to a tension-controlled section based on d_t is equal to the following:

$$\rho_t = \frac{C}{bd_t f_y} \tag{6.40}$$

The following relationship between ρ and ρ_t is obtained from Equations (6.39) and (6.40):

$$\rho = \left(\frac{d_t}{d}\right)\rho_t \tag{6.41}$$

The preceding discussion assumes the inner layer of reinforcement yields, which is advantageous for design purposes. Reinforcement located a distance equal to or greater than d_{ty} from the extreme compression fiber yields where d_{ty} is calculated by Equation (6.14) or by Equation (6.15) for Grade 60 reinforcement (see Section 6.4.2 of this publication). Unless reinforcement is provided on the sides of a beam, it is common for all layers of reinforcement to yield in normally proportioned beams.

Rectangular Sections with Tension and Compression Reinforcement

For rectangular sections where it has been determined that the section is not tension-controlled (that is, $\varepsilon_t < \varepsilon_{ty} + 0.003$), compression reinforcement may be added to the section so that it becomes tension-controlled (see Section 6.4.2 of this publication and Figure 6.9).

A common design method in this situation is to set $\varepsilon_t = \varepsilon_{ty} + 0.003$, which means the section is designed at the tension-controlled net tensile strain limit. The next step is to determine the nominal flexural strength, $M_{n,t}$, corresponding to $\varepsilon_t = \varepsilon_{ty} + 0.003$ using Equation (6.11):

$$M_{n,t} = A_s f_y\left(d - \frac{a}{2}\right) = \rho f_y\left(d - \frac{0.59\rho f_y}{f'_c}\right)bd^2 \tag{6.42}$$

where $a = \rho d f_y / 0.85 f'_c$.

For a section with a single layer of tension reinforcement, the reinforcement ratio to be used in Equation (6.42) is the following:

$$\rho = \rho_t = \frac{A_{s,t}}{bd} = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{0.003}{\varepsilon_{ty} + 0.006}\right) \tag{6.43}$$

where $A_{s,t}$ is determined by Equation (6.37).

In situations where compression reinforcement is required, it is common for two layers of tension reinforcement to be needed. In this case, the reinforcement ratio to be used in Equation (6.42) is the following [see Equation (6.41)]:

$$\rho = \rho_t\left(\frac{d_t}{d}\right) = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{0.003}{\varepsilon_{ty} + 0.006}\right)\left(\frac{d_t}{d}\right) \tag{6.44}$$

The required nominal flexural strength resisted by the compression reinforcement, M'_n, is the difference between the required nominal flexural strength $(M_n = M_u / \phi)$ and $M_{n,t}$:

$$M'_n = \frac{M_u}{\phi} - M_{n,t} \tag{6.45}$$

where $\phi = 0.9$.

In general, the stress in the compression reinforcement, f_s', can be determined by the following equation:

$$f_s' = 0.003E_s\left(1 - \frac{d'}{c_t}\right) \le f_y \tag{6.46}$$

where $c_t = 0.003d / \varepsilon_t = 0.003d / (\varepsilon_{ty} + 0.006)$. Compression reinforcement located a distance equal to or less than d_y' from the extreme compression fiber yields where d_y' is determined by Equation (6.17) or (6.18) [see Section 6.4.2 of this publication].

The required area of compression reinforcement, A_s', is determined from strength design:

$$A_s' = \frac{M_n'}{f_s'(d - d')} \tag{6.47}$$

Finally, the total required area of tension reinforcement, A_s, is the summation of the area of reinforcement corresponding to the tension-controlled strain limit and A_s' assuming $f_s' = f_y$:

$$A_s = \rho bd + \frac{M_n'}{f_y(d - d')} \tag{6.48}$$

where ρ is determined by Equation (6.43) or (6.44) for a single layer or two layers of tension reinforcement, respectively, and M_n' is determined by Equation (6.45).

T-Beams and Inverted L-Beams with Tension Reinforcement

Nominal Flexural Strength – Flange in Tension. The flange of a T-beam or an inverted L-beam is in tension at locations of negative moment in a continuous system (see Section 6.4.2 of this publication).

Once the effective flange width, b_f, is determined (see Figure 6.10), the required area of flexural reinforcement, A_s, is calculated using Equation (6.33) for a rectangular section with a single layer of reinforcement (see Figure 6.11). If needed, the contribution of compression reinforcement in the web can be included where A_s and A_s' are determined by Equations (6.48) and (6.47), respectively.

The minimum reinforcement requirements in ACI 9.6.1.2 are also applicable [see Equation (6.35)].

Nominal Flexural Strength – Flange in Compression. At locations of positive moment, a portion of the flange or the entire flange of a T-beam or inverted L-beam is in compression. Assuming the section is tension-controlled with rectangular section behavior (that is, $a \le h$), a can be determined by the following equation (see Figure 6.12):

$$a = \frac{a_1 d - \sqrt{(a_1 d)^2 - (2a_1 M_u / \phi)}}{a_1} \tag{6.49}$$

where $a_1 = 0.85 f_c' b_f$ and $\phi = 0.9$.

If a determined by Equation (6.49) is less than or equal to h, the initial assumption that the section behaves as a rectangular section is correct and A_s can be determined by Equation (6.33).

If it is found that $a > h$, the initial assumption of rectangular behavior is not correct, and the compressive zone is T- or L-shaped (see Figure 6.13). In such cases, the next step is to determine the area of reinforcement, A_{sf}, and the design strength, ϕM_{n1}, corresponding to the overhanging flange(s):

$$A_{sf} = \frac{0.85 f_c'(b_f - b_w)h}{f_y} \tag{6.50}$$

$$M_{n1} = A_{sf} f_y \left(d - \frac{h}{2} \right) \tag{6.51}$$

The moment strength of the beam web, M_{u2}, is determined by subtracting the design strength ϕM_{n1} from M_u at the section:

$$M_{u2} = M_u - \phi M_{n1} \tag{6.52}$$

The area of flexural reinforcement, A_{sw}, required to resist M_{u2} is determined by Equations (6.34) and (6.33) where $M_u = M_{u2}$ and $b = b_w$.

The total required area of flexural reinforcement is the sum of the areas corresponding to the contributions from the overhanging flange(s) and the web:

$$A_s = A_{sf} + A_{sw} \tag{6.53}$$

Once A_s is determined by Equation (6.53), the assumption that the section is tension-controlled needs to be checked, that is, it must be verified that $A_{sw} \le A_{s,t}$ where $A_{s,t}$ is determined by Equation (6.37) or (6.38) with $b = b_w$. If the section is not tension-controlled, compression reinforcement may be added to the section to make it tension-controlled or the dimensions of the beam must be modified.

The minimum reinforcement requirements in ACI 9.6.1.2 are also applicable [see Equation (6.35)].

6.5.2 Required Shear Reinforcement

The required area A_v and spacing s of shear reinforcement perpendicular to the axis of a member is determined by Equation (6.8), which can be rewritten as follows:

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi f_{yt} d} \tag{6.54}$$

The nominal shear strength provided by the concrete, V_c, is obtained from Table 6.4 and the nominal shear strength provided by the shear reinforcement (stirrups), V_s, is calculated by Equation (6.29).

A summary of the shear requirements in ACI 9.6.3, 9.7.6, and 22.5 is given in Table 6.5 assuming the effects from any torsional moments, if present, need not be considered. Note that these requirements are not applicable to the beam types in ACI Table 9.6.3.1, which are cases where the minimum shear reinforcement requirements of ACI 9.6.3.4 need not be satisfied if $V_u \le \phi V_c$. Also, "Along the length" in Table 6.5 refers to the maximum spacing of legs of shear reinforcement along the length of the member and "Across the width" refers to the maximum spacing of legs of shear reinforcement across the width of the member (see Figure 6.19).

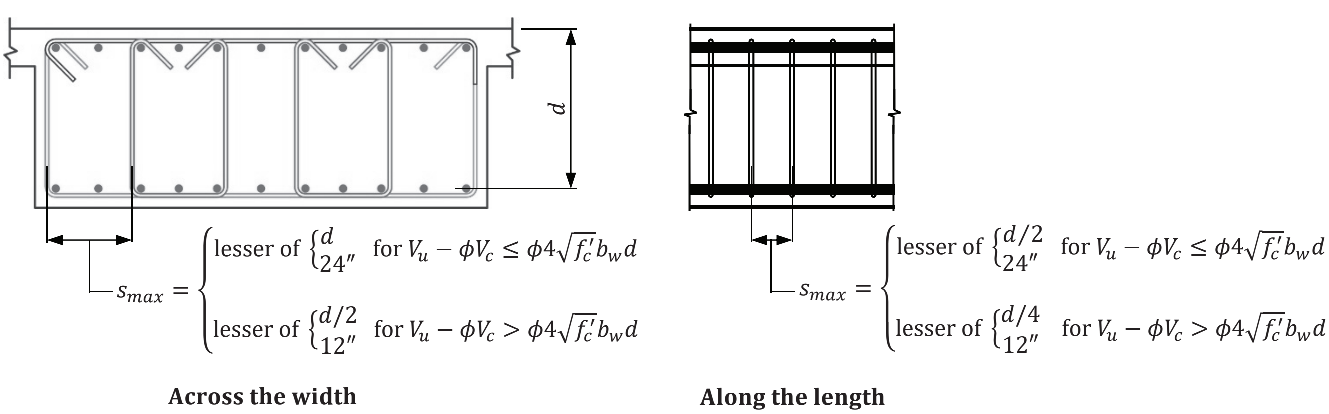


Figure 6.19 Maximum spacing requirements for legs of shear reinforcement.

Table 6.5 Shear Reinforcement Requirements for Beams*

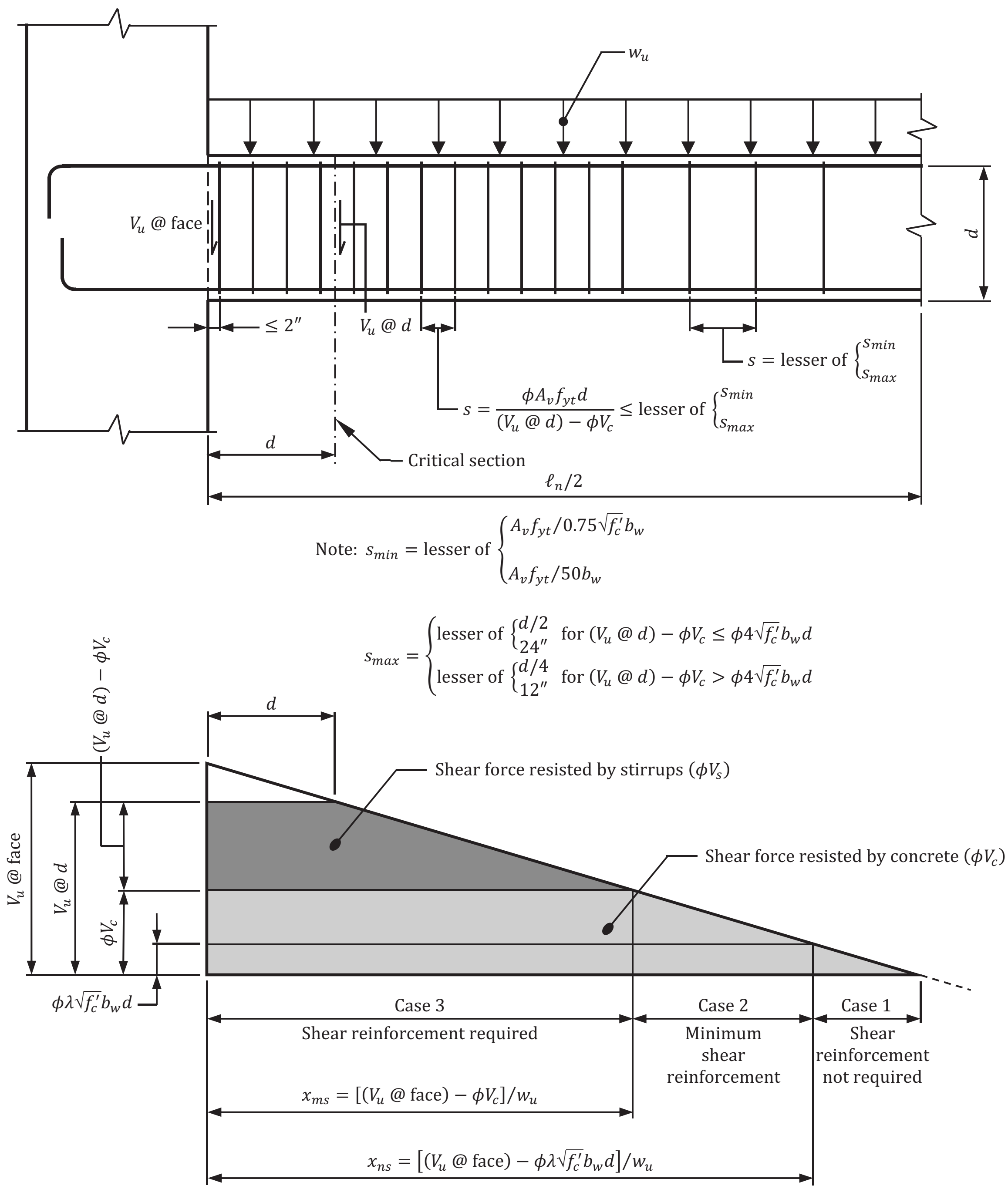
Case	Required Shear Strength, V_u	Required Shear Reinforcement, A_v	Stirrup spacing, s	
			Required	Maximum
1	$V_u \le \phi\lambda\sqrt{f'_c} b_w d$	None	—	—
2	$\phi\lambda\sqrt{f'_c} b_w d < V_u \le \phi V_c$	$A_{v,min} = \text{greater of} \begin{cases} \dfrac{0.75\sqrt{f'_c} b_w s}{f_{yt}} \\ \dfrac{50 b_w s}{f_{yt}} \end{cases}$	$s_{min} = \text{lesser of} \begin{cases} \dfrac{A_v f_{yt}}{0.75\sqrt{f'_c} b_w} \\ \dfrac{A_v f_{yt}}{50 b_w} \end{cases}$	• Along the length: $s_{max} = \dfrac{d}{2} \le 24 \text{ in.}$ • Across the width: $s_{max} = d \le 24 \text{ in.}$
3	$V_u > \phi V_c$	$A_v = \text{greater of} \begin{cases} \dfrac{(V_u - \phi V_c)s}{\phi f_{yt} d} \\ A_{v,min} \end{cases}$	$s = \text{lesser of} \begin{cases} \dfrac{\phi A_v f_{yt} d}{V_u - \phi V_c} \\ s_{min} \end{cases}$	• For $V_u - \phi V_c \le \phi 4\sqrt{f'_c} b_w d$: – Along the length: $s_{max} = \dfrac{d}{2} \le 24 \text{ in.}$ – Across the width: $s_{max} = d \le 24 \text{ in.}$ • For $V_u - \phi V_c > \phi 4\sqrt{f'_c} b_w d$: – Along the length: $s_{max} = \dfrac{d}{4} \le 12 \text{ in.}$ – Across the width: $s_{max} = \dfrac{d}{2} \le 12 \text{ in.}$

**Minimum reinforcement requirements in this table are applicable in all cases except for those in ACI Table 9.6.3.1.*

Prior to calculating the required shear reinforcement, it is important to check that Equation (6.28) is satisfied (ACI 22.5.1.2). The size of the section must be increased where $V_u > \phi(V_c + 8\sqrt{f'_c} b_w d)$.

For economy, stirrup size and spacing (along the length and across the width of the member) are established at the critical section first. At sections away from the critical section, the spacing can be increased, but the stirrup bar size and number of legs should be the same as those at the critical section. Stirrup spacing should be changed as few times as possible over the required length; it is good practice to provide no more than 3 different stirrup spacings where required (although 2 different spacings are preferred, if possible or practical) with the first stirrup located no more than 2 in. from the face of support. Larger stirrup bars at wider spacing are usually more cost-effective than smaller bars at closer spacing; the latter require disproportionally higher costs for fabrication and placement.

The segments along the span where shear reinforcement is and is not required based on the three cases in Table 6.5 are identified in Figure 6.20 for a reinforced concrete beam subjected to a factored uniformly distributed gravity load, w_u. In this case, two stirrup spacings are provided along the length of the member assuming the three requirements in ACI 9.4.3.2 pertaining to the location of the critical section for shear are satisfied: (1) the spacing s corresponds to the factored shear force at the critical section located a distance d from the face of the support (V_u @ d), and (2) the spacing s_{min} corresponds to that based on minimum shear reinforcement ($A_{v,min}$).



Case	V_u	Shear Reinforcement
1	$V_u \leq \phi\lambda\sqrt{f_c'}b_w d$	Not required
2	$\phi\lambda\sqrt{f_c'}b_w d < V_u \leq \phi V_c$	$A_{v,min} =$ greater of $\begin{cases} 0.75\sqrt{f_c'}b_w s/f_{yt} \\ 50b_w s/f_{yt} \end{cases}$
3	$(V_u @ d) > \phi V_c$	$A_v = \dfrac{[(V_u @ d) - \phi V_c]s}{\phi f_{yt} d} \geq A_{v,min}$

Figure 6.20 Segments along the span of a reinforced concrete beam where shear reinforcement is and is not required.

To facilitate determination of the required size and spacing of the shear reinforcement, values of $V_u - \phi V_c$ are given in Table 6.6 using the equations in Table 6.5 for stirrup spacings $s = d / n_s$ where n_s is taken as 2, 3, and 4 (which provides a practical range for s). These values are based on Grade 60 stirrups with 2 legs and are independent of member size and concrete compressive strength. For stirrup configurations with more than 2 legs, the tabulated values can be increased accordingly (for example, for a stirrup configuration with 4 legs, the corresponding value of $V_u - \phi V_c$ is equal to two times the tabulated value). A stirrup size and spacing is selected from Table 6.6 that provides a value of $V_u - \phi V_c$ equal to or slightly greater than that required from analysis. For example, assume from analysis the required $V_u - \phi V_c = 30 \text{ kips}$. From Table 6.6, #4 U-stirrups (2 legs) spaced at $d / 2$ provides $V_u - \phi V_c = 36 \text{ kips} > 30 \text{ kips}$ where d is the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement in the beam.

Table 6.6 Values of $V_u - \phi V_c$ for Grade 60 Stirrups*

Stirrup spacing, s	#3 U-stirrups	#4 U-stirrups	#5 U-stirrups
$d / 2$	19.8	36.0	55.8
$d / 3$	29.7	54.0	83.7
$d / 4$	39.6	72.0	111.6

**Tabulated values are for 2 legs of vertical shear reinforcement. Multiply the tabulated values by $(n_{s\ell} / 2)$ for stirrup arrangements with other than 2 legs where $n_{s\ell}$ is the total number of stirrup legs provided across the width of the member.*

The distance from the face of the support where minimum shear reinforcement can be used is determined by setting the factored shear force, V_u, equal to the design shear strength of the concrete, ϕV_c. The following equation is applicable for the beam in Figure 6.20:

$$(V_u \text{ @ face}) - w_u x_{ms} = \phi V_c \tag{6.55}$$

where x_{ms} is the distance from the face of the support to the section where minimum shear reinforcement can be used (that is, the distance to the onset of Case 2). Solving for x_{ms} results in the following:

$$x_{ms} = \frac{(V_u \text{ @ face}) - \phi V_c}{w_u} \tag{6.56}$$

Similarly, the distance, x_{ns}, from the face of the support to the section where shear reinforcement is no longer required (that is, the distance to the onset of Case 1) can be determined from the following equation:

$$x_{ns} = \frac{(V_u \text{ @ face}) - \phi\lambda\sqrt{f'_c}b_w d}{w_u} \tag{6.57}$$

In general, the following equation can be used to determine the distance, x, from the face of the support to the section where the stirrup configuration can be spaced at s along the span for a given area of shear reinforcement, A_v, where s is greater than that determined at the critical section:

$$x = \frac{(V_u \text{ @ face}) - \phi\left(\frac{A_v f_{yt} d}{s} + V_c\right)}{w_u} \tag{6.58}$$

It is important to ensure the value of s used in Equation (6.58) is less than or equal to the governing s_{min} and s_{max} values given in Table 6.5. For example, the distance from the face of the support where $s = d / 2$ can be used is determined as follows:

$$x_{(s=d/2)} = \frac{(V_u \text{ @ face}) - \phi\left(2A_v f_{yt} + V_c\right)}{w_u} \tag{6.59}$$

6.5.3 Required Torsion Reinforcement

Transverse Reinforcement

The required area A_t and spacing s of transverse torsional reinforcement perpendicular to the axis of a member are calculated by Equation (6.9) where T_n is determined by the first of the equations in Equation (6.30):

$$\frac{A_t}{s} = \frac{T_u}{2\phi \cot\theta A_o f_{yt}} \tag{6.60}$$

As noted previously, A_t is the area of one leg of the outermost closed stirrups, which have a center-to-center spacing of s along the length of the member. Regardless of the total number of stirrup legs across the width of the beam, only the outermost legs resist torsional effects.

The maximum spacing of transverse torsional reinforcement is equal to the lesser of $p_h / 8$ and 12 in. (ACI 9.7.6.3.3).

The minimum area of transverse reinforcement for the combined effects of shear and torsion is covered in Section 6.5.4 of this publication.

Longitudinal Reinforcement

The longitudinal reinforcement required for torsion, A_ℓ, is also calculated by Equation (6.9) where T_n is determined using both of the equations in Equation (6.30):

$$A_\ell = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right) \cot^2\theta \geq A_{\ell,min} \tag{6.61}$$

In this equation, A_t / s is calculated by Equation (6.60).

The minimum area of longitudinal reinforcement, $A_{\ell,min}$, is given in ACI 9.6.4.3:

$$A_{\ell,min} = \text{lesser of} \begin{cases} \dfrac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\dfrac{A_t}{s}\right) p_h \left(\dfrac{f_{yt}}{f_y}\right) \\[2ex] \dfrac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\dfrac{25b_w}{f_{yt}}\right) p_h \left(\dfrac{f_{yt}}{f_y}\right) \end{cases} \tag{6.62}$$

It is permitted to reduce A_ℓ in the flexural compression zones of a beam by an area equal to $M_u / 0.9df_y$ where M_u is the factored bending moment occurring at the section simultaneously with T_u (ACI 9.5.4.5). The reduced area of longitudinal reinforcement must not be taken less than $A_{\ell,min}$ determined by Equation (6.62).

6.5.4 Reinforcement Requirements for Combined Flexure, Shear, and Torsion

The amounts of transverse and longitudinal reinforcement required to resist the combined effects from bending moments, shear forces, and torsional moments are determined in accordance with ACI 9.5.4.3.

The total required area of transverse reinforcement per stirrup leg is equal to the required area of transverse reinforcement for shear [Equation (6.54)] plus the required area of transverse reinforcement for torsion [Equation (6.60)]. Where torsional reinforcement is required, the minimum amount of transverse reinforcement per leg is determined by the following equation (ACI 9.6.4.2):

$$\frac{A_v}{2s} + \frac{A_t}{s} \geq \text{ greater of } \begin{cases} 0.375\sqrt{f_c'}b_w / f_{yt} \\ \\ 25b_w / f_{yt} \end{cases} \tag{6.63}$$

Longitudinal torsional reinforcement, A_ℓ, is added to that required for flexure, A_s. If torsional reinforcement is required, A_ℓ must be distributed around the perimeter of the beam, so an area of steel equal to $A_\ell / 4$ is added to each face of the beam (ACI 9.7.5.1). Thus, at the top and bottom faces, the total required area of reinforcement is equal to $A_s^- + (A_\ell / 4)$ and $A_s^+ + (A_\ell / 4)$, respectively. Similarly, at least $A_\ell / 4$ must be provided on the side faces or must be added to any other longitudinal reinforcement on the side faces.

Appendix B in Reference 10 contains tables to facilitate torsion design for edge (spandrel) beams. Torsional section properties are given for common slab thicknesses and edge beam sizes. Also included are the corresponding values for threshold torsion, cracking torsion, and required transverse and longitudinal reinforcement. No calculations are required to obtain any of these items for a given beam size and slab thickness.

6.6 Reinforcement Detailing

6.6.1 Concrete Cover

Reinforcing bars are placed in beams with a minimum concrete cover to protect them from weather, fire, and other effects. Minimum cover requirements are given in ACI 20.5.1 (ACI 9.7.1.1). For beams with transverse reinforcement that enclose the longitudinal reinforcing bars, concrete cover is measured from the surface of the concrete to the outer edge of the transverse reinforcement (see Figure 6.21). The minimum cover to the transverse reinforcement is equal to 1.5 in. for reinforcing bars in beams not exposed to weather or in contact with the ground (ACI Table 20.5.1.3.1).

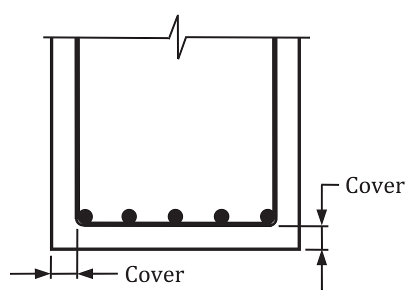


Figure 6.21 Concrete cover for beams.

Cover requirements for beams subjected to fatigue in structures designed to be watertight (such as tanks) or in structures subjected to very aggressive exposures (such as concrete exposed to moisture or external sources of chlorides) are given in ACI 24.3.5.

6.6.2 Flexural Reinforcement Spacing

Minimum Spacing of Flexural Reinforcing Bars

Minimum clear spacing of parallel reinforcing bars in a single horizontal layer is given in ACI 25.2 (ACI 9.7.2.1). These limits have been established primarily so concrete can flow readily into the spaces between adjoining reinforcing bars.

Minimum spacing requirements are given in Figure 6.22 where d_{agg} is the nominal maximum aggregate size in the concrete mixture; c_s is the clear cover to the stirrup; d_s is the diameter of the stirrup; r is the inside bend radius of the stirrup (which is equal to $2d_s$ for #3, #4, and #5 stirrups, and $3d_s$ for #6, #7, and #8 stirrups; see ACI Table 25.3.2); and d_b is the overall diameter of the longitudinal reinforcing bars (the overall diameter of a reinforcing bar includes the deformations on the bar; see Table 6.7). The minimum clear spacing between parallel reinforcing bars placed in 2 or more horizontal layers is 1 in. (ACI 25.2.2; see Figure 6.22). In such cases, longitudinal reinforcement in the upper layers must be placed directly above reinforcement in the bottom layer.

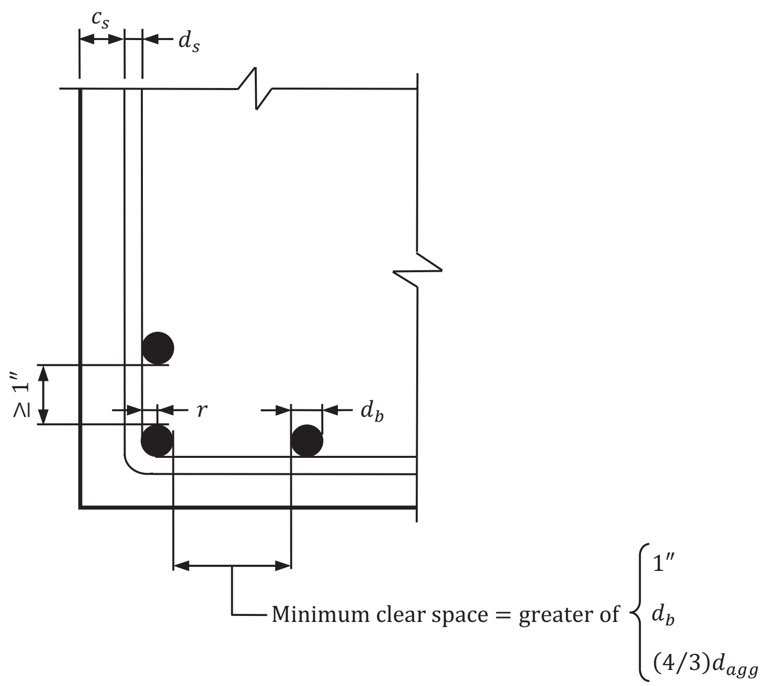


Figure 6.22 Minimum clear spacing requirements for reinforcing bars.

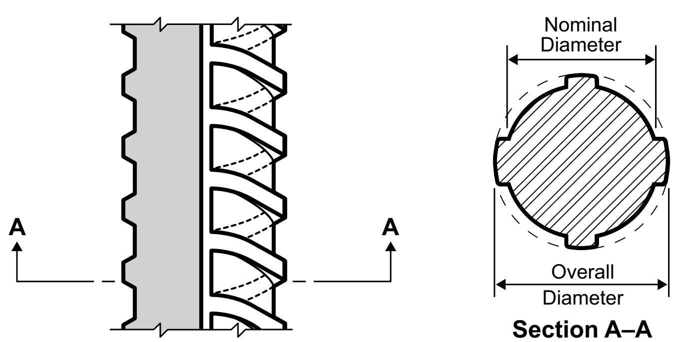


Table 6.7 Overall Reinforcing Bar Diameter

Bar Size	Approximate Diameter to Deformations (in.)
#3	7/16
#4	9/16
#5	11/16
#6	7/8

(table continued on next page)

Table 6.7 Overall Reinforcing Bar Diameter (cont.)

Bar Size	Approximate Diameter to Deformations (in.)
#7	1
#8	1-1/8
#9	1-1/4
#10	1-7/16
#11	1-5/8
#14	1-7/8
#18	2-1/2

The following equation can be used to determine the maximum number of reinforcing bars, n_{max}, that can fit in a single layer based on the minimum spacing limits in ACI 25.2.1:

$$n_{max} = \frac{b_w - 2(c_s + d_s + r)}{(\text{clear space}) + d_b} + 1 \qquad (6.64)$$

In this equation, "clear space" refers to the greater of the three minimum clear spaces given in Figure 6.22. The value of n_{max} determined by Equation (6.64) is rounded down to the next whole number.

The maximum number of bars that can fit in a single layer based on the minimum spacing requirements in ACI 25.2.1 are given in Table 6.8 based on the following:

- Overall bar diameter is used for the longitudinal reinforcement (Table 6.7)
- Clear cover to the stirrup $c_s = 1.5$ in.
- Nominal maximum aggregate size $d_{agg} = 3/4$ in.
- #3 stirrups used for #4, #5, and #6 longitudinal bars and #4 stirrups used for #7 and larger longitudinal bars

Table 6.8 Maximum Number of Reinforcing Bars Permitted in a Single Layer*

Bar Size	Beam Width (in.)												
	12	14	16	18	20	22	24	26	28	30	36	42	48
#4	5	6	7	8	10	11	12	14	15	16	20	24	28
#5	4	5	7	8	9	10	11	13	14	15	19	22	26
#6	4	5	6	7	8	9	10	11	12	14	17	20	23
#7	3	4	5	6	7	8	9	10	11	12	15	18	21
#8	3	4	5	6	7	7	8	9	10	11	14	16	19
#9	3	4	4	5	6	7	8	8	9	10	12	15	17
#10	2	3	4	5	5	6	7	7	8	9	11	13	15
#11	2	3	3	4	5	5	6	7	7	8	10	11	13

**Overall bar diameter is used for the longitudinal reinforcement (Table 6.7)*
Clear cover to the stirrup $c_s = 1.5$ in.
Nominal maximum aggregate size $d_{agg} = 3/4$ in.
#3 stirrups are used for #4, #5, and #6 longitudinal bars and #4 stirrups are used for #7 and larger longitudinal bars

Maximum Spacing of Flexural Reinforcing Bars for Crack Control

Maximum center-to center spacing of reinforcing bars is given in ACI 24.3 (ACI 9.7.2.2). The intent of these requirements is to control flexural cracking. In general, a larger number of finer cracks are preferable to a few wide cracks mainly for reasons of durability and appearance.

The maximum center-to-center bar spacing, s, is determined by the equations in ACI Table 24.3.2 (ACI 24.3.2). The following equation is applicable to deformed reinforcing bars:

$$s \leq \text{lesser of} \begin{cases} 15\left(\dfrac{40{,}000}{f_s}\right) - 2.5c_c \\ \\ 12\left(\dfrac{40{,}000}{f_s}\right) \end{cases} \tag{6.65}$$

In this equation, c_c is the least distance from the surface of the flexural reinforcement to the tension face of the section. For example, for a beam with a 1.5-in. concrete cover to #4 stirrups, $c_c = 1.5 + 0.5 = 2.0$ in. The term f_s is the calculated stress in the flexural reinforcement closet to the tension face at service loads (that is, f_s is equal to the unfactored bending moment at the section divided by the product of the area of reinforcement and the internal moment arm). In lieu of calculating f_s, it is permitted to be taken as $2f_y / 3$ (ACI 24.3.2.1). For Grade 60 reinforcement, $f_s = 2 \times 60{,}000 / 3 = 40{,}000$ psi. Assuming $c_c = 2.0$ in., the maximum center-to-center bar spacing $s = 10$ in. from Equation (6.65). Note that s is independent of the size of the longitudinal reinforcing bars in the section.

The following equation can be used to determine the minimum number of reinforcing bars, n_{min}, required in a single layer to satisfy the crack control requirements of ACI 24.3.2:

$$n_{min} = \frac{b_w - 2c_c - d_b}{s} + 1 \tag{6.66}$$

In this equation, s is determined by Equation (6.65). The calculated value of n_{min} is rounded up to the next whole number.

The minimum number of bars that can fit in a single layer based on the crack control requirements of ACI 24.3.2 are given in Table 6.9 based on the following:

- Grade 60 reinforcement with $f_s = 40{,}000$ psi
- Overall bar diameter is used for the longitudinal reinforcement (Table 6.7)
- Least distance from the surface of the flexural reinforcement to the tension face of the section $c_c = 2.0$ in.

Table 6.9 Minimum Number of Reinforcing Bars Required in a Single Layer*

Beam Width (in.)												
12	14	16	18	20	22	24	26	28	30	36	42	48
2	2	3	3	3	3	3	4	4	4	5	5	6

**Grade 60 reinforcement with* $f_s = 40{,}000$ psi
Overall bar diameter is used for the longitudinal reinforcement (Table 6.7)
Least distance from the surface of the flexural reinforcement to the tension face of the section $c_c = 2.0$ in.

Distribution of Tension Reinforcement in Flanges of T-Beams

Requirements for the control of cracking in the flanges of T-beams in tension (that is, at negative bending regions) are given in ACI 24.3.4 (see Figure 6.11). The beam flexural reinforcement in the flange (slab), A_s, must be uniformly distributed over a width equal to the lesser of the effective flange width, b_f, defined in ACI 6.3.2 (see Figure 6.10) and $\ell_n / 10$ where ℓ_n is the length of the clear span of the beam measured face-to-face of supports (see Figure 6.23).

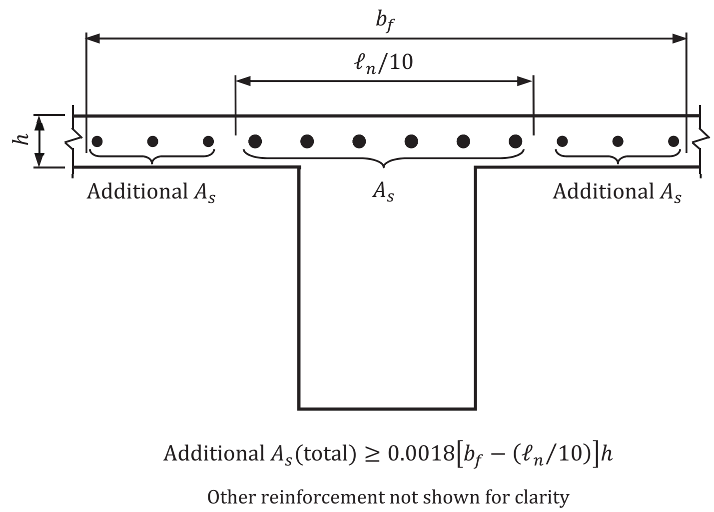


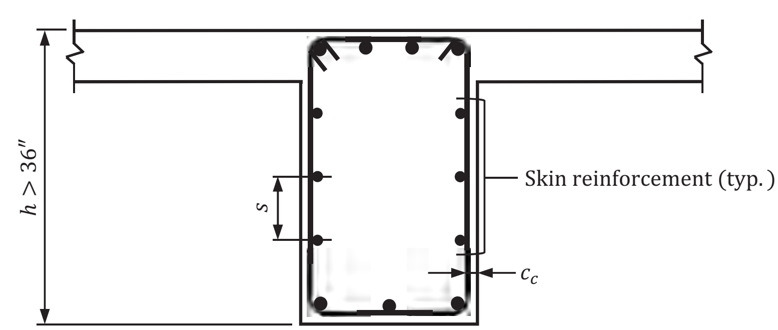
Figure 6.23 Distribution of tension reinforcement in the flange of a T-beam.

Additional reinforcement satisfying ACI 24.4.3.1 (shrinkage and temperature reinforcement) must be provided in the outer portions of the flange where $b_f > \ell_n / 10$.

Crack Control Reinforcement in Deep Flexural Members

For beams with an overall depth, h, greater than 36 in., longitudinal skin reinforcement must be uniformly distributed on both faces of the beam for a distance equal to at least $h / 2$ from the tension face to help control the width of the cracks that can form on these faces (ACI 9.7.2.3). The maximum center-to-center spacing of the skin reinforcement is equal to that required for crack control in ACI 24.3.2 where c_c is the clear cover from the side face to the edge of the skin reinforcement.

The extent of the skin reinforcement is illustrated in ACI Figure R9.7.2.3 at positive and negative bending regions along the span of the beam. The area of skin reinforcement is not specified in ACI 9.7.2.3; however, according to ACI R9.7.2.3, #3 to #5 bars are typically provided. An alternate configuration is shown in Figure 6.24 where it is assumed skin reinforcement is provided over the entire span.



$$s \leq \text{lesser of} \begin{cases} 15\left(\dfrac{40{,}000}{f_s}\right) - 2.5c_c \\ 12\left(\dfrac{40{,}000}{f_s}\right) \end{cases}$$

Figure 6.24 Skin reinforcement for deep flexural members.

It is permitted to include the skin reinforcement when calculating the nominal moment strength of the beam, M_n, provided a strain compatibility analysis of the section is performed.

6.6.3 Selection of Flexural Reinforcement

The size and spacing of the reinforcing bars at a critical section for flexure must be determined based on the required area of reinforcement and the minimum and maximum spacing requirements given in Section 6.6.2 of this publication.

Selecting the number of reinforcing bars in a single layer within the limits of Table 6.8 and Table 6.9 provides automatic conformity with the minimum and maximum spacing requirements in ACI 25.2 and 24.3, respectively, given the assumptions noted in those tables.

In general, overall cost savings is usually achieved by specifying the largest practical longitudinal bar sizes that satisfy both strength and serviceability requirements. Using the same bar sizes and lengths wherever possible also has a positive impact on cost savings.

6.6.4 Development of Flexural Reinforcement

Overview

Development of deformed reinforcement in beams for flexure must be in accordance with ACI 25.4 (ACI 9.7.1.2). The calculated tensile or compressive force in the longitudinal flexural reinforcement at each section in a beam must be developed on each side of that section by embedment length, hooks, headed deformed bars, mechanical devices, or any combination thereof (ACI 9.7.3.1, ACI 25.4.1.1).

It is important to note the following provisions given in ACI 25.4.1.2, 25.4.1.3, and 25.4.1.4 when determining development lengths:

- Hooked and headed portions of a reinforcing bar must not be used to develop the reinforcing bar in compression.
- Development lengths do not require a strength reduction factor, ϕ.
- Values of $\sqrt{f_c'}$ used to calculate development lengths are limited to 100 psi.

Information on how to determine the required development lengths for flexural reinforcement in a beam is given below.

Development of Deformed Bars in Tension

Provisions for the development of deformed reinforcing bars in tension are given in ACI 25.4.2. The tension development length, ℓ_d, is determined using the equations in ACI 25.4.2.3 or 25.4.2.4 in conjunction with the modification factors in ACI 25.4.2.5. The requirements of ACI 25.4.2.3 are based on those in ACI 25.4.2.4, so the latter requirements are covered first.

<u>Method 1 – ACI 25.4.2.4</u>

The development length in tension of a deformed reinforcing bar, ℓ_d, is determined by ACI Equation (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \tag{6.67}$$

The terms in Equation (6.67) are as follows (see ACI Table 25.4.2.5):

- *Modification factor for lightweight concrete,* λ. This factor reflects the lower tensile strength of lightweight concrete:

$$\lambda = \begin{cases} 0.75 \text{ for lightweight concrete} \\ \\ 1.0 \text{ for normalweight concrete} \end{cases} \tag{6.68}$$

- *Reinforcement grade factor,* ψ_g. This factor accounts for the effect of reinforcement yield strength on the required development length:

$$\psi_g = \begin{cases} 1.0 \text{ for Grade 40 or Grade 60} \\ \\ 1.15 \text{ for Grade 80} \\ \\ 1.3 \text{ for Grade 100} \end{cases} \tag{6.69}$$

- *Reinforcement coating factor,* ψ_e. This factor accounts for the reduced bond strength between the concrete and epoxy-coated or zinc and epoxy dual-coated reinforcing bars:

$$\psi_e = \begin{cases} 1.5 \text{ for epoxy-coated or zinc and epoxy dual-coated bars with clear} \\ \text{cover} < 3d_b \text{ or clearing spacing} < 6d_b \\ \\ 1.2 \text{ for epoxy-coated or zinc and epoxy dual-coated bars for all other conditions} \\ \\ 1.0 \text{ for uncoated or zinc-coated (galvanized) bars} \end{cases} \tag{6.70}$$

- *Reinforcement size factor,* ψ_s. This factor reflects the more favorable performance of smaller diameter reinforcing bars:

$$\psi_s = \begin{cases} 1.0 \text{ for \#7 and larger bars} \\ \\ 0.8 \text{ for \#6 and smaller bars} \end{cases} \tag{6.71}$$

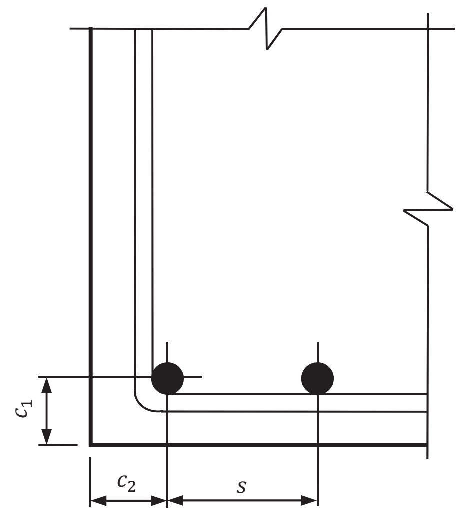
- *Casting position factor,* ψ_t. This factor reflects the adverse effects that can occur to the top horizontal reinforcement in a member due to vertical migration of water and mortar, which collect on the underside of the bars during placement of the concrete:

$$\psi_t = \begin{cases} 1.3 \text{ where more than 12 in. of fresh concrete is placed below the horizontal reinforcement} \\ \\ 1.0 \text{ in all other cases} \end{cases} \tag{6.72}$$

According to the footnote in ACI Table 25.4.2.5, $\psi_t \psi_e$ need not exceed 1.7.

- *Spacing or cover dimension*, c_b. This term is defined as follows (see Figure 6.25):

$$c_b = \text{lesser of} \begin{cases} \text{the distance from the center of a bar to the nearest concrete surface (lesser of } c_1 \text{ and } c_2) \\ \\ \text{one-half the center-to-center spacing of the bars being developed } (s\ /\ 2) \end{cases} \tag{6.73}$$



$$c_b = \text{lesser of} \begin{cases} c_1 \\ c_2 \\ s/2 \end{cases}$$

Figure 6.25 Spacing or cover dimension, c_b.

- *Transverse reinforcement index,* K_{tr}. The transverse reinforcement index represents the role of confining reinforcement across the potential splitting planes: larger amounts of confining reinforcement reduces the potential for splitting failure, thereby reducing the overall required development length. This index is determined by ACI Equation (25.4.2.4b):

$$K_{tr} = \frac{40A_{tr}}{sn} \tag{6.74}$$

where $A_{tr} =$ total cross-sectional area of all transverse reinforcement within a spacing s that crosses the potential plane of splitting through the reinforcement being developed

$s =$ center-to-center spacing of the transverse reinforcement

$n =$ number of bars being developed along the plane of splitting

It is permitted to conservatively use $K_{tr} = 0$ if transverse reinforcement is present or required. For reinforcing bars with $f_y \geq 80{,}000$ psi spaced closer than 6.0 in. on center, transverse reinforcement must be provided along the development or lap splice length such that $K_{tr} \geq 0.5d_b$ (ACI 9.7.1.4 and 25.4.2.2).

The confining term $(c_b + K_{tr})/d_b$ must be taken less than or equal to 2.5 in Equation (6.67) [ACI 25.4.2.4]. It has been shown that when this term is less than 2.5, splitting failures are likely to occur. A pullout failure of the reinforcement is more likely when this term is greater than 2.5, so an increase in the anchorage capacity due to an increase in cover or amount of confining reinforcement is not likely.

The tension development length ℓ_d determined by ACI Equation (25.4.2.4a) is permitted to be reduced in cases where the flexural reinforcement is greater than that required from analysis, except for the six cases in ACI 25.4.10.2 (ACI 25.4.10.1). The reduction factor applied to ℓ_d is equal to the required area of flexural reinforcement divided by the provided area of reinforcement. The reduced development length must not be taken less than 12 in.

Values of ℓ_d for deformed bars in beams are given in Table 6.10 based on the following assumptions:

- Normalweight concrete with $f_c' = 4{,}000$ psi
- Uncoated Grade 60 reinforcement
- $K_{tr} = 0$
- Concrete cover controls the determination of c_b
- $c_b = (d_b / 2) + d_s + 1.5$ in.
- #3 stirrups are used for #4, #5, and #6 longitudinal bars and #4 stirrups are used for #7 and larger longitudinal bars

Calculated values of ℓ_d are rounded up to the next whole number. Values of ℓ_d for beams with coated bars can be obtained by multiplying the tabulated values by the appropriate value of ψ_e in Equation (6.70). Similarly, values of ℓ_d for beams made of lightweight concrete can be obtained by dividing the tabulated values by 0.75 in accordance with Equation (6.68).

Table 6.10 Tension Development Length, ℓ_d, for Deformed Bars in Beams in Accordance with ACI 25.4.2.4*

Bar Size	ℓ_d (in.)	
	Top	Other
#4	15	12
#5	19	15
#6	23	18
#7	33	25
#8	37	29
#9	46	36
#10	57	44
#11	68	53

**Normalweight concrete with* $f_c' = 4{,}000$ psi
Uncoated Grade 60 reinforcement
$K_{tr} = 0$
Concrete cover controls the determination of c_b
$c_b = (d_b / 2) + d_s + 1.5$ in.
#3 stirrups used for #4, #5, and #6 longitudinal bars and #4 stirrups used for #7 and larger longitudinal bars

Tabulated values for ℓ_d based on the requirements of ACI 25.4.2.4 for a variety of other conditions are given in Reference 9.

<u>Method 2 – ACI 25.4.2.3</u>

The method in ACI 25.4.2.3 to determine ℓ_d is based on the requirements given in ACI 25.4.2.4 and preselected values of the confining term $(c_b + K_{tr})/d_b$. The two spacing and cover cases in ACI Table 25.4.2.3 are given in Table 6.11.

Table 6.11 Tension Development Length, ℓ_d, for Deformed Bars in Beams in Accordance with ACI 25.4.2.3

Spacing and Cover		#6 and Smaller Bars	#7 and Larger Bars
Case 1	Condition 1 1. Clear spacing of bars being developed or lap spliced $\geq d_b$ 2. Clear cover $\geq d_b$ 3. Stirrups or ties throughout ℓ_d not less than the required minimum Condition 2 1. Clear spacing of bars being developed or lap spliced $\geq 2d_b$ 2. Clear cover $\geq d_b$	$\left(\dfrac{f_y \psi_t \psi_e \psi_g}{25\lambda\sqrt{f'_c}}\right) d_b$	$\left(\dfrac{f_y \psi_t \psi_e \psi_g}{20\lambda\sqrt{f'_c}}\right) d_b$
Case 2	Other conditions	$\left(\dfrac{3f_y \psi_t \psi_e \psi_g}{50\lambda\sqrt{f'_c}}\right) d_b$	$\left(\dfrac{3f_y \psi_t \psi_e \psi_g}{40\lambda\sqrt{f'_c}}\right) d_b$

In order to determine ℓ_d using the equations in Case 1, the spacing, cover, and confinement of the bars being developed must meet all the requirements under Condition 1 or Condition 2 (see Figure 6.26). It is presumed that these conditions occur frequently in practice. When the requirements of either Condition 1 or Condition 2 are met, it is assumed $(c_b + K_{tr}) / d_b = 1.5$; that value is substituted into Equation (6.67), which results in the equations in the first row of Table 6.11 for Case 1.

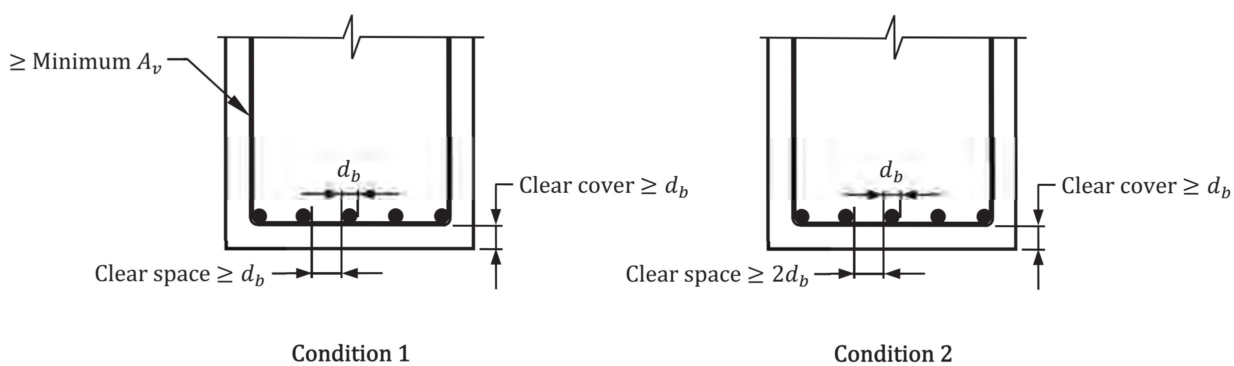


Figure 6.26 Spacing, cover, and confinement conditions of ACI 25.4.2.3.

Case 2 is applicable where the requirements in either Condition 1 or Condition 2 in Case 1 are not satisfied; in this case, it is assumed $(c_b + K_{tr}) / d_b = 1.0$. Tension development lengths, ℓ_d, must be the greater of the values determined by the equations in Table 6.11 and 12 in.

Tabulated values for ℓ_d based on the requirements of ACI 25.4.2.3 are given in Reference 9 for a variety of conditions.

Development of Standard Hooks in Tension

Hooks are provided at the ends of reinforcing bars to provide additional anchorage where development length cannot be attained with straight bars. Standard hook geometry for development of deformed bars in tension is given in ACI Table 25.3.1 (ACI 25.3.1).

The development length of a deformed reinforcing bar in tension with a standard hook, ℓ_{dh}, is given in ACI 25.4.3.1:

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f'_c}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{6.75}$$

This development length is measured from the critical section to the outside face of the hook (see Figure 4.10). The modification factors in Equation (6.75) are given in Table 6.12 (see ACI Table 25.4.3.2).

Table 6.12 Modification Factors for Development of Hooked Bars in Tension

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Confining reinforcement, ψ_r	For #11 and smaller bars with $A_{th} \geq 0.4A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For #11 and smaller hooked bars terminating inside a column core with side cover normal to the plane of the hook ≥ 2.5 in. or with side cover normal to the plane of the hook $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c\,/\,15{,}000) + 0.6$
	$f'_c \geq 6{,}000$ psi	1.0

The term A_{th} is the total cross-sectional area of ties or stirrups confining the hooked bars and A_{hs} is the total cross-sectional area of the hooked bars being developed at the same critical section. The term s is the center-to-center spacing of the hooked bars, which have a nominal diameter d_b. Detailing requirements for A_{th} are given in ACI 25.4.3.3 (see ACI Figures R25.4.3.3a and R25.4.3.3b for confining reinforcement placed parallel and perpendicular to hooked bars being developed, respectively).

The detailing requirements in ACI 25.4.3.4 apply to hooked bars at discontinuous ends of members (that is, at ends of simply-supported members, at the free ends of cantilevers, and at exterior joints where members do not extend beyond the joint) where both side cover and top (or bottom) cover to the hook is less than 2.5 in.

Values of ℓ_{dh} for hooked deformed bars in beams based on normalweight concrete with $f'_c = 4{,}000$ psi, uncoated Grade 60 reinforcement, center-to-center hooked bar spacing $s \geq 6d_b$, and side cover normal to the plane of the hook $\geq 6d_b$ are given in Table 6.13. Calculated values of ℓ_{dh} are rounded up to the next whole number.

Table 6.13 Tension Development Length, ℓ_{dh}, for Hooked Deformed Bars in Beams*

Bar Size	ℓ_{dh} (in.)
#4	6
#5	8
#6	10
#7	13
#8	15
#9	18
#10	22
#11	25

*Normalweight concrete with $f_c' = 4{,}000$ psi
Uncoated Grade 60 reinforcement
Center-to-center hooked bar spacing $s \geq 6d_b$
Side cover normal to the plane of the hook $\geq 6d_b$

Development of Headed Deformed Bars in Tension

Provisions for the development of headed deformed bars in tension are given in ACI 25.4.4. Examples of headed reinforcing bars are given in Figure 4.11.

Headed deformed bars are permitted to be used only when the conditions of ACI 25.4.4.1 are satisfied:

(a) Bar must conform to ACI 20.2.1.6
(b) Bar size must be #11 or smaller
(c) Net bearing area of head, A_{brg}, must be at least $4A_b$ where A_b is the area of the bar
(d) Concrete must be normalweight
(e) Clear cover to the bar must be greater than or equal to $2d_b$ where d_b is the nominal diameter of the bar
(f) Center-to-center spacing of the bars must be greater than or equal to $3d_b$

The development length of a headed deformed reinforcing bar in tension, ℓ_{dt}, is given in ACI 25.4.4.2:

$$\ell_{dt} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_p \psi_o \psi_c}{75\sqrt{f_c'}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{6.76}$$

This development length is measured from the critical section to the bearing face of the head (see ACI Figures R25.4.4.2a and R25.4.4.2b). The modification factors in Equation (6.76) are given in Table 6.14 (see ACI Table 25.4.4.3).

Table 6.14 Modification Factors for Development of Headed Bars in Tension

Modification Factor	Condition	Value of Factor
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Parallel tie reinforcement, ψ_p	For #11 and smaller bars with $A_{tt} \geq 0.3A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For headed bars (1) terminating inside a column core with side cover to bar ≥ 2.5 in. or (2) with side cover to bar $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c / 15{,}000) + 0.6$
	$f'_c \geq 6{,}000$ psi	1.0

The term ψ_p accounts for confining effects provided by transverse reinforcement oriented parallel to the development length of headed bars. Confinement requirements for headed negative moment reinforcement in a beam terminated in a joint are given in ACI 25.4.4.6. At beam-column joints, the total cross-sectional area of transverse reinforcement, A_{tt}, must be located within $8d_b$ of the centerline of the headed bar toward the middle of the joint where d_b is the nominal diameter of the headed bar [see ACI Figure R25.4.4.4(a)]. Where A_{tt} is greater than or equal to $0.3A_{hs}$ or where the center-to center spacing of the headed bars is greater than or equal to $6d_b$, $\psi_p = 1.0$. The term A_{hs} is the total cross-sectional area of the headed bars being developed.

For other than beam-column joints, A_{tt} must not be considered and $\psi_p = 1.0$ provided $s \geq 6d_b$ (ACI 25.4.4.5).

Values of ℓ_{dt} for headed deformed bars in beams based on normalweight concrete with $f'_c = 4{,}000$ psi, uncoated Grade 60 reinforcement, center-to-center headed bar spacing $s \geq 6d_b$, and side cover to the bar $\geq 6d_b$ are given in Table 6.15. Calculated values of ℓ_{dt} are rounded up to the next whole number.

Table 6.15 Tension Development Length, ℓ_{dt}, for Headed Deformed Bars in Beams*

Bar Size	ℓ_{dt} (in.)
#4	6
#5	6
#6	8
#7	9
#8	11
#9	14
#10	16
#11	19

**Normalweight concrete with* $f'_c = 4{,}000$ psi
Uncoated Grade 60 reinforcement
$s \geq 6d_b$
Side cover normal to the bar $\geq 6d_b$

Development of Mechanically Anchored Deformed Bars in Tension

Any mechanical attachment or device capable of developing f_y of the deformed bars is permitted to be used provided it is approved by the building official in accordance with ACI 1.10 (ACI 25.4.5). The use of mechanical devices that do not meet the requirements in ACI 20.2.1.6 or that are not developed in accordance with ACI 25.4.4 may be used provided test results are available that demonstrate the ability of the head and bar system to develop or anchor the required force in the bar.

Development of Positive and Negative Flexural Reinforcement

Flexural reinforcement must be properly developed or anchored in a reinforced concrete beam so that the beam performs as intended in accordance with the strength design method. Critical sections for development of flexural reinforcement occur at the following (ACI 9.7.3.2):

(1) Points of maximum stress (that is, sections of maximum bending moment)

(2) Points along the span where adjacent reinforcement is terminated because it is no longer required to resist flexure

In continuous beams subjected to uniform gravity loads, the maximum positive and negative bending moments typically occur near midspan and at the faces of the supports, respectively. Positive and negative flexural reinforcing bars must be developed or anchored on both sides of these critical sections (ACI 9.7.3.1).

The required area of flexural reinforcement at a critical section can be determined using the methods given in Section 6.5 of this publication.

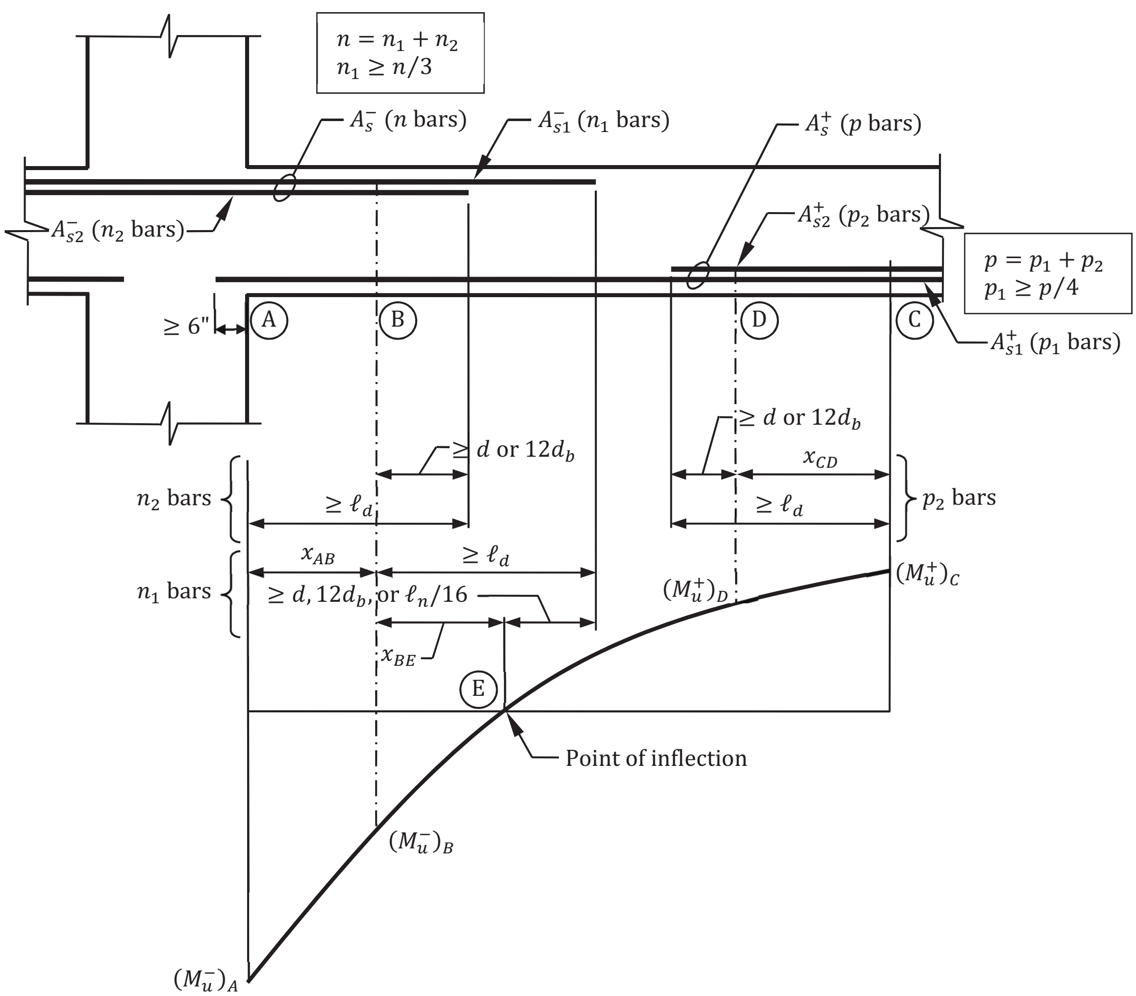


Figure 6.27 Development of flexural reinforcement in a beam.

For the continuous beam in Figure 6.27, the total required area of negative reinforcement for the maximum negative factored bending moment, $(M_u^-)_A$, at critical section A is equal to A_s^-, and the total number of reinforcing bars at this location is equal to n. Similarly, the total required area of positive reinforcement for the maximum positive factored bending moment, $(M_u^+)_C$, at critical section C is equal to A_s^+, and the total number of reinforcing bars at this location is equal to p. Requirements for the development of these two sets of reinforcing bars are discussed next.

Negative Flexural Reinforcement

Assume a portion of A_s^- is cut off at section B where it is no longer required for flexural strength. This makes section B a critical section. At this location, the reinforcement has an area equal to A_{s2}^- and the number of reinforcing bars is equal to n_2. The area of the remaining portion of reinforcing bars is equal to $A_{s1}^- = A_s^- - A_{s2}^-$ and the corresponding number of reinforcing bars is equal to $n_1 = n - n_2$. This reinforcement must be able to resist the negative factored bending moment $(M_u^-)_B$ at section B.

Because section B is a critical section, n_1 bars must be adequately developed to the right of this section. This is achieved by extending the bars a minimum distance of ℓ_d past section B as shown in Figure 6.27 where ℓ_d is the development length in tension of a deformed bar, which is determined in accordance with ACI 25.4.2 (ACI 9.7.3.4).

According to ACI 9.7.3.8.4, at least one-third of the total negative reinforcement provided at a support must have an embedment length equal to the greater of d, $12d_b$, and $\ell_n / 16$ past the point of inflection where ℓ_n is the clear span length measured face-to-face of supports (which in this case are columns). Therefore, to satisfy this requirement, which accounts for possible shifting of the bending moment diagram at the point of inflection due to the approximate bending moment diagram customarily used in design, $n_1 \geq n / 3$. The minimum length of n_1 bars to the right of critical section A is equal to the following (ACI 9.7.3.4 and 9.7.3.8.4):

$$\text{Minimum length of } n_1 \text{ bars} = \text{greater of} \begin{cases} x_{AB} + \ell_d \\ \text{greater of} \begin{cases} x_{AB} + x_{BE} + d \\ x_{AB} + x_{BE} + 12d_b \\ x_{AB} + x_{BE} + (\ell_n/16) \end{cases} \end{cases} \tag{6.77}$$

In these equations, x_{AB} is the distance from section A to the theoretical cutoff point at section B and x_{BE} is the distance from section B to the point of inflection at section E.

Because section A is a critical section, the bars cutoff at section B must be developed a distance equal to at least ℓ_d beyond that section. Additionally, ACI 9.7.3.3 stipulates these bars must extend beyond the point where they are no longer required a distance equal to at least the greater of d and $12d_b$ (except at supports of simply-supported spans and at free ends of cantilevers). The minimum length of n_2 bars to the right of critical section A is equal to the following:

$$\text{Minimum length of } n_2 \text{ bars} = \text{greater of} \begin{cases} \ell_d \\ \text{greater of} \begin{cases} x_{AB} + d \\ x_{AB} + 12d_b \end{cases} \end{cases} \tag{6.78}$$

Flexural tension reinforcement is not permitted to be terminated in a tension zone unless one of the following conditions are satisfied (ACI 9.7.3.5):

(1) At the cutoff point, $V_u \leq 2\phi V_n/3$ where ϕV_n is the design shear strength of the section at that point (see Section 6.4.3 of this publication).

(2) For #11 and smaller bars, continuing reinforcement provides at least double the area required for flexure at the cutoff point and $V_u \leq 3\phi V_n/4$.

(3) Stirrup or hoop area in excess of that required for shear and torsion is provided along each terminated bar over a distance equal to $3d / 4$ from the cutoff point. The excess area of the stirrups or hoops must be greater than or equal to $60b_w s / f_{yt}$ where the center-to-center spacing must be less than or equal to $d / (8\beta_b)$ and $\beta_b = \text{(area of reinforcement cut off)/(total area of tension reinforcement at the section)}$.

The development of the negative reinforcing bars to the left of section A depends on its location within the framing system. At interior joints, like the one depicted in Figure 6.27, development is achieved by continuing the negative reinforcing bars into the span to the left of the column. In an end span where the beam terminates at an edge column, a standard hook is provided at the ends of the negative reinforcing bars.

Positive Flexural Reinforcement

Assume a portion of A_s^+ is cut off at section D. This makes section D a critical section. At this location, the reinforcement has an area equal to A_{s2}^+ and the number of reinforcing bars is equal to p_2. The area of the remaining portion of reinforcing bars is equal to $A_{s1}^+ = A_s^+ - A_{s2}^+$ and the corresponding number of reinforcing bars is equal to $p_1 = p - p_2$. This reinforcement must be able to resist the negative factored bending moment $(M_u^+)_D$ at section D.

The bars cut off at section D must be developed to a distance greater than or equal to ℓ_d beyond the critical section C. Like in the case of the negative reinforcement, these bars must extend beyond the point they are no longer required by a distance equal to the greater of d and $12d_b$ (ACI 9.7.3.3). The minimum length of p_2 bars to the left of critical section C is equal to the following:

$$\text{Minimum length of } p_2 \text{ bars} = \text{greater of} \begin{cases} \ell_d \\ \text{greater of} \begin{cases} x_{CD} + d \\ x_{CD} + 12d_b \end{cases} \end{cases} \tag{6.79}$$

In this equation, x_{CD} is the distance between critical section C and the theoretical cutoff point located at section D.

The requirements of ACI 9.7.3.5 pertaining to reinforcement terminated in a tension zone are also applicable in this case.

According to ACI 9.7.3.8.2, at least one-fourth of the positive reinforcement must extend at least 6 in. into the support (except at simple supports; see ACI 9.7.3.8.1). Thus, $p_1 \geq p/4$. Note that if a beam is part of the LFRS, this reinforcement must be anchored to develop f_y at the support.

The diameter of the positive reinforcement is limited at points of inflection (and at simple supports) in accordance with ACI 9.7.3.8.3; this requirement helps to ensure the bars are developed in a length short enough such that the moment capacity is greater than the applied moment over the entire length of the beam (see ACI Figure R9.7.3.8.3):

$$\ell_d \leq \begin{cases} \dfrac{1.3M_n}{V_u} + \ell_a \text{ if the end of the reinforcement is confined by a compressive reaction} \\ \\ \dfrac{M_n}{V_u} + \ell_a \text{ if the end of the reinforcement is not confined by a compressive reaction} \end{cases} \tag{6.80}$$

In this equation, M_n is the nominal flexural strength of the beam section assuming all the reinforcement at that section is stressed to f_y, V_u is the factored shear force at the section, and ℓ_a is the embedment length of the reinforcement beyond the inflection point, which is the greater of d and $12d_b$. Additional information on this topic can be found in ACI R9.7.3.8.3.

6.6.5 Splices of Deformed Reinforcement

Overview

Splices of deformed reinforcement in beams must be in accordance with ACI 25.5 (ACI 9.7.1.3). The primary reasons for splicing reinforcement are based on (1) restrictions related to transporting the reinforcing bars to the construction site and (2) limitations related to handling and placing the reinforcing bars in the field.

Lap splices, mechanical splices, and welded splices are common types of splices for flexural reinforcement.

Lap Splices

Lap splices are usually the most economical type of splice. There are basically two types of lap splices: contact and noncontact. In a contact lap splice, the bars are generally in contact over a specified length and are tied together [see Figure 4.13(a)]. In a noncontact lap splice, the bars are not in contact, and the center-to-center spacing of the bars being spliced must not exceed the limits indicated in Figure 4.13(b) (see ACI 25.5.1.3). Contact lap splices are usually preferred because the bars are tied together and are less likely to displace during construction. The minimum clear spacing between a contact lap splice and adjacent splices or bars is the same as that for individual bars (ACI 25.5.1.2; see Figure 6.22).

Lap splices must be provided at locations away from maximum stress (that is, maximum bending moment) and should be staggered wherever possible. Reinforcing bars provided for negative bending moments (top bars) are spliced near the midspan of a member, and reinforcing bars provided for positive bending moments (bottom bars) are spliced over the supports.

The required tension lap splice length, ℓ_{st}, depends on the tension development length of the bars, ℓ_d, the area of reinforcement provided over the length of the splice, and the percentage of reinforcement spliced at any one location. Because experimental data on lap splices with #14 and #18 bars are sparse, the use of tension lap splices for these bars are prohibited except as permitted in ACI 25.5.5.3 (ACI 25.5.1.1).

Lap splices are classified as Class A or Class B. The length of a lap splice is given as a multiple of ℓ_d (ACI 25.5.2.1):

$$\text{Class A lap splice length: } \ell_{st} = 1.0\ell_d \geq 12 \text{ in.} \tag{6.81}$$

$$\text{Class B lap splice length: } \ell_{st} = 1.3\ell_d \geq 12 \text{ in.} \tag{6.82}$$

When determining ℓ_d to be used in determining ℓ_{st}, the 12-in. minimum length specified in ACI 25.4.2.1(b) and the excess reinforcement modification factor of ACI 25.4.10.1 are not applicable (ACI 25.5.1.4). Also, for reinforcing bars with $f_y \geq 80{,}000$ psi spaced closer than 6 in. on center, transverse reinforcement must be provided along the splice length such that $K_{tr} \geq 0.5d_b$ (ACI 25.5.1.5).

The default splice classification for a lap splice is Class B. If both of the following conditions are satisfied, a Class A splice is permitted (see ACI Table 25.5.2.1):

- $A_{s,provided} / A_{s,required} \geq 2.0$ over the entire splice length where $A_{s,provided}$ and $A_{s,required}$ are the area of reinforcement provided at the splice location and the area of reinforcement required by analysis at the splice location, respectively
- Less than or equal to 50 percent of the reinforcement is spliced within the required lap splice length

Where bars of different size are lap-spliced in tension, ℓ_{st} is equal to the greater of (1) ℓ_d of the larger bar and (2) ℓ_{st} of the smaller bar (ACI 25.5.2.2).

Mechanical Splices

Mechanical splices are, in general, a complete assembly of a coupler, a coupling sleeve, or an end-bearing sleeve, including any additional material or components, required to accomplish the splicing of the reinforcing bars. According to ACI 25.5.7.1, mechanical splices must develop in tension or compression at least $1.25f_y$ of the bar. This is to ensure yielding occurs in the reinforcing bar adjacent to the mechanical splice prior to the failure of the splice.

Illustrated in Figure 4.14 are three of the more popular types of mechanical splices. The cold-swaged coupling sleeve in Figure 4.14(a) uses a hydraulic swaging press with special dies to deform the sleeve around the ends of the spliced bars to produce positive mechanical interlock with the reinforcing bars. The shear screw coupling sleeve in Figure 4.14(b) consists of a coupling sleeve with shearhead screws, which are designed to shear off at a specified torque. The reinforcing bars are inserted to meet at the center of the coupling sleeve and then the screws are tightened. The tightening process embeds the pointed screws into the bars. The non-upset straight thread coupler in Figure 4.14(c) consists of a coupler with internal straight threads at each end that joins the two reinforcing bars with matching external threads. Additional information on these and other mechanical splices can be found in Reference 8.

Mechanical splices can be advantageous in a number of situations, including the following:

- When long lap splices are needed
- When lap splices cause reinforcement congestion
- Where spacing of the flexural reinforcement is insufficient to permit lap splices

Mechanical splices need not be staggered unless the splices are in tension tie members (that is, members having an axial tensile force sufficient to create tension over the cross section, a level of stress in the reinforcement such that every bar should be fully effective, and limited concrete cover on all sides) where the minimum stagger between splices in adjacent bars must be at least 30 in. in accordance with ACI 25.5.7.4 (ACI 25.5.7.3).

Welded Splices

The use of welded splices are permitted to be used, provided they conform to the requirements of ACI 26.6.4 (ACI 25.5.7). Like mechanical splices, a full welded splice, which is generally intended for #6 and larger bars, must be able to develop at least $1.25f_y$ of the bar.

Welded splices need not be staggered unless the splices are in tension tie members.

6.6.6 Longitudinal Torsional Reinforcement

Where it has been determined the effects of torsion must be considered (see Section 6.3.1 of this publication), the detailing requirements in ACI 9.7.5 must be satisfied for the longitudinal torsional reinforcement. A summary of the requirements in ACI 9.7.5.1 and 9.7.5.2 is given in Figure 6.28. In the figure, s is the center-to-center spacing of the required combined transverse reinforcement. The longitudinal torsional reinforcement is combined with that required for flexure (see Section 6.5.4 of this publication).

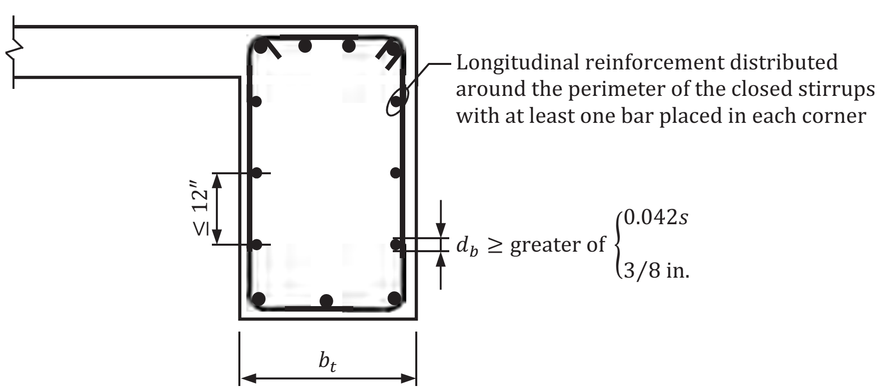


Figure 6.28 Details for longitudinal torsional reinforcement.

Longitudinal torsional reinforcement must be provided for a distance equal to at least $b_t + d$ beyond the point is required (ACI 9.7.5.3). The term b_t is the width of that part of the cross-section that contains the closed stirrups resisting torsion (see Figure 6.28). This distance is larger than that used for shear reinforcement and flexure reinforcement because torsional diagonal cracks develop in a helical form around a member. Also, the longitudinal reinforcement must be fully developed for tension at both ends of the member; proper anchorage is especially important at the ends where the largest torsional moments usually occur (ACI 9.7.5.4).

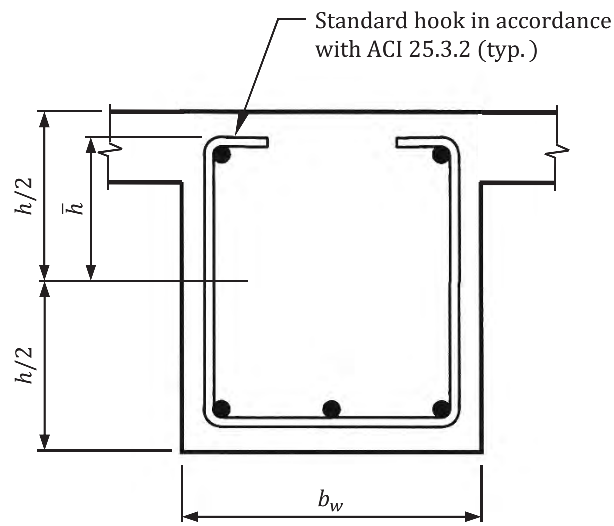
6.6.7 Transverse Reinforcement

Overview

The detailing requirements of ACI 9.7.6 must be satisfied for transverse reinforcement required for shear or for shear and torsion. Where shear and torsion must be considered, the most restrictive detailing requirements of ACI 25.7 apply (ACI 9.7.6.1.1 and 9.7.6.1.2).

Shear Reinforcement

Shear reinforcement must be properly anchored and developed to be fully effective. Requirements for the development of stirrups are given in ACI 25.7.1 and are illustrated in Figure 6.29.



$$\bar{h} \geq \frac{0.014 d_b f_{yt}}{\lambda \sqrt{f_c'}} \quad \text{for \#6, \#7, and \#8 stirrups with } f_{yt} > 40{,}000 \text{ psi}$$

Figure 6.29 Anchorage details for U-stirrups.

In general, the stirrups must extend as close to the compression and tension surfaces of the member as cover and other constraints permit and must extend at least a distance d from the extreme compression fiber (ACI 25.7.1.1). Each bend in the continuous portion of a single- or multiple-leg U-stirrup and each bend in a closed stirrup must enclose a longitudinal bar (ACI 25.7.1.2). Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops are given in ACI Table 25.3.2.

The beam depth that must be provided to satisfy the minimum embedment length for #6, #7, and #8 stirrup bars with $f_{yt} > 40{,}000$ psi is given in Table 6.16 [ACI 25.7.1.3(b)]. The beam depths in the table are based on normal-weight concrete with $f_c' = 4{,}000$ psi, Grade 60 stirrup bars, and a 1.5-in. cover to the stirrup hook. Beams with overall depths less than those in Table 6.16 are not permitted to use #6, #7, and #8 stirrups with $f_{yt} > 40{,}000$ psi. No minimum depth requirements are given for beams with (1) #5 stirrup bars and smaller of any grade and (2) #6, #7, and #8 stirrups with $f_{yt} \leq 40{,}000$ psi.

Table 6.16 Minimum Beam Depth to Anchor #6, #7, and #8 Stirrups*

Stirrup Bar	Minimum Beam Depth, h (in.)
#6	23
#7	27
#8	30

**Normalweight concrete with* $f_c' = 4{,}000$ psi
Grade 60 stirrup bars
1.5-in. cover to the stirrup hooks

Beams must have sufficient width, b_w, to allow for bend radii at corners of stirrups. Minimum beam widths corresponding to stirrup bar size are given in Table 6.17.

Table 6.17 Minimum Beam Width for Stirrup Development

Stirrup Bar	Minimum Beam Width, b_w (in.)
#3	10
#4	12
#5	14
#6	18
#7	20
#8	22

Closed stirrups consisting of pairs of U-stirrups are permitted to be used except for torsion or integrity reinforcement (ACI 25.7.1.7). The legs of the stirrups must be lap spliced with a splice length equal to at least $1.3\ell_d$ but not less than 12 in. where the tension development length, ℓ_d, is determined in accordance with ACI 25.4.2 (see Figure 6.30). In cases where the required lap length cannot fit within a member that has a depth of at least 18 in., pairs of U-stirrups can still be used, provided the force in each leg is equal to or less than 9,000 lb. For Grade 60 reinforcement, only a #3 stirrup satisfies this requirement: $\text{force in stirrup leg} = 0.11 \times 60{,}000 = 6{,}600 \text{ lb.}$

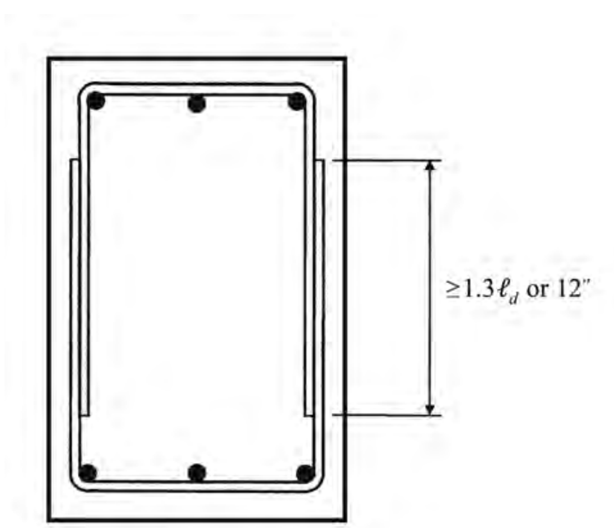


Figure 6.30 A closed stirrup formed by pairs of U-stirrups.

Torsion Reinforcement

Requirements for stirrups subjected to torsional moments are given in ACI 25.7.1.6. As noted previously, closed stirrups must be used in such cases because cracking can occur on all faces of a beam. The corners of a beam are susceptible to spalling due to the inclined compressive stresses occurring due to torsion. Where spalling is not restrained by an adjacent slab, both ends of a stirrup must terminate with 135-degree standard hooks around longitudinal bars in the section [ACI 25.7.1.6(a)]. Spalling is essentially prevented where a slab is present on one or both sides of a beam web, so 90-degree standard hooks in accordance with ACI 25.7.1.3 are permitted in such cases [ACI 25.7.1.5(b)]. Details for transverse torsion reinforcement where spalling is unrestrained and restrained are given in Figure 6.31.

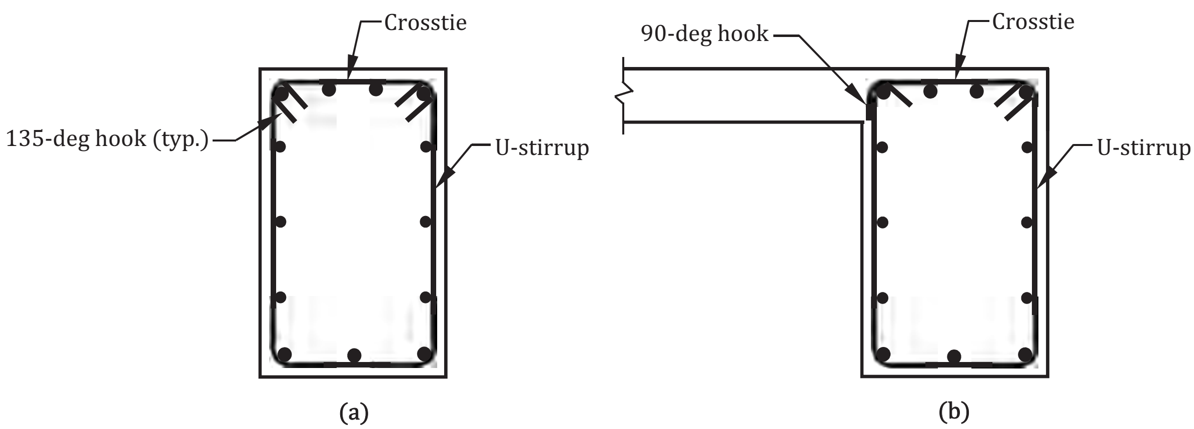


Figure 6.31 Details for transverse torsion reinforcement.
(a) Spalling is unrestrained. (b) Spalling is restrained.

Two-piece closed stirrups are permitted to be used (ACI 25.7.1.6.1). The closed stirrup in Figure 6.31(a) consists of a U-stirrup with 135-degree hooks at both ends and a crosstie with 135-degree hooks at both ends. The 135-degree hooks are required on both ends of both pieces of reinforcement because spalling is unrestrained. Similarly, Figure 6.31(b) consists of a U-stirrup with 135-degree hooks at both ends and a crosstie with a 135-degree hook on one end and a 90-degree hook on the other end. The 90-degree hook of the crosstie is located on the side adjacent to the slab, which provides restraint against spalling. Using two-piece closed stirrups are preferred because closed, one-piece stirrups make it difficult to place the longitudinal reinforcement in the beam where the reinforcement is built in place (as opposed to preassembled cages).

Like longitudinal reinforcement required for torsion, transverse torsional reinforcement must extend a distance of at least $b_t + d$ beyond the point it is required by analysis (ACI 9.7.6.3.2).

6.6.8 Structural Integrity Reinforcement

Structural integrity reinforcement requirements for cast-in-place beams are given in ACI 9.7.7. The purpose of this reinforcement is to improve the redundancy and ductility in the structure so that in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be localized and the structure will have a higher probability of maintaining overall stability.

A summary of the structural integrity reinforcement requirements is given in Table 6.18.

Table 6.18 Structural Integrity Reinforcement Requirements for Beams

Requirement		ACI Section No.
Perimeter Beams	1. At least one-quarter of the maximum positive moment reinforcement, but not less than 2 bars, must be continuous; 2. At least one-sixth of the negative moment reinforcement at a support, but not less than 2 bars, must be continuous; and, 3. Longitudinal structural integrity reinforcement must be enclosed by closed stirrups in accordance with ACI 25.7.1.6 or by hoops along the clear span of the beam.	9.7.7.1
Other Than Perimeter Beams	1. At least one-quarter of the maximum positive moment reinforcement, but not less than 2 bars, must be continuous, or 2. Longitudinal reinforcement must be enclosed by closed stirrups in accordance with ACI 25.7.1.6 or by hoops along the clear span of the beam.	9.7.7.2
Longitudinal structural integrity reinforcement must pass through the region bounded by the longitudinal reinforcement in the column.		9.7.7.3
Longitudinal structural integrity reinforcement at noncontinuous supports must be anchored to develop f_y at the face of the support.		9.7.7.4
Splices for longitudinal structural integrity reinforcement must occur at or near a support for positive moment reinforcement and at or near midspan for negative moment reinforcement.		9.7.7.5
Splices for longitudinal structural integrity reinforcement must be mechanical or welded splices in accordance with ACI 25.5.7 or Class B tension lap splices in accordance with ACI 25.5.2.		9.7.7.6

6.6.9 Flexural Reinforcement Requirements for SDC B

For beams in buildings assigned to SDC B that are part of an ordinary moment frame resisting seismic load effects, the detailing requirements in ACI 18.3.2 must be satisfied. Two continuous flexural bars must be provided at both the top and bottom faces of the beam. The continuous bottom bars must have an area equal to at least 25 percent the maximum area of the bottom bars along the span and must be anchored to develop f_y in tension at the face of the support.

These provisions are intended to improve continuity and improve lateral force resistance and structural integrity.

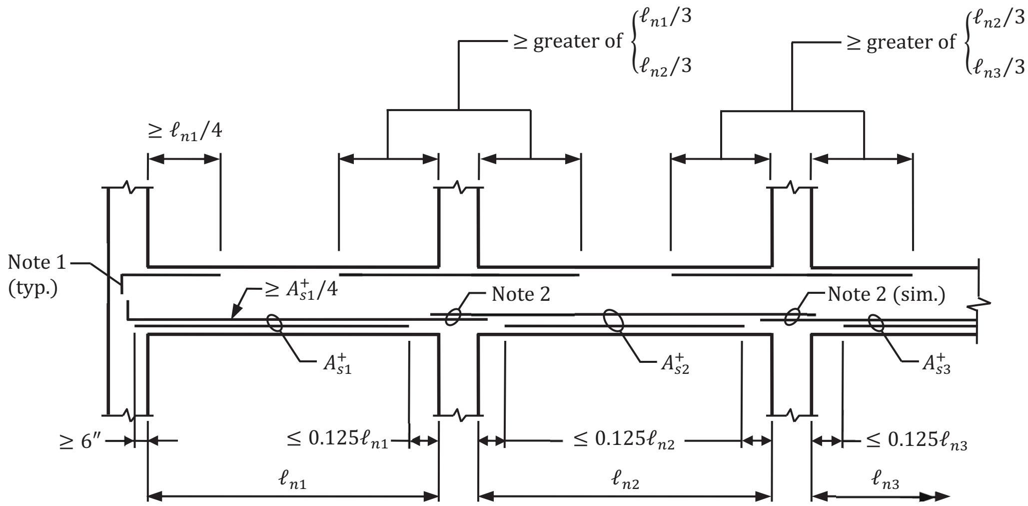
6.6.10 Recommended Flexural Reinforcement Details

Recommended flexural reinforcement details for beams based on the requirements covered in the previous sections of this chapter, including the structural integrity requirements of ACI 9.7.7, are given in Figure 6.32. The bar lengths given in the figure are based on a beam subjected to uniformly distributed gravity loads where the shape of the moment diagram is known; these lengths can be used for beams designed using the simplified method of analysis in ACI 6.5 (see Section 6.3.1 of this publication). For beams subjected to the effects from other types of loads, required bar lengths must be determined by calculations. Also, for beams that are part of an ordinary moment frame in buildings assigned to SDC B, the requirements of ACI 18.3.2 must also be satisfied.

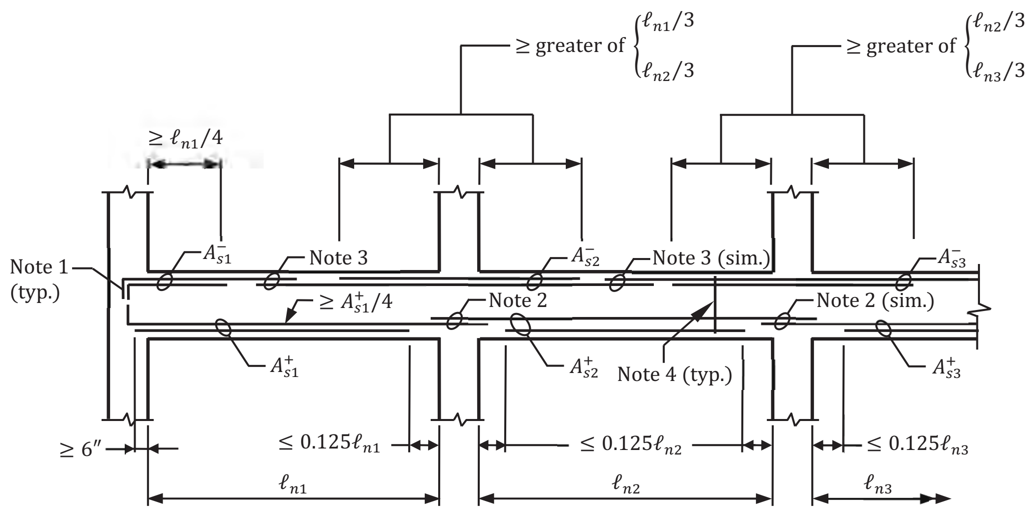
6.7 Deflections

6.7.1 Overview

Two methods can be used for controlling beam deflections: (1) providing a beam depth greater than or equal to the minimum beam depth prescribed in ACI 9.3.1 (see Section 6.2.1 of this publication) and (2) providing a beam depth



Beams Other Than Perimeter Beams



Perimeter Beams

Notes

1. Reinforcement to be anchored to develop f_y at the face of the support. Standard hooks are shown in this figure.
2. At least the larger of ($A_{s1}^{+}/4$) or ($A_{s2}^{+}/4$), but not less than 2 bars, must be continuous or spliced with Class B tension lap splices or mechanical or welded splices.
3. At least the larger of ($A_{s1}^{-}/6$) or ($A_{s2}^{-}/6$), but not less than 2 bars, must be continuous or spliced with Class B tension lap splices or mechanical or welded splices.
4. Closed stirrups in accordance with ACI Section 25.7.1.6 or hoops must be provided along the clear span.
5. Where the requirements in Note 2 are not satisfied for beams other than perimeter beams, closed stirrups in accordance with ACI Section 25.7.1.6 or hoops along the clear span must be provided.

Figure 6.32 Recommended flexural reinforcement details for beams.

based on calculated deflections and deflection limitations (ACI 9.3.2). Immediate and time-dependent deflections can be calculated for any reinforced concrete beam in accordance with ACI 24.2 regardless of whether the minimum depth requirements of ACI 9.3.1 are satisfied or not. However, the provisions of ACI 24.2 must be used where members are supporting elements likely to be damaged by relatively large deflections.

Calculated deflections must be less than or equal to the limiting deflections in ACI 24.2.2 in order to satisfy serviceability requirements.

Procedures for determining deflections of beams are given below (these procedures can also be used to determine deflections of one-way slabs). Although numerous methods are available to determine deflections, it is important to note that these methods can only estimate deflections within an accuracy range of 20 to 40 percent. This is primarily due to the variability in the constituent materials of concrete and to tolerances in construction. This range of accuracy should be considered carefully when calculating deflections for deflection-sensitive members.

6.7.2 Immediate Deflections

Uncracked Sections

For beams loaded such that maximum bending moment at service loads, M_a, is less than or equal to $2M_{cr} / 3$, the section is assumed to be uncracked and the immediate deflection of the beam can be calculated using any elastic method of analysis. In such cases, the gross moment of inertia, I_g, of the section (neglecting the longitudinal reinforcement) can be used in the deflection calculations.

The cracking moment, M_{cr}, is determined by ACI Equation (24.2.3.5):

$$M_{cr} = \frac{f_r I_g}{y_t} \tag{6.83}$$

where y_t is the distance from the centroidal axis of the gross section, neglecting longitudinal reinforcement, to the tension face of the member and the modulus of rupture, f_r, is determined by ACI Equation (19.2.3.1):

$$f_r = 7.5\lambda\sqrt{f_c'} \tag{6.84}$$

The modification factor λ is determined by Table 6.2 or Table 6.3 in this publication.

The cracking moment is multiplied by 2/3 to account for (1) restraint that can reduce the effective cracking moment and (2) reduced tensile strength of concrete during construction that can lead to cracking, which could subsequently have an impact on service deflections.

Cracked Sections

Once a section cracks, the gross moment of inertia can no longer be used to determine immediate deflections. Instead, the cracked moment of inertia, I_{cr}, must be used. In general, I_{cr} is determined by transforming the longitudinal reinforcing steel into an equivalent area of concrete (see Figure 6.33); the transformation is attained by multiplying the area of reinforcement, A_s, by the modular ratio $n = E_s / E_c$ where E_s is the modulus of elasticity of the reinforcing steel (ACI 20.2.2.2) and E_c is the modulus of elasticity of the concrete (ACI 19.2.2). The concrete below the neutral axis, which is located a distance equal to kd from the extreme compression fiber, is cracked and is ignored in the analysis.

Taking moments of areas about the neutral axis results in the following equation:

$$b \times kd \times \frac{kd}{2} = nA_s \times (d - kd) \tag{6.85}$$

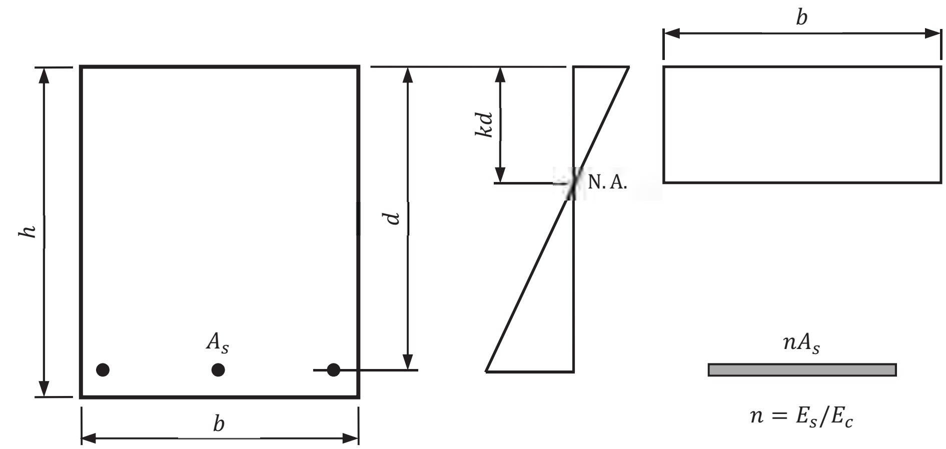
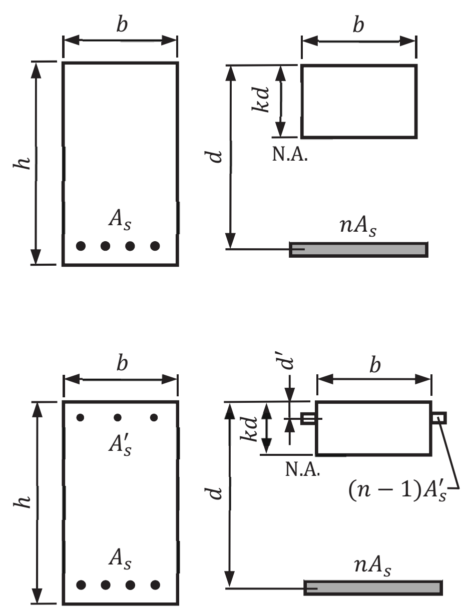


Figure 6.33 Cracked transformed section of a rectangular beam with tension reinforcement.



$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1}$$

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2$$

$$kd = \frac{\sqrt{2a_1 d\left(1 + \frac{a_2 d'}{d}\right) + (1 + a_2)^2} - (1 + a_2)}{a_1}$$

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2 + (n-1)A'_s(kd - d')^2$$

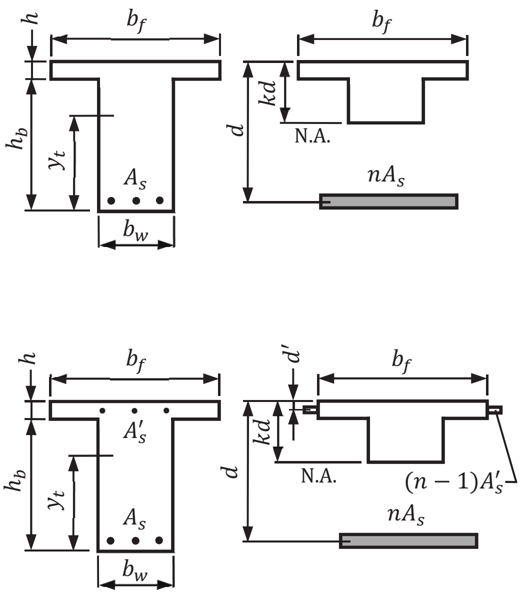
Notes:
$n = E_s/E_c$
$I_g = bh^3/12$
$a_1 = b/nA_s$
$a_2 = (n-1)A'_s/nA_s$

Figure 6.34 Cracked section properties for rectangular reinforced concrete beams.

Equation (6.85) can be solved for kd:

$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1} \tag{6.86}$$

where $a_1 = b / nA_s$.



$$kd = \frac{\sqrt{a_3(2d + ha_4) + (1 + a_4)^2} - (1 + a_4)}{a_3}$$

$$I_{cr} = \frac{(b_f - b_w)h^3}{12} + \frac{b_w(kd)^3}{3} + (b_f - b_w)h\left(kd - \frac{h}{2}\right)^2 + nA_s(d - kd)^2$$

$$kd = \frac{\sqrt{a_3(2d + ha_4 + 2a_2d') + (1 + a_2 + a_4)^2} - (1 + a_2 + a_4)}{a_3}$$

$$I_{cr} = \frac{(b_f - b_w)h^3}{12} + \frac{b_w(kd)^3}{3} + (b_f - b_w)h\left(kd - \frac{h}{2}\right)^2$$

$$+nA_s(d - kd)^2 + (n - 1)A'_s(kd - d')^2$$

Notes:

$n = E_s/E_c$

$y_t = [(b_f - b_w)(h_b + 0.5h)h + 0.5b_w(h_b + h)^2]/[(b_f - b_w)h + b_w(h_b + h)]$

$I_g = [(b_f - b_w)h^3/12] + (b_f - b_w)h(h_b + 0.5h - y_t)^2 + [b_w(h_b + h)^3/12] + b_w(h_b + h)[y_t - 0.5(h_b + h)]^2$

$a_2 = (n - 1)A'_s/nA_s$

$a_3 = b_w/nA_s$

$a_4 = h(b_f - b_w)/nA_s$

Figure 6.35 Cracked section properties for flanged reinforced concrete beams.

Thus, I_{cr} is equal to the following:

$$I_{cr} = \frac{b(kd)^3}{12} + nA_s(d - kd)^2 \tag{6.87}$$

where kd is determined by Equation (6.86).

Similar equations can be derived for rectangular sections with tension and compression reinforcement and for flanged sections. Cracked section properties for rectangular and flanged reinforced concrete beams are given in Figure 6.34 and Figure 6.35, respectively.

Effective Moment of Inertia

It is evident from the preceding discussion that two different moments of inertia would be required for calculating immediate deflections (that is, I_g and I_{cr}). To eliminate the need for two moments of inertia, the effective moment of inertia, I_e, can be used in lieu of a more comprehensive analysis (see ACI Table 24.2.3.5):

$$I_e = \begin{cases} I_g \text{ for } M_a \le 2M_{cr}/3 \\[2ex] \dfrac{I_{cr}}{1 - \left(\dfrac{2M_{cr}/3}{M_a}\right)^2\left(1 - \dfrac{I_{cr}}{I_g}\right)} \text{ for } M_a > 2M_{cr}/3 \end{cases} \tag{6.88}$$

In this equation, M_a is the maximum moment in the beam due to service loads at the stage deflections are calculated.

For continuous beams, it is permitted to calculate immediate deflections using I_e as the average of the values calculated by Equation (6.88) at the critical positive and negative moment sections (ACI 24.2.3.6). It is also permitted to calculate deflections using I_e at midspan for prismatic beams with simple and continuous spans, and at the support for cantilevers (ACI 24.2.3.7).

Approximate Immediate Deflections

The following equation, which is in ACI 318-83 Commentary Section 9.5.4.2, can be used to determine immediate deflection, Δ_i, at the tips of cantilevers and at the midspan of simply supported and continuous members:

$$\Delta_i = \frac{5KM_a\ell^2}{48E_cI_e} \tag{6.89}$$

In this equation, M_a is the support bending moment at service loads for cantilevers and the midspan bending moment at service loads for simply supported and continuous beams. Values of the deflection coefficient, K, are given in Table 6.19 for beams subjected to uniformly distributed service loads, w, and different span conditions.

Table 6.19 Deflection Coefficient, *K*

Span Condition	K
Cantilever	2.0
Simple	1.0
Continuous	$1.2 - 0.2(M_o / M_a)$ where $M_o = w\ell^2 / 8$

6.7.3 Time-Dependent Deflections

In one-way flexural members, creep and shrinkage due to sustained loads cause time-dependent deflections that can be 2 to 3 times greater than the immediate deflections. Not accounting for time-dependent deflections can result in a significant underestimation of total deflection.

Time-dependent deflections are determined by multiplying the immediate deflection, Δ_i, due to sustained loads by the factor λ_Δ, which is determined by ACI Equation (24.2.4.1.1) [ACI 24.2.4.1.1]:

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'} \tag{6.90}$$

In this equation, ξ is the time-dependent factor for sustained loads given in ACI Table 24.2.4.1.3 (see Table 6.20) and $\rho' = A_s' / bd$ is the reinforcement ratio for any compression reinforcement in the section. The term $(1 + 50\rho')$ accounts for the effect of compression reinforcement in reducing time-dependent deflections. It is permitted to calculate ρ' at midspan for simple and continuous spans, and at the support for cantilevers (ACI 24.2.4.1.2).

Table 6.20 Time-Dependent Factor for Sustained Loads, ξ

Sustained Load Duration (Months)	ξ
3	1.0
6	1.2
12	1.4
60 or more	2.0

Only the dead load and a portion of the sustained live load (if applicable) need to be included in the determination of Δ_i when calculating the time-dependent deflection due to creep and shrinkage, Δ_{cs}:

$$\Delta_{cs} = \lambda_\Delta \Delta_i = \left(\frac{\xi}{1+50\rho'}\right)\Delta_i \tag{6.91}$$

6.7.4 Maximum Permissible Calculated Deflections

Deflections calculated using the methods in Sections 6.7.2 and 6.7.3 of this publication must be less than or equal to the maximum permissible deflections given in ACI Table 24.2.2 (see Table 6.21).

Table 6.21 Maximum Permissible Calculated Deflections

Member	Condition	Deflection to be Considered	Deflection Limitation
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to the maximum of L_r, S, and R	ℓ / 180(1)
Floors		Immediate deflection due to L	ℓ / 360
Roof or floors	Supporting or attached to nonstructural elements likely to be damaged by large deflections	The part of the total deflection occurring after attachment of nonstructural elements(2)	ℓ / 480(3)
	Supporting or attached to nonstructural elements not likely to be damaged by large deflections		ℓ / 240(4)

(1) This limit is not intended to safeguard against ponding. Ponding must be checked by deflection calculations, including (1) additional deflections due to ponded water and (2) consideration of time-dependent effects of sustained load, camber, construction tolerances, and reliability of provisions for drainage.
(2) The time-dependent deflection is to be determined in accordance with ACI 24.2.4; this deflection is permitted to be reduced by the amount of deflection calculated to occur before attachment of nonstructural elements. This amount must be calculated based on accepted engineering data relating time-dependent characteristics of members similar to those being considered.
(3) This limit is permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.
(4) This limit must not exceed the tolerance provided for the nonstructural elements.

The limitations given in Table 6.20 relate only to supported or attached nonstructural elements. Where structural members are likely to be affected by deflection or deformation of the members to which they are attached in such a way as to adversely affect the strength of the structure, these deflections and the resulting forces must be considered explicitly in the analysis and design of the structural members, as required by ACI 24.2.1.

6.8 Design Procedure

The design procedure in Figure 6.36 can be used in the design and detailing of rectangular beam sections with one layer of longitudinal tension reinforcement. Included in the figure are the section numbers, table numbers, and figure numbers where specific information on that topic can be found in this chapter.

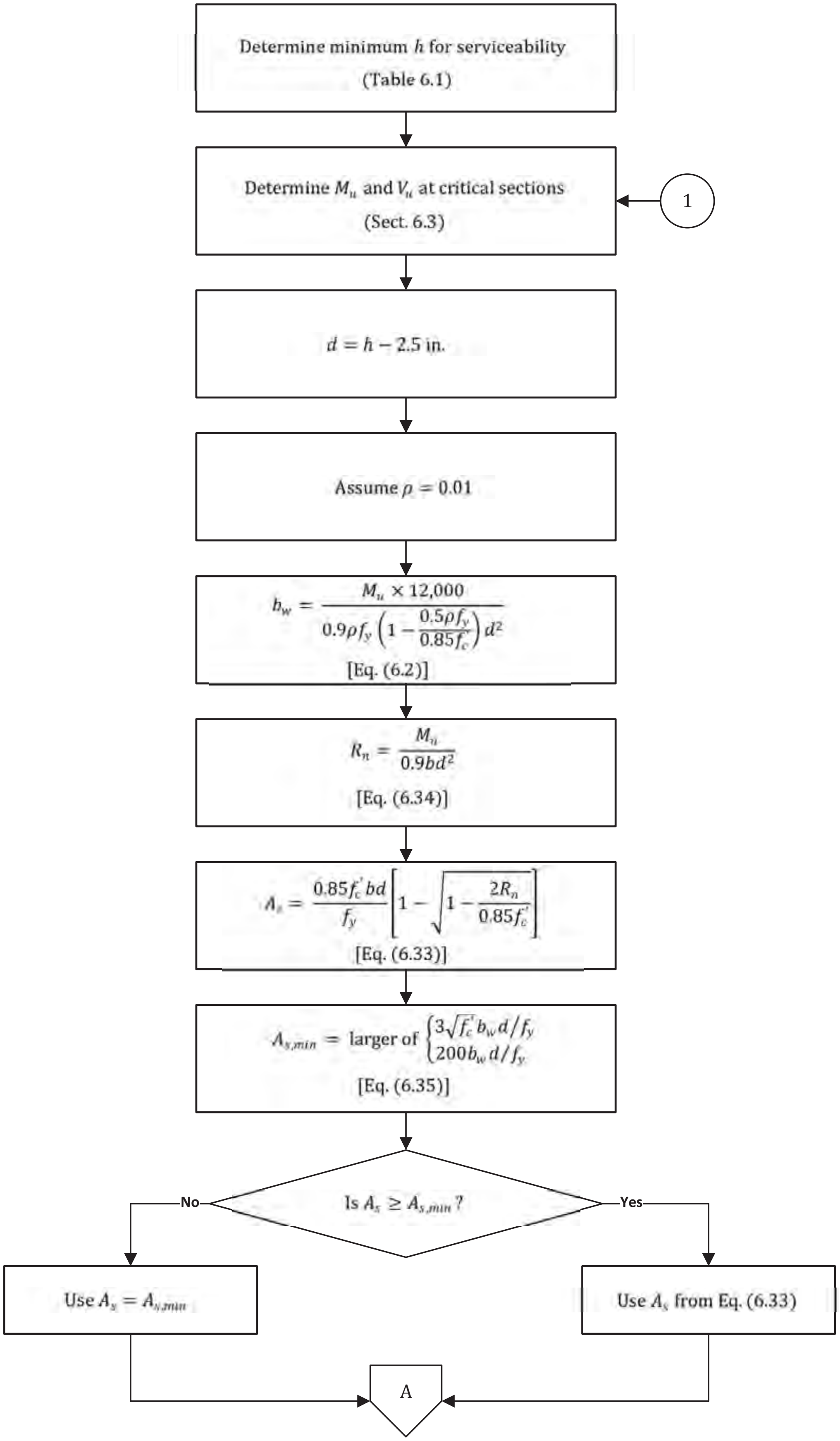


Figure 6.36 Design procedure for beams.

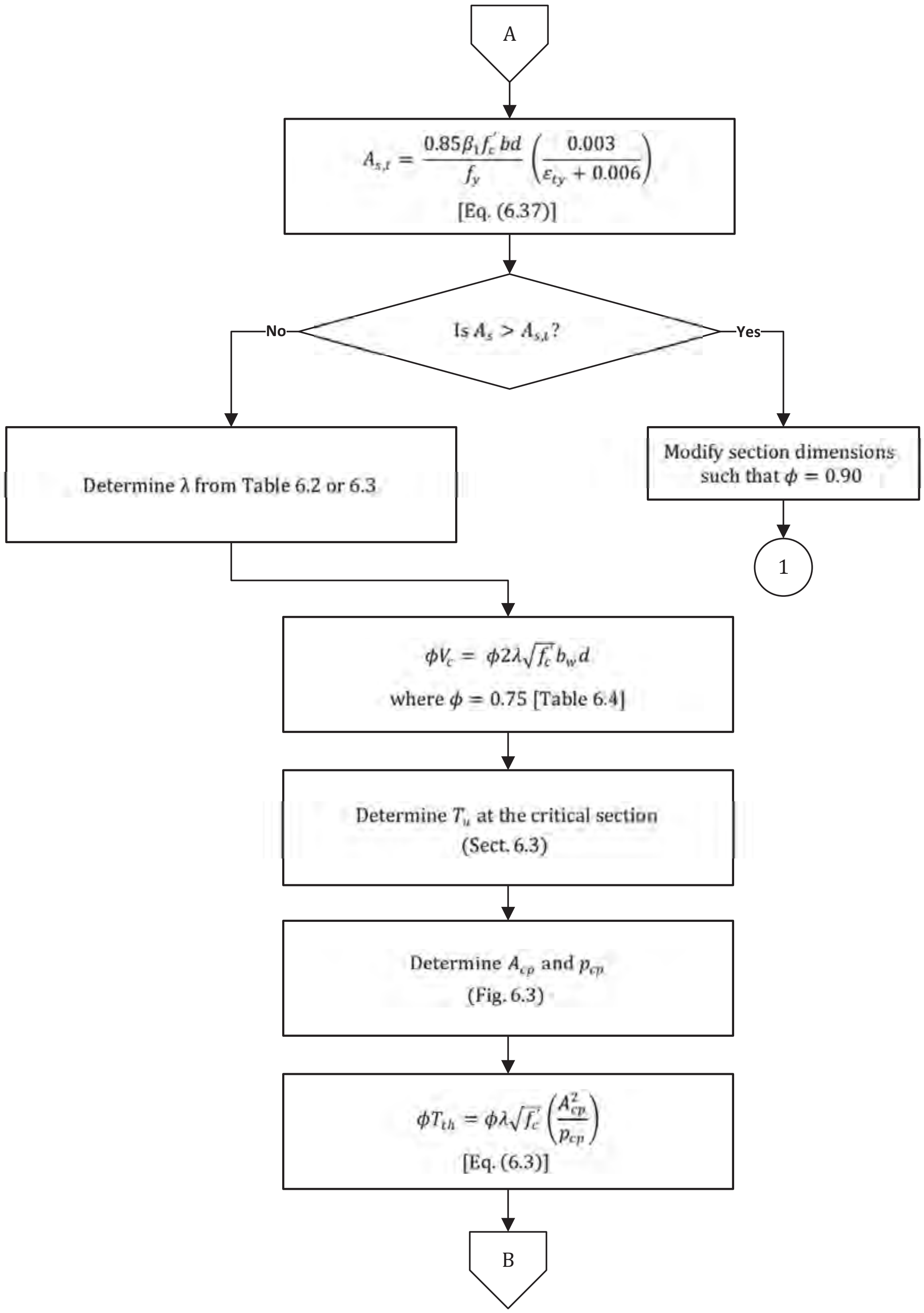


Figure 6.36 Design procedure for beams. (cont.)

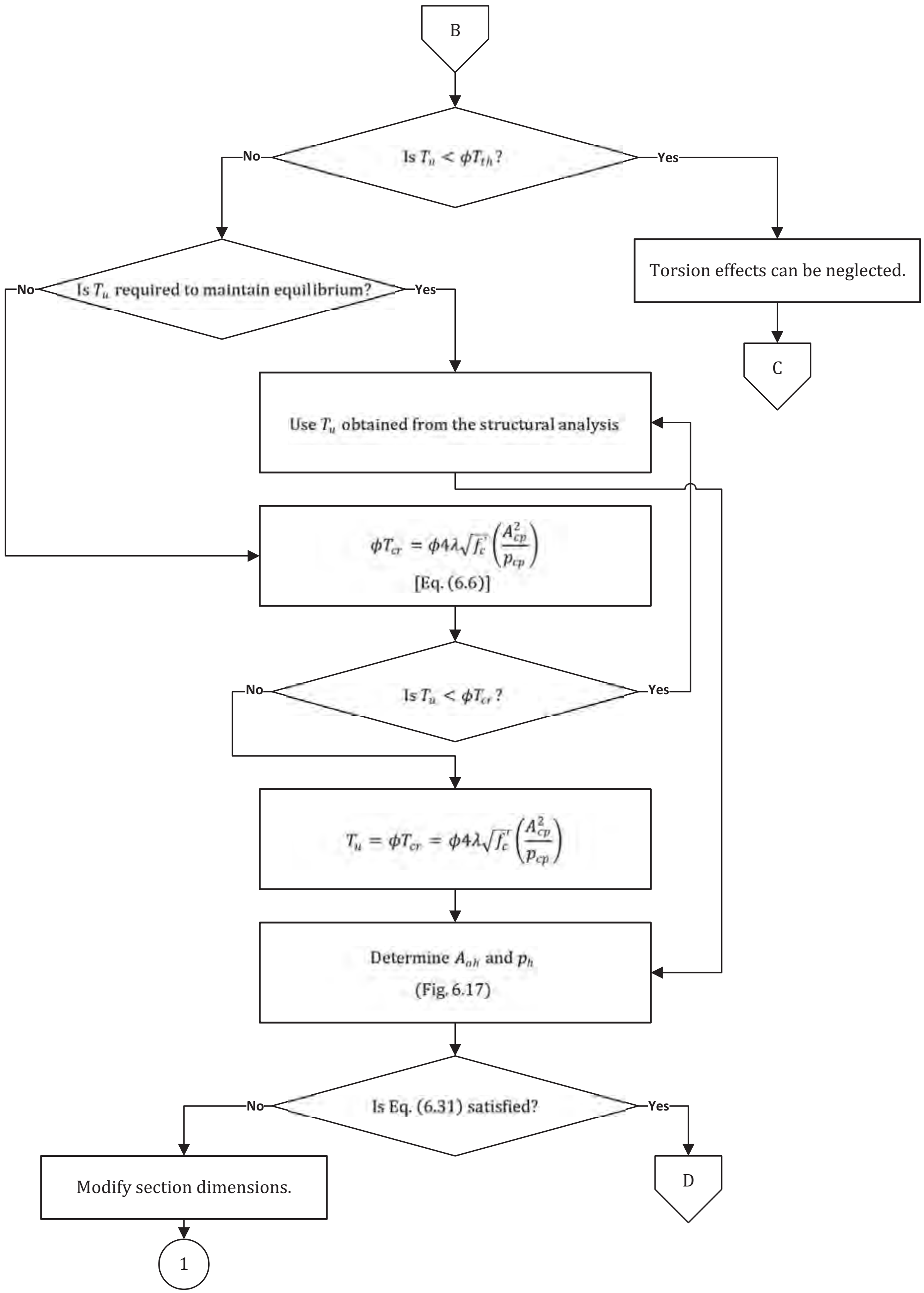


Figure 6.36 Design procedure for beams. (cont.)

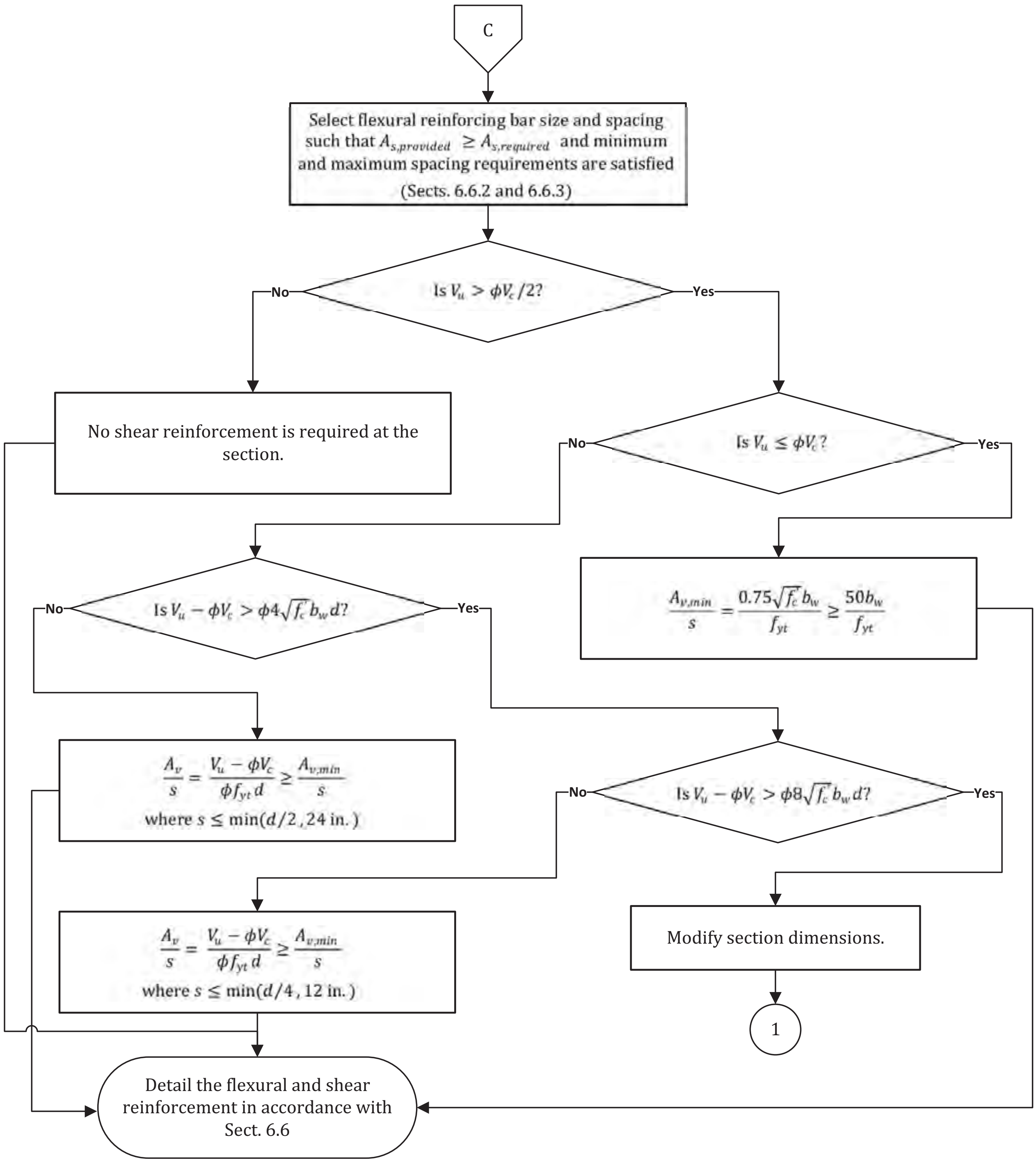


Figure 6.36 Design procedure for beams. (cont.)

D

$$A_o = 0.85A_{oh}$$

$$\theta = 45 \text{ degrees}$$

$$\frac{A_t}{s} = \frac{T_u}{2\phi \cot\theta \, A_o f_{yt}}$$

[Eq. (6.60)]

$$\frac{A_v}{2s} + \frac{A_t}{s} \geq \text{greater of} \begin{cases} 0.375\sqrt{f_c'}\, b_w / f_{yt} \\ 25 b_w / f_{yt} \end{cases}$$

[Eq. (6.63)]*

$$A_{\ell,min} \geq \text{lesser of} \begin{cases} \dfrac{5\sqrt{f_c'}\, A_{cp}}{f_y} - \left(\dfrac{A_t}{s}\right) p_h \dfrac{f_{yt}}{f_y} \\ \dfrac{5\sqrt{f_c'}\, A_{cp}}{f_y} - \left(\dfrac{25 b_w}{s}\right) p_h \dfrac{f_{yt}}{f_y} \end{cases}$$

[Eq. (6.62)]

$$A_\ell = \frac{A_t}{s} p_h \left(\frac{f_{yt}}{f_y}\right) \cot^2\theta \geq A_{\ell,min}$$

Combine A_ℓ and A_s
(Sect. 6.5.4)

Detail the longitudinal and transverse reinforcement in accordance with Sect. 6.6

*Determine $\dfrac{A_v}{2s}$ using path C in this flowchart

Figure 6.36 Design procedure for beams. (cont.)

6.9 Examples

6.9.1 Example 6.1 – Determination of Beam Size: Building #1 (Framing Option C), Beam is Not Part of the LFRS, SDC A

Determine the size of a typical beam along column line 4 in Building #1, Framing Option C, at a typical floor level assuming the beam is not part of the LFRS (see Figure 1.1). Also assume a 7.0-in. slab, 24 in. by 24 in. columns, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the beam depth based on serviceability requirements ACI 9.3.1.1

Assume the beam is not supporting or attached to partitions or other construction likely to be damaged by large deflections. Thus, deflections in accordance with ACI 24.2 need not be calculated.

Because the length is the same for all spans, minimum beam depth is based on the support condition of one end continuous:

$$\text{Beam depth} = \frac{\ell}{18.5} = \frac{23.5 \times 12}{18.5} = 15.2 \text{ in.}$$ Table 6.1

For formwork economy, determine minimum depth for the beams in the east-west direction because the span lengths are longer than those in the north-south direction:

$$\text{Beam depth} = \frac{\ell}{18.5} = \frac{25.0 \times 12}{18.5} = 16.2 \text{ in.}$$

Use a 24.0-in.-deep beam, which is adequate for the beams in both directions (see Example 5.3 for the determination of the minimum slab thickness, which depends on the beam sizes).

Step 2 – Determine the beam width based on strength requirements Sect. 6.2.2

Assuming $A_s = 0.01 b_w d$, the required b_w can be determined by the following equation with $f_c' = 4{,}000$ psi and Grade 60 reinforcement:

$$b_w = \frac{24.4 M_u}{d^2}$$ Eq. (6.1)

The largest M_u along the spans is used to determine b_w.

The factored bending moments in the beam due to the weight of the slab and the superimposed dead and live loads are determined in Step 4 of Example 5.9 for this beam-supported two-way slab using the Direct Design Method (see Table 5.32). These factored bending moments are based on ACI Eq. (5.3.1b): $U = 1.2D + 1.6L$. From Table 5.32, the largest M_u in the beam occurs at the face of the first interior column and is equal to 137.3 ft-kips.

Conservatively estimate the factored dead load of the beam web as 1.0 kip/ft.

The limitations of the simplified method of analysis in ACI 6.5.1 are satisfied for this beam, and the maximum factored bending moment due to the weight of the beam stem also occurs at the face of the first interior column:

$$M_u = \frac{w_u \ell_n^2}{10} = \frac{1.0 \times (23.5 - 2.0)^2}{10} = 46.2 \text{ ft-kips}$$ Figure 4.3

Total $M_u = 137.3 + 46.2 = 183.5$ ft-kips

Approximate $d = 24.0 - 2.5 = 21.5$ in.

Therefore,

$$b_w = \frac{24.4 \times 183.5}{21.5^2} = 9.7 \text{ in.}$$

Unless the width is restricted for architectural reasons, a 10.0-in.-wide beam is impractical to use. Any functional beam width wider than 9.7 in. satisfies strength requirements. Use a 28.0-in.-wide beam, which is 4.0 in. wider than the 24.0-in. column it frames into (see Sect. 6.2.3 of this publication on guidelines for sizing beams for economy).

Check the assumption regarding the weight of the beam web:

$$w_{u(stem)} = \frac{1.2 \times 28.0 \times (24.0 - 7.0)}{144 \times 1{,}000} \times 150 = 0.6 \text{ kip/ft} < 1.0 \text{ kip/ft}$$

Use a 28.0 in. by 24.0 in. beam.

6.9.2 Example 6.2 – Determination of Flexural Reinforcement: Beam in Building #1 (Framing Option C), Beam is Not Part of the LFRS, SDC A, Single Layer of Tension Reinforcement

Determine the required flexural reinforcement for the 28.0 in. by 24.0 in. beam along column line 4 in Building #1, Framing Option C, at a typical floor level assuming the beam is not part of the LFRS (see Figure 1.1). Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 6.1.

Step 1 – Determine the factored bending moments along the span

The factored bending moments in the beam due to the weight of the slab, the superimposed dead load, and the live load on the slab are determined in Step 4 of Example 5.9 for this beam-supported two-way slab (see Table 5.32). These factored bending moments are based on ACI Eq. (5.3.1b): $U = 1.2D + 1.6L$.

Given a 7.0-in.-thick slab, the factored dead load of the beam web is equal to 0.6 kip/ft (see Example 6.1).

The limitations of the simplified method of analysis in ACI 6.5.1 are satisfied for this beam. The factored bending moments due to the dead load of the beam web are determined using Figure 4.3 and are given in Table 6.22 where $\ell_n = 23.5 - 2.0 = 21.5$ ft for all spans.

Table 6.22 Factored Bending Moments due to Dead Load of Beam Web (ft-kips)

End Span			Interior Span	
Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
$\frac{w_u \ell_n^2}{16} = -17.3$	$\frac{w_u \ell_n^2}{14} = 19.8$	$\frac{w_u \ell_n^2}{10} = -27.7$	$\frac{w_u \ell_n^2}{16} = 17.3$	$\frac{w_u \ell_n^2}{11} = -25.2$

Total design bending moments are given in Table 6.23 (see Table 5.32).

Table 6.23 Design Bending Moments (ft-kips) for the Beam in Example 6.2

End Span			Interior Span	
Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
$-31.9 - 17.3 = -49.2$	$114.9 + 19.8 = 134.7$	$-137.3 - 27.7 = -165.0$	$70.2 + 17.3 = 87.5$	$-130.9 - 25.2 = -156.1$

Step 2 – Determine the effective flange width, b_f, of the beam ACI 6.3.2.1

The effective flange width, b_f, of the beam is equal to the following:

$$b_f = b_w + \text{ least of } \begin{cases} 16h = 16 \times 7.0 = 112.0 \text{ in.} \\ s_w = (25.0 \times 12) - 28.0 = 272.0 \text{ in.} \\ \ell_n / 4 = (23.5 - 2.0) \times 12 / 4 = 64.5 \text{ in.} \end{cases}$$

Figure 6.10

$$= 28.0 + 64.5 = 92.5 \text{ in.}$$

Step 3 – Determine the required flexural reinforcement at the critical sections

At negative moment critical sections (flange in tension), required A_s is determined by the following assuming a single layer of tension reinforcement:

$$R_n = \frac{M_u}{\phi b d^2} \quad \text{Eq. (6.34)}$$

$$A_s = \frac{0.85 f'_c b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] \quad \text{Eq. (6.33)}$$

where $b = b_w = 28.0$ in. Figure 6.11

At positive moment critical sections (flange in compression), determine the depth of the equivalent stress block, a, assuming rectangular section behavior using the largest positive M_u in Table 6.23 (which results in the largest a):

$$a = \frac{a_1 d - \sqrt{(a_1 d)^2 - (2 a_1 M_u / \phi)}}{a_1}$$

Eq. (6.49)

$$= \frac{(314.5 \times 21.5) - \sqrt{(314.5 \times 21.5)^2 - (2 \times 314.5 \times 134.7 \times 12 / 0.9)}}{314.5} = 0.27 \text{ in.} < h = 7.0 \text{ in.}$$

where $a_1 = 0.85 f'_c b_f = 0.85 \times 4 \times 92.5 = 314.5$ kips/in.

Because $a < h$, the section behaves as a rectangular section and Eqs. (6.34) and (6.33) can be used to determine A_s with $b = b_f = 92.5$ in. at positive moment sections (see Figure 6.12).

A summary of the required reinforcement is given in Table 6.24. It is evident that minimum reinforcement is required at all sections. Also, all sections are tension-controlled because $A_s < A_{s,t}$.

Table 6.24 Required Flexural Reinforcement for the Beam in Example 6.2

Location		M_u (ft-kips)	R_n (psi)	A_s (in^2)*
End span	Exterior negative	−49.2	51	2.01
	Positive	134.7	42	2.01
	First interior negative	−165.0	170	2.01
Interior span	Positive	87.5	27	2.01
	Negative	−156.1	161	2.01

*$Min.\ A_s = 200b_w d / f_y = 2.01$ in.^{2} [Eq. (6.35)]

$$Max.\ A_{s,t} = 0.018bd = \begin{cases} 0.018 \times 28.0 \times 21.5 = 10.8 \text{ in.}^2 \text{ at negative critical sections} \\ 0.018 \times 92.5 \times 21.5 = 35.8 \text{ in.}^2 \text{ at positive critical sections} \end{cases}$$ [Eq. (6.38)]

Step 4 – Select the flexural reinforcement Sect. 6.6.3

Select the flexural reinforcement in a single layer based on the minimum and maximum spacing requirements in ACI 25.2 and 24.3, respectively.

- Negative reinforcement

  $b_f = 92.5 \text{ in.} > \ell_n / 10 = 21.5 \times 12 / 10 = 25.8 \text{ in.}$

  Because $b_f > \ell_n / 10$ and $\ell_n / 10 \cong b_w$, distribute the required A_s within b_w. Figure 6.23

  For the 28.0-in.-wide beam:

  Minimum number of reinforcing bars $= 4$ Table 6.9

  Try 5-#6 bars ($A_{s,provided} = 2.20 \text{ in.}^2 > A_{s,required} = 2.01 \text{ in.}^2$).

  Maximum number of #6 bars in a 28-in.-wide section $= 12 > 5$ Table 6.8

  The reinforcement provided in the slab is at least equal to the shrinkage and temperature reinforcement prescribed in ACI 24.4.3.1 (see Table 5.33 in Example 5.9); this reinforcement satisfies the requirement in ACI 24.3.4 for additional longitudinal reinforcement in the outer portions of the flange where $b_f > \ell_n / 10$.

- Positive reinforcement

  Strength and spacing requirements at the positive critical sections are satisfied using 5-#6 bars where the width of the beam is equal to 28.0 in.

Use 5-#6 bars at all critical sections.

6.9.3 Example 6.3 – Determination of Shear Reinforcement: Beam in Building #1 (Framing Option C), Beam is Not Part of the LFRS, SDC A

Determine the required shear reinforcement for the beam along column line 4 in Building #1, Framing Option C, at a typical floor level assuming the beam is not part of the LFRS (see Figure 1.1). Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 6.1 and 6.2.

Step 1 – Determine the factored shear forces along the span

It is determined in Step 4 of Example 5.9 that $\alpha_{f1}\ell_2 / \ell_1 > 1.0$ for the interior beams in the north-south direction. Therefore, these beams must be designed to resist 100 percent of the shear forces in this slab system (see Sect. 5.3.4 of this publication).

The tributary area that must be used to determine V_u for the beam in this example is given in Figure 6.37, which is the same for all beams along any interior column line in the north-south direction (see Sect. 5.3.4 of this publication).

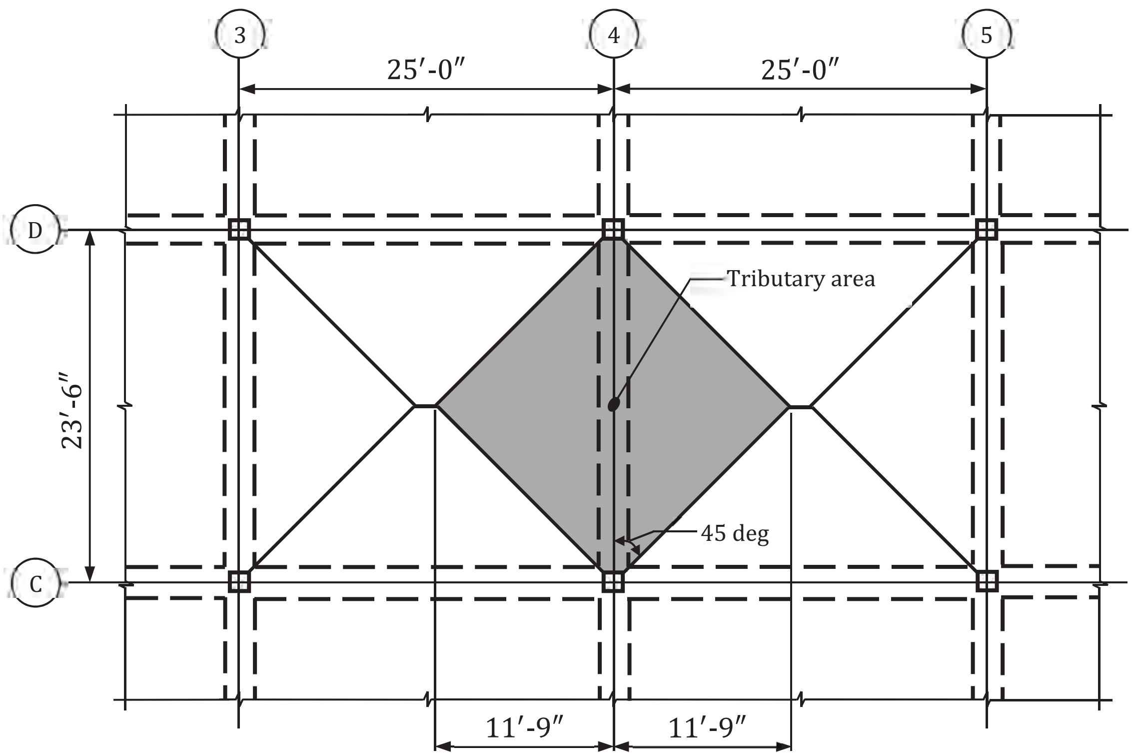


Figure 6.37 Tributary area for shear forces on the beam in Example 6.3.

Determine the factored dead and live loads on the beam:

$$\text{Slab dead load} = (7.0 / 12) \times 150.0 \times (11.75 + 11.75) / 1{,}000 = 2.1 \text{ kips/ft}$$

$$\text{Dead load of beam web} = [28.0 \times (24.0 - 7.0) / 144] \times 150.0 / 1{,}000 = 0.50 \text{ kip/ft}$$

For simplicity, the uniformly distributed dead load of the beam web is transformed into a triangular distributed load. The shear force at the face of the support due to the uniformly distributed dead load is equal to $0.50 \times 21.5 / 2 = 5.4$ kips. The triangular distributed load that produces a shear force of 5.4 kips at the face of the support is equal to $4 \times 5.4 / 21.5 = 1.0$ kip/ft.

$$\text{Superimposed dead load} = 10.0 \times (11.75 + 11.75) / 1{,}000 = 0.24 \text{ kip/ft}$$

$$\text{Live load} = 65.0 \times (11.75 + 11.75) / 1{,}000 = 1.5 \text{ kips/ft}$$

$$w_u = [1.2 \times (2.1 + 1.0 + 0.24)] + (1.6 \times 1.5) = 6.4 \text{ kips/ft} \qquad \text{Table 3.3}$$

The maximum shear force occurs at the face of the first interior support in an end span and is obtained from equilibrium considering the 6.4-kips/ft triangular load and the 49.2 ft-kip and 165.0 ft-kip negative moments at the exterior support and the face of the first interior support, respectively (see Figure 6.38 and Table 6.24). It is assumed for sim-

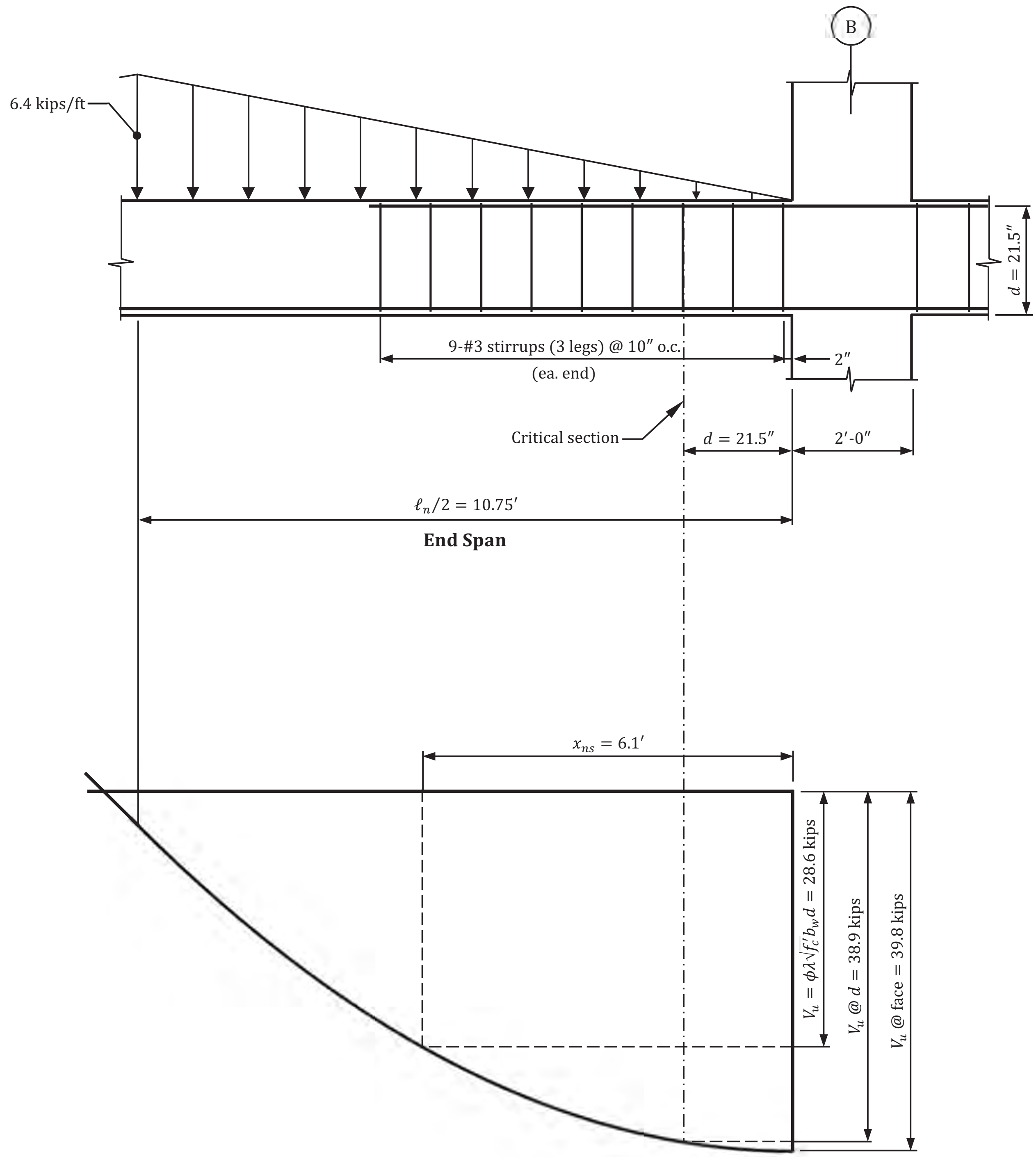


Figure 6.38 Partial shear diagram for the beam in Example 6.3.

plicity that the triangular load is equal to zero at the faces of the supports instead of at the centers of the columns; this assumption has a negligible impact on the results.

$$V_u \text{ @ face} = \left[\frac{1}{2} \times \left(\frac{6.4 \times 21.5}{2}\right)\right] + \left(\frac{165.0 - 49.2}{21.5}\right) = 39.8 \text{ kips}$$

The 3 conditions in ACI 9.4.3.2 are satisfied, so the critical section for shear can be taken a distance $d = 21.5$ in. $= 1.79$ ft from the face of the support:

$$V_u \ @\ d = 39.8 - \left[\left(\frac{6.4 \times 1.79}{10.75}\right) \times \left(\frac{1.79}{2}\right)\right] = 38.9 \text{ kips}$$

Step 2 – Determine the required shear reinforcement

Assuming at least minimum shear reinforcement is provided, ϕV_c can be determined from the following equation with the axial force $N_u = 0$:

$$\phi V_c = \phi 2\lambda\sqrt{f_c'}b_w d = 0.75 \times 2 \times 1.0\sqrt{4{,}000} \times 28.0 \times 21.5 / 1{,}000 = 57.1 \text{ kips}$$ Table 6.4

where $\lambda = 1.0$ for normalweight concrete. Table 6.2

Because $\phi V_c = 57.1 \text{ kips} > V_u = 38.9 \text{ kips} > \phi\lambda\sqrt{f_c'}b_w d = 28.6 \text{ kips}$, provide minimum shear reinforcement:

$$\frac{A_{v,min}}{s} = \text{ greater of } \begin{cases} 0.75\sqrt{f_c'}b_w / f_{yt} = 0.75\sqrt{4{,}000} \times 28.0 / 60{,}000 = 0.022 \text{ in.}^2/\text{in.} \\ \\ 50b_w / f_{yt} = 50 \times 28.0 / 60{,}000 = 0.023 \text{ in.}^2/\text{in.} \end{cases}$$ Table 6.5

For $V_u - \phi V_c < \phi 4\sqrt{f_c'}b_w d$:

Maximum spacing $s_{max} = \text{lesser of } d / 2 = 10.8 \text{ in. or } 24.0 \text{ in.}$ along the length of the beam. Figure 6.19

Assuming $s = 10$ in., $A_{v,min}$ is equal to the following:

$$A_{v,min} = 0.023 \times 10.0 = 0.23 \text{ in.}^2$$

Across the width of the beam, maximum spacing $s_{max} = \text{lesser of } d = 21.5 \text{ in. or } 24.0 \text{ in.}$ Figure 6.19

Assuming #4 U-stirrups (2 outer legs) with 1.5-in. cover:

$$s = 28.0 - (2 \times 1.5) - 0.5 = 24.5 \text{ in.} > s_{max} = 21.5 \text{ in.}$$

Therefore, 3 stirrup legs must be provided to satisfy maximum spacing requirements across the width of the beam.

Use #3 stirrups with 3 legs spaced at 10 in. on center ($A_{v,provided} = 3 \times 0.11 = 0.33 \text{ in.}^2 > A_{v,min} = 0.23 \text{ in.}^2$).

Stirrups are no longer required at the section where $V_u = \phi\lambda\sqrt{f_c'}b_w d = 28.6$ kips. The length from the face of the support where stirrups are no longer required, x_{ns}, can be obtained from the following equation (see Figure 6.38):

$$28.6 = 39.8 - \left(\frac{6.4x_{ns}}{10.75}\right)\left(\frac{x_{ns}}{2}\right) \rightarrow x_{ns} = 6.1 \text{ ft}$$

Use 9-#3 stirrups (3 legs) spaced at 10 in. on center at both ends of the beam with the first stirrup located 2 in. from the face of the column.

6.9.4 Example 6.4 – Determination of Reinforcement Details: Beam in Building #1 (Framing Option C), Beam is Not Part of the LFRS, SDC A

Determine the required reinforcement details for the beam along column line 4 in Building #1, Framing Option C, at a typical floor level assuming the beam is not part of the LFRS (see Figure 1.1). Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 6.1 through 6.3.

Step 1 – Determine the flexural reinforcement details

It is determined in Example 6.2 that 5-#6 bars are required at the negative and positive critical sections in the beam. This number of reinforcing bars satisfies the minimum and maximum spacing requirements in ACI 25.2 and 24.3, respectively (see Step 4 in Example 6.2).

Beyond the point of inflection in the moment diagram (that is, at the section where $M_u = 0$), the negative reinforcement is no longer required for strength. The maximum length to the point of inflection occurs in the end span (see Figure 6.39):

$$\left(\frac{6.4x_{pi}}{10.75}\right)\left(\frac{x_{pi}}{2}\right)\left(\frac{x_{pi}}{3}\right) - 39.8x_{pi} + 165.0 = 0 \rightarrow x_{pi} = 4.35 \text{ ft}$$

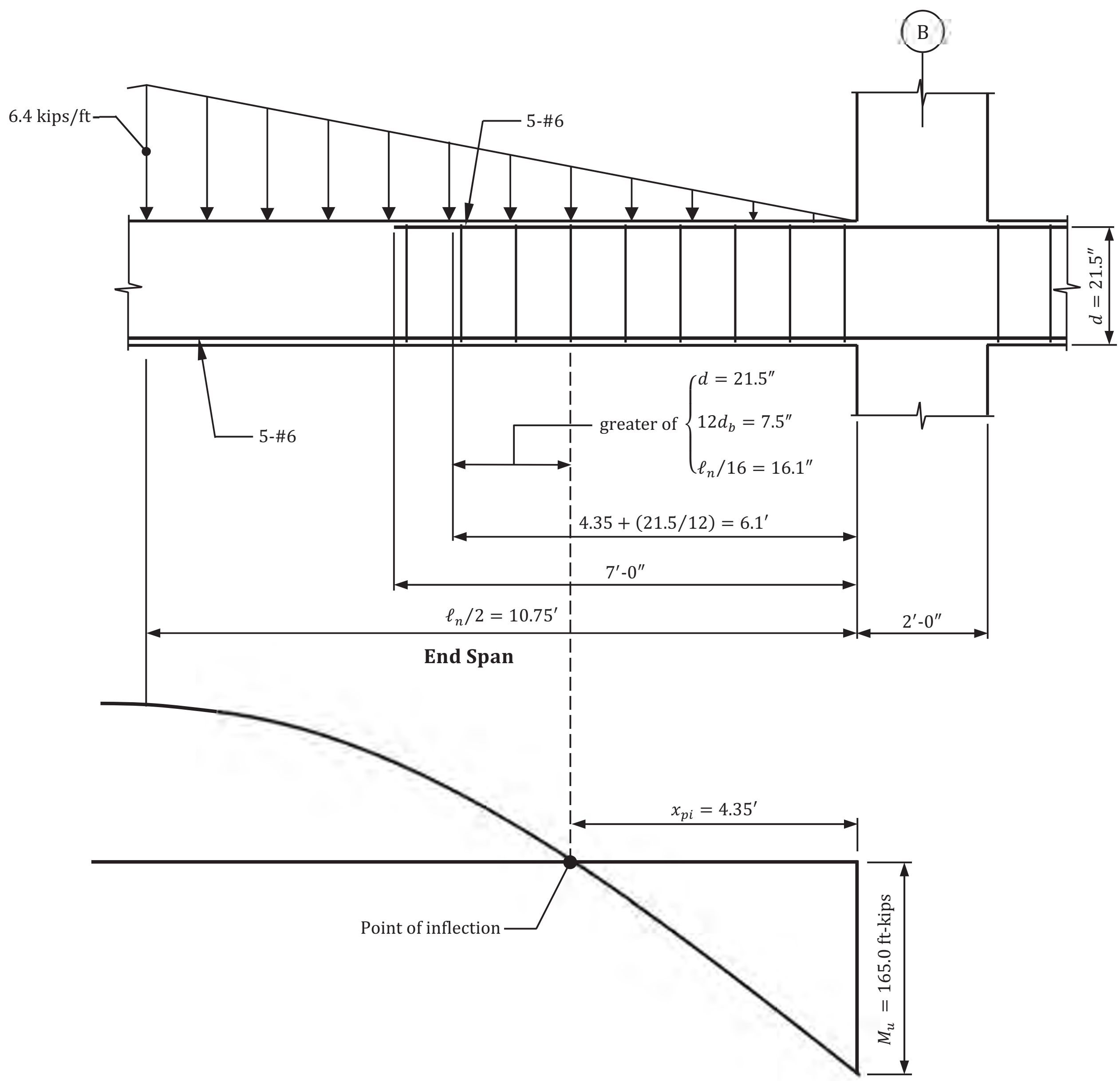


Figure 6.39 Location of the point of inflection for the beam in Example 6.4

The negative reinforcement must have an embedment length beyond the point of inflection equal to the greater of the following:

$$\text{Embedment length} = \text{greater of} \begin{cases} d = 21.5 \text{ in.} \\ 12d_b = 12 \times 0.75 = 9.0 \text{ in.} \\ \ell_n / 16 = 21.5 \times 12 / 16 = 16.1 \text{ in.} \end{cases}$$ ACI 9.7.3.8.4

Therefore, the 5-#6 top bars are permitted to be cut off a distance equal to $4.35 + (21.5 / 12) = 6.1 \text{ ft}$ from the face of the support. However, from Example 6.3, 9-#3 stirrups (3 legs) spaced at 10 in. on center must be provided at both ends of the beam with the first stirrup located 2 in. from the face of the column. This means the last stirrup is located $(80.0 + 2.0) / 12 = 6.8 \text{ ft}$ from the face of the support. Therefore, provide a 7 ft-0 in. embedment length for the 5-#6 top bars. This reinforcement is not terminated in a tension zone (see Figure 6.39) and can be conservatively used at all support locations.

For simpler detailing, the 5-#6 bottom bars are made continuous instead of cutting off a portion of them, the latter of which is shown in Figure 6.32. According to Note 2 in that figure, a Class B tension lap splice is required over the supports. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #6 reinforcing bars, $\psi_s = 0.8$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + (d_b)_{stirrup} + 0.5(d_b)_{long.} = 1.5 + 0.375 + (0.5 \times 0.75) = 2.3 \text{ in.} \\ \dfrac{s}{2} = \dfrac{28.0 - [2 \times (1.5 + 0.375)] - 0.75}{2 \times 4} = 2.9 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.3 + 0) / 0.75 = 3.1 > 2.5 \text{ use } 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{2.5} \right) \times 0.75 = 17.1 \text{ in.} > 12.0 \text{ in.}$$

$\text{Class B lap splice length} = 1.3\ell_d = 1.3 \times 17.1 = 22.2 \text{ in.}$ ACI Table 25.5.2.1

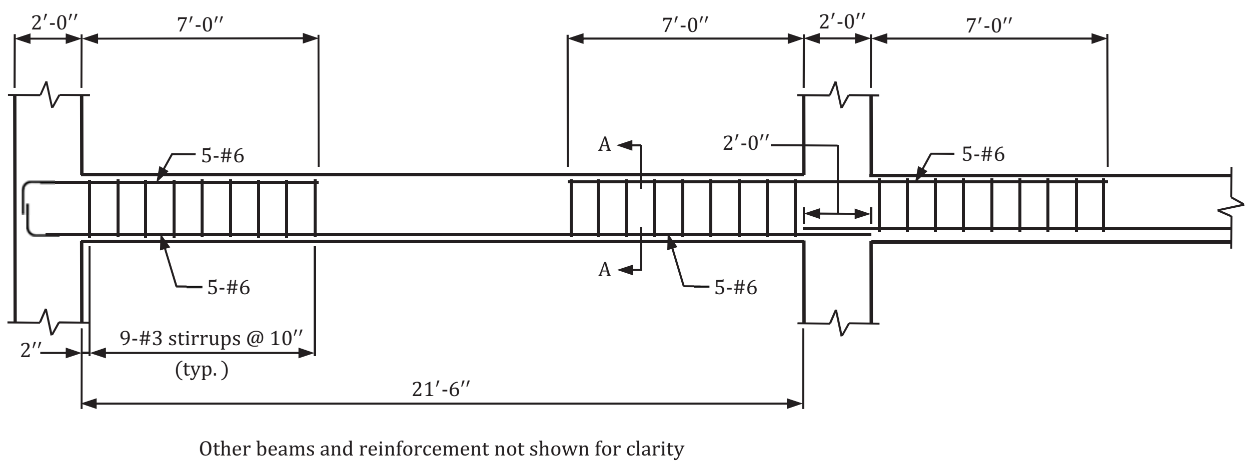
Provide a 2 ft-0 in. lap splice length.

Step 2 – Determine the shear reinforcement details

It is determined in Example 6.3 that 9-#3 stirrups with 3 legs spaced at 10 in. on center are adequate for shear strength requirements. This arrangement consists of (1) a U-stirrup extending across the effective width of the beam $[\text{which is equal to } b_w - (2 \times \text{cover})]$ with 90-degree hooks at each end and (2) a crosstie with 90-degree and 135-degree hooks at the ends tied to the top and bottom longitudinal reinforcement.

The minimum beam width required for development of the #3 stirrups is 10.0 in. < 28.0 in. Table 6.17

Reinforcement details for the beam are given in Figure 6.40. The details satisfy the structural integrity requirements in ACI 9.7.7 for beams other than perimeter beams and can be used for all beams along the interior column lines in the north-south direction that are not part of the LFRS.



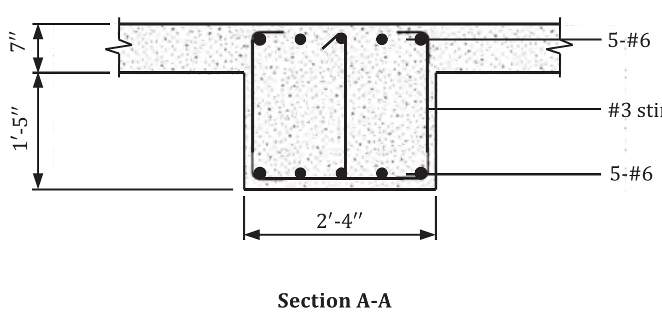


Figure 6.40 Reinforcement details for the beam in Examples 6.1 through 6.4.

6.9.5 Example 6.5 – Determination of Deflections: Beam in Building #1 (Framing Option C), Beam is Not Part of the LFRS, SDC A

Determine the immediate and time-dependent deflections for the beam in the end span along column line 4 in Building #1, Framing Option C, at a typical floor level assuming the beam is not part of the LFRS (see Figure 1.1). Assume the beam is not supporting or attached to nonstructural elements likely to be damaged by large deflections and 30 percent of the live load is sustained. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 6.1 through 6.4.

Step 1 – Determine the service loads and bending moments

$$\text{Slab dead load} = (7.0 / 12) \times 150.0 = 87.5 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$$

$$q_D = 87.5 + 10.0 = 97.5 \text{ lb/ft}^2$$

$$\text{Dead load of beam web} = w_D = [28.0 \times (24.0 - 7.0) / 144] \times 150.0 / 1{,}000 = 0.50 \text{ kip/ft}$$

$$q_L = 65.0 \text{ lb/ft}^2$$

The Direct Design Method can be used to determine the bending moments in the beam due to the weight of the slab, the superimposed dead load, and the live load (see Steps 3 and 4 of Example 5.9).

$$M_{oD} = \frac{q_D \ell_2 \ell_n^2}{8} = \frac{97.5 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 140.8 \text{ ft-kips}$$

$$M_{oL} = \frac{q_L \ell_2 \ell_n^2}{8} = \frac{65.0 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 93.9 \text{ ft-kips}$$

The bending moment coefficients at the critical sections in the beam in the end span are given in Table 5.32.

Exterior negative:

$$0.10 M_{oD} = 0.10 \times 140.8 = 14.1 \text{ ft-kips}$$

$$0.10 M_{oL} = 0.10 \times 93.9 = 9.4 \text{ ft-kips}$$

Positive:

$$0.36 M_{oD} = 0.36 \times 140.8 = 50.7 \text{ ft-kips}$$

$$0.36 M_{oL} = 0.36 \times 93.9 = 33.8 \text{ ft-kips}$$

First interior negative:

$$0.43 M_{oD} = 0.43 \times 140.8 = 60.5 \text{ ft-kips}$$

$$0.43 M_{oL} = 0.43 \times 93.9 = 40.4 \text{ ft-kips}$$

The bending moments due to the weight of the beam stem can be determined using the simplified method in ACI 6.5.1 (see Step 1 in Example 6.2).

Exterior negative:

$$M_D = \frac{w_D \ell_n^2}{16} = \frac{0.50 \times 21.5^2}{16} = 14.5 \text{ ft-kips}$$

Positive:

$$M_D = \frac{w_D \ell_n^2}{14} = \frac{0.50 \times 21.5^2}{14} = 16.5 \text{ ft-kips}$$

First interior negative:

$$M_D = \frac{w_D \ell_n^2}{10} = \frac{0.50 \times 21.5^2}{10} = 23.1 \text{ ft-kips}$$

The sustained positive and negative bending moments are the following where 30 percent of the live load is sustained:

$$M_{sus}^{+} = 50.7 + 16.5 + (0.3 \times 33.8) = 77.3 \text{ ft-kips}$$

At the exterior support: $M_{sus}^{-} = 14.1 + 14.5 + (0.3 \times 9.4) = 31.4$ ft-kips

At the first interior support: $M_{sus}^{-} = 60.5 + 23.1 + (0.3 \times 40.4) = 95.7$ ft-kips

Step 2 – Determine the material properties of the concrete and reinforcing steel

$f_r = 7.5\lambda\sqrt{f_c'} = 7.5 \times 1.0\sqrt{4{,}000} = 474$ psi — Eq.(6.84)

where $\lambda = 1.0$ for normalweight concrete. — Table 6.2

$E_c = 57{,}000\sqrt{f_c'} = 57{,}000\sqrt{4{,}000} = 3{,}605{,}000$ psi for normalweight concrete — ACI Eq. (19.2.2.1b)

$E_s = 29{,}000{,}000$ psi — ACI 20.2.2.2

Modular ratio $n = E_s / E_c = 8.0$

Step 3 – Determine the gross and cracked moments of inertia

Effective flange width of the beam $b_f = 92.5$ in. — Step 2, Example 6.2

- Positive moment section

  The gross section of the beam at the positive moment section is shown in Figure 6.41.

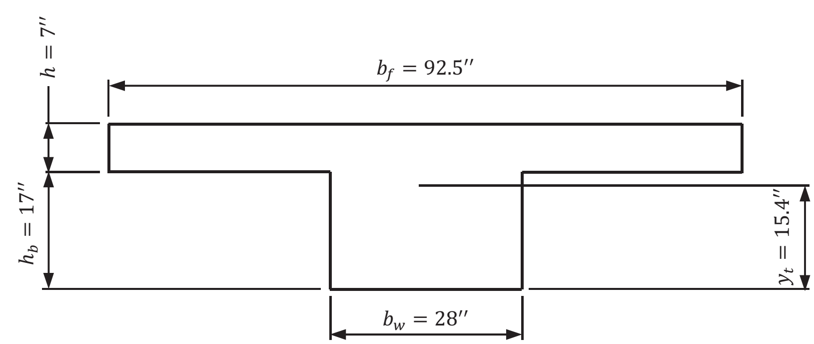


Figure 6.41 Gross section of the beam at a positive moment section.

The gross section properties of the beam are the following:

$$y_t = \frac{(b_f - b_w)(h_b + 0.5h)h + 0.5b_w(h_b + h)^2}{(b_f - b_w)h + b_w(h_b + h)}$$

Figure 6.35

$$= \frac{\{(92.5 - 28.0) \times [17.0 + (0.5 \times 7.0)] \times 7.0\} + [0.5 \times 28.0 \times (17.0 + 7.0)^2]}{[(92.5 - 28.0) \times 7.0] + [28.0 \times (17.0 + 7.0)]} = 15.4 \text{ in.}$$

$$I_g = \frac{1}{12}(b_f - b_w)h^3 + (b_f - b_w)h(h_b + 0.5h - y_t)^2 + \frac{1}{12}b_w(h_b + h)^3 + b_w(h_b + h)[y_t - 0.5(h_b + h)]^2$$

$$= \left[\frac{1}{12} \times (92.5 - 28.0) \times 7.0^3\right] + \left[(92.5 - 28.0) \times 7.0 \times (17.0 + 3.5 - 15.4)^2\right] + \left(\frac{1}{12} \times 28.0 \times 24.0^3\right)$$

$$+\{28.0 \times (17.0 + 7.0) \times [15.4 - (0.5 \times 24.0)]^2\}$$

$$= 53{,}612 \text{ in.}^4$$

From Example 6.2, 5-#6 bottom bars are required $(A_s = 2.20 \text{ in.}^2)$.

Assuming a rectangular compression area, the cracked section properties are the following (see Figure 6.42).

$$a_1 = \frac{b_f}{nA_s} = \frac{92.5}{8.0 \times 2.20} = 5.3 \text{ / in.}$$ Figure 6.34

$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1} = \frac{\sqrt{(2 \times 5.3 \times 21.5) + 1} - 1}{5.3} = 2.7 \text{ in.} < h = 7.0 \text{ in.}$$

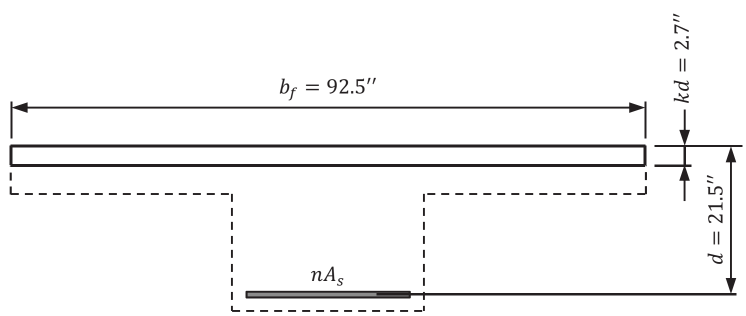


Figure 6.42 Cracked transformed section of the beam at a positive moment section.

Therefore, the assumption of a rectangular section is correct.

$$I_{cr} = \frac{b_f(kd)^3}{3} + nA_s(d - kd)^2 = \frac{92.5 \times 2.7^3}{3} + [(8.0 \times 2.20) \times (21.5 - 2.7)^2] = 6{,}827 \text{ in.}^4$$ Figure 6.34

- Negative moment section

  The negative moment section of the beam is the 28.0 by 24.0 in. beam web. From Example 6.2, 5-#6 top bars are required at the exterior and first interior sections $(A_s = 2.20 \text{ in.}^2)$.

$$I_g = \frac{1}{12}b_w(h_b + h)^3 = \frac{1}{12} \times 28.0 \times (17.0 + 7.0)^3 = 32{,}256 \text{ in.}^4$$ Figure 6.34

$$a_1 = \frac{b_w}{nA_s} = \frac{28.0}{8.0 \times 2.20} = 1.6 \text{ / in.}$$

$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1} = \frac{\sqrt{(2 \times 1.6 \times 21.5) + 1} - 1}{1.6} = 4.6 \text{ in.}$$

$$I_{cr} = \frac{b_w(kd)^3}{3} + nA_s(d-kd)^2 = \frac{28.0 \times 4.6^3}{3} + \left[(8.0 \times 2.20) \times (21.5 - 4.6)^2\right] = 5{,}935 \text{ in.}^4$$

Step 4 – Determine the effective moment of inertia

The effective moment of inertia, I_e, is determined by Eq. (6.88) for dead, sustained, and dead plus live load cases.

- Positive moment section

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474 \times 53{,}612}{15.4 \times 12{,}000} = 137.5 \text{ ft-kips}$$

$$M_D^+ = 50.7 + 16.5 = 67.2 \text{ ft-kips} < 2M_{cr} / 3 = 91.7 \text{ ft-kips}$$

Therefore, $(I_e)_D^+ = I_g = 53{,}612 \text{ in.}^4$

$$M_{sus}^+ = 77.3 \text{ ft-kips} < 2M_{cr} / 3 = 91.7 \text{ ft-kips}$$

Therefore, $(I_e)_{sus}^+ = I_g = 53{,}612 \text{ in.}^4$

$$M_{D+L}^+ = 50.7 + 16.5 + 33.8 = 101.0 \text{ ft-kips} > 2M_{cr} / 3 = 91.7 \text{ ft-kips}$$

Therefore,

$$(I_e)_{D+L}^+ = \frac{I_{cr}}{1 - \left(\dfrac{2M_{cr}/3}{M_{D+L}^+}\right)^2 \left(1 - \dfrac{I_{cr}}{I_g}\right)} = \frac{6{,}827}{1 - \left[\left(\dfrac{91.7}{101.0}\right)^2 \times \left(1 - \dfrac{6{,}827}{53{,}612}\right)\right]} = 24{,}326 \text{ in.}^4 < I_g \qquad \text{Eq. (6.88)}$$

- Negative moment section

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474 \times 32{,}256}{12.0 \times 12{,}000} = 106.2 \text{ ft-kips}$$

At the exterior support:

$$M_D^- = 14.1 + 14.5 = 28.6 \text{ ft-kips} < 2M_{cr} / 3 = 70.8 \text{ ft-kips}$$

Therefore, $(I_e)_D^- = I_g = 32{,}256 \text{ in.}^4$

$$M_{sus}^- = 31.4 \text{ ft-kips} < 2M_{cr} / 3 = 70.8 \text{ ft-kips}$$

Therefore, $(I_e)_{sus}^- = I_g = 32{,}256 \text{ in.}^4$

$$M_{D+L}^- = 14.1 + 14.5 + 9.4 = 38.0 \text{ ft-kips} < 2M_{cr} / 3 = 70.8 \text{ ft-kips}$$

Therefore, $(I_e)_{D+L}^- = I_g = 32{,}256 \text{ in.}^4$

At the first interior support:

$$M_D^- = 60.5 + 23.1 = 83.6 \text{ ft-kips} > 2M_{cr} / 3 = 70.8 \text{ ft-kips}$$

Therefore,

$$(I_e)_D^- = \frac{I_{cr}}{1 - \left(\dfrac{2M_{cr}/3}{M_D^-}\right)^2 \left(1 - \dfrac{I_{cr}}{I_g}\right)} = \frac{5{,}935}{1 - \left[\left(\dfrac{70.8}{83.6}\right)^2 \times \left(1 - \dfrac{5{,}935}{32{,}256}\right)\right]} = 14{,}310 \text{ in.}^4 < I_g$$

$$M_{sus}^{-} = 95.7 \text{ ft-kips} > 2M_{cr} / 3 = 70.8 \text{ ft-kips}$$

Therefore,

$$(I_e)_{sus}^{-} = \frac{I_{cr}}{1 - \left(\dfrac{2M_{cr}/3}{M_{sus}^{-}}\right)^2 \left(1 - \dfrac{I_{cr}}{I_g}\right)} = \frac{5{,}935}{1 - \left[\left(\dfrac{70.8}{95.7}\right)^2 \times \left(1 - \dfrac{5{,}935}{32{,}256}\right)\right]} = 10{,}725 \text{ in.}^4 < I_g$$

$$M_{D+L}^{-} = 60.5 + 23.1 + 40.4 = 124.0 \text{ ft-kips} > 2M_{cr} / 3 = 70.8 \text{ ft-kips}$$

Therefore,

$$(I_e)_{D+L}^{-} = \frac{I_{cr}}{1 - \left(\dfrac{2M_{cr}/3}{M_{D+L}^{-}}\right)^2 \left(1 - \dfrac{I_{cr}}{I_g}\right)} = \frac{5{,}935}{1 - \left[\left(\dfrac{70.8}{124.0}\right)^2 \times \left(1 - \dfrac{5{,}935}{32{,}256}\right)\right]} = 8{,}086 \text{ in.}^4 < I_g$$

- Average effective moment of inertia

  Dead load:

  $$(I_e)_D = \frac{53{,}612 + 32{,}256 + 14{,}310}{3} = 33{,}393 \text{ in.}^4$$ ACI 24.2.3.6

  Sustained load:

  $$(I_e)_{sus} = \frac{53{,}612 + 32{,}256 + 10{,}725}{3} = 32{,}198 \text{ in.}^4$$

  Dead plus live load:

  $$(I_e)_{D+L} = \frac{24{,}326 + 32{,}256 + 8{,}086}{3} = 21{,}556 \text{ in.}^4$$

Step 5 – Determine the immediate deflections and check the maximum permissible deflection for live loads

Assume the loads due to the dead load of the slab, the superimposed dead load, and the live load are uniformly distributed over the span instead of the actual triangular load distribution. The following equation can be used to determine immediate deflections:

$$\Delta_i = \frac{5KM_a\ell^2}{48E_cI_e}$$ Eq. (6.89)

For continuous members:

$$K = 1.2 - 0.2(M_o / M_a)$$ Table 6.19

where $M_o = w\ell^2 / 8$ and M_a is the service load moment at midspan. Also assume $K = 1.2$.

- Dead load

  $$(\Delta_i)_D = \frac{5KM_D^{+}\ell^2}{48E_c(I_e)_D} = \frac{5 \times 1.2 \times 67.2 \times 12{,}000 \times (21.5 \times 12)^2}{48 \times 3{,}605{,}000 \times 33{,}393} = 0.06 \text{ in.}$$

- Sustained load

  $$(\Delta_i)_{sus} = \frac{5KM_{sus}^{+}\ell^2}{48E_c(I_e)_{sus}} = \frac{5 \times 1.2 \times 77.3 \times 12{,}000 \times (21.5 \times 12)^2}{48 \times 3{,}605{,}000 \times 32{,}198} = 0.07 \text{ in.}$$

- Dead plus live load

$$(\Delta_i)_{D+L} = \frac{5KM^+_{D+L}\ell^2}{48E_c(I_e)_{D+L}} = \frac{5\times 1.2\times 101.0\times 12{,}000\times(21.5\times 12)^2}{48\times 3{,}605{,}000\times 21{,}556} = 0.13 \text{ in.}$$

- Live load

$$(\Delta_i)_L = (\Delta_i)_{D+L} - (\Delta_i)_D = 0.13 - 0.06 = 0.07 \text{ in.}$$

For a floor member not supporting or attached to nonstructural elements likely to be damaged by large deflections, the maximum permissible immediate live load deflection is equal to the following:

$$(\Delta_i)_{L|max} = \ell / 360 = (21.5\times 12)/360 = 0.72 \text{ in.} > (\Delta_i)_L = 0.07 \text{ in.}$$ Table 6.21

Step 6 – Determine the time-dependent deflections and check the maximum permissible deflection

Assuming a 60-month duration of loading with $\rho' = 0$:

$$\lambda_\Delta = \frac{\xi}{1+50\rho'} = \frac{2.0}{1+0} = 2.0$$ Eq. (6.90), Table 6.20

The time-dependent deflection due to creep and shrinkage is equal to the following:

$$\Delta_{cs} = \lambda_\Delta(\Delta_i)_{sus} = 2.0\times 0.07 = 0.14 \text{ in.}$$ Eq. (6.91)

$$\text{Total deflection} = \Delta_{cs} + (\Delta_i)_L = 0.14 + 0.07 = 0.21 \text{ in.} < \ell / 240 = 1.1 \text{ in.}$$ Table 6.21

The 28.0 in. by 24.0 in. beam is adequate for deflection.

6.9.6 Example 6.6 – Determination of Flexural Reinforcement: Beam in Building #1 (Framing Option C), Beam is Part of the LFRS, SDC A, Multiple Layers of Tension Reinforcement

Determine the required flexural reinforcement for the beam in an exterior span along column line 4 in Building #1, Framing Option C, at the second-floor level assuming the beam is part of the LFRS and the dimensions of the beam are limited to 12 in. by 24 in. (see Figure 1.1). Assume the live load at this floor level is equal to 175 lb/ft^2 and the columns are 24.0 in. by 24.0 in. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the factored bending moments, M_u along the span

It can be determined using the requirements in ACI 8.3.1.2 that a 7.0-in.-thick slab is adequate for serviceability based on the 12 in. by 24 in. beams. Also, the Direct Design Method can be used to determine the bending moments in the column and middle strips of the slab (see Sect. 5.3.4 of this publication).

$$\text{Slab dead load} = (7.0/12)\times 150.0 = 87.5 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$$

$$q_D = 87.5 + 10.0 = 97.5 \text{ lb/ft}^2$$

$$\text{Dead load of beam web} = w_D = [12.0\times(24.0-7.0)/144]\times 150.0/1{,}000 = 0.21 \text{ kip/ft}$$

$$q_L = 175.0 \text{ lb/ft}^2$$

According to the Direct Design Method, the beams in this example must resist 85 percent of the column strip bending moments due to the weight of the slab, the superimposed dead load, and the live load.

$$M_{oD} = \frac{q_D \ell_2 \ell_n^2}{8} = \frac{97.5 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 140.8 \text{ ft-kips}$$

$$M_{oL} = \frac{q_L \ell_2 \ell_n^2}{8} = \frac{175.0 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 252.8 \text{ ft-kips}$$

The bending moments due to weight of the beam stem can be determined using the simplified method in ACI 6.5.1.

A summary of the service bending moments due to the dead and live loads at the critical sections of the beam in the end span is given in Table 6.25.

Table 6.25 Bending Moments (ft-kips) due to Service Dead and Live Loads at the Second-Floor Level for the Beam in the End Span in Example 6.6

Exterior Negative	Positive	First Interior Negative
$0.13M_{oD} + \frac{w_D \ell_n^2}{16} = -24.4$ $0.13M_{oL} = -32.9$	$0.36M_{oD} + \frac{w_D \ell_n^2}{14} = 57.6$ $0.36M_{oL} = 91.0$	$0.43M_{oD} + \frac{w_D \ell_n^2}{10} = -70.3$ $0.43M_{oL} = -108.7$

The bending moments in the beam due to wind loads are given in Table 6.26. The "plus-minus" sign preceding the tabulated values signifies the wind loads can act in both the north direction and the south direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the east elevation of the building).

Table 6.26 Bending Moments (ft-kips) due to Wind Loads at the Second-Floor Level for the Beam in the End Span in Example 6.6

Exterior Negative	Positive	First Interior Negative
± 39.2	—	± 38.9

The design bending moments from the governing load combinations are given in Table 6.27.

Table 6.27 Design Bending Moments (ft-kips) at the Second-Floor Level for the Beam in the End Span in Example 6.6

Load Combination		Exterior Negative	Positive	First Interior Negative
$1.2D + 1.6L$		-81.9	214.7	-258.3
$1.2D + 1.0W + 0.5L$	SSR	-6.5	114.6	-177.6
	SSL	-84.9	114.6	-99.8
$0.9D + 1.0W$	SSR	17.2	51.8	-102.2
	SSL	-61.2	51.8	-24.4

Step 2 – Determine the effective flange width, b_f, of the beam ACI 6.3.2.1

The effective flange width, b_f, is equal to the following:

$$b_f = b_w + \text{ least of } \begin{cases} 16h = 16 \times 7.0 = 112.0 \text{ in.} \\ s_w = (25.0 \times 12) - 12.0 = 288.0 \text{ in.} \\ \ell_n / 4 = (23.5 - 2.0) \times 12 / 4 = 64.5 \text{ in.} \end{cases}$$

Figure 6.10

$$= 12.0 + 64.5 = 76.5 \text{ in.}$$

Step 3 – Determine the required flexural reinforcement at the critical sections

At negative moment critical sections (flange in tension), required A_s is determined by the following where $d = 24.0 - 2.5 = 21.5$ in. for beams with one layer of tension reinforcement:

$$R_n = \frac{M_u}{\phi b d^2} \qquad \text{Eq. (6.34)}$$

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] \qquad \text{Eq. (6.33)}$$

where $b = b_w = 12.0$ in. Figure 6.11

At positive moment critical sections (flange in compression), determine the depth of the equivalent stress block, a, assuming rectangular section behavior using the largest positive M_u in Table 6.27 (which results in the largest a):

$$a = \frac{a_1 d - \sqrt{(a_1 d)^2 - (2 a_1 M_u / \phi)}}{a_1}$$

Eq. (6.49)

$$= \frac{(260.1 \times 20.5) - \sqrt{(260.1 \times 20.5)^2 - (2 \times 260.1 \times 214.7 \times 12 / 0.9)}}{260.1} = 0.54 \text{ in.} < h = 7.0 \text{ in.}$$

where $a_1 = 0.85 f_c' b_f = 0.85 \times 4 \times 76.5 = 260.1$ kips/in. and $d = 24.0 - 3.5 = 20.5$ in. (where it is assumed 2 layers of tension reinforcement are provided because the beam is relatively narrow).

Because $a < h$, the section behaves as a rectangular section.

Determine if the reinforcing steel in the inner layer located d_2 from the extreme compression fiber yields assuming a 1.5-in. clear cover to #4 stirrups, #8 longitudinal bars, and a 1.0-in. clear spacing between the layers of longitudinal reinforcement:

$$c = a / \beta_1 = 0.54 / 0.85 = 0.64 \text{ in.}$$

$$\varepsilon_s = 0.003\left(\frac{d_2}{c} - 1\right) = 0.003 \times \left[\frac{24.0 - 1.5 - 0.5 - 1.0 - 1.0 - (1.0 / 2)}{0.64} - 1\right] = 0.088 > \varepsilon_{ty} = 60 / 29{,}000 = 0.0021$$

Therefore, the reinforcing steel in the inner layer yields and A_s can be determined using Eqs. (6.34) and (6.33) with $b = b_f = 76.5$ in. and $d = 24.0 - 1.5 - 0.5 - 1.0 - (1.0 / 2) = 20.5$ in.

A summary of the required reinforcement is given in Table 6.28. It is evident that all sections are tension-controlled because $A_s < A_{s,t}$.

Table 6.28 Required Flexural Reinforcement for the Beam in Example 6.6

Location	M_u (ft-kips)	R_n (psi)	A_s (in^2)*
Exterior negative	−84.9	204	0.91
Positive	214.7	89	2.36
First interior negative	−258.3	621	2.97

$$*Min.\ A_s = 200 b_w d / f_y = \begin{cases} 200 \times 12.0 \times 21.5 / 60{,}000 = 0.86 \text{ in.}^2 \text{ at negative critical sections} \\ 200 \times 12.0 \times 20.5 / 60{,}000 = 0.82 \text{ in.}^2 \text{ at positive critical sections} \end{cases} \quad \text{[Eq. (6.35)]}$$

$$Max.\ A_{s,t} = 0.018 bd = \begin{cases} 0.018 \times 12.0 \times 21.5 = 4.64 \text{ in.}^2 \text{ at negative critical sections} \\ 0.018 \times 76.5 \times 20.5 = 28.2 \text{ in.}^2 \text{ at positive critical sections} \end{cases} \quad \text{[Eq. (6.38)]}$$

Step 4 – Select the flexural reinforcement Sect. 6.6.3

Select the flexural reinforcement based on the minimum and maximum spacing requirements in ACI 25.2 and 24.3, respectively.

- Negative reinforcement

  $b_f = 76.5 \text{ in.} > \ell_n / 10 = 21.5 \times 12 / 10 = 25.8 \text{ in.}$

  Because $b_f > \ell_n / 10$, distribute the required negative A_s within $\ell_n / 10$. Figure 6.23

  Exterior negative:

  With 1.5-in.-cover, #4 stirrups, Grade 60 reinforcing bars, and $f_s = 2f_y / 3 = 40{,}000$ psi, $c_c = 1.5 + 0.5 = 2.0$ in. and the maximum center-to-center bar spacing is equal to the following:

$$s = \text{ lesser of } \begin{cases} 15\left(\dfrac{40{,}000}{40{,}000}\right) - (2.5 \times 2.0) = 10.0 \text{ in.} \\ 12\left(\dfrac{40{,}000}{40{,}000}\right) = 12.0 \text{ in.} \end{cases} \quad \text{Eq. (6.65)}$$

  Use 4-#5 bars ($A_{s,provided} = 1.24 \text{ in.}^2 > 0.91 \text{ in.}^2$; $s = [26.0 - (11/16)]/3 = 8.4 \text{ in.} < s_{max} = 10.0$ in.). Table 6.7

  The reinforcement provided in the slab is at least equal to the shrinkage and temperature reinforcement in ACI 24.4.3.1 (see Table 5.33 in Example 5.9); this reinforcement satisfies the requirement in ACI 24.3.4 for additional longitudinal reinforcement in the outer portions of the flange where $b_f > \ell_n / 10$.

  First interior negative:

  Use 4-#8 bars ($A_{s,provided} = 3.16 \text{ in.}^2 > 2.97 \text{ in.}^2$; $s = (26.0 - 1.128)/3 = 8.3 \text{ in.} < s_{max} = 10.0$ in.). Table 6.7

- Positive reinforcement

  Minimum number of reinforcing bars required within a 12.0-in. width $= 2$ Table 6.9

  Maximum number of #7 bars in a 12-in.-wide section $= 3 > 2$ Table 6.8

Use 4-#7 bars in 2 layers with 2 bars per layer ($A_{s,provided} = 2.40 \text{ in.}^2 > 2.36 \text{ in.}^2$).

The 4-#7 bars are also adequate for the 17.2 ft-kip positive moment at the exterior support for SSR (see Table 6.27).

Comments. Instead of using 4-#7 bars in 2 layers for the positive reinforcement, 3-#9 bars in a single layer can be used ($A_{s,provided} = 3.00 \text{ in.}^2 > 2.97 \text{ in.}^2$), which satisfies minimum and maximum spacing requirements. The primary purpose of this example is to illustrate the design procedure with multiple layers of tension reinforcement.

6.9.7 Example 6.7 – Determination of Shear Reinforcement: Beam in Building #1 (Framing Option C), Beam is Part of the LFRS, SDC A

Determine the required shear reinforcement for the beam in an exterior span along column line 4 in Building #1, Framing Option C, at the second-floor level assuming the beam is part of the LFRS and the dimensions of the beam are limited to 12 in. by 24 in. (see Figure 1.1). Assume the live load at this floor level is equal to175 lb/ft^2. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 6.6.

Step 1 – Determine the factored shear forces along the span

It can be determined that $\alpha_{f1}\ell_2 / \ell_1 > 1.0$ for the interior beams in the north-south direction. Therefore, these beams must be designed to resist 100 percent of the shear forces in this slab system (see Sect. 5.3.4 of this publication).

For the beam in this example, the tributary area is given in Figure 6.37; this tributary area is the same for all beams along any interior column line in the north-south direction. In addition to the shear forces due to the gravity loads, the beam must resist the shear forces due to the wind loads.

Determine the factored dead and live loads on the beam:

$$\text{Slab dead load} = (7.0 / 12) \times 150.0 \times 23.5 / 1{,}000 = 2.1 \text{ kips/ft}$$

$$\text{Dead load of beam web} = [12.0 \times (24.0 - 7.0) / 144] \times 150.0 / 1{,}000 = 0.21 \text{ kip/ft}$$

For simplicity, the uniformly distributed dead load of the beam web is transformed into a triangular distributed load. The shear force at the face of the support due to the uniformly distributed dead load is equal to $0.21 \times 21.5 / 2 = 2.3$ kips. The triangular distributed load that produces a shear force of 2.3 kips at the face of the support is equal to $4 \times 2.3 / 21.5 = 0.43$ kip/ft.

$$\text{Superimposed dead load} = 10.0 \times 23.5 / 1{,}000 = 0.24 \text{ kip/ft}$$

$$\text{Live load} = 175.0 \times 23.5 / 1{,}000 = 4.1 \text{ kip/ft}$$

The maximum shear force occurs at the first interior support in an end span for the load combination $U = 1.2D + 1.6L$ (see Figure 6.43 and Table 6.27). It is assumed for simplicity that the triangular load is equal to zero at the faces of the supports instead of at the centers of the columns; this assumption has negligible impact on the results.

$$w_u = [1.2 \times (2.1 + 0.43 + 0.24)] + (1.6 \times 4.1) = 9.9 \text{ kips/ft}$$ Table 3.3

$$V_u \text{ @ face} = \left[\frac{1}{2} \times \left(\frac{9.9 \times 21.5}{2}\right)\right] + \left(\frac{258.3 - 81.9}{21.5}\right) = 61.4 \text{ kips}$$

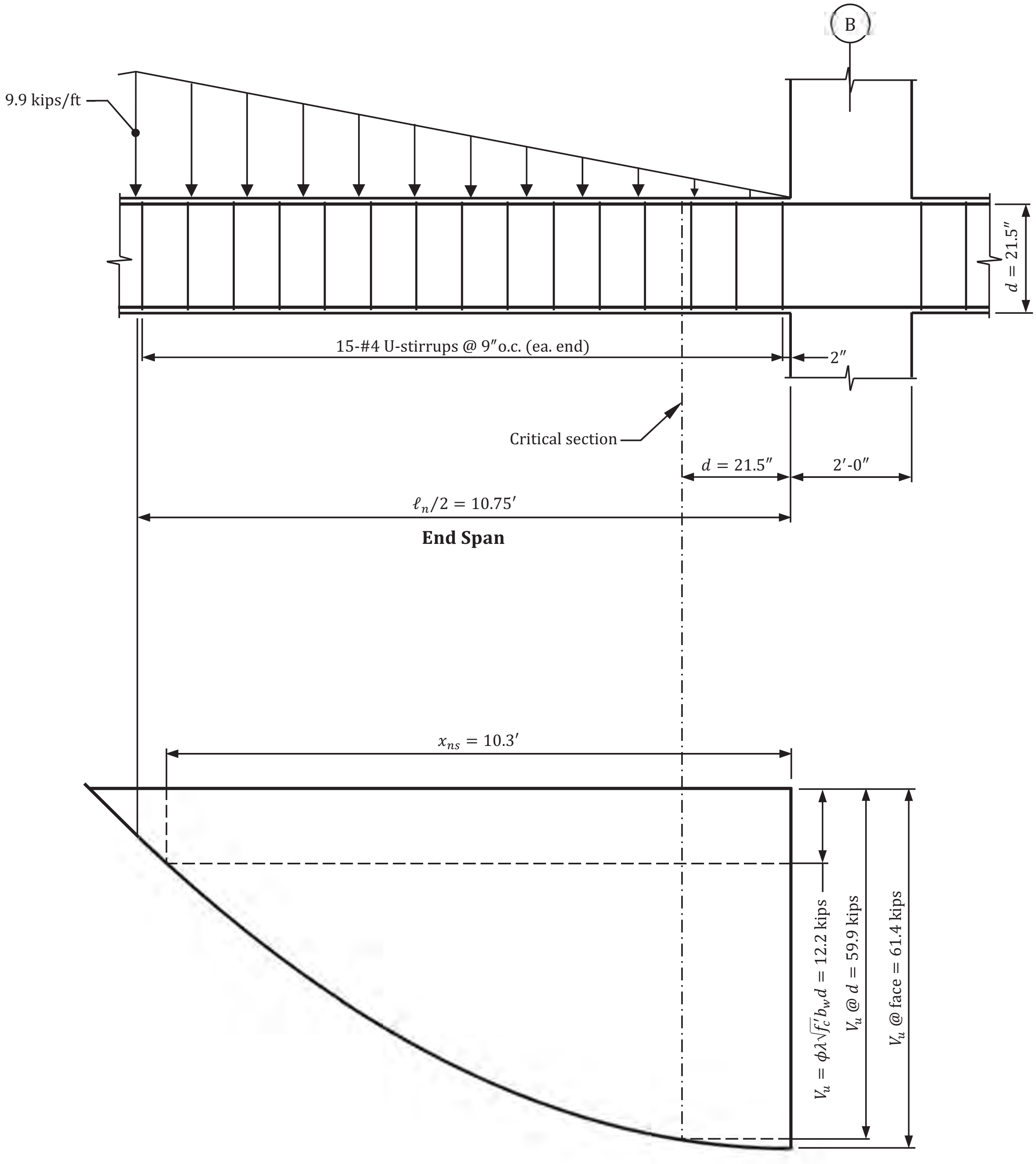


Figure 6.43 Partial shear diagram for the beam in Example 6.7.

The 3 conditions in ACI 9.4.3.2 are satisfied, so the critical section for shear can be taken a distance $d = 21.5$ in. from the face of the support (see Figure 6.4):

$$V_u \text{ @ } d = 61.4 - \left[\left(\frac{9.9 \times 1.79}{10.75}\right) \times \left(\frac{1.79}{2}\right)\right] = 59.9 \text{ kips}$$

Step 2 – Determine the required shear reinforcement

Assuming at least minimum shear reinforcement is provided, ϕV_c can be determined from the following equation with the axial force $N_u = 0$:

$\phi V_c = \phi 2\lambda\sqrt{f'_c} b_w d = 0.75 \times 2 \times 1.0\sqrt{4{,}000} \times 12.0 \times 21.5 / 1{,}000 = 24.5$ kips Table 6.4

where $\lambda = 1.0$ for normalweight concrete. Table 6.2

$V_u - \phi V_c = 59.9 - 24.5 = 35.4$ kips

For $V_u - \phi V_c = 35.4$ kips $< \phi 4\sqrt{f'_c} b_w d = 49.0$ kips:

Maximum spacing $s_{max} = d / 2 = 10.8$ in. along the length of the beam. Figure 6.19

#4 U-stirrups spaced at $d / 2$ provides $V_u - \phi V_c = 36.0$ kips > 35.4 kips Table 6.6

$$\frac{A_v}{s} = \frac{2 \times 0.20}{10} = 0.040 \text{ in.}^2 / \text{in.}$$

Table 6.5

$$> \frac{A_{v,min}}{s} = \text{ greater of } \begin{cases} 0.75\sqrt{f'_c} b_w / f_{yt} = 0.75\sqrt{4{,}000} \times 12.0 / 60{,}000 = 0.0095 \text{ in.}^2/\text{in.} \\ 50 b_w / f_{yt} = 50 \times 12.0 / 60{,}000 = 0.010 \text{ in.}^2/\text{in.} \end{cases}$$

Across the width of the beam, maximum spacing $s_{max} = d = 21.5$ in. Figure 6.19

For #4 U-stirrups with 1.5-in. cover:

$s = 12.0 - (2 \times 1.5) - 0.5 = 8.5$ in. $< s_{max} = 21.5$ in.

Use #4 U-stirrups spaced at 10 in. on center $(A_{v,provided} = 2 \times 0.20 = 0.40 \text{ in.}^2)$.

Stirrups are no longer required at the section where $V_u = \phi\lambda\sqrt{f'_c} b_w d = 12.2$ kips. The length from the face of the support where stirrups are no longer required, x_{ns}, can be obtained from the following equation (see Figure 6.43):

$$12.2 = 61.4 - \left(\frac{9.9 x_{ns}}{10.75}\right)\left(\frac{x_{ns}}{2}\right) \rightarrow x_{ns} = 10.3 \text{ ft}$$

For simpler detailing, use 15-#4 U-stirrups spaced at 9 in. on center at both ends of the beam with the first stirrup located 2 in. from the face of the column.

Comments. This shear reinforcement arrangement can be used for all beams in all spans along the column lines in the north-south direction for the frames that are part of the LFRS.

Instead of using Table 6.6 to determine the size and spacing of the shear reinforcement, use Case 3 in Table 6.5. Assuming #4 U-stirrups, the required spacing is equal to the following:

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (2 \times 0.20) \times 60 \times 21.5}{35.4} = 10.9 \text{ in.} > s_{max} = \frac{d}{2} = 10.8 \text{ in., use } s = 10.0 \text{ in.}$$

A 9.0-in. stirrup spacing is used at each end in lieu of specifying a uniform stirrup spacing across the entire length of the beam because the latter results in a spacing that is not convenient to measure in the field.

6.9.8 Example 6.8 – Determination of Reinforcement Details: Beam in Building #1 (Framing Option C), Beam is Part of the LFRS, SDC A

Determine the required reinforcement details for the beam in an exterior span along column line 4 in Building #1, Framing Option C, at the second-floor level assuming the beam is part of the LFRS and the dimensions of the beam are limited to 12 in. by 24 in. (see Figure 1.1). Assume the live load at this floor level is equal to 175 lb/ft^2. Also assume normalweight concrete with $f_c' = 4{,}000 \text{ psi}$ and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 6.6 and 6.7.

Step 1 – Determine the flexural reinforcement details

It is determined in Example 6.6 that the following reinforcement is required at the critical sections:

- Exterior negative: 4-#5 bars within 26.0 in.
- Positive: 4-#7 bars (2 layers)
- First interior negative: 4-#8 bars within 26.0 in.

The provided number of reinforcing bars at all critical sections satisfies the minimum and maximum spacing requirements in ACI 25.2 and 24.3, respectively (see Step 4 in Example 6.6).

The top reinforcing bars are made continuous over the entire length of the end span because stirrups are required over the entire length and the beam is subjected to wind loads.

A Class B tension lap splice is required near midspan. According to ACI 25.5.2.2, where bars of different size are lap spliced in tension, the required lap splice length is based on the greater of the tension development length ℓ_d of the larger bar and the tension lap splice length ℓ_{st} of the smaller bar.

The tension development length of the 4-#8 bars is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$

ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #8 reinforcing bars, $\psi_s = 1.0$

For more than 12 in. of fresh concrete cast below the negative reinforcement, $\psi_t = 1.3$

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + (d_b)_{stirrup} + 0.5(d_b)_{long.} = 1.5 + 0.50 + (0.5 \times 1.0) = 2.5 \text{ in.} \\ \dfrac{s}{2} = \dfrac{12.0 - [2 \times (1.5 + 0.50)] - 1.0}{2} = 3.5 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$$(c_b + K_{tr}) / d_b = (2.5 + 0) / 1.0 = 2.5$$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.3 \times 1.0 \times 1.0 \times 1.0}{2.5} \right) \times 1.0 = 37.0 \text{ in.} > 12.0 \text{ in.}$$

Similarly, for the 4-#5 bars:

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.3 \times 1.0 \times 0.8 \times 1.0}{2.5} \right) \times 0.625 = 18.5 \text{ in.} > 12.0 \text{ in.}$$

Class B lap splice length of the #5 bars $= 1.3\ell_d = 1.3 \times 18.5 = 24.1$ in. ACI Table 25.5.2.1

Therefore, the tension development length of the #8 bars govern.

Use a 3 ft-2 in. lap splice length for the top reinforcing bars in the end span.

For simpler detailing, the 4-#7 bottom bars are made continuous instead of cutting off a portion of them, the later of which is shown in Figure 6.32. According to Note 2 in that figure, a Class B tension lap splice is required over the supports. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$

ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #7 reinforcing bars, $\psi_s = 1.0$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + (d_b)_{stirrup} + 0.5(d_b)_{long.} = 1.5 + 0.50 + (0.5 \times 0.875) = 2.4 \text{ in.} \\ \\ \dfrac{s}{2} = \dfrac{12.0 - [2 \times (1.5 + 0.5)] - 0.875}{2} = 3.6 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.4 + 0) / 0.875 = 2.7 > 2.5$ use 2.5

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} \right) \times 0.875 = 24.9 \text{ in.} > 12.0 \text{ in.}$$

Class B lap splice length $= 1.3\ell_d = 1.3 \times 24.9 = 32.4$ in. ACI Table 25.5.2.1

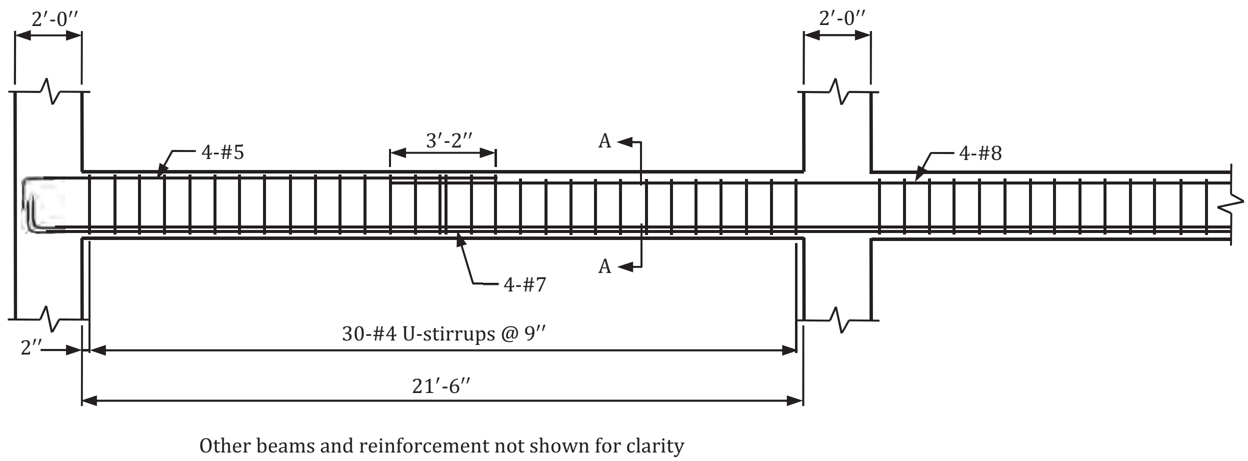
Provide a 2 ft-9 in. lap splice length.

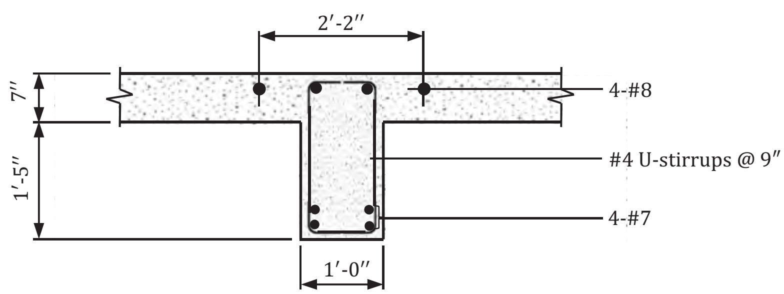
Step 2 – Determine the shear reinforcement details

It is determined in Example 6.7 that 15-#4 U-stirrups spaced at 9 in. on center are adequate for shear strength requirements. The U-stirrups extends over the effective width of the beam $[\text{which is equal to } b_w - (2 \times \text{cover})]$ with 90-degree hooks at each end and are tied to the top and bottom longitudinal reinforcement.

Minimum b_w for #4 stirrup development $= 12.0\ \text{in.}$, which is equal to the provided b_w. Table 6.17

Reinforcement details for the beam are given in Figure 6.44. The details satisfy the structural integrity requirements in ACI 9.7.7 for beams other than perimeter beams and can be used for all beams along the interior column lines in the north-south direction that are part of the LFRS.





Section A-A

Figure 6.44 Reinforcement details for the beam in Examples 6.6 through 6.8.

6.9.9 Example 6.9 – Determination of Deflections: Beam in Building #1 (Framing Option C), Beam is Part of the LFRS, SDC A, Includes Compression Reinforcement

Determine the immediate and time-dependent deflections for the beam along column line 4 in Building #1, Framing Option C, at the second-floor level assuming the beam is part of the LFRS and the dimensions of the beam are limited to 12 in. by 24 in. (see Figure 1.1). Assume the live load at this floor level is equal to175 lb/ft^2, the beam is not supporting or attached to nonstructural elements likely to be damaged by large deflections, and 75 percent of the live load is sustained. Also assume normalweight concrete with $f'_c = 4{,}000\ \text{psi}$ and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 6.6 through 6.8.

Step 1 – Determine the service loads and bending moments

The service dead and live load bending moments are given in Table 6.25 for the beam in the end span (see Step 1 of Example 6.6) and are repeated in Table 6.29 for convenience.

Table 6.29 Bending Moments (ft-kips) due to Service Dead and Live Loads at the Second-Floor Level for the Beam in the End Span in Example 6.9

Exterior Negative	Positive	First Interior Negative
$M_D^- = -24.4$ $M_L^- = -32.9$	$M_D^+ = 57.6$ $M_L^+ = 91.0$	$M_D^- = -70.3$ $M_L^- = -108.7$

The sustained positive and negative bending moments are the following where 75 percent of the live load is sustained:

$M_{sus}^+ = 57.6 + (0.75 \times 91.0) = 125.9$ ft-kips

At the exterior support: $M_{sus}^- = 24.4 + (0.75 \times 32.9) = 49.1$ ft-kips

At the first interior support: $M_{sus}^- = 70.3 + (0.75 \times 108.7) = 151.8$ ft-kips

Step 2 – Determine the material properties of the concrete and reinforcing steel

$f_r = 7.5\lambda\sqrt{f_c'} = 7.5 \times 1.0\sqrt{4{,}000} = 474$ psi Eq. (6.84)

where $\lambda = 1.0$ for normalweight concrete. Table 6.2

$E_c = 57{,}000\sqrt{f_c'} = 57{,}000\sqrt{4{,}000} = 3{,}605{,}000$ psi for normalweight concrete ACI Eq. (19.2.2.1b)

$E_s = 29{,}000{,}000$ psi ACI 20.2.2.2

Modular ratio $n = E_s / E_c = 8.0$

Step 3 – Determine the gross and cracked moments of inertia

Effective flange width of the beam $b_f = 76.5$ in. Step 2, Example 6.6

- Positive moment section

  The gross section of the beam at the positive moment section is shown in Figure 6.45.

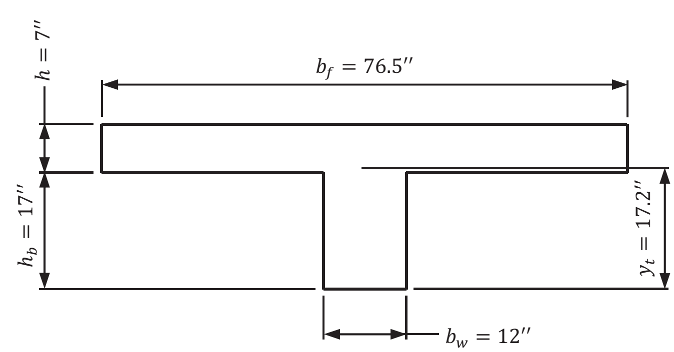


Figure 6.45 Gross section of the beam at a positive moment section.

The gross section properties of the beam are the following:

$$y_t = \frac{(b_f - b_w)(h_b + 0.5h)h + 0.5b_w(h_b + h)^2}{(b_f - b_w)h + b_w(h_b + h)}$$

Figure 6.35

$$= \frac{\{(76.5 - 12.0) \times [17.0 + (0.5 \times 7.0)] \times 7.0\} + [0.5 \times 12.0 \times (17.0 + 7.0)^2]}{[(76.5 - 12.0) \times 7.0] + [12.0 \times (17.0 + 7.0)]} = 17.2 \text{ in.}$$

$$I_g = \frac{1}{12}(b_f - b_w)h^3 + (b_f - b_w)h(h_b + 0.5h - y_t)^2 + \frac{1}{12}b_w(h_b + h)^3 + b_w(h_b + h)[y_t - 0.5(h_b + h)]^2$$

$$= \left[\frac{1}{12} \times (76.5 - 12.0) \times 7.0^3\right] + \left[(76.5 - 12.0) \times 7.0 \times (17.0 + 3.5 - 17.2)^2\right] + \left(\frac{1}{12} \times 12.0 \times 24.0^3\right)$$

$$+\{12.0 \times (17.0 + 7.0) \times [17.2 - (0.5 \times 24.0)]^2\}$$

$$= 28{,}372 \text{ in.}^4$$

From Example 6.6, 4-#7 bottom bars are required $(A_s = 2.40 \text{ in.}^2)$.

Assuming a rectangular compression area, the cracked section properties are the following (see Figure 6.46).

$$a_1 = \frac{b_f}{nA_s} = \frac{76.5}{8.0 \times 2.40} = 4.0 / \text{in.}$$

Figure 6.34

$$d = 24.0 - 1.5 - 0.5 - 0.875 - (1.0/2) = 20.6 \text{ in.}$$

$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1} = \frac{\sqrt{(2 \times 4.0 \times 20.6) + 1} - 1}{4.0} = 3.0 \text{ in.} < h = 7.0 \text{ in.}$$

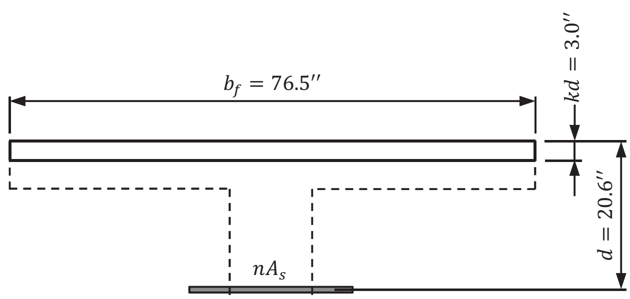


Figure 6.46 Cracked transformed section of the beam at a positive moment section.

Therefore, the assumption of a rectangular section is correct.

$$I_{cr} = \frac{b_f(kd)^3}{3} + nA_s(d - kd)^2 = \frac{76.5 \times 3.0^3}{3} + [(8.0 \times 2.40) \times (20.6 - 3.0)^2] = 6{,}636 \text{ in.}^4$$

Figure 6.34

- Negative moment section

  The negative moment section of the beam is the 12.0 by 24.0 in. beam web. From Example 6.6, 4-#5 top bars are required at the exterior support $(A_s = 1.24 \text{ in.}^2)$ and 4-#8 bars are required at the first interior support $(A_s = 3.16 \text{ in.}^2)$.

$$I_g = \frac{1}{12}b_w(h_b + h)^3 = \frac{1}{12} \times 12.0 \times (17.0 + 7.0)^3 = 13{,}824 \text{ in.}^4$$ Figure 6.34

At the exterior support:

$$a_1 = \frac{b_w}{nA_s} = \frac{12.0}{8.0 \times 1.24} = 1.2 / \text{in.}$$

$$d = 24.0 - 1.5 - 0.5 - (0.625 / 2) = 21.7 \text{ in.}$$

$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1} = \frac{\sqrt{(2 \times 1.2 \times 21.7) + 1} - 1}{1.2} = 5.2 \text{ in.}$$

$$I_{cr} = \frac{b_w(kd)^3}{3} + nA_s(d - kd)^2 = \frac{12.0 \times 5.2^3}{3} + \left[(8.0 \times 1.24) \times (21.7 - 5.2)^2\right] = 3{,}263 \text{ in.}^4$$

At the first exterior support:

$$a_1 = \frac{b_w}{nA_s} = \frac{12.0}{8.0 \times 3.16} = 0.5 / \text{in.}$$

$$d = 24.0 - 1.5 - 0.5 - (1.0 / 2) = 21.5 \text{ in.}$$

$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1} = \frac{\sqrt{(2 \times 0.5 \times 21.5) + 1} - 1}{0.5} = 7.5 \text{ in.}$$

$$I_{cr} = \frac{b_w(kd)^3}{3} + nA_s(d - kd)^2 = \frac{12.0 \times 7.5^3}{3} + \left[(8.0 \times 3.16) \times (21.5 - 7.5)^2\right] = 6{,}642 \text{ in.}^4$$

Step 4 – Determine the effective moment of inertia

The effective moment of inertia, I_e, is determined by Eq. (6.88) for dead, sustained, and dead plus live load cases.

- Positive moment section

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474 \times 28{,}372}{17.2 \times 12{,}000} = 65.2 \text{ ft-kips}$$

$$M_D^+ = 57.6 \text{ ft-kips} > 2M_{cr} / 3 = 43.5 \text{ ft-kips}$$

Therefore,

$$(I_e)_D^+ = \frac{I_{cr}}{1 - \left(\dfrac{2M_{cr}/3}{M_D^+}\right)^2 \left(1 - \dfrac{I_{cr}}{I_g}\right)} = \frac{6{,}636}{1 - \left[\left(\dfrac{43.5}{57.6}\right)^2 \times \left(1 - \dfrac{6{,}636}{28{,}372}\right)\right]} = 11{,}786 \text{ in.}^4 < I_g$$ Eq. (6.88)

$$M_{sus}^+ = 125.9 \text{ ft-kips} > 2M_{cr} / 3 = 43.5 \text{ ft-kips}$$

Therefore,

$$(I_e)_{sus}^+ = \frac{I_{cr}}{1 - \left(\dfrac{2M_{cr}/3}{M_{sus}^+}\right)^2 \left(1 - \dfrac{I_{cr}}{I_g}\right)} = \frac{6{,}636}{1 - \left[\left(\dfrac{43.5}{125.9}\right)^2 \times \left(1 - \dfrac{6{,}636}{28{,}372}\right)\right]} = 7{,}305 \text{ in.}^4 < I_g$$

$$M_{D+L}^+ = 57.6 + 91.0 = 148.6 \text{ ft-kips} > 2M_{cr} / 3 = 43.5 \text{ ft-kips}$$

Therefore,

$$(I_e)^+_{D+L} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^+_{D+L}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{6,636}{1-\left[\left(\dfrac{43.5}{148.6}\right)^2\times\left(1-\dfrac{6,636}{28,372}\right)\right]} = 7{,}102 \text{ in.}^4 < I_g$$

- Negative moment section

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474\times 13,824}{12.0\times 12,000} = 45.5 \text{ ft-kips}$$

At the exterior support:

$$M^-_D = 24.4 \text{ ft-kips} < 2M_{cr}/3 = 30.3 \text{ ft-kips}$$

Therefore, $(I_e)^-_D = I_g = 13,824 \text{ in.}^4$

$$M^-_{sus} = 49.4 \text{ ft-kips} > 2M_{cr}/3 = 30.3 \text{ ft-kips}$$

Therefore,

$$(I_e)^-_{sus} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^-_{sus}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{3,263}{1-\left[\left(\dfrac{30.3}{49.1}\right)^2\times\left(1-\dfrac{3,263}{13,824}\right)\right]} = 4,602 \text{ in.}^4 < I_g$$

$$M^-_{D+L} = 24.4 + 32.9 = 57.3 \text{ ft-kips} > 2M_{cr}/3 = 30.3 \text{ ft-kips}$$

Therefore,

$$(I_e)^-_{D+L} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^-_{sus}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{3,263}{1-\left[\left(\dfrac{30.3}{57.3}\right)^2\times\left(1-\dfrac{3,263}{13,824}\right)\right]} = 4{,}149 \text{ in.}^4 < I_g$$

At the first interior support:

$$M^-_D = 70.3 \text{ ft-kips} > 2M_{cr}/3 = 30.3 \text{ ft-kips}$$

Therefore,

$$(I_e)^-_D = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^-_D}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{6,642}{1-\left[\left(\dfrac{30.3}{70.3}\right)^2\times\left(1-\dfrac{6,642}{13,824}\right)\right]} = 7,352 \text{ in.}^4 < I_g$$

$$M^-_{sus} = 151.8 \text{ ft-kips} > 2M_{cr}/3 = 30.3 \text{ ft-kips}$$

Therefore,

$$(I_e)^-_{sus} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^-_{sus}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{6,642}{1-\left[\left(\dfrac{30.3}{151.8}\right)^2\times\left(1-\dfrac{6,642}{13,824}\right)\right]} = 6,782 \text{ in.}^4 < I_g$$

$$M^-_{D+L} = 70.3 + 108.7 = 179.0 \text{ ft-kips} > 2M_{cr}/3 = 30.3 \text{ ft-kips}$$

Therefore,

$$(I_e)^{-}_{D+L} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^{-}_{sus}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{6,642}{1-\left[\left(\dfrac{30.3}{179.0}\right)^2\times\left(1-\dfrac{6,642}{13,824}\right)\right]} = 6,742 \text{ in.}^4 < I_g$$

- Average effective moment of inertia

Dead load:

$$(I_e)_D = \frac{11,786+13,824+7,352}{3} = 10,987 \text{ in.}^4$$ ACI 24.2.3.6

Sustained load:

$$(I_e)_{sus} = \frac{7,305+4,602+6,782}{3} = 6,230 \text{ in.}^4$$

Dead plus live load:

$$(I_e)_{D+L} = \frac{7,102+4,149+6,742}{3} = 5,998 \text{ in.}^4$$

Step 5 – Determine the immediate deflections and check the maximum permissible deflection for live loads

Assume the loads due to the dead load of the slab, the superimposed dead load, and the live load are uniformly distributed over the span instead of the actual triangular load distribution. The following equation can be used to determine immediate deflections:

$$\Delta_i = \frac{5KM_a\ell^2}{48E_cI_e}$$ Eq. (6.89)

For continuous members:

$$K = 1.2 - 0.2(M_o / M_a)$$ Table 6.19

where $M_o = w\ell^2/8$ and M_a is the service load moment at midspan. Also assume $K = 1.2$.

- Dead load

$$(\Delta_i)_D = \frac{5KM^{+}_D\ell^2}{48E_c(I_e)_D} = \frac{5\times1.2\times57.6\times12,000\times(21.5\times12)^2}{48\times3,605,000\times10,987} = 0.15 \text{ in.}$$

- Sustained load

$$(\Delta_i)_{sus} = \frac{5KM^{+}_{sus}\ell^2}{48E_c(I_e)_{sus}} = \frac{5\times1.2\times125.9\times12,000\times(21.5\times12)^2}{48\times3,605,000\times6,230} = 0.56 \text{ in.}$$

- Dead plus live load

$$(\Delta_i)_{D+L} = \frac{5KM^{+}_{D+L}\ell^2}{48E_c(I_e)_{D+L}} = \frac{5\times1.2\times148.6\times12,000\times(21.5\times12)^2}{48\times3,605,000\times5,998} = 0.69 \text{ in.}$$

- Live load

$$(\Delta_i)_L = (\Delta_i)_{D+L} - (\Delta_i)_D = 0.69 - 0.15 = 0.54 \text{ in.}$$

For a floor member not supporting or attached to nonstructural elements likely to be damaged by large deflections, the maximum permissible immediate live load deflection is equal to the following:

$$(\Delta_i)_{L|max} = \ell / 360 = (21.5 \times 12) / 360 = 0.72 \text{ in.} > (\Delta_i)_L = 0.54 \text{ in.}$$ Table 6.21

Step 6 – Determine the time-dependent deflections and check the maximum permissible deflection

Assuming a 60-month duration of loading with $\rho' = 0$:

$$\lambda_\Delta = \frac{\xi}{1+50\rho'} = \frac{2.0}{1+0} = 2.0$$ Eq. (6.90), Table 6.20

The time-dependent deflection due to creep and shrinkage is equal to the following:

$$\Delta_{cs} = \lambda_\Delta (\Delta_i)_{sus} = 2.0 \times 0.56 = 1.1 \text{ in.}$$ Eq. (6.91)

$$\text{Total deflection} = \Delta_{cs} + (\Delta_i)_L = 1.1 + 0.54 = 1.6 \text{ in.} > \ell / 240 = 1.1 \text{ in.}$$ Table 6.21

The 12.0 in. by 24.0 in. beam is not adequate for time-dependent deflections.

Step 7 – Recalculate the deflections using compression reinforcement

Recalculate the deflections including the contribution of compression reinforcement in the section. For purposes of analysis, assume the 4-#8 top bars extend across the entire length of the beam.

- **Step 7a – Determine the cracked moments of inertia**

  Positive moment section

  Assuming a rectangular compression area, the cracked section properties are the following:

$$a_1 = \frac{b_f}{nA_s} = \frac{76.5}{8.0 \times (4 \times 0.60)} = 4.0 / \text{in.}$$ Figure 6.34

$$a_2 = \frac{(n-1)A_s'}{nA_s} = \frac{(8.0-1) \times (4 \times 0.79)}{8.0 \times (4 \times 0.60)} = 1.2$$

$$d' = 1.5 + 0.5 + (1.0 / 2) = 2.5 \text{ in.}$$

$$kd = \frac{\sqrt{2a_1 d\left(1 + \frac{a_2 d'}{d}\right) + (1+a_2)^2} - (1+a_2)}{a_1}$$

$$= \frac{\sqrt{\left[(2 \times 4.0 \times 20.6) \times \left(1 + \frac{1.2 \times 2.5}{20.6}\right)\right] + (1+1.2)^2} - (1+1.2)}{4.0} = 2.9 \text{ in.} < h = 7.0 \text{ in.}$$

Therefore, the assumption of a rectangular section is correct.

$$I_{cr} = \frac{b_f (kd)^3}{3} + nA_s (d-kd)^2 + (n-1)A_s'(kd-d')^2$$ Figure 6.34

$$= \frac{76.5 \times 2.9^3}{3} + [(8.0 \times 2.40) \times (20.6 - 2.9)^2] + [(8.0-1) \times 3.16 \times (2.9-2.5)^2] = 6{,}641 \text{ in.}^4$$

Negative moment section

At the exterior and first interior supports:

$$a_1 = \frac{b_w}{nA_s} = \frac{12.0}{8.0 \times (4 \times 0.79)} = 0.5 \text{ / in.}$$ Figure 6.34

$$a_2 = \frac{(n-1)A_s'}{nA_s} = \frac{(8.0-1) \times (4 \times 0.60)}{8.0 \times (4 \times 0.79)} = 0.7$$

$$d' = 1.5 + 0.5 + (0.875 / 2) = 2.4 \text{ in.}$$

$$kd = \frac{\sqrt{2a_1 d\left(1 + \frac{a_2 d'}{d}\right) + (1+a_2)^2} - (1+a_2)}{a_1}$$

$$= \frac{\sqrt{\left[(2 \times 0.5 \times 21.5) \times \left(1 + \frac{0.7 \times 2.4}{21.5}\right)\right] + (1+0.7)^2} - (1+0.7)}{0.5} = 6.8 \text{ in.}$$

$$I_{cr} = \frac{b_w (kd)^3}{3} + nA_s(d-kd)^2 + (n-1)A_s'(kd-d')^2$$

Figure 6.34

$$= \frac{12.0 \times 6.8^3}{3} + [(8.0 \times 3.16) \times (21.5 - 6.8)^2] + [(8.0-1) \times 2.40 \times (6.8-2.4)^2] = 7{,}046 \text{ in.}^4$$

- **Step 7b – Determine the effective moment of inertia**

Positive moment section

$$(I_e)_D^+ = \frac{I_{cr}}{1 - \left(\frac{2M_{cr}/3}{M_D^+}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)} = \frac{6{,}641}{1 - \left[\left(\frac{43.5}{57.6}\right)^2 \times \left(1 - \frac{6{,}641}{28{,}372}\right)\right]} = 11{,}792 \text{ in.}^4 < I_g$$ Eq. (6.88)

$$(I_e)_{sus}^+ = \frac{I_{cr}}{1 - \left(\frac{2M_{cr}/3}{M_{sus}^+}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)} = \frac{6{,}641}{1 - \left[\left(\frac{43.5}{125.9}\right)^2 \times \left(1 - \frac{6{,}641}{28{,}372}\right)\right]} = 7{,}309 \text{ in.}^4 < I_g$$

$$(I_e)_{D+L}^+ = \frac{I_{cr}}{1 - \left(\frac{2M_{cr}/3}{M_{D+L}^+}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)} = \frac{6{,}641}{1 - \left[\left(\frac{43.5}{148.6}\right)^2 \times \left(1 - \frac{6{,}641}{28{,}372}\right)\right]} = 7{,}108 \text{ in.}^4 < I_g$$

Negative moment sections

At the exterior support:

$$(I_e)_D^- = 13{,}824 \text{ in.}^4$$

$$(I_e)^-_{sus} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^-_{sus}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{7{,}046}{1-\left[\left(\dfrac{30.3}{49.1}\right)^2\times\left(1-\dfrac{7{,}046}{13{,}824}\right)\right]} = 8{,}664 \text{ in.}^4 < I_g$$

$$(I_e)^-_{D+L} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^-_{sus}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{7{,}046}{1-\left[\left(\dfrac{30.3}{57.3}\right)^2\times\left(1-\dfrac{7{,}046}{13{,}824}\right)\right]} = 8{,}166 \text{ in.}^4 < I_g$$

At the first interior support:

$$(I_e)^-_{D} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^-_{D}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{7{,}046}{1-\left[\left(\dfrac{30.3}{70.3}\right)^2\times\left(1-\dfrac{7{,}046}{13{,}824}\right)\right]} = 7{,}752 \text{ in.}^4 < I_g$$

$$(I_e)^-_{sus} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^-_{sus}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{7{,}046}{1-\left[\left(\dfrac{30.3}{151.8}\right)^2\times\left(1-\dfrac{7{,}046}{13{,}824}\right)\right]} = 7{,}186 \text{ in.}^4 < I_g$$

$$(I_e)^-_{D+L} = \frac{I_{cr}}{1-\left(\dfrac{2M_{cr}/3}{M^-_{sus}}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} = \frac{7{,}046}{1-\left[\left(\dfrac{30.3}{179.0}\right)^2\times\left(1-\dfrac{7{,}046}{13{,}824}\right)\right]} = 7{,}146 \text{ in.}^4 < I_g$$

<u>Average effective moment of inertia</u>

Dead load:

$$(I_e)_D = \frac{11{,}792+13{,}824+7{,}752}{3} = 11{,}123 \text{ in.}^4$$ ACI 24.2.3.6

Sustained load:

$$(I_e)_{sus} = \frac{7{,}309+8{,}664+7{,}186}{3} = 7{,}720 \text{ in.}^4$$

Dead plus live load:

$$(I_e)_{D+L} = \frac{7{,}108+8{,}166+7{,}146}{3} = 7{,}473 \text{ in.}^4$$

- **Step 7c – Determine the immediate deflections and check the maximum permissible deflection for live loads**

Dead load:

$$(\Delta_i)_D = \frac{5KM^+_D\ell^2}{48E_c(I_e)_D} = \frac{5\times1.2\times57.6\times12{,}000\times(21.5\times12)^2}{48\times3{,}605{,}000\times11{,}123} = 0.14 \text{ in.}$$

Sustained load:

$$(\Delta_i)_{sus} = \frac{5KM^+_{sus}\ell^2}{48E_c(I_e)_{sus}} = \frac{5\times1.2\times125.9\times12{,}000\times(21.5\times12)^2}{48\times3{,}605{,}000\times7{,}720} = 0.45 \text{ in.}$$

Dead plus live load:

$$(\Delta_i)_{D+L} = \frac{5KM^+_{D+L}\ell^2}{48E_c(I_e)_{D+L}} = \frac{5\times 1.2\times 148.6\times 12{,}000\times(21.5\times 12)^2}{48\times 3{,}605{,}000\times 7{,}473} = 0.55 \text{ in.}$$

Live load:

$$(\Delta_i)_L = (\Delta_i)_{D+L} - (\Delta_i)_D = 0.55 - 0.14 = 0.41 \text{ in.}$$

For a floor member not supporting or attached to nonstructural elements likely to be damaged by large deflections, the maximum permissible immediate live load deflection is equal to the following:

$$(\Delta_i)_{L|max} = \ell \,/\, 360 = (21.5\times 12)\,/\,360 = 0.72 \text{ in.} > (\Delta_i)_L = 0.41 \text{ in.}$$ Table 6.21

- **Step 7d – Determine the time-dependent deflections and check the maximum permissible deflection**

  Assuming a 60-month duration of loading:

$$\rho' = A'_s \,/\, b_f d = (4\times 0.79)\,/\,(76.5\times 20.6) = 0.0020$$ ACI 24.2.4.1.2

$$\lambda_\Delta = \frac{\xi}{1+50\rho'} = \frac{2.0}{1+(50\times 0.0020)} = 1.8$$ Eq. (6.90), Table 6.20

  The time-dependent deflection due to creep and shrinkage is equal to the following:

$$\Delta_{cs} = \lambda_\Delta(\Delta_i)_{sus} = 1.8\times 0.45 = 0.81 \text{ in.}$$ Eq. (6.91)

$$\text{Total deflection} = \Delta_{cs} + (\Delta_i)_L = 0.81 + 0.41 = 1.2 \text{ in.} > \ell\,/\,240 = 1.1 \text{ in.}$$ Table 6.21

  The 12.0 in. by 24.0 in. beam is not adequate for time-dependent deflections.

Comments. The deflection calculations are determined assuming a uniformly distributed load equal to the maximum value of the actual triangular load, which is conservative. As such, it is safe to conclude this beam is adequate for time-dependent deflections when the contribution of the compression reinforcement is included.

6.9.10 Example 6.10 – Determination of Joist Size: Joist in Building #2, Joist is Not Part of the LFRS, SDC C

Determine the size of a typical joist in Building #2 at a typical floor level where the joist is not part of the LFRS (see Figure 1.2). Assume a 4.5-in.-thick slab. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2.

Step 1 – Determine the joist depth based on serviceability requirements — ACI 9.3.1.1

Assume the joist (beam) is not supporting or attached to partitions or other construction likely to be damaged by large deflections. Thus, deflections in accordance with ACI 24.2 need not be calculated.

The minimum joist depth is based on the end spans (longest spans) with a support condition of one end continuous:

$$\text{Joist depth} = \frac{\ell}{18.5} = \frac{42.5\times 12}{18.5} = 27.6 \text{ in.}$$ Table 6.1

Use a 24-in.-deep pan form, which results in an overall depth equal to $24.0 + 4.5 = 28.5 \text{ in.} > 27.6 \text{ in.}$

Step 2 – Determine the joist width

Where 53-in.-wide pan forms are used, a 7.0-in.-wide joist rib is typically specified, resulting in a 5-ft center-to-center spacing of the ribs (see Figure 1.2). The 7.0-in. width occurs at the bottom of the rib, and the width typically increases at a 12 to 1 slope to the underside of the slab.

The adequacy of the 7.0-in.-wide joist rib is checked for moment strength in Example 6.11.

Use a 7.0 in. by 28.5 in. joist.

6.9.11 Example 6.11 – Determination of Flexural Reinforcement: Joist in Building #2, Joist Not Part of the LFRS, SDC C

Determine the required flexural reinforcement for a typical joist in Building #2 at a typical floor level assuming the joist is not part of the LFRS (see Figure 1.2). Also assume 18.0 in. by 32.0 in. edge columns, 28.0 in. by 28.0 in corner columns, 32.0-in.-wide interior beams, 30.0-in.-wide edge beams, normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Example 6.10.

Step 1 – Determine the factored bending moments along the span

$$\text{Weight of joist system} = \frac{150.0}{5.0 \times 144} \times \{(60.0 \times 4.5) + [2 \times (2 \times 24.0 / 2)] + (7.0 \times 24.0)\} = 101.3 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 20 \text{ lb/ft}^2$$

$$\text{Live load} = 100 \text{ lb/ft}^2$$

Check if the limitations of the simplified method of analysis in ACI 6.5.1 are satisfied for this system:

- Members are prismatic
- Loads are uniformly distributed
- $L = 100 \text{ lb/ft}^2 < 3D = 3 \times (101.3 + 20.0) = 363.9 \text{ lb/ft}^2$
- There are 3 spans, which is greater than 2 spans
- $\text{Longer span/shorter span} = 42.5 / 35.5 = 1.2$

Therefore, the simplified method of analysis in ACI 6.5.1 can be used to determine the bending moments and shear forces.

$$w_u = [1.2 \times (101.3 + 20.0) \times 5] + (1.6 \times 100.0 \times 5) = 1{,}528 \text{ lb/ft}$$ Table 3.3

$$\text{End span: } \ell_{n,1} = 42.5 - [28.0 / (2 \times 12)] - (2.0 / 12) - [32.0 / (2 \times 12)] = 39.8 \text{ ft}$$

$$\text{Interior span: } \ell_{n,2} = 35.5 - (32.0 / 12) = 32.8 \text{ ft}$$

$$\ell_{n,avg} = (\ell_{n,1} + \ell_{n,2}) / 2 = 36.3 \text{ ft}$$

The factored bending moments are given in Table 6.30. Figure 4.3

Table 6.30 Factored Bending Moments for a Typical Joist in Example 6.11 (ft-kips)

End Span			Interior Span	
Exterior Negative	**Positive**	**First Interior Negative**	**Positive**	**Interior Negative**
$\frac{w_u \ell_{n,1}^2}{24} = -100.9$	$\frac{w_u \ell_{n,1}^2}{14} = 172.9$	$\frac{w_u \ell_{n,avg}^2}{10} = -201.3$	$\frac{w_u \ell_{n,2}^2}{16} = 102.7$	$\frac{w_u \ell_{n,avg}^2}{11} = -183.0$

Step 2 – Determine the effective flange width of the joist ACI 6.3.2.1

The effective flange width, b_f, is equal to the following:

$$b_f = b_w + \text{ least of } \begin{cases} 16h = 16 \times 4.5 = 72.0 \text{ in.} \\ s_w = 53.0 \text{ in.} \\ \ell_n / 4 = 32.8 \times 12 / 4 = 98.4 \text{ in.} \end{cases} = 7.0 + 53.0 = 60.0 \text{ in.}$$ Figure 6.10

Step 3 – Determine the required flexural reinforcement at the critical sections

At negative moment critical sections (flange in tension), required A_s is determined by the following assuming a single layer of tension reinforcement:

$$R_n = \frac{M_u}{\phi b d^2}$$ Eq. (6.34)

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right]$$ Eq. (6.33)

where $b = b_w = 7.0$ in. Figure 6.11

At positive moment critical sections (flange in compression), determine the depth of the equivalent stress block, a, assuming rectangular section behavior using the largest positive M_u in Table 6.30 (which results in the largest a):

$$a = \frac{a_1 d - \sqrt{(a_1 d)^2 - (2 a_1 M_u / \phi)}}{a_1}$$

$$= \frac{(204.0 \times 26.0) - \sqrt{(204.0 \times 26.0)^2 - (2 \times 204.0 \times 172.9 \times 12 / 0.9)}}{204.0} = 0.44 \text{ in.} < h = 4.5 \text{ in.}$$ Eq. (6.49)

where $a_1 = 0.85 f_c' b_f = 0.85 \times 4 \times 60.0 = 204.0$ kips/in.

Because $a < h$, the section behaves as a rectangular section and Eqs. (6.34) and (6.33) can be used to determine A_s with $b = b_f = 60.0$ in. (see Figure 6.12).

A summary of the required reinforcement is given in Table 6.31. All sections are tension-controlled because $A_s < A_{s,t}$.

Table 6.31 Required Flexural Reinforcement for the Joist in Example 6.11

Location		M_u (ft-kips)	R_n (psi)	A_s (in^2)*
End span	Exterior negative	–100.9	284	0.90
	Positive	172.9	57	1.49
	First interior negative	–201.3	567	1.89
Interior span	Positive	102.7	34	0.88
	Negative	–183.0	516	1.71

*$Min.\ A_s = 200 b_w d / f_y = 0.61$ in.^{2} [Eq. (6.35)]

$$Max.\ A_{s,t} = 0.018bd = \begin{cases} 0.018 \times 7.0 \times 26.0 = 3.28 \text{ in.}^2 \text{ at negative critical sections} \\ 0.018 \times 60.0 \times 26.0 = 28.1 \text{ in.}^2 \text{ at positive critical sections} \end{cases}$$ [Eq. (6.38)]

Step 4 – Select the flexural reinforcement

Select the flexural reinforcement in a single layer based on the minimum and maximum spacing requirements in ACI 25.2 and 24.3, respectively.

- Negative reinforcement

  $b_f = 60.0 \text{ in.} > \ell_n / 10 = 39.8 \times 12 / 10 = 47.8 \text{ in.}$

  Because $b_f > \ell_n / 10$, distribute the required A_s within $\ell_n / 10$. Figure 6.23

  Exterior negative:

  With 1.5-in. cover, Grade 60 reinforcing bars, and $f_s = 2f_y / 3 = 40{,}000$ psi, the maximum center-to-center bar spacing is equal to the following:

$$s = \text{ lesser of } \begin{cases} 15\left(\dfrac{40{,}000}{40{,}000}\right) - (2.5 \times 1.5) = 11.3 \text{ in.} \\ 12\left(\dfrac{40{,}000}{40{,}000}\right) = 12.0 \text{ in.} \end{cases}$$ Eq. (6.65)

  Required reinforcement within the 48.0-in. width $= 0.9 / (48.0 / 12) = 0.23$ in.^{2}/ft

  Use #4 @ 10 in. on center $[A_{s,provided} = (12/10) \times 0.20 = 0.24 \text{ in.}^2/\text{ft}]$.

  This reinforcement also satisfies the requirement in ACI 24.3.4 for additional longitudinal reinforcement in the outer portions of the flange where $b_f > \ell_n / 10$.

  First interior negative:

  Required reinforcement within the 48.0-in. width: $1.89 / (48.0 / 12) = 0.47$ in.^{2}/ft

  Use #5 @ 8 in. on center $[A_{s,provided} = (12/8) \times 0.31 = 0.47 \text{ in.}^2/\text{ft}]$.

  This reinforcement also satisfies the requirement in ACI 24.3.4 for additional longitudinal reinforcement in the outer portions of the flange where $b_f > \ell_n / 10$.

- Positive reinforcement

  Use 2-#8 bars $(A_{s,provided} = 1.58 \text{ in.}^2 > 1.49 \text{ in.}^2)$.

Assuming a 0.75-in. maximum aggregate size:

$$\text{Minimum clear space} = \begin{cases} 1.0 \text{ in.} \\ d_b = 1.0 \text{ in.} \\ (4/3)d_{agg} = 1.0 \text{ in.} \end{cases}$$ Figure 6.22

Check minimum clear space:

$$s = 7.0 - [2 \times (1.5 + 1.125)] = 1.75 \text{ in.} > 1.0 \text{ in.}$$ Table 6.7

Comments. The reactions at the ends of the joists in the end spans cause torsional moments on the edge beams. No adjustment of moments in accordance with ACI 22.7.3.3 has been considered in this example. See Example 6.18 for the design of the joists considering redistribution of torsional moments.

6.9.12 Example 6.12 – Determination of Shear Reinforcement: Joist in Building #2, Joist is Not Part of the LFRS, SDC C

Determine the required shear reinforcement for a typical joist in Building #2 at a typical floor level assuming the joist is not part of the LFRS (see Figure 1.2). Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Examples 6.10 and 6.11.

Step 1 – Determine the factored shear forces along the span

The simplified method of analysis can be used to determine the shear forces for this system (see Step 1 in Example 6.10).

$$w_u = [1.2 \times (101.3 + 20.0) \times 5] + (1.6 \times 100.0 \times 5) = 1{,}528 \text{ lb/ft}$$

The maximum shear force occurs at the face of the first interior support:

$$V_u \text{ @ face} = \frac{1.15 w_u \ell_{n,1}}{2} = \frac{1.15 \times 1{,}528 \times 39.8}{2 \times 1{,}000} = 35.0 \text{ kips}$$

The 3 conditions in ACI 9.4.3.2 are satisfied, so the critical section for shear can be taken a distance $d = 26.0$ in. from the face of the transverse beam (see Figure 6.4):

$$V_u \text{ @ } d = 35.0 - \left(\frac{1{,}528 \times 26.0}{12 \times 1{,}000}\right) = 31.7 \text{ kips}$$

Step 2 – Determine the required shear reinforcement

Assuming at least minimum shear reinforcement is provided, ϕV_c can be determined from the following equation with the axial force $N_u = 0$:

$$\phi V_c = \phi 2\lambda\sqrt{f_c'} b_w d = 0.75 \times 2 \times 1.0\sqrt{4{,}000} \times 7.0 \times 26.0 / 1{,}000 = 17.3 \text{ kips}$$ Table 6.4

where $\lambda = 1.0$ for normalweight concrete. Table 6.2

$$V_u - \phi V_c = 31.7 - 17.3 = 14.4 \text{ kips}$$

For $V_u - \phi V_c = 14.4 \text{ kips} < \phi 4\sqrt{f_c'} b_w d = 34.6 \text{ kips}$:

Maximum spacing $s_{max} = d / 2 = 13.0 \text{ in.}$ along the length of the joist. Figure 6.19

Because of the narrow width of the joist, provide single-leg stirrups. Figure 6.16

A single-leg #4 stirrup spaced at $d / 2$ provides $V_u - \phi V_c = 36.0 / 2 = 18.0 \text{ kips} > 14.4 \text{ kips}$ Table 6.6

$$\frac{A_v}{s} = \frac{0.20}{13} = 0.015 \text{ in.}^2 / \text{in.}$$

Table 6.5

$$> \frac{A_{v,min}}{s} = \text{ greater of } \begin{cases} 0.75\sqrt{f_c'} b_w / f_{yt} = 0.75\sqrt{4{,}000} \times 7.0 / 60{,}000 = 0.0055 \text{ in.}^2/\text{in.} \\ 50 b_w / f_{yt} = 50 \times 7.0 / 60{,}000 = 0.0058 \text{ in.}^2/\text{in.} \end{cases}$$

Use #4 single-leg stirrups spaced at 12 in. on center ($A_{v,provided} = 0.20 \text{ in.}^2$).

Stirrups are no longer required at the section where $V_u = \phi \lambda \sqrt{f_c'} b_w d = 8.6 \text{ kips}$. The length from the face of the support where stirrups are no longer required, x_{ns}, can be obtained from the following equation:

$$x_{ns} = \frac{35.0 - 8.6}{1{,}528 / 1{,}000} = 17.3 \text{ ft} \qquad \text{Eq. (6.58)}$$

Use 19-#4 single-leg stirrups spaced at 12.0 in. on center at both ends of the joist with the first stirrup located 2 in. from the face of the transverse beam.

Comments. The shear calculations were performed using $b_w = 7.0 \text{ in.}$, which is the minimum width of the sloping web. An average of the top and bottom rib widths can be used in the calculations, but because the slope is 12 to 1, little is gained by doing so.

The reactions at the ends of the joists in the end spans cause torsional moments on the edge beams. No adjustment of shear forces in accordance with ACI 22.7.3.3 have been considered in this example. See Example 6.18 for the design of the joists considering redistribution of torsional moments.

6.9.13 Example 6.13 – Determination of Reinforcement Details: Joist in Building #2, Joist is Not Part of the LFRS, SDC C

Determine the required reinforcement details for a typical joist in Building #2 at a typical floor level assuming the joist is not part of the LFRS (see Figure 1.2). Also assume normalweight concrete with $f_c' = 4{,}000 \text{ psi}$ and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Examples 6.10 through 6.12.

Step 1 – Determine the flexural reinforcement details

It is determined in Example 6.11 that the following reinforcement is required at the critical sections:

- Exterior negative: #4 @ 10 in. on center
- Positive: 2-# 8 bars
- First interior negative: #5 @ 8 in. on center

The provided number of reinforcing bars at all critical sections satisfies the minimum and maximum spacing requirements in ACI 25.2 and 24.3, respectively (see Step 4 in Example 6.11).

The lengths of the reinforcing bars can be determined by Figure 6.32 for beams other than perimeter beams because the joist is subjected to uniformly distributed gravity loads.

For simpler detailing, the 2-#8 bottom bars are made continuous instead of cutting off a portion of them, the latter of which is shown in Figure 6.32. According to Note 2 in that figure, a Class B tension lap splice is required over the supports. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t\psi_e\psi_s\psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #8 reinforcing bars, $\psi_s = 1.0$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + 0.5(d_b)_{long.} = 1.5 + (0.5 \times 1.0) = 2.0 \text{ in.} \\ \\ \dfrac{s}{2} = \dfrac{7.0 - (2 \times 1.5) - 1.0}{2} = 1.5 \text{ in.} \end{cases}$$

Set $K_{tr} = 0.$ ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (1.5 + 0) / 1.0 = 1.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60,000}{1.0\sqrt{4,000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{1.5} \right) \times 1.0 = 47.4 \text{ in.} > 12.0 \text{ in.}$$

$\text{Class B lap splice length} = 1.3\ell_d = 1.3 \times 47.4 = 61.6 \text{ in.}$ ACI Table 25.5.2.1

Provide a 5 ft-2 in. lap splice length.

Flexural reinforcement details are given in Figure 6.47.

Step 2 – Determine the shear reinforcement details

It is determined in Example 6.12 that 20-#4 single-leg stirrups spaced at 12.0 in. on center are adequate for shear strength requirements, which is bent into the profile shown in Figure 6.16. It is attached to one of the bottom bars of the joist and has a standard hook at the top.

Shear reinforcement details for the joist in the end span are given in Figure 6.48.

The details satisfy the structural integrity requirements in ACI 9.7.7 for beams other than perimeter beams and can be used for all joists.

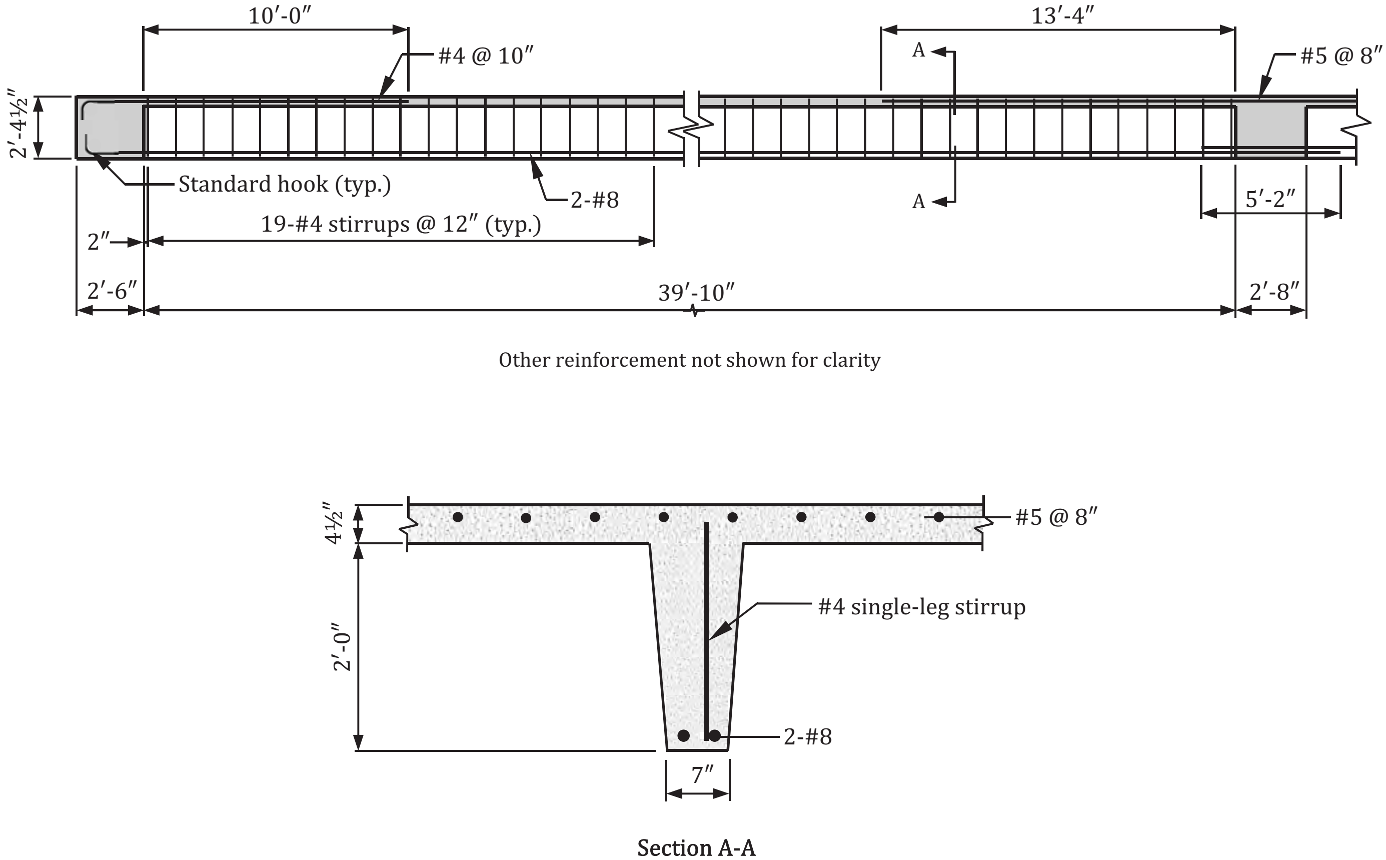


Figure 6.47 Flexural reinforcement details for the joist in Example 6.13.

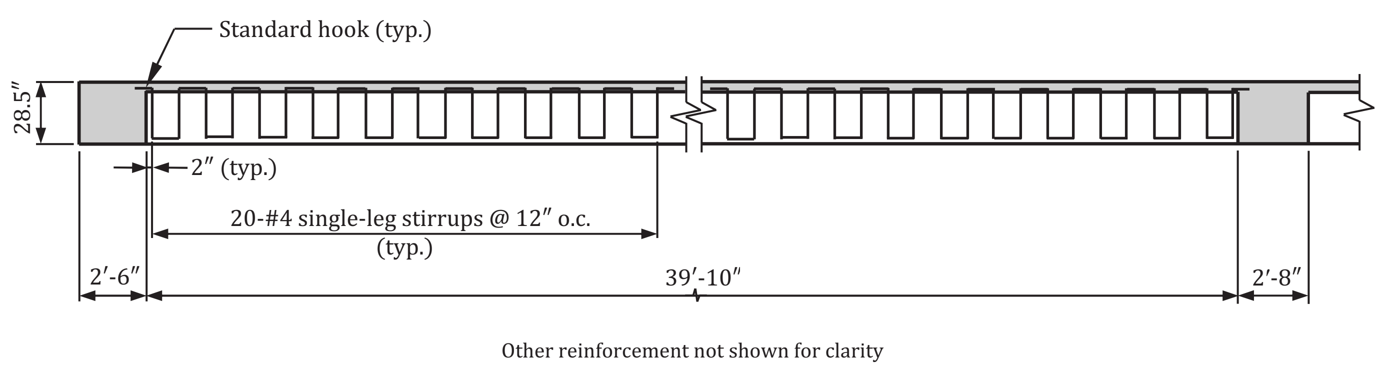


Figure 6.48 Shear reinforcement details for the joist in Example 6.13.

6.9.14 Example 6.14 – Determination of Deflections: Joist in Building #2, Typical Floor, Joist is Not Part of the LFRS, SDC C

Determine the immediate and time-dependent deflections for a typical joist in an end span in Building #2 at a typical floor level assuming the joist is not part of the LFRS (see Figure 1.2). Assume the joist is not supporting or attached to nonstructural elements likely to be damaged by large deflections and 30 percent of the live load is sustained. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Examples 6.10 through 6.13.

Step 1 – Determine the service loads and bending moments

$$\text{Weight of joist system} = 101.3\ \text{lb/ft}^2$$

$$\text{Superimposed dead load} = 20\ \text{lb/ft}^2$$

$$w_D = (101.3 + 20) \times 5 = 607\ \text{lb/ft}$$

$$\text{Live load} = 100\ \text{lb/ft}^2$$

$$w_L = 100.0 \times 5 = 500\ \text{lb/ft}$$

The simplified method of analysis can be used to determine the bending moments for this system (see Step 1 in Example 6.10).

A summary of the service dead and live load bending moments is given in Table 6.32.

Table 6.32 Service Dead and Live Load Bending Moments (ft-kips) for the Joist in an End Span in Example 6.14

Exterior Negative	Positive	First Interior Negative
$\frac{w_D \ell_{n,1}^2}{24} = -40.1$	$\frac{w_D \ell_{n,1}^2}{14} = 68.7$	$\frac{w_D \ell_{n,avg}^2}{10} = -80.0$
$\frac{w_L \ell_{n,1}^2}{24} = -33.0$	$\frac{w_L \ell_{n,1}^2}{14} = 56.6$	$\frac{w_L \ell_{n,avg}^2}{10} = -65.9$

The sustained positive and negative bending moments are the following:

$$M_{sus}^{+} = 68.7 + (0.3 \times 56.6) = 85.7\ \text{ft-kips}$$

At the exterior support: $M_{sus}^{-} = 40.1 + (0.3 \times 33.0) = 50.0$ ft-kips

At the first interior support: $M_{sus}^{-} = 80.0 + (0.3 \times 65.9) = 99.8$ ft-kips

Step 2 – Determine the material properties of the concrete and reinforcing steel

$$f_r = 7.5\lambda\sqrt{f_c'} = 7.5 \times 1.0\sqrt{4{,}000} = 474\ \text{psi}$$ Eq. (6.84)

where $\lambda = 1.0$ for normalweight concrete. Table 6.2

$$E_c = 57{,}000\sqrt{f_c'} = 57{,}000\sqrt{4{,}000} = 3{,}605{,}000\ \text{psi for normalweight concrete}$$ ACI Eq. (19.2.2.1b)

$$E_s = 29{,}000{,}000\ \text{psi}$$ ACI 20.2.2.2

Modular ratio $n = E_s / E_c = 8.0$

Step 3 – Determine the gross and cracked moments of inertia

Effective flange width of the joist $b_f = 60.0$ in. Step 2, Example 6.11

- Positive moment section

  The gross section of the joist at the positive moment section is shown in Figure 6.49.

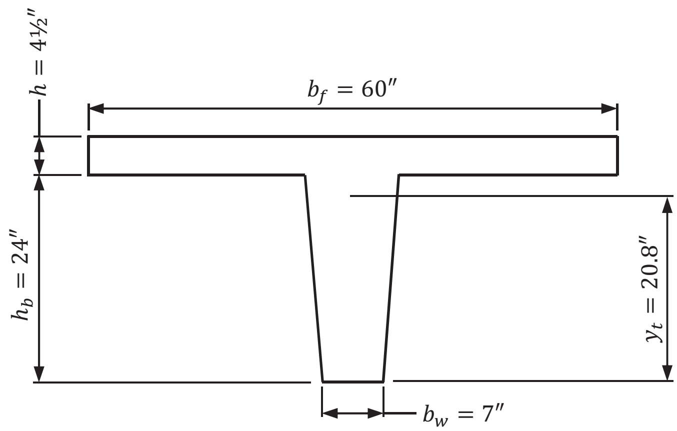


Figure 6.49 Gross section of the joist at a positive moment section.

The gross section properties of the joist are the following (not including the sloped portions of the web):

$$y_t = \frac{(b_f - b_w)(h_b + 0.5h)h + 0.5b_w(h_b + h)^2}{(b_f - b_w)h + b_w(h_b + h)}$$

Figure 6.35

$$= \frac{\{(60.0 - 7.0) \times [24.0 + (0.5 \times 4.5)] \times 4.5\} + [0.5 \times 7.0 \times (24.0 + 4.5)^2]}{[(60.0 - 7.0) \times 4.5] + [7.0 \times (24.0 + 4.5)]} = 20.8 \text{ in.}$$

$$I_g = \frac{1}{12}(b_f - b_w)h^3 + (b_f - b_w)h(h_b + 0.5h - y_t)^2 + \frac{1}{12}b_w(h_b + h)^3 + b_w(h_b + h)[y_t - 0.5(h_b + h)]^2$$

$$= \left[\frac{1}{12} \times (60.0 - 7.0) \times 4.5^3\right] + \{(60.0 - 7.0) \times 4.5 \times [24.0 + (0.5 \times 4.5) - 20.8]^2\} + \left(\frac{1}{12} \times 7.0 \times 28.5^3\right)$$

$$+\{7.0 \times 28.5 \times [20.8 - (0.5 \times 28.5)]^2\}$$

$$= 29{,}549 \text{ in.}^4$$

From Example 6.11, 2-#8 bottom bars are required $(A_s = 1.58 \text{ in.}^2)$.

Assuming a rectangular compression area, the cracked section properties are the following (see Figure 6.50).

$$a_1 = \frac{b_f}{nA_s} = \frac{60.0}{8.0 \times 1.58} = 4.8 / \text{in.}$$

Figure 6.34

$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1} = \frac{\sqrt{(2 \times 4.8 \times 26.0) + 1} - 1}{4.8} = 3.1 \text{ in.} < h = 4.5 \text{ in.}$$

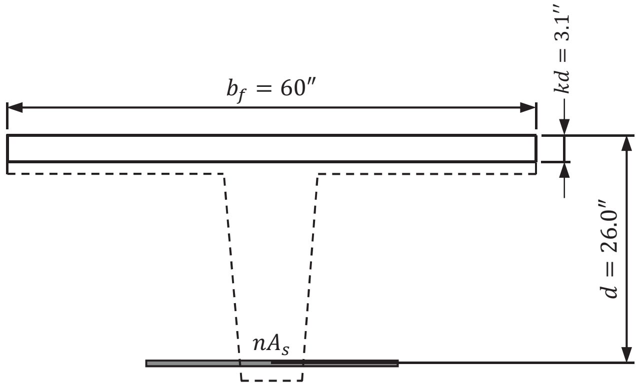


Figure 6.50 Cracked transformed section of the joist at a positive moment section.

Therefore, the assumption of a rectangular section is correct.

$$I_{cr} = \frac{b_f(kd)^3}{3} + nA_s(d - kd)^2 = \frac{60.0 \times 3.1^3}{3} + [(8.0 \times 1.58) \times (26.0 - 3.1)^2] = 7{,}224 \text{ in.}^4$$ Figure 6.34

- Negative moment section

  The negative moment section of the beam is the 7.0 in. by 28.5 in. joist web. From Example 6.11, #4 top bars at 10 in. on center are required at the exterior support [$A_s = 0.20 \times (60.0 / 10) = 1.20 \text{ in.}^2$] and #5 top bars at 8 in. on center are required at the first interior support [$A_s = 0.31 \times (60.0 / 8.0) = 2.33 \text{ in.}^2$].

$$I_g = \frac{1}{12} b_w (h_b + h)^3 = \frac{1}{12} \times 7.0 \times (24.0 + 4.5)^3 = 13{,}504 \text{ in.}^4$$ Figure 6.34

At the exterior support:

$$a_1 = \frac{b_w}{nA_s} = \frac{7.0}{8.0 \times 1.20} = 0.7 / \text{in.}$$

$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1} = \frac{\sqrt{(2 \times 0.7 \times 26.0) + 1} - 1}{0.7} = 7.3 \text{ in.}$$

$$I_{cr} = \frac{b_w(kd)^3}{3} + nA_s(d - kd)^2 = \frac{7.0 \times 7.3^3}{3} + \left[(8.0 \times 1.20) \times (26.0 - 7.3)^2\right] = 4{,}265 \text{ in.}^4$$

At the first interior support:

$$a_1 = \frac{b_w}{nA_s} = \frac{7.0}{8.0 \times 2.33} = 0.4 / \text{in.}$$

$$kd = \frac{\sqrt{2a_1 d + 1} - 1}{a_1} = \frac{\sqrt{(2 \times 0.4 \times 26.0) + 1} - 1}{0.4} = 9.2 \text{ in.}$$

$$I_{cr} = \frac{b_w(kd)^3}{3} + nA_s(d - kd)^2 = \frac{7.0 \times 9.2^3}{3} + \left[(8.0 \times 2.33) \times (26.0 - 9.2)^2\right] = 7{,}078 \text{ in.}^4$$

Step 4 – Determine the effective moment of inertia

The effective moment of inertia, I_e, is determined by Eq. (6.88) for dead, sustained, and dead plus live load cases.

- Positive moment section

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474 \times 29{,}549}{20.8 \times 12{,}000} = 56.1 \text{ ft-kips}$$

$$M_D^+ = 68.7 \text{ ft-kips} > 2M_{cr}/3 = 37.4 \text{ ft-kips}$$

Therefore,

$$(I_e)_D^+ = \frac{I_{cr}}{1 - \left(\frac{2M_{cr}/3}{M_D^+}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)} = \frac{7{,}224}{1 - \left[\left(\frac{37.4}{68.7}\right)^2 \times \left(1 - \frac{7{,}224}{29{,}549}\right)\right]} = 9{,}308 \text{ in.}^4 < I_g \qquad \text{Eq. (6.88)}$$

$$M_{sus}^+ = 85.7 \text{ ft-kips} > 2M_{cr}/3 = 37.4 \text{ ft-kips}$$

Therefore,

$$(I_e)_{sus}^+ = \frac{I_{cr}}{1 - \left(\frac{2M_{cr}/3}{M_{sus}^+}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)} = \frac{7{,}224}{1 - \left[\left(\frac{37.4}{85.7}\right)^2 \times \left(1 - \frac{7{,}224}{29{,}549}\right)\right]} = 8{,}438 \text{ in.}^4 < I_g$$

$$M_{D+L}^+ = 68.7 + 56.6 = 125.3 \text{ ft-kips} > 2M_{cr}/3 = 37.4 \text{ ft-kips}$$

Therefore,

$$(I_e)_{D+L}^+ = \frac{I_{cr}}{1 - \left(\frac{2M_{cr}/3}{M_{D+L}^+}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)} = \frac{7{,}224}{1 - \left[\left(\frac{37.4}{125.3}\right)^2 \times \left(1 - \frac{7{,}224}{29{,}549}\right)\right]} = 7{,}745 \text{ in.}^4 < I_g \qquad \text{Eq. (6.88)}$$

- Negative moment section

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474 \times 13{,}504}{(28.5/2) \times 12{,}000} = 37.4 \text{ ft-kips}$$

At the exterior support:

$$M_D^- = 40.1 \text{ ft-kips} > 2M_{cr}/3 = 25.0 \text{ ft-kips}$$

Therefore,

$$(I_e)_D^- = \frac{I_{cr}}{1 - \left(\frac{2M_{cr}/3}{M_D^-}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)} = \frac{4{,}265}{1 - \left[\left(\frac{25.0}{40.1}\right)^2 \times \left(1 - \frac{4{,}265}{13{,}504}\right)\right]} = 5{,}810 \text{ in.}^4 < I_g$$

$$M_{sus}^- = 50.0 \text{ ft-kips} > 2M_{cr}/3 = 25.0 \text{ ft-kips}$$

Therefore,

$$(I_e)^-_{sus} = \frac{I_{cr}}{1-\left(\frac{2M_{cr}/3}{M^-_{sus}}\right)^2\left(1-\frac{I_{cr}}{I_g}\right)} = \frac{4,265}{1-\left[\left(\frac{25.0}{50.0}\right)^2 \times \left(1-\frac{4,265}{13,504}\right)\right]} = 5{,}145 \text{ in.}^4 < I_g$$

$M^-_{D+L} = 40.1 + 33.0 = 73.1$ ft-kips $> 2M_{cr}/3 = 25.0$ ft-kips

Therefore,

$$(I_e)^-_{D+L} = \frac{I_{cr}}{1-\left(\frac{2M_{cr}/3}{M^-_{D+L}}\right)^2\left(1-\frac{I_{cr}}{I_g}\right)} = \frac{4,265}{1-\left[\left(\frac{25.0}{73.1}\right)^2 \times \left(1-\frac{4,265}{13,504}\right)\right]} = 4,636 \text{ in.}^4 < I_g$$

At the first interior support:

$M^-_D = 80.0$ ft-kips $> 2M_{cr}/3 = 25.0$ ft-kips

Therefore,

$$(I_e)^-_{D} = \frac{I_{cr}}{1-\left(\frac{2M_{cr}/3}{M^-_{D}}\right)^2\left(1-\frac{I_{cr}}{I_g}\right)} = \frac{7,078}{1-\left[\left(\frac{25.0}{80.0}\right)^2 \times \left(1-\frac{7,078}{13,504}\right)\right]} = 7,423 \text{ in.}^4 < I_g$$

$M^-_{sus} = 99.8$ ft-kips $> 2M_{cr}/3 = 25.0$ ft-kips

Therefore,

$$(I_e)^-_{sus} = \frac{I_{cr}}{1-\left(\frac{2M_{cr}/3}{M^-_{sus}}\right)^2\left(1-\frac{I_{cr}}{I_g}\right)} = \frac{7,078}{1-\left[\left(\frac{25.0}{99.8}\right)^2 \times \left(1-\frac{7,078}{13,504}\right)\right]} = 7,296 \text{ in.}^4 < I_g$$

$M^-_{D+L} = 80.0 + 65.9 = 145.9$ ft-kips $> 2M_{cr}/3 = 25.0$ ft-kips

Therefore,

$$(I_e)^-_{D+L} = \frac{I_{cr}}{1-\left(\frac{2M_{cr}/3}{M^-_{D+L}}\right)^2\left(1-\frac{I_{cr}}{I_g}\right)} = \frac{7,078}{1-\left[\left(\frac{25.0}{145.9}\right)^2 \times \left(1-\frac{7,078}{13,504}\right)\right]} = 7{,}178 \text{ in.}^4 < I_g$$

- Average effective moment of inertia

Dead load:

$$(I_e)_D = \frac{9,308 + 5,810 + 7,423}{3} = 7{,}514 \text{ in.}^4$$ ACI 24.2.3.6

Sustained load:

$$(I_e)_{sus} = \frac{8,438 + 5,145 + 7,296}{3} = 6,960 \text{ in.}^4$$

Dead plus live load:

$$(I_e)_{D+L} = \frac{7,745 + 4,636 + 7,178}{3} = 6{,}520 \text{ in.}^4$$

Step 5 – Determine the immediate deflections and check the maximum permissible deflection for live loads

The following equation can be used to determine immediate deflections:

$$\Delta_i = \frac{5KM_a\ell^2}{48E_cI_e} \qquad \text{Eq. (6.89)}$$

For continuous members:

$$K = 1.2 - 0.2(M_o / M_a) \qquad \text{Table 6.19}$$

where $M_o = w\ell^2 / 8$ and $M_a = w\ell^2 / 14$. Therefore, $K = 1.2 - [0.2 \times (14 / 8)] = 0.85$.

- Dead load

$$(\Delta_i)_D = \frac{5KM_D^+\ell^2}{48E_c(I_e)_D} = \frac{5 \times 0.85 \times 68.7 \times 12{,}000 \times (39.8 \times 12)^2}{48 \times 3{,}605{,}000 \times 7{,}514} = 0.62 \text{ in.}$$

- Sustained load

$$(\Delta_i)_{sus} = \frac{5KM_{sus}^+\ell^2}{48E_c(I_e)_{sus}} = \frac{5 \times 0.85 \times 85.7 \times 12{,}000 \times (39.8 \times 12)^2}{48 \times 3{,}605{,}000 \times 6{,}960} = 0.83 \text{ in.}$$

- Dead plus live load

$$(\Delta_i)_{D+L} = \frac{5KM_{D+L}^+\ell^2}{48E_c(I_e)_{D+L}} = \frac{5 \times 0.85 \times 125.3 \times 12{,}000 \times (39.8 \times 12)^2}{48 \times 3{,}605{,}000 \times 6{,}520} = 1.3 \text{ in.}$$

- Live load

$$(\Delta_i)_L = (\Delta_i)_{D+L} - (\Delta_i)_D = 1.3 - 0.62 = 0.68 \text{ in.}$$

For a floor member not supporting or attached to nonstructural elements likely to be damaged by large deflections, the maximum permissible immediate live load deflection is equal to the following:

$$(\Delta_i)_{L|max} = \ell / 360 = (39.8 \times 12) / 360 = 1.3 \text{ in.} > (\Delta_i)_L = 0.68 \text{ in.} \qquad \text{Table 6.21}$$

Step 6 – Determine the time-dependent deflections and check the maximum permissible deflection

Assuming a 60-month duration of loading with $\rho' = 0$:

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'} = \frac{2.0}{1 + 0} = 2.0 \qquad \text{Eq. (6.90), Table 6.20}$$

The time-dependent deflection due to creep and shrinkage is equal to the following:

$$\Delta_{cs} = \lambda_\Delta(\Delta_i)_{sus} = 2.0 \times 0.83 = 1.7 \text{ in.} \qquad \text{Eq. (6.91)}$$

$$\text{Total deflection} = \Delta_{cs} + (\Delta_i)_L = 1.7 + 0.68 = 2.4 \text{ in.} > \ell / 240 = 2.0 \text{ in.} \qquad \text{Table 6.21}$$

The joist is not adequate for time-dependent deflections.

Comments. It is assumed in this example that the contribution of the compression reinforcement is zero (that is, $\rho' = 0$). It can be shown the joist is adequate for time-dependent deflections when the compression reinforcement is included in the analysis.

6.9.15 Example 6.15 – Determination of Beam Size: Edge Beam in Building #2, Beam is Not Part of the LFRS, SDC C

Determine the size of an edge beam in Building #2 at the second-floor level where the beam is not part of the LFRS (see Figure 1.2). Assume $24 + 4.5 \times 7 + 53$ joists, 28 in. by 28 in. corner columns, and 18 in. by 32 in. edge columns. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Example 6.10.

Step 1 – Determine the beam depth based on serviceability requirements ACI 9.3.1.1

Assume the beam is not supporting or attached to partitions or other construction likely to be damaged by large deflections. Thus, deflections in accordance with ACI 24.2 need not be calculated.

The minimum beam depth is based on the end spans with a support condition of one end continuous:

$$\text{Beam depth} = \frac{\ell}{18.5} = \frac{30.0 \times 12}{18.5} = 19.5 \text{ in.}$$ Table 6.1

Because the beams are part of a wide-module joist system where the overall depth of the joists are 28.5 in., the depth of the beams is taken as 28.5 in. for formwork economy.

Step 2 – Determine the beam width

Assuming an initial estimate of $A_s = 0.01 b_w d$ with $f_c' = 4{,}000$ psi and Grade 60 reinforcement, the required b_w is determined by the following equation:

$$b_w = \frac{24.4 M_u}{d^2}$$ Eq. (6.1)

The largest M_u along the spans is used to determine b_w.

$$\text{Weight of joist system} = \frac{150.0}{5.0 \times 144} \times \{(60.0 \times 4.5) + [2 \times (2 \times 24.0 / 2)] + (7.0 \times 24.0)\} = 101.3 \text{ lb/ft}^2$$

$$\text{Superimposed dead load} = 20 \text{ lb/ft}^2$$

$$\text{Live load} = 100 \text{ lb/ft}^2$$

Conservatively estimate the factored dead load of the beam web as 1.0 kip/ft.

$$w_u = \left\{1.2 \times \left[(101.3 + 20.0) \times \left(\frac{42.5}{2} + \frac{28.0}{2 \times 12}\right)\right] / 1{,}000 + 1.0\right\} + \left[1.6 \times 100.0 \times \left(\frac{42.5}{2} + \frac{28.0}{2 \times 12}\right)\right] / 1{,}000$$ Table 3.3

$$= 7.9 \text{ kip/ft}$$

The limitations of the simplified method of analysis in ACI 6.5.1 are satisfied for this beam (see Step 1 in Example 6.16).

Exterior spans: $\ell_n = 30.0 - (28.0 / 24) - (32.0 / 24) = 27.5$ ft

Interior spans: $\ell_n = 30.0 - (32.0 / 12) = 27.3$ ft

Maximum M_u occurs at the face of the first interior column in an end span:

$$M_u = \frac{w_u \ell_n^2}{10} = \frac{7.9 \times [(27.5 + 27.3) / 2]^2}{10} = 593.1 \text{ ft-kips}$$ Figure 4.3

Approximate $d = 28.5 - 2.5 = 26.0$ in.

Therefore,

$$b_w = \frac{24.4 \times 593.1}{26.0^2} = 21.4 \text{ in.}$$

For formwork economy, use a 30.0-in.-wide beam, which is 2.0 in. wider than the 28.0-in. corner column it frames into (see Sect. 6.2.3 of this publication on guidelines for sizing beams for economy).

Use a 30.0 in. by 28.5 in. beam.

6.9.16 Example 6.16 – Determination of Flexural Reinforcement: Edge Beam in Building #2, Beam is Not Part of the LFRS, SDC C

Determine the required flexural reinforcement for an edge beam in Building #2 at the second-floor level where the beam is not part of the LFRS (see Figure 1.2). Assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Example 6.15.

Step 1 – Determine the factored bending moments along the span

$$\text{Weight of joist system} = \frac{150.0}{5.0 \times 144} \times \{(60.0 \times 4.5) + [2 \times (2 \times 24.0 / 2)] + (7.0 \times 24.0)\} = 101.3 \text{ lb/ft}^2$$

$$\text{Weight of beam web} = \frac{30.0 \times 24.0}{144} \times 150.0 = 750 \text{ lb/ft}$$

$$\text{Superimposed dead load} = 20 \text{ lb/ft}^2$$

$$\text{Live load} = 100 \text{ lb/ft}^2$$

$$w_u = \left[1.2 \times (101.3 + 20.0) \times \left(\frac{42.5}{2} + \frac{28.0}{2 \times 12}\right)\right] / 1{,}000 + (1.2 \times 750 / 1{,}000) + \left[1.6 \times 100.0 \times \left(\frac{42.5}{2} + \frac{28.0}{2 \times 12}\right)\right] / 1{,}000$$

$$= 7.8 \text{ kip/ft}$$

Check if the limitations of the simplified method of analysis in ACI 6.5.1 are satisfied for this system:

- Members are prismatic
- Loads are uniformly distributed
- $L = 100 \text{ psf} < 3D = 3 \times [101.3 + 20.0 + (750 / 22.4)] = 464.4 \text{ psf}$
- There are 6 spans, which is greater than 2 spans
- All spans have the same length

Therefore, the simplified method of analysis in ACI 6.5.1 can be used to determine the bending moments and shear forces.

Exterior spans: $\ell_{n,1} = 30.0 - (28.0 / 24) - (32.0 / 24) = 27.5$ ft

Interior spans: $\ell_{n,2} = 30.0 - (32.0 / 12) = 27.3$ ft

$\ell_{n,avg} = (\ell_{n,1} + \ell_{n,2}) / 2 = (27.5 + 27.3) / 2 = 27.4$ ft

The factored bending moments are given in Table 6.33. Figure 4.3

Table 6.33 Factored Bending Moments for an Edge Beam in Example 6.16 (ft-kips)

End Span			Interior Span	
Exterior Negative	**Positive**	**First Interior Negative**	**Positive**	**Interior Negative**
$\frac{w_u \ell^2_{n,1}}{16} = -368.7$	$\frac{w_u \ell^2_{n,1}}{14} = 421.3$	$\frac{w_u \ell^2_{n,avg}}{10} = -585.6$	$\frac{w_u \ell^2_{n,2}}{16} = 363.3$	$\frac{w_u \ell^2_{n,avg}}{11} = -532.4$

Step 2 – Determine the effective flange width of the beam ACI 6.3.2.1

For an edge beam, the effective flange width, b_f, is equal to the following:

$$b_f = b_w + \text{ least of } \begin{cases} 6h = 6 \times 4.5 = 27.0 \text{ in.} \\ s_w / 2 = [(42.5 \times 12) - 16.0 - (32.0 / 2)] / 2 = 239.0 \text{ in.} \\ \ell_n / 12 = 27.5 \times 12 / 12 = 27.5 \text{ in.} \end{cases} = 30.0 + 27.0 = 57.0 \text{ in.}$$

Figure 6.10

Step 3 – Determine the required flexural reinforcement at the critical sections

At negative moment critical sections (flange in tension), required A_s is determined by the following assuming a single layer of tension reinforcement:

$$R_n = \frac{M_u}{\phi b d^2}$$ Eq. (6.34)

$$A_s = \frac{0.85 f'_c b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right]$$ Eq. (6.33)

where $b = b_w = 30.0$ in. Figure 6.11

At positive moment critical sections (flange in compression), determine the depth of the equivalent stress block, a, assuming rectangular section behavior using the largest positive M_u in Table 6.33 (which results in the largest a):

$$a = \frac{a_1 d - \sqrt{(a_1 d)^2 - (2a_1 M_u / \phi)}}{a_1}$$

Eq. (6.33)

$$= \frac{(193.8 \times 26.0) - \sqrt{(193.8 \times 26.0)^2 - (2 \times 193.8 \times 421.3 \times 12 / 0.9)}}{193.8} = 1.1 \text{ in.} < h = 4.5 \text{ in.}$$

where $a_1 = 0.85 f'_c b_f = 0.85 \times 4 \times 57.0 = 193.8$ kips/in.

Because $a < h$, the section behaves as a rectangular section and Eqs. (6.34) and (6.33) can be used to determine A_s with $b = b_f = 57.0$ in. (see Figure 6.12).

A summary of the required reinforcement is given in Table 6.34. All sections are tension-controlled because $A_s < A_{s,t}$.

Table 6.34 Required Flexural Reinforcement for the Edge Beam in Example 6.16

Location		M_u (ft-kips)	R_n (psi)	A_s (in^2)*
End span	Exterior negative	–368.7	242	3.27
	Positive	421.3	146	3.68
	First interior negative	–585.6	385	5.33
Interior span	Positive	363.3	126	3.17
	Negative	–532.4	350	4.81

*Min. $A_s = 200 b_w d / f_y = 2.60 \text{ in.}^2$ [Eq. (6.35)]

$$Max.\ A_{s,t} = 0.018bd = \begin{cases} 0.018 \times 30.0 \times 26.0 = 14.0 \text{ in.}^2 \text{ at negative critical sections} \\ 0.018 \times 57.0 \times 26.0 = 26.7 \text{ in.}^2 \text{ at positive critical sections} \end{cases}$$ [Eq. (6.38)]

It is determined in Example 6.18 that torsional effects must be considered for this edge beam. Thus, the flexural reinforcement in Table 6.34 must be combined with the longitudinal torsional reinforcement, which is determined in Example 6.18. The size and spacing of the total required longitudinal reinforcement are given in Example 6.19.

6.9.17 Example 6.17 – Determination of Shear Reinforcement: Edge Beam in Building #2, Beam is Not Part of the LFRS, SDC C

Determine the required shear reinforcement for an edge beam in Building #2 at the second-floor level where the beam is not part of the LFRS (see Figure 1.2). Assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Examples 6.15 and 6.16.

Step 1 – Determine the factored shear forces along the span

The maximum shear force occurs at the first interior support in an end span for the load combination $U = 1.2D + 1.6L$ (see Figure 4.3).

$$w_u = \left[1.2 \times (101.3 + 20.0) \times \left(\frac{42.5}{2} + \frac{28.0}{2 \times 12}\right)\right] / 1{,}000 + (1.2 \times 750 / 1{,}000) + \left[1.6 \times 100.0 \times \left(\frac{42.5}{2} + \frac{28.0}{2 \times 12}\right)\right] / 1{,}000$$

$$= 7.8 \text{ kips/ft}$$

$$V_u \text{ @ face} = 1.15 w_u \ell_{n1} / 2 = 1.15 \times 7.8 \times 27.5 / 2 = 123.3 \text{ kips}$$

The 3 conditions in ACI 9.4.3.2 are satisfied, so the critical section for shear can be taken a distance $d = 26.0$ in. from the face of the support (see Figure 6.4):

$$V_u \text{ @ } d = 123.3 - [7.8 \times (26.0 / 12)] = 106.4 \text{ kips}$$

Step 2 – Determine the required transverse reinforcement for shear

Assuming at least minimum shear reinforcement is provided, ϕV_c is determined from the following with the axial force $N_u = 0$:

$\phi V_c = \phi 2\lambda\sqrt{f'_c} b_w d = 0.75 \times 2 \times 1.0\sqrt{4{,}000} \times 30.0 \times 26.0 / 1{,}000 = 74.0 \text{ kips}$ Table 6.4

where $\lambda = 1.0$ for normalweight concrete. Table 6.2

$V_u - \phi V_c = 106.4 - 74.0 = 32.4 \text{ kips}$

For $V_u - \phi V_c = 32.4 \text{ kips} < \phi 4\sqrt{f'_c} b_w d = 148.0 \text{ kips:}$

Maximum spacing $s_{max} = d / 2 = 13.0 \text{ in.}$ along the length of the beam. Figure 6.19

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi f_{yt} d} = \frac{32.4}{0.75 \times 60 \times 26.0} = 0.0277 \text{ in.}^2/\text{in.}$$

It is determined in Example 6.18 that torsional effects must be considered for this edge beam. Thus, the shear reinforcement calculated in this example must be combined with the transverse torsional reinforcement, which is determined in Example 6.18. The size and spacing of the total required transverse reinforcement are given in Example 6.19.

6.9.18 Example 6.18 – Determination of Torsion Reinforcement: Edge Beam in Building #2, Second-Floor Level, Beam is Not Part of the LFRS, SDC C

Determine the required torsion reinforcement for an edge beam in Building #2 at the second-floor level where the beam is not part of the LFRS (see Figure 1.2). Assume normalweight concrete with $f'_c = 4{,}000 \text{ psi}$ and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Examples 6.15 through 6.17.

Step 1 – Determine the factored torsional moments along the span

The wide-module joist system framing into the edge beam produces a uniform torsional load along the length of the beam. The torsional load is caused by the shear forces and bending moments transferred from the joists to the beam (see Figure 6.51). The dead load of the joists and the superimposed dead load and live load on the joists must be considered:

$$w_u = [1.2 \times (101.3 + 20.0)) + (1.6 \times 100.0) = 305.6 \text{ lb/ft}^2$$

The factored shear and bending moments at the end of the joists per foot width of joists are determined using Figure 4.3:

$$V_u = w_u \ell_n / 2 = (305.6 \times 39.8) / (2 \times 1{,}000) = 6.1 \text{ kips/ft}$$

$$M_u = w_u \ell_n^2 / 24 = (305.6 \times 39.8^2) / (24 \times 1{,}000) = 20.2 \text{ ft-kips/ft}$$

The total uniform torsional load on the edge beam is equal to the sum of the torsional loads due to V_u and M_u :

$$t_u = [6.1 \times (15.0 / 12)] + 20.2 = 27.8 \text{ ft-kips/ft}$$

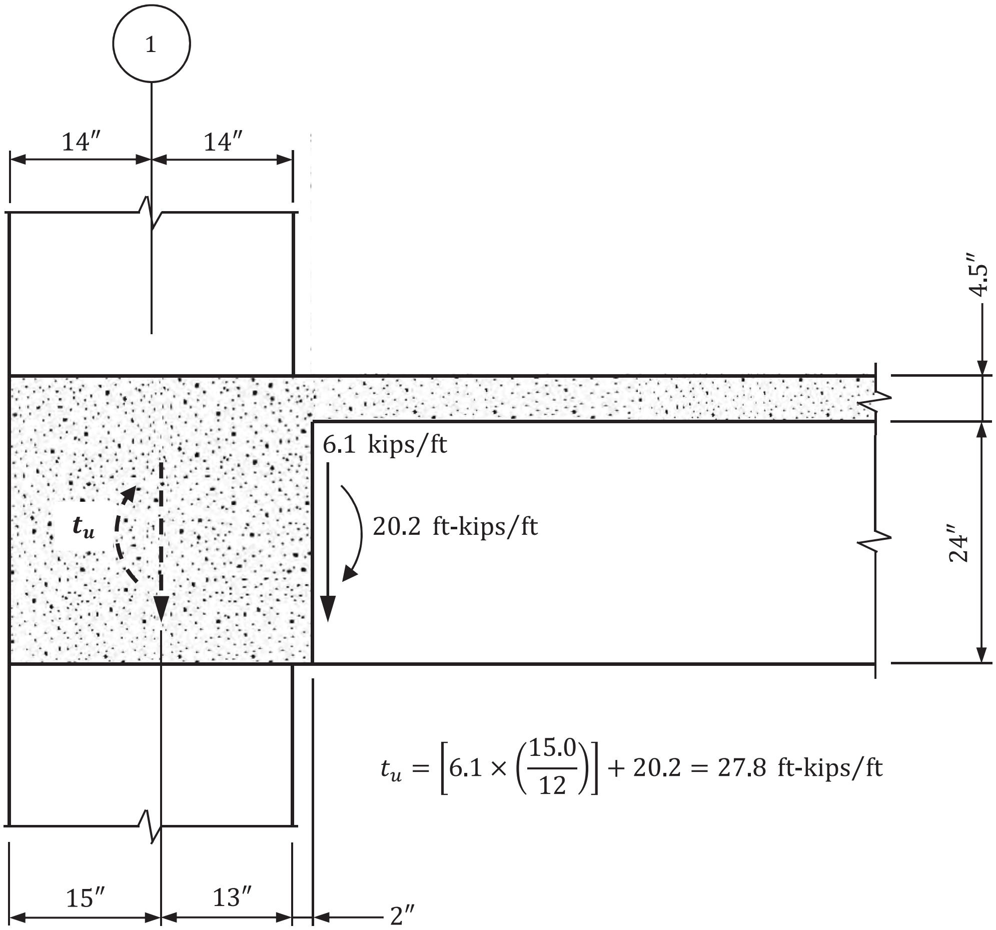


Figure 6.51 Reactions on the edge beam in Example 6.18.

Assuming the ends of the edge beam are fixed against rotation by the columns, the factored torsional moment at the face of each end of the beam due to t_u is equal the following:

$$T_u = t_u \ell_n / 2 = 27.8 \times 27.5 / 2 = 382.3 \text{ ft-kips}$$

where the longest clear span is used to calculate T_u.

$$T_u \ @ \ d = 382.3 - [27.8 \times (26.0 / 12)] = 322.1 \text{ ft-kips}$$ ACI 9.4.4.3

Step 2 – Determine if torsional effects must be considered

Torsion can be neglected where the factored torsional moment from analysis is less than the threshold torsion:

$$T_u < \phi T_{th} = \phi \lambda \sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}} \right)$$ Eq. (6.3)

Determine the overhanging flange width, b_e, permitted to be used in the calculation of A_{cp} and p_{cp}:

$$b_e = \text{least of} \begin{cases} h_b = 28.5 - 4.5 = 24.0 \text{ in.} \\ 4h = 4 \times 4.5 = 18.0 \text{ in.} \end{cases}$$ Eq. (6.4)

Torsional section properties including the overhanging flange:

$$A_{cp} = b_w(h_b + h) + hb_e = [30.0 \times (24.0 + 4.5)] + (4.5 \times 18.0) = 936 \text{ in.}^2$$ Figure 6.3

$$p_{cp} = 2(h_b + h + b_w + b_e) = 2 \times (24.0 + 4.5 + 30.0 + 18.0) = 153 \text{ in.}$$

$$A_{cp}^2 / p_{cp} = 5{,}726 \text{ in.}$$

Torsional section properties without the overhanging flange:

$$A_{cp} = b_w(h_b + h) = 30.0 \times (24.0 + 4.5) = 855 \text{ in.}^2$$

$$p_{cp} = 2(h_b + h + b_w) = 2 \times (24.0 + 4.5 + 30.0) = 117 \text{ in.}$$

$$A_{cp}^2 / p_{cp} = 6{,}248 \text{ in.}$$

Because A_{cp}^2 / p_{cp} for the beam with the overhanging flange is less than that for the beam without the overhanging flange, use $A_{cp}^2 / p_{cp} = 6{,}248$ in. [ACI (9.2.4.4(b)].

$$T_u = 322.1 \text{ ft-kips} > \phi T_{th} = 0.75 \times 1.0 \times \sqrt{4{,}000} \times 6{,}248 / 12{,}000 = 24.7 \text{ ft-kips}$$

Therefore, torsional effects must be considered.

Because the edge beam is part of indeterminate system in which redistribution of internal forces can occur following torsional cracking, the maximum factored torsional moment at the critical section need not exceed the cracking torsional moment:

$$T_u = \phi T_{cr} = \phi 4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right) = 0.75 \times 1.0 \times 4 \times \sqrt{4{,}000} \times 6{,}248 / 12{,}000 = 98.8 \text{ ft-kips}$$ ACI 22.7.3.2

This redistribution is equivalent to reducing the factored distributed torsional loading on the edge beam to the following:

$$t_u = \frac{2 \times 98.8}{27.5 - (2 \times 26.0 / 12)} = 8.5 \text{ ft-kips/ft}$$

Step 3 – Adjust the bending moments and shear forces in the joists in the end spans

The bending moments in the joists in the end spans must be adjusted in accordance with ACI 22.7.3.3.

The bending moments at the critical sections in an end span are given in Table 6.30 in Example 6.11 and are reproduced in Figure 6.52 per foot width of joist (that is, the tabulated values of the bending moments are divided by the 5.0-ft spacing between joists).

Prior to adjustment, the torsional moment at the centroid of the edge beam is equal to 27.8 ft-kips/ft, which is the bending moment in the joists at this location (see Step 1 and Figure 6.51). Due to redistribution of the factored torsional moments, the torsional moment is reduced to 8.5 ft-kips/ft at the centroid of the edge beam (see Step 2). Therefore, to account for redistribution of the torsional moments, a torsional moment equal to $27.8 - 8.5 = 19.3$ ft-kips/ft must be applied at the centroid of the edge beam in the opposite direction to the 27.8 ft-kip/ft bending moment (see Figure 6.52). One-half of the 19.3 ft-kips/ft torsional moment is carried over to the centroid of the first interior beam.

The design bending moments in the joists per foot width of joist after adjustment are obtained at the critical sections by algebraically combining the bending moments prior to adjustment with those due to redistribution (see Figure 6.52).

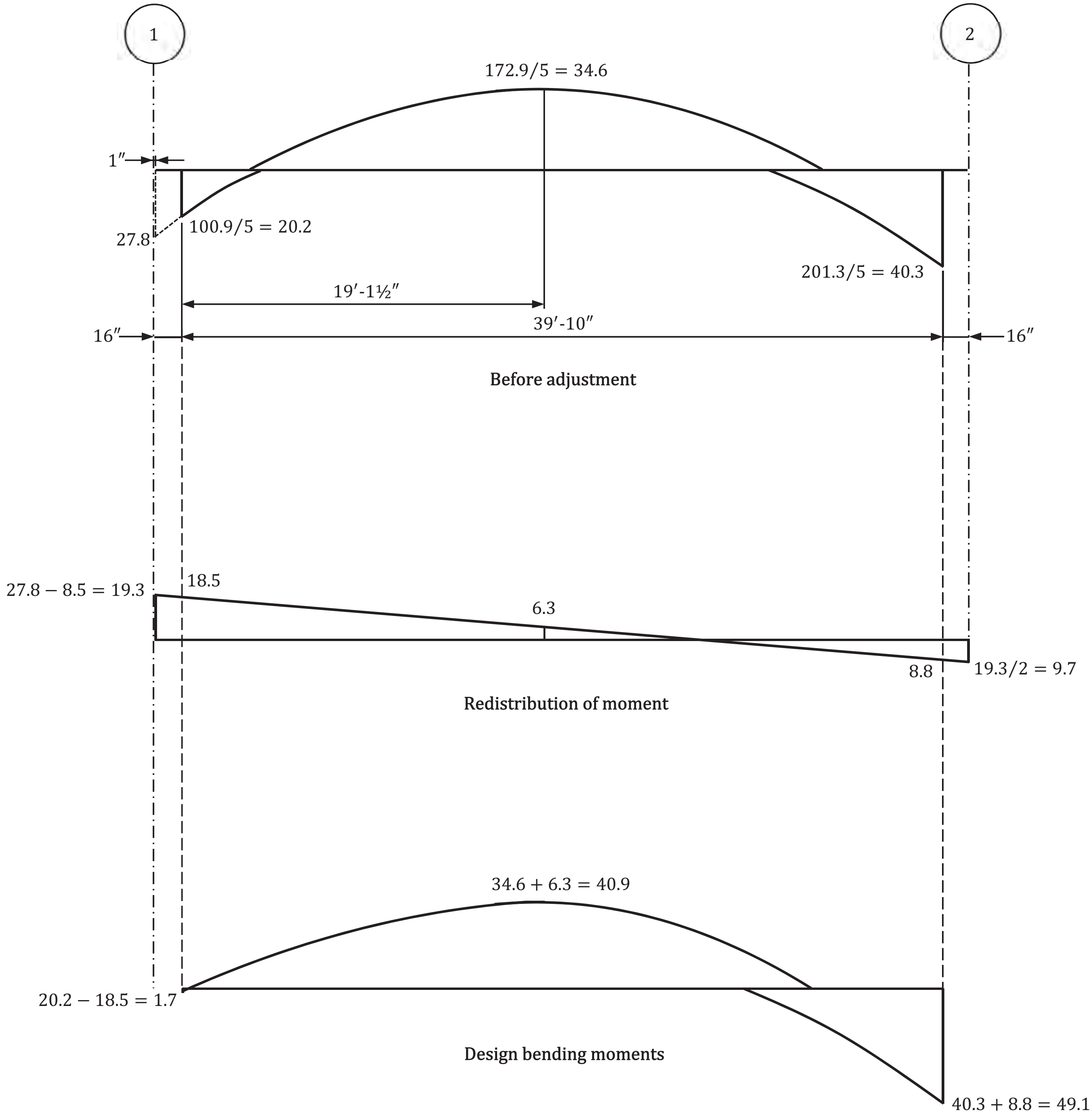


Figure 6.52 Bending moments in a joist in an end span (ft-kips/ft).

The required area of flexural reinforcement based on the design bending moments in Figure 6.52 are given in Table 6.35; the numbers in parentheses under the A_s column are the required areas of flexural reinforcement prior to adjustment of the torsional moments (see Example 6.11). As expected, the flexural reinforcement decreases at the exterior negative critical section and increases at the other two critical sections. The number and size of the reinforcing bars are also given in Table 6.35, along with the reinforcing bars selected in Example 6.11, which are given in parentheses. At the exterior negative critical section, 8-#4 bars are used to satisfy spacing requirements.

Table 6.35 Required Flexural Reinforcement for the Joist in the End Span in Example 6.18

Location	M_u (ft-kips)	R_n (psi)	A_s (in^2)*	Reinforcement
Exterior negative	$5 \times (-1.7) = -8.5$	24	0.61 (0.90)	#4 @ 10″ (#4 @ 10″)
Positive	$5 \times 40.9 = 204.5$	67	1.77 (1.49)	2-#9 (2-#8)
First interior negative	$5 \times (-49.1) = -245.5$	692	2.37 (1.89)	#5 @ 6″ (#5 @ 8″)

*$Min.\ A_s = 200 b_w d / f_y = 0.61\ \text{in.}^2$ [Eq. (6.35)]

$$Max.\ A_{s,t} = 0.018bd = \begin{cases} 0.018 \times 7.0 \times 26.0 = 3.28\ \text{in.}^2 \text{ at negative critical sections} \\ \\ 0.018 \times 60.0 \times 26.0 = 28.1\ \text{in.}^2 \text{ at positive critical sections} \end{cases} \quad \text{Eq. [(6.38)]}$$

The maximum design shear force after redistribution occurs at the face of the first interior support:

$$V_u \text{ @ face} = \left(\frac{1{,}528 \times 39.8^2}{2 \times 1{,}000} + 245.5 - 8.5 \right) / 39.8 = 36.4 \text{ kips}$$

$$V_u \text{ @ } d = 36.4 - \left(\frac{1{,}528 \times 26.0}{12 \times 1{,}000} \right) = 33.1 \text{ kips}$$

Assuming at least minimum shear reinforcement is provided, ϕV_c can be determined from the following equation with the axial force $N_u = 0$:

$\phi V_c = \phi 2 \lambda \sqrt{f'_c} b_w d = 0.75 \times 2 \times 1.0 \sqrt{4{,}000} \times 7.0 \times 26.0 / 1{,}000 = 17.3 \text{ kips}$ Table 6.4

where $\lambda = 1.0$ for normalweight concrete. Table 6.2

$V_u - \phi V_c = 33.1 - 17.3 = 15.8 \text{ kips}$

For $V_u - \phi V_c = 15.8 \text{ kips} < \phi 4 \sqrt{f'_c} b_w d = 34.6 \text{ kips}$:

Maximum spacing $s_{max} = d/2 = 13.0$ in. along the length of the joist. Figure 6.19

Because of the narrow width of the joist, provide single-leg stirrups. Figure 6.16

A single-leg #4 stirrup spaced at $d/2$ provides $V_u - \phi V_c = 36.0/2 = 18.0 \text{ kips} > 15.8 \text{ kips}$ Table 6.6

$$\frac{A_v}{s} = \frac{0.20}{13} = 0.015 \text{ in.}^2/\text{in.}$$

Table 6.5

$$> \frac{A_{v,min}}{s} = \text{ greater of } \begin{cases} 0.75\sqrt{f'_c} b_w / f_{yt} = 0.75\sqrt{4{,}000} \times 7.0 / 60{,}000 = 0.0055 \text{ in.}^2/\text{in.} \\ \\ 50 b_w / f_{yt} = 50 \times 7.0 / 60{,}000 = 0.0058 \text{ in.}^2/\text{in.} \end{cases}$$

Use #4 single-leg stirrups spaced at 12 in. on center $(A_{v,provided} = 0.20 \text{ in.}^2)$, which is the same shear reinforcement determined prior to redistribution.

Step 4 – Check the adequacy of the cross-sectional dimensions of the beam

Assuming 1.5-in. clear cover to #4 closed stirrups:

$$x_1 = b_w - 2c - d_s = 30.0 - (2 \times 1.5) - 0.5 = 26.5 \text{ in.}$$ Figure 6.17

$$y_1 = h_b + h - 2c - d_s = 24.0 + 4.5 - (2 \times 1.5) - 0.5 = 25.0 \text{ in.}$$

$$A_{oh} = x_1 y_1 = 26.5 \times 25.0 = 662.5 \text{ in.}^2$$

$$p_h = 2(x_1 + y_1) = 2 \times (26.5 + 25.0) = 103.0 \text{ in.}$$

From Step 2, $T_u = 98.8$ ft-kips at the critical section.

From Step 1 in Example 6.17, $V_u = 106.4$ kips at the critical section.

From Step 2 in Example 6.17, $V_c = 74.0 / 0.75 = 98.7$ kips.

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} = \sqrt{\left(\frac{106.4 \times 1{,}000}{30.0 \times 26.0}\right)^2 + \left(\frac{98.8 \times 12{,}000 \times 103.0}{1.7 \times 662.5^2}\right)^2} = 213.1 \text{ psi}$$

$$< \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) = 0.75 \times \left(\frac{98.7 \times 1{,}000}{30.0 \times 26.0} + 8\sqrt{4{,}000}\right) = 474.4 \text{ psi}$$ Eq. (6.31)

Therefore, the cross-sectional dimensions are adequate.

Step 5 – Determine the required transverse reinforcement for torsion

$$\frac{A_t}{s} = \frac{T_u}{2\phi \cot\theta A_o f_{yt}} = \frac{98.8 \times 12{,}000}{2 \times 0.75 \times \cot 45 \times (0.85 \times 662.5) \times 60{,}000} = 0.0234 \text{ in.}^2/\text{in.}$$ Eq. (6.60)

Step 6 – Determine the required longitudinal reinforcement for torsion

$$A_\ell = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right) \cot^2\theta = 0.0234 \times 103.0 \times \left(\frac{60}{60}\right) \times \cot^2 45 = 2.41 \text{ in.}^2$$ Eq. (6.61)

$$A_{\ell,min} = \text{lesser of} \begin{cases} \dfrac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\dfrac{A_t}{s}\right) p_h \left(\dfrac{f_{yt}}{f_y}\right) = \dfrac{5\sqrt{4{,}000} \times 855}{60{,}000} - (0.0234 \times 103.0 \times 1.0) = 2.10 \text{ in.}^2 \\ \dfrac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\dfrac{25 b_w}{f_{yt}}\right) p_h \left(\dfrac{f_{yt}}{f_y}\right) = \dfrac{5\sqrt{4{,}000} \times 855}{60{,}000} - \left[\left(\dfrac{25 \times 30.0}{60{,}000}\right) \times 103.0 \times 1.0\right] = 3.22 \text{ in.}^2 \end{cases}$$ Eq. (6.62)

Use $A_\ell = 2.41 \text{ in.}^2$

The transverse and longitudinal for torsion are combined with the transverse reinforcement required for shear and the longitudinal reinforcement required for flexure (see Example 6.19).

6.9.19 Example 6.19 – Design for Combined Flexure, Shear, and Torsion: Edge Beam in Building #2, Beam is Not Part of the LFRS, SDC C

Design the edge beam in Building #2 at the second-floor level for the combined effects due to flexure, shear, and torsion assuming the beam is not part of the LFRS (see Figure 1.2). Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Examples 6.15 through 6.18.

Step 1 – Determine the total transverse reinforcement

The total required area of transverse reinforcement per stirrup leg is equal to that required for shear (Example 6.17) plus that required for torsion (Example 6.18):

$$\frac{A_v}{2s} + \frac{A_t}{s} = \frac{0.0277}{2} + 0.0234 = 0.0373 \text{ in.}^2/\text{in.}$$

$$> \text{greater of} \begin{cases} 0.375\sqrt{f_c'}b_w \,/\, f_{yt} = 0.375 \times \sqrt{4{,}000} \times 30.0 \,/\, 60{,}000 = 0.0119 \text{ in.}^2/\text{in.} \\ 25b_w \,/\, f_{yt} = 25 \times 30.0 \,/\, 60{,}000 = 0.0125 \text{ in.}^2/\text{in.} \end{cases}$$

For #4 closed stirrups, the required spacing at the critical section is equal to the following:

$$s = \frac{A_b}{\dfrac{A_v}{2s} + \dfrac{A_t}{s}} = \frac{0.20}{0.0373} = 5.4 \text{ in.}$$

It is determined in Example 6.17 that $V_u - \phi V_c = 32.4 \text{ kips} < \phi 4\sqrt{f_c'}b_w d = 148.0 \text{ kips}$.

Therefore, the maximum spacing of the transverse reinforcement is equal to the following:

$$s_{max} = \text{lesser of} \begin{cases} d \,/\, 2 = 26.0 \,/\, 2 = 13.0 \text{ in.} \\ p_h \,/\, 8 = 103.0 \,/\, 8 = 12.9 \text{ in.} \\ 12.0 \text{ in.} \end{cases}$$

Determine the location where the maximum stirrup spacing can be used.

Try a section located 6.0 ft from the face of the support:

$$V_u = 123.3 - (7.8 \times 6.0) = 76.5 \text{ kips}$$

$$\frac{A_v}{2s} = \frac{V_u - \phi V_c}{2\phi f_{yt} d} = \frac{76.5 - 74.0}{2 \times 0.75 \times 60 \times 26.0} = 0.0011 \text{ in.}^2/\text{in.}$$

$$T_u = 98.8 - [8.5 \times (6.0 - 2.167)] = 66.2 \text{ ft-kips}$$

$$\frac{A_t}{s} = \frac{T_u}{2\phi \cot\theta A_o f_{yt}} = \frac{66.2 \times 12{,}000}{2 \times 0.75 \times \cot 45 \times (0.85 \times 662.5) \times 60{,}000} = 0.0157 \text{ in.}^2/\text{in.}$$

$$\frac{A_v}{2s}+\frac{A_t}{s}=0.0011+0.0157=0.0168\text{ in.}^2/\text{in.}>A_{v,min}=0.0125\text{ in.}^2/\text{in.}$$

$$s=\frac{A_b}{\dfrac{A_v}{2s}+\dfrac{A_t}{s}}=\frac{0.20}{0.0168}=11.9\text{ in.}$$

Therefore, the closed #4 stirrups can be spaced at 12.0 in. on center at a distance equal to approximately 6.0 ft from the face of the column.

The distance from the face of the support where transverse reinforcement is no longer required for shear is determined from the following equation (see Figure 6.53):

$$x_{ns|shear}=\frac{(V_u\text{ @ face})-\phi\lambda\sqrt{f_c'}b_wd}{w_u}=\frac{123.3-(0.75\times1.0\times\sqrt{4{,}000}\times30.0\times26.0/1{,}000)}{7.8}=11.1\text{ ft} \qquad \text{Eq. (6.57)}$$

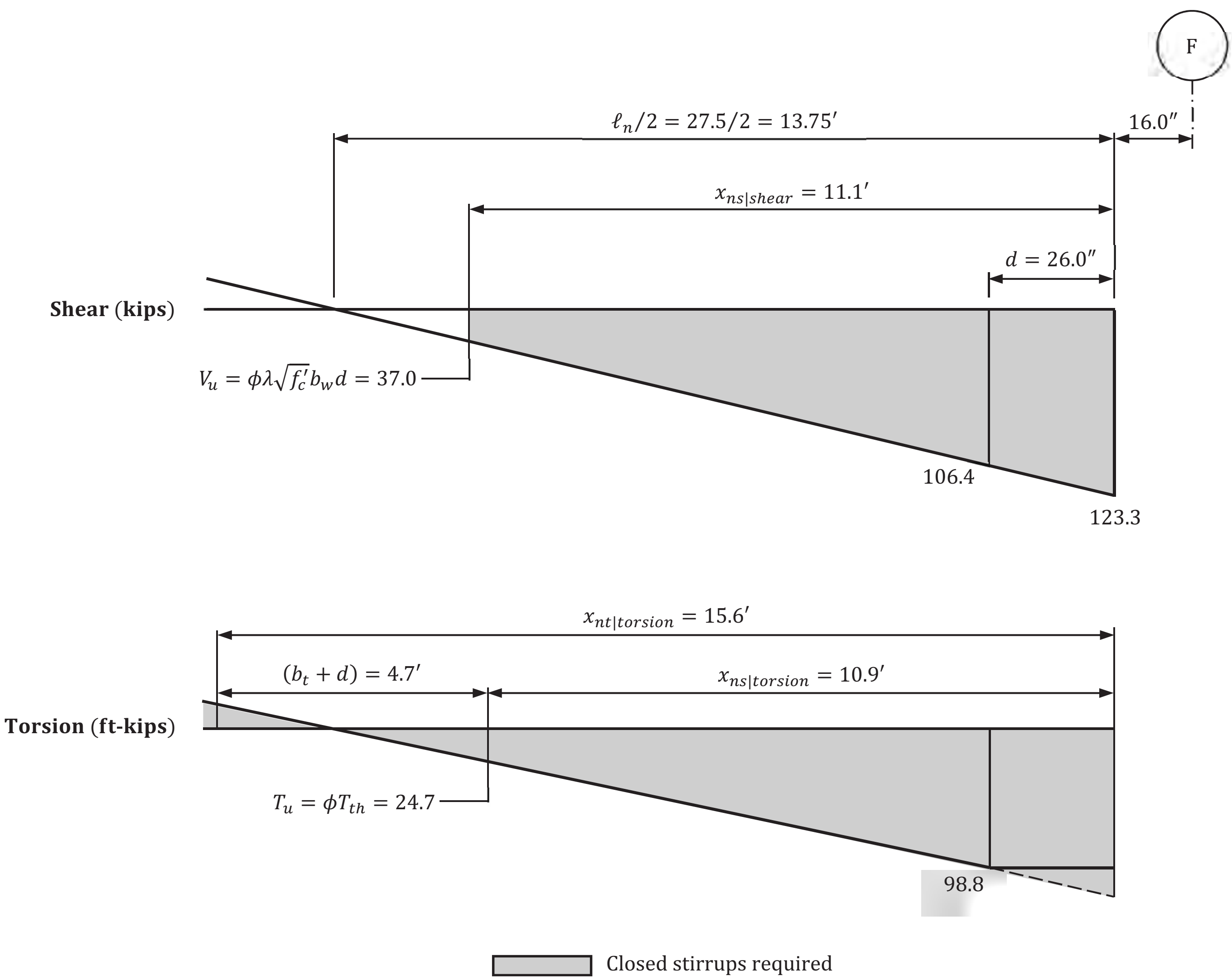


Figure 6.53 Shear and torsion diagrams in an end span for the edge beam in Example 6.19.

Similarly, the distance from the face of the support where transverse reinforcement is no longer required for torsion is determined from the following equation (see Figure 6.53):

$$x_{ns|torsion} = \left[\frac{(T_u \ @ \ d) - \phi T_{th}}{t_u} + d\right] = \left(\frac{98.8 - 24.7}{8.5} + \frac{26.0}{12}\right) = 10.9 \text{ ft}$$

The transverse reinforcement for torsion must extend a distance $(b_t + d)$ past the location it is no longer required:

$$x_{nt|torsion} = x_{ns|torsion} + (b_t + d) = 10.9 + \left(\frac{30.0}{12} + \frac{26.0}{12}\right) = 15.6 \text{ ft} > \frac{\ell_n}{2} = \frac{27.5}{2} = 13.75 \text{ ft} \qquad \text{ACI 9.7.6.3.2}$$

Therefore, torsion requirements govern $(x_{nt|torsion} > x_{ns|shear})$. Because $x_{nt|torsion} > \ell_n / 2$, closed stirrups are required over the entire length of the beam.

Step 2 – Determine the total longitudinal reinforcement

The required flexural reinforcement at the critical sections is given in Table 6.34.

From Step 6 in Example 6.18, $A_\ell = 2.41 \text{ in.}^2$ This reinforcement must be distributed around the perimeter of the beam with a maximum spacing of 12.0 in. and must be combined with that required for flexure. Thus, assign $2.41 / 4 = 0.60 \text{ in.}^2$ to each face.

Use 2-#5 bars on each side face of the beam $(A_{s,provided} = 0.62 \text{ in.}^2 > 0.60 \text{ in.}^2)$

Assuming #9 top and bottom longitudinal reinforcement with #4 closed stirrups and 1.5-in. clear cover:

$$\text{Clear spacing between bars on each side face} = \{28.5 - [2 \times (1.5 + 0.5 + 1.128 + 0.625)]\} / 3 = 7.0 \text{ in.}$$

$$s = \begin{cases} 7.0 + 0.625 = 7.6 \text{ in.} < 12.0 \text{ in. (center-to-center spacing of the \#5 bars)} \\ 7.0 + (0.625 / 2) + (1.128 / 2) = 7.9 \text{ in.} < 12.0 \text{ in. (center-to-center spacing of the \#5 and \#9 bars)} \end{cases} \qquad \text{Figure 6.28}$$

Check the requirement of ACI 9.7.5.2:

$$d_b = 0.625 \text{ in.} > 0.042s = 0.042 \times 11.0 = 0.46 \text{ in. and } 3/8 \text{ in.} \qquad \text{Figure 6.28}$$

where the largest spacing of the transverse reinforcement, s, is used to check this requirement.

The remaining longitudinal reinforcement for torsion is distributed equally between the top and bottom of the section: $0.5 \times [2.41 - (2 \times 0.62)] = 0.59 \text{ in.}^2$

- Exterior negative:

  $\text{Total longitudinal reinforcement} = 3.27 + 0.59 = 3.86 \text{ in.}^2$

  Use 4-#9 bars $(A_{s,provided} = 4.00 \text{ in.}^2 > 3.86 \text{ in.}^2$ and $A_{s,min} = 200 b_w d / f_y = 2.60 \text{ in.}^2)$.

- Positive:

  $\text{Total longitudinal reinforcement} = 3.68 + 0.59 = 4.27 \text{ in.}^2$

  Use 5-#9 bars $(A_{s,provided} = 5.00 \text{ in.}^2 > 4.27 \text{ in.}^2)$.

- First interior negative:

  $\text{Total longitudinal reinforcement} = 5.33 + 0.59 = 5.92 \text{ in.}^2$

  Use 6-#9 bars $(A_{s,provided} = 6.00 \text{ in.}^2 > 5.92 \text{ in.}^2)$.

The provided number of bars at all locations also satisfy maximum and minimum spacing requirements (see Table 6.8 and Table 6.9, respectively).

6.9.20 Example 6.20 – Determination of Reinforcement Details: Edge Beam in Building #2, Beam is Not Part of the LFRS, SDC C

Determine the required reinforcement details for the edge beam in Building #2 at the second-floor level assuming the beam is not part of the LFRS (see Figure 1.2). Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Examples 6.15 through 6.19.

Step 1 – Determine the flexural reinforcement details

It is determined in Example 6.19 that the following reinforcement is required at the critical sections in the end span:

- Exterior negative: 4-#9 bars
- Positive: 5-# 9 bars
- First interior negative: 6-#9 bars

Also, 2-#5 bars are required on each side face.

The provided number of reinforcing bars at all critical sections satisfies the minimum and maximum spacing requirements in ACI 25.2 and 24.3, respectively (see Step 2 in Example 6.19).

It is determined in Example 6.19 that the transverse torsion reinforcement must be provided over the entire span of the beam. Consequently, the longitudinal torsional reinforcement must also be provided over the entire span (ACI 9.7.5.3). The lengths of the reinforcing bars in Figure 6.32 are not applicable in this example because of torsion. Also, the 6-#9 top bars required at the first interior support are made continuous over the entire span.

A Class B tension lap splice is required over the supports for the 5-#9 bottom bars. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #9 reinforcing bars, $\psi_s = 1.0$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + (d_b)_{stirrup} + 0.5(d_b)_{long.} = 1.5 + 0.5 + (0.5 \times 1.128) = 2.6 \text{ in.} \\ \\ \dfrac{s}{2} = \dfrac{30.0 - (2 \times 1.5) - 1.128}{2 \times 4} = 3.2 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.6 + 0) / 1.128 = 2.3 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.3} \right) \times 1.128 = 34.9 \text{ in.} > 12.0 \text{ in.}$$

Class B lap splice length $= 1.3\ell_d = 1.3 \times 34.9 = 45.4$ in. ACI Table 25.5.2.1

Provide a 4 ft-0 in. lap splice length.

Step 2 – Determine the transverse reinforcement details

It is determined in Example 6.19 that #4 closed stirrups must be provided over the entire span of the beam.

Assuming only 2 outer stirrup legs are provided, check the spacing across the width of the beam:

$$s = 30.0 - (2 \times 1.5) - 0.5 = 26.5 \text{ in.} > s_{max} = \text{lesser of} \begin{cases} d = 26.0 \text{ in.} \\ \\ 24.0 \text{ in.} \end{cases} \text{for } V_u - \phi V_c < \phi 4\sqrt{f'_c} b_w d$$

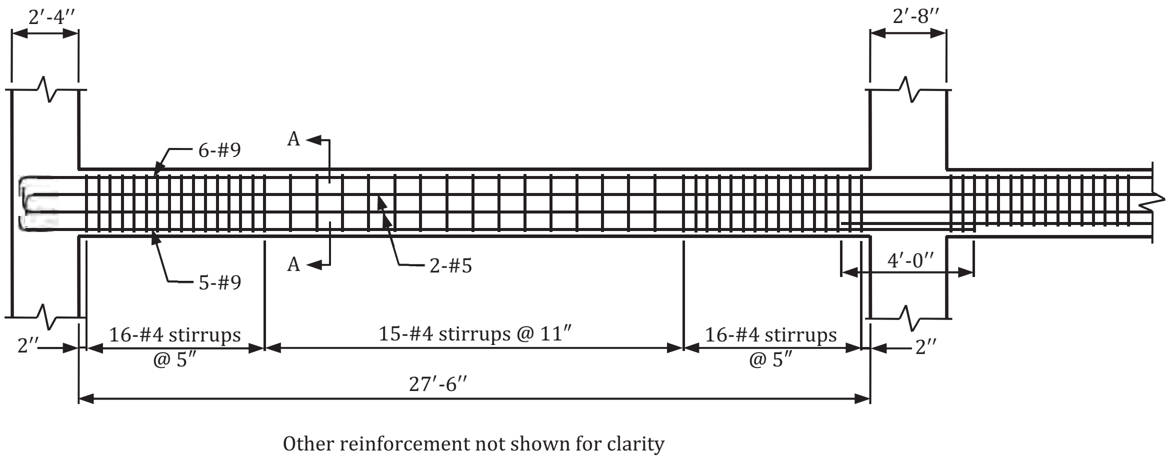
Figure 6.19

Therefore, provide a #4 crosstie to satisfy the requirements for spacing of the stirrups across the width of the beam.

Based on the calculations in Example 6.19, provide the following closed stirrup arrangements over the length of the beam:

- 16-#4 closed stirrups with 1-#4 crosstie spaced at 5.0 in. on center with the first closed stirrup located 2.0 in. from the face of the column
- 15-#4 closed stirrups with 1-#4 crosstie spaced at 11.0 in. on center within the center portion of the beam

Reinforcement details for the beam in the end span are given in Figure 6.54. The details satisfy the structural integrity requirements in ACI 9.7.7 for perimeter beams and can be used for all edge beams.



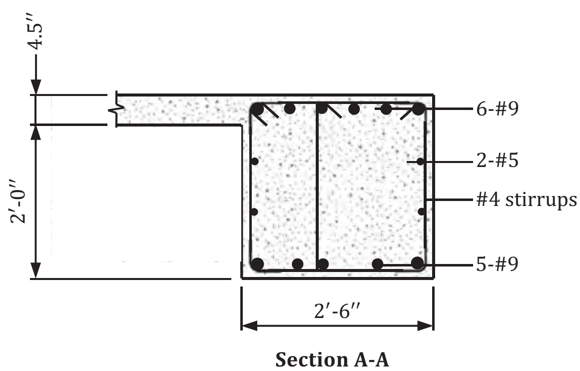


Figure 6.54 Reinforcement details for the edge beam in Examples 6.15 through 6.20.

Chapter 7
COLUMNS

7.1 Overview

The primary function of a column is to support axial forces from floor and roof gravity loads and to transfer those forces to a structural element below (such as a transfer beam or a foundation system). Columns not part of the lateral force-resisting system (LFRS) of the building resist primarily axial compression forces due to gravity loads; bending moments and shear forces from gravity loads must also be resisted, but these reactions are usually relatively small, especially for interior columns. Columns in moment frames designated to part of the LFRS must be designed for the combined effects of flexure and axial forces and the accompanying shear forces. Slenderness effects may need to be considered in the design of a column regardless of whether the column is part of the LFRS or not.

The design and detailing of cast-in-place concrete columns with nonprestressed reinforcement (longitudinal and transverse) are covered in this chapter. Provisions for columns are given in ACI Chapter 10, which are applicable to members in buildings assigned to Seismic Design Category (SDC) A and B.

7.2 Dimensional Limits

No minimum sizes are explicitly required in ACI 318 for columns in buildings assigned to SDC A through C. However, there are some practical constructability issues that need to be considered, which have a direct impact on the cross-sectional dimensions. It is important to ensure that the longitudinal bars can fit within the cross-section considering the minimum cover requirements of ACI 20.5.1 and the minimum spacing requirements of ACI 25.2. The cross-sectional dimensions and the longitudinal bar sizes must be selected to minimize reinforcement congestion, especially at beam-column or slab-column joints. It is also important to verify that the requirements for transverse reinforcement (ties or spirals) are satisfied. Therefore, it is recommended to use at least a 12-in. column dimension for rectangular columns and at least a 12-in. diameter for circular columns.

For columns that have cross-sectional dimensions larger than required to resist the factored loads (like those in the upper floors of a building), it is permitted to calculate the minimum longitudinal reinforcement ratio based on the required cross-sectional area rather than the provided cross-sectional area (ACI 10.3.1.2). The provided area of longitudinal reinforcement in such cases must not be less than 0.5 percent of the actual cross-sectional area of the column. This requirement is not applicable to columns that are part of a special moment frame or that must be designed in accordance with ACI 18.14 (columns not part of the seismic-force-resisting system).

Where columns are built integrally with concrete walls, the cross-sectional area to be used in the design of the column must not exceed an area equal to $(b_1+3)(b_2+3)$ where b_1 and b_2 are the cross-sectional dimensions (in inches) of the column core measured to the outside edges of the transverse reinforcement (ACI 10.3.1.3; see Figure 7.1).

In cases where ACI 10.3.1.2 or 10.3.1.3 have been applied, the actual cross-sectional dimensions of the column must be used in the structural analysis of the building and in the design of any members that interact with the column (ACI 10.3.1.5).

7.3 Required Strength

7.3.1 Analysis Methods

Overview

The analysis methods in ACI Chapter 6 in conjunction with the factored load combinations in ACI Chapter 5 are to be used to calculate required strength (see ACI 10.4.1.2 and 10.4.1.1, respectively).

Axial forces, bending moments, shear forces, and torsional moments on a reinforced concrete column, including the effects of slenderness where applicable (ACI 6.2.5.3), can be determined by the following analysis methods in ACI 6.2.3.

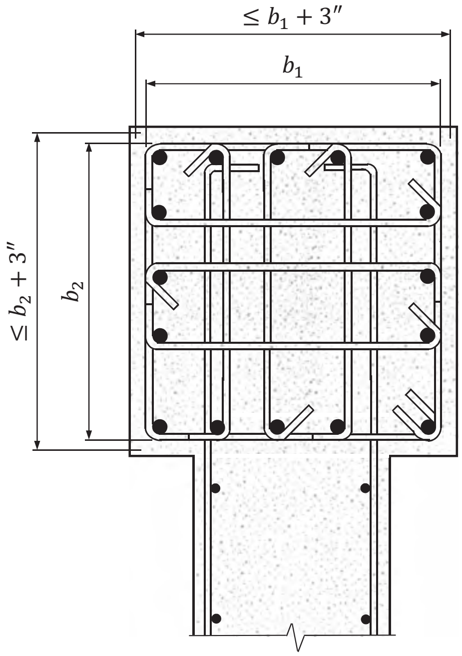


Figure 7.1 Reinforced concrete column built integrally with a concrete wall.

Linear Elastic First-Order Analysis

In a linear elastic first-order analysis, reactions in the structural members are determined based on the undeformed geometry of the structure; slenderness (second-order) effects, which can have a significant influence on the design strength of columns, are not explicitly accounted for in the analysis. Where applicable, slenderness effects in columns may be determined by the magnified moment method in ACI 6.6.4 using the results from the first-order analysis (see Section 7.3.3 of this publication).

Linear Elastic Second-Order Analysis

In a linear elastic second-order analysis, it is assumed the structure remains elastic and second-order effects are inherently accounted for in the analysis (that is, equations of equilibrium are satisfied using the deformed shape of the structure). The effects of cracking along the length of the members and creep must be considered in the analysis (see ACI 6.7.1.1). Where applicable, slenderness effects on columns may be determined by the moment magnification method for nonsway frames in ACI 6.6.4.5 using the results from the second-order analysis (ACI 6.7.1.2). The method for nonsway frames is applicable because a second-order analysis inherently accounts for the relative displacements of the ends of the members.

Inelastic Analysis

In an inelastic analysis, nonlinear stress-strain response of the materials in the structure and compatibility of deformations must be considered (ACI 6.8.1.1). Equilibrium must be satisfied in the undeformed configuration where an inelastic first-order analysis is used or in the deformed configuration where an inelastic second-order analysis is used. It must be demonstrated that the results from an inelastic analysis are in substantial agreement with results from tests (ACI 6.8.1.2). Like in the case of a linear elastic second-order analysis, slenderness effects on columns are permitted to be determined by the moment magnification method for nonsway frames in ACI 6.6.4.5 using the results from the inelastic analysis (ACI 6.8.1.3).

Finite Element Analysis

A finite element analysis conforming to the provisions of ACI 6.9 can be used to analyze essentially any reinforced concrete structure provided an appropriate model is constructed (ACI 6.9).

The moment magnification procedure tends to give more conservative results than those obtained from a more refined analysis. Regardless of the analysis method, the maximum factored bending moments, M_u, at the ends of the members of a frame must be limited to ensure the structure is stable. According to ACI 6.2.5.3, M_u of any structural member obtained from an analysis including second-order effects due to slenderness must not exceed 140 percent of the moment obtained from a first-order analysis.

Section Properties

When performing a first-order or an elastic second-order lateral load analysis, the section properties in ACI 6.6.3.1.1 may be used in lieu of a more sophisticated analysis to account for member cracking and other effects. The reduced moments of inertia in ACI Table 6.6.3.1.1(a) and the alternative moments of inertia in ACI Table 6.6.3.1.1(b) for various structural members are given in Table 7.1 and Table 7.2, respectively. The reduced moments of inertia in Table 7.1 are based on an analysis where strength-level (factored) loads are used. For an analysis based on service-level loads, it is permitted to use reduced moments of inertia equal to 1.4 times the values in Table 7.1 provided the moments of inertia do not exceed the gross moments of inertia I_g (ACI 6.6.3.2.2).

Table 7.1 Moments of Inertia to Use in a Linear Elastic First-Order or Second-Order Analysis Using Strength-Level Loads

Member	Moment of Inertia
Columns	$0.70I_g$
Walls—uncracked	$0.70I_g$
Walls—cracked	$0.35I_g$
Beams	$0.35I_g$
Flat plates and flat slabs	$0.25I_g$

Table 7.2 Alternative Moments of Inertia to Use in a Linear Elastic First-Order or Second-Order Analysis Using Strength-Level or Service-Level Loads

Member	Moment of Inertia*
Columns and walls	$0.35I_g \le I = \left(0.80 + \frac{25A_{st}}{A_g}\right)\left(1 - \frac{M_u}{P_u h} - \frac{0.5P_u}{P_o}\right)I_g \le 0.875I_g$
Beams, flat plates, and flat slabs	$0.25I_g \le I = \left(0.10 + 25\rho\right)\left(1.2 - \frac{0.2b_w}{d}\right)I_g \le 0.5I_g$

**For a service-level load analysis, use service-level axial force and moment effects in the equation for columns and walls.*

The more refined equations for reduced moments of inertia in Table 7.2 include the (1) factored axial force , P_u, and factored moment, M_u, on a column for the load combination under consideration, (2) nominal axial strength at zero eccentricity, P_o, determined by ACI Equation (22.4.2.2), and (3) area of longitudinal reinforcement, A_{st}. These equations are applicable for strength-level and service-level loading, even though they are presented in terms of factored load effects. For service-level analysis, the factored load effects are replaced by the corresponding service-level load effects.

Regardless of the equations used to determine moment of inertias, the gross area of the section, A_g, is to be used in the analysis. Also, the cross-sectional area for calculation of shear deformations is $b_w h$.

In lieu of the requirements of ACI 6.6.3.1, it is permitted to assume in a first-order factored lateral load analysis that all the members in a structure have a reduced moment of inertia equal to 50 percent of the gross moment of inertia I_g (ACI 6.6.3.1.2). A more detailed analysis considering the effective stiffness of all members is also permitted.

7.3.2 Factored Axial Force and Moment

Axial forces and bending moments at the ends of a column are obtained from an analysis of the structure. When designing a reinforced concrete column, it is not always obvious which load combination for combined factored axial force (P_u) and bending moment (M_u) governs. Therefore, each applicable factored load combination must be considered in design (ACI 10.4.2.1).

7.3.3 Slenderness Effects

Overview

Columns can generally be classified as "short columns" and "slender columns." The term "short column" is often used to indicate a column with a strength equal to that computed for its cross-section. In such cases, strength can be represented by an interaction diagram, which is constructed based on the geometric and material properties of the section (see Figure 7.2 and Section 7.4.3 of this publication). If a column does not deflect laterally, any combination of axial force and bending moment falling outside of the interaction curve is assumed to cause failure. This is commonly referred to as a material failure. For purposes of design, a column has adequate strength when all the factored combinations of axial force and bending moment obtained from an elastic, first-order analysis of the frame fall within or on the design strength interaction curve.

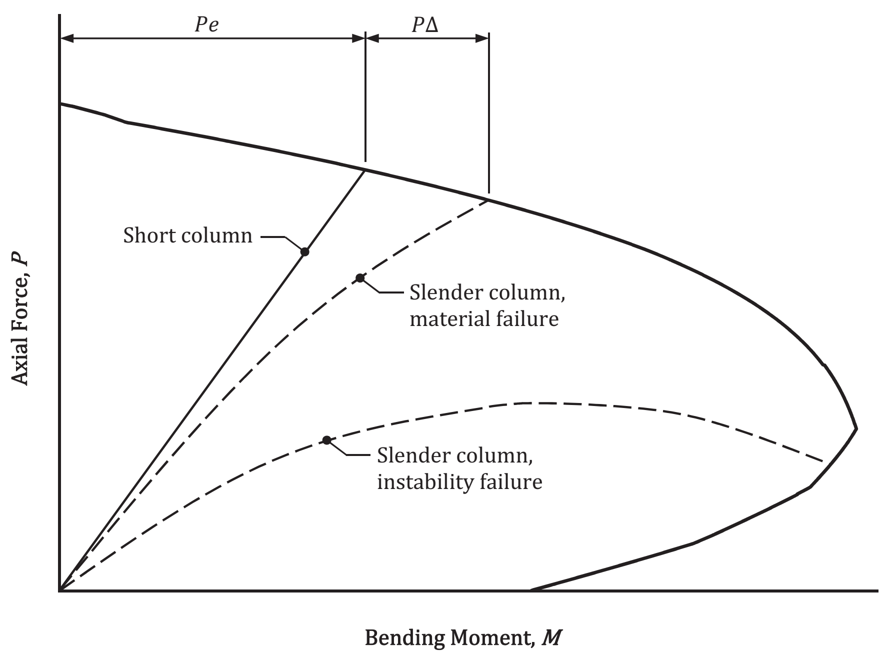


Figure 7.2 Impact of slenderness effects on design strength of columns.

A "slender column" is defined as a column whose strength is reduced by second-order deformations due to horizontal displacements. Secondary effects caused by horizontal deflection are commonly referred to as P-delta effects. Consider the idealized column in Figure 7.3, which is part of a frame. The bending moments at the ends of the column are equal to $M = Pe$ where P is the axial force on the column and e is the corresponding eccentricity. Because of the applied loading, the column displaces horizontally by an amount equal to Δ. This deflection causes an additional (or secondary) moment, which is equal to $P\Delta$. Thus, the total moment in the column is equal to the moment caused by the applied loading (Pe) plus the moment caused by the horizontal deflection of the member $(P\Delta)$. This total moment and its effect on column strength is depicted in Figure 7.2 (denoted as "Slender column, material failure" in the figure).

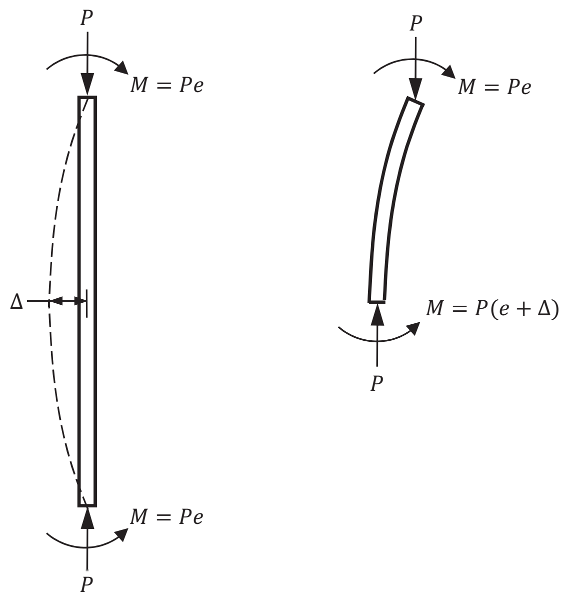


Figure 7.3 P-delta effects in a column.

In the case of very slender columns, it is possible for the deflection Δ to increase indefinitely with an increase in the axial force P. This type of failure is known as "instability failure," which generally occurs at an axial force less than that corresponding to material failure of the section (see Figure 7.2).

Slenderness effects can be neglected in columns with slenderness ratios less than or equal to the limiting values in ACI 6.2.5. Information on how to determine slenderness ratios and how to design for the effects of slenderness are covered below.

Columns in Nonsway and Sway Frames

Sway in buildings due to lateral or other forces can have a substantial influence on second-order effects in columns. It is common for secondary effects to increase with increasing sway. Distinguishing between columns in nonsway frames and those in sway frames can usually be done by comparing the total lateral stiffness of the columns in a story to that of the bracing elements. Moment frame buildings are typically laterally flexible, and the columns are more susceptible to secondary effects than those in a frame with structural walls.

Methods to determine whether a column is in a nonsway frame (braced against sidesway) or in a sway frame (not braced against sidesway) are given in ACI 6.2.5.1 and 6.6.4.3. The conditions for columns in a nonsway frame are given in Table 7.3.

Table 7.3 Conditions for Columns in a Nonsway Frame (Braced Against Sidesway)

ACI Section No.	Condition
6.2.5.1	Bracing elements (for example, moment frames or walls) resisting lateral movement of a story have a total stiffness ≥ 12 times the gross lateral stiffness of the columns in that story in the direction of analysis.
6.6.4.3	Increase in column end moments due to second-order effects ≤ 5 percent of the first-order end moments.
	Where factored loads are used to calculate the lateral deflection of a frame, stability index, Q, for a story satisfies the following equation: $Q = \dfrac{\Sigma P_u \Delta_o}{V_{us} \ell_c} \leq 0.05$ where ΣP_u = total factored vertical load in the story Δ_o = first-order relative lateral deflection between the top and bottom of the story V_{us} = factored horizontal story shear in a story ℓ_c = length of the columns measured center-to-center of the joints in the frame Where service loads are used to calculate the lateral deflection of a frame, stability index, Q, for a story satisfies the following equation: $Q = \dfrac{1.2\Sigma(P_D + P_L)(1.4\Delta_s)}{V_s \ell_c} \leq 0.05$ where $\Sigma(P_D + P_L)$ = total service dead and live axial loads in the story Δ_s = first-order, service-level relative lateral deflection between the top and bottom of the story V_s = service-level story shear

The numerator $\Sigma P_u \Delta_o$ in the equation $Q = \Sigma P_u \Delta_o / V_{us} \ell_c$ is the moment in a story due to the axial loads ΣP_u being displaced an amount Δ_o due to the horizontal story shear, V_{us} (see Figure 7.4). The sum of the factored axial loads in the story, ΣP_u, must correspond to the lateral loading case for which this sum is the greatest. The denominator $V_{us} \ell_c$ in the equation for Q is the overall moment in a story due to V_{us}.

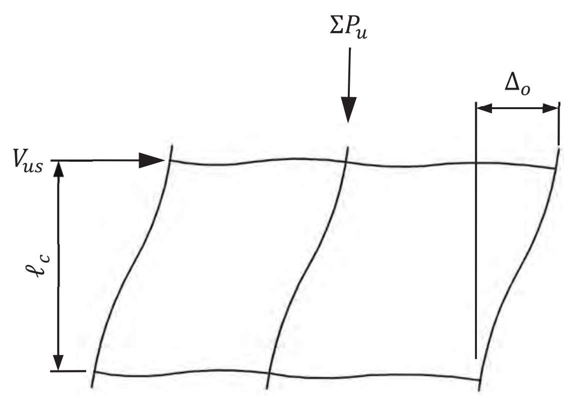


Figure 7.4 Definition of stability index, Q.

Where the conditions in Table 7.3 are not met, the columns are considered to be in a sway frame (that is, in a frame not braced against sidesway). In general, a frame may contain both nonsway and sway columns and stories.

Consideration of Slenderness Effects

Whether slenderness effects need to be considered or not for nonsway and sway frames depends on the slenderness ratio of the column, $k\ell_u / r$. The terms in the slenderness ratio are defined in Table 7.4 (see ACI 6.2.5).

Table 7.4 Slenderness Ratio, $k\ell_u / r$

$k =$ effective length factor	Nonsway Frames	$k \leq 1.0$
		Determine k using ACI Figure R6.2.5.1(a) or use $k = 1.0$
	Sway Frames	$k \geq 1.0$
		Determine k using ACI Figure R6.2.5.1(b)
$\ell_u =$ unsupported length	Unsupported length, ℓ_u, is the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction of analysis.	
$r =$ radius of gyration	Radius of gyration, r, is permitted to be calculated by the following: $r = \sqrt{I_g / A_g}$ $r = 0.30h$ for rectangular columns where h is the dimension of the column in the direction of analysis $r = 0.25h$ for circular columns where h is the diameter of the column	

The effective length factor, k, is determined based on the stiffness ratio, Ψ, at the ends of the column. This ratio is determined by dividing the sum of the stiffnesses of the columns at a joint by the sum of the stiffnesses of the flexural members framing into that joint in the direction of analysis:

$$\Psi = \frac{\sum (EI / \ell_c)_{columns}}{\sum (EI / \ell)_{beams}} \tag{7.1}$$

In this equation, E is the modulus of elasticity of the concrete, which is determined in accordance with ACI 19.2.2; I is the moment of inertia of the column or flexural member; ℓ_c is the length of the columns measured center-to-center of the joints in the frame; and ℓ is the length of the flexural members measured center-to-center of the joints.

The stiffness ratio Ψ is determined at both the top and bottom of a column. The chart in ACI Figure R6.2.5.1(a) for nonsway frames and in ACI Figure R6.2.5.1(b) for sway frames can be used to graphically obtain k by drawing a line from the stiffness ratio Ψ_A at the top of the column to the stiffness ratio Ψ_B at the bottom of the column. The value of k is obtained from the chart at the location where the line crosses the k-axis. The charts in ACI Figure R6.2.5.1 are based on the equations in Table 7.5.

Table 7.5 Equations for Stiffness Ratio, Ψ

Nonsway frames	$\Psi = \frac{-2k}{\pi} \tan\left(\frac{\pi}{2k}\right)$
Sway frames	$\Psi = \frac{6k}{\pi} \cot\left(\frac{\pi}{2k}\right)$

The unsupported length of a column, ℓ_u, is the clear distance between floor slabs, beams, or other elements capable of providing lateral support in the direction of analysis. For example, the beam in Figure 7.5 provides lateral support to the top of the column in the direction parallel to the x-axis, and the unsupported length is equal to $(\ell_u)_1$, which

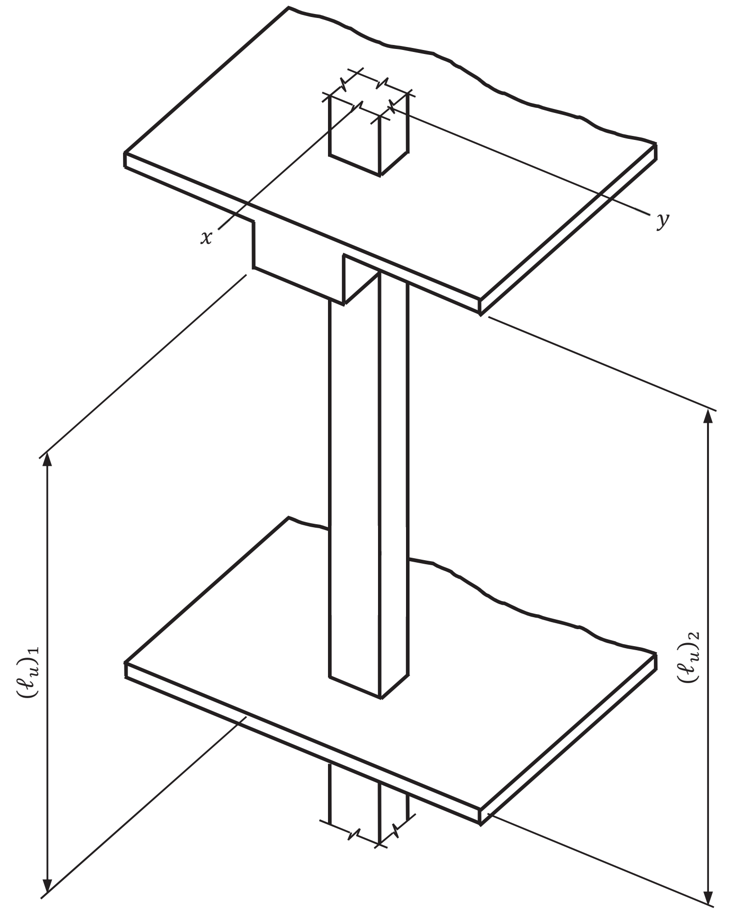


Figure 7.5 Unsupported column length with beams and slabs.

is the distance from the bottom of the beam to the top of the slab at the bottom of the column. This beam does not provide lateral support to the column parallel to the *y*-direction, so the unsupported length $(\ell_u)_2$ is the distance from the bottom of the slab at the top of the column to the top of the slab at the bottom of the column.

The unsupported column length for columns with capitals or haunches is illustrated in Figure 7.6 for the case of a column capital. In this case, ℓ_u is measured from the top of the slab at the bottom of the column to the lower extremity of the capital.

Slenderness limits are given in ACI 6.2.5.1(a) for columns not braced against sidesway and in ACI 6.2.5.1(b) for columns braced against sidesway. Slenderness effects are permitted to be neglected when the equations in Table 7.6 are satisfied.

Table 7.6 Limits for Slenderness Effects

Column Bracing Condition	Slenderness Ratios Where Slenderness is Permitted to Be Neglected
Not braced against sidesway (sway)	$\frac{k\ell_u}{r} \leq 22$
Braced against sidesway (nonsway)	$\frac{k\ell_u}{r} \leq \text{ lesser of } \begin{cases} 34 + 12(M_1/M_2)\ * \\ 40 \end{cases}$

**Ratio M_1 / M_2 is negative if a column is bent in single curvature and is positive if a column is bent in double curvature.*

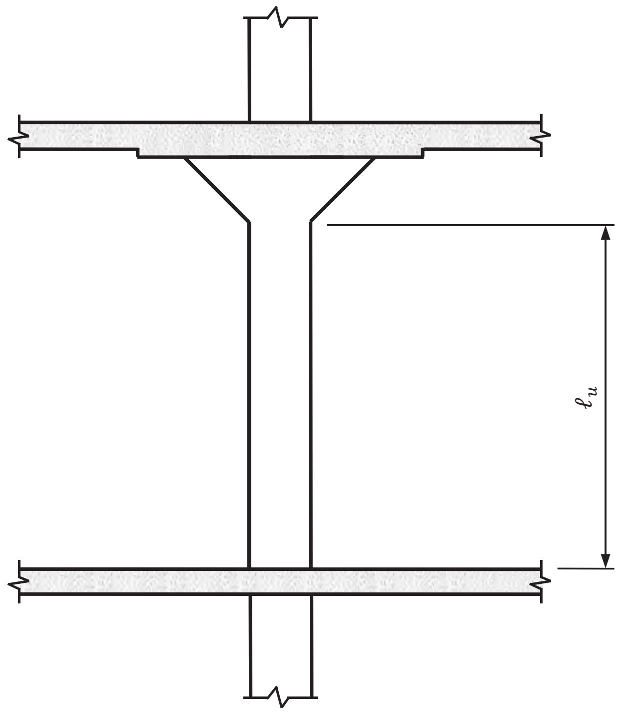


Figure 7.6 Unsupported length of a column with a capital.

Because columns braced against sidesway are more stable when bent in double curvature, the limiting slenderness ratio is larger than that for columns bent in single curvature.

The limiting values of the slenderness ratios in Table 7.6 are based on a 5 percent increase in moments due to slenderness effects, which is considered to be acceptable.

Minimum column dimensions so that slenderness effects need not be considered as a function of the unsupported column length, ℓ_u, are given in Table 7.7. The minimum dimensions for columns braced against sidesway are based on $k = 1.0$ for columns bent in double curvature where $M_1 / M_2 = +1.0$. For columns not braced against sidesway, the minimum column dimensions are based on $k = 1.5$. Tabulated minimum column dimensions are rounded up to the next whole even numbers.

Table 7.7 Minimum Column Dimensions to Neglect Slenderness

Unsupported Column Length, ℓ_u (ft)	Column Bracing Condition	Minimum Column Dimension (in.)	
		Rectangular (c_1)	Circular
7	Braced against sidesway	8	10
	Not braced against sidesway	20	24
8	Braced against sidesway	8	10
	Not braced against sidesway	22	28
9	Braced against sidesway	10	12
	Not braced against sidesway	26	30
10	Braced against sidesway	10	12
	Not braced against sidesway	28	34

Minimum column dimensions to neglect slenderness for columns braced against sidesway are small; the dimensions are, at most, equal to the minimum practical dimensions given in Section 7.2 of this publication. Therefore, slenderness effects typically do not need to be considered for columns braced against sidesway in buildings with typical story heights. In taller buildings, shear walls or a combination of shear walls and moment frames are often utilized, which essentially brace the columns against sidesway.

Columns not braced against sidesway most often occur in low-rise buildings without walls where the lateral displacements are usually relatively small. Thus, second-order effects are typically negligible even if column dimensions are smaller than the minimum dimensions in Table 7.7.

Slenderness effects usually need to be considered where the cross-sectional dimensions of a column are limited due to architectural or other constraints and/or the unsupported length of the column is relatively long.

Moment Magnification Method

Overview

The moment magnification method in ACI 6.6.4 is an approximate method that accounts for slenderness effects by magnifying the bending moments at the ends of a column obtained from a first-order analysis. In general, a slender column must be designed to resist the combined effects from factored axial compression forces and magnified bending moments, which are obtained by multiplying the first-order factored bending moments by a moment magnifier that is a function of the factored axial force and the critical buckling load of the column. The column is adequate where all the factored load combinations for combined axial force and magnified bending moment fall within or on the design strength interaction diagram, which is obtained from a strain compatibility analysis of the section.

Nonsway Frames

Provisions for slender columns in nonsway frames are given in ACI 6.6.4.5. Columns must be designed for combinations of factored axial forces, P_u, and factored magnified moments, M_c, where M_c is determined by ACI Equation (6.6.4.5.1):

$$M_c = \delta M_2 \qquad (7.2)$$

In this equation, M_2 is the larger of (1) the two factored column end moments from a first-order analysis and (2) minimum moment $M_{2,min} = P_u(0.6 + 0.03h)$ where h is the dimension of the column in the direction of analysis (ACI 6.6.4.5.4).

The magnification factor, δ, is determined by ACI Equation (6.6.4.5.2):

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0 \qquad (7.3)$$

The moment factor, C_m, relates the actual moment diagram to an equivalent uniform moment diagram. It is assumed in the moment magnifier method that the maximum moment is at or near the mid-height of the column. If the maximum moment occurs at one end of the column, design is based on an equivalent uniform moment $C_m M_2$ (that is, $C_m M_2$ is assumed to act at both ends of the column). This leads to the same maximum moment at or near mid-height of the column when magnified.

The value of C_m depends on whether transverse loads are applied between the supports of the column or not (see Table 7.8). In ACI Equation (6.6.4.5.3a) for the case of columns without transverse loads applied between supports, M_1 / M_2 is the ratio of the smaller to the larger factored end moment; this ratio is negative if the column is bent in single curvature and positive if the column is bent in double curvature (see Figure 7.7). For columns with transverse loads applied between supports, it is possible for the maximum moment to occur at a section away from the end of the member; thus, $C_m = 1.0$ in this case [ACI Equation (6.6.4.5.3b)].

Table 7.8 Values of C_m

Column Transverse Load Condition	C_m
Transverse loads not applied between supports	$0.6 - 0.4(M_1 / M_2)$
Transverse loads applied between supports	1.0

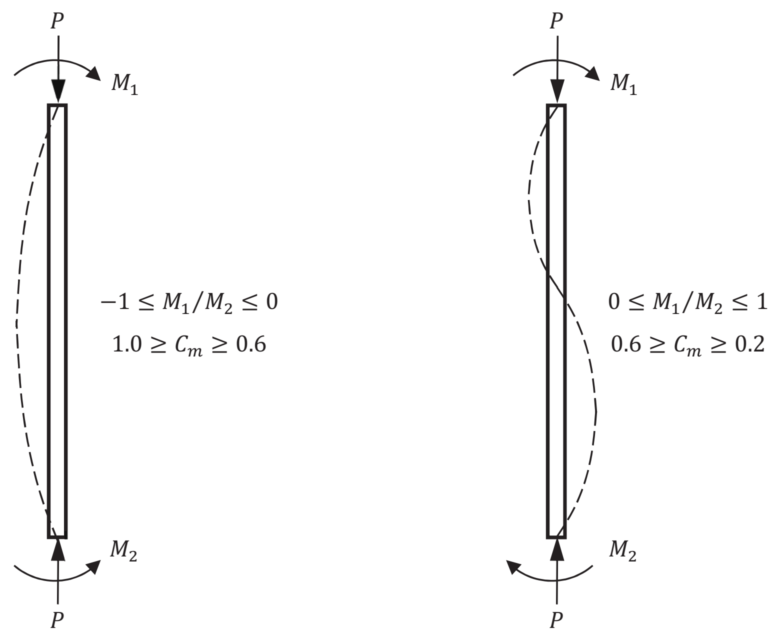


Figure 7.7 Values of C_m for columns without transverse loads applied between supports.

In cases where $M_{2,min} > M_2$, the factored column end moments M_1 and M_2 from analysis are to be used in ACI Equation (6.6.4.5.3a) to determine C_m (ACI 6.6.4.5.4). Alternatively, C_m is permitted to be taken equal to 1.0.

The critical buckling load, P_c, is determined by ACI Equation (6.6.4.4.2):

$$P_c = \frac{\pi^2 (EI)_{eff}}{(k\ell_u)^2} \tag{7.4}$$

The effective flexural stiffness of a column, $(EI)_{eff}$, is permitted to be calculated by ACI Equation (6.6.4.4.4a), (6.6.4.4.4b), or (6.6.4.4.4c) [ACI 6.6.4.4.4; see Table 7.9]. The numerators in these equations represent the short-term column stiffness. Creep effects due to sustained loads are accounted for by dividing the short-term stiffness by $(1 + \beta_{dns})$ where β_{dns} is the ratio of the maximum factored sustained axial load to the maximum factored axial load associated with the same load combination. This factor is typically equal to the factored axial dead load divided by the total factored axial load on a column.

Table 7.9 Effective Flexural Stiffness, *(EI)_{eff}*

ACI Equation No.	$(EI)_{eff}$
6.6.4.4.4a	$\dfrac{0.4E_cI_g}{1+\beta_{dns}}$
6.6.4.4.4b	$\dfrac{0.2E_cI_g+E_sI_{se}}{1+\beta_{dns}}$
6.6.4.4.4c	$\dfrac{E_cI\,*}{1+\beta_{dns}}$

$*0.35I_g \le I = \left(0.80+\dfrac{25A_{st}}{A_g}\right)\left(1-\dfrac{M_u}{P_uh}-\dfrac{0.5P_u}{P_o}\right)I_g \le 0.875I_g$ *(see Table 7.2)*

ACI Equation (6.6.4.4.4a) is a simplified version of ACI Equation (6.6.4.4.4b) and does not directly account for longitudinal reinforcement in the column like the latter equation does through the term I_{se}, which is the moment of inertia of the longitudinal reinforcement about the centroidal axis of the cross-section in the direction of analysis. Thus, this equation is not as accurate as ACI Equation (6.6.4.4.4b), especially in columns with greater amounts of longitudinal reinforcement.

ACI Equation (6.6.4.4.4b) was originally derived for small eccentricity ratios $(M_u \,/\, P_uh)$ and large axial load ratios $(P_u \,/\, P_o)$ and represents the lower limit of the practical range of stiffness values, especially for columns with larger amounts of longitudinal reinforcement. ACI Equation (6.6.4.4.4c) usually yields more accurate values of the effective flexural stiffness than those determined by the other two equations where I in this equation is the moment of inertia for columns and walls in ACI Table 6.6.3.1.1(b).

The 0.75 factor in the denominator of Equation (7.3) is a stiffness reduction factor similar to the one derived for the reduced moment of inertia, which is equal to 0.875. Additional information on the development of this stiffness factor is given in ACI R6.6.4.5.2.

Sway Frames

Provisions for slender columns in sway frames are given in ACI 6.6.4.6. Columns must be designed for combinations of factored axial forces P_u and factored total moments M_1 and M_2 at the ends of the column, which are the summation of the following:

1. Unmagnified factored moments M_{1ns} and M_{2ns} due to loads causing no appreciable sidesway (like gravity loads), and
2. Magnified factored moments δ_sM_{1s} and δ_sM_{2s} due to loads causing appreciable sidesway (like wind and earthquake loads) [see ACI Equations (6.6.4.6.1a) and (6.6.4.6.1b)].

Therefore,

$$M_1 = M_{1ns} + \delta_sM_{1s} \tag{7.5}$$

$$M_2 = M_{2ns} + \delta_sM_{2s} \tag{7.6}$$

The moments $M_{1ns,}M_{2ns,}M_{1s},$ and M_{2s} are determined from a first-order elastic analysis of the structure using reduced section properties of the members in accordance with ACI Table 6.6.3.1.1(a) [see Table 7.1].

The moment magnification factor, δ_s, for sway frames, is determined by either a linear elastic second-order elastic analysis (see Section 7.3.1 of this publication) or by ACI Equation (6.6.4.6.2a) or (6.6.4.6.2b) [ACI 6.6.4.6.2]:

$$\delta_s = \frac{1}{1-Q} \geq 1.0 \tag{7.7}$$

$$\delta_s = \frac{1}{1-\dfrac{\Sigma P_u}{0.75\Sigma P_c}} \geq 1.0 \tag{7.8}$$

Equation (7.7) is the solution of an infinite series representing the iterative P-delta analysis and closely predicts the magnitude of second-order moments in a sway frame where $\delta_s \leq 1.5$. Equation (7.8) or a linear elastic second-order elastic analysis in accordance with ACI 6.7 must be used in cases where δ_s calculated by Equation (7.7) exceeds 1.5. The stability index, Q, is determined by ACI Equation (6.6.4.4.1) [see Table 7.3] and is based on deflections obtained from a first-order elastic analysis using reduced section properties of the members.

Equation (7.8) was part of the moment magnifier method that appeared in previous editions of ACI 318. The factored axial loads, P_u, are summed over the entire story, and the critical buckling loads, P_c, are summed over the sway-resisting columns in that story. Like in the case of nonsway frames, the effective stiffness $(EI)_{eff}$ used in the calculation of P_c is determined by ACI 6.6.4.4.4. For sway frames, the term β_{ds}, which is the ratio of the maximum factored sustained story shear to the maximum factored story shear, is used in the denominator of the equations of $(EI)_{eff}$ instead of β_{dns}. Note that β_{ds} is normally equal to zero because wind and seismic loads are not sustained loads. An example of a sustained lateral load is lateral earth pressure.

The magnified moments at the ends of a column must be considered in the design of the beams framing into the columns (ACI 6.6.4.6.3). Beam stiffness plays a key role in the stability of columns, and this provision ensures the beams will have adequate strength to resist the magnified column moments.

It is possible for the maximum moment in a slender column in a sway frame to occur at a section away from its ends. It is permitted to magnify such moments using the provisions for nonsway frames in ACI 6.6.4.5 where C_m is calculated using the total moments M_1 and M_2 determined by Equations (7.5) and (7.6) [ACI 6.6.4.6.4].

7.3.4 Required Shear Strength for Columns in Buildings Assigned to Seismic Design Category B

For certain columns in buildings assigned to SDC B that are part of an ordinary moment frame resisting seismic load effects, the shear strength requirements of ACI 18.3.3 must be satisfied in addition to all other applicable requirements. In cases where the unsupported length of a column, ℓ_u, is less than or equal to five times the plan dimension of the column, c_1, in the direction of analysis, the design shear strength of the column must be taken as the lesser of the following:

- The shear force, V_u, associated with the development of nominal moment strengths, M_n, of the column at each restrained end of the unsupported length due to reverse curvature bending (see Figure 7.8). In the figure, M_{nt} and M_{nb} are the nominal flexural strengths at the top and bottom of the column, respectively. These flexural strengths must be calculated for the factored axial force, P_u, consistent with the direction of analysis resulting in the highest flexural strength. Sidesway to the right and sidesway to the left must both be considered.
- The maximum shear force obtained from the design load combinations of ACI Chapter 5 including the earthquake effect, E, with $\Omega_o E$ substituted for E in the load combinations. The term Ω_o is the overstrength factor for ordinary moment frames of reinforced concrete, which is equal to 3 (see ASCE/SEI Table 12.2-1).

The provisions for the design shear strength are intended to provide additional capacity to resist shear in relatively squat columns, which are vulnerable to shear failure under earthquake loading.

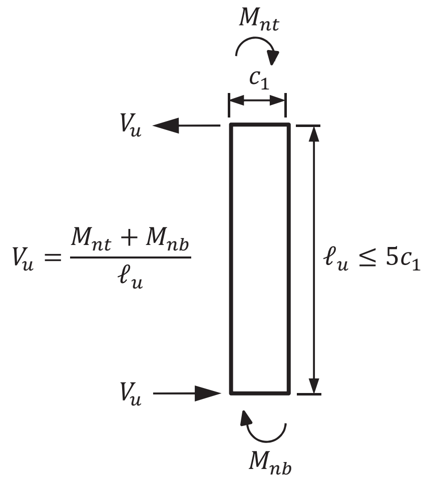


Figure 7.8 Design shear strength of reinforced concrete columns in buildings assigned to SDC B.

7.4 Design Strength

7.4.1 General

For each applicable factored load combination in ACI Table 5.3.1, the following equations must be satisfied at any section in a column (ACI 10.5.1.1):

$$\phi P_n \geq P_u \quad (7.9)$$

$$\phi M_n \geq M_u \quad (7.10)$$

$$\phi V_n \geq V_u \quad (7.11)$$

$$\phi T_n \geq T_u \quad (7.12)$$

Strength reduction factors, ϕ, are determined in accordance with ACI 21.2. A summary of the strength reduction factors pertinent to the design of reinforced concrete columns is given in Table 7.10. The strain used to define a compression-controlled section, ε_{ty}, is equal to the specified yield strength of the reinforcement, f_y, divided by the modulus of elasticity of the reinforcing steel, E_s, which can be taken as 29,000,000 psi for any grade of reinforcement (ACI 20.2.2.2).

Table 7.10 Strength Reduction Factors, ϕ

Net Tensile Strain, ε_t	Classification	Strength Reduction Factor, ϕ	
		Type of Transverse Reinforcement	
		Spirals Conforming to ACI 25.7.3	Other
$\varepsilon_t \leq \varepsilon_{ty}$	Compression-controlled	0.75	0.65
$\varepsilon_{ty} < \varepsilon_t \leq \varepsilon_{ty} + 0.003$	Transition*	$0.75 + \frac{0.15(\varepsilon_t - \varepsilon_{ty})}{0.003}$	$0.65 + \frac{0.25(\varepsilon_t - \varepsilon_{ty})}{0.003}$
$\varepsilon_t \geq \varepsilon_{ty} + 0.003$	Tension-controlled	0.90	0.90

**Sections classified as transition are permitted to use ϕ corresponding to compression-controlled sections.*

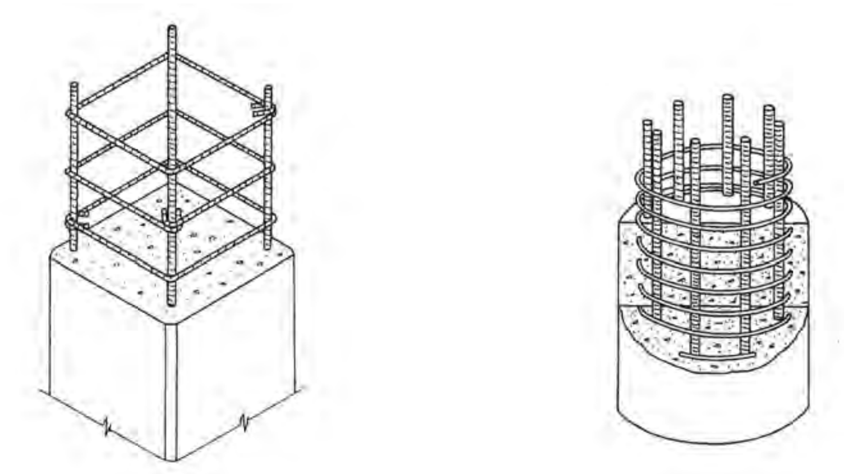
For columns subjected primarily to uniaxial compression forces, Equation (7.9) is applicable. For columns subjected to combined moment and axial forces, Equations (7.9) and (7.10) must both be satisfied for all applicable factored load combinations. The shear and torsional strength equations [Equations (7.11) and (7.12), respectively] must be satisfied for any column.

Methods to determine the nominal strengths P_n, combined P_n and M_n, V_n, and T_n are given in Sections 7.4.2, 7.4.3, 7.4.4, and 7.4.5 of this publication, respectively.

7.4.2 Nominal Axial Strength

Nominal Axial Compressive Strength

The nominal axial compressive strength, P_n, of a nonslender, reinforced concrete column is determined in accordance with ACI 22.4.2. For nonprestressed columns with transverse reinforcement consisting of (1) ties conforming to ACI 22.4.2.4 and (2) spirals conforming to ACI 22.4.2.5 (see Figure 7.9), P_n must be less than or equal to the maximum nominal axial compressive strength, $P_{n,max}$, which is determined using the equations in ACI Table 22.4.2.1 and the nominal axial strength at zero eccentricity, P_o, determined by ACI Equation (22.4.2.2):



(a) tied column

(b) spiral column

Figure 7.9 Reinforced concrete columns. (a) With tied transverse reinforcement. (b) With spiral transverse reinforcement.

- For columns with ties:

$$P_n \leq P_{n,max} = 0.80P_o = 0.80[0.85f_c'(A_g - A_{st}) + f_yA_{st}] \tag{7.13}$$

- For columns with spirals:

$$P_n \leq P_{n,max} = 0.85P_o = 0.85[0.85f_c'(A_g - A_{st}) + f_yA_{st}] \tag{7.14}$$

These equations account for any accidental eccentricities that may exist in a column that were not accounted for in analysis. In the case of tied columns, the eccentricity is approximately 10 percent of the column depth, while for spiral columns, the eccentricity is approximately 5 percent.

When calculating $P_{n,max}$, the value of the specified yield strength of the longitudinal reinforcement, f_y, is limited to 80,000 psi even though f_y for the longitudinal reinforcement used in the column may be larger than that (ACI 22.4.2.1).

Nominal Axial Tensile Strength

The nominal axial tensile strength of a nonprestressed, reinforced concrete column, P_{nt}, must be less than or equal to the maximum nominal axial tensile strength, $P_{nt,max}$, which is determined by ACI Equation (22.4.3.1):

$$P_{nt} \leq P_{nt,max} = f_y A_{st} \tag{7.15}$$

It is evident that the entire tensile force must be resisted by the longitudinal reinforcement in a column.

7.4.3 Nominal Strength of Columns Subjected to Moment and Axial Forces

Overview

The nominal strength of a reinforced concrete column subjected to both moment and axial force is determined using equilibrium, strain compatibility, and the design assumptions in ACI 22.2, which are summarized in Table 7.11.

Table 7.11 Design Assumptions for Concrete and Nonprestressed Reinforcement

Material	Assumptions
Concrete	1. Maximum strain in the extreme concrete compression fiber is assumed equal to 0.003.
	2. Tensile strength of concrete is neglected in flexural and axial strength calculations.
	3. The relationship between concrete compressive stress and strain is to be represented by a rectangular, trapezoidal, parabolic, or other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.
	4. The equivalent rectangular concrete stress distribution in accordance with ACI 22.2.2.4.1 through 22.2.2.4.3 satisfies the requirement of ACI 22.2.2.3 for the relationship between concrete compressive stress and strain.
	5. A concrete stress of $0.85f_c'$ is to be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross-section and a line parallel to the neutral axis located a distance a from the fiber of maximum compressive strain where $a = \beta_1 c$. The distance from the fiber of maximum compressive strain to the neutral axis c is to be measured perpendicular to the neutral axis. Values of β_1 are as follows (ACI Table 22.2.2.4.3): • For 2,500 psi $\leq f_c' \leq$ 4,000 psi : $\beta_1 = 0.85$ • For 4,000 psi $< f_c' <$ 8,000 psi : $\beta_1 = 0.85 - [0.05(f_c' - 4{,}000) / 1{,}000]$ • For $f_c' \geq 8{,}000$ psi : $\beta_1 = 0.65$
Nonprestressed reinforcement	1. Deformed reinforcement used to resist tensile or compressive forces must conform to ACI 20.2.1.
	2. The stress-strain relationship and the modulus of elasticity of the deformed reinforcement are to be idealized in accordance with ACI 20.2.2.1 and 20.2.2.2.

Rectangular Sections

The general principles and assumptions of the strength design method in Table 7.11 are applied to the rectangular reinforced concrete column in Figure 7.10. For purposes of discussion, it is assumed the longitudinal reinforcement in layer 1 is located closest to the extreme compression fiber of the section.

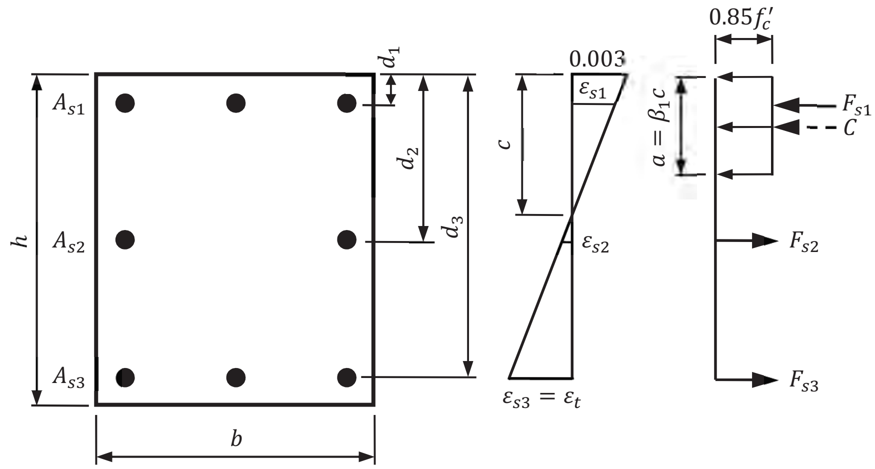


Figure 7.10 Rectangular reinforced concrete column subjected to moment and axial compression.

The nominal axial strength, P_n, and the nominal flexural strength, M_n, of the column in Figure 7.10 can be obtained for a given strain, ε_t, using the equations in Figure 7.11 where ε_t is the net tensile strain in the extreme layer of tension reinforcement at nominal strength, excluding strains due to creep, shrinkage, and temperature.

Circular Sections

The general principles and assumptions of the strength design method in Table 7.11 are applied to the circular reinforced concrete column in Figure 7.12. For purposes of discussion, it is assumed the longitudinal reinforcement in layer 1 is located closest to the extreme compression fiber of the section. For circular sections, the shape of the compression zone is related to a segment of a circle (see Figure 7.13).

The nominal axial strength, P_n, and the nominal flexural strength, M_n, of the column in Figure 7.12 can be obtained for a given strain, ε_t, using the equations in Figure 7.14.

Interaction Diagrams

An interaction diagram for a reinforced concrete column is a collection of P_n and M_n values determined for a series of strain distributions using the methods above for rectangular and circular sections. Nominal strengths for a given strain distribution represent a single point on the interaction diagram. Such diagrams are useful in establishing the adequacy of a column subjected to a combination of axial forces and bending moments.

Nominal and design strength interaction diagrams for a tied, rectangular reinforced concrete column with a symmetrical distribution of longitudinal reinforcement are given in Figure 7.15. The portions of the diagrams corresponding to compressive-controlled sections, tension-controlled sections, and sections in the transition zone are identified in the figure.

The design strength for combined moment and axial forces must be greater than or equal to the corresponding required strengths to satisfy strength requirements [see Equations (7.9) and (7.10)]. Design strength values are obtained by multiplying the nominal strength values with the strength reduction factors in ACI 21.2 (see Section 7.4.1 of this publication).

Factored axial force-bending moment combinations falling on or within the boundaries of the design strength interaction diagram can be safely carried by the column. This column is adequate for the combination denoted by Point 1 in Figure 7.15, but it is not adequate for the combination denoted by Point 2.

Interaction diagrams about the major axis and the minor axis of a rectangular column can be different; this must be taken into consideration when designing the column. For circular reinforced concrete columns with 6 longitudinal bars or less, the design flexural strength depends on the bar arrangement, and the column must be checked using the

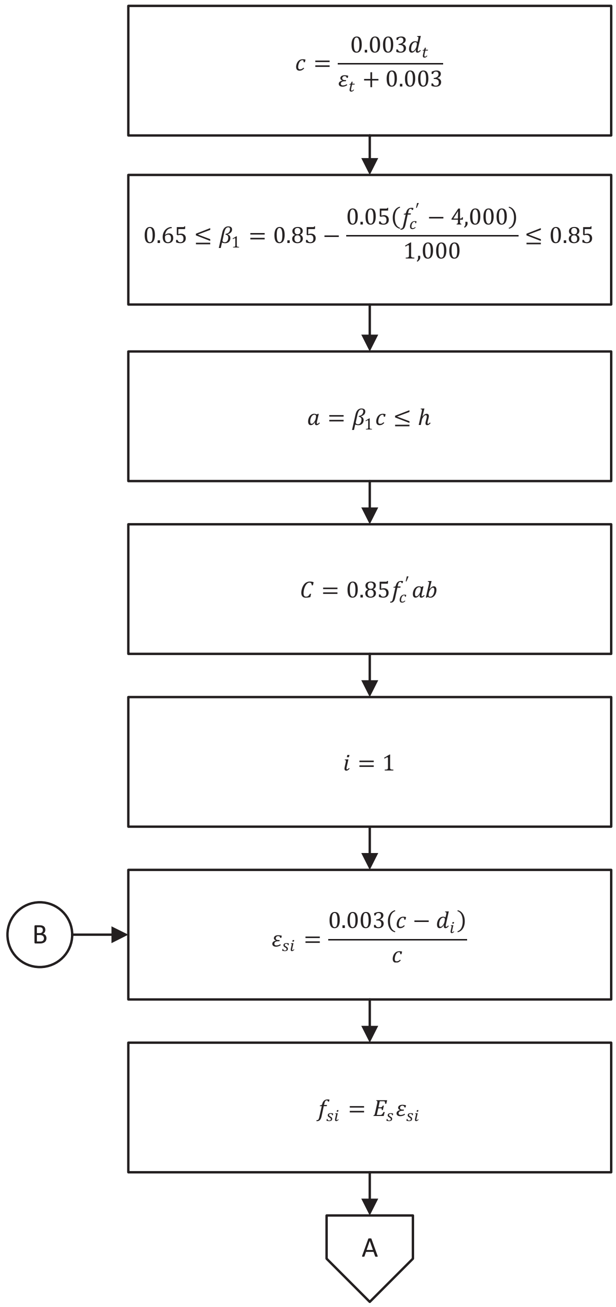


Figure 7.11 Combined flexural and axial strength of a rectangular reinforced concrete column.

arrangement that gives minimum design flexural strength. Utilizing 8 or more longitudinal bars essentially eliminates the need to determine different design flexural strengths based on bar arrangement.

Reference 16 contains 900 design strength interaction diagrams for the following:

- Tied, rectangular columns ranging in size from 12 to 48 in., inclusive;
- Tied and spiral circular columns ranging in diameter from 12 to 48 in., inclusive;
- Grade 60 and Grade 80 longitudinal reinforcement;
- Concrete compressive strengths from 4,000 psi to 14,000 psi, inclusive; and,
- Longitudinal reinforcement ratios from 1 percent to less than 2.5 percent

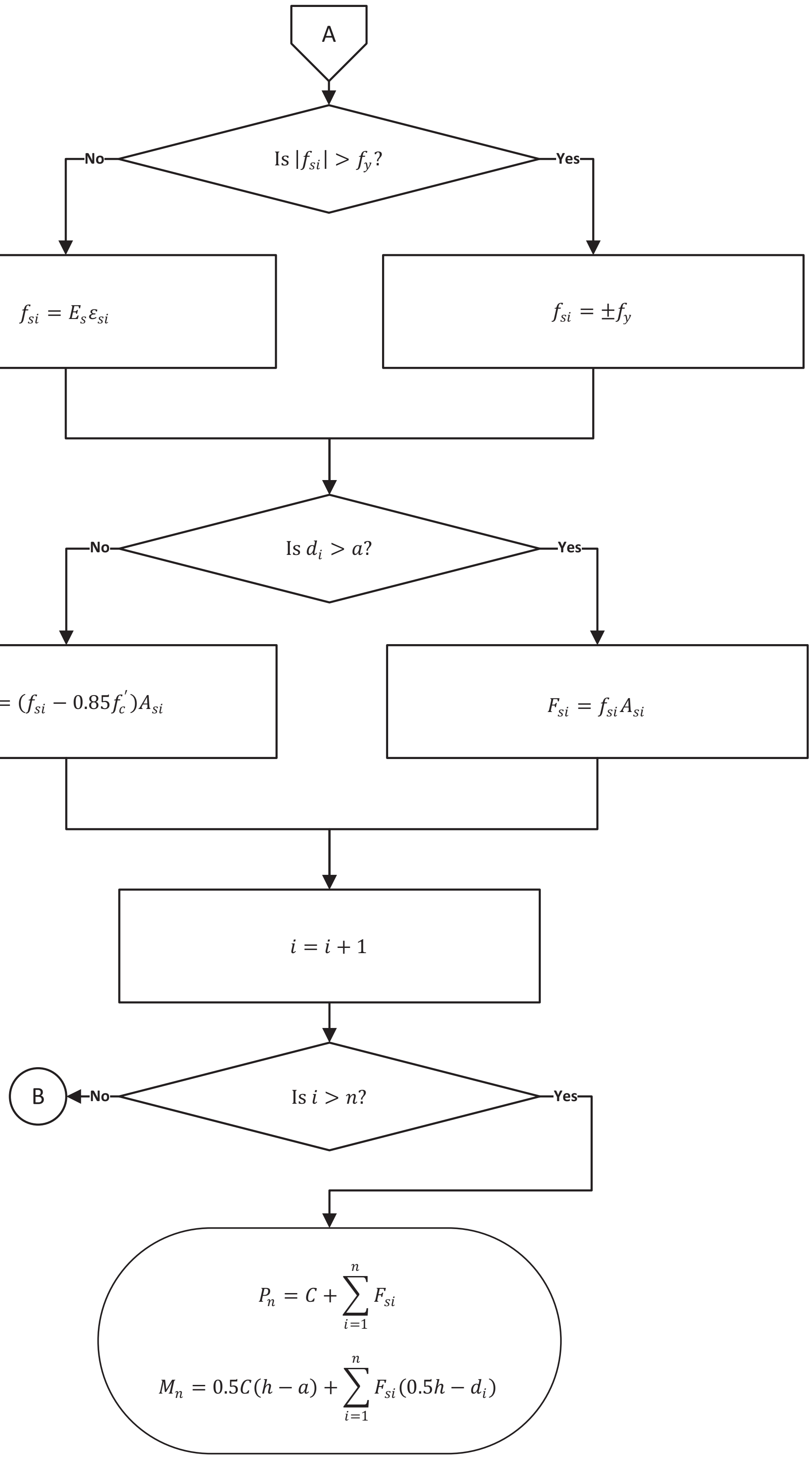


Figure 7.11 (cont.) Combined flexural and axial strength of a rectangular reinforced concrete column.

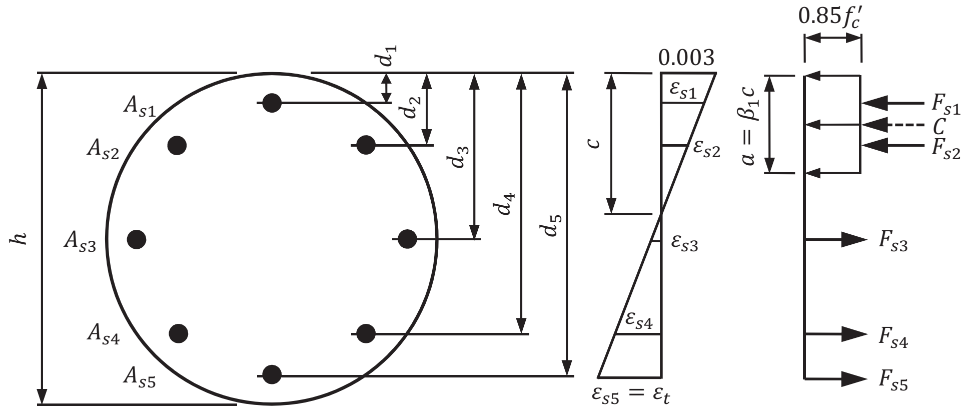


Figure 7.12 Circular reinforced concrete column subjected to moment and axial compression.

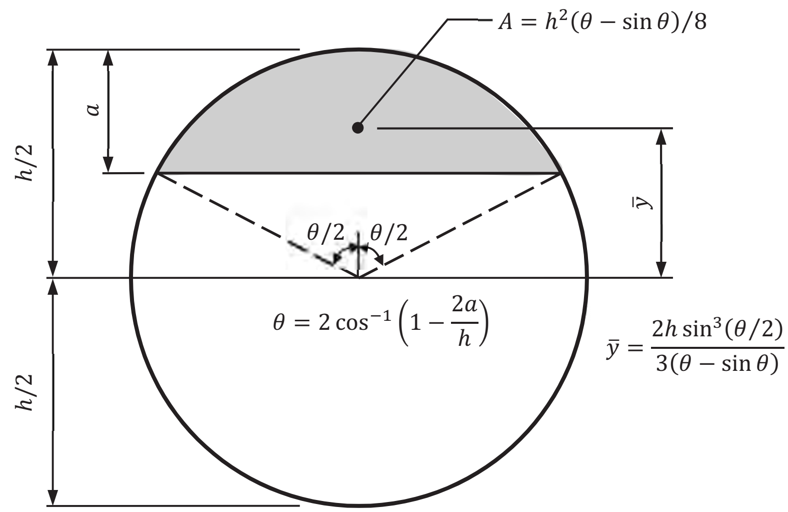


Figure 7.13 Section properties of the compression zone for a circular reinforced concrete column.

Biaxial Loading

Overview

Biaxial bending of columns occurs where loading causes bending simultaneously about both principal axes. This is often encountered in corner columns. The strength of an axially loaded column subjected to bending moments about both principal axes can be represented by a biaxial strength interaction surface (see Figure 7.16). This surface is formed by a series of uniaxial interaction diagrams drawn radially from the vertical axis. Intermediate interaction diagrams between the angle θ equal to zero degrees (uniaxial bending about the x-axis) and 90 degrees (uniaxial bending about the y-axis) are obtained by varying the angle of the neutral axis for assumed strain configurations.

A biaxial strength interaction surface is constructed by performing a series of involved and time-consuming strain compatibility analyses. For example, the strains and forces for a rectangular column subjected to an axial compression force (P_u) and biaxial bending $(M_{u,y} = P_u e_x$ and $M_{u,x} = P_u e_y)$ are given in Figure 7.17. Designing a column for this loading condition can be a very lengthy process without the use of a computer program. However, conservative results for columns subjected to biaxial bending can be obtained using some approximate methods.

$$c = \frac{0.003 d_t}{\varepsilon_t + 0.003}$$

$$0.65 \le \beta_1 = 0.85 - \frac{0.05(f_c' - 4{,}000)}{1{,}000} \le 0.85$$

$$a = \beta_1 c \le h$$

$$\theta = 2\cos^{-1}\left(1 - \frac{2a}{h}\right)$$

$$\bar{y} = \frac{2h\sin^3(\theta/2)}{3(\theta - \sin\theta)}$$

$$C = \frac{0.85 f_c' h^2}{8}(\theta - \sin\theta)$$

$$i = 1$$

B

$$\varepsilon_{si} = \frac{0.003(c - d_i)}{c}$$

$$f_{si} = E_s \varepsilon_{si}$$

A

Figure 7.14 Combined flexural and axial strength of a circular reinforced concrete column.

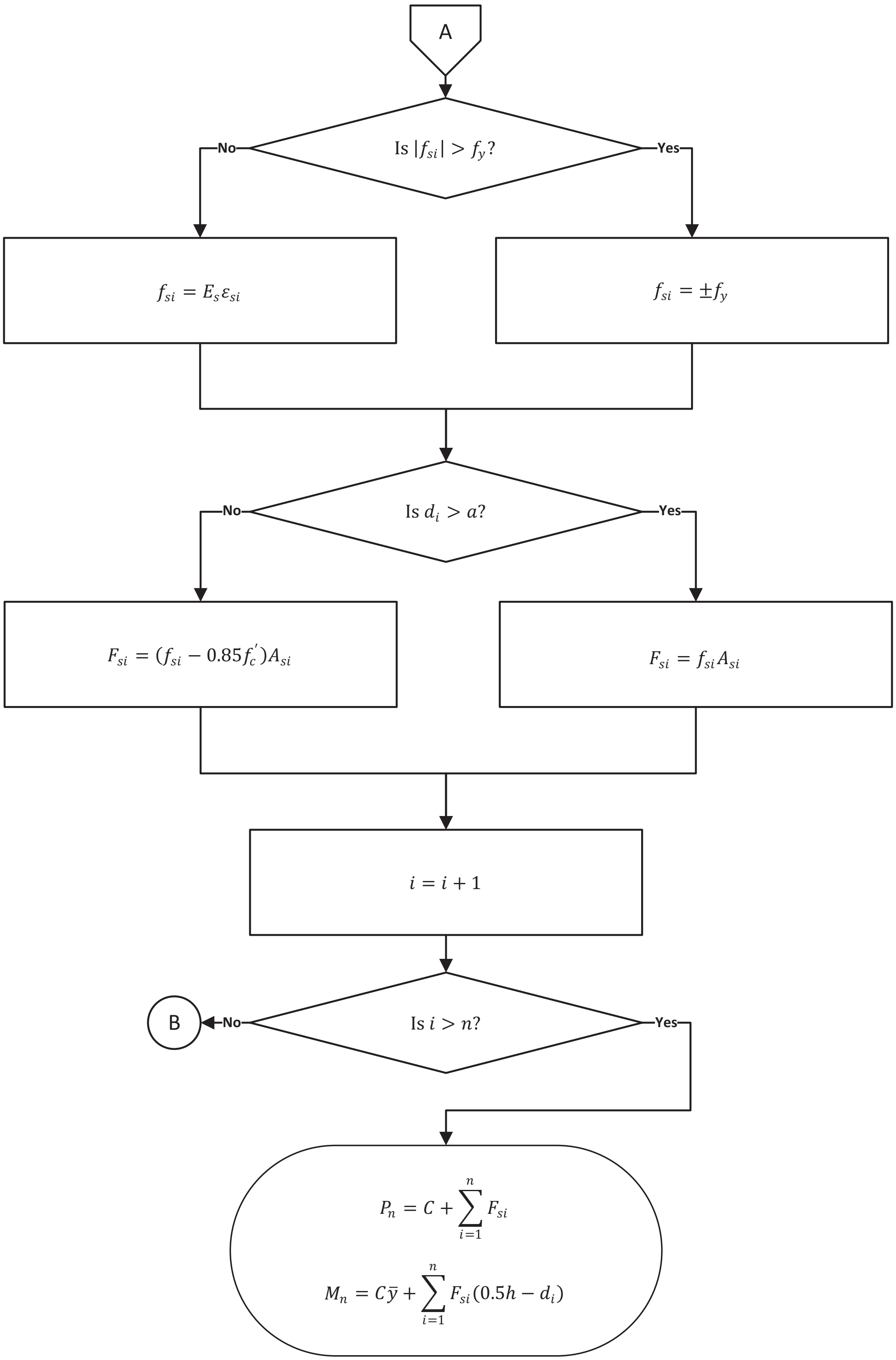


Figure 7.14 (cont.) Combined flexural and axial strength of a circular reinforced concrete column.

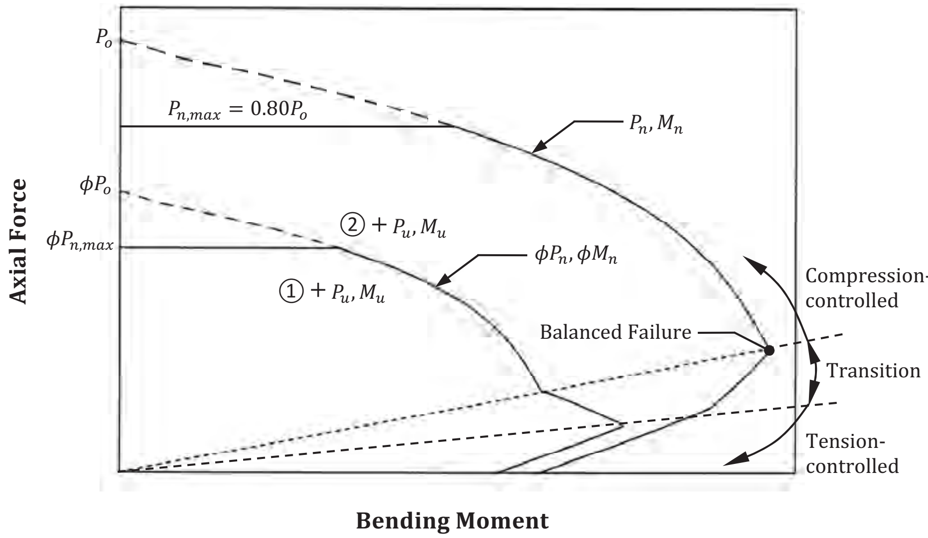


Figure 7.15 Nominal and design strength interaction diagrams.

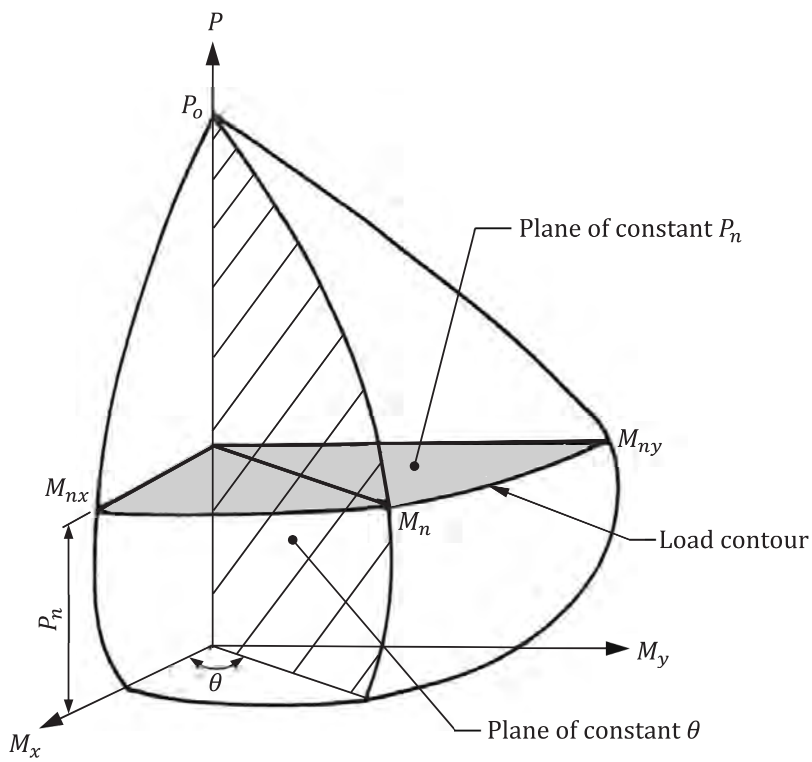


Figure 7.16 Biaxial strength interaction surface.

Approximate Method for Columns with Equal Bending Moment Capacities About Both Principal Axes

For circular columns or for square columns with the same longitudinal reinforcement on each face, the uniaxial bending capacity about the *x*-axis, M_{nox}, is the same as that about the *y*-axis, M_{noy}. A horizontal slice through the biaxial interaction surface of a square column at an axial compression force $P_u = \phi P_n$ is shown in Figure 7.18 where the dash-dot line represents the nominal moment strengths and the solid line represents the design moment

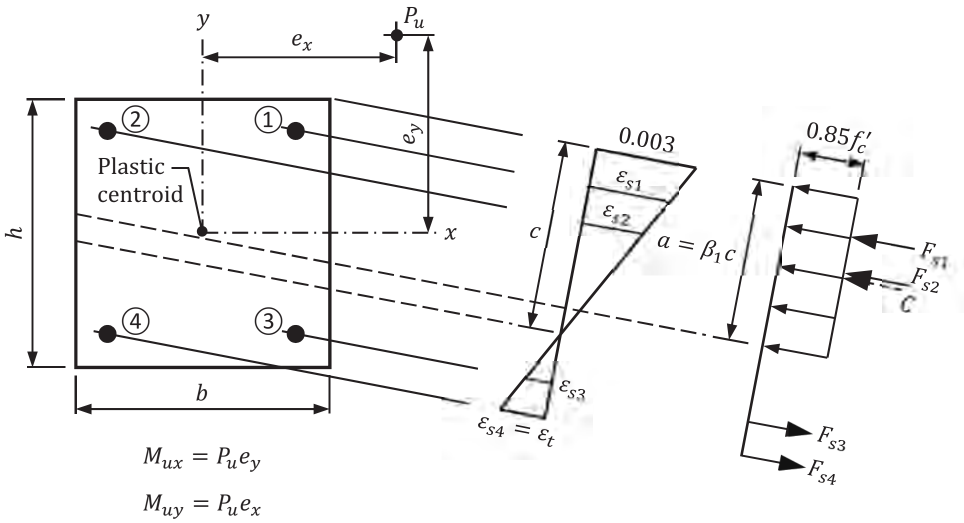


Figure 7.17 Rectangular column subjected to biaxial bending.

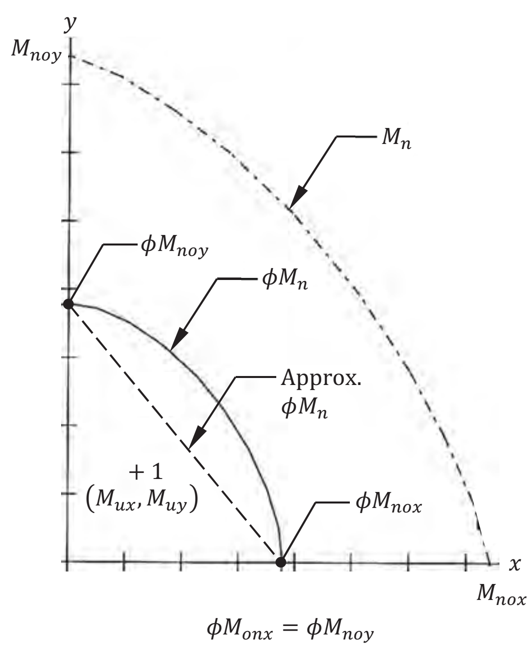


Figure 7.18 Biaxial capacity approximation for circular columns and square columns with the same longitudinal reinforcement on each face.

strengths. The moments M_{nox} and M_{noy} can be determined using the methods described above or by any other computer software or design aid.

The straight dashed line in Figure 7.18 connects the uniaxial design moment strengths $\phi M_{nox} = \phi M_{noy}$. This line always lies inside the actual biaxial capacity curve, so it can be used as a conservative (but not overly conservative) representation of the actual capacity diagram. Therefore, a circular column or a square column with the same longitudinal reinforcement on each face subjected to a factored axial compression force P_u is adequate for any combination of M_{ux} and M_{uy} that lies within the approximate biaxial capacity surface (an example of an adequate bending moment combination is represented by point 1 in Figure 7.18).

The approximate capacity surface can be represented by the following equation:

$$\frac{M_{ux}}{\phi M_{nox}}+\frac{M_{uy}}{\phi M_{noy}}=1 \tag{7.16}$$

Because $\phi M_{nox}=\phi M_{noy}$, Equation (7.16) can be rewritten as follows:

$$\frac{M_{ux}+M_{uy}}{\phi M_{nox}}=1 \tag{7.17}$$

Therefore, for a given P_u, a column is adequate where $M_u=M_{ux}+M_{uy}\leq \phi M_{nox}=\phi M_{noy}$.

Reciprocal Load Method

The Reciprocal Load Method provides a simple and conservative estimate of the strength of a column under biaxial loading conditions (Reference 17). The nominal axial strength, P_{ni}, corresponding to eccentricities about both principal axes of a column is calculated by the following equation:

$$\frac{1}{P_{ni}}=\frac{1}{P_{nx}}+\frac{1}{P_{ny}}-\frac{1}{P_o} \tag{7.18}$$

where $P_{nx}=$ nominal axial strength when the column is subjected to the uniaxial moment $M_{nx}=M_{ux}/\phi$

$P_{ny}=$ nominal axial strength when the column is subjected to the uniaxial moment $M_{ny}=M_{uy}/\phi$

$P_o=$ nominal axial strength at zero eccentricity

Determination of the items needed to calculate P_{ni} is relatively straightforward. The nominal axial strengths P_{nx} and P_{ny} can be obtained from uniaxial nominal strength strain compatibility analyses, and P_o is determined by ACI Equation (22.4.2.2).

The design axial strength ϕP_{ni} is obtained by multiplying P_{ni} determined by Equation (7.18) with the appropriate strength reduction factor ϕ based on the strain in the reinforcing bars farthest from the compression face of the column. The column is adequate where the design axial strength ϕP_{ni} is greater than or equal to the factored axial force P_u.

The Reciprocal Load Method produces reasonably accurate results when flexure does not govern the design of a column, that is, when P_{nx} and P_{ny} are greater than the axial strength corresponding to balanced failure. In cases where flexure governs, another method, like the Load Contour Method discussed below, should be utilized.

Load Contour Method

For columns with unequal bending moment capacities about the principal axes and where flexure governs the design (that is, where axial forces are relatively small), the Load Contour Method can be used to determine biaxial strength (Reference 17). The load contour (that is, the horizontal slice through the biaxial interaction surface) corresponding to a nominal axial compression force, P_n, can be approximated by the following nondimensional interaction equation:

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\alpha}+\left(\frac{M_{ny}}{M_{noy}}\right)^{\beta}=1 \tag{7.19}$$

In this equation, $M_{nx}=M_{ux}/\phi$ and $M_{ny}=M_{uy}/\phi$ are the nominal biaxial moments that occur simultaneously on the column, and M_{nox} and M_{noy} are the nominal uniaxial moment strengths about the *x*-axis and *y*-axis, respectively. The exponents α and β are a function of the amount, distribution, and location of the longitudinal reinforcement in the column; the dimensions of the column; and the strength and elastic properties of the concrete and reinforcing steel. It is demonstrated in Reference 17 that it is reasonably accurate to assume $\alpha = \beta$. Interaction curves for various $\alpha = \beta$ are given in Figure 7.19. Included is the straight line corresponding to $\alpha = \beta = 1$, which results in the following interaction equation:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} = 1 \tag{7.20}$$

This equation always yields conservative results because it underestimates the capacity of the column. The column is adequate for the applied axial force, $P_n = P_u / \phi$, and the applied bending moments, $M_{nx} = M_{ux} / \phi$ and $M_{ny} = M_{uy} / \phi$ that lie within the approximate biaxial capacity surface, that is, where the following equation is satisfied:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} \leq 1.0 \tag{7.21}$$

The PCA Load Contour Method is an extension of the Load Contour Method and information on this method can be found in Reference 18.

Slenderness effects must also be considered in the design of columns subjected to biaxial loading, where applicable. A review of several methods to evaluate slenderness effects in columns subjected to axial compression forces and biaxial bending is given in Reference 19.

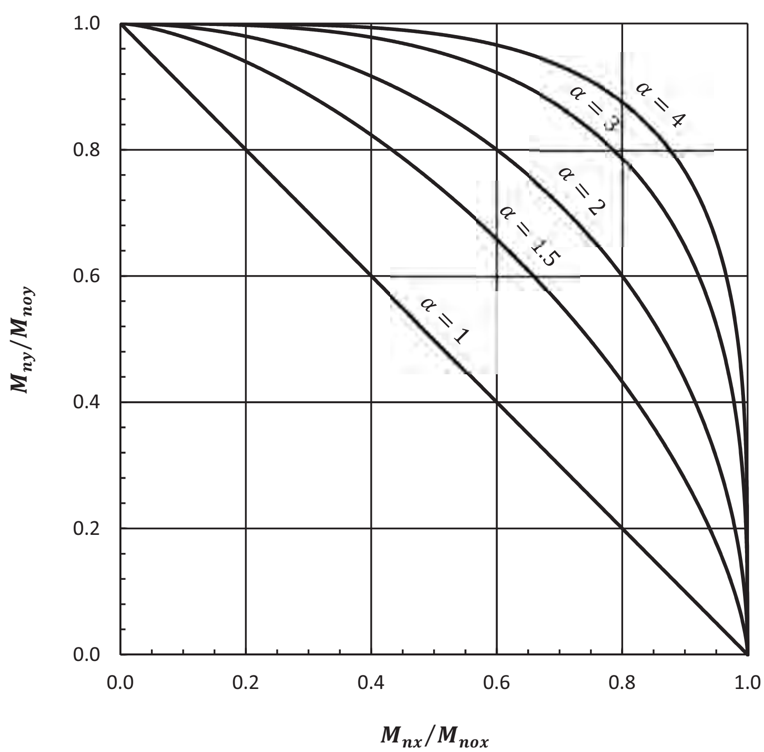


Figure 7.19 Interaction curves for the Load Contour Method.

7.4.4 Nominal Shear Strength

Overview

The nominal one-way shear strength, V_n, at a section in a column is determined in accordance with ACI 22.5 (ACI 10.5.3.1):

$$V_n = V_c + V_s \tag{7.22}$$

where V_c is the nominal shear strength provided by concrete and V_s is the nominal shear strength provided by shear reinforcement.

Nominal Shear Strength Provided by Concrete

For nonprestressed members, V_c is determined by the applicable equation in ACI Table 22.5.5.1 (ACI 22.5.5.1). The equations in that table are given in Table 7.12 along with the maximum permitted nominal shear strength $5\lambda\sqrt{f_c'}b_w d$ specified in ACI 22.5.5.1.1.

Table 7.12 Nominal Shear Strength Provided by Concrete, V_c

Area of Shear Reinforcement, A_v*	V_c**
$\geq A_{v,min}$	Either of $\left\{\begin{array}{l} \left(2\lambda\sqrt{f_c'} + \dfrac{N_u}{6A_g}\right)b_w d \\ \left[8\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \dfrac{N_u}{6A_g}\right]b_w d \end{array}\right. \leq 5\lambda\sqrt{f_c'}b_w d$
$< A_{v,min}$	$\left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \dfrac{N_u}{6A_g}\right]b_w d \leq 5\lambda\sqrt{f_c'}b_w d$

*Minimum area of shear reinforcement, $A_{v,min}$, is determined by ACI 10.6.2.2
**$N_u / 6A_g \leq 0.05f_c'$ (ACI 22.5.5.1.2)

The minimum area of shear reinforcement in a column, $A_{v,min}$, is determined in accordance with ACI 10.6.2.2 [see Equation (7.26) below].

The modification factor, λ, reflects the reduced mechanical properties of lightweight concrete relative to normal-weight concrete of the same compressive strength is given in Table 7.13 based on equilibrium density and in Table 7.14 based on composition of aggregates in the concrete mix (see ACI 19.2.4).

Table 7.13 Values of λ Based on Equilibrium Density, w_c

Equilibrium Density, w_c	λ
$w_c \leq 100 \text{ lb/ft}^3$	0.75
$100 \text{ lb/ft}^3 < w_c \leq 135 \text{ lb/ft}^3$	$0.0075w_c \leq 1.0$
$w_c > 135 \text{ lb/ft}^3$	1.0

Table 7.14 Values of λ Based on Composition of Aggregates

Concrete	Composition of Aggregates	λ
All-lightweight	Fine: ASTM C330 Coarse: ASTM C330	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330 and C33 Coarse: ASTM C330	0.75 to 0.85(1)
Sand-lightweight	Fine: ASTM C33 Coarse: ASTM C330	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33 Coarse: Combination of ASTM C330 and ASTM C33	0.85 to 1.0(2)

(1) Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.
(2) Linear interpolation from 0.85 to 1.0 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of aggregate.

The size effect modification factor, λ_s, accounts for the phenomenon indicated in test results that the shear strength attributed to concrete in members without shear reinforcement does not increase in direct proportion with member depth. This factor is determined by ACI Equation (22.5.5.1.3) [ACI 22.5.5.1.3]:

$$\lambda_s = \sqrt{\frac{2}{1+(d/10)}} \leq 1.0 \tag{7.23}$$

It is evident from Equation (7.23) that λ_s is less than 1.0 for members with $d > 10.0$ in.

The term ρ_w is equal to the area of flexural reinforcement, A_s, at the section divided by $b_w d$ where b_w is the width of a rectangular column perpendicular to the direction of analysis or the diameter of a circular column [ACI 22.5.2.2(b)]. The effective depth d can be obtained from a strain compatibility analysis for each applicable load combination; in the case of circular columns, it is permitted to take d as 80 percent of the diameter of the column [ACI 22.5.2.2(a)]. According to ACI R22.5.5.1, A_s may be taken as the sum of the areas of the longitudinal reinforcement located more than two-thirds of the overall member depth away from the extreme compression fiber.

According to the first Note in ACI Table 22.5.5.1, the axial force, N_u, which has the units of pounds, is to be taken as positive for compression forces acting on the gross area of the column, A_g, and is to be taken as negative for tension forces.

Values of $\sqrt{f'_c}$ used to calculate V_c are limited to 100 psi (ACI 22.5.3.1; the exception in ACI 22.5.3.2 is not applicable to columns). This limitation on f'_c is primarily due to the fact that there is a lack of test data and practical experience with concrete having compressive strengths greater than 10,000 psi.

In rare occasions where axial tension acts on a column, V_c can be taken as zero.

To minimize the likelihood of diagonal compression failure in the concrete and to limit the extent of cracking, the cross-sectional dimensions of a section must be selected to satisfy ACI Equation (22.5.1.2) [ACI 22.5.1.2]:

$$V_u \leq \phi(V_c + 8\sqrt{f'_c} b_w d) \tag{7.24}$$

Nominal Shear Strength Provided by Shear Reinforcement

The nominal shear strength of the transverse reinforcement is determined by ACI Equation (22.5.8.5.3):

$$V_s = \frac{A_v f_{yt} d}{s} \tag{7.25}$$

where s is the center-to-center spacing of ties or the spiral pitch. For columns with rectilinear ties and crossties, A_v is equal to the area of all the ties and crossties within the spacing s in the direction of analysis (ACI 22.5.8.5.5). In the case of circular ties or spirals, A_v is equal to two times the area of the tie or spiral within the spacing s (ACI 22.5.8.5.6).

A minimum area of shear reinforcement, $A_{v,min}$, must be provided at sections where $V_u > 0.5\phi V_c$ (ACI 10.6.2.1):

$$A_{v,min} = \text{greater of} \begin{cases} \dfrac{0.75\sqrt{f'_c} b_w s}{f_{yt}} \\ \\ \dfrac{50 b_w s}{f_{yt}} \end{cases} \tag{7.26}$$

For columns subjected to relatively small factored shear forces, the concrete alone may be sufficient to satisfy shear strength requirements. When more than just the concrete shear strength is needed, V_s is typically calculated using the requirements in ACI 10.7.6, 25.7.2, and 25.7.3 for transverse bar size and spacing. If shear strength requirements are not satisfied based on those requirements, the amount and/or spacing of the transverse reinforcement must be adjusted appropriately, or the compressive strength of the concrete can be increased, although the latter is generally not the most effective way to increase shear strength. In cases where the shear demand is very large, the size of the column may need to be increased.

Biaxial Shear Strength

Reinforced concrete columns may be subjected to the effects from biaxial shear forces (this commonly occurs in corner columns). For symmetrically reinforced circular columns, the nominal one-way shear strength is the same along any axis of the member, and shear strength can be evaluated based on the requirements outlined above using the resultant of the shear forces acting on the section.

For rectangular sections, it is not practical to determine the nominal shear strength based on the direction of the resultant shear force. As an alternative, the biaxial shear force requirements of ACI 22.5.1.10 must be satisfied.

Required shear stresses $v_{u,x} = V_{u,x} / b_w d$ and $v_{u,y} = V_{u,y} / b_w d$ are calculated in the orthogonal x and y directions of the section where $V_{u,x}$ and $V_{u,y}$ are the factored shear forces in the x and y directions, respectively, b_w is the cross-sectional dimension of the column perpendicular to the direction of analysis, and d is the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement in the direction of analysis (see Figure 7.20).

Nominal shear stresses $v_{n,x}$ and $v_{n,y}$ in the x and y directions are determined by dividing the nominal shear strength, V_n, calculated in accordance with Equation (7.22) by the applicable b_w and d in each direction.

According to ACI 22.5.1.10, interaction of orthogonal shear forces need not be considered where one (or both) of the following equations is satisfied:

$$\frac{v_{u,x}}{\phi v_{n,x}} \le 0.5 \tag{7.27}$$

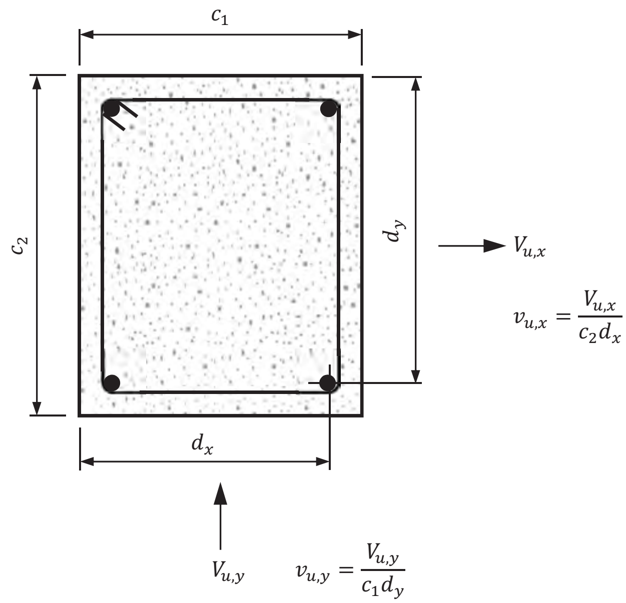


Figure 7.20 Determination of biaxial shear stresses on a rectangular reinforced concrete column.

$$\frac{v_{u,y}}{\phi v_{n,y}} \leq 0.5 \tag{7.28}$$

In cases where $v_{u,x} / \phi v_{n,x} > 0.5$ and $v_{u,y} / \phi v_{n,y} > 0.5$, then interaction effects must be considered and ACI Equation (22.5.1.11) must be satisfied:

$$\frac{v_{u,x}}{\phi v_{n,x}} + \frac{v_{u,y}}{\phi v_{n,y}} \leq 1.5 \tag{7.29}$$

Where Equation (7.29) is not satisfied, the cross-sectional dimensions of the column, the amount of shear reinforcement, or both must be increased accordingly. The compressive strength of the concrete can also be increased but doing so does has a smaller impact than increasing the other items.

7.4.5 Nominal Torsional Strength

Columns in building structures are rarely subjected to torsional moments. In cases where such effects must be considered, the provisions in ACI Chapter 9 must be satisfied (ACI 10.5.4). These requirements are covered in Sections 6.4.4, 6.5.3, and 6.5.4 of this publication for beams.

7.5 Reinforcement Limits

7.5.1 Longitudinal Reinforcement

For columns supporting only gravity loads or that are part of an ordinary or intermediate moment frame, the minimum and maximum areas of longitudinal reinforcement, A_{st}, are given in ACI 10.6.1.1, which must be satisfied regardless of the type of transverse reinforcement used in the column:

- Minimum $A_{st} = 0.01A_g$
- Maximum $A_{st} = 0.08A_g$

where A_g is the gross cross-sectional area of the column.

The 1 percent lower limit is meant to provide resistance to any bending moments not accounted for in the analysis, such as those caused by construction tolerances or misalignments. It is also meant to help reduce creep and shrinkage in the concrete under sustained compression stresses. The upper limits help minimize congestion and the development of higher shear stresses, although a 1 to 2 percent reinforcement ratio, $\rho_g = A_{st} / A_g$, is generally more efficient and cost effective (see Reference 7). At locations where lap splices are used, the longitudinal reinforcement ratio may be equal to two times that at sections away from the lap splice, and that ratio must be less than or equal to 0.08.

As noted in Section 7.2 of this publication, a column is permitted to be designed of sufficient size to carry the required factored loads and additional concrete is permitted to be added to the section without having to increase the minimum longitudinal reinforcement to satisfy the 1 percent limit in ACI 10.6.1.1 (ACI 10.3.1.2). It is assumed the additional concrete does not carry any load; however, the actual column size must be included in the structural analysis to correctly account for its actual stiffness. The columns in the upper stories of a building where the factored loads are usually relatively small can typically be designed using a reinforcement ratio less than 1 percent based on this provision.

7.5.2 Shear Reinforcement

Minimum shear reinforcement requirements for columns are given in ACI 10.6.2 and Equation (7.26). For columns with rectilinear ties and crossties, A_v is equal to the area of all the ties and crossties within the spacing s in the direction of analysis (ACI 22.5.8.5.5). In the case of circular ties or spirals, A_v is equal to two times the area of the tie or spiral within the spacing s (ACI 22.5.8.5.6).

7.6 Sizing the Cross-Section

7.6.1 Axial Compression

For columns subjected primarily to axial compression forces, the cross-sectional dimension of a column can be obtained by setting the total factored axial compression force, P_u, equal to the design axial load strength, $\phi P_{n,max}$, given in ACI Table 22.4.2.1 for columns with ties conforming to ACI 22.4.2.4 and with spirals conforming to ACI 22.4.2.5 [see Equations (7.13) and (7.14) in this publication]:

- For tied columns:

$$A_g \geq \frac{P_u}{\phi 0.80[0.85 f_c'(1-\rho_g) + f_y \rho_g]}, \quad \phi = 0.65 \tag{7.30}$$

- For spiral columns:

$$A_g \geq \frac{P_u}{\phi 0.85[0.85 f_c'(1-\rho_g) + f_y \rho_g]}, \quad \phi = 0.75 \tag{7.31}$$

where $\rho_g = A_{st} / A_g$.

A preliminary column size can be obtained from these equations for a given P_u assuming a value of ρ_g, which as noted previously, should be in the range of 1 to 2 percent to achieve overall economy.

The charts in Appendix B of Reference 16 can be used to quickly determine a preliminary size of a nonslender, tied, square column for a given factored axial compression force P_u with the following:

- Grade 60 or Grade 80 longitudinal reinforcement;
- Concrete compressive strengths from 4,000 psi to 14,000 psi; and,
- Longitudinal reinforcement ratios from 1 to 2 percent

The required gross column area, A_g, is also provided in the charts, which can be used to obtain preliminary sizes for rectangular or circular tied columns. A modification factor is given in the appendix to acquire preliminary sizes for spiral columns.

The use of high-strength longitudinal reinforcement can have an impact on the required size of a column, although the impact is generally not significant because the concrete is resisting most of the compression force. However, when high-strength longitudinal reinforcement is combined with high-strength concrete, less congestion in the structure and more usable space in the building may be possible.

7.6.2 Combined Moment and Axial Force

Columns that are part of the LFRS in a moment frame resisting the effects from lateral forces are typically subjected to combined moment and axial forces. Because of the inherent complexity of this situation, a preliminary column size is often determined based on axial compression forces from gravity loads only and then adjusted later, if required, to account for the effects due to the bending moments.

As noted in Section 7.4.3 of this publication, Appendix C in Reference 16 can be used to facilitate selection of a preliminary column size for columns subjected to combined moment and axial forces.

7.6.3 Slenderness Effects

Minimum column dimensions so that slenderness effects need not be considered as a function of the unsupported column length, ℓ_u, are given in Table 7.7 of this publication.

7.7 Determination of Required Reinforcement

7.7.1 Required Longitudinal Reinforcement

Axial Compression

For nonslender reinforced concrete columns subjected to primarily uniaxial compression forces, the required area of longitudinal reinforcement, A_{st}, can be obtained by using the appropriate equations in ACI Table 22.4.2.1 for tied and spiral columns. Where P_u, A_g, f'_c, and f_y are known, the following equation can be used to determine the longitudinal reinforcement ratio $\rho_g = A_{st} / A_g$:

$$\rho_g = \frac{(P_u / \phi\phi_c A_g) - 0.85f'_c}{f_y - 0.85f'_c}$$

$$\geq \rho_{g,min} = 0.01 \tag{7.32}$$

$$\leq \rho_{g,max} = 0.08$$

In this equation, the strength reduction, ϕ, is equal to 0.65 for tied columns and 0.75 for spiral columns, and the term ϕ_c is equal to 0.80 for tied columns and 0.85 for spiral columns. As noted previously, overall economy is typically achieved by using a longitudinal reinforcement ratio between approximately 0.01 and 0.02.

The charts in Appendix B of Reference 16 can be used to determine ρ_g for nonslender, tied and spiral columns.

Combined Moment and Axial Force

The required longitudinal reinforcement can be acquired from an interaction diagram for columns subjected to combined moment and axial force.

The design strength interaction diagrams in Appendix C of Reference 16 can be used to quickly obtain the longitudinal reinforcement for a wide range of rectangular and circular column sizes, longitudinal bar configurations, and material properties.

7.7.2 Required Shear Reinforcement

For a given A_v, the spacing of the transverse reinforcement in a column, s, can be determined by the following equation, which is based on Equation (7.25):

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} \tag{7.33}$$

Maximum spacing of shear reinforcement in columns is given in ACI 10.7.6.5.2 and is summarized in Table 7.15.

Table 7.15 Maximum Spacing of Shear Reinforcement in Columns

V_s (lb)	Maximum Spacing, s (in.)
$\leq 4\sqrt{f'_c} b_w d$	Lesser of $\begin{cases} d/2 \\ 24 \text{ in.} \end{cases}$
$> 4\sqrt{f'_c} b_w d$	Lesser of $\begin{cases} d/4 \\ 12 \text{ in.} \end{cases}$

For a given s, A_v can be determined from the following equation:

$$A_v = \frac{(V_u - \phi V_c)s}{\phi f_{yt} d} \tag{7.34}$$

For columns with rectilinear ties and crossties, A_v is equal to the area of all the ties and crossties within the spacing s in the direction of analysis (ACI 22.5.8.5.5). In the case of circular ties or spirals, A_v is equal to two times the area of the tie or spiral within the spacing s (ACI 22.5.8.5.6).

7.8 Reinforcement Detailing

7.8.1 Concrete Cover

Reinforcing bars are placed in columns with a minimum concrete cover to protect them from weather, fire, and other effects. Minimum cover requirements are given in ACI 20.5.1 (ACI 10.7.1.1). For columns with ties and spirals, concrete cover is measured from the surface of the concrete to the outer edge of the transverse reinforcement (see Figure 7.21 for a rectangular column with ties and crossties). The minimum cover to the transverse reinforcement is equal to 1.5 in. for reinforcing bars in columns not exposed to weather or in contact with the ground (ACI Table 20.5.1.3.1).

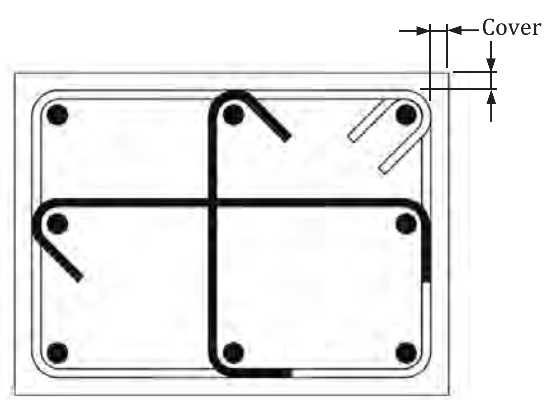


Figure 7.21 Concrete cover for columns.

7.8.2 Minimum Number of Longitudinal Bars

The minimum number of longitudinal bars that must be provided in a column is given in ACI 10.7.3.1 and is based on the type of transverse reinforcement (see Table 7.16).

Table 7.16 Minimum Number of Longitudinal Bars in a Reinforced Concrete Column

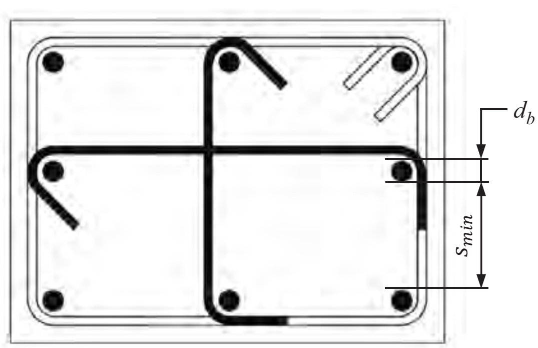
Type of Transverse Reinforcement	Minimum Number of Longitudinal Bars
Triangular ties	3
Rectangular or circular ties	4
Spirals	6

7.8.3 Spacing of Longitudinal Bars

Minimum clear spacing between the longitudinal bars in a column must conform to the requirements of ACI 25.2 (ACI 10.7.2.1). The purpose of these requirements is to ensure concrete can easily flow between the longitudinal bars. The minimum clear spacing, s_{min}, between longitudinal bars at any section in a column, including at locations where lap splices are used, is determined by the following equation (see ACI 25.2.3 and Figure 7.22):

$$s_{min} = \text{greater of} \begin{cases} 1.5 \text{ in.} \\ 1.5 d_b \\ (4/3) d_{agg} \end{cases} \tag{7.35}$$

In this equation, d_b is the diameter of the longitudinal bars in the column and d_{agg} is the nominal maximum size of coarse aggregate in the concrete mix.



$$s_{min} = \text{greater of} \begin{cases} 1.5 \text{ in.} \\ 1.5 d_b \\ (4/3) d_{agg} \end{cases}$$

Figure 7.22 Minimum clear spacing requirements for longitudinal bars in columns.

Minimum face dimensions of rectangular, tied columns with bearing splices, normal lap splices, and tangential lap splices based on the minimum clear spacing requirements of ACI 25.2.3 are given in Table 7.17 (see Figure 7.23 for the splice types). The face dimensions are rounded up to the nearest inch and are based on a (1) 1.5-in. clear cover to #3 ties for #5 through #10 longitudinal bars and #4 ties for #11, #14, and #18 longitudinal bars and (2) a maximum aggregate size of 1.0 in.

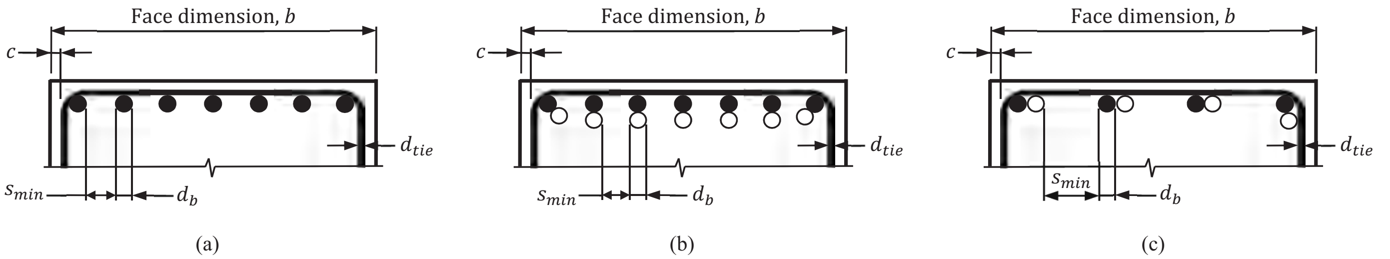


Figure 7.23 Column splices for columns with longitudinal bars arranged in a rectangular pattern. (a) Bearing. (b) Normal lap. (c) Tangential lap.

Table 7.17 Minimum Face Dimensions (Inches) of Rectangular, Tied Columns

Splice Type	Bar Size	Number of Bars Per Face								
		2	3	4	5	6	7	8	9	10
Bearing	#5	7	9	11	13	15	18	20	22	24
	#6	7	9	12	14	16	18	21	23	25
	#7	7	10	12	15	17	19	22	24	26
	#8	8	10	13	15	18	20	23	25	28
	#9	8	11	14	17	19	22	25	28	31
	#10	9	12	15	18	21	25	28	31	34
	#11	9	13	16	20	24	27	31	34	38
	#14	10	15	19	23	27	32	36	40	44
	#18	12	18	24	29	35	41	46	52	58
Normal Lap	#5	8	10	12	14	16	18	21	23	25
	#6	8	11	13	15	17	20	22	24	26
	#7	9	11	13	16	18	21	23	25	28
	#8	9	12	14	17	19	22	24	27	29
	#9	10	13	15	18	21	24	27	29	32
	#10	10	14	17	20	23	26	29	33	36
	#11	11	15	18	22	25	29	33	36	40
	#14	13	17	21	25	30	34	38	42	47
	#18	16	21	27	32	38	44	49	55	61
Tangential Lap	#5	8	10	13	16	19	21	24	27	30
	#6	8	11	14	17	20	23	26	29	32
	#7	8	12	15	18	21	25	28	31	34
	#8	9	12	16	19	23	26	30	33	37
	#9	9	13	17	21	25	29	33	37	41
	#10	10	14	19	23	28	32	37	41	46
	#11	11	16	21	26	31	36	40	45	50
	#14	12	18	24	30	36	42	48	54	60
	#18	15	23	30	38	46	54	62	70	78

The equations in Table 7.18 can be used to determine minimum face dimensions for any rectangular, tied column. The items in Table 7.18 for normal lap splices are defined in Figure 7.24.

Table 7.18 Equations to Determine Minimum Face Dimensions (Inches) of Rectangular, Tied Columns

Splice Type	Minimum Face Dimension, b*
Bearing	$b = 2(c + d_{tie}) + nd_b + (n-1)s_{min}$
Normal Lap	$b = 2(c + d_{tie}) + (n-2)d_b + (n-3)s_{min} + 2\bar{s}$ where $\bar{s} = (s_{min} + d_b)\cos\theta + \bar{a}$ $\theta = \arcsin[\bar{b} / (s_{min} + d_b)]$ $\bar{a} = d_b / \sqrt{2}$ $\bar{b} = (1 - 1/\sqrt{2})d_b$
Tangential Lap	$b = 2(c + d_{tie}) + (2n-1)d_b + (n-1)s_{min}$

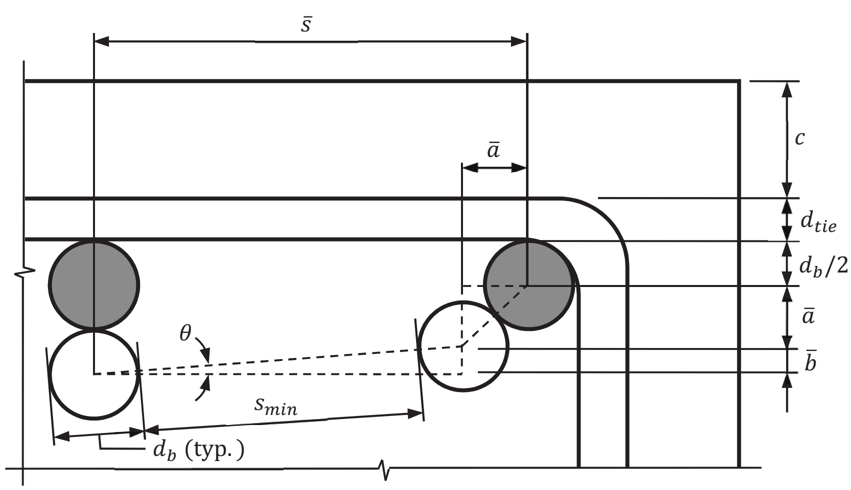
*c = clear cover to ties

d_{tie} = diameter of tie bar

n = number of longitudinal bars per face

d_b = diameter of longitudinal bar

$s_{min} = \max[1.5 \text{ in.}, 1.5d_b, (4/3)d_{agg}]$



$\bar{a} = d_b/\sqrt{2}$ $\quad$ $\theta = \arcsin[\bar{b}/(s_{min} + d_b)]$

$\bar{b} = (1 - 1/\sqrt{2})d_b$ $\quad$ $\bar{s} = (s_{min} + d_b)\cos\theta + \bar{a}$

$$s_{min} = \text{greater of} \begin{cases} 1.5 \text{ in.} \\ 1.5d_b \\ (4/3)\, d_{agg} \end{cases}$$

Figure 7.24 Items for normal lap splices.

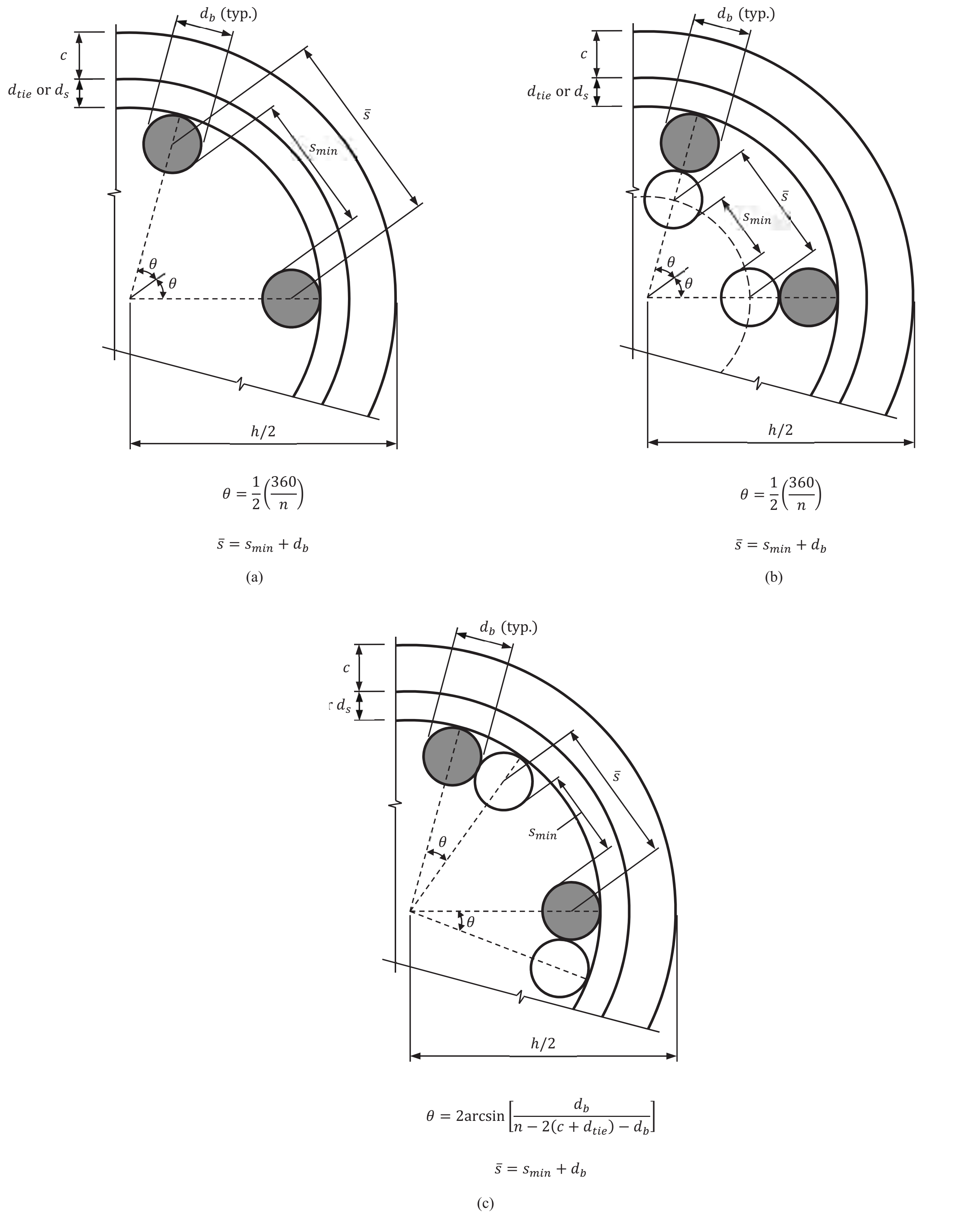


Figure 7.25 Column splices for columns with longitudinal bars arranged in a circular pattern. (a) Bearing. (b) Normal lap. (c) Tangential lap.

The maximum number of longitudinal bars in a column where the bars are arranged in a circle with bearing splices, normal lap splices, and tangential lap splices is given in Table 7.19 (see Figure 7.25 for the splice types where d_{tie} is the tie diameter and d_s is the spiral diameter). In addition to satisfying the minimum clear spacing requirements of ACI 25.2.3, the requirements for minimum and maximum area of longitudinal reinforcement in ACI 10.6.1.1 are also satisfied at sections away from lap splice locations where all the bars are spliced. However, in many cases, either the provided reinforcement ratio is much larger than the recommended maximum value of 2 percent or there is an inordinately large number of smaller longitudinal bars. The number of bars have been rounded down to the nearest whole number and are based on a 1.5-in. clear cover to #4 ties or spirals.

Table 7.19 Maximum Number of Longitudinal Bars in Columns with Longitudinal Bars Arranged in a Circle

Splice Type	*h* (in.)	Bar Size								
		#5	#6	#7	#8	#9	#10	#11	#14	#18
Bearing	12	10	9	9	8	7	6	5*	4*	—
	14	13	12	11	11	9	8	7	5*	—
	16	16	15	14	13	11	10	9	7	4*
	18	19	18	17	16	14	12	11	8	5*
	20	22	21	19	18	16	14	12	10	6
	22	25	24	22	21	18	16	14	11	7
	24	28	26	25	23	20	18	16	13	9
	26	31	29	27	26	23	20	18	14	10
	28	34	32	30	28	25	22	20	16	11
	30	37	35	33	31	27	24	21	17	13
	32	40	38	35	33	29	26	23	19	14
	34	43	40	38	36	32	28	25	20	15
	36	46	43	41	38	34	30	27	22	16
	38	49	46	43	41	36	32	28	23	17
	40	52	49	46	43	38	34	30	25	18
	42	55	51	49	46	41	36	32	26	19
	44	58	54	51	48	43	38	34	28	20
	46	61	57	54	51	45	40	36	29	22
	48	64	60	57	54	47	42	37	31	23
Normal Lap	12	8	7	6	6	4*	—	—	—	—
	14	11	10	9	8	7	5*	4*	—	—
	16	14	13	12	11	9	7	6	4*	—
	18	17	16	14	13	11	9	8	6	—
	20	20	19	17	16	13	11	10	7	4*
	22	23	21	20	18	16	13	12	9	5*
	24	26	24	22	21	18	15	13	10	7
	26	29	27	25	23	20	17	15	12	8

(table continued on next page)

Table 7.19 Maximum Number of Longitudinal Bars in Columns with Longitudinal Bars Arranged in a Circle (cont.)

Splice Type	h (in.)	Bar Size								
		#5	#6	#7	#8	#9	#10	#11	#14	#18
Normal Lap (cont.)	28	32	30	28	26	22	19	17	13	9
	30	35	33	30	28	25	21	19	15	10
	32	38	35	33	31	27	23	21	16	11
	34	41	38	36	33	29	25	22	18	12
	36	44	41	38	36	31	27	24	19	13
	38	47	44	41	38	34	29	26	21	15
	40	50	47	44	41	36	31	28	22	16
	42	53	49	46	43	38	33	30	24	17
	44	56	52	49	46	40	35	31	25	18
	46	59	55	52	48	42	37	33	27	19
	48	62	58	54	51	45	39	35	28	20
Tangential Lap	12	8	7	6	6	5*	4*	4*	—	—
	14	10	9	8	7	6	6	5*	4*	—
	16	12	11	10	9	8	7	6	5*	—
	18	15	13	12	11	10	8	7	6	4*
	20	17	15	14	13	11	10	9	7	5*
	22	19	18	16	15	13	11	10	8	5*
	24	22	20	18	17	14	13	11	9	6
	26	24	22	20	18	16	14	13	10	7
	28	26	24	22	20	18	16	14	11	8
	30	28	26	24	22	19	17	15	12	9
	32	31	28	26	24	21	18	16	13	10
	34	33	30	28	26	22	20	18	14	10
	36	35	32	30	27	24	21	19	16	11
	38	38	34	31	29	26	23	20	17	12
	40	40**	36	33	31	27	24	21	18	13
	42	42**	38	35	33	29	25	23	19	14
	44	44**	41	37	34	30	27	24	20	14
	46	47**	43	39	36	32	28	25	21	15
	48	49**	45	41	38	34	30	27	22	16

**Indicates the tabulated value is not applicable to columns where a minimum of 6 longitudinal bars are required.*
***Indicates the longitudinal reinforcement ratio is less than 1 percent.*
A dash (—) indicates the maximum number of longitudinal bars is less than 4.

The equations in Table 7.20 can be used to determine the maximum number of longitudinal bars for any column where the longitudinal reinforcement is arranged in a circle.

Table 7.20 Equations to Determine the Maximum Number of Longitudinal Bars in a Column with Longitudinal Bars Arranged in a Circle

Splice Type	Maximum Number of Longitudinal Bars, n*
Bearing	$n = \dfrac{180}{\arcsin\left[\dfrac{s_{min} + d_b}{h - 2(c + d_{tie}) - d_b}\right]}$
Normal Lap	$n = \dfrac{180}{\arcsin\left[\dfrac{s_{min} + d_b}{h - 2(c + d_{tie}) - 3d_b}\right]}$
Tangential Lap	$n = \dfrac{180}{\arcsin\left[\dfrac{s_{min} + d_b}{h - 2(c + d_{tie}) - d_b}\right] + \arcsin\left[\dfrac{d_b}{h - 2(c + d_{tie}) - d_b}\right]}$

*c = clear cover to ties or to spiral
d_{tie} = diameter of tie bar (or, diameter of spiral, d_s)
d_b = diameter of longitudinal bar
$s_{min} = \max[1.5 \text{ in.}, 1.5d_b, (4/3)d_{agg}]$

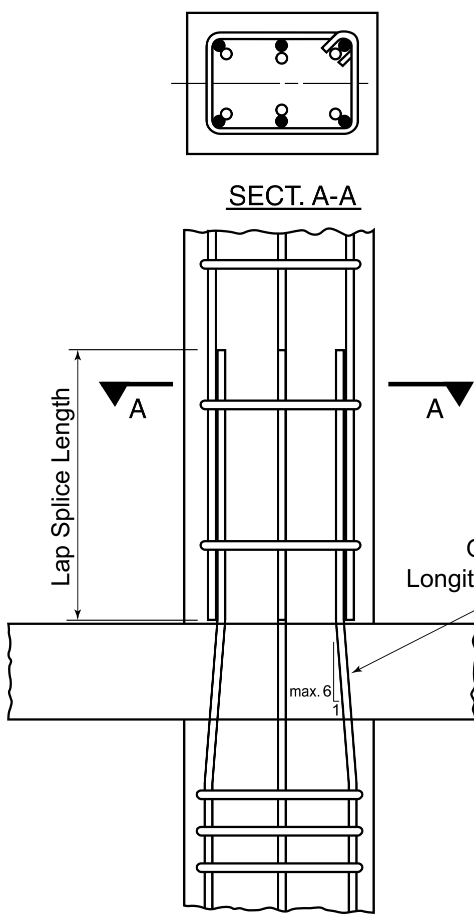


Figure 7.26 Offset bent longitudinal reinforcement in a column.

7.8.4 Offset Bent Longitudinal Reinforcement

Offset bent longitudinal reinforcement is typically provided at locations where the longitudinal bars are spliced in a column (see Figure 7.26). According to ACI 10.7.4.1, the slope of the inclined portion of an offset bent bar relative to the longitudinal axis of the column must not exceed 1 to 6, and the portions of the bar above and below the offset must be parallel to the axis of the column.

In cases where the face of a column is offset by 3 in. or more, offset bent longitudinal reinforcement is not permitted (ACI 10.7.4.2). Instead, the offset longitudinal reinforcing bars on each face must be lap spliced with separate dowel bars. Shown in Figure 7.27 is the case where separate dowel bars must be provided at a location where a change in column cross-section occurs.

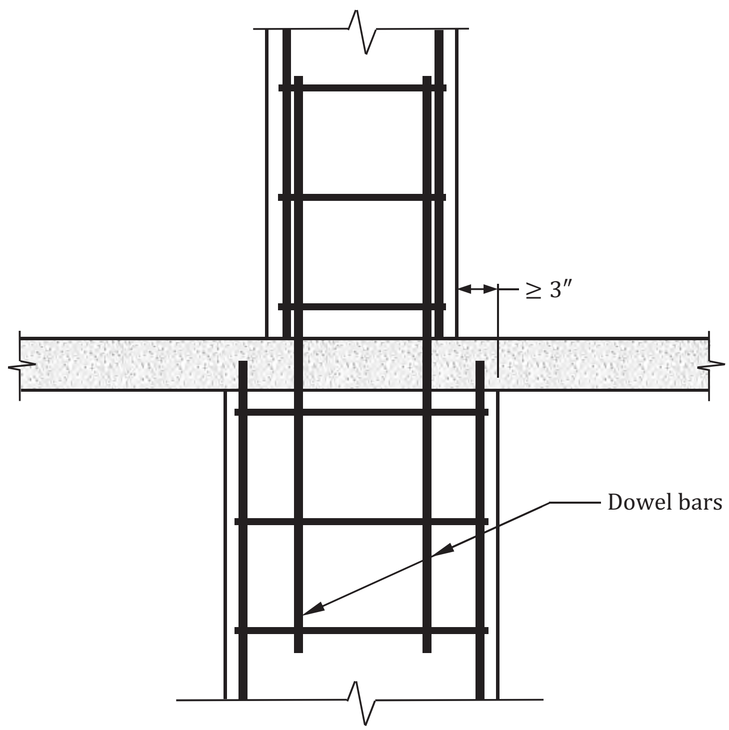


Figure 7.27 Separate dowel bars at offset column faces.

7.8.5 Splices of Longitudinal Reinforcement

Overview

Splice requirements for longitudinal reinforcement in columns are given in ACI 10.7.5 for lap splices, mechanical splices, butt-welded splices, and end-bearing splices. Splice lengths must be provided that satisfy the factored load combinations on a column. Where the stress in all the longitudinal bars due to factored loads is compressive, all the splice types listed above may be used. All but end-bearing splices are permitted when the longitudinal bar stress is tensile. Requirements for splices are given in ACI 25.5 (ACI 10.7.5.1.3).

Lap Splices

Lap splices, which are the most popular and usually the most economical type of splices used in columns, are permitted to occur immediately above the top of the slab for columns that are not part of special moment frames (see Figure 7.26). This is the preferred location for ease of construction: The longitudinal bars from the column below extend above the slab a distance greater than or equal to the required lap splice length, and the longitudinal bars in the column above are tied to these bars after the floor below has been constructed.

Lap splices are not permitted for bars larger than #11 bars except in the case of compression lap splices where #14 or #18 bars are permitted to be spliced to #11 or smaller bars (ACI 25.5.1.1 and 25.5.5.3).

The type of lap splice that must be used (compressive or tensile) depends on the stress in the longitudinal bars due to the factored load combinations (see Figure 7.28). Required lap splice lengths for the three zones indicated in Figure 7.28 are given in Table 7.21.

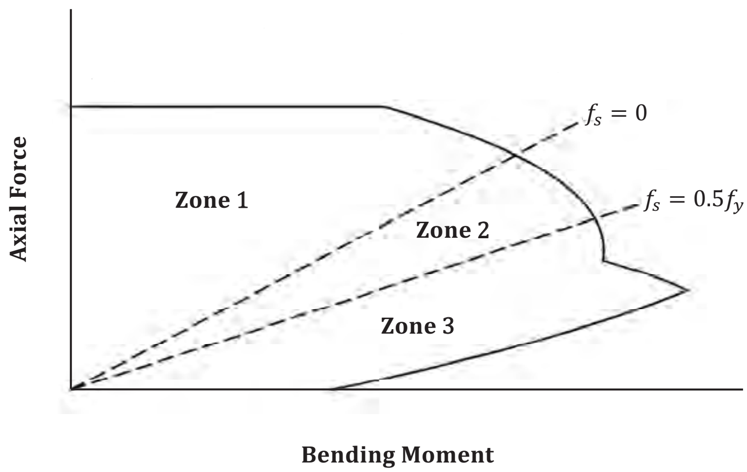


Figure 7.28 Splice requirements for reinforced concrete columns.

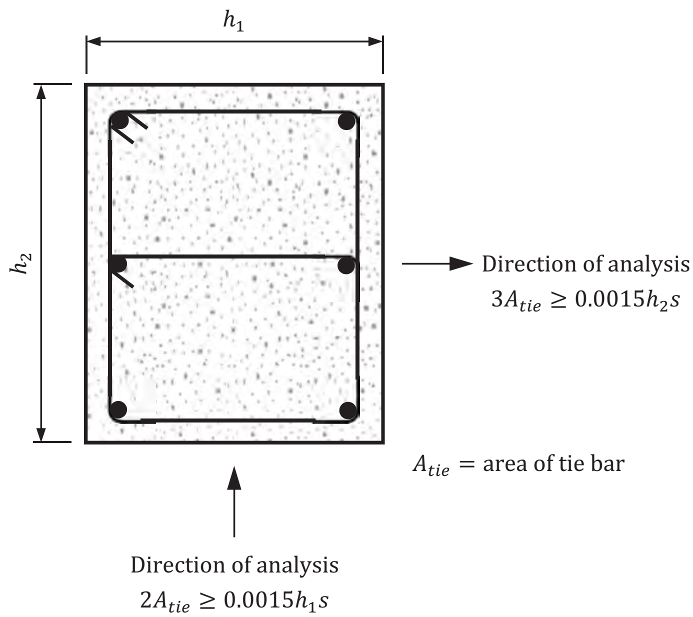


Figure 7.29 Application of ACI 10.7.5.2.1(a).

Table 7.21 Required Lap Splices Lengths

Zone	Stress in Longitudinal Bars	Minimum Lap Splice Length		Notes	ACI Section No.
1	All longitudinal bars are in compression	$f_y \le 60{,}000$ psi	$\ell_{sc} = 0.0005 f_y d_b \ge 12$ in.	1–7	10.7.5.2.1 25.5.5.1
		$60{,}000 \text{ psi} < f_y \le 80{,}000$ psi	$\ell_{sc} = (0.0009 f_y - 24) d_b \ge 12$ in.		
		$f_y > 80{,}000$ psi	$\ell_{sc} = \text{greater of} \begin{cases} (0.0009 f_y - 24) d_b \\ \ell_{st} \text{ calculated by ACI 25.5.2.1} \end{cases}$		
2	Stress in the longitudinal bars on the tension face is tensile and $\le 0.5 f_y$	Class A tension lap splice $\ell_{st} = \ell_d \ge 12$ in. Class B tension lap splice length $\ell_{st} = 1.3\ell_d \ge 12$ in.		7–10	10.7.5.2.2 25.5.2.1
3	Stress in the longitudinal bars on the tension face is tensile and $> 0.5 f_y$	Class B tension lap splice length $\ell_{st} = 1.3\ell_d \ge 12$ in.		7–10	10.7.5.2.2 25.5.2.1

Notes:

1. For tied columns where ties throughout the lap splice length have an effective area $\ge 0.0015hs$ in both directions, ℓ_{sc} is permitted to be multiplied by 0.83. Tie legs perpendicular to h are considered in calculating the effective tie area [ACI 10.7.5.2.1(a); see Figure 7.29]. For spiral columns where spirals satisfying ACI 25.7.3 are provided throughout the lap splice length, ℓ_{sc} is permitted to be multiplied by 0.75 [ACI 10.7.5.2.1(b)].
2. For $f'_c < 3{,}000$ psi, ℓ_{sc} must be increased by one-third [ACI 25.5.5.1].
3. Compression lap splices must not be used for bars larger than #11 except as permitted in ACI 25.5.5.3 [ACI 25.5.5.2].
4. Compression lap splices of #14 and #18 bars to #11 or smaller bars are permitted in accordance with ACI 25.5.5.4 (see Note 5) [ACI 25.5.5.3].
5. Where bars of different size are lap spliced in compression, ℓ_{sc} is the longer of (a) ℓ_{dc} of the larger bar calculated by ACI 25.4.9.1 (see below) and (b) ℓ_{sc} of the smaller bar calculated by ACI 25.5.5.1 [ACI 25.5.5.4].

$$\ell_{dc} = \text{greater of} \begin{cases} (f_y \psi_r / 50\lambda\sqrt{f'_c}) d_b \\ 0.0003 f_y \psi_r d_b \\ 8 \text{ in.} \end{cases}$$

where λ and ψ_r are given in ACI Table 25.4.9.3 (see Table 7.22)

6. ℓ_{st} is determined by ACI Table 25.5.2.1 where ℓ_d is determined by ACI 25.4.2.1(a) [ACI 25.5.2.1].
7. Reduction of development length in accordance with ACI 25.4.10.1 is not permitted in calculating lap splice lengths (ACI 25.5.1.4).
8. Requirements for Class A and Class B tension lap splices are given in ACI Table 10.7.5.2.2 (see Table 7.23).
9. Tension development length ℓ_d is determined in accordance with ACI 25.4.2.1(a).
10. Where bars of different size are lap spliced together, ℓ_{st} is the greater of (a) ℓ_d of the larger bar and (b) ℓ_{st} of the smaller bar (ACI 25.5.2.2).

Table 7.22 Modification Factors for Deformed Bars in Compression

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Confining reinforcement, Ψ_r	Longitudinal reinforcement enclosed in the following: 1. A spiral 2. A circular continuously wound tie with $d_b \geq 0.25$ in. and a pitch ≤ 4 in. 3. #4 ties in accordance with ACI 25.7.2 spaced ≤ 4 in. on center 4. Hoops in accordance with ACI 25.7.4 spaced ≤ 4 in. on center	0.75
	Other	1.0

Table 7.23 Requirements for Class A and Class B Tension Lap Splices

Stress in the longitudinal bars on the tension face	Splice Details	Splice Type	Tension Lap Splice Length, ℓ_{st}*
$\leq 0.5f_y$	There are ≤ 50 percent of the longitudinal bars spliced at any section and the lap splices on adjacent bars are staggered by at least ℓ_d (see Figure 7.30)	Class A	$\ell_{st} = \text{longer of} \begin{cases} 1.0\ell_d \\ 12 \text{ in.} \end{cases}$
	Other	Class B	$\ell_{st} = \text{longer of} \begin{cases} 1.3\ell_d \\ 12 \text{ in.} \end{cases}$
$> 0.5f_y$	All cases	Class B	$\ell_{st} = \text{longer of} \begin{cases} 1.3\ell_d \\ 12 \text{ in.} \end{cases}$

**Tension development length ℓ_d is determined in accordance with ACI 25.4.2.1(a).*

Provisions for the development of deformed reinforcing bars in tension are given in ACI 25.4.2. The tension development length, ℓ_d, is determined using the provisions of ACI 25.4.2.3 or 25.4.2.4 in conjunction with the modification factors in ACI 25.4.2.5. The requirements of ACI 25.4.2.3 are based on those in ACI 25.4.2.4, so the latter requirements are covered first.

Method 1 – ACI 25.4.2.4

The development length in tension of a deformed reinforcing bar, ℓ_d, is determined by ACI Equation (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \tag{7.36}$$

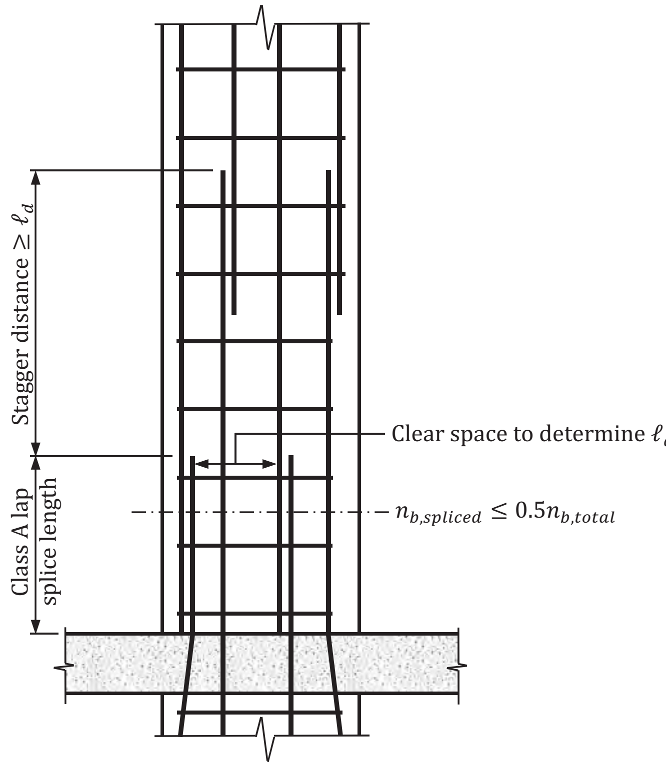


Figure 7.30 Class A Tension Lap Splice

The factors in Equation (7.36) are given in ACI Table 25.4.2.5; these factors are given in Table 7.24 along with definitions for the other terms in Equation (7.36). The confining term $(c_b + K_{tr})/d_b$ must be taken less than or equal to 2.5 in Equation (7.36) [ACI 25.4.2.4].

Table 7.24 Factors for Development of Deformed Bars in Tension

Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Reinforcement grade, ψ_g	Grade 40 or Grade 60	1.0
	Grade 80	1.15
	Grade 100	1.3
Epoxy, ψ_e^*	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover $< 3d_b$ or clear spacing $< 6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size, ψ_s	#7 and larger bars	1.0
	#6 and smaller bars	0.8

(table continued on next page)

Table 7.24 Factors for Development of Deformed Bars in Tension (cont.)

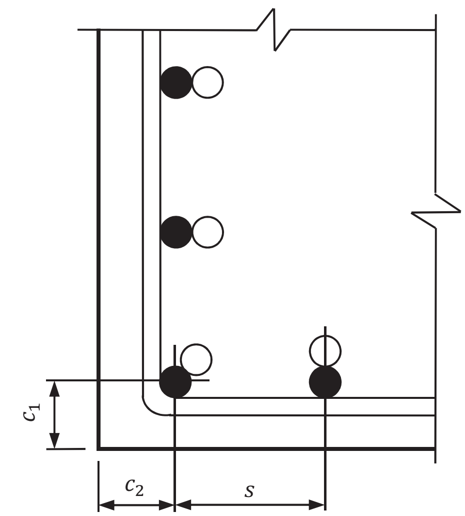
Factor	Condition	Value of Factor
Casting position, ψ_t^*	More than 12 in. of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0
Spacing or cover dimension, c_b	All	Lesser of: 1. The distance from the center of a longitudinal bar to the nearest concrete surface. 2. One-half the center-to-center spacing of the longitudinal bars being developed (see Figure 7.31).
Transverse reinforcement index, K_{tr}	All	$K_{tr} = \frac{40A_{tr}}{sn}$ ** where: 1. A_{tr} = total cross-sectional area of all transverse reinforcement within a spacing s that crosses the potential plane of splitting through the longitudinal reinforcement being developed 2. s = center-to-center spacing of the transverse reinforcement 3. n = number of bars being developed along the plane of splitting

**The product $\psi_t \psi_e$ need not exceed 1.7.*

***It is permitted to conservatively use $K_{tr} = 0$ if transverse reinforcement is present or required. For reinforcing bars with $f_y \geq 80{,}000$ psi spaced closer than 6.0 in. on center, transverse reinforcement must be provided along the development length such that $K_{tr} \geq 0.5d_b$ (ACI 25.4.2.2).*

Method 2 – ACI 25.4.2.3

The method given in ACI 25.4.2.3 to determine ℓ_d is based on the requirements given in ACI 25.4.2.4 and preselected values of the confining term $(c_b + K_{tr})/d_b$. Two sets of spacing and cover cases are given in ACI Table 25.4.2.3 (see Table 7.25). The modifications factors in these equations are obtained from ACI Table 25.4.2.5 (see Table 7.24).



$$c_b = \text{lesser of} \begin{cases} c_1 \\ c_2 \\ s/2 \end{cases}$$

Figure 7.31 Spacing or cover dimension, c_b.

Table 7.25 Tension Development Length, ℓ_d, for Deformed Bars in Accordance with ACI 25.4.2.3

<table>
<tr><th colspan="2">Spacing and Cover</th><th>#6 and Smaller Bars</th><th>#7 and Larger Bars</th></tr>
<tr><td rowspan="2">Case 1</td><td>Condition 1
1. Clear spacing of bars being developed or lap spliced $\geq d_b$
2. Clear cover $\geq d_b$
3. Stirrups or ties throughout ℓ_d not less than the required minimum</td><td rowspan="2">$\left(\dfrac{f_y \psi_t \psi_e \psi_g}{25\lambda\sqrt{f_c'}}\right) d_b$</td><td rowspan="2">$\left(\dfrac{f_y \psi_t \psi_e \psi_g}{20\lambda\sqrt{f_c'}}\right) d_b$</td></tr>
<tr><td>Condition 2
1. Clear spacing of bars being developed or lap spliced $\geq 2d_b$
2. Clear cover $\geq d_b$</td></tr>
<tr><td>Case 2</td><td>Other conditions</td><td>$\left(\dfrac{3f_y \psi_t \psi_e \psi_g}{50\lambda\sqrt{f_c'}}\right) d_b$</td><td>$\left(\dfrac{3f_y \psi_t \psi_e \psi_g}{40\lambda\sqrt{f_c'}}\right) d_b$</td></tr>
</table>

End-Bearing Splices

End-bearing splices, which are also referred to as compression-only mechanical splices, are permitted to be used to splice the longitudinal bars in a column where the force in all the bars is compressive for all load combinations (ACI 10.7.5.3). Compression forces are transmitted from one longitudinal bar to another by bearing of square-cut ends held in concentric contact by a suitable proprietary device. Tolerances for the bar ends are given in ACI 25.5.6.3.

End-bearing splices are permitted to be used only in columns where closed ties, closed stirrups, spirals, or hoops are provided (ACI 25.5.6.2). This ensures minimum shear resistance is provided in the member.

According to ACI 10.7.5.3.1, end-bearing splices must have a tensile strength of at least 25 percent of f_y of the continuing longitudinal reinforcement area on each face of a column. This is typically achieved by either staggering the location of the end-bearing splices or adding additional longitudinal reinforcement through the splice locations.

Additional information and various types of end-bearing splices can be found in Reference 9.

Mechanical and Welded Splices

Mechanical or welded splices are permitted in columns and must meet the requirements of ACI 25.5.7. These types of splices may be used in both tension and compression.

A variety of proprietary mechanical devices are available that can be used to splice longitudinal reinforcing bars in a column (see Reference 9). These types of splices are commonly specified at locations where inordinately long lap splices would be required or where lap splices would cause congestion. Mechanical splices must be able to develop in tension or compression at least 125 percent of the yield strength of the longitudinal bars.

Welded splices must also be able to develop $1.25f_y$ of the longitudinal bars. These types of splices are primarily intended for #6 bars and larger. Welding of reinforcing bars must conform to the requirements of ACI 26.6.4.

Mechanical or welded splices in columns need not be staggered (ACI 25.5.7.3).

7.8.6 Transverse Reinforcement

Overview

The main purposes of transverse reinforcement in columns are to provide lateral support of the longitudinal reinforcement and to provide shear resistance where required. The maximum spacing requirements in ACI 25.7.2 for ties and in ACI 25.7.3 for spirals are meant to achieve these purposes. Also included in these sections are minimum spacing requirements, which help to ensure concrete can easily flow between the transverse reinforcing bars.

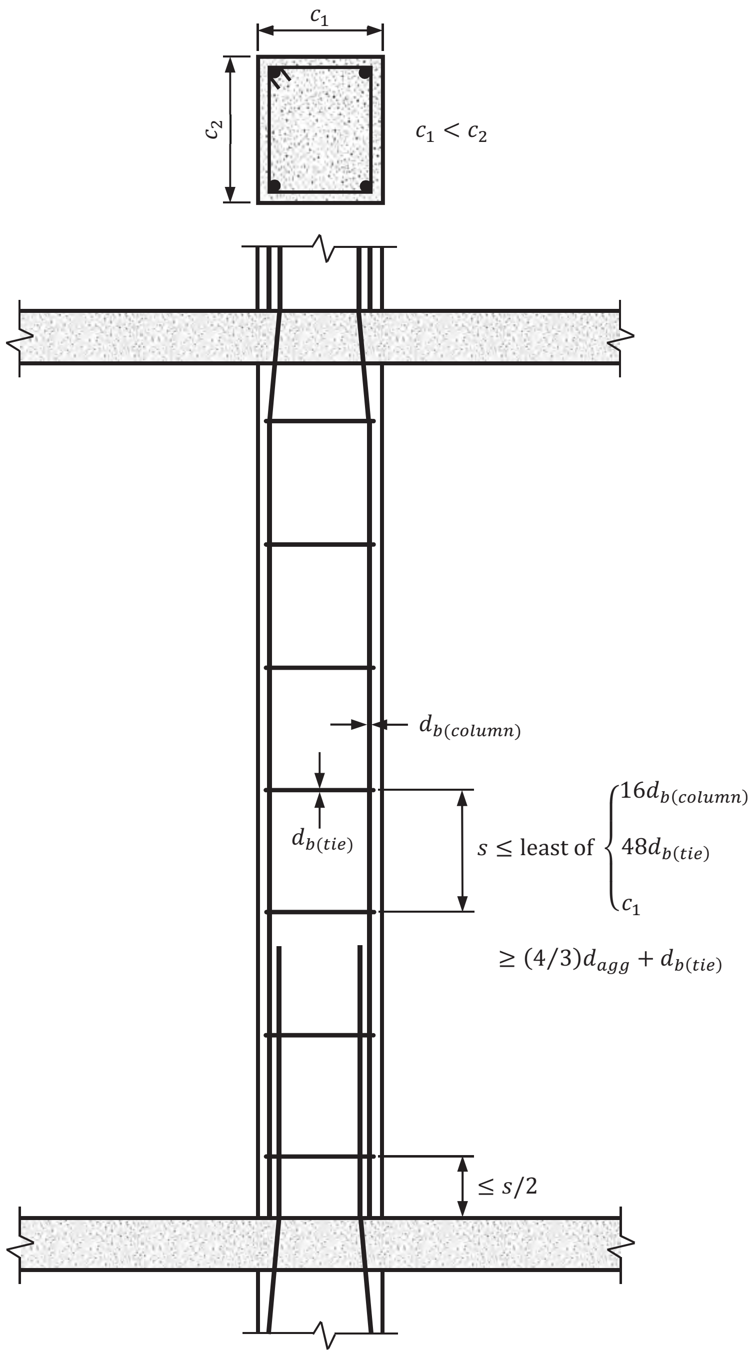
The types of transverse reinforcement typically used in columns are (1) ties (rectilinear or circular), (2) spirals, or (3) hoops. Requirements for size and spacing of ties and spirals are covered below.

Tie Reinforcement

Requirements for columns with tie reinforcement are given in ACI 10.7.6 and 25.7.2. Ties consist of a closed loop of deformed reinforcing bars [see Figure 7.9(a) for rectilinear ties]. Minimum inside bend diameters and standard hook geometry for ties are given in ACI Table 25.3.2. Information for 180-degree hooks is included in that table, but such hooks are not recommended mainly due to the difficulties associated with placing them in the field.

The requirements for minimum and maximum tie spacing (ACI 25.7.2.1) and minimum tie bar size based on the size of the longitudinal bars in the column (ACI 25.7.2.2) are given in Figure 7.32. Tabulated values of the maximum center-to-center tie spacing are also provided in Figure 7.32 for various column longitudinal bar sizes; the smallest cross-sectional dimension of the column must be compared to the appropriate tabulated value and the lesser of the two is to be used for the tie spacing.

The provisions in ACI 25.7.2.3 pertain to rectilinear tie configurations and the maximum clear spacing permitted between laterally supported longitudinal bars in a column. Corner bars and every other longitudinal bar in the sec-



Tie Bar	Column Long. Bar	Max. s (in.)*
#3	#5	10
	#6	12
	#7	14
	#8	16
	#9	18
	#10	18
#4	#11	22
	#14	24
	#18	24

*Max. $s \leq c_1$ must also be considered

Figure 7.32 Tie requirements for reinforced concrete columns.

tion must have lateral support provided by the corner of a tie that has a hook of not more than 135 degrees where the clear spacing between the longitudinal bars on each side is less than or equal to 6 in. This is illustrated in Figure 7.33(a) for the case of a square, tied column with 16 longitudinal bars. Because the clear spacing between the bars is less than 6 in., crossties with 90-degree and 135-degree hooks on each end are provided around every other longitudinal bar. The clear space between longitudinal bars is greater than 6 in. for the column in Figure 7.33(b). In this case, crossties are required around every longitudinal bar in accordance with ACI 25.7.2.3(b) [see ACI Figure R25.7.2.3a].

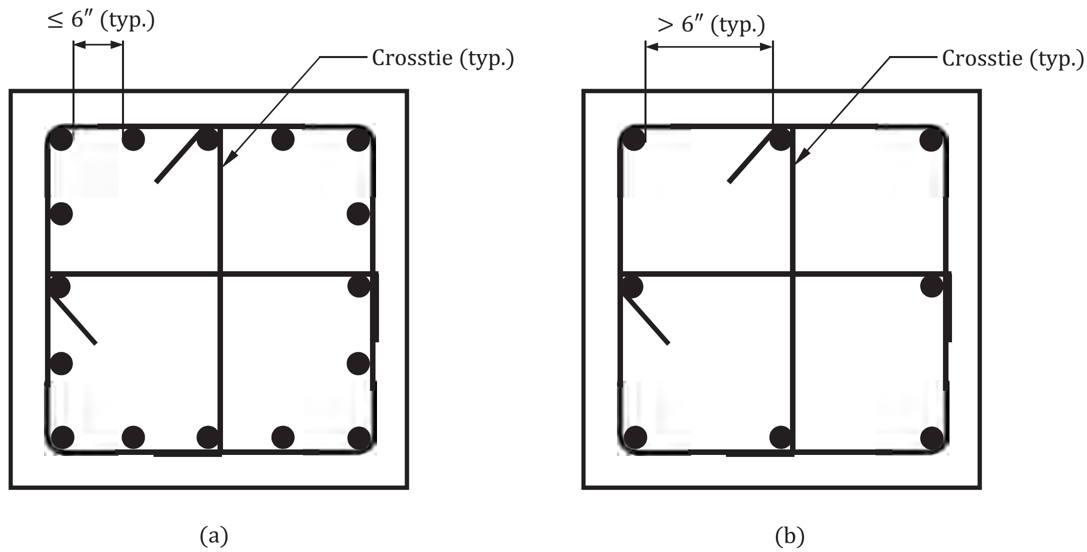


Figure 7.33 Lateral support requirements for longitudinal bars in tied columns.

Additional requirements for lateral support of longitudinal bars with ties are given in ACI 10.7.6.2. These requirements are given in Figure 7.34.

Circular ties may be used in cases where the longitudinal bars are located around the perimeter of a circle, like in circular or square columns (ACI 25.7.2.4). Anchorage requirements for circular ties are given in ACI 25.7.2.4.1 (see ACI Figure R25.7.2.4.1).

Spiral Reinforcement

Requirements for columns with spiral reinforcement are given in ACI 10.7.6.3 and 25.7.3. Standard spiral sizes are #3 to #5, and the clear spacing between consecutive turns on a spiral must not exceed 3 in. or be less than the greater of 1 in. or (4/3) times the diameter of the largest aggregate size, d_{agg}, used in the concrete mix.

The minimum volumetric spiral reinforcement ratio, ρ_s, is given by ACI Equation (25.7.3.3):

$$\rho_s \geq 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}} \tag{7.37}$$

In this equation, A_{ch} is the area of the column core enclosed by the spiral, which is equal to $\pi D_{ch}^2 / 4$ where D_{ch} is the diameter of the column core measured to the outside edges of the spiral reinforcement (see Figure 7.35).

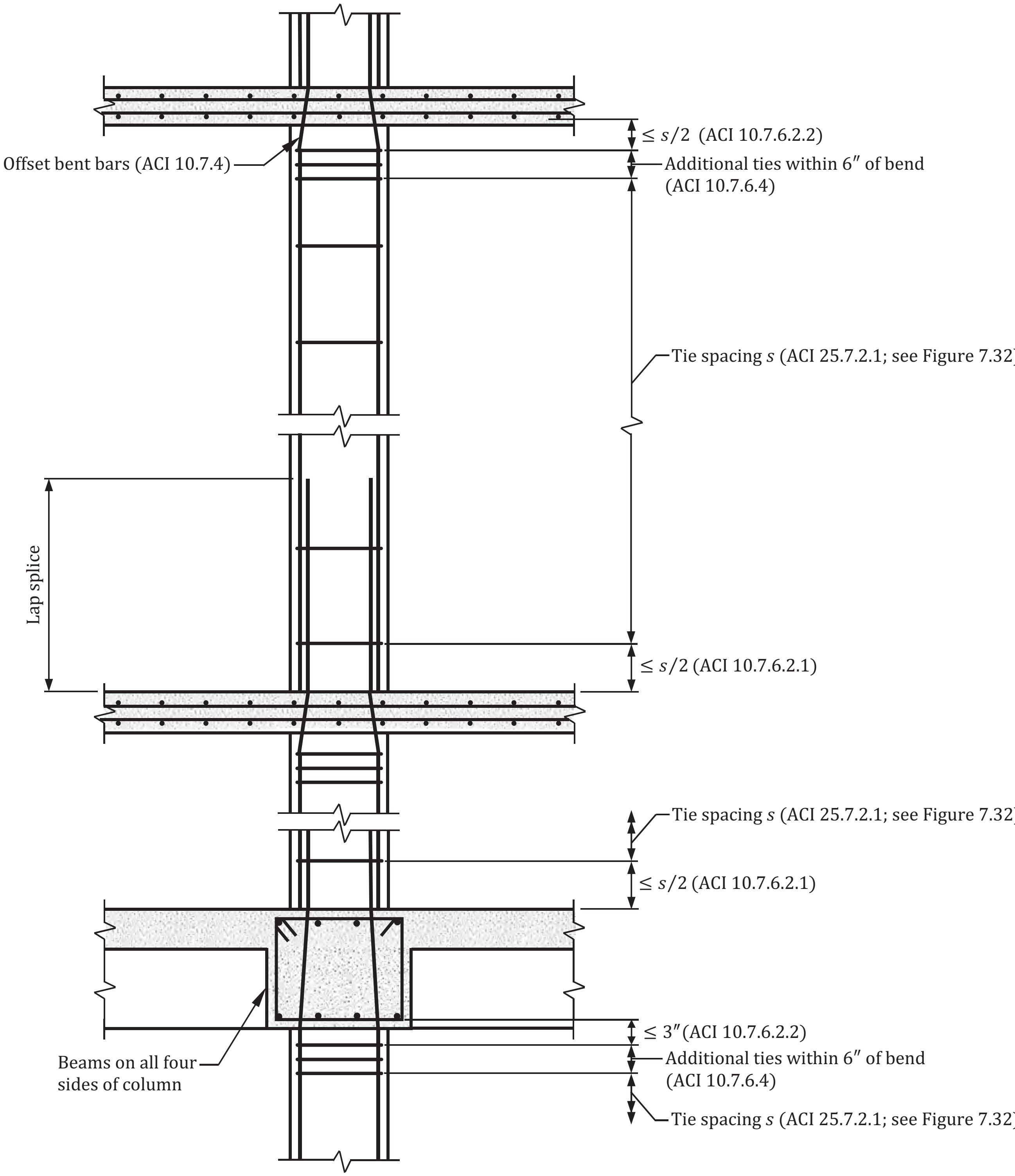


Figure 7.34 Tie and splice details for reinforced concrete columns.

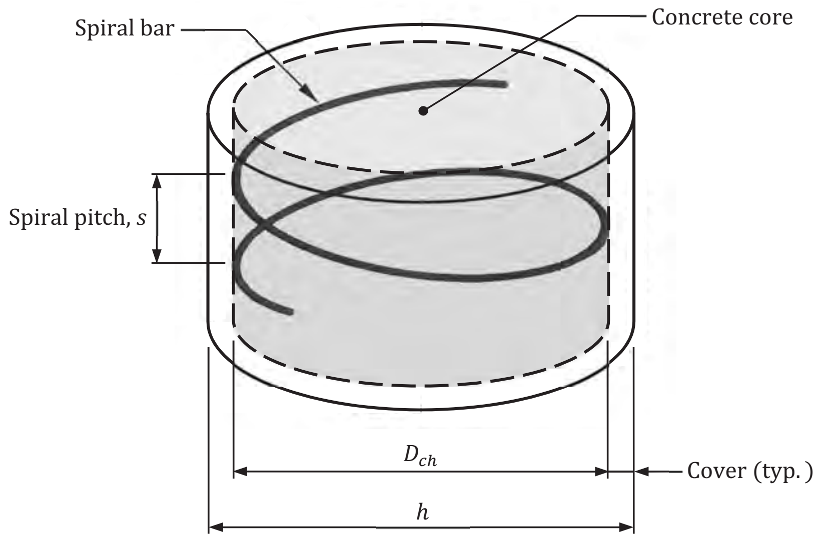


Figure 7.35 Spiral reinforcement.

The volumetric spiral reinforcement ratio, ρ_s, is equal to the following:

$$\rho_s = \frac{\text{Volume of spiral reinforcement}}{\text{Volume of concrete core}} = \frac{A_{bs}\pi D_{ch}}{(\pi D_{ch}^2 / 4)s} = \frac{4A_{bs}}{D_{ch}s} \tag{7.38}$$

where A_{bs} is the area of the spiral reinforcing bar and s is the center-to-center spacing of the spirals (which is commonly referred to as the spiral pitch).

An equation for s can be obtained by substituting Equation (7.38) into Equation (7.37):

$$s = \frac{8.9A_{bs}}{D_{ch}\left(\frac{A_g}{A_{ch}} - 1\right)\left(\frac{f'_c}{f_{yt}}\right)} \tag{7.39}$$

The limitations in ACI 25.7.3.1 must also be considered when determining s.

Recommended standard spirals for circular columns with various reinforcement yield strengths and concrete compressive strengths are given in Table 7.26.

Table 7.26 Recommended Standard Spirals for Circular Columns

f_{yt} (psi)	f'_c (psi)	Column Size (in.)	Spiral Size and Pitch
60,000	4,000	12 – 24	#3 @ 2″
		26 – 48	#3 @ 2-1/4″
	7,000	12 – 16	#4 @ 2″
		18 – 48	#4 @ 2-1/4″
	10,000	20 – 48	#5 @ 2-1/2″
	14,000	36 – 48	#5 @ 1-3/4″

(table continued on next page)

Table 7.26 Recommended Standard Spirals for Circular Columns (cont.)

f_{yt} (psi)	f'_c (psi)	Column Size (in.)	Spiral Size and Pitch
80,000	4,000	12 – 20	#3 @ 2-3/4″
		22 – 48	#3 @ 3″
	7,000	12, 14	#4 @ 2-1/2″
		16 – 38	#4 @ 3″
		40 – 48	#4 @ 3-1/4″
	10,000	20 – 34	#5 @ 3-1/4″
		36 – 48	#5 @ 3-1/2″
	14,000	36 – 48	#5 @ 2-1/2″
100,000	4,000	12 – 48	#3 @ 3-1/4″
	7,000	12 – 48	#4 @ 3-1/2″
	10,000	20 – 48	#5 @ 3-1/2″
	14,000	36 – 48	#5 @ 3″

Spirals must be anchored at each end by providing one and one-half extra turns of the spiral bar (ACI 25.7.3.4; see ACI Figure R25.7.3.4).

Requirements for spiral splices are given in ACI 25.7.3.5. Mechanical or welded splices in accordance with ACI 25.5.7 are permitted, as are lap splices in accordance with ACI 25.7.3.6 for $f_{yt} \leq 60{,}000$ psi. Lap splice lengths must be the greater of 12 in. or the lap length determined by ACI Table 25.7.3.6, which is equal to a multiple of the diameter of the spiral bar. Lap splice lengths are given in ACI Table 25.7.3.6 for uncoated bars without hooks and for coated bars with and without hooks and are based on f_{yt} equal to 60,000 psi regardless of the actual yield strength of the bar [see ACI 25.7.3.5(b)]:

- Uncoated or zinc-coated galvanized deformed bars without hooks: Lap length $= 48d_b \geq 12$ in.
- Epoxy-coated or zinc and epoxy dual coated deformed bars without hooks: Lap length $= 72d_b \geq 12$ in.
- Epoxy-coated or zinc and epoxy dual coated deformed bars with a standard hook conforming to ACI 25.3.2: Lap length $= 48d_b \geq 12$ in.

Requirements for lateral support of longitudinal bars using spirals at the top and bottom of a story are given in ACI 10.7.6.3 and are summarized in Figure 7.36.

7.9 Connections to Foundations

7.9.1 Overview

Vertical and horizontal forces must be transferred at the interface between a column and a foundation (ACI 16.3). Vertical compression forces are transferred by bearing on the concrete or by a combination of bearing and interface reinforcement. Tension forces must be transferred entirely by reinforcement, which may consist of extended longitudinal bars, dowels, anchor bolts, or mechanical connectors. Horizontal forces are transferred using the shear-friction provisions of ACI 22.9 or other appropriate methods.

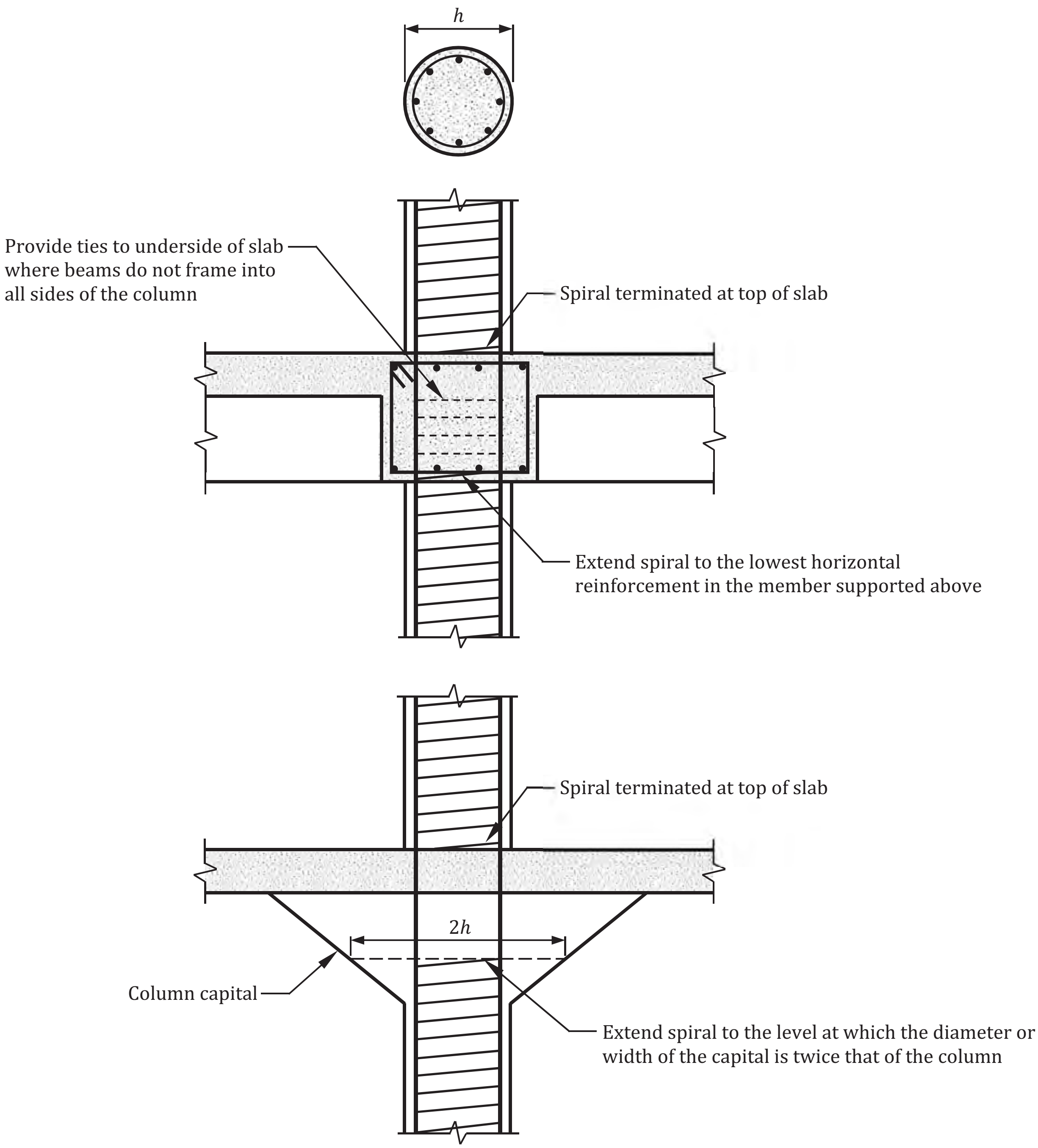


Figure 7.36 Lateral support of longitudinal bars using spirals.

7.9.2 Vertical Transfer

Compression

Where vertical compression forces are transferred to a foundation, bearing strength requirements of ACI 22.8 must be satisfied for both the column and the foundation (ACI 16.3.3.4). For bearing on a column, the factored bearing force, B_u, must be less than or equal to the design bearing strength, ϕB_n, where the nominal bearing strength, B_n, is given in ACI Table 22.8.3.2:

$$B_u \leq \phi B_n = \phi 0.85 f_c' A_1 \tag{7.40}$$

In this equation, ϕ is equal to 0.65 for bearing (ACI Table 21.2.1) and A_1 is the gross area of the column.

For bearing on a foundation wider on all sides than the loaded area, the following equation must be satisfied:

$$B_u \leq \phi B_n = (\phi 0.85 f_c' A_1)\sqrt{A_2 / A_1} \leq 2(\phi 0.85 f_c' A_1) \tag{7.41}$$

The term A_2 is the area of the lower base of the largest frustrum of a pyramid, cone, or tapered wedge contained wholly within the foundation and having for its upper base the loaded area A_1 and having side slopes of 1 vertical to 2 horizontal. Areas A_1 and A_2 are given in Figure 7.37 for the case of a column supported by a spread footing.

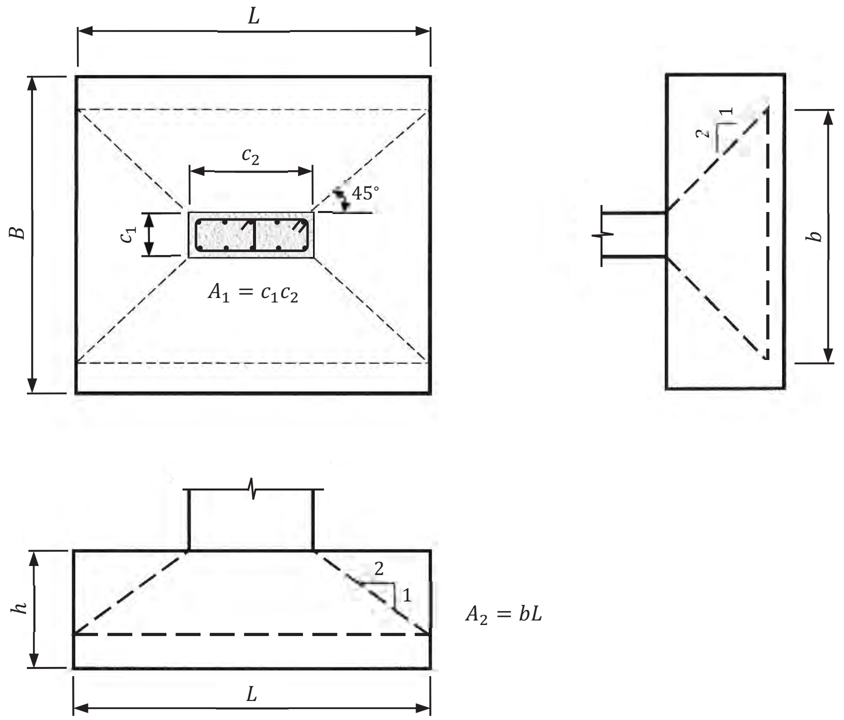


Figure 7.37 Determination of areas for bearing strength.

Where $B_u > \phi B_n$, the excess compression stress from the column must be transferred by reinforcement to the foundation. The required area of interface reinforcement, A_s, is determined by the following equation:

$$A_s = \frac{B_u - \phi B_n}{\phi f_y} \geq A_{s,min} = 0.005 A_g \tag{7.42}$$

In this equation, A_g is the gross area of the column. The minimum amount of interface reinforcement, $A_{s,min}$, must be provided even where $B_u \leq \phi B_n$ (ACI 16.3.4.1).

Dowel bars emanating from the foundation are the type of interface reinforcement commonly used because of ease of construction. The dowel bars are set in the foundation prior to casting the foundation concrete and are subsequently spliced to the column longitudinal bars. Dowels should be positioned so as not to interfere with the longitudinal bars or the tie hooks in the column.

Illustrated in Figure 7.38 are dowel bars across the interface between a reinforced concrete column and spread footing where all the longitudinal bars in the column are in compression for all factored load combinations (Zone 1 in Figure 7.28). The dowel bars must extend into the footing at least a compression development length, ℓ_{dc}, determined in accordance with ACI 25.4.9.2 (see Section 7.8.5 of this publication). It is not required to provide dowel bars for all the longitudinal bars in such cases.

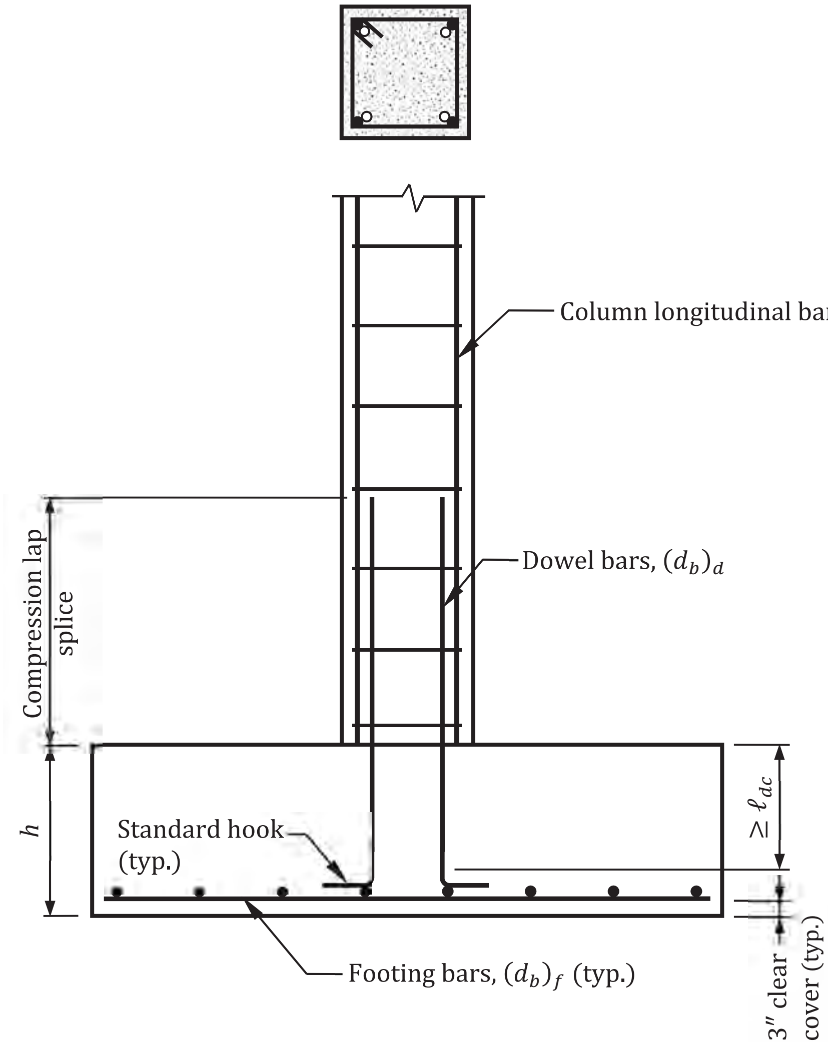


Figure 7.38 Dowel bars at the interface of a reinforced concrete column and footing where all the longitudinal bars in the column are in compression.

Standard hooks are typically provided at the ends of the dowel bars, which are tied to the reinforcing bars in the footing; this detail is far more economical than terminating the dowel bars above the footing reinforcing bars. The hooked portion of the dowels cannot be considered effective for developing the dowels bars in compression (ACI 25.4.1.2). The following equation must be satisfied to ensure that the dowel bars are adequately developed into the footing:

$$h \geq \ell_{dc} + r + (d_b)_d + 2(d_b)_f + \text{cover} \tag{7.43}$$

In this equation, r is the radius of the dowel bar bend (see ACI Table 25.3.1, which contains minimum inside bend diameters for bars with standard 90-degree hooks). Where Equation (7.43) is not satisfied, either a greater number of smaller dowel bars can be used (if possible) or the depth of the footing must be increased.

The information above pertaining to the development of dowel bars in spread footings is also applicable to the development of dowel bars in pile caps. For columns supported on drilled piers or caissons, the dowel bars in the drilled pier or caisson usually do not have hooks at the ends.

The dowel bars must also be fully developed in the column; this is typically achieved by lap splicing the dowel bars to the longitudinal bars in the column (see Figure 7.38). Where the dowel bars are the same size as the column bars, the minimum compression lap splice length is used (see Table 7.21). Where the dowel bars are different in size than the column bars, the compression lap splice length must satisfy the requirements of ACI 25.5.5.4. Lap splices of #14 and #18 longitudinal bars in compression for all factored load combinations with #11 and smaller dowel bars are permitted provided the requirements of ACI 25.5.5.3 are satisfied (ACI 16.3.5.4).

Tension

Tension forces transferred from a column to a foundation must be resisted entirely by reinforcement across the interface (ACI 16.3.1.2(b) and 16.3.5.2), and dowel bars must be provided for all the longitudinal bars in the column. In such cases, factored load combinations fall within Zones 2 or 3 of the design strength interaction diagram (see Figure 7.28).

Tensile anchorage of the dowel bars into a footing or pile cap is typically accomplished by providing 90-degree standard hooks at the ends of the dowel bars with the development length of the hooked bar, ℓ_{dh}, determined in accordance with ACI 25.4.3 (see Figure 7.39):

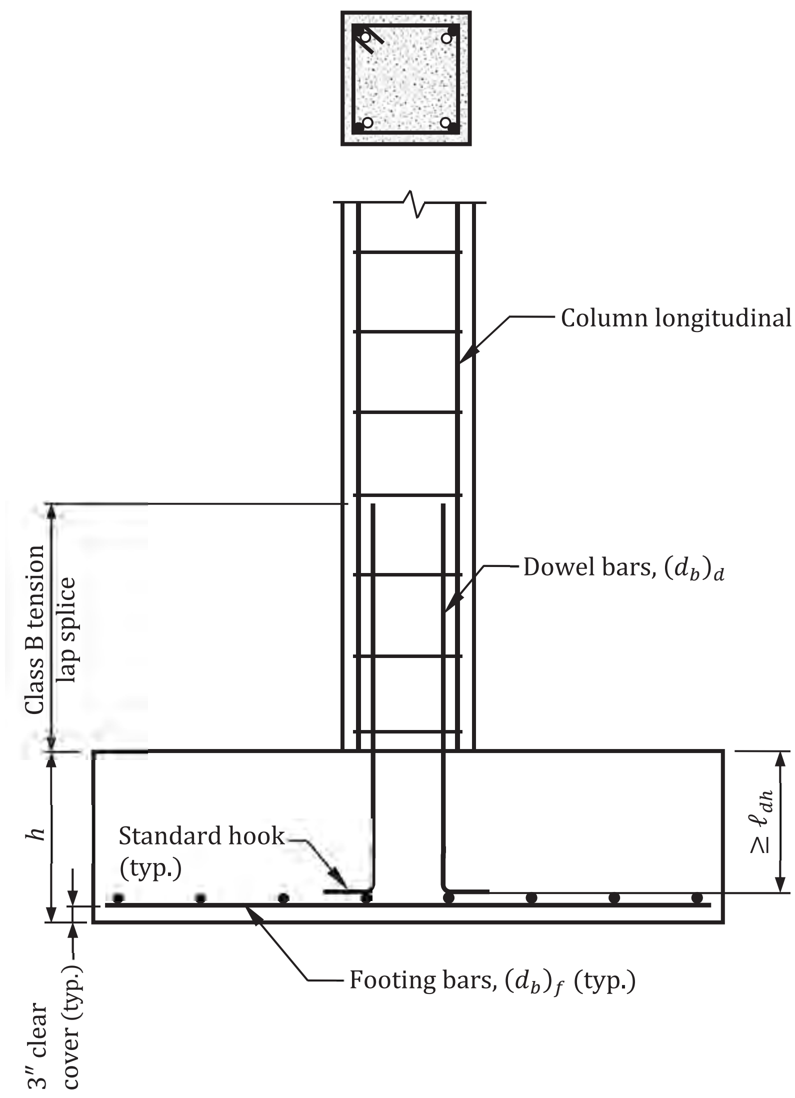


Figure 7.39 Dowel bars at the interface of a reinforced concrete column and footing where the longitudinal bars in the column are in compression and tension.

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f'_c}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{7.44}$$

This development length is measured from the critical section to the outside face of the hook (see Figure 4.10 of this publication). The modification factors in Equation (7.44) are given in ACI Table 25.4.3.2 (see Table 7.27).

Table 7.27 Modification Factors for Development of Hooked Bars in Tension

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Confining reinforcement, ψ_r	For #11 and smaller bars with $A_{th} \geq 0.4A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For #11 and smaller hooked bars 1. terminating inside a column core with side cover normal to the plane of the hook ≥ 2.5 in. or 2. with side cover normal to the plane of the hook $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c / 15{,}000) + 0.6$
	$f'_c \geq 6{,}000$ psi	1.0

The confining reinforcement factor, Ψ_r, is typically equal to 1.6 for hooked dowel bars in footings because confining reinforcement is usually not provided in such cases.

The following equation must be satisfied to ensure the dowel bars are adequately developed into the footing (considering the hooked portion of the dowel bars can be developed in tension):

$$h \geq \ell_{dh} + 2(d_b)_f + \text{cover} \tag{7.45}$$

Where Equation (7.45) is not satisfied, the depth of the footing usually must be increased.

For development of the dowel bars into the column, a tension lap splice or a mechanical connection in accordance with ACI 25.5.2 or 25.5.7, respectively, must be provided between the dowel bars and the longitudinal bars. In the case of lap splices, a Class B tension lap splice is required (see Table 7.21 and Figure 7.39).

7.9.3 Horizontal Transfer

The shear-friction method of ACI 22.9 is permitted to be used to determine the nominal shear strength, V_n, at the contact surface between the column and the foundation (ACI 16.3.3.5). The required area of reinforcement A_{vf} across the interface between the column and the foundation is determined by the following equation, which is applicable to shear-friction reinforcement perpendicular to the interface (ACI 22.9.4.2):

$$A_{vf} \geq \frac{V_u}{\phi f_y \mu} \tag{7.46}$$

In this equation, V_u is the factored shear force due to the lateral force effects at the interface, the strength reduction factor ϕ is equal to 0.75, and μ is the coefficient of friction, which is obtained from ACI Table 22.9.4.2 (see Table 7.28).

Table 7.28 Coefficient of Friction, μ

Contact Surface Condition	μ
Concrete placed monolithically	1.4λ
Concrete placed against hardened concrete that is clean, free of laitance, and intentionally roughened to a full amplitude of approximately ¼ in.	1.0λ
Concrete placed against hardened concrete that is clean, free of laitance, and not intentionally roughened	0.6λ

Upper limits on shear-friction strength are given in ACI Table 22.9.4.4 (see Table 7.29). The term A_c is the area of concrete resisting V_u. For example, where a column is supported by a footing, A_c is equal to the gross area of the column. Where the concrete strengths of the column and the foundation are different, the smaller of the two must be used in these equations (ACI 22.9.4.4).

Table 7.29 Maximum V_n Across the Assumed Shear Plane

Contact Surface Condition	Maximum $V_n = V_u / \phi$*
Normalweight concrete placed monolithically or placed against hardened concrete intentionally roughened to a full amplitude of approximately ¼ in.	Least of $\begin{cases} 0.2f'_c A_c \\ (480 + 0.08f'_c)A_c \\ 1{,}600A_c \end{cases}$
Other cases	Least of $\begin{cases} 0.2f'_c A_c \\ 800A_c \end{cases}$

*A_c = *area of concrete resisting* V_u

The required amount of shear friction reinforcement, A_{vf}, is permitted to be determined by Equation (7.46) based on a net force equal to $(V_u / \phi) - C_{net}$ where C_{net} is a permanent net compression force transmitted across the assumed shear plane (ACI 22.9.4.5). According to ACI 22.9.4.6, the area of reinforcement required to resist a net factored tension force across the assumed shear plane is to be added to the area of reinforcement required for shear friction that crosses the assumed shear plane. For columns transmitting a bending moment across the shear plane, the flexural compression and tension forces are in equilibrium and do not change the resultant compression force, $A_{vf} f_y$, acting across the shear plane or the shear-friction resistance. Thus, it is not necessary to provide additional reinforcement to resist the flexural tension stresses, unless the required flexural tension reinforcement exceeds the amount of shear-transfer reinforcement provided in the flexural tension zone.

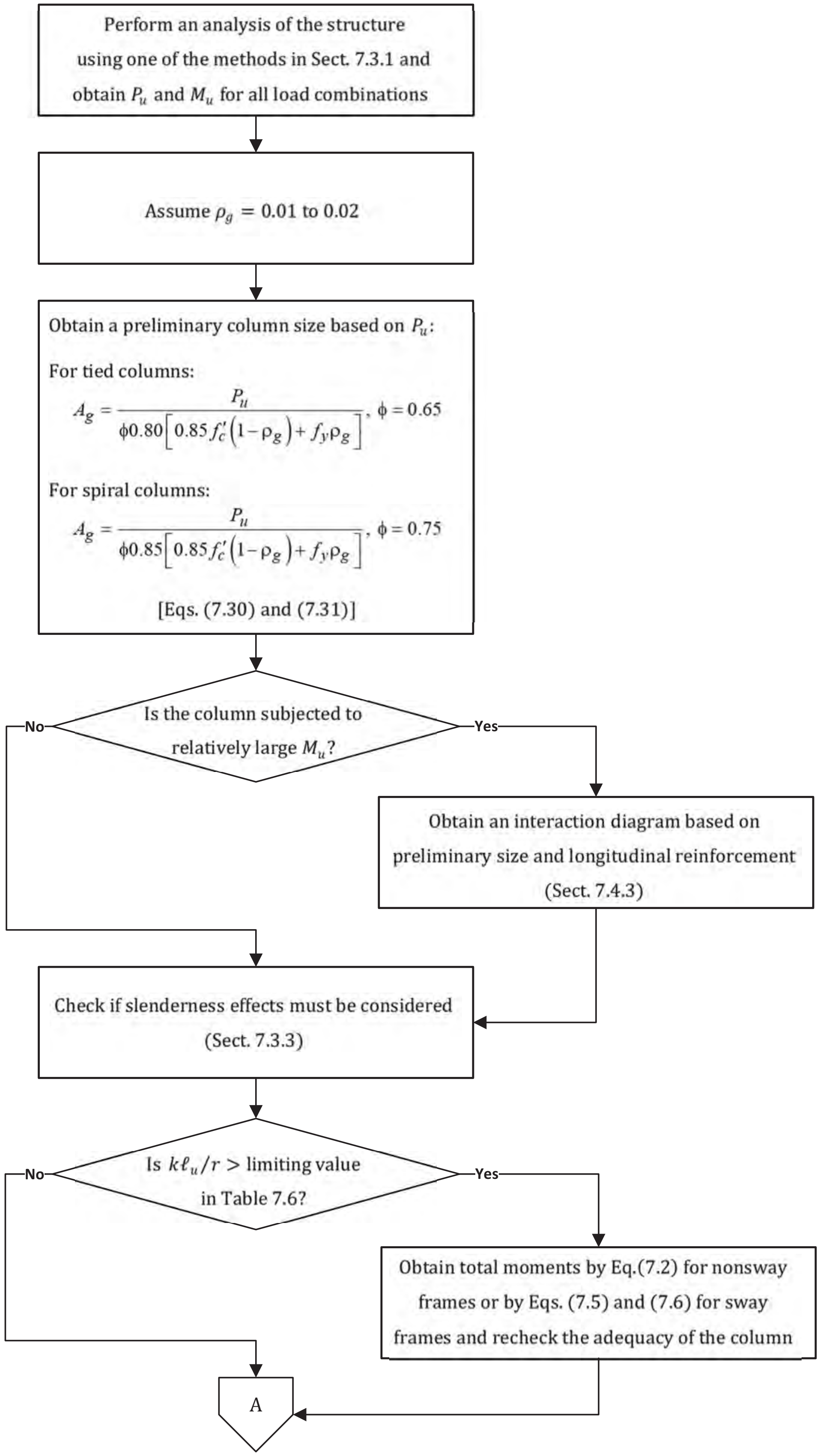


Figure 7.40 Design procedure for columns.

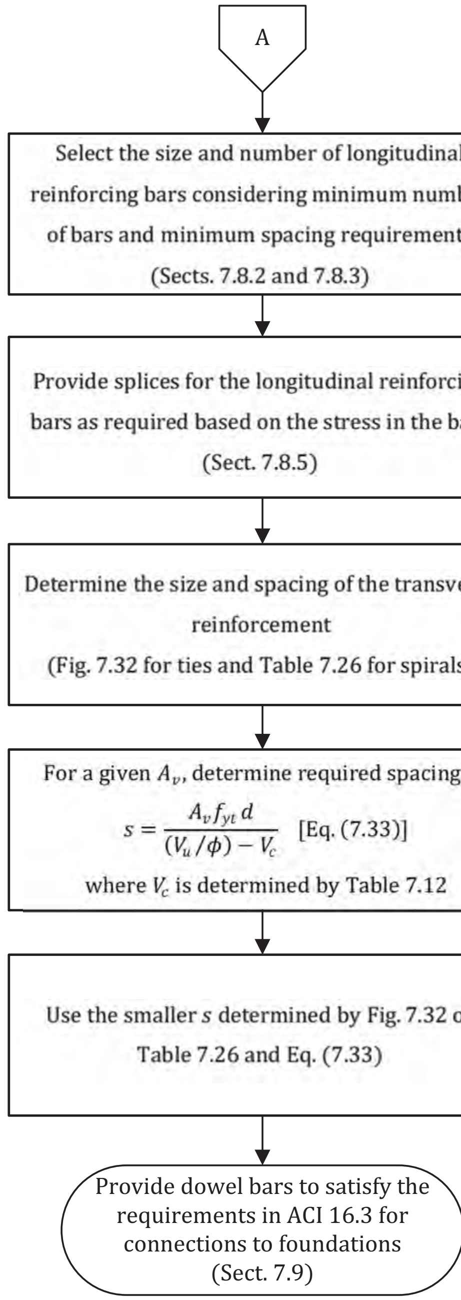


Figure 7.40 Design procedure for columns (cont.).

Full tension anchorage of the shear-friction reinforcement must be provided into the foundation and into the column. The lengths of these bars are determined in the same way as those for vertical transfer where tension forces are present.

The area of the dowel bars is usually determined initially based on the requirements for vertical transfer. That area is compared with the area required for horizontal transfer, and the larger of the two is provided at the interface.

7.10 Design Procedure

The design procedure in Figure 7.40 can be used in the design and detailing of reinforced concrete columns subjected to axial compression forces and uniaxial bending. Included in the figure are the section numbers, table numbers, and figure numbers where specific information in this chapter can be found.

7.11 Examples

7.11.1 Example 7.1 – Determination of Preliminary Column Size: Building #1 (Framing Option B), Rectangular, Tied Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine a preliminary size for column C3 in the first story of Building #1, Framing Option B assuming a rectangular tied column (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the factored axial forces Table 3.3

$$\text{Dead load of slab} = (8.5\,/\,12)\times 150.0 = 106.3\ \text{lb/ft}^2$$

Roof: $\text{Superimposed dead load} = 12\ \text{lb/ft}^2$

$\text{Live load} = 20\ \text{lb/ft}^2$

Floors: $\text{Superimposed dead load} = 10\ \text{lb/ft}^2$

$\text{Live load} = 15\ \text{lb/ft}^2\ \text{(nonreducible partitions)} + 50\ \text{lb/ft}^2$

$$\text{Tributary area to column} = 25.0\times 23.5 = 587.5\ \text{sq ft}$$

This column is not part of the LFRS, and the bending moments due to gravity load effects are negligible.

A summary of the axial forces for column C3 in the first story is given in Table 7.30.

Table 7.30 Summary of Axial Forces for Column C3

Load Case		Axial Force (kips)
Dead (D)		342.7
Roof live (L_r)		11.8
Live (L)		152.8
Load Combination		
ACI Eq. (5.3.1a)	$1.4D$	479.8
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$	661.6
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$	506.5

Step 2 – Determine the gross area of the column

Assuming a 1 percent reinforcement ratio for the longitudinal reinforcement $(\rho_g = 0.01)$, the required gross area of a tied column is the following using the largest P_u in Table 7.30:

$$A_g = \frac{P_u}{\phi 0.80[0.85 f_c'(1-\rho_g) + f_y\rho_g]} = \frac{661.6}{0.65\times 0.80\times\{[0.85\times 4\times(1-0.01)] + (60\times 0.01)\}} = 320.8\ \text{in.}^2 \quad \text{Eq. (7.30)}$$

With a 1 percent reinforcement ratio, an 18-in. square column is adequate $(A_g = 324.0\ \text{in.}^2)$.

For a 2 percent reinforcement ratio, the required A_g is equal to 280.7 in.^2, which corresponds to a 17-in. square column.

An 18-in. square column is selected.

Comments. It is shown in Example 5.2 that an 8.5-in.-thick slab is adequate for serviceability and shear strength requirements based on 24-in. square columns. The required slab thickness based on 18-in. columns is equal to $\ell_n / 33 = [(25.0 \times 12) - 18.0] / 33 = 8.6 \text{ in.}$ It can be shown that two-way shear requirements are not satisfied with 18-in. columns and an 8.5-in-thick slab. The 18-in. columns can be used, however, if shear reinforcement in the slab is provided.

7.11.2 Example 7.2 – Determination of Longitudinal Reinforcement: Building #1 (Framing Option B), Rectangular, Tied Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine the required longitudinal reinforcement for column C3 in the first story of Building #1, Framing Option B assuming an 18-in. square tied column (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f'_c = 4{,}000 \text{ psi}$, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 7.1.

Step 1 – Determine the required area of longitudinal reinforcement

Based on an 18-in. square tied column, the required reinforcement ratio, ρ_g, is equal to the following:

$$\rho_g = \frac{(P_u / \phi\phi_c A_g) - 0.85 f'_c}{f_y - 0.85 f'_c} = \frac{[661.6 / (0.65 \times 0.80 \times 18.0^2)] - (0.85 \times 4)}{60 - (0.85 \times 4)} = 0.01 < 0.08 \qquad \text{Eq. (7.32)}$$

$$A_{st} = 0.01 \times 18.0^2 = 3.24 \text{ in.}^2$$

Select 4-#9 bars $(A_{st,provided} = 4.00 \text{ in.}^2 > 3.24 \text{ in.}^2;\ \rho_g = 0.012)$.

Step 2 – Check the minimum number of longitudinal bars and the minimum face dimension of the column

For rectangular, tied columns:

Minimum number of longitudinal bars with rectangular ties $= 4$ Table 7.16

Minimum face dimension $= 10 \text{ in.}$ for 2-#9 bars per face and normal lap splices $< 18 \text{ in.}$ Table 7.17

Use 4-#9 bars.

7.11.3 Example 7.3 – Determination of Transverse Reinforcement: Building #1 (Framing Option B), Rectangular, Tied Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine the required transverse reinforcement for column C3 in the first story of Building #1, Framing Option B assuming an 18-in. square tied column (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f'_c = 4{,}000 \text{ psi (maximum aggregate size} = 0.75 \text{ in.)}$, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.1 and 7.2.

Step 1 – Determine the required tie bar size ACI 25.7.2.2

Because the shear force due to the factored gravity loads is negligible, the required tie bar size is determined using ACI 25.7.2.2.

For #9 longitudinal bars, use #3 ties.

Step 2 – Determine the required tie bar spacing ACI 25.7.2.1

Because the shear force due to the factored gravity loads is negligible, the required tie bar spacing is determined using ACI 25.7.2.1.

$$s \le \text{ lesser of} \begin{cases} 16d_{b(column)} = 16 \times 1.128 = 18.1 \text{ in.} \\ 48d_{b(tie)} = 48 \times 0.375 = 18.0 \text{ in.} \\ c_1 = 18.0 \text{ in.} \end{cases}$$

$$\ge (4/3)d_{agg} + d_{b(tie)} = [(4/3) \times 0.75] + 0.375 = 1.375 \text{ in.}$$

Use #3 ties spaced at 18 in. on center; this matches the value in Figure 7.32.

7.11.4 Example 7.4 – Determination of Dowel Reinforcement at the Foundation: Building #1 (Framing Option B), Rectangular, Tied Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine the required dowel reinforcement for column C3 in the first story of Building #1, Framing Option B assuming an 18-in. square tied column supported by a $2'\text{-}0'' \times 11'\text{-}6'' \times 11'\text{-}6''$ spread footing (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f_c' = 4{,}000$ psi (maximum aggregate size $= 0.75$ in.), and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.1, 7.2, and 7.3.

Step 1 – Check the bearing loads on the column and footing ACI 16.3.3.4

- Bearing strength of the column:

  $B_u = 661.6 \text{ kips} < \phi B_n = \phi 0.85 f_c' A_1 = 0.65 \times 0.85 \times 4 \times 18.0^2 = 716.0 \text{ kips}$ Eq. (7.40)

- Bearing strength of the footing:

  $\phi B_n = \phi 0.85 f_c' A_1 \sqrt{A_2 / A_1} \le 2\phi 0.85 f_c' A_1$ Eq. (7.41)

  Footing thickness $h = 24.0$ in.

  Horizontal projection for a 2:1 slope $= 2 \times 24.0 = 48.0$ in. Figure 7.37

  Projected length $b = 48.0 + 18.0 + 48.0 = 114.0 \text{ in.} < 11.5 \times 12 = 138.0 \text{ in.}$

  Therefore, $A_2 = b^2 = 114.0^2 = 12{,}996.0 \text{ in.}^2$

  $\sqrt{A_2 / A_1} = \sqrt{12{,}996.0 / 18.0^2} = 6.3 > 2.0;$ use 2.0

  $B_u = 661.6 \text{ kips} < \phi B_n = 2 \times 716.0 = 1{,}432.0 \text{ kips}$

Step 2 – Determine the required interface reinforcement ACI 16.3.4.1

Because the design bearing strengths are adequate for the column and footing, provide minimum area of reinforcement across the interface:

$A_{s,min} = 0.005A_g = 0.005 \times 18.0^2 = 1.62 \text{ in.}^2$

Provide 4-#6 dowel bars ($A_{s,provided} = 1.76 \text{ in.}^2 > 1.62 \text{ in.}^2$).

Step 3 – Check the development of the dowel bars in the footing

Determine the compression development length, ℓ_{dc}, of the #6 dowel bars:

$$\ell_{dc} = \text{ larger of } \begin{cases} (f_y \psi_r \,/\, 50\lambda\sqrt{f'_c})d_b \\ 0.0003 f_y \psi_r d_b \\ 8 \text{ in.} \end{cases}$$

Table 7.21, Note 5

$\psi_r = 1.0$ Table 7.22

$\lambda = 1.0$ for normalweight concrete Table 7.13

Therefore,

$$\ell_{dc} = \text{ larger of } \begin{cases} [(60{,}000 \times 1.0) \,/\, (50 \times 1.0 \times \sqrt{4{,}000})] \times 0.75 = 14.2 \text{ in.} \\ 0.0003 \times 60{,}000 \times 1.0 \times 0.75 = 13.5 \text{ in.} \\ 8 \text{ in.} \end{cases}$$

Assuming two layers of #9 bars in the footing with a clear cover of 3 in., determine the minimum footing thickness for the development of the dowel bars:

$$\text{Minimum } h = \ell_{dc} + r + (d_b)_d + 2(d_b)_f + \text{cover}$$

Eq. (7.43)

$$= 14.2 + (6 \times 0.75 \,/\, 2) + 0.75 + (2 \times 1.128) + 3.0 = 22.5 \text{ in.}$$

Because the minimum $h = 22.5$ in. is less than the provided footing thickness of 24.0 in., the #6 hooked dowel bars can be fully developed in the footing for compression.

Step 4 – Determine the development length of the dowel bars in the column

The dowel bars are lap spliced to the longitudinal reinforcement in the column. Because the dowel bars are smaller in diameter than the longitudinal bars, the compression lap splice length, ℓ_{sc}, must be greater than or equal to the larger of the following:

1. Development length in compression, ℓ_{dc}, of the #9 longitudinal bars:

$$\ell_{dc} = \text{ larger of } \begin{cases} (f_y \psi_r \,/\, 50\lambda\sqrt{f'_c})d_b = [(60{,}000 \times 1.0) \,/\, (50 \times 1.0 \times \sqrt{4{,}000})] \times 1.128 = 21.4 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 1.128 = 20.3 \text{ in.} \\ 8.0 \text{ in.} \end{cases}$$

2. Compression lap splice length, ℓ_{sc}, of the #6 dowel bars:

$$\ell_{sc} = \text{ greater of } \begin{cases} 0.0005 f_y d_b = 0.0005 \times 60{,}000 \times 0.75 = 22.5 \text{ in.} \\ \\ 12.0 \text{ in.} \end{cases}$$ Table 7.21

Provide a lap splice length of 2 ft-0 in.

Negligible horizonal forces are transferred from the column to the footing so horizontal force transfer is not investigated.

Reinforcement details for this column are given in Figure 7.41.

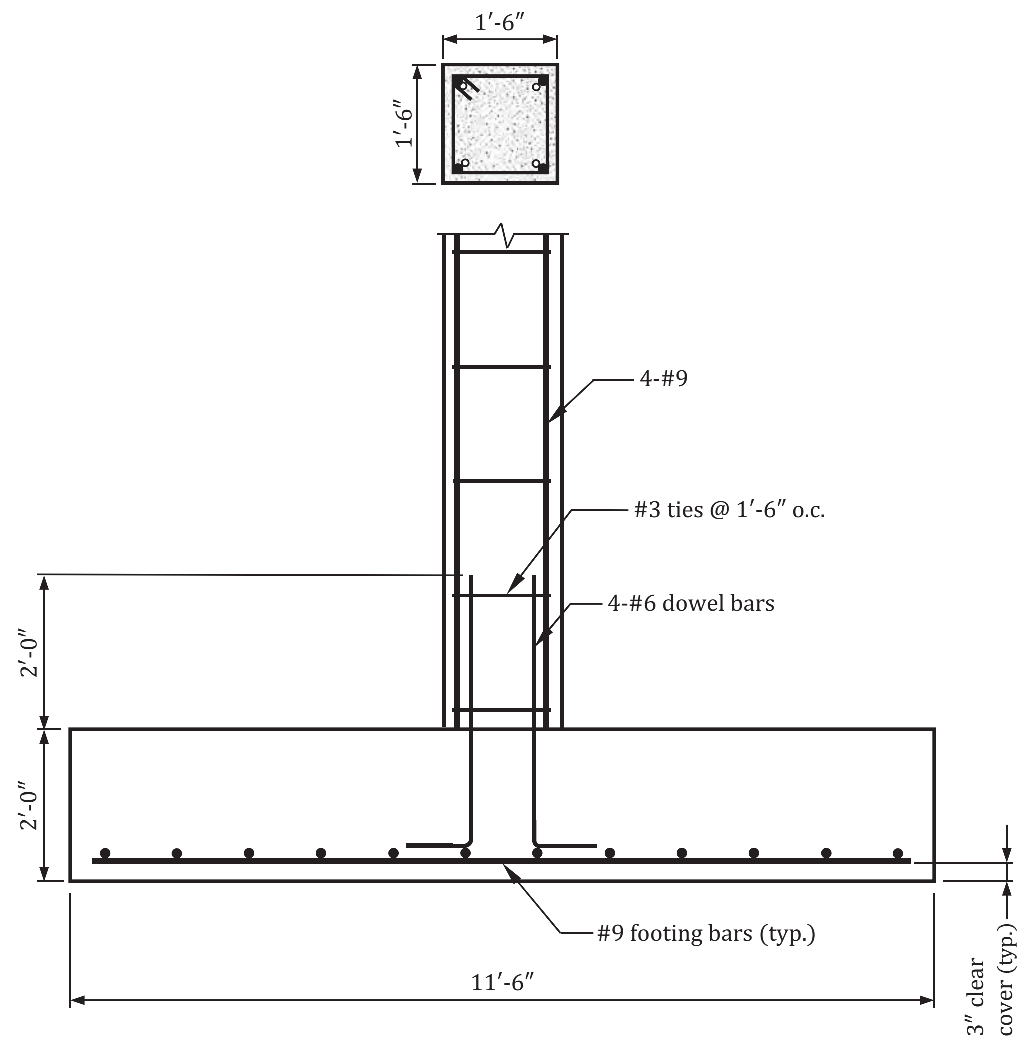


Figure 7.41 Reinforcement details for column C3 in Examples 7.1 through 7.4.

7.11.5 Example 7.5 – Determination of Preliminary Column Size: Building #1 (Framing Option B), Circular, Tied Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine a preliminary size for column C3 in the first story of Building #1, Framing Option B assuming the column is circular with transverse reinforcement consisting of ties (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f'_c = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the factored axial forces Table 3.3

Dead load of slab $= (8.5 / 12) \times 150.0 = 106.3 \text{ lb/ft}^2$

Roof: Superimposed dead load $= 12 \text{ lb/ft}^2$

Live load $= 20 \text{ lb/ft}^2$

Floors: Superimposed dead load $= 10 \text{ lb/ft}^2$

Live load $= 15 \text{ lb/ft}^2$ (nonreducible partitions) $+ 50 \text{ lb/ft}^2$

Tributary area to column $= 25.0 \times 23.5 = 587.5$ sq ft

This column is not part of the LFRS, and the bending moments due to gravity load effects are negligible.

A summary of the axial forces for column C3 in the first story is given in Table 7.30 in Example 7.1.

Step 2 – Determine the gross area of the column

Assuming a 1 percent reinforcement ratio for the longitudinal reinforcement $(\rho_g = 0.01)$, the required gross area of a tied column is the following using the largest P_u in Table 7.30:

$$A_g = \frac{P_u}{\phi 0.80[0.85 f_c'(1-\rho_g) + f_y \rho_g]} = \frac{661.6}{0.65 \times 0.80 \times \{[0.85 \times 4 \times (1-0.01)] + (60 \times 0.01)\}} = 320.8 \text{ in.}^2 \qquad \text{Eq. (7.30)}$$

With a 1 percent reinforcement ratio, a 22-in. circular column is adequate $(A_g = 380.1 \text{ in.}^2)$.

For a 2 percent reinforcement ratio, the required A_g is equal to 280.7 in.^2, which corresponds approximately to a 20-in. circular column.

A 20-in. circular column is selected.

7.11.6 Example 7.6 – Determination of Longitudinal Reinforcement: Building #1 (Framing Option B), Circular, Tied Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine the required longitudinal reinforcement for column C3 in the first story of Building #1, Framing Option B assuming a 20-in. circular tied column (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 7.5.

Step 1 – Determine the required area of longitudinal reinforcement

Based on a 20-in. circular tied column, the required reinforcement ratio, ρ_g, is equal to the following:

$$\rho_g = \frac{(P_u / \phi\phi_c A_g) - 0.85 f_c'}{f_y - 0.85 f_c'} = \frac{[661.6 / (0.65 \times 0.80 \times \pi \times 20.0^2 / 4)] - (0.85 \times 4)}{60 - (0.85 \times 4)} = 0.012 < 0.08 \qquad \text{Eq. (7.32)}$$

$$A_{st} = 0.012 \times \pi \times 20^2 / 4 = 3.77 \text{ in.}^2$$

Select 4-#9 bars $(A_{st,provided} = 4.00 \text{ in.}^2 > 3.77 \text{ in.}^2;\ \rho_g = 0.013)$.

Step 2 – Check the minimum and maximum number of longitudinal bars

For circular, tied columns:

Minimum number of longitudinal bars with circular ties $= 4$ Table 7.16

Maximum number of #9 bars $= 13$ for a 20-in. circular column and normal lap splices Table 7.19

Use 4-#9 bars.

Comments. It can be shown that the size and spacing of the circular ties and the number and size of the dowel bars for the 20-in. circular tied column are the same as those determined in Examples 7.3 and 7.4 for the 18-in. square tied column (see Figure 7.42).

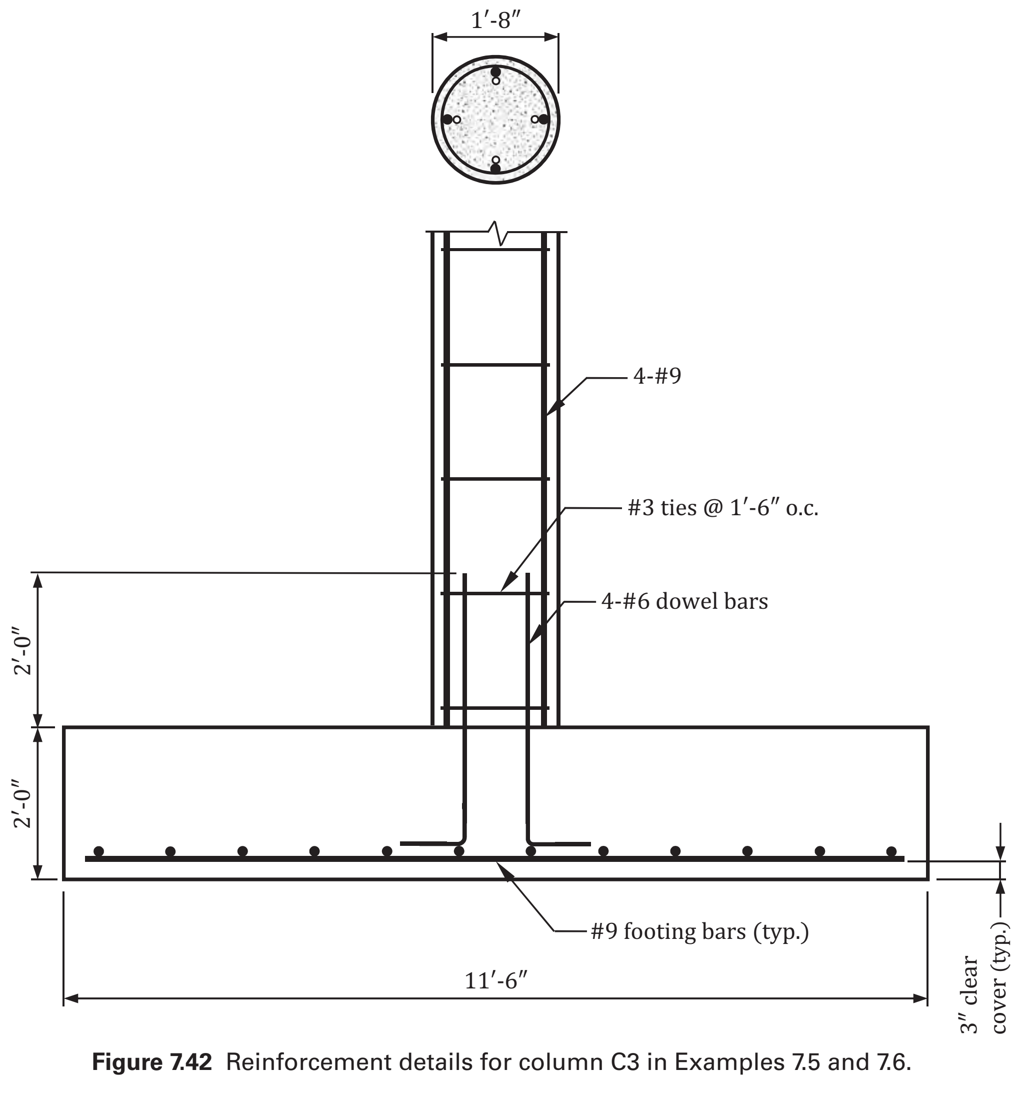


Figure 7.42 Reinforcement details for column C3 in Examples 7.5 and 7.6.

7.11.7 Example 7.7 – Determination of Preliminary Column Size: Building #1 (Framing Option B), Circular, Spiral Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine a preliminary size for column C3 in the first story of Building #1, Framing Option B assuming a circular spiral column (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f'_c = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the factored axial forces Table 3.3

$\text{Dead load of slab} = (8.5 / 12) \times 150.0 = 106.3 \text{ lb/ft}^2$

Roof: $\text{Superimposed dead load} = 12 \text{ lb/ft}^2$

$\text{Live load} = 20 \text{ lb/ft}^2$

Floors: $\text{Superimposed dead load} = 10 \text{ lb/ft}^2$

$\text{Live load} = 15 \text{ lb/ft}^2 \text{ (nonreducible partitions)} + 50 \text{ lb/ft}^2$

$\text{Tributary area to column} = 25.0 \times 23.5 = 587.5 \text{ sq ft}$

This column is not part of the LFRS, and the bending moments due to gravity load effects are negligible.

A summary of the axial forces for column C3 in the first story is given in Table 7.30 in Example 7.1.

Step 2 – Determine the gross area of the column

Assuming a 1 percent reinforcement ratio for the longitudinal reinforcement $(\rho_g = 0.01)$, the required gross area of a spiral column is the following using the largest P_u in Table 7.30:

$$A_g = \frac{P_u}{\phi 0.85[0.85 f_c'(1-\rho_g) + f_y \rho_g]} = \frac{661.6}{0.75 \times 0.85 \times \{[0.85 \times 4 \times (1-0.01)] + (60 \times 0.01)\}} = 261.7 \text{ in.}^2 \quad \text{Eq. (7.31)}$$

With a 1 percent reinforcement ratio, a 19-in. circular column is adequate $(A_g = 283.5 \text{ in.}^2)$.

For a 2 percent reinforcement ratio, the required A_g is equal to 229.0 in.^2, which corresponds approximately to an 18-in. circular column.

An 18-in. circular column is selected.

7.11.8 Example 7.8 – Determination of Longitudinal Reinforcement: Building #1 (Framing Option B), Circular, Spiral Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine the required longitudinal reinforcement for column C3 in the first story of Building #1, Framing Option B assuming an 18-in. circular spiral column (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 7.7.

Step 1 – Determine the required area of longitudinal reinforcement

Based on an 18-in. circular spiral column, the required reinforcement ratio, ρ_g, is equal to the following:

$$\rho_g = \frac{(P_u/\phi\phi_c A_g) - 0.85 f_c'}{f_y - 0.85 f_c'} = \frac{[661.6 / (0.75 \times 0.85 \times \pi \times 18.0^2 / 4)] - (0.85 \times 4)}{60 - (0.85 \times 4)} = 0.012 < 0.08 \quad \text{Eq. (7.32)}$$

$A_{st} = 0.012 \times \pi \times 18.0^2 / 4 = 3.05 \text{ in.}^2$

Select 7-#6 bars $(A_{st,provided} = 3.08 \text{ in.}^2 > 3.05 \text{ in.}^2;\ \rho_g = 0.012)$.

Step 2 – Check the minimum and maximum number of longitudinal bars

For circular, spiral columns:

Minimum number of longitudinal bars with spirals $= 6 < 7$ — Table 7.16

Maximum number of #6 bars $= 16$ for an 18-in. circular column and normal lap splices > 7 — Table 7.19

Use 7-#6 bars.

7.11.9 Example 7.9 – Determination of Transverse Reinforcement: Building #1 (Framing Option B), Circular, Spiral Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine the required transverse reinforcement for column C3 in the first story of Building #1, Framing Option B assuming an 18-in. circular spiral column (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f_c' = 4{,}000$ psi (maximum aggregate size $= 0.75$ in.), and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.7 and 7.8.

Step 1 – Determine the required spiral bar size — ACI 25.7.3.2

Because the shear force due to the factored gravity loads is negligible, the required spiral bar size is determined using ACI 25.7.3.2.

Spiral bars must have a diameter of at least 3/8 in., so use a #3 spiral.

Step 2 – Determine the required spiral bar spacing — ACI 25.7.3.1, 25.7.3.3

Because the shear force due to the factored gravity loads is negligible, the required spiral bar spacing is determined using ACI 25.7.3.3.

$$s = \frac{8.9A_{bs}}{D_{ch}\left(\dfrac{A_g}{A_{ch}} - 1\right)\left(\dfrac{f_c'}{f_{yt}}\right)} = \frac{8.9 \times 0.11}{(18.0 - 3.0) \times \left(\dfrac{18.0^2}{15.0^2} - 1\right) \times \left(\dfrac{4}{60}\right)} = 2.2 \text{ in.} < 3.0 + 0.375 = 3.375 \text{ in.}$$

Eq. (7.39)

$$\geq \text{ greater of } \begin{cases} 1.0 + 0.375 = 1.375 \text{ in.} \\ (4/3)d_{agg} + d_b = 1.0 + 0.375 = 1.375 \text{ in.} \end{cases}$$

Use a #3 spiral at a 2-in. pitch; this matches the value in Table 7.26.

7.11.10 Example 7.10 – Determination of Dowel Reinforcement at the Foundation: Building #1 (Framing Option B), Circular, Spiral Column is Not Part of the LFRS, SDC A, Column Subjected to Primarily Axial Forces

Determine the required dowel reinforcement for column C3 in the first story of Building #1, Framing Option B assuming an 18-in. circular spiral column supported by a $2'\text{-}0'' \times 11'\text{-}6'' \times 11'\text{-}6''$ spread footing (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f_c' = 4{,}000$ psi (maximum aggregate size $= 0.75$ in.), and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.7, 7.8, and 7.9.

Step 1 – Check the bearing loads on the column and footing ACI 16.3.3.4

- Bearing strength of the column:

$$B_u = 661.6 \text{ kips} > \phi B_n = \phi 0.85 f'_c A_1 = 0.65 \times 0.85 \times 4 \times \pi \times 18.0^2 / 4 = 562.4 \text{ kips} \quad \text{Eq. (7.40)}$$

Therefore, transfer the excess compression force by interface reinforcement:

$$A_s = \frac{B_u - \phi B_n}{\phi f_y} = \frac{661.6 - 562.4}{0.65 \times 60} = 2.54 \text{ in.}^2 > A_{s,min} = 0.005 A_g = 0.005 \times \pi \times 18.0^2 / 4 = 1.27 \text{ in.}^2 \quad \text{Eq. (7.42)}$$

- Bearing strength of the footing:

$$\phi B_n = \phi 0.85 f'_c A_1 \sqrt{A_2 / A_1} \leq 2\phi 0.85 f'_c A_1 \quad \text{Eq. (7.41)}$$

Footing thickness $h = 24.0$ in.

Horizontal projection for a 2:1 slope $= 2 \times 24.0 = 48.0$ in.

Projected diameter of cone $b = 48.0 + 18.0 + 48.0 = 114.0 \text{ in.} < 11.5 \times 12 = 138.0 \text{ in.}$

Therefore, $A_2 = \pi b^2 / 4 = 10{,}207.0 \text{ in.}^2$

$$\sqrt{A_2 / A_1} = \sqrt{10{,}207.0 / (\pi \times 18.0^2 / 4)} = 6.3 > 2.0; \text{ use } 2.0$$

$$B_u = 661.6 \text{ kips} < \phi B_n = 2 \times 562.4 = 1{,}124.8 \text{ kips}$$

Step 2 – Determine the required interface reinforcement ACI 16.3.4.1

It is determined in Step 1 that the required area of interface reinforcement is $A_s = 2.54 \text{ in.}^2$

Provide 7-#6 dowel bars ($A_{s,provided} = 3.08 \text{ in.}^2 > 2.54 \text{ in.}^2$).

Step 3 – Check the development of the dowel bars in the footing

Determine the compression development length, ℓ_{dc}, of the #6 dowel bars:

$$\ell_{dc} = \text{ larger of } \begin{cases} (f_y \psi_r / 50\lambda\sqrt{f'_c}) d_b \\ 0.0003 f_y \psi_r d_b \\ 8 \text{ in.} \end{cases} \quad \text{Table 7.21, Note 5}$$

$\psi_r = 1.0$ (the dowel bars are not enclosed by any transverse reinforcement in the footing) Table 7.22

$\lambda = 1.0$ for normalweight concrete Table 7.13

$$\ell_{dc} = \text{ larger of } \begin{cases} [(60{,}000 \times 1.0) / (50 \times 1.0 \times \sqrt{4{,}000})] \times 0.75 = 14.2 \text{ in.} \\ 0.0003 \times 60{,}000 \times 0.75 \times 0.75 = 10.1 \text{ in.} \\ 8 \text{ in.} \end{cases}$$

Assuming two layers of #9 bars in the footing with a clear cover of 3 in., determine the minimum footing thickness for the development of the #6 dowel bars:

$$\text{Minimum } h = \ell_{dc} + r + (d_b)_d + 2(d_b)_f + \text{cover}$$

Eq. (7.43)

$$= 14.2 + (6 \times 0.75 / 2) + 0.75 + (2 \times 1.128) + 3.0 = 22.5 \text{ in.}$$

Because the minimum $h = 22.5$ in. is less than the provided footing thickness of 24.0 in., the #6 hooked dowel bars can be fully developed in the footing for compression.

Step 4 – Determine the development length of the dowel bars in the column

The #6 dowel bars are lap spliced to the #6 longitudinal reinforcement in the column. Because the dowel bars are the same size as the longitudinal bars, the compression lap splice length, ℓ_{sc}, must be greater than or equal to the following:

$$\ell_{sc} = \text{ greater of } \begin{cases} 0.0005 f_y d_b = 0.0005 \times 60{,}000 \times 0.75 = 22.5 \text{ in.} \\ \\ 12.0 \text{ in.} \end{cases}$$

Table 7.21

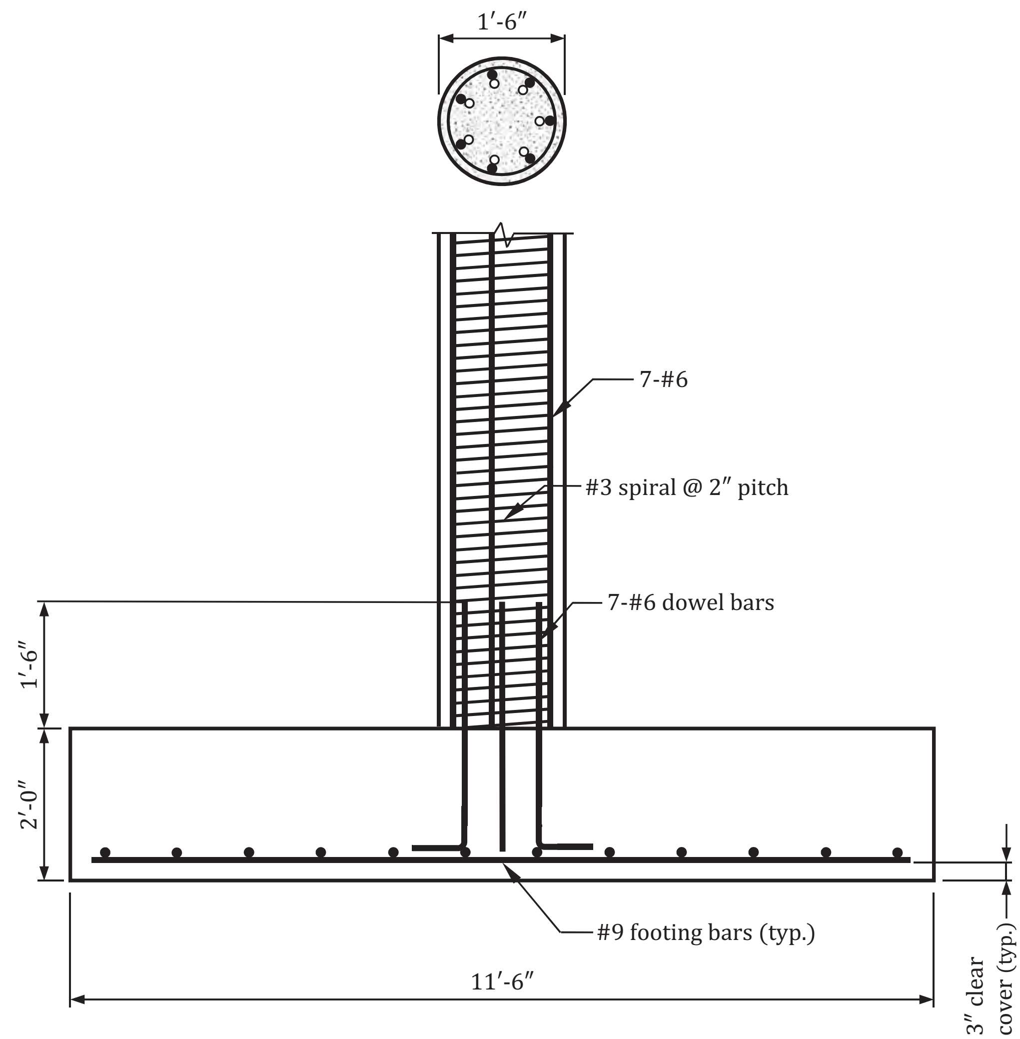


Figure 7.43 Reinforcement details for column C3 in Examples 7.7 through 7.10.

For spiral columns, ℓ_{sc} is permitted to be multiplied by 0.75:

$\text{Lap splice length} = 0.75 \times 22.5 = 16.9 \text{ in.}$ ACI 10.7.5.2.1(b)

Provide a lap splice length of 1 ft-6 in.

Negligible horizonal forces are transferred from the column to the footing, so horizontal force transfer is not investigated.

Reinforcement details for this column are given in Figure 7.43.

7.11.11 Example 7.11 – Construction of Nominal and Design Strength Interaction Diagrams: Building #1 (Framing Option B), Rectangular, Tied Column, Grade 60 Longitudinal Reinforcement

Determine nominal and design strength interaction diagrams for the 18-in. square tied column in Examples 7.1 through 7.4. The column is reinforced with 4-#9 longitudinal bars and a clear cover of 1.5 in. is provided to #3 ties (see Figure 7.44). Assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

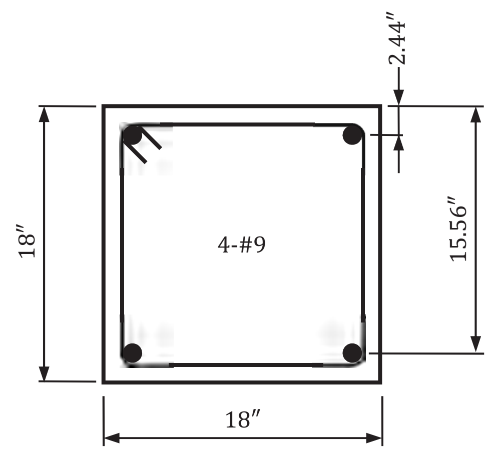


Figure 7.44 Reinforced concrete column in Example 7.11.

Nominal and design axial forces and bending moments are determined at the following points:

A. Pure axial compression
B. Balanced point
C. Stress in reinforcement farthest from compression face $= 0$
D. Stress in reinforcement farthest from compression face $= 0.5 f_y$
E. Pure bending

Calculations are provided for pure axial compression and the balanced point; a summary of the results is given for all the other points. In the calculations, it is assumed compression is positive.

A. Pure axial compression

The nominal axial compressive strength for a tied column is determined by the following:

$$P_{n,max} = 0.80 P_o = 0.80[0.85 f_c'(A_g - A_{st}) + f_y A_{st}]$$

Eq. (7.13)

$$= 0.80\{[0.85 \times 4 \times (18.0^2 - 4.0)] + (60 \times 4.0)\} = 1{,}062.4 \text{ kips}$$

The section is compression-controlled, so $\phi = 0.65$. Table 7.10

$\phi P_{n,max} = 0.65 \times 1{,}062.4 = 690.6$ kips

B. Balanced point

Step 1 – Determine the neutral axis depth

Table 7.11

Balanced failure of the section occurs when the maximum strain in the concrete is equal to 0.003 and the strain in the reinforcement farthest from the compression face is $\varepsilon_t = \varepsilon_{ty} = f_y / E_s = 60 / 29{,}000 = 0.00207$ (Note: ACI 21.2.2.1 permits $\varepsilon_{ty} = 0.002$ for Grade 60 deformed reinforcement, but that value is not used in this example). The neutral axis depth, c, is determined by the following equation:

$$c = \frac{0.003d_2}{\varepsilon_t + 0.003} = \frac{0.003 \times 15.56}{0.00207 + 0.003} = 9.21 \text{ in.}$$

Figure 7.10

Step 2 – Determine the depth of the equivalent stress block

$a = \beta_1 c = 0.85 \times 9.21 = 7.83$ in. Table 7.11

Step 3 – Determine the resultant compression force

$C = 0.85 f_c' ab = 0.85 \times 4 \times 7.83 \times 18.0 = 479.2$ kips

Step 4 – Determine the strain in the reinforcement

$$\varepsilon_{si} = \frac{0.003(c - d_i)}{c}$$

Layer 1: $\varepsilon_{s1} = \dfrac{0.003 \times (9.21 - 2.44)}{9.21} = 0.00221$

Layer 2: $\varepsilon_{s2} = \dfrac{0.003 \times (9.21 - 15.56)}{9.21} = -0.00207$

Step 5 – Determine the stress in the reinforcement

$f_{si} = E_s \varepsilon_{si}$

Layer 1: $f_{s1} = 29{,}000 \times 0.00221 = 64.1 \text{ ksi} > 60.0 \text{ ksi, use } 60.0 \text{ ksi}$

Layer 2: $f_{s2} = 29{,}000 \times (-0.00207) = -60.0$ ksi

Step 6 – Determine the force in the reinforcement

Where $d_i < a$: $F_{si} = (f_{si} - 0.85 f_c') A_{si}$

Where $d_i > a$: $F_{si} = f_{si} A_{si}$

Layer 1 $(d_1 = 2.44 \text{ in.} < a = 7.83 \text{ in.})$: $F_{s1} = (f_{s1} - 0.85 f_c') A_{s1} = [60.0 - (0.85 \times 4.0)] \times 2.0 = 113.2$ kips

Layer 2 $(d_2 = 15.56 \text{ in.} > a = 7.83 \text{ in.})$: $F_{s2} = f_{s2} A_{s2} = -60.0 \times 2.0 = -120.0$ kips

Step 7 – Determine the nominal and design axial strengths

$$P_n = C + \Sigma F_{si} = 479.2 + (113.2 - 120.0) = 472.4 \text{ kips}$$

$$\phi P_n = 0.65 \times 472.4 = 307.1 \text{ kips}$$

Step 8 – Determine the nominal and design flexural strengths

$$M_n = 0.5C(h - a) + \Sigma F_{si}(0.5h - d_i)$$

$$= \{[0.5 \times 479.2 \times (18.0 - 7.83)] + [113.2 \times (9.0 - 2.44)] + [(-120.0) \times (9.0 - 15.56)]\} / 12 = 330.5 \text{ ft-kips}$$

$$\phi M_n = 0.65 \times 330.5 = 214.8 \text{ ft-kips}$$

C. Stress in reinforcement farthest from the compression face $= 0$

$$c = \frac{0.003 d_2}{\varepsilon_t + 0.003} = \frac{0.003 \times 15.56}{0 + 0.003} = 15.56 \text{ in.}$$

$$a = \beta_1 c = 0.85 \times 15.56 = 13.23 \text{ in.}$$

$$C = 0.85 f_c' ab = 0.85 \times 4 \times 13.23 \times 18.0 = 809.7 \text{ kips}$$

The strains, stresses, and forces at each layer of reinforcement are given in Table 7.31.

Table 7.31 Summary of Reinforcement Strains, Stresses, and Forces for Stress in Reinforcement Farthest from Compression Face = 0

d_{si} (in.)	ε_{si}	f_{si} (ksi)	F_{si} (kips)
2.44	0.00253	60.0	113.2
15.56	0	0	0

$$P_n = C + \Sigma F_{si} = 809.7 + 113.2 = 922.9 \text{ kips}$$

$$\phi P_n = 0.65 \times 922.9 = 599.9 \text{ kips}$$

$$M_n = 0.5C(h - a) + \Sigma F_{si}(0.5h - d_i)$$

$$= \{[0.5 \times 809.7 \times (18.0 - 13.23)] + [113.2 \times (9.0 - 2.44)]\} / 12 = 222.8 \text{ ft-kips}$$

$$\phi M_n = 0.65 \times 222.8 = 144.8 \text{ ft-kips}$$

D. Stress in reinforcement farthest from the compression face = $0.5 f_y$

$$c = \frac{0.003 d_2}{\varepsilon_t + 0.003} = \frac{0.003 \times 15.56}{(0.00207 / 2) + 0.003} = 11.57 \text{ in.}$$

$$a = \beta_1 c = 0.85 \times 11.57 = 9.83 \text{ in.}$$

The strains, stresses, and forces at each layer of reinforcement are given in Table 7.32.

Table 7.32 Summary of Reinforcement Strains, Stresses, and Forces for Stress in Reinforcement Farthest from Compression Face = $0.5f_y$

d_{si} (in.)	ε_{si}	f_{si} (ksi)	F_{si} (kips)
2.44	0.00237	60.0	113.2
15.56	−0.00104	−30.0	−60.0

$$P_n = C + \Sigma F_{si} = 601.6 + (113.2 - 60.0) = 654.8 \text{ kips}$$

$$\phi P_n = 0.65 \times 654.8 = 425.6 \text{ kips}$$

$$M_n = 0.5C(h - a) + \Sigma F_{si}(0.5h - d_i)$$

$$= \{[0.5 \times 601.8 \times (18.0 - 9.83)] + [113.2 \times (9.0 - 2.44)] + [(-60.0) \times (9.0 - 15.56)]\} / 12 = 299.6 \text{ ft-kips}$$

$$\phi M_n = 0.65 \times 299.6 = 194.7 \text{ ft-kips}$$

E. Pure bending

There is no closed-form solution to determine the depth of the neutral axis for the case of pure bending. From trial-and-error, it is found that $c = 2.38$ in.

$$a = \beta_1 c = 0.85 \times 2.38 = 2.02 \text{ in.}$$

$$C = 0.85 f_c' ab = 0.85 \times 4 \times 2.02 \times 18.0 = 123.6 \text{ kips}$$

The strains, stresses, and forces at each layer of reinforcement are given in Table 7.33.

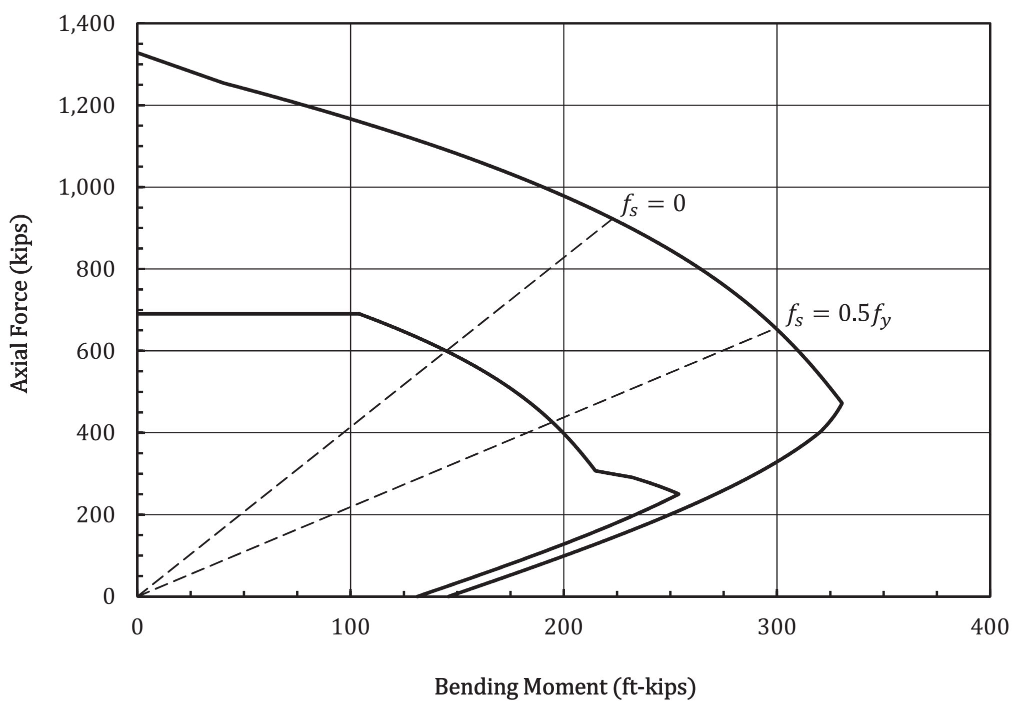


Figure 7.45 Nominal and design strength interaction diagrams for the column in Example 7.11.

Table 7.33 Summary of Reinforcement Strains, Stresses, and Forces for Pure Bending

d_{si} (in.)	ε_{si}	f_{si} (ksi)	F_{si} (kips)
2.44	−0.00007	−2.0	−4.0
15.56	−0.01661	−60.0	−120.0

$P_n = C + \Sigma F_{si} = 123.6 + (-4.0 - 120.0) = -0.4 \text{ kips} \cong 0$ (difference due to roundoff)

$$M_n = 0.5C(h - a) + \Sigma F_{si}(0.5h - d_i)$$

$$= \{[0.5 \times 123.6 \times (18.0 - 2.02)] + [(-2.0) \times (9.0 - 2.44)] + [(-120.0) \times (9.0 - 15.56)]\} / 12 = 146.8 \text{ ft-kips}$$

$$\phi M_n = 0.9 \times 146.8 = 132.1 \text{ ft-kips}$$

The nominal and design strength interaction diagrams for this column are given in Figure 7.45.

7.11.12 Example 7.12 – Construction of Nominal and Design Strength Interaction Diagrams: Building #1 (Framing Option B), Rectangular, Tied Column, Grade 100 Longitudinal Reinforcement

Determine nominal and design strength interaction diagrams for the 18-in. square tied column in Examples 7.1 through 7.4. The column is reinforced with 4-#9 longitudinal bars and a clear cover of 1.5 in. is provided to #3 ties (see Figure 7.44). Assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 100 reinforcement.

Nominal and design axial forces and bending moments are determined at the following points:

A. Pure axial compression
B. Balanced point
C. Stress in reinforcement farthest from compression face $= 0$
D. Stress in reinforcement farthest from compression face $= 0.5f_y$
E. Pure bending

Calculations are provided for pure axial compression and the balanced point; a summary of the results is given for all the other points. In the calculations, it is assumed compression is positive.

A. Pure axial compression

The nominal axial compressive strength for a tied column is determined by the following:

$$P_{n,max} = 0.80P_o = 0.80[0.85f'_c(A_g - A_{st}) + f_yA_{st}]$$

Eq. (7.13)

$$= 0.80\{[0.85 \times 4 \times (18.0^2 - 4.0)] + (80 \times 4.0)\} = 1{,}126.4 \text{ kips}$$

Note that $f_y = 80$ ksi is used in the equation for $P_{n,max}$ in accordance with the requirement in ACI 22.4.2.1, which limits f_y to 80 ksi for members subjected to pure axial compression.

The section is compression-controlled, so $\phi = 0.65$. Table 7.10

$$\phi P_{n,max} = 0.65 \times 1{,}126.4 = 732.2 \text{ kips}$$

B. Balanced point

Step 1 – Determine the neutral axis depth

Table 7.11

Balanced failure of the section occurs when the maximum strain in the concrete is equal to 0.003 and the strain in the reinforcement farthest from the compression face is $\varepsilon_t = \varepsilon_{ty} = f_y / E_s = 100 / 29{,}000 = 0.00345$. The neutral axis depth, c, is determined by the following equation:

$$c = \frac{0.003 d_2}{\varepsilon_t + 0.003} = \frac{0.003 \times 15.56}{0.00345 + 0.003} = 7.24 \text{ in.}$$

Figure 7.10

Step 2 – Determine the depth of the equivalent stress block

$$a = \beta_1 c = 0.85 \times 7.24 = 6.15 \text{ in.}$$

Table 7.11

Step 3 – Determine the resultant compression force

$$C = 0.85 f_c' ab = 0.85 \times 4 \times 6.15 \times 18.0 = 376.4 \text{ kips}$$

Step 4 – Determine the strain in the reinforcement

$$\varepsilon_{si} = \frac{0.003(c - d_i)}{c}$$

Layer 1: $\varepsilon_{s1} = \dfrac{0.003 \times (7.24 - 2.44)}{7.24} = 0.00199$

Layer 2: $\varepsilon_{s2} = \dfrac{0.003 \times (7.24 - 15.56)}{7.24} = -0.00345$

Step 5 – Determine the stress in the reinforcement

$$f_{si} = E_s \varepsilon_{si}$$

Layer 1: $f_{s1} = 29{,}000 \times 0.00199 = 57.7 \text{ ksi}$

Layer 2: $f_{s2} = 29{,}000 \times (-0.00345) = -100.0 \text{ ksi}$

Step 6 – Determine the force in the reinforcement

Where $d_i < a: \ F_{si} = (f_{si} - 0.85 f_c') A_{si}$

Where $d_i > a: \ F_{si} = f_{si} A_{si}$

Layer 1 $(d_1 = 2.44 \text{ in.} < a = 6.15 \text{ in.})$: $F_{s1} = (f_{s1} - 0.85 f_c') A_{s1} = [57.7 - (0.85 \times 4.0)] \times 2.0 = 108.6 \text{ kips}$

Layer 2 $(d_2 = 15.56 \text{ in.} > a = 7.83 \text{ in.})$: $F_{s2} = f_{s2} A_{s2} = -100.0 \times 2.0 = -200.0 \text{ kips}$

Step 7 – Determine the nominal and design axial strengths

$$P_n = C + \Sigma F_{si} = 376.4 + (108.6 - 200.0) = 285.0 \text{ kips}$$

$$\phi P_n = 0.65 \times 285.0 = 185.3 \text{ kips}$$

Step 8 – Determine the nominal and design flexural strengths

$$M_n = 0.5C(h - a) + \Sigma F_{si}(0.5h - d_i)$$

$$= \{[0.5 \times 376.4 \times (18.0 - 6.15)] + [108.6 \times (9.0 - 2.44)] + [(-200.0) \times (9.0 - 15.56)]\} / 12 = 354.6 \text{ ft-kips}$$

$$\phi M_n = 0.65 \times 354.6 = 230.5 \text{ ft-kips}$$

C. Stress in reinforcement farthest from the compression face = 0

$$c = \frac{0.003 d_2}{\varepsilon_t + 0.003} = \frac{0.003 \times 15.56}{0 + 0.003} = 15.56 \text{ in.}$$

$$a = \beta_1 c = 0.85 \times 15.56 = 13.23 \text{ in.}$$

$$C = 0.85 f_c' ab = 0.85 \times 4 \times 13.23 \times 18.0 = 809.7 \text{ kips}$$

The strains, stresses, and forces at each layer of reinforcement are given in Table 7.34.

Table 7.34 Summary of Reinforcement Strains, Stresses, and Forces for Stress in Reinforcement Farthest from Compression Face = 0

d_{si} (in.)	ε_{si}	f_{si} (ksi)	F_{si} (kips)
2.44	0.00253	73.4	140.0
15.56	0	0	0

$$P_n = C + \Sigma F_{si} = 809.7 + 140.0 = 949.7 \text{ kips}$$

$$\phi P_n = 0.65 \times 949.7 = 617.3 \text{ kips}$$

$$M_n = 0.5C(h - a) + \Sigma F_{si}(0.5h - d_i)$$

$$= \{[0.5 \times 809.7 \times (18.0 - 13.23)] + [140.0 \times (9.0 - 2.44)]\} / 12 = 237.5 \text{ ft-kips}$$

$$\phi M_n = 0.65 \times 237.5 = 154.4 \text{ ft-kips}$$

D. Stress in reinforcement farthest from the compression face = $0.5f_y$

$$c = \frac{0.003 d_2}{\varepsilon_t + 0.003} = \frac{0.003 \times 15.56}{(0.00345 / 2) + 0.003} = 9.88 \text{ in.}$$

$$a = \beta_1 c = 0.85 \times 9.88 = 8.40 \text{ in.}$$

$$C = 0.85 f_c' ab = 0.85 \times 4 \times 8.40 \times 18.0 = 514.1 \text{ kips}$$

The strains, stresses, and forces at each layer of reinforcement are given in Table 7.35.

Table 7.35 Summary of Reinforcement Strains, Stresses, and Forces for Stress in Reinforcement Farthest from Compression Face = $0.5f_y$

d_{si} (in.)	ε_{si}	f_{si} (ksi)	F_{si} (kips)
2.44	0.00226	65.5	124.2
15.56	−0.00172	−49.9	−99.8

$$P_n = C + \Sigma F_{si} = 514.1 + (124.2 - 99.8) = 538.5 \text{ kips}$$

$$\phi P_n = 0.65 \times 538.5 = 350.0 \text{ kips}$$

$$M_n = 0.5C(h - a) + \Sigma F_{si}(0.5h - d_i)$$

$$= \{[0.5 \times 514.1 \times (18.0 - 8.40)] + [124.2 \times (9.0 - 2.44)] + [(-99.8) \times (9.0 - 15.56)]\} / 12 = 328.1 \text{ ft-kips}$$

$$\phi M_n = 0.65 \times 328.1 = 213.3 \text{ ft-kips}$$

E. Pure bending

There is no closed-form solution to determine the depth of the neutral axis for the case of pure bending. From trial-and-error, it is found that $c = 3.19$ in.

$$a = \beta_1 c = 0.85 \times 3.19 = 2.71 \text{ in.}$$

$$C = 0.85 f_c' ab = 0.85 \times 4 \times 2.71 \times 18.0 = 165.9 \text{ kips}$$

The strains, stresses, and forces at each layer of reinforcement are given in Table 7.36.

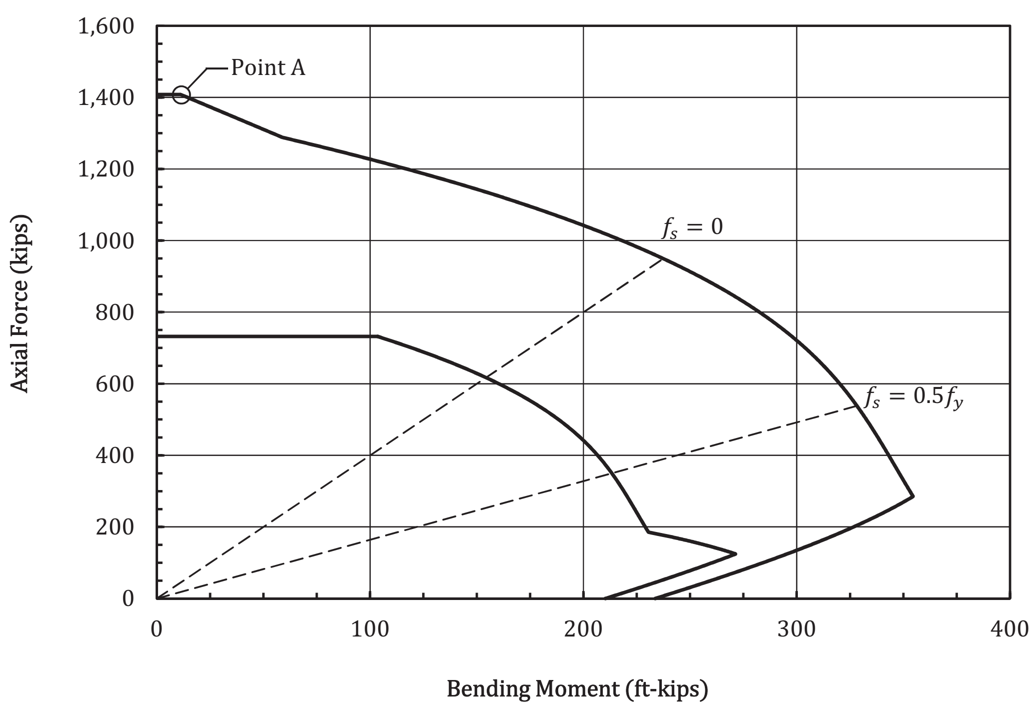


Figure 7.46 Nominal and design strength interaction diagrams for the column in Example 7.12.

Table 7.36 Summary of Reinforcement Strains, Stresses, and Forces for Pure Bending

d_{si} (in.)	ε_{si}	f_{si} (ksi)	F_{si} (kips)
2.44	0.00071	20.6	34.4
15.56	−0.01163	−100.0	−200.0

$$P_n = C + \Sigma F_{si} = 165.9 + (34.4 - 200.0) = 0.3 \text{ kips} \cong 0 \text{ (difference due to roundoff)}$$

$$M_n = 0.5C(h-a) + \Sigma F_{si}(0.5h - d_i)$$

$$= \{[0.5 \times 165.9 \times (18.0 - 2.71)] + [34.4 \times (9.0 - 2.44)] + [(-200.0) \times (9.0 - 15.56)]\} / 12 = 233.8 \text{ ft-kips}$$

$$\phi M_n = 0.9 \times 233.8 = 210.4 \text{ ft-kips}$$

The nominal and design strength interaction diagrams for this column are given in Figure 7.46. Point A on the nominal strength interaction diagram represents the maximum nominal axial force $P_o = 1,408.0$ kips, which corresponds to the 80 ksi limit for pure axial compression in ACI 22.4.2.1. A horizontal line connects that point to the point where $P_o = 1,408.0$ kips and $M_n = 0$.

7.11.13 Example 7.13 – Construction of Nominal and Design Strength Interaction Diagrams: Building #1 (Framing Option B), Circular, Tied Column, Grade 60 Longitudinal Reinforcement

Determine nominal and design strength interaction diagrams for the 20-in. circular, tied column in Examples 7.5 and 7.6. The column is reinforced with 4-#9 longitudinal bars and a clear cover of 1.5 in. is provided to #3 ties (see Figure 7.47). Assume normalweight concrete with $f_c' = 4,000$ psi and Grade 60 reinforcement.

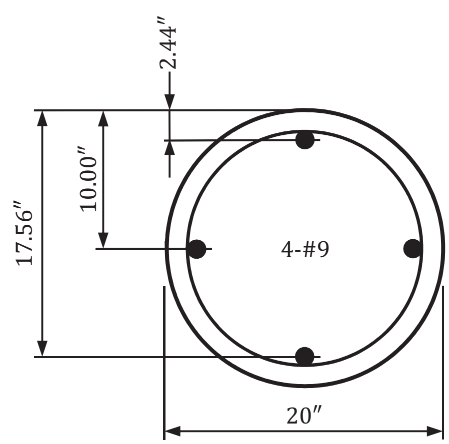


Figure 7.47 Reinforced concrete column in Example 7.13.

Nominal and design axial forces and bending moments are determined at the following points:

A. Pure axial compression
B. Balanced point
C. Stress in reinforcement farthest from compression face $= 0$
D. Stress in reinforcement farthest from compression face $= 0.5f_y$
E. Pure bending

Calculations are provided for pure axial compression and the balanced point; a summary of the results is given for all the other points. In the calculations, it is assumed compression is positive.

A. Pure axial compression

The nominal axial compressive strength for a tied column is determined by the following:

$$P_{n,max} = 0.80P_o = 0.80[0.85f_c'(A_g - A_{st}) + f_y A_{st}]$$

Eq. (7.13)

$$= 0.80\{[0.85 \times 4 \times (314.2 - 4.0)] + (60 \times 4.0)\} = 1{,}035.7 \text{ kips}$$

where $A_g = \pi \times 20.0^2 / 4 = 314.2$ in.^{2}

The section is compression-controlled, so $\phi = 0.65.$ Table 7.10

$$\phi P_{n,max} = 0.65 \times 1{,}035.7 = 673.2 \text{ kips}$$

B. Balanced point

Step 1 – Determine the neutral axis depth

Table 7.11

Balanced failure of the section occurs when the maximum strain in the concrete is equal to 0.003 and the strain in the reinforcement farthest from the compression face is $\varepsilon_t = \varepsilon_{ty} = f_y / E_s = 60 / 29{,}000 = 0.00207$ (Note: ACI 21.2.2.1 permits $\varepsilon_{ty} = 0.002$ for Grade 60 deformed reinforcement, but that value is not used in this example). The neutral axis depth, $c,$ is determined by the following equation:

$$c = \frac{0.003d_3}{\varepsilon_t + 0.003} = \frac{0.003 \times 17.56}{0.00207 + 0.003} = 10.39 \text{ in.}$$

Figure 7.12

Step 2 – Determine the depth of the equivalent stress block

$$a = \beta_1 c = 0.85 \times 10.39 = 8.83 \text{ in.}$$

Table 7.11

Step 3 – Determine the section properties of the compression zone

$$\theta = 2\cos^{-1}\left(1 - \frac{2a}{h}\right) = 2 \times \cos^{-1}\left(1 - \frac{2 \times 8.83}{20.0}\right) = 2.907 \text{ rad}$$

Figure 7.13

$$\overline{y} = \frac{2h\sin^3(\theta / 2)}{3(\theta - \sin\theta)} = \frac{2 \times 20.0 \times \sin^3(2.907 / 2)}{3 \times (2.907 - \sin 2.907)} = 4.88 \text{ in.}$$

Step 4 – Determine the resultant compression force

$$C = \frac{0.85f_c'h^2}{8}(\theta - \sin\theta) = \frac{0.85 \times 4 \times 20.0^2}{8} \times (2.907 - \sin 2.907) = 454.7 \text{ kips}$$

Step 5 – Determine the strain in the reinforcement

$$\varepsilon_{si} = \frac{0.003(c - d_i)}{c}$$

Layer 1: $\varepsilon_{s1} = \dfrac{0.003 \times (10.39 - 2.44)}{10.39} = 0.00230$

Layer 2: $\varepsilon_{s2} = \dfrac{0.003 \times (10.39 - 10.00)}{10.39} = 0.00011$

Layer 3: $\varepsilon_{s3} = \dfrac{0.003 \times (10.39 - 17.56)}{10.39} = -0.00207$

Step 6 – Determine the stress in the reinforcement

$f_{si} = E_s \varepsilon_{si}$

Layer 1: $f_{s1} = 29{,}000 \times 0.00230 = 66.7 \text{ ksi} > 60.0 \text{ ksi, use } 60.0 \text{ ksi}$

Layer 2: $f_{s2} = 29{,}000 \times 0.00011 = 3.2 \text{ ksi}$

Layer 3: $f_{s3} = 29{,}000 \times (-0.00207) = -60.0 \text{ ksi}$

Step 7 – Determine the force in the reinforcement

$\text{Where } d_i < a: \; F_{si} = (f_{si} - 0.85 f_c')A_{si}$

$\text{Where } d_i > a: \; F_{si} = f_{si} A_{si}$

Layer 1 $(d_1 = 2.44 \text{ in.} < a = 8.83 \text{ in.})$: $F_{s1} = (f_{s1} - 0.85 f_c')A_{s1} = [60.0 - (0.85 \times 4.0)] \times 1.0 = 56.6 \text{ kips}$

Layer 2 $(d_2 = 10.00 \text{ in.} > a = 8.83 \text{ in.})$: $F_{s2} = f_{s2} A_{s2} = 3.2 \times 2.0 = 6.4 \text{ kips}$

Layer 3 $(d_3 = 17.56 \text{ in.} > a = 8.83 \text{ in.})$: $F_{s3} = f_{s3} A_{s3} = -60.0 \times 1.0 = -60.0 \text{ kips}$

Step 8 – Determine the nominal and design axial strengths

$P_n = C + \Sigma F_{si} = 454.7 + (56.6 + 6.4 - 60.0) = 457.7 \text{ kips}$

$\phi P_n = 0.65 \times 457.7 = 297.5 \text{ kips}$

Step 9 – Determine the nominal and design flexural strengths

$$M_n = C\overline{y} + \Sigma F_{si}(0.5h - d_i)$$

$$= \{(454.7 \times 4.88) + [56.6 \times (10.0 - 2.44)] + [6.4 \times (10.0 - 10.0)] + [(-60.0) \times (10.0 - 17.56)]\} / 12 = 258.4 \text{ ft-kips}$$

$\phi M_n = 0.65 \times 258.4 = 168.0 \text{ ft-kips}$

C. Stress in reinforcement farthest from the compression face = 0

$$c = \frac{0.003 d_3}{\varepsilon_t + 0.003} = \frac{0.003 \times 17.56}{0 + 0.003} = 17.56 \text{ in.}$$

$a = \beta_1 c = 0.85 \times 17.56 = 14.93 \text{ in.}$

$$\theta = 2\cos^{-1}\left(1 - \frac{2a}{h}\right) = 2 \times \cos^{-1}\left(1 - \frac{2 \times 14.93}{20.0}\right) = 4.173 \text{ rad}$$ Figure 7.13

$$\overline{y} = \frac{2h\sin^3(\theta / 2)}{3(\theta - \sin\theta)} = \frac{2 \times 20.0 \times \sin^3(4.173 / 2)}{3 \times (4.173 - \sin 4.173)} = 1.75 \text{ in.}$$

$$C = \frac{0.85 f_c' h^2}{8}(\theta - \sin\theta) = \frac{0.85 \times 4 \times 20.0^2}{8} \times (4.173 - \sin 4.173) = 855.3 \text{ kips}$$

The strains, stresses, and forces at each layer of reinforcement are given in Table 7.37.

Table 7.37 Summary of Reinforcement Strains, Stresses, and Forces for Stress in Reinforcement Farthest from Compression Face = 0

d_{si} (in.)	ε_{si}	f_{si} (ksi)	F_{si} (kips)
2.44	0.00258	60.0	56.6
10.00	0.00129	37.5	68.2
17.56	0	0	0

$$P_n = C + \Sigma F_{si} = 855.3 + 56.6 + 68.2 = 980.1 \text{ kips}$$

$$\phi P_n = 0.65 \times 980.1 = 637.1 \text{ kips}$$

$$M_n = C\overline{y} + \Sigma F_{si}(0.5h - d_i)$$

$$= \{(855.3 \times 1.75) + [56.6 \times (10.0 - 2.44)] + [68.2 \times (10.0 - 10.0)] + [0 \times (10.0 - 17.56)]\} / 12 = 160.4 \text{ ft-kips}$$

$$\phi M_n = 0.65 \times 160.4 = 104.3 \text{ ft-kips}$$

D. Stress in reinforcement farthest from the compression face = $0.5f_y$

$$c = \frac{0.003 d_3}{\varepsilon_t + 0.003} = \frac{0.003 \times 17.56}{(0.00207 / 2) + 0.003} = 13.06 \text{ in.}$$

$$a = \beta_1 c = 0.85 \times 13.06 = 11.10 \text{ in.}$$

$$\theta = 2\cos^{-1}\left(1 - \frac{2a}{h}\right) = 2 \times \cos^{-1}\left(1 - \frac{2 \times 11.10}{20.0}\right) = 3.362 \text{ rad}$$ Figure 7.13

$$\overline{y} = \frac{2h\sin^3(\theta / 2)}{3(\theta - \sin\theta)} = \frac{2 \times 20.0 \times \sin^3(3.362 / 2)}{3 \times (3.362 - \sin 3.362)} = 3.66 \text{ in.}$$

$$C = \frac{0.85 f_c' h^2}{8}(\theta - \sin\theta) = \frac{0.85 \times 4 \times 20.0^2}{8} \times (3.362 - \sin 3.362) = 608.7 \text{ kips}$$

The strains, stresses, and forces at each layer of reinforcement are given in Table 7.38.

Table 7.38 Summary of Reinforcement Strains, Stresses, and Forces for Stress in Reinforcement Farthest from Compression Face = $0.5f_y$

d_{si} (in.)	ε_{si}	f_{si} (ksi)	F_{si} (kips)
2.44	0.00244	60.0	56.6
10.00	0.00070	20.4	34.0
17.56	−0.00104	−30.0	−30.0

$$P_n = C + \Sigma F_{si} = 608.7 + (56.6 + 34.0 - 30.0) = 669.3 \text{ kips}$$

$$\phi P_n = 0.65 \times 669.3 = 435.1 \text{ kips}$$

$$M_n = C\overline{y} + \Sigma F_{si}(0.5h - d_i)$$

$$= \{(608.7 \times 3.66) + [56.6 \times (10.0 - 2.44)] + [34.0 \times (10.0 - 10.0)] + [(-30.0) \times (10.0 - 17.56)]\} / 12 = 240.2 \text{ ft-kips}$$

$$\phi M_n = 0.65 \times 240.2 = 156.1 \text{ ft-kips}$$

E. Pure bending

There is no closed-form solution to determine the depth of the neutral axis for the case of pure bending. From trial-and-error, it is found that $c = 4.52$ in.

$$a = \beta_1 c = 0.85 \times 4.52 = 3.84 \text{ in.}$$

$$\theta = 2\cos^{-1}\left(1 - \frac{2a}{h}\right) = 2 \times \cos^{-1}\left(1 - \frac{2 \times 3.84}{20.0}\right) = 1.814 \text{ rad}$$ Figure 7.13

$$\overline{y} = \frac{2h\sin^3(\theta / 2)}{3(\theta - \sin\theta)} = \frac{2 \times 20.0 \times \sin^3(1.814 / 2)}{3 \times (1.814 - \sin 1.814)} = 7.73 \text{ in.}$$

$$C = \frac{0.85 f_c' h^2}{8}(\theta - \sin\theta) = \frac{0.85 \times 4 \times 20.0^2}{8} \times (1.814 - \sin 1.814) = 143.4 \text{ kips}$$

The strains, stresses, and forces at each layer of reinforcement are given in Table 7.39.

Table 7.39 Summary of Reinforcement Strains, Stresses, and Forces for Pure Bending

d_{si} (in.)	ε_{si}	f_{si} (ksi)	F_{si} (kips)
2.44	0.00138	40.0	36.6
10.00	−0.00364	−60.0	−120.0
17.56	−0.00865	−60.0	−60.0

$$P_n = C + \Sigma F_{si} = 143.4 + (36.6 - 120.0 - 60.0) = 0$$

$$M_n = C\overline{y} + \Sigma F_{si}(0.5h - d_i)$$

$$= \{(143.4 \times 7.73) + [36.6 \times (10.0 - 2.44)] + [(-120.0) \times (10.0 - 10.0)] + [(-60.0) \times (10.0 - 17.56)]\} / 12 = 153.2 \text{ ft-kips}$$

$\phi M_n = 0.9 \times 153.2 = 137.9$ ft-kips

The nominal and design strength interaction diagrams for this column are given in Figure 7.48.

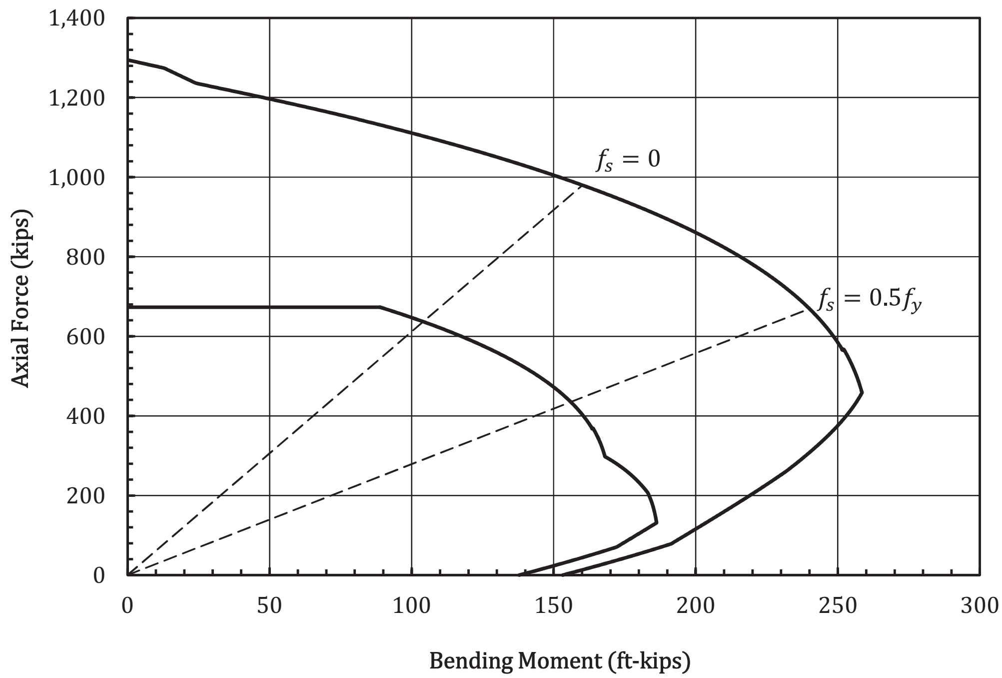


Figure 7.48 Nominal and design strength interaction diagrams for the column in Example 7.13.

7.11.14 Example 7.14 – Determination of Preliminary Column Size: Building #1 (Framing Option C), Rectangular, Tied Column is Part of the LFRS, SDC A, Column Subjected to Uniaxial Bending and Axial Forces

Determine a preliminary size for column C1 in the first story of Building #1, Framing Option C for wind forces in the north-south direction assuming a rectangular tied column (see Figure 1.1). Also assume a 7.0-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the factored axial forces and bending moments Table 3.3

$$\text{Dead load of slab} = (7.0 / 12) \times 150.0 = 87.5 \text{ lb/ft}^2$$

$$\text{Dead load of beam webs} = \frac{(24.0 - 7.0) \times 28.0 \times 150.0}{144 \times 1{,}000} \times [23.5 + (25.0 / 2)] = 17.9 \text{ kips}$$

Roof: Superimposed dead load $= 12$ lb/ft^2

Live load $= 20$ lb/ft^2

Floors: Superimposed dead load $= 10$ lb/ft^2

Live load $= 15$ lb/ft^2 (nonreducible partitions) $+ 50$ lb/ft^2

$$\text{Tributary area to column} = 23.5 \times (25.0 / 2) = 293.8 \text{ sq ft}$$

This column is part of the LFRS and the bending moments due to gravity load effects are small compared to the bending moments due to wind load effects in the north-south direction, so the former are not considered.

A summary of the axial forces and bending moments for column C1 in the first story from an analysis of the structure is given in Table 7.40 for sidesway to the right (SSR) and sidesway to the left (SSL).

Table 7.40 Summary of Axial Forces and Bending Moments for Column C1

Load Case			Axial Force (kips)	Bending Moment (ft-kips)	
				Top	Bottom
Dead (D)			233.3	0	0
Roof live (L_r)			5.9	0	0
Live (L)			76.4	0	0
Wind (W)			0	±20.4	±63.9
Load Combination					
ACI Eq. (5.3.1a)	$1.4D$		326.6	0	0
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		405.2	0	0
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		327.6	0	0
	$1.2D + 1.6L_r + 0.5W$	SSR	289.4	−10.2	−32.0
		SSL	289.4	10.2	32.0
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L + 0.5L_r$	SSR	321.1	−20.4	−63.9
		SSL	321.1	20.4	63.9
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	SSR	210.0	−20.4	−63.9
		SSL	210.0	20.4	63.9

Step 2 – Determine the gross area of the column

A preliminary column size is initially determined based on factored axial force only.

Assuming a 2 percent reinforcement ratio for the longitudinal reinforcement $(\rho_g = 0.02)$, the required gross area of a tied column is the following using the largest P_u in Table 7.40:

$$A_g = \frac{P_u}{\phi 0.80[0.85 f_c'(1-\rho_g) + f_y \rho_g]} = \frac{405.2}{0.65 \times 0.80 \times \{[0.85 \times 4 \times (1-0.02)] + (60 \times 0.02)\}} = 171.9 \text{ in.}^2 \qquad \text{Eq. (7.30)}$$

A 14-in. square column has a gross area $A_g = 196.0 \text{ in.}^2$

However, a 20-in. square column is selected for the following reasons: (1) the column is subjected to bending moments that have not been accounted for in the initial design, (2) it has not been determined whether the column is part of a nonsway or sway frame and whether slenderness effects need to be considered or not and (3) preliminary analysis indicates that at least 20-in. square columns are needed to provide sufficient overall lateral stiffness for the building. The column size can be adjusted accordingly once more calculations are performed.

7.11.15 Example 7.15 – Determination of Nonsway or Sway Frame: Building #1 (Framing Option C), Rectangular, Tied Column is Part of the LFRS, SDC A

Determine whether the first story of Building #1, Framing Option C is a nonsway or sway frame assuming all the columns are 20-in. square (see Figure 1.1). Also assume a 7.0-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 7.14 for a summary of the loads at the various levels.

Step 1 – Determine the total loads in the first story

$$\text{Total area per level} = 151.67 \times 95.67 = 14{,}510 \text{ sq ft}$$

- Roof: $P_D = (87.5 + 12.0) \times 14{,}510 / 1{,}000 = 1{,}444$ kips
  $P_{L_r} = 20.0 \times 14{,}510 / 1{,}000 = 290$ kips
- Typical floor: $P_D = (87.5 + 10.0) \times 14{,}510 / 1{,}000 = 1{,}415$ kips
  $P_L = 65.0 \times 14{,}510 / 1{,}000 = 943$ kips

$$\text{Dead load of columns (total)} = \frac{20.0 \times 20.0 \times 150.0}{144 \times 1{,}000} \times 60.0 \times 35 = 875 \text{ kips}$$

$$\text{Dead load of beam webs (total)} = \frac{(24.0 - 7.0) \times 28.0 \times 150.0}{144 \times 1{,}000} \times [(28 \times 21.83) + (30 \times 23.33)] \times 5 = 3{,}251 \text{ kips}$$

$$\text{Total } P_D = 1{,}444 + (4 \times 1{,}415) + 875 + 3{,}251 = 11{,}230 \text{ kips}$$

$$\text{Total } P_L = 4 \times 943 = 3{,}772 \text{ kips}$$

The total factored load must correspond to the lateral loading case for which it is a maximum. In this example, ACI Eq. (5.3.1d) produces the largest load:

$$\Sigma P_u = 1.2 P_D + 1.0 P_W + 0.5 P_L + 0.5 P_{L_r} = 15{,}507 \text{ kips}$$

Step 2 – Determine the stability index for the first story

An elastic analysis of the structure was performed with the building subjected to the north-south wind forces in Table 3.10 assuming the columns are fixed at the base. The reduced moments of inertia in Table 7.1 were used for the columns, beams, and slabs. The following results were obtained from the analysis:

Factored horizontal shear in the first story: $V_{us} = 232.9$ kips

Relative lateral deflection between the top and bottom of the first story: $\Delta_o = 0.10$ in.

The stability index, Q, is determined from the following equation:

$$Q = \frac{\Sigma P_u \Delta_o}{V_{us} \ell_c} = \frac{15{,}507 \times 0.10}{232.9 \times (14.0 \times 12)} = 0.04 < 0.05 \qquad \text{Table 7.3}$$

Because $Q < 0.05$, the frame in the first story is a nonsway frame (see ACI 6.6.4.3).

7.11.16 Example 7.16 – Check if Slenderness Effects Must be Considered: Building #1 (Framing Option C), Rectangular, Tied Column is Part of the LFRS, SDC A, Nonsway Frame

Check if slenderness effects must be considered for column C1 in the first story of Building #1, Framing Option C assuming the column is 20-in. square (see Figure 1.1). Also assume a 7.0-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.14 and 7.15.

It is determined in Example 7.15 that the frame in the first story is nonsway in the north-south direction. Therefore, slenderness effects can be neglected when the following equation is satisfied:

$$\frac{k\ell_u}{r} \le \begin{cases} 34 + 12(M_1 / M_2) \\ 40 \end{cases} \qquad \text{Table 7.6}$$

For all load combinations including wind, $M_1 / M_2 = 10.2 / 32.0 = 20.4 / 63.9 = 0.32.$ Table 7.40

The column is bent in double curvature for all load combinations; therefore, slenderness can be neglected where the following is satisfied:

$$\frac{k\ell_u}{r} \le \begin{cases} 34 + (12 \times 0.32) = 38 \\ 40 \end{cases}$$

Assuming the effective length factor, k, is equal to the 1.0 (which is the maximum value for nonsway frames; see Table 7.4):

$$\frac{k\ell_u}{r} = \frac{1.0 \times [(14.0 \times 12) - (24.0 / 2)]}{0.3 \times 20.0} = 26 < 38$$

Therefore, slenderness effects need not be considered.

7.11.17 Example 7.17 – Determination of Longitudinal Reinforcement: Building #1 (Framing Option C), Rectangular, Tied Column is Part of the LFRS, SDC A, Nonsway Frame, Column Subjected to Uniaxial Bending and Axial Forces

Determine the required longitudinal reinforcement for column C1 in the first story of Building #1, Framing Option C assuming a 20-in. square tied column (see Figure 1.1). Also assume a 7.0-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.14 through 7.16.

Step 1 – Determine the required area of longitudinal reinforcement Eq. (7.32)

Based on a 20-in. square tied column, the required reinforcement ratio, ρ_g, is equal to the following for the largest factored axial force in Table 7.40, which occurs without any appreciable bending moments:

$$\rho_g = \frac{(P_u / \phi\phi_c A_g) - 0.85 f_c'}{f_y - 0.85 f_c'} = \frac{[405.2 / (0.65 \times 0.80 \times 20.0^2)] - (0.85 \times 4)}{60 - (0.85 \times 4)} < 0$$

Therefore, use the minimum required $\rho_g = 0.01$:

$A_{st} = 0.01 \times 20.0^2 = 4.00 \text{ in.}^2$

Select 4-#9 bars ($A_{st,provided} = 4.00 \text{ in.}^2$).

Step 2 – Check if the selected longitudinal reinforcement is adequate for all load combinations

The design strength interaction diagram for this column reinforced with 4-#9 bars is given in Figure 7.49. Also shown in the figure are load combination points for the factored axial forces and bending moments in Table 7.40.

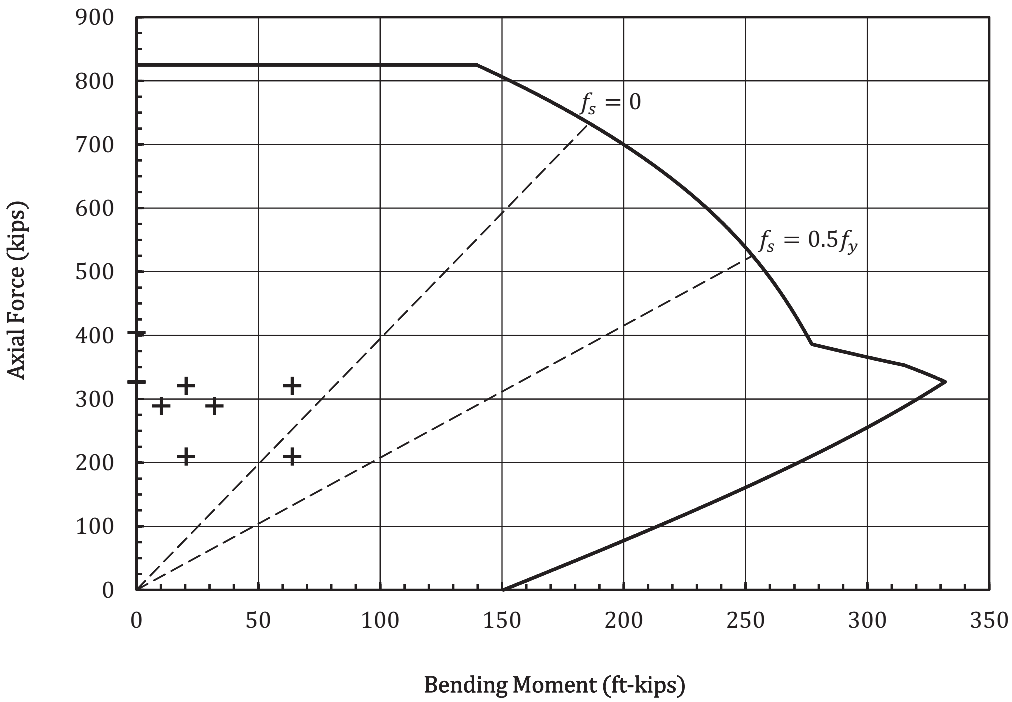


Figure 7.49 Design strength interaction diagram for column C1.

It is evident from the figure that the column is adequate for all factored load combinations.

Step 3 – Check the minimum number of longitudinal bars and the minimum face dimension of the column

For rectangular, tied columns:

Minimum number of longitudinal bars with rectangular ties $= 4$ — Table 7.16

Minimum face dimension $= 10$ in. for 2-#9 bars per face and normal lap splices < 20 in. — Table 7.17

Use 4-#9 bars.

Comments. It is evident from Figure 7.49 that the 20-in. column with 4-#9 bars possesses significantly more strength than required based on the factored load combinations. However, as noted in Example 7.14, 20-in. columns are needed primarily for lateral stiffness of the moment frames. Furthermore, the column is also part of the LFRS in the east-west direction and must also be designed for combined gravity and wind load effects for wind in that direction. Decreasing the amount of longitudinal reinforcement below 1 percent in accordance with ACI 10.3.1.2 is not done in this example.

7.11.18 Example 7.18 – Determination of Transverse Reinforcement: Building #1 (Framing Option C), Rectangular, Tied Column is Part of the LFRS, SDC A, Nonsway Frame, Column Subjected to Uniaxial Bending and Axial Forces

Determine the required transverse reinforcement for column C1 in the first story of Building #1, Framing Option C assuming a 20-in. square tied column (see Figure 1.1). Also assume a 7.0-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.14 through 7.17.

Step 1 – Determine the required tie bar size ACI 25.7.2.2

The required tie bar size is determined using ACI 25.7.2.2.

For #9 longitudinal bars, use #3 ties.

Step 2 – Determine the required tie bar spacing ACI 25.7.2.1

The required tie bar spacing is determined using ACI 25.7.2.1; the tie bar size and spacing will then be checked to determine if they are adequate for shear strength requirements.

$$s \leq \text{ lesser of} \begin{cases} 16d_{b(column)} = 16 \times 1.128 = 18.1 \text{ in.} \\ \\ 48d_{b(tie)} = 48 \times 0.375 = 18.0 \text{ in.} \\ \\ c_1 = 20.0 \text{ in.} \end{cases}$$

$$\geq (4\,/\,3)d_{agg} + d_{b(tie)} = [(4\,/\,3) \times 0.75] + 0.375 = 1.375 \text{ in.}$$

Try #3 ties spaced at 18 in. on center.

Step 3 – Check the adequacy of the tie bar size and spacing for shear ACI 22.5

Maximum factored shear force, V_u, is obtained from ACI Eqs. (5.3.1d) and (5.3.1f) based on the factored bending moment due to the wind load effects and is equal to 6.5 kips (see Table 7.40 and Figure 7.50).

Check if the design shear strength of the concrete, ϕV_c, alone is adequate, which is determined based on whether $A_{v,min}$ is provided or not (see Table 7.12).

$$A_{v,min} = \text{greater of} \begin{cases} \dfrac{0.75\sqrt{f_c'}b_w s}{f_{yt}} = \dfrac{0.75 \times \sqrt{4{,}000} \times 20.0 \times 18.0}{60{,}000} = 0.29 \text{ in.}^2 \\ \\ \dfrac{50b_w s}{f_{yt}} = \dfrac{50 \times 20.0 \times 18.0}{60{,}000} = 0.30 \text{ in.}^2 \end{cases} \qquad \text{Eq. (7.26)}$$

The minimum area of shear reinforcement is greater than $A_v = 2 \times 0.11 = 0.22 \text{ in.}^2$ provided by the #3 ties spaced 18 in. on center. Because V_u is relatively small, it is not necessary to increase the tie bar size and/or decrease the spacing, so V_c is determined based on $A_v < A_{v,min}$:

$$V_c = \left[8\lambda_s \lambda (\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d \qquad \text{Table 7.12}$$

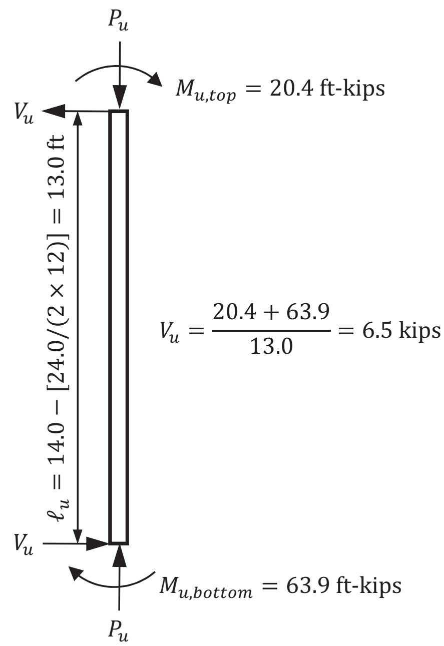


Figure 7.50 Factored shear force for column C1.

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (17.56 / 10)}} = 0.85$$ Eq. (7.23)

$\lambda = 1.0$ for normalweight concrete Table 7.13

$\rho_w = \frac{A_s}{b_w d} = \frac{2 \times 1.00}{20.0 \times 17.56} = 0.0057$ (there are 2-#9 bars located more than two-thirds of the overall member depth from the extreme compression fiber).

Use minimum $N_u = 210.0$ kips corresponding to ACI Eq. (5.3.1f) because this results in minimum V_c.

$N_u / 6A_g = 210{,}000 / (6 \times 20.0^2) = 87.5 \text{ psi} < 0.05 f_c' = 200.0 \text{ psi}$ ACI 22.5.5.1.2

$$V_c = \{[8 \times 0.85 \times 1.0 \times (0.0057)^{1/3} \times \sqrt{4{,}000}] + 87.5\} \times 20.0 \times 17.56 / 1{,}000 = 57.7 \text{ kips}$$

$$< 5\lambda\sqrt{f_c'} b_w d = 111.1 \text{ kips}$$

$\phi V_c = 0.75 \times 57.7 = 43.3 \text{ kips} > V_u = 6.5 \text{ kips}$

Also, $V_u = 6.5 \text{ kips} < 0.5\phi V_c = 21.7 \text{ kips},$ so minimum shear reinforcement is not required. ACI 10.6.2.1

Therefore, use #3 ties spaced at 18 in. on center.

7.11.19 Example 7.19 – Determination of Dowel Reinforcement at the Foundation: Building #1 (Framing Option C), Rectangular, Tied Column is Part of the LFRS, SDC A, Nonsway Frame, Column Subjected to Uniaxial Bending and Axial Forces

Determine the required dowel reinforcement for column C1 in the first story of Building #1, Framing Option C assuming a 20-in. square tied column supported by a $2'\text{-}6'' \times 8'\text{-}6'' \times 8'\text{-}6''$ spread footing (see Figure 1.1). Also assume a 7.0-in.-thick slab, normalweight concrete with $f_c' = 4{,}000 \text{ psi (maximum aggregate size} = 0.75 \text{ in.)}$, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.14 through 7.18.

Step 1 – Check the bearing stresses on the column and footing ACI 16.3.3.4

Bearing strength of the column.

Factored bearing stress, b_u, is determined for pure compression using the factored axial force determined by ACI Eqs. (5.3.1b) and for compression plus bending using the factored axial force and bending moment determined by ACI Eqs. (5.3.1d) and (5.3.1f) [see Table 7.40].

- ACI Eq. (5.3.1b):

$$\text{Axial compression stress} = b_u = \frac{P_u}{A_1} = \frac{405.2 \times 1{,}000}{20.0^2} = 1{,}013 \text{ psi}$$

- ACI Eq. (5.3.1d):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{321.1 \times 1{,}000}{20.0^2} = 803 \text{ psi}$$

$$\text{Bending stress (compression and tension)} = \frac{6M_u}{c_1^3} = \frac{6 \times 63.9 \times 12{,}000}{20.0^3} = 575 \text{ psi}$$

$$\text{Maximum compression stress} = 803 + 575 = 1{,}378 \text{ psi}$$

$$\text{Minimum compression stress} = 803 - 575 = 228 \text{ psi}$$

- ACI Eq. (5.3.1f):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{210.0 \times 1{,}000}{20.0^2} = 525 \text{ psi}$$

$$\text{Bending stress (compression and tension)} = \frac{6M_u}{c_1^3} = \frac{6 \times 63.9 \times 12{,}000}{20.0^3} = 575 \text{ psi}$$

$$\text{Maximum compression stress} = 525 + 575 = 1{,}100 \text{ psi}$$

$$\text{Maximum tension stress} = 525 - 575 = -50 \text{ psi}$$

Design bearing strength:

$$\phi b_n = \phi 0.85 f_c' = 0.65 \times 0.85 \times 4{,}000 = 2{,}210 \text{ psi}$$

Tension forces cannot be transferred through bearing at the interface, so dowel bars will be provided that match the size of the longitudinal bars in the column. This reinforcement ensures that both the compression and tension forces are adequately transferred from the column into the footing.

Bearing strength of the footing.

There is no need to check the bearing strength of the footing because interface reinforcement must be provided due to the net tension stress from the factored axial force and bending moment determined by ACI Eq. (5.3.1f).

Step 2 – Determine the required interface reinforcement ACI 16.3.4.1

Try 4-#9 dowel bars. This reinforcement matches the longitudinal reinforcement in the column and ensures that both the compression and tension forces are adequately transferred through the interface.

Check minimum area of reinforcement across the interface:

$$A_{s,min} = 0.005A_g = 0.005 \times 20.0^2 = 2.00 \text{ in.}^2 < A_{s,provided} = 4.00 \text{ in.}^2$$

Step 3 – Check the development of the dowel bars in the footing

It is evident from Figure 7.49 that the stress in the longitudinal reinforcement farthest from the extreme compression face is tensile for one of the load combinations (that is, the load combination falls within Zone 2 of the interaction diagram; see Figure 7.28). Therefore, the dowel bars must be developed for tension into the footing (and into the column).

A standard 90-degree hook will be provided at the ends of the #9 dowel bars. Determine the tension development length, ℓ_{dh}, of the #9 dowel bars terminating in a hook:

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \quad \text{Eq. (7.44)}$$

$\psi_e = 1.0$ for uncoated bars Table 7.27

$\psi_r = 1.6$ (confining reinforcement not provided)

$\psi_o = 1.0$ (side cover normal to hook $> 6d_b$)

$\psi_c = (f_c' / 15{,}000) + 0.6 = (4{,}000 / 15{,}000) + 0.6 = 0.9$

$\lambda = 1.0$ for normalweight concrete

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} = \left(\dfrac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.9}{55 \times 1.0 \times \sqrt{4{,}000}}\right) \times 1.128^{1.5} = 29.8 \text{ in.} \\ 8d_b = 8 \times 1.128 = 9.0 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

Assuming two layers of #9 flexural bars in the footing with a clear cover of 3 in., determine the minimum footing thickness for the development of the dowel bars:

$$\text{Minimum } h = \ell_{dh} + 2(d_b)_f + \text{ cover} = 29.8 + (2 \times 1.128) + 3.0 = 35.1 \text{ in.} \quad \text{Eq. (7.45)}$$

The minimum $h = 35.1$ in. is greater than the provided footing thickness of 30.0 in. Thus, increase the footing thickness to 36.0 in. to accommodate development of the #9 dowel bars.

Step 4 – Determine the development length of the dowel bars in the column

The dowel bars must be lap spliced to the longitudinal reinforcement in the column using a tension lap splice. Because all the dowel bars are spliced at the same location, a Class B tension lap splice is required (ACI Table 25.5.2.1).

Development length in tension, ℓ_d, of the #9 bars:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \quad \text{Eq. (7.36)}$$

$\psi_t = 1.0$ Table 7.24

$\psi_e = 1.0$ for uncoated bars

$\psi_s = 1.0$ for #9 bars

$\psi_g = 1.0$ for Grade 60 reinforcement

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + (d_b)_{tie} + 0.5(d_b)_{long.} = 1.5 + 0.375 + (0.5 \times 1.128) = 2.4 \text{ in.} \\ \dfrac{s}{2} = \dfrac{20.0 - (2 \times 1.5) - (2 \times 0.375) - 1.128}{2} = 7.6 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.4 + 0) / 1.128 = 2.1 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60,000}{1.0 \times \sqrt{4,000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.1} \right) \times 1.128 = 38.2 \text{ in.} > 12.0 \text{ in.}$$

$$\text{Class B lap splice length} = 1.3\ell_d = 1.3 \times 38.2 = 49.7 \text{ in.}$$

Provide a lap splice length of 4 ft-2 in.

Step 5 – Check horizontal force transfer

It is determined in Example 7.18 that $V_u = 6.5$ kips is transferred horizontally between the column and footing. The required area of shear-friction reinforcement is determined as follows assuming the surface between the column and footing has not been intentionally roughened:

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{V_u}{\phi f_y (0.6\lambda)} = \frac{6.5}{0.75 \times 60 \times (0.6 \times 1.0)} = 0.24 \text{ in.}^2 \quad \text{Eq. (7.46), Table 7.28}$$

The 4-#9 dowel bars provides 4.00 in.^{2} across the interface, which is greater than the required amount of 0.24 in.^{2}

Check the upper shear limit:

$$V_u = 6.5 \text{ kips} < \text{ least of } \begin{cases} \phi 0.2 f'_c A_c = 0.75 \times 0.2 \times 4 \times 20.0^2 = 240.0 \text{ kips} \\ \\ \phi 800 A_c = 0.75 \times 800 \times 20.0^2 / 1{,}000 = 240.0 \text{ kips} \end{cases}$$ Table 7.29

Reinforcement details for this column are given in Figure 7.51.

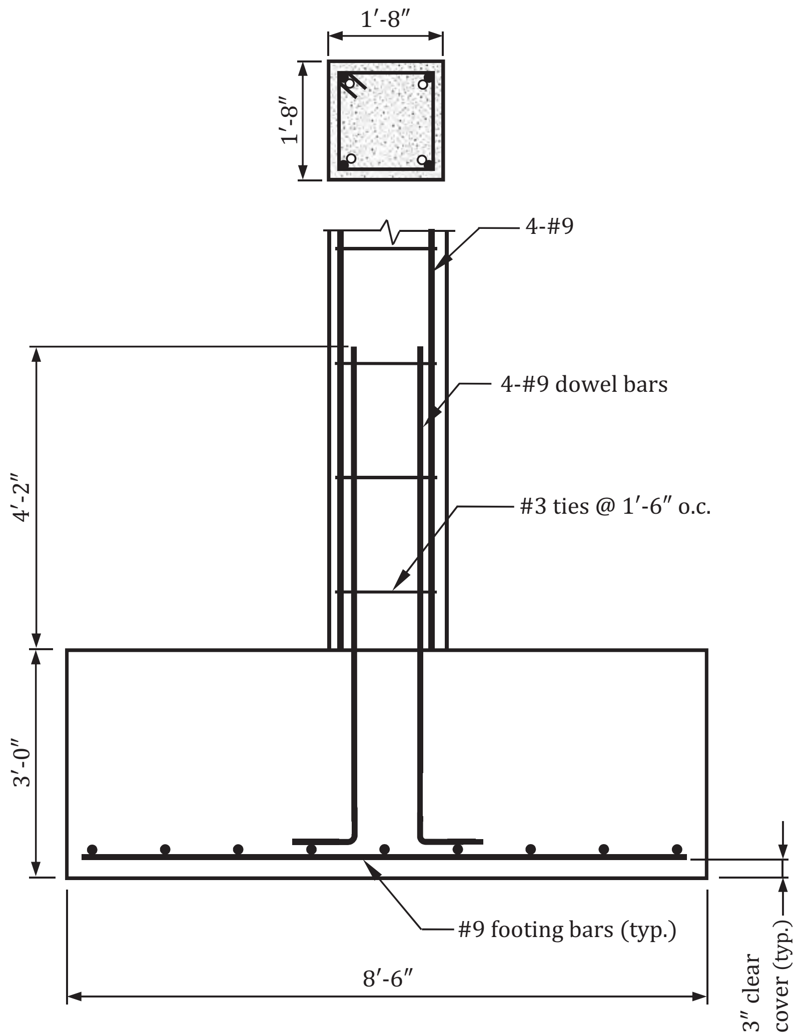


Figure 7.51 Reinforcement details for column C1 in Examples 7.14 through 7.19.

7.11.20 Example 7.20 – Determination of Longitudinal Reinforcement: Building #1 (Framing Option C), Rectangular, Tied Column is Part of the LFRS, SDC A, Nonsway Frame, Column Subjected to Biaxial Bending and Axial Forces

Determine the required longitudinal reinforcement for column E1 in the first story of Building #1, Framing Option C assuming a 20-in. square tied column (see Figure 1.1). Also assume a 7.0-in.-thick slab, normalweight concrete with $f'_c = 4{,}000$ psi (maximum aggregate size $= 0.75$ in.), and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the required area of longitudinal reinforcement

This corner column is subjected to axial forces and biaxial bending, so there is no direct way of determining the required area of longitudinal reinforcement.

Therefore, initially try the minimum required $\rho_g = 0.01$:

$A_{st} = 0.01 \times 20.0^2 = 4.00$ in.^{2}

Select 4-#9 bars ($A_{st,provided} = 4.00$ in.^{2}).

Step 2 – Check if the selected longitudinal reinforcement is adequate for all load combinations

An elastic analysis of the structure was performed with the building subjected to the north-south and east-west wind forces in Table 3.10 assuming the columns are fixed at the base. The critical wind loading for a corner column occurs when 75 percent of the wind forces in both directions are applied simultaneously to the structure (Load Case 3 in ASCE/SEI Fig. 27.3-8). The reduced moments of inertia in Table 7.1 were used for the columns, beams, and slabs in the analysis.

A summary of the axial forces and bending moments for column E1 in the first story is given in Table 7.41 for sidesway to the right (SSR) and sidesway to the left (SSL) in both principal directions.

Table 7.41 Summary of Axial Forces and Bending Moments for Column E1

Load Case			Axial Force (kips)	Bending Moment (ft-kips)			
				North-South		East-West	
				Top	Bottom	Top	Bottom
Dead (D)			161.5	–25.2	15.5	27.7	–17.1
Roof live (L_r)			2.9	—	—	—	—
Live (L)			38.3	–11.0	6.8	11.9	–7.3
Wind (W)			±10.1	±8.5	±43.7	±4.4	±24.5
Load Combination							
ACI Eq. (5.3.1a)	$1.4D$		226.1	–35.3	21.7	38.8	–23.9
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		256.5	–47.8	29.5	52.3	–32.2
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		217.6	–35.7	22.0	39.2	–24.2
	$1.2D + 1.6L_r + 0.5W$	SSR	193.4	–34.5	–3.3	31.0	–32.8
		SSL	203.5	–26.0	40.5	35.4	–8.3
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L + 0.5L_r$	SSR	204.3	–44.2	–21.7	34.8	–48.7
		SSL	224.5	–27.2	65.7	43.6	0.3
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	SSR	135.3	–31.2	–29.8	20.5	–39.9
		SSL	155.5	–14.2	57.7	29.3	9.1

It is determined in Example 7.15 that the first story of this building is nonsway. It can also be determined that slenderness effects need not be considered.

With 4-#9 longitudinal bars, this column has the same bending moment capacity about both principal axes. Thus, the approximate method depicted in Figure 7.18 can be used to determine the adequacy of the column for each load combination in Table 7.41.

To illustrate the design procedure, the column will be checked for the factored axial force and bending moments obtained from ACI Eq. (5.3.1d) for sidesway to the right.

At an axial force $P_u = 204.3$ kips, the uniaxial design moment strengths are the following (see Figure 7.52):

$(\phi M_{no})_{NS} = (\phi M_{no})_{EW} = 274.0$ ft-kips

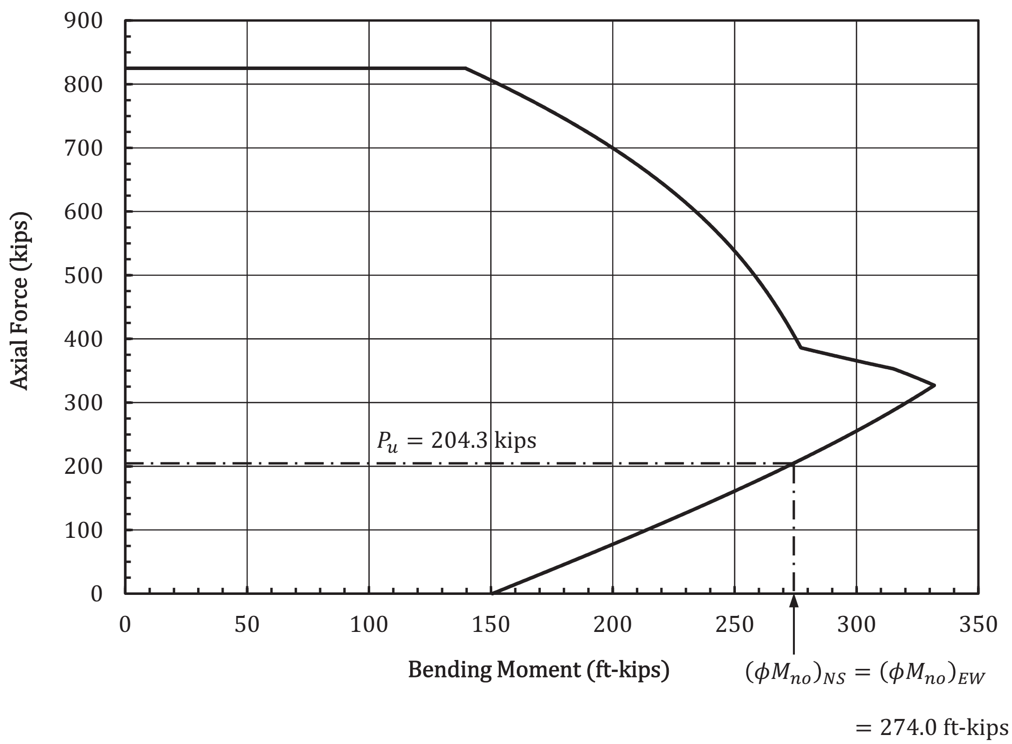


Figure 7.52 Uniaxial design strength interaction diagram for column E1.

The approximate biaxial strength diagram corresponding to $P_u = 204.3$ kips is given in Figure 7.53. It is evident from the figure that the column is adequate for the applied factored biaxial moments.

It can be determined in a similar fashion that the column is adequate for all the other load combinations.

Use 4-#9 longitudinal bars.

Comments. For comparison purposes, Reference 14 was used to check the adequacy of this column. As expected, the column was found to be adequate.

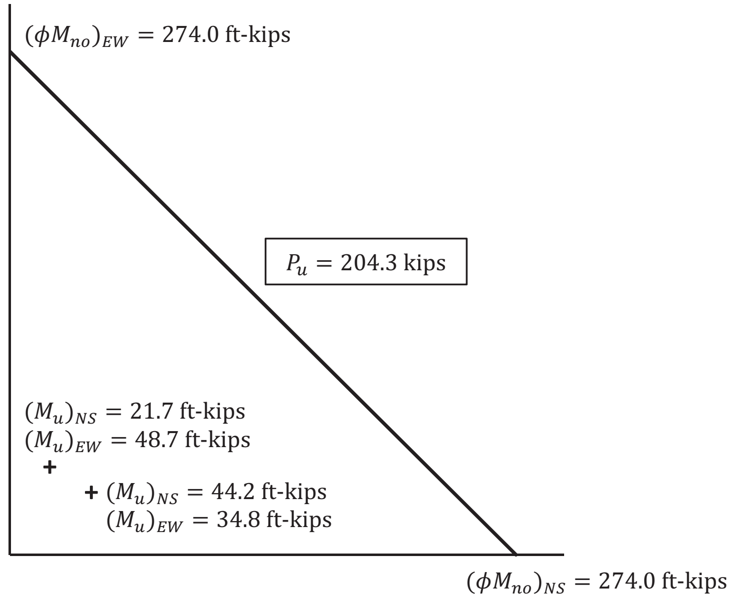


Figure 7.53 Approximate biaxial strength diagram for column E1.

7.11.21 Example 7.21 – Determination of Transverse Reinforcement: Building #1 (Framing Option C), Rectangular, Tied Column is Part of the LFRS, SDC A, Nonsway Frame, Column Subjected to Biaxial Bending and Axial Forces

Determine the required transverse reinforcement for column E1 in the first story of Building #1, Framing Option C assuming a 20-in. square tied column (see Figure 1.1). Also assume a 7.0-in.-thick slab, normalweight concrete with $f_c' = 4{,}000$ psi (maximum aggregate size $= 0.75$ in.), and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 7.20.

Step 1 – Determine the required tie bar size ACI 25.7.2.2

The required tie bar size is determined using ACI 25.7.2.2.

For #9 longitudinal bars, use #3 ties.

Step 2 – Determine the required tie bar spacing ACI 25.7.2.1

The required tie bar spacing is determined using ACI 25.7.2.1; the tie bar size and spacing will then be checked to determine if they are adequate for shear strength requirements.

$$s \le \text{ lesser of} \begin{cases} 16d_{b(column)} = 16 \times 1.128 = 18.1 \text{ in.} \\ 48d_{b(tie)} = 48 \times 0.375 = 18.0 \text{ in.} \\ c_1 = 20.0 \text{ in.} \end{cases}$$

$$\ge (4/3)d_{agg} + d_{b(tie)} = [(4/3) \times 0.75] + 0.375 = 1.375 \text{ in.}$$

Try #3 ties spaced at 18 in. on center.

Step 3 – Check the adequacy of the tie bar size and spacing for shear ACI 22.5

Maximum factored shear force, V_u, is obtained from ACI Eq. (5.3.1d) based on the factored bending moments at the top and bottom of the column (see Table 7.41):

- North-south wind and SSL:

$$(V_u)_{NS} = \frac{27.2 + 65.7}{14.0 - \left(\frac{24.0}{2 \times 12}\right)} = 7.2 \text{ kips}$$

- East-west wind and SSR:

$$(V_u)_{EW} = \frac{34.8 + 48.7}{14.0 - \left(\frac{24.0}{2 \times 12}\right)} = 6.4 \text{ kips}$$

Check if the design shear strength of the concrete, ϕV_c, alone is adequate, which is determined based on whether $A_{v,min}$ is provided or not (see Table 7.12).

$$A_{v,min} = \text{greater of} \begin{cases} \dfrac{0.75\sqrt{f_c'}b_w s}{f_{yt}} = \dfrac{0.75 \times \sqrt{4{,}000} \times 20.0 \times 18.0}{60{,}000} = 0.29 \text{ in.}^2 \\ \\ \dfrac{50 b_w s}{f_{yt}} = \dfrac{50 \times 20.0 \times 18.0}{60{,}000} = 0.30 \text{ in.}^2 \end{cases}$$ Eq. (7.26)

The minimum area of shear reinforcement is greater than $A_v = 0.15$ in.^{2} provided by the #3 ties spaced 18 in. on center. Because V_u is relatively small in both directions, it is not necessary to increase the tie bar size and/or decrease the spacing, so V_c is determined based on $A_v < A_{v,min}$:

$$V_c = \left[8\lambda_s \lambda (\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g}\right] b_w d$$ Table 7.12

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (17.56 / 10)}} = 0.85$$ Eq. (7.23)

where $d = 20.0 - 1.5 - 0.375 - (1.128 / 2) = 17.56$ in.

$\lambda = 1.0$ for normalweight concrete Table 7.13

$\rho_w = \dfrac{A_s}{b_w d} = \dfrac{2 \times 1.00}{20.0 \times 17.56} = 0.0057$ (there are 2-#9 bars located more than two-thirds of the overall member depth

from the extreme compression fiber).

- North-south wind:
  Use axial force $N_u = 224.5$ kips, which corresponds to ACI Eq. (5.3.1d) and sidesway to the left.

  $N_u / 6A_g = 224{,}500 / (6 \times 20.0^2) = 93.5 \text{ psi} < 0.05 f_c' = 200.0 \text{ psi}$ ACI 22.5.5.1.2

$$(V_c)_{NS} = \{[8 \times 0.85 \times 1.0 \times (0.0057)^{1/3} \times \sqrt{4,000}] + 93.5\} \times 20.0 \times 17.56 / 1,000 = 59.8 \text{ kips}$$

$$< 5\lambda\sqrt{f_c'}b_w d = 111.1 \text{ kips}$$

$$(\phi V_c)_{NS} = 0.75 \times 59.8 = 44.9 \text{ kips} > (V_u)_{NS} = 7.2 \text{ kips}$$

- East-west wind:

  Use axial force $N_u = 204.3$ kips, which corresponds to ACI Eq. (5.3.1d) and sidesway to the right.

$$N_u / 6A_g = 204,300 / (6 \times 20.0^2) = 85.1 \text{ psi} < 0.05f_c' = 200.0 \text{ psi}$$ ACI 22.5.5.1.2

$$(V_c)_{EW} = \{[8 \times 0.85 \times 1.0 \times (0.0057)^{1/3} \times \sqrt{4,000}] + 85.1\} \times 20.0 \times 17.56 / 1,000 = 56.9 \text{ kips}$$

$$< 5\lambda\sqrt{f_c'}b_w d = 111.1 \text{ kips}$$

$$(\phi V_c)_{EW} = 0.75 \times 56.9 = 42.7 \text{ kips} > (V_u)_{EW} = 6.4 \text{ kips}$$

Check if interaction of shear forces acting along both the north-south and east-west directions must be considered (see ACI 22.5.1.10):

$$(V_u)_{NS} / (\phi V_c)_{NS} = 7.2 / 44.9 = 0.16 < 0.5$$

$$(V_u)_{EW} / (\phi V_c)_{EW} = 6.4 / 42.7 = 0.15 < 0.5$$

Because the ratios are less than 0.5, interaction of shear forces are permitted to be neglected.

Therefore, use #3 ties spaced at 18 in. on center.

7.11.22 Example 7.22 – Determination of Longitudinal Reinforcement: Building #1 (Framing Option C), Rectangular, Tied Column is Part of the LFRS, SDC A, Nonsway Frame, Column Subjected to Uniaxial Bending and Axial Forces, Slenderness Effects

Determine the required longitudinal reinforcement for column C1 in the first story of Building #1, Framing Option C assuming a 12-in. square tied column must be used for this column only due to architectural reasons (see Figure 1.1). Also assume a 7.0-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f_c' = 4,000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the factored axial forces — Table 3.3

$$\text{Dead load of slab} = (7.0 / 12) \times 150.0 = 87.5 \text{ lb/ft}^2$$

$$\text{Dead load of beam webs} = \frac{(24.0 - 7.0) \times 28.0 \times 150.0}{144 \times 1,000} \times [23.5 + (25.0 / 2)] = 17.9 \text{ kips}$$

Roof: $\text{Superimposed dead load} = 12 \text{ lb/ft}^2$

$\text{Live load} = 20 \text{ lb/ft}^2$

Floors: Superimposed dead load $= 10\ \text{lb/ft}^2$

Live load $= 15\ \text{lb/ft}^2$ (nonreducible partitions) $+ 50\ \text{lb/ft}^2$

Tributary area to column $= 23.5 \times (25.0 / 2) = 293.8$ sq ft

This column is part of the LFRS for wind in the north-south direction and the bending moments due to gravity load effects are negligible.

A summary of the axial forces and bending moments for column C1 in the first story from an analysis of the structure is given in Table 7.42 for sidesway to the right (SSR) and sidesway to the left (SSL).

Table 7.42 Summary of Axial Forces and Bending Moments for Column C1

Load Case			Axial Force (kips)	Bending Moment (ft-kips)	
				Top	Bottom
Dead (D)			233.3	0	0
Roof live (L_r)			5.9	0	0
Live (L)			76.4	0	0
Wind (W)			0	±4.5	±6.3
Load Combination					
ACI Eq. (5.3.1a)	$1.4D$		326.6	0	0
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		405.2	0	0
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		327.6	0	0
	$1.2D + 1.6L_r + 0.5W$	SSR	289.4	2.3	–3.2
		SSL	289.4	–2.3	3.2
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L + 0.5L_r$	SSR	321.1	4.5	–6.3
		SSL	321.1	–4.5	6.3
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	SSR	210.0	4.5	–6.3
		SSL	210.0	–4.5	6.3

Step 2 – Check if slenderness effects must be considered

It is determined in Example 7.15 that the frame in the first story is nonsway (which is still valid even though the size of this one column has been reduced). Therefore, slenderness effects can be neglected when the following equation is satisfied:

$$\frac{k\ell_u}{r} \le \begin{cases} 34 + 12(M_1 / M_2) \\ \\ 40 \end{cases} \qquad \text{Table 7.6}$$

For all load combinations including wind, $M_1 / M_2 = 2.3 / 3.2 = 4.5 / 6.3 = 0.72.$ Table 7.42

The column is bent in double curvature for all load combinations; therefore, slenderness can be neglected where the following is satisfied:

$$\frac{k\ell_u}{r} \leq \begin{cases} 34 + (12 \times 0.72) = 42.6 \\ \\ 40 \end{cases}$$

Assuming the effective length factor, k, is equal to the 1.0 (which is the maximum value for nonsway frames; see Table 7.4):

$$\frac{k\ell_u}{r} = \frac{1.0 \times [(14.0 \times 12) - (24.0 / 2)]}{0.3 \times 12.0} = 43.3 > 40$$

Therefore, slenderness effects must be considered.

Step 3 – Determine the magnified moments ACI 6.6.4.5

Magnified moments, M_c, for each load combination are determined for this nonsway frame by the following equation:

$$M_c = \delta M_2 \qquad \text{Eq. (7.2)}$$

A summary of the magnification factors, δ, is given in Table 7.43 for each load combination with nonzero values of M_1 and M_2. It is assumed that the column is reinforced with 4-#9 longitudinal bars ($\rho_g = 0.028$) and #3 ties. The results for sidesway to the right and sidesway to the left are equal because the gravity bending moments are equal to zero.

Table 7.43 Magnified Moments for Column C1

Load Combination		β_{dns}	$(EI)_{eff}$ (kip-in^2)	P_c (kips)	C_m	δ	$M_{2,min}$ (ft-kips)	M_c (ft-kips)
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5W$	0.97	1,418,949	575.5	0.31	1.00	23.2	23.2
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L + 0.5L_r$	0.87	1,494,829	606.2	0.31	1.06	25.7	27.2
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	1.00	1,397,665	566.8	0.31	1.00	16.8	16.8

It is evident from Table 7.43 that $\delta < 1.4$ in all load combinations, which means the structural system is stable and need not be revised (ACI 6.2.6). Also, the minimum moment $M_{2,min} = P_u(0.6 + 0.03h)$ is greater than M_2 in all load combinations (see Table 7.42).

Sample calculations for the load combination corresponding to ACI Eq. (5.3.1d) are as follows:

$$E_c = w_c^{1.5} 33 \sqrt{f_c'} = 150.0^{1.5} \times 33 \times \sqrt{4{,}000} / 1{,}000 = 3{,}834 \text{ ksi} \qquad \text{ACI Eq. (19.2.2.1a)}$$

$$\beta_{dns} = \frac{1.2D}{1.2D + 1.0W + 0.5L + 0.5L_r} = \frac{1.2 \times 233.3}{321.1} = 0.87$$

Effective flexural stiffness, $(EI)_{eff}$, can be calculated by any of the equations in Table 7.9. ACI Eq. (6.6.4.4.4b) is used in this example:

$$(EI)_{eff} = \frac{(0.2E_cI_g + E_sI_{se})}{1+\beta_{dns}}$$

$$I_g = 12.0^4 / 12 = 1{,}728 \text{ in.}^4$$

The moment of inertia of the reinforcement about the centroidal axis of the column, I_{se}, is determined by multiplying the area of the reinforcing bars by the square of the distance from the centroid of the reinforcing bars to the centroid of the column (the moment of inertia of a reinforcing bar about its centroidal axis is negligible):

$$I_{se} = 2\times(2\times1.00)\times[(12.0/2) - 1.5 - 0.375 - (1.128/2)]^2 = 50.7 \text{ in.}^4$$

$$(EI)_{eff} = \frac{(0.2\times3{,}834\times1{,}728) + (29{,}000\times50.7)}{1+0.87} = 1{,}494{,}829 \text{ kip-in.}^2$$

$$P_c = \frac{\pi^2(EI)_{eff}}{(k\ell_u)^2} = \frac{\pi^2\times1{,}494{,}829}{[1.0\times(14.0-1.0)\times12]^2} = 606.2 \text{ kips}$$ Eq. (7.4)

For the column bent in double curvature:

$$C_m = 0.6 - 0.4(M_1/M_2) = 0.6 - [0.4\times(4.5/6.3)] = 0.31$$ Table 7.8

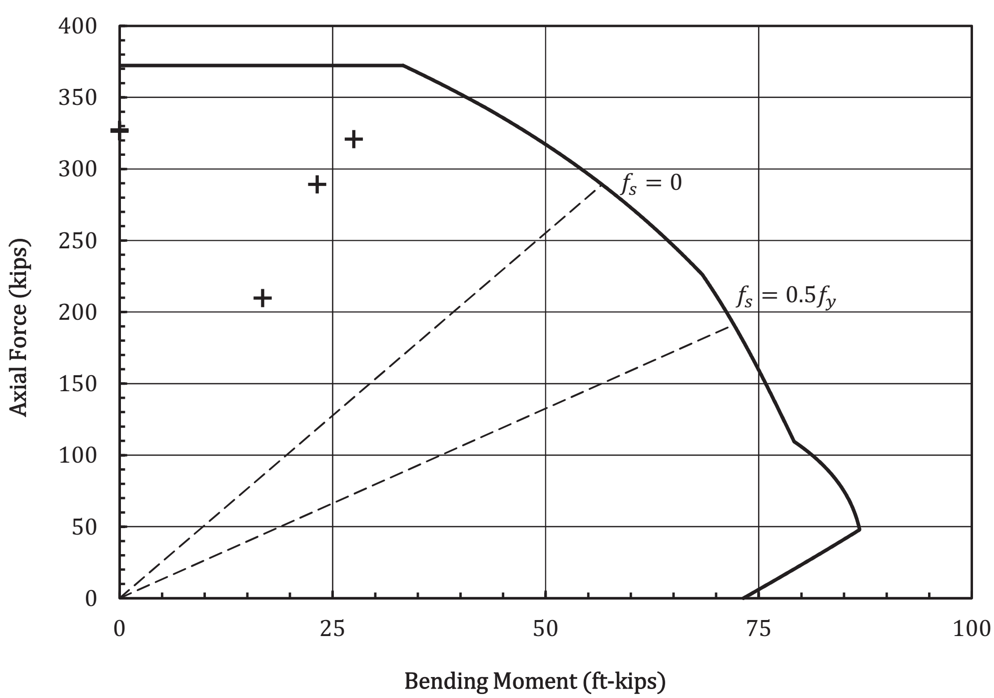


Figure 7.54 Design strength interaction diagram for column C1.

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} = \frac{0.31}{1 - \dfrac{321.1}{0.75 \times 606.2}} = 1.06 \qquad \text{Eq. (7.3)}$$

$$M_{2,min} = P_u(0.6 + 0.03h) = 321.1 \times [0.6 + (0.03 \times 12.0)] / 12 = 25.7 \text{ ft-kips}$$

$$M_c = \delta M_{2,min} = 1.06 \times 25.7 = 27.2 \text{ ft-kips}$$

Step 4 – Check the adequacy of the assumed longitudinal reinforcement

The design strength interaction diagram for column C1 is given in Figure 7.54. Also shown in the figure are the load combination points for the factored axial forces P_u in Table 7.42 and the corresponding factored bending moments M_c in Table 7.43 where applicable. Because all load combinations fall within the design strength interaction diagram, the column is adequate.

Use 4-#9 longitudinal bars.

7.11.23 Example 7.23 – Determination of Nonsway or Sway Frame: Building #1 (Framing Option B), Rectangular, Tied Column is Part of the LFRS, SDC A

Determine whether the first story of Building #1, Framing Option B is a nonsway or sway frame assuming all the columns are 24-in. square (see Figure 1.1). Also assume an 8.5-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f'_c = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the total loads in the first story

$$\text{Total area per level} = 152.0 \times 96.0 = 14{,}592 \text{ sq ft}$$

$$\text{Dead load of slab} = (8.5 / 12) \times 150.0 = 106.3 \text{ lb/ft}^2$$

Roof: $P_D = (106.3 + 12.0) \times 14{,}592 / 1{,}000 = 1{,}726$ kips

$P_{L_r} = 20.0 \times 14{,}592 / 1{,}000 = 292$ kips

Typical floor: $P_D = (106.3 + 10.0) \times 14{,}592 / 1{,}000 = 1{,}697$ kips

$P_L = 65.0 \times 14{,}592 / 1{,}000 = 949$ kips

$$\text{Dead load of columns (total)} = \frac{24.0 \times 24.0 \times 150.0}{144 \times 1{,}000} \times 60.0 \times 35 = 1{,}260 \text{ kips}$$

$$\text{Dead load of beam webs (total)} = \frac{(24.0 - 8.5) \times 28.0 \times 150.0}{144 \times 1{,}000} \times [(8 \times 21.5) + (12 \times 23.0)] \times 5 = 1{,}013 \text{ kips}$$

$$\text{Total } P_D = 1{,}726 + (4 \times 1{,}697) + 1{,}260 + 1{,}013 = 10{,}787 \text{ kips}$$

$$\text{Total } P_L = 4 \times 949 = 3{,}796 \text{ kips}$$

The total factored load must correspond to the lateral loading case for which it is a maximum. In this example, ACI Eq. (5.3.1d) produces the largest load:

$$\Sigma P_u = 1.2P_D + 1.0P_W + 0.5P_L + 0.5P_{L_r} = 14{,}988 \text{ kips}$$

Step 2 – Determine the stability index for the first story

An elastic analysis of the structure was performed with the building subjected to the north-south wind forces in Table 3.10 assuming the columns are fixed at the base. The reduced moments of inertia in Table 7.1 were used for the columns, beams, and slabs. The following results were obtained from the analysis:

Factored horizontal shear in the first story: $V_{us} = 232.9$ kips

Relative lateral deflection between the top and bottom of the first story: $\Delta_o = 0.29$ in.

The stability index, Q, is determined from the following equation:

$$Q = \frac{\Sigma P_u \Delta_o}{V_{us} \ell_c} = \frac{14,988 \times 0.29}{232.9 \times (14.0 \times 12)} = 0.11 > 0.05$$ Table 7.3

Because $Q > 0.05$, the frame in the first story is a sway frame (see ACI 6.6.4.3).

7.11.24 Example 7.24 – Check if Slenderness Effects Must be Considered: Building #1 (Framing Option B), Rectangular, Tied Column is Part of the LFRS, SDC A, Sway Frame

Check if slenderness effects must be considered for column D1 in the first story of Building #1, Framing Option B assuming the column is 24-in. square (see Figure 1.1). Also assume an 8.5-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f_c' = 4,000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 7.23.

It is determined in Example 7.23 that the frame in the first story is a sway frame. Therefore, slenderness effects can be neglected when the following equation is satisfied:

$$\frac{k\ell_u}{r} \leq 22$$ Table 7.6

The effective length factor, k, is determined from ACI Fig. R6.2.5 for sway frames. In the analysis, the columns are assumed to be fixed at the base, so stiffness ratio $\Psi_B = 1.0$. At the top of the column, Ψ_A is determined using the ratio of the stiffness of the columns to the stiffness of the beams. The reduced moments of inertia in Table 7.1 are used in calculating the stiffnesses.

$$I_{col} = \frac{0.7 \times 24.0^4}{12} = 19,354 \text{ in.}^4$$

$$E_c = w_c^{1.5} 33 \sqrt{f_c'} = 150.0^{1.5} \times 33 \times \sqrt{4,000} \,/\, 1,000 = 3,834 \text{ ksi}$$ ACI Eq. (19.2.2.1a)

Column below the second-floor level:

$$\frac{E_c I_{col}}{\ell_c} = \frac{3,834 \times 19,354}{14.0 \times 12} = 442 \times 10^3 \text{ in.-kips}$$

Column above the second-floor level:

$$\frac{E_c I_{col}}{\ell_c} = \frac{3,834 \times 19,354}{11.5 \times 12} = 538 \times 10^3 \text{ in.-kips}$$

$$I_{beam} = \frac{0.35 \times 28.0 \times 24.0^3}{12} = 11{,}290 \text{ in.}^4$$

$$\frac{E_c I_{beam}}{\ell_b} = \frac{3{,}834 \times 11{,}290}{23.5 \times 12} = 154 \times 10^3 \text{ in.-kips}$$

Therefore,

$$\Psi_A = \frac{\Sigma EI_{col} / \ell_c}{\Sigma EI_{beam} / \ell_b} = \frac{442 + 538}{2 \times 154} = 3.2 \qquad \text{Eq. (7.1)}$$

From the alignment chart in ACI Fig. R6.2.5 for sway frames with $\Psi_A = 3.2$ and $\Psi_B = 1.0,$ k is approximately equal to 1.55.

$$\frac{k\ell_u}{r} = \frac{1.55 \times [(14.0 \times 12) - (24.0 / 2)]}{0.3 \times 24.0} = 33.6 > 22$$

Therefore, slenderness effects must be considered.

7.11.25 Example 7.25 – Determination of Longitudinal Reinforcement: Building #1 (Framing Option B), Rectangular, Tied Column is Part of the LFRS, SDC A, Sway Frame, Column Subjected to Uniaxial Bending and Axial Forces, Slenderness Effects

Determine the required longitudinal reinforcement for column D1 in the first story of Building #1, Framing Option B assuming a 24-in. square tied column (see Figure 1.1). Also assume an 8.5-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f'_c = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.23 and 7.24.

Step 1 – Determine the factored axial forces and bending moments

$$\text{Dead load of slab} = (8.5 / 12) \times 150.0 = 106.3 \text{ lb/ft}^2$$

$$\text{Dead load of beam webs} = \frac{(24.0 - 8.5) \times 28.0 \times 150.0}{144 \times 1{,}000} \times 23.5 = 10.6 \text{ kips}$$

Roof: $\text{Superimposed dead load} = 12 \text{ lb/ft}^2$

$\text{Live load} = 20 \text{ lb/ft}^2$

Floors: $\text{Superimposed dead load} = 10 \text{ lb/ft}^2$

$\text{Live load} = 15 \text{ lb/ft}^2 \text{ (nonreducible partitions)} + 50 \text{ lb/ft}^2$

$$\text{Tributary area to column} = 23.5 \times (25.0 / 2) = 293.8 \text{ sq ft}$$

This column is part of the LFRS.

A summary of the axial forces and bending moments for column D1 in the first story from an analysis of the structure is given in Table 7.44 for sidesway to the right (SSR) and sidesway to the left (SSL).

Table 7.44 Summary of Axial Forces and Bending Moments for Column D1

Load Case			Axial Force (kips)	Bending Moment (ft-kips)	
				Top	Bottom
Dead (D)			224.4	−4.6	2.8
Roof live (L_r)			5.9	—	—
Live (L)			76.4	−3.8	2.4
Wind (W)			±1.9	±69.1	±232.1
Load Combination					
ACI Eq. (5.3.1a)	$1.4D$		314.2	−6.4	3.9
ACI Eq. (5.3.1b)	$1.2D+1.6L+0.5L_r$		394.5	−11.6	7.2
ACI Eq. (5.3.1c)	$1.2D+1.6L_r+0.5L$		316.9	−7.4	4.6
	$1.2D+1.6L_r+0.5W$	SSR	277.8	−40.1	−112.7
		SSL	279.7	29.0	119.4
ACI Eq. (5.3.1d)	$1.2D+1.0W+0.5L+0.5L_r$	SSR	308.5	−76.5	−227.5
		SSL	312.3	61.7	236.7
ACI Eq. (5.3.1f)	$0.9D+1.0W$	SSR	200.1	−73.2	−229.6
		SSL	203.9	65.0	234.6

Step 2 – Determine the total moments including slenderness effects — ACI 6.6.4.6

The total moments, M_2, at the end of the column are determined by the following equation:

$$M_2 = M_{2ns} + \delta_s M_{2s} \quad \text{Eq. (7.6)}$$

These moments are used because they are greater than the M_1 moments determined by Eq. (7.5).

In this example, the nonsway moments M_{2ns} are due to gravity load effects and the sway moments M_{2s} are due to wind load effects. These moments are given in Table 7.45 for all load combinations.

Table 7.45 Summary of Nonsway and Sway Moments (ft-kips) for Column D1

Load Combination			M_1	M_2	M_{1ns}	M_{2ns}	M_{1s}	M_{2s}
ACI Eq. (5.3.1a)	$1.4D$		3.9	−6.4	3.9	−6.4	—	—
ACI Eq. (5.3.1b)	$1.2D+1.6L+0.5L_r$		7.2	−11.6	7.2	−11.6	—	—
ACI Eq. (5.3.1c)	$1.2D+1.6L_r+0.5L$		4.6	−7.4	4.6	−7.4	—	—
	$1.2D+1.6L_r+0.5W$	SSR	−40.1	−112.7	−5.5	3.4	−34.6	−116.1
		SSL	29.0	119.4	−5.5	3.4	34.6	116.1

(table continued on next page)

Table 7.45 Summary of Nonsway and Sway Moments (ft-kips) for Column D1 (cont.)

Load Combination			M_1	M_2	M_{1ns}	M_{2ns}	M_{1s}	M_{2s}
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L + 0.5L_r$	SSR	−76.5	−227.5	−7.4	4.6	−69.1	−232.1
		SSL	61.7	236.7	−7.4	4.6	69.1	232.1
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	SSR	−73.2	−229.6	−4.1	2.5	−69.1	−232.1
		SSL	65.0	234.6	−4.1	2.5	69.1	232.1

The following equation is used to determine the moment magnification factor, δ_s, for sway frames:

$$\delta_s = \frac{1}{1-Q} \geq 1.0 \qquad \text{Eq. (7.7)}$$

The stability index, Q, is determined by ACI Eq. (6.6.4.4.1) for each load combination that includes W.

A summary of the slenderness calculations for column D1 is given in Table 7.46.

Table 7.46 Summary of Slenderness Calculations for Column D1

Load Combination			ΣP_u (kips)	Δ_o (in.)	V_{us} (kips)	Q	δ_s	M_2 (ft-kips)
ACI Eq. (5.3.1a)	$1.4D$		—	—	—	—	—	−6.4
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		—	—	—	—	—	−11.6
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		—	—	—	—	—	−7.4
	$1.2D + 1.6L_r + 0.5W$	SSR	13,412	0.15	116.5	0.10	1.11	−125.5
		SSL	13,412	0.15	116.5	0.10	1.11	132.3
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L + 0.5L_r$	SSR	14,988	0.29	232.9	0.11	1.12	−255.4
		SSL	14,988	0.29	232.9	0.11	1.12	264.6
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	SSR	9,708	0.29	232.9	0.07	1.08	−248.2
		SSL	9,708	0.29	232.9	0.07	1.08	253.2

Sample calculations for the load combination in ACI Eq. (5.3.1d) for sidesway to the right are as follows:

$\Sigma P_u = 1.2P_D + 1.0P_W + 0.5P_L + 0.5P_{L_r} = 14{,}988$ kips — Example 7.23, Step 1

$\Delta_o = 0.29$ in. — Example 7.23, Step 2

$V_{us} = 232.9$ kips — Example 7.23, Step 2

$$Q = \frac{\Sigma P_u \Delta_o}{V_{us}\ell_c} = \frac{14{,}988 \times 0.29}{232.9 \times (14.0 \times 12)} = 0.11$$

$$\delta_s = \frac{1}{1-Q} = \frac{1}{1-0.11} = 1.12$$

$M_2 = M_{2ns} + \delta_s M_{2s} = 4.6 + [1.12 \times (-232.1)] = -255.4$ ft-kips

It can be determined based on the requirements in ACI 6.6.4.6.4 that maximum moments occur at the ends of the columns.

Step 3 – Determine the required area of longitudinal reinforcement

A 1 percent reinforcement ratio is initially selected for this column. Therefore, $A_{st} = 0.01 \times 24.0^2 = 5.76$ in.^{2}

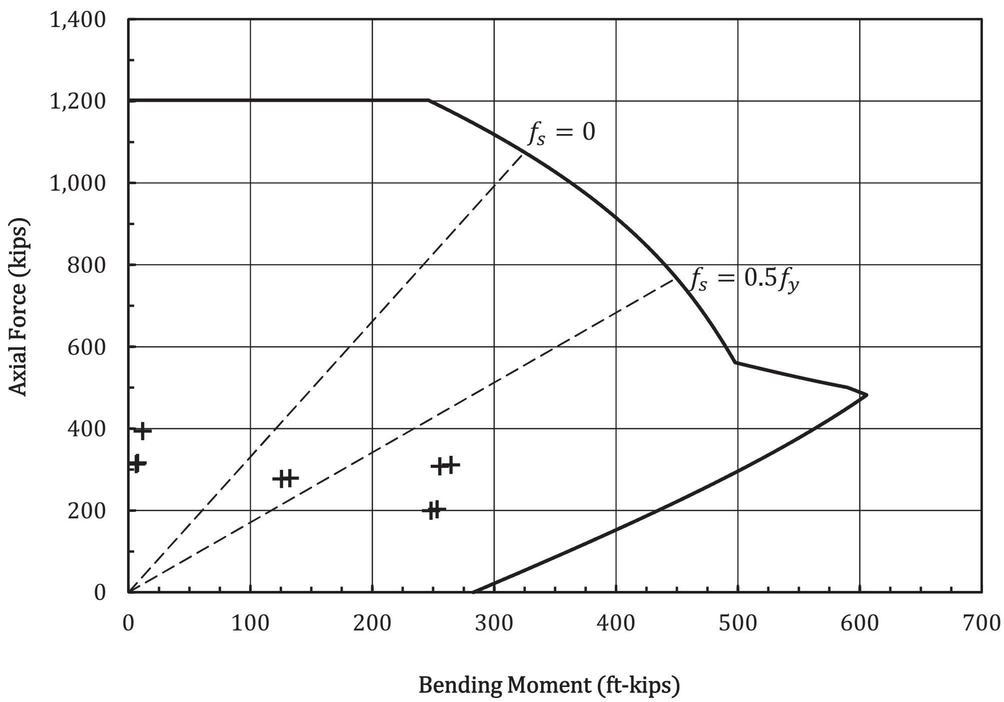


Figure 7.55 Design strength interaction diagram for column D1.

Try 4-#11 bars ($A_{st,provided} = 6.24$ in.^{2}).

The design strength interaction diagram for column D1 reinforced with 4-#11 bars is given in Figure 7.55. Also shown in the figure are the load combination points for factored axial forces P_u in Table 7.44 and the corresponding total bending moments M_2 in Table 7.46. The column is adequate because all the load combination points fall within the design strength interaction diagram.

Use 4-#11 bars.

7.11.26 Example 7.26 – Determination of Transverse Reinforcement: Building #1 (Framing Option B), Rectangular, Tied Column is Part of the LFRS, SDC A, Sway Frame, Column Subjected to Uniaxial Bending and Axial Forces, Slenderness Effects

Determine the required transverse reinforcement for column D1 in the first story of Building #1, Framing Option B assuming a 24-in. square tied column (see Figure 1.1). Also assume an 8.5-in.-thick slab, 28.0 by 24.0 in. beams, normalweight concrete with $f'_c = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.23 through 7.25.

Step 1 – Determine the required tie bar size ACI 25.7.2.2

The required tie bar size is determined using ACI 25.7.2.2.

For #11 longitudinal bars, use #4 ties.

Step 2 – Determine the required tie bar spacing ACI 25.7.2.1

The required tie bar spacing is determined using ACI 25.7.2.1; the tie bar size and spacing will then be checked to determine if they are adequate for shear strength requirements.

$$s \leq \text{ lesser of } \begin{cases} 16d_{b(column)} = 16 \times 1.41 = 22.6 \text{ in.} \\ 48d_{b(tie)} = 48 \times 0.5 = 24.0 \text{ in.} \\ c_1 = 24.0 \text{ in.} \end{cases}$$

$$\geq (4/3)d_{agg} + d_{b(tie)} = [(4/3) \times 0.75] + 0.375 = 1.375 \text{ in.}$$

Try #4 ties spaced at 22 in. on center.

Step 3 – Check the adequacy of the tie bar size and spacing for shear ACI 22.5

Maximum factored shear force, V_u, is obtained from ACI Eq. (5.3.1d) for sidesway to the right and must include the effects from slenderness (see Tables 7.42 and 7.43):

$$V_u = \frac{1.12 \times (76.5 + 227.5)}{14.0 - [24.0 / (2 \times 12)]} = 26.2 \text{ kips}$$

Check if the design shear strength of the concrete, ϕV_c, alone is adequate, which is determined based on whether $A_{v,min}$ is provided or not (see Table 7.12).

$$A_{v,min} = \text{greater of } \begin{cases} \dfrac{0.75\sqrt{f'_c}b_w s}{f_{yt}} = \dfrac{0.75 \times \sqrt{4,000} \times 24.0 \times 22.0}{60,000} = 0.42 \text{ in.}^2 \\ \dfrac{50b_w s}{f_{yt}} = \dfrac{50 \times 24.0 \times 22.0}{60,000} = 0.44 \text{ in.}^2 \end{cases} \qquad \text{Eq. (7.26)}$$

The minimum area of shear reinforcement is greater than $A_v = 2 \times 0.20 = 0.40$ in.^{2} provided by the #4 ties spaced 22 in. on center. Determine V_c based on $A_v < A_{v,min}$:

$$V_c = \left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right]b_w d \qquad \text{Table 7.12}$$

$$\lambda_s = \sqrt{\frac{2}{1 + (d/10)}} = \sqrt{\frac{2}{1 + (21.3/10)}} = 0.80 \qquad \text{Eq. (7.23)}$$

where $d = 24.0 - 1.5 - 0.5 - (1.41/2) = 21.3$ in.

$\lambda = 1.0$ for normalweight concrete Table 7.13

$\rho_w = \dfrac{A_s}{b_w d} = \dfrac{2 \times 1.56}{24.0 \times 21.3} = 0.0061$ (there are 2-#11 bars located more than two-thirds of the overall member depth from the extreme compression fiber).

Use $N_u = 308.5$ kips, which corresponds to ACI Eq. (5.3.1d).

$N_u / 6A_g = 308{,}500 / (6 \times 24.0^2) = 89.3 \text{ psi} < 0.05 f_c' = 200.0 \text{ psi}$ ACI 22.5.5.1.2

$$V_c = \{[8 \times 0.80 \times 1.0 \times (0.0061)^{1/3} \times \sqrt{4{,}000}] + 89.3\} \times 24.0 \times 21.3 / 1{,}000 = 83.5 \text{ kips}$$

$$< 5\lambda\sqrt{f_c'}b_w d = 161.7 \text{ kips}$$

$$\phi V_c = 0.75 \times 83.5 = 62.6 \text{ kips} > V_u = 26.2 \text{ kips}$$

Also, $V_u = 26.2 \text{ kips} < 0.5\phi V_c = 31.3$ kips, so minimum shear reinforcement need not be provided (ACI 10.6.2.1).

Therefore, use #4 ties spaced at 22 in. on center.

7.11.27 Example 7.27 – Determination of Dowel Reinforcement at the Foundation: Building #1 (Framing Option B), Rectangular, Tied Column is Part of the LFRS, SDC A, Sway Frame, Column Subjected to Uniaxial Bending and Axial Forces, Slenderness Effects

Determine the required dowel reinforcement for column D1 in the first story of Building #1, Framing Option B assuming a 24-in. square tied column supported by a mat foundation with a thickness of 4 ft-0 in. (see Figure 1.1). Also assume an 8.5-in.-thick slab, normalweight concrete with $f_c' = 4{,}000$ psi (maximum aggregate size = 0.75 in.), and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 7.23 through 7.26.

Step 1 – Check the bearing stresses on the column and mat foundation ACI 16.3.3.4

Bearing strength of the column.

Factored bearing stress, b_u, is determined for compression plus bending using the factored axial forces and bending moments determined by ACI Eqs. (5.3.1b) and (5.3.1d) [see Tables 7.41 and 7.43].

- ACI Eq. (5.3.1b):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{394.5 \times 1{,}000}{24.0^2} = 685 \text{ psi}$$

$$\text{Bending stress (compression and tension)} = \frac{6M_u}{c_1^3} = \frac{6 \times 7.2 \times 12{,}000}{24.0^3} = 38 \text{ psi}$$

$$\text{Maximum compression stress} = 685 + 38 = 723 \text{ psi}$$

$$\text{Minimum compression stress} = 685 - 38 = 647 \text{ psi}$$

- ACI Eq. (5.3.1d):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{312.3 \times 1{,}000}{24.0^2} = 542 \text{ psi}$$

$$\text{Bending stress (compression and tension)} = \frac{6M_u}{c_1^3} = \frac{6 \times 264.6 \times 12{,}000}{24.0^3} = 1{,}378 \text{ psi}$$

$$\text{Maximum compression stress} = 542 + 1{,}378 = 1{,}920 \text{ psi}$$

$$\text{Maximum tension stress} = 542 - 1{,}378 = -836 \text{ psi}$$

Design bearing strength:

$$\phi b_n = \phi 0.85 f_c' = 0.65 \times 0.85 \times 4{,}000 = 2{,}210 \text{ psi}$$

Tension forces cannot be transferred through bearing at the interface, so dowel bars will be provided that match the size of the longitudinal bars in the column. This reinforcement ensures that both the compression and tension forces are adequately transferred from the column into the footing.

Bearing strength of the mat foundation.

There is no need to check the bearing strength of the mat foundation because interface reinforcement must be provided due to the net tension stress from the factored axial force and bending moment determined by ACI Eq. (5.3.1d).

Step 2 – Determine the required interface reinforcement ACI 16.3.4.1

Try 4-#11 dowel bars. This reinforcement matches the longitudinal reinforcement in the column and ensures that both the compression and tension forces are adequately transferred through the interface.

Check minimum area of reinforcement across the interface:

$$A_{s,min} = 0.005A_g = 0.005 \times 24.0^2 = 2.88 \text{ in.}^2 < A_{s,provided} = 6.24 \text{ in.}^2$$

Step 3 – Check the development of the dowel bars in the footing

It is evident from Figure 7.55 that the stress in the longitudinal reinforcement farthest from the extreme compression face is tensile for a number of load combinations (that is, the load combinations fall within Zones 2 and 3 of the interaction diagram; see Figure 7.28). Therefore, the dowel bars must be developed for tension into the footing (and into the column).

A standard 90-degree hook will be provided at the ends of the #11 dowel bars. Determine the tension development length, ℓ_{dh}, of the #11 dowel bars terminating in a hook:

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \qquad \text{Eq. (7.44)}$$

$\psi_e = 1.0$ for uncoated bars Table 7.27

$\psi_r = 1.6$ (confining reinforcement not provided)

$\psi_o = 1.0$ (side cover normal to hook $> 6d_b$)

$\psi_c = (f'_c \,/\, 15{,}000) + 0.6 = (4{,}000 \,/\, 15{,}000) + 0.6 = 0.9$

$\lambda = 1.0$ for normalweight concrete

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f'_c}}\right) d_b^{1.5} = \left(\dfrac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.9}{55 \times 1.0 \times \sqrt{4{,}000}}\right) \times 1.41^{1.5} = 41.6 \text{ in.} \\ 8d_b = 8 \times 1.41 = 11.3 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

Assuming two layers of #10 bars in the mat foundation with a clear cover of 3 in., determine the minimum mat thickness for the development of the dowel bars:

$$\text{Minimum } h = \ell_{dh} + 2(d_b)_f + \text{cover} = 41.6 + (2 \times 1.27) + 3.0 = 47.1 \text{ in.} \qquad \text{Eq. (7.45)}$$

The minimum $h = 47.1$ in. is less than the provided mat thickness of 48.0 in. Therefore, the #11 dowel bars can be adequately developed in the mat.

Step 4 – Determine the development length of the dowel bars in the column

The dowel bars must be lap spliced to the longitudinal reinforcement in the column using a tension lap splice. Because all the dowel bars are spliced at the same location, a Class B tension lap splice is required (ACI Table 25.5.2.1).

Development length in tension, ℓ_d, of the #11 bars:

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f'_c}}\frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\right) d_b \geq 12.0 \text{ in.} \qquad \text{Eq. (7.36)}$$

$\psi_t = 1.0$ Table 7.24

$\psi_e = 1.0$ for uncoated bars

$\psi_s = 1.0$ for #11 bars

$\psi_g = 1.0$ for Grade 60 reinforcement

$$c_b = \text{lesser of} \begin{cases} \text{cover} + (d_b)_{tie} + 0.5(d_b)_{long.} = 1.5 + 0.5 + (0.5 \times 1.41) = 2.7 \text{ in.} \\ \dfrac{s}{2} = \dfrac{24.0 - (2 \times 1.5) - (2 \times 0.5) - 1.41}{2} = 9.3 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) \,/\, d_b = (2.7 + 0) \,/\, 1.41 = 1.9 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0 \times \sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{1.9} \right) \times 1.41 = 52.8 \text{ in.} > 12.0 \text{ in.}$$

$$\text{Class B lap splice length} = 1.3\ell_d = 1.3 \times 52.8 = 68.6 \text{ in.}$$

Provide a lap splice length of 5 ft-9 in.

Step 5 – Check horizontal force transfer

It is determined in Example 7.26 that $V_u = 26.2$ kips is transferred horizontally between the column and mat foundation. The required area of shear-friction reinforcement is determined as follows assuming the surface between the column and the mat has not been intentionally roughened:

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{V_u}{\phi f_y (0.6\lambda)} = \frac{26.2}{0.75 \times 60 \times (0.6 \times 1.0)} = 0.97 \text{ in.}^2 \qquad \text{Eq. (7.46), Table 7.28}$$

The 4-#11 dowel bars provides 6.24 in.^{2} across the interface, which is greater than the required amount of 0.97 in.^{2}

Check the upper shear limit:

$$V_u = 26.2 \text{ kips} < \begin{cases} \phi 0.2 f'_c A_c = 0.75 \times 0.2 \times 4 \times 24.0^2 = 345.6 \text{ kips} \\ \phi 800 A_c = 0.75 \times 800 \times 24.0^2 / 1{,}000 = 345.6 \text{ kips} \end{cases} \qquad \text{Table 7.29}$$

Reinforcement details for this column are given in Figure 7.56.

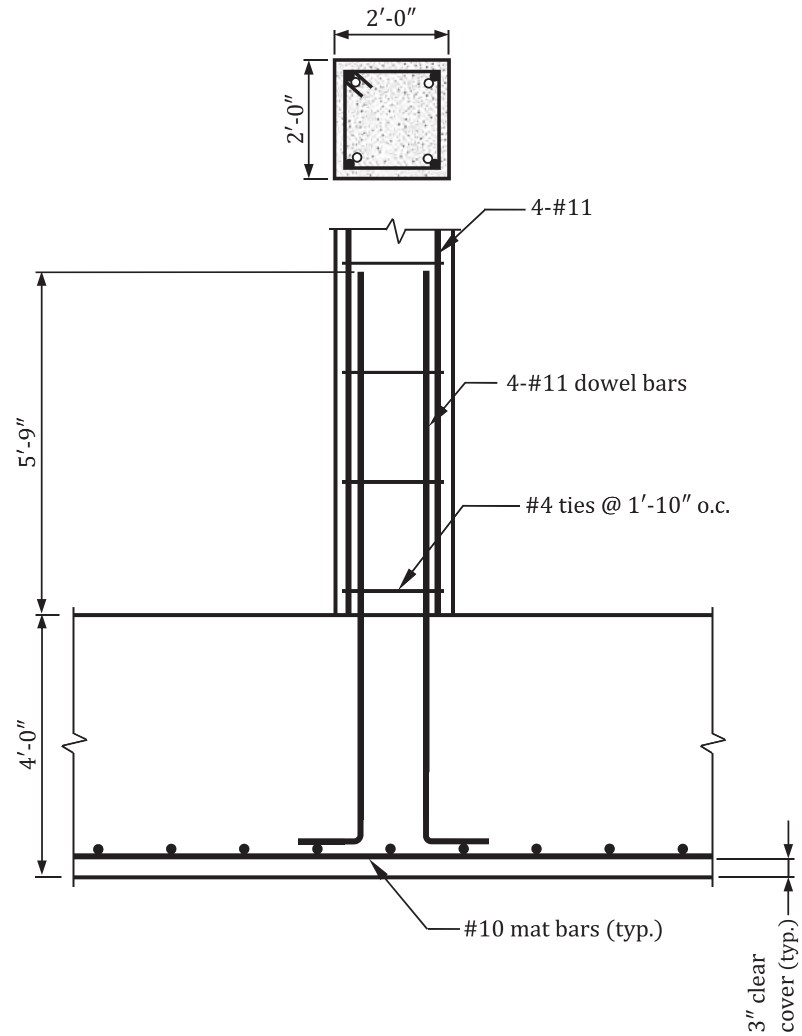


Figure 7.56 Reinforcement details for column D1 in Examples 7.23 through 7.27.

Chapter 8
WALLS

8.1 Overview

A wall is defined as a vertical structural element with a horizontal length-to-thickness ratio greater than 3, which is designed to resist axial forces, lateral forces, or both (ACI 2.3). Structural walls are designed to resist the effects from vertical forces and lateral forces in the plane of the wall (that is, in the direction parallel to the horizontal length of the wall) and are commonly referred to as shear walls.

According to Reference 2, a load-bearing (or, bearing) concrete wall is a wall supporting more than 200 pounds per linear foot of vertical load in addition to its own weight. A nonload-bearing (or, nonbearing) wall is any wall that is not a load-bearing wall; such walls support essentially their own weight and possibly out-of-plane loads like those from wind.

Axial forces, bending moments, and shear forces are resisted by one layer or two layers of vertical and horizontal reinforcement in a wall.

The design and detailing of cast-in-place concrete walls with nonprestressed reinforcement are covered in this chapter. Provisions for walls are given in ACI Chapter 11 and are applicable to walls in buildings assigned to Seismic Design Category (SDC) A, B, and C. Cantilever retaining walls are not covered here; see Reference 20 for comprehensive design and detailing requirements for these types of walls. Specific design recommendations for cast-in-place walls constructed with insulating concrete forms are given in the references in ACI R11.1.6.

8.2 Design Limits

8.2.1 Minimum Wall Thickness

Minimum thicknesses of bearing walls, nonbearing walls, and exterior basement and foundation walls are given in ACI 11.3.1.1 (see Figure 8.1 for bearing and nonbearing walls). These minimum thicknesses need not be applied to bearing walls and exterior basement and foundation walls designed by the provisions in ACI 11.5.2 or analyzed by the alternative analysis method in ACI 11.8.

8.2.2 Intersecting Elements

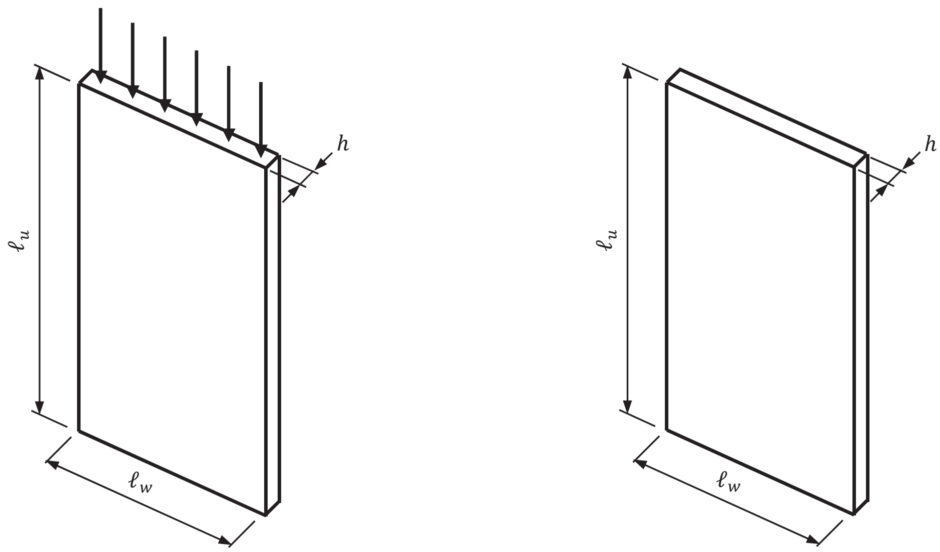
For cast-in-place concrete walls where the factored axial force, P_u, exceeds $0.2f_c'A_g$, the portion of the wall within the thickness of the floor system must have a specified concrete compressive strength of at least $0.8f_c'$ of the wall (ACI 11.2.4.2). The 0.8 factor reflects the reduced confinement in floor-wall joints compared with floor-column joints for members subjected to gravity loads.

8.3 Required Strength

8.3.1 Analysis Methods

The analysis methods in ACI Chapter 6 in conjunction with the factored load combinations in ACI Chapter 5 are to be used to calculate required strength (see ACI 11.4.1.2 and 11.4.1.1, respectively).

Axial forces, bending moments, and shear forces can be determined by any of the analysis methods in ACI 6.2.3. When performing a first-order or an elastic second-order lateral load analysis, the section properties in ACI 6.6.3.1.1 may be used in lieu of a more sophisticated analysis to account for member cracking and other effects. The reduced moments of inertia in ACI Table 6.6.3.1.1(a) and the alternative moments of inertia in ACI Table 6.6.3.1.1(b) for various structural members are given in Table 8.1 and Table 8.2, respectively. The reduced moments of inertia in Table 8.1 are based on an analysis where strength-level (factored) loads are used. For an analysis based on service-level loads, it is permitted to use reduced moments of inertia equal to 1.4 times the values in Table 8.1 provided the moments of inertia do not exceed the gross moments of inertia I_g (ACI 6.6.3.2.2).



$$h \geq \text{greater of} \begin{cases} \text{lesser of} \begin{cases} \ell_w/25 \\ \ell_u/25 \end{cases} \\ 4'' \end{cases}$$

Bearing wall*

$$h \geq \text{greater of} \begin{cases} \text{lesser of} \begin{cases} \ell_w/30 \\ \ell_u/30 \end{cases} \\ 4'' \end{cases}$$

Nonbearing wall

*Applicable to walls designed by ACI 11.5.3 only

Figure 8.1 Minimum thicknesses for bearing and nonbearing walls.

Table 8.1 Moments of Inertia to Use in a Linear Elastic First-Order or Second-Order Analysis Using Strength-Level Loads

Member	Moment of Inertia
Columns	$0.70I_g$
Walls — uncracked	$0.70I_g$
Walls — cracked	$0.35I_g$
Beams	$0.35I_g$
Flat plates and flat slabs	$0.25I_g$

Table 8.2 Alternative Moments of Inertia to Use in a Linear Elastic First-Order or Second-Order Analysis Using Strength-Level or Service-Level Loads

Member	Moment of Inertia*
Columns and walls	$0.35I_g \leq I = \left(0.80 + \frac{25A_{st}}{A_g}\right)\left(1 - \frac{M_u}{P_u h} - \frac{0.5P_u}{P_o}\right)I_g \leq 0.875I_g$
Beams, flat plates, and flat slabs	$0.25I_g \leq I = \left(0.10 + 25\rho\right)\left(1.2 - \frac{0.2b_w}{d}\right)I_g \leq 0.5I_g$

**For a service-level load analysis, use service-level axial force and moment effects in the equation for columns and walls.*

The more refined equations for reduced moments of inertia in Table 8.2 include the (1) factored axial force , P_u, and factored moment, M_u, on a wall for the load combination under consideration, (2) nominal axial strength at zero eccentricity, P_o, determined by ACI Equation (22.4.2.2), and (3) area of longitudinal (vertical) reinforcement, A_{st}. These equations are applicable for strength-level and service-level loading, even though they are presented in terms of factored load effects. For service-level analysis, the factored load effects are replaced by the corresponding service-level load effects.

Regardless of the equations used to determine moments of inertia, the gross area of the section, A_g, is to be used in the analysis. Also, the cross-sectional area for calculation of shear deformations in a wall is $\ell_w h$.

In lieu of the requirements of ACI 6.6.3.1, it is permitted to assume in a first-order factored lateral load analysis that all the members in a structure have a reduced moment of inertia equal to 50 percent of the gross moment of inertia I_g (ACI 6.6.3.1.2). A more detailed analysis considering the effective stiffness of all members is also permitted.

8.3.2 Factored Axial Force, Moment, and Shear

Walls must be designed for factored axial forces, bending moments, and shear forces due to in-plane and/or out-of-plane forces, including the effects of slenderness where appropriate, for each applicable load combination (ACI 11.4.1.4, 11.4.2, and 11.4.3). In-plane and out-of-plane forces and their effects on a bearing wall are illustrated in Figure 8.2 and Figure 8.3, respectively.

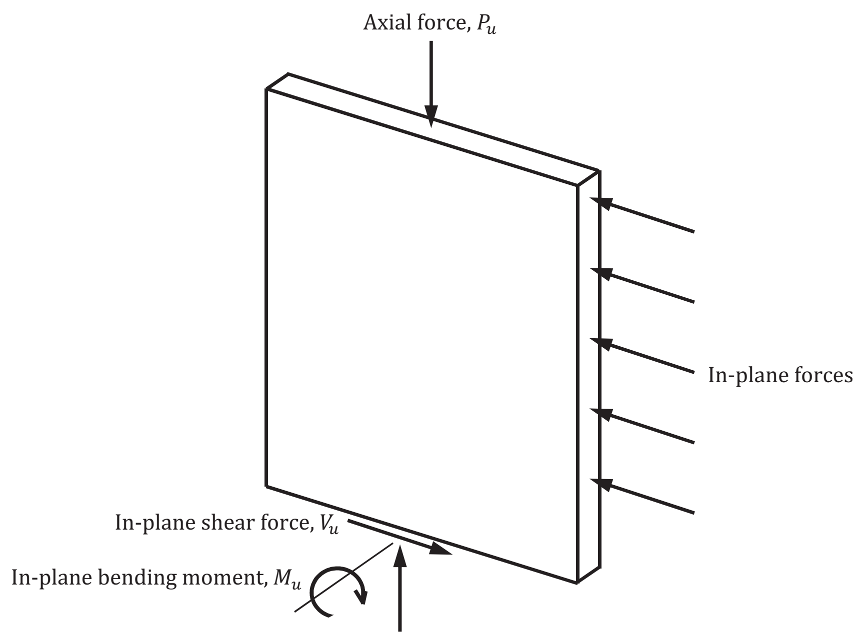


Figure 8.2 In-plane forces on a bearing wall.

8.3.3 Slenderness Effects

Overview

Like columns, walls must be designed for the effects of slenderness where applicable. Determining whether slenderness effects need to be considered or not depends on whether the frame is nonsway or sway. According to ACI 6.6.4.3, a frame is considered to be nonsway where the stability index, Q, is less than 0.05 where Q is determined by ACI Equation (6.6.4.4.1):

$$Q = \frac{\Sigma P_u \Delta_o}{V_{us} \ell_c} \tag{8.1}$$

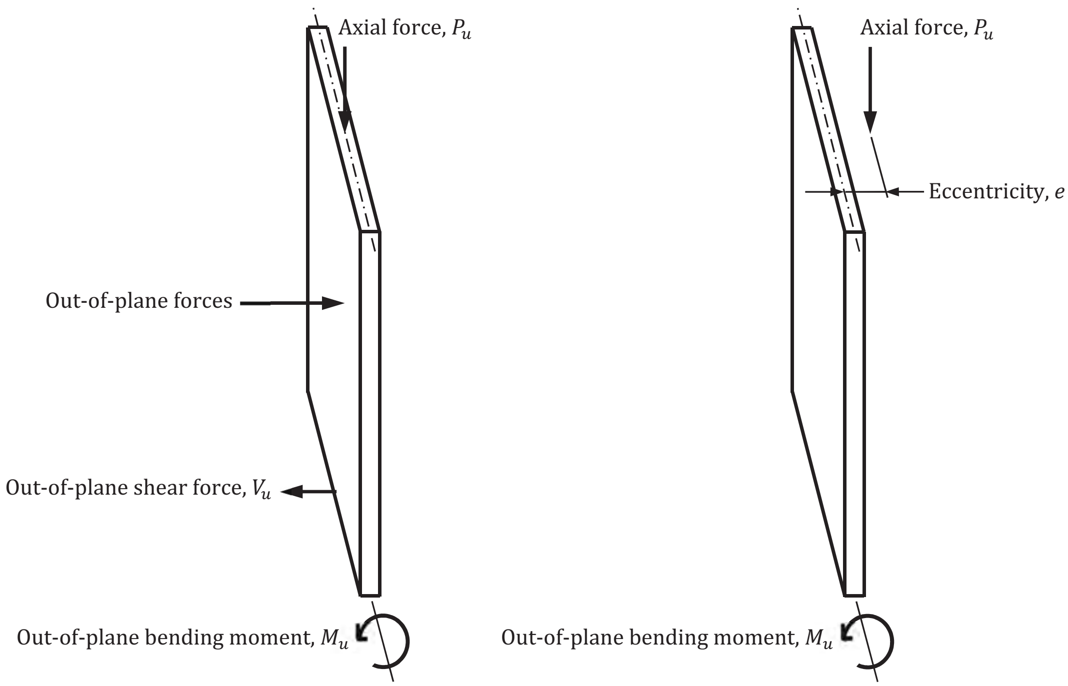


Figure 8.3 Out-of-plane forces on a bearing wall.

where ΣP_u = total factored vertical load in the story

Δ_o = first-order relative lateral deflection between the top and bottom of the story

V_{us} = factored horizontal story shear in a story

ℓ_c = length of the walls measured center-to-center of the joints in the frame

More often than not, the frame in the direction of analysis can be considered nonsway where structural walls are used to resist the lateral forces in that direction.

Whether slenderness effects need to be considered or not for nonsway and sway frames depends on the slenderness ratio of the wall, $k\ell_u / r$, in the direction of analysis. The terms in the slenderness ratio are defined in Table 8.3 (see ACI 6.2.5).

Table 8.3 Slenderness Ratio, $k\ell_u / r$

k = effective length factor	Nonsway Frames	$k \leq 1.0$
		Determine k using ACI Figure R6.2.5(a) or use $k = 1.0$
	Sway Frames	$k \geq 1.0$
		Determine k using ACI Figure R6.2.5(b)
ℓ_u = unsupported length	Unsupported length, ℓ_u, is the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction of analysis.	
r = radius of gyration	Radius of gyration, r, is permitted to be calculated by the following: 1. $r = \sqrt{I_g / A_g}$ 2. $r = 0.30\bar{h}$ for rectangular walls where $\bar{h}$ is the dimension of the wall in the direction of analysis (that is, either h for slenderness about the minor axis or ℓ_w for slenderness about the major axis)	

Slenderness effects are permitted to be neglected when the equations in Table 8.4 are satisfied (ACI 6.2.5).

Table 8.4 Limits for Slenderness Effects

Wall Bracing Condition	Slenderness Ratios Where Slenderness is Permitted to Be Neglected
Not braced against sidesway (sway)	$\frac{k\ell_u}{r} \leq 22$
Braced against sidesway (nonsway)	$\frac{k\ell_u}{r} \leq \text{ lesser of } \begin{cases} 34 + 12(M_1 \,/\, M_2)\,* \\ 40 \end{cases}$

Ratio $M_1 \,/\, M_2$ is negative if a wall is bent in single curvature and is positive if a wall is bent in double curvature.

It is more likely the out-of-plane bending moments in a wall would need to be magnified than the in-plane bending moments: In the direction parallel to the length of the wall, slenderness effects can usually be neglected because the radius of gyration of the wall, r, in that direction is relatively large, which results in a slenderness ratio less than the limits in Table 8.4. In the direction parallel to the thickness of the wall, the slenderness ratio is more likely to be greater than the prescribed slenderness limits, which means factored bending moments about the minor axis of the wall would need to be magnified.

The following methods are permitted to be used to account for slenderness effects (ACI 11.4.1.3):

- Moment magnification method (ACI 6.6.4)
- Linear elastic second-order analysis (ACI 6.7)
- Inelastic analysis (ACI 6.8)
- Alternative method for out-of-plane slender wall analysis (ACI 11.8)

The moment magnification method and the alternative method are covered below. Information on the linear elastic second-order analysis and the inelastic analysis is given in Section 7.3.1 of this publication.

Moment Magnification Method

Details of the moment magnification method are given in Section 7.3.3 of this publication. When determining the critical buckling load, P_c, by ACI Equation (6.6.4.4.2), the effective stiffness, $(EI)_{eff}$, determined by ACI Equation (6.6.4.4.4c) is used for walls:

$$(EI)_{eff} = \frac{E_c I}{1 + \beta_{dns}} \tag{8.2}$$

In this equation, E_c is the modulus of elasticity of the concrete determined in accordance with ACI 19.2.2; I is the moment of inertia of the wall determined by the equation for columns and walls in Table 8.2; and β_{dns} is the ratio of the maximum factored sustained axial load to the maximum factored axial load associated with the same load combination, which is typically equal to the factored axial dead load divided by the total factored axial load on a wall.

As noted previously, frames are typically nonsway when structural walls are part of the frame; in such cases, the wall is to be designed using the magnified moments, M_c, determined by ACI Equation (6.6.4.5.1) where applicable.

Alternative Method for Out-of-Plane Slender Wall Analysis

Limitations

The design procedure given in ACI 11.8 can be used for the out-of-plane design of slender, reinforced concrete walls provided the following limitations are satisfied (ACI 11.8.1.1):

1. The cross-section of the wall is constant over the entire height.
2. The wall is tension-controlled for out-of-plane moment effects.
3. The design flexural strength of the wall, ϕM_n, is greater than or equal to the cracking moment, M_{cr}, where $M_{cr} = f_r I_g / y_t$ and $f_r = 7.5\lambda\sqrt{f_c'}$ (see ACI 19.2.3).
4. The factored axial force, P_u, at the mid-height section of the wall is less than or equal to $0.06 f_c' A_g$.
5. The calculated out-of-plane deflection due to service loads, Δ_s, including P-delta effects, is less than or equal to $\ell_c / 150$ where ℓ_c is the length of the wall measured center-to-center of the joints.

When one or more of these five limitations are not satisfied, this method cannot be used. For example, walls with relatively large window and door openings are not considered to have a constant cross-section over their full height, so the alternative method is not applicable because the first limitation is not satisfied.

Modeling

The following modeling requirements must be satisfied when using the alternative method (ACI 11.8.2):

1. The wall must be analyzed as a simply supported, axially loaded member subjected to a uniformly distributed lateral load with the maximum moment and deflection occurring at the mid-height of the wall (see Figure 8.4).

2. Concentrated gravity loads, P_{ug}, applied to the wall above any section are assumed to be distributed over a width equal to the bearing width plus a width on each side that increases at a slope of 2 vertical to 1 horizontal, but not extending beyond the spacing of the concentrated loads or the edges of the wall panel (see Figure 8.5).

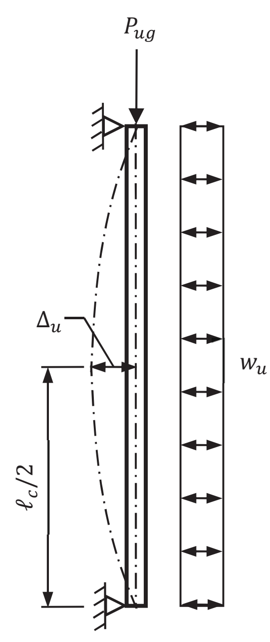
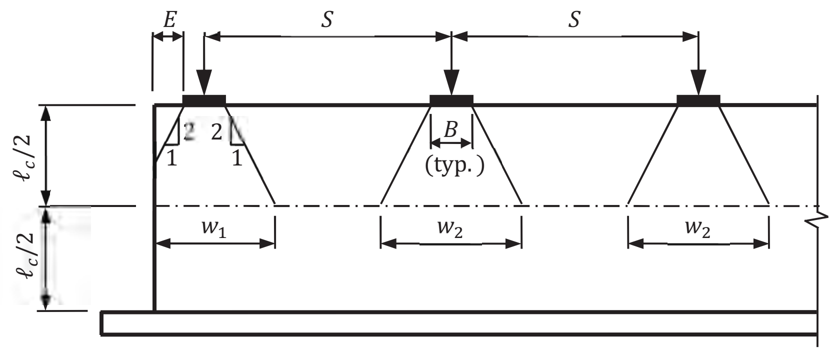


Figure 8.4 Wall analysis in accordance with ACI 11.8.



$$w_1 = B + (\ell_c/4) + E \le S$$

$$w_2 = B + (\ell_c/2) \le S$$

Figure 8.5 Distribution of concentrated loads in the alternative design method for walls.

Factored Moment

The following strength equation must be satisfied at the mid-height of a wall:

$$\phi M_n \ge M_u \tag{8.3}$$

Two methods to determine M_u, which includes the effects due to slenderness, are given in ACI 11.8.3: (1) by iterative calculation and (2) by direct calculation.

<u>Iterative Calculation Method</u>

In the iterative calculation method, the factored moment, M_u, consists of two parts [ACI Equation (11.8.3.1a)]:

$$M_u = M_{ua} + P_u \Delta_u \tag{8.4}$$

The moment M_{ua} is the maximum factored moment at the mid-height of the wall due to (a) lateral loads and/or (b) vertical factored loads, P_u, applied at an eccentricity from the centroid of the wall. Note that M_{ua} does not include P-delta effects. The deflection Δ_u is the total deflection at the mid-height of the wall due to the factored loads. This deflection is determined by ACI Equation (11.8.3.1b):

$$\Delta_u = \frac{5 M_u \ell_c^2}{(0.75) 48 E_c I_{cr}} \tag{8.5}$$

In this equation, ℓ_c is the length of the wall, measured center-to-center of joints, and I_{cr} is the moment of inertia of the cracked wall section transformed to concrete, which is determined by ACI Equation (11.8.3.1c):

$$I_{cr} = \frac{E_s}{E_c}\left(A_s + \frac{P_u h}{2 f_y d}\right)(d-c)^2 + \frac{\ell_w c^3}{3} \tag{8.6}$$

In this equation, E_s is the modulus of elasticity of the reinforcing steel, E_c is the modulus of elasticity of the concrete, A_s is the area of vertical reinforcement in the wall, d is the distance from the extreme compression fiber to the centroid of A_s, c is the distance from the extreme compression fiber to the neutral axis, and ℓ_w is the length of the wall. The ratio E_s / E_c must be taken greater than or equal to 6 in this equation.

In the strength design method, the neutral axis depth, c, is related to the depth of the equivalent rectangular stress block, a, by $c = a / \beta_1$. In general, a is the force in the tension reinforcement $(A_s f_y)$ divided by the equivalent compressive stress $(0.85f_c')$ times the width of the section. In the alternative design method, an effective area of longitudinal reinforcement, $A_{se,w}$, is used in the determination of a and c, and is calculated by the following equation:

$$A_{se,w} = A_s + \frac{P_u h}{2f_y d} \tag{8.7}$$

This effective area of longitudinal reinforcement is used in the determination of I_{cr} [see the first term in parentheses in Equation (8.6)].

Therefore, the depth of the equivalent stress block for bending about the minor axis of the wall can be determined by the following equation:

$$a = \frac{A_s f_y + (P_u h / 2d)}{0.85 f_c' \ell_w} \tag{8.8}$$

Also, the distance from the extreme compression fiber to the neutral axis is the following:

$$c = \frac{A_s f_y + (P_u h / 2d)}{0.85 f_c' \ell_w \beta_1} \tag{8.9}$$

It is evident from Equation (8.5) that the deflection Δ_u is a function of M_u. However, M_u is a function of Δ_u, which is apparent from Equation (8.4). This clearly illustrates the iterative nature inherent to this method. Thus, M_u can be determined by assuming a value of Δ_u and then performing several calculation iterations until convergence occurs.

Direct Calculation Method

In the direct calculation method, M_u is determined by ACI Equation (11.8.3.1d):

$$M_u = \frac{M_{ua}}{1 - \dfrac{5P_u \ell_c^2}{(0.75)48E_c I_{cr}}} \tag{8.10}$$

Regardless of which method is used to determine M_u, ϕM_n is determined by the following equation:

$$\phi M_n = \phi A_{se,w} f_y \left(d - \frac{a}{2}\right) \tag{8.11}$$

Out-of-Plane Deflection

In addition to satisfying strength requirements, the following deflection requirement must be satisfied: $\Delta_s \le \ell_c / 150$, which is the fifth limitation in ACI 11.8.1.1.

The maximum deflection due to service loads, Δ_s, which includes second-order effects, depends on the magnitude of the service load moment, M_a [ACI Equation (11.8.4.2)]:

$$M_a = M_{sa} + P_s \Delta_s \tag{8.12}$$

The terms M_{sa} and P_s are the first-order, service-level bending moment and axial load at the mid-height of the wall, respectively.

Out-of-plane deflections increase rapidly when M_a exceeds two-thirds of the cracking moment, M_{cr}. Thus, Δ_s is determined by one of the following two equations in ACI Table 11.8.4.1:

For $M_a \leq 2M_{cr} / 3$:

$$\Delta_s = \left(\frac{M_a}{M_{cr}}\right)\Delta_{cr} \tag{8.13}$$

For $M_a > 2M_{cr} / 3$:

$$\Delta_s = \frac{2\Delta_{cr}}{3} + \left[\frac{M_a - (2M_{cr} / 3)}{M_n - (2M_{cr} / 3)}\right]\left(\Delta_n - \frac{2\Delta_{cr}}{3}\right) \tag{8.14}$$

In these equations, Δ_{cr} and Δ_n are the mid-height deflections corresponding to the cracking moment M_{cr} and the nominal flexural strength M_n, respectively, and are determined as follows [see ACI Equations (11.8.4.3a) and (11.8.4.3b)]:

$$\Delta_{cr} = \frac{5M_{cr}\ell_c^2}{48E_cI_g} \tag{8.15}$$

$$\Delta_n = \frac{5M_n\ell_c^2}{48E_cI_{cr}} \tag{8.16}$$

Because Δ_s is a function of M_a [see Equation (8.13) or (8.14)] and M_a is a function of Δ_s [see Equation (8.12)], there is no closed-form solution for Δ_s. A value of Δ_s is obtained by iteration, that is, an initial value of Δ_s is assumed and calculations are performed until convergence occurs.

Service-level load combinations are not given in ACI Chapter 5 but are discussed in Appendix CC of Reference 3. For structures subjected to wind, the following service-level load combination is recommended for calculating service-level lateral deflections of walls and frames:

$$D + 0.5L + W_a \tag{8.17}$$

where W_a are the effects due to wind forces obtained from serviceability wind speeds, which are given in Figures CC.2-1, CC.2-2, CC.2-3, and CC.2-4 for 10-year, 25-year, 50-year, and 100-year mean recurrence intervals, respectively. Using wind speeds based on more than a 100-year mean recurrence interval in checking lateral deflections is excessively conservative.

For slender walls subjected to strength-level earthquake effects, E, the following load combination can be used to evaluate service-level lateral deflections:

$$D + 0.5L + 0.7E \tag{8.18}$$

8.4 Design Strength

8.4.1 General

For each applicable factored load combination in ACI Table 5.3.1, the following equations must be satisfied at any section in a wall (ACI 11.5.1.1):

$$\phi P_n \geq P_u \tag{8.19}$$

$$\phi M_n \geq M_u \tag{8.20}$$

$$\phi V_n \geq V_u \tag{8.21}$$

Strength reduction factors, ϕ, are determined in accordance with ACI 21.2. A summary of the strength reduction factors pertinent to the design of reinforced concrete walls is given in Table 8.5 for transverse reinforcement other than spirals. The strain used to define a compression-controlled section, ε_{ty}, is equal to the specified yield strength of the reinforcement, f_y, divided by the modulus of elasticity of the reinforcing steel, E_s, which can be taken as 29,000,000 psi for any grade of reinforcement (ACI 20.2.2.2).

Table 8.5 Strength Reduction Factors, ϕ

Net Tensile Strain, ε_t	Classification	Strength Reduction Factor, ϕ*
$\varepsilon_t \leq \varepsilon_{ty}$	Compression-controlled	0.65
$\varepsilon_{ty} < \varepsilon_t \leq \varepsilon_{ty} + 0.003$	Transition**	$0.65 + \dfrac{0.25(\varepsilon_t - \varepsilon_{ty})}{0.003}$
$\varepsilon_t \geq \varepsilon_{ty} + 0.003$	Tension-controlled	0.90

** Applicable to transverse reinforcement other than spirals.*
***Sections classified as transition are permitted to use ϕ corresponding to compression-controlled sections.*

For bearing walls subjected primarily to uniaxial compression forces, Equation (8.19) is applicable. For bearing walls subjected to combined axial forces and bending moment (in-plane or out-of-plane), Equations (8.19) and (8.20) must both be satisfied for all applicable factored load combinations (ACI 11.5.2.1). Alternatively, bearing walls subjected to axial forces and out-of-plane bending moments are permitted to be designed using the simplified design method in ACI 11.5.3 provided the limitations of that method are satisfied (see Section 8.4.3 of this publication).

For nonbearing walls, Equation (8.20) must be satisfied where M_n is determined in accordance with the flexural strength requirements of ACI 22.3 (ACI 11.5.2.2).

Equation (8.21) must be satisfied for both in-plane and out-of-plane shear forces on a wall.

Methods to determine the nominal strengths P_n, combined P_n and M_n, and V_n, are given in Sections 8.4.2, 8.4.3, and 8.4.4, respectively.

8.4.2 Nominal Axial Strength

Nominal Axial Compressive Strength

The nominal axial compressive strength, P_n, of a nonslender, reinforced concrete wall is determined in accordance with ACI 22.4.2. The nominal axial compressive strength, P_n, must be less than or equal to the maximum nominal axial compressive strength, $P_{n,max}$, which is determined using ACI Table 22.4.2.1 and the nominal axial strength at zero eccentricity, P_o, determined by ACI Equation (22.4.2.2):

$$P_n \leq P_{n,max} = 0.80P_o = 0.80[0.85f_c'(A_g - A_{st}) + f_yA_{st}] \quad (8.22)$$

When calculating $P_{n,max}$, the value of the specified yield strength of the longitudinal reinforcement, f_y, is limited to 80,000 psi even though f_y for the longitudinal (vertical) reinforcement used in the wall may be larger than that (ACI 22.4.2.1).

Nominal Axial Tensile Strength

The nominal axial tensile strength of a nonprestressed, reinforced concrete wall, P_{nt}, must be less than or equal to the maximum nominal axial tensile strength, $P_{nt,max}$, which is determined by ACI Equation (22.4.3.1):

$$P_{nt} \leq P_{nt,max} = f_yA_{st} \quad (8.23)$$

It is evident that the entire tensile force must be resisted by the longitudinal reinforcement in a wall.

8.4.3 Nominal Strength of Walls Subjected to Moment and Axial Forces

Overview

The nominal strength of a reinforced concrete wall subjected to both moment and axial force is determined using equilibrium, strain compatibility, and the design assumptions in ACI 22.2, which are summarized in Table 8.6.

Table 8.6 Design Assumptions for Concrete and Nonprestressed Reinforcement

Material	Assumptions
Concrete	1. Maximum strain in the extreme concrete compression fiber is assumed equal to 0.003.
	2. Tensile strength of concrete is neglected in flexural and axial strength calculations.
	3. The relationship between concrete compressive stress and strain is to be represented by a rectangular, trapezoidal, parabolic, or other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.
	4. The equivalent rectangular concrete stress distribution in accordance with ACI 22.2.2.4.1 through 22.2.2.4.3 satisfies the requirement of ACI 22.2.2.3 for the relationship between concrete compressive stress and strain.
	5. A concrete stress of $0.85f_c'$ is to be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross-section and a line parallel to the neutral axis located a distance a from the fiber of maximum compressive strain where $a = \beta_1 c$. The distance from the fiber of maximum compressive strain to the neutral axis c is to be measured perpendicular to the neutral axis. Values of β_1 are as follows: • For 2,500 psi $\leq f_c' \leq$ 4,000 psi : $\beta_1 = 0.85$ • For 4,000 psi $< f_c' <$ 8,000 psi : $\beta_1 = 0.85 - [0.05(f_c' - 4{,}000)/1{,}000]$ • For $f_c' \geq 8{,}000$ psi : $\beta_1 = 0.65$
Nonprestressed reinforcement	1. Deformed reinforcement used to resist tensile or compressive forces must conform to ACI 20.2.1.
	2. The stress-strain relationship and the modulus of elasticity of the deformed reinforcement are to be idealized in accordance with ACI 20.2.2.1 and 20.2.2.2.

Rectangular Sections

The general principles and assumptions of the strength design method in Table 8.6 are applied to the rectangular reinforced concrete wall given in Figure 8.6. For purposes of discussion, it is assumed the longitudinal reinforcement in layer 1 is located closest to the extreme compression fiber of the section.

The equations in Figure 7.11 of this publication for rectangular column sections can be used to determine the nominal axial strength, P_n, and the nominal flexural strength, M_n, of the rectangular wall in Figure 8.6 for a given strain, ε_t, which is the net tensile strain in the extreme layer of tension reinforcement at nominal strength, excluding strains due to creep, shrinkage, and temperature.

I-, T-, and L-Shaped Sections

For walls with flanges where the flanges are in compression, the nominal axial and flexural strengths depend on whether the depth of the equivalent stress block, a, is less than or greater than the flange thickness.

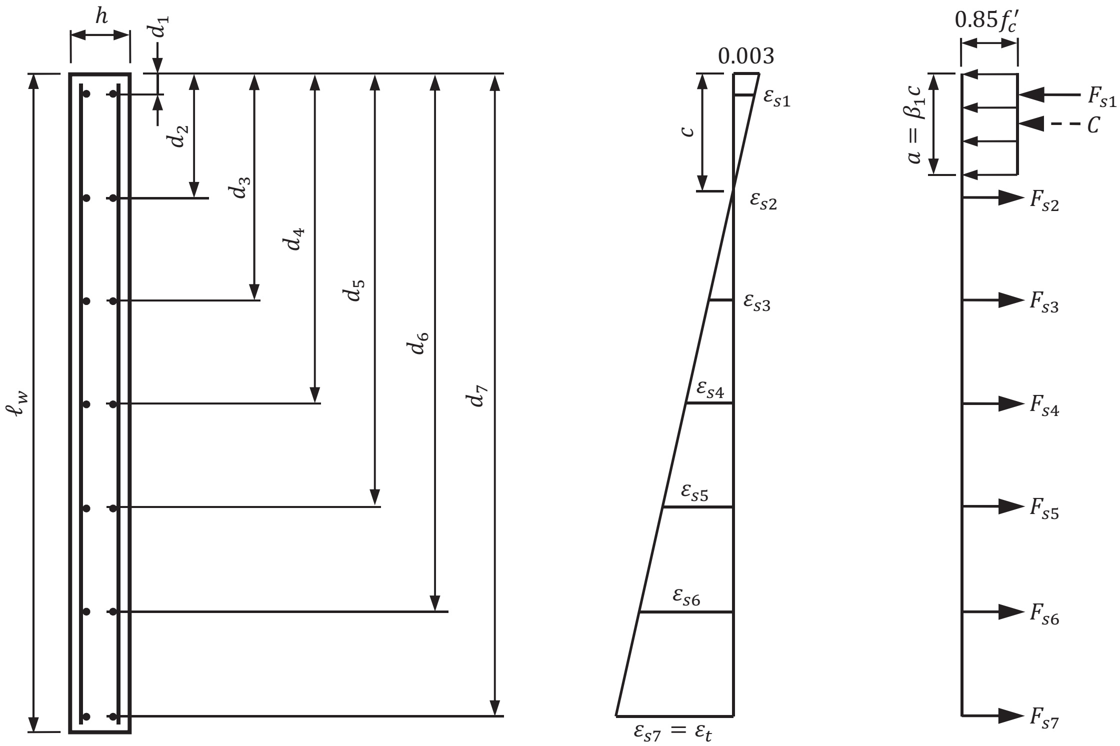


Figure 8.6 Reinforced concrete wall subjected to moment and axial compression.

Where a is less than or equal to the flange thickness, P_n and M_n can be determined using the equations in Figure 7.11 for a rectangular wall section.

Where a is greater than the flange thickness, C and M_n are determined based on a T- or L-shaped compression zone. Consider the I-shaped wall section in Figure 8.7 where the equivalent stress block falls in the web. The resultant compression force, C, can be determined by the following equation:

$$C = 0.85f_c'[b_f t_f + (a - t_f)h] \tag{8.24}$$

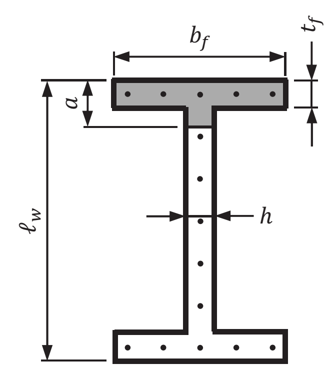


Figure 8.7 An I-shaped wall where the equivalent stress block falls in the web.

The strains, stresses, and forces in the longitudinal reinforcement and P_n are determined using the expressions in Figure 7.11. The nominal flexural strength is determined by the following equation:

$$M_n = C\overline{y} + \sum_{i=1}^{n} F_{si}\left(\frac{\ell_w}{2} - d_i\right) \tag{8.25}$$

where the distance from the centroid of the wall to the centroid of the compression zone, $\overline{y}$, is equal to the following:

$$\overline{y} = \frac{b_f t_f(\ell_w - t_f) + h(a - t_f)(\ell_w - a - t_f)}{2[b_f t_f + (a - t_f)h]} \tag{8.26}$$

Similar expressions can be derived for L-shaped compression zones.

It is assumed in the above discussion that the entire width of the flange is effective in resisting the combined axial force and bending moment. Although not specifically required in ACI Chapter 11, it may be warranted to use a flange width less than the actual flange width in the analysis. A method to determine the portion of the flange considered to be effective for walls subjected to earthquake effects is given in ACI 18.10.5.2; that effective flange width can be used for any wall with flanges in lieu of performing a more refined analysis. Another option is to completely disregard the flanges and to design the wall as a rectangular section of length ℓ_w and thickness h.

Simplified Design Method

The simplified design method in ACI 11.5.3 can be used to determine the nominal axial compressive strength, P_n, of a wall where the following two limitations are satisfied:

1. The wall has a solid, rectangular cross-section.
2. The resultant of all applicable factored loads falls within the middle third of the wall thickness.

The second limitation is illustrated in Figure 8.8 for the case of a factored axial force, P_u, acting at an eccentricity, e, less than or equal to $h / 6$. In general, the total eccentricity caused by all applicable factored load effects, including those from lateral loads, must be less than or equal to $h / 6$.

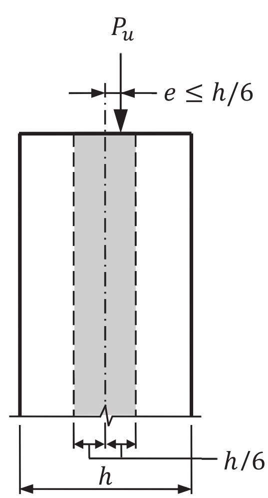


Figure 8.8 Eccentricity limitation in the simplified design method.

ACI Equation (11.5.3.1) can be used to determine P_n when both limitations are satisfied:

$$P_n = 0.55 f_c' A_g \left[1 - \left(\frac{k\ell_c}{32h}\right)^2\right] \tag{8.27}$$

In this equation, A_g is the gross area of the wall and k is the effective length factor given in ACI Table 11.5.3.2 (see Table 8.7).

Table 8.7 Effective Length Factor for Walls Designed by the Simplified Design Method

Boundary Conditions		k
Walls braced top and bottom against lateral translation	Restrained against rotation at one or both ends (top, bottom, or both)	0.8
	Unrestrained against rotation at both ends	1.0
Walls not braced against lateral translation		2.0

A value of k equal to 0.8 implies the end of the wall is attached to a member with a flexural stiffness at least equal to that of the wall in the direction of analysis. A value of k equal to 2.0 would be applicable, for example, to free-standing (cantilever) walls or to walls connected to diaphragms that undergo significant deflections when subjected to lateral loads.

Equation (8.27) takes into consideration both eccentricity and slenderness effects. In order to satisfy strength requirements, the design strength, ϕP_n, must be greater than or equal to the factored axial force, P_u, where $\phi = 0.65$ for compression-controlled sections.

It is inherently assumed in the simplified design method that the walls are designed considering P_u as a concentric axial compression force. As such, it is best suited for relatively short walls subjected to vertical loads. Because the eccentricity must not exceed $h / 6$, the application of this method becomes very limited when lateral loads need to be considered.

8.4.4 Nominal Shear Strength

In-Plane Shear

For walls subjected to in-plane shear forces, the nominal shear strength, V_n, at a section is determined in accordance with ACI 11.5.4.2 and 11.5.4.3 (ACI 11.5.4.1):

$$V_n = (\alpha_c \lambda \sqrt{f_c'} + \rho_t f_{yt}) A_{cv} \leq 8\sqrt{f_c'} A_{cv} \tag{8.28}$$

The term $\alpha_c \lambda \sqrt{f_c'} A_{cv}$ is the nominal shear strength of the concrete and $\rho_t f_{yt} A_{cv}$ is the nominal shear strength of the transverse (horizontal) reinforcement in the wall.

The coefficient α_c defines the relative contribution of the concrete strength to the nominal shear strength and is determined by the following:

$$\alpha_c = \begin{cases} 3 \text{ for } h_w / \ell_w \leq 1.5 \\ 2\left[1 + \left(2 - \dfrac{h_w}{\ell_w}\right)\right] \text{ for } 1.5 < h_w / \ell_w < 2.0 \\ 2 \text{ for } h_w / \ell_w \geq 2.0 \end{cases} \tag{8.29}$$

For walls where a net tension force is determined for the entire wall section, α_c is calculated by ACI Equation (11.5.4.4) [ACI 11.5.4.4]:

$$\alpha_c = 2\left(1 + \frac{N_u}{500 A_g}\right) \geq 0.0 \tag{8.30}$$

The axial tension force, N_u, is negative and has the units of pounds.

The modification factor, λ, reflects the reduced mechanical properties of lightweight concrete relative to normal-weight concrete of the same compressive strength is given in Table 8.8 based on equilibrium density and in Table 8.9 based on composition of aggregates in the concrete mixture (see ACI 19.2.4).

Table 8.8 Values of λ Based on Equilibrium Density, w_c

Equilibrium Density, w_c	λ
$w_c \leq 100\ \text{lb/ft}^3$	0.75
$100\ \text{lb/ft}^3 < w_c \leq 135\ \text{lb/ft}^3$	$0.0075 w_c \leq 1.0$
$w_c > 135\ \text{lb/ft}^3$	1.0

Table 8.9 Values of λ Based on Composition of Aggregates

Concrete	Composition of Aggregates	λ
All-lightweight	Fine: ASTM C330 Coarse: ASTM C330	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330 and C33 Coarse: ASTM C330	0.75 to 0.85(1)
Sand-lightweight	Fine: ASTM C33 Coarse: ASTM C330	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33 Coarse: Combination of ASTM C330 and ASTM C33	0.85 to 1.0(2)

(1) Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.
(2) Linear interpolation from 0.85 to 1.0 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of aggregate.

The term ρ_t is the ratio of the distributed transverse (horizontal) reinforcement to the gross area of the concrete area perpendicular to that reinforcement.

The area A_{cv} is the gross area of the cross-section of the wall resisting the shear force. For a rectangular wall with no openings, $A_{cv} = h\ell_w$. Illustrated in Figure 8.9 is the area A_{cv} to be used in Equation (8.28) where the ends of the wall are enlarged or frame into columns (this is often referred to as a "barbell" wall because of its shape).

For walls with one or more openings, the area of the opening(s) must not be included in A_{cv} (see Figure 8.9).

For walls with $h_w / \ell_w < 2$, the strut-and-tie method in ACI Chapter 23 may be used to design for in-plane shear forces instead of the method described above (ACI 11.5.4.1).

Out-of-Plane Shear

For walls subjected to out-of-plane shear forces, the nominal shear strength, V_n, is determined by the one-way shear strength requirements in ACI 22.5 (ACI 11.5.5.1). Because shear reinforcement is generally not provided to satisfy one-way out-of-plane shear requirements, $V_n = V_c$ where the nominal shear strength of the concrete, V_c, is determined by the following equation from ACI Table 22.5.5.1 where $A_v < A_{v,min}$:

$$V_c = \left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d \leq 5\lambda\sqrt{f_c'}b_w d \qquad (8.31)$$

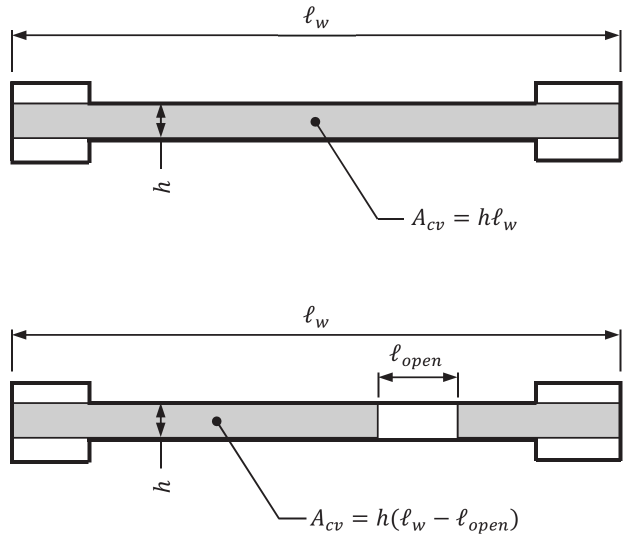


Figure 8.9 Determination of A_{cv}

The size effect modification factor, λ_s, accounts for the phenomenon indicated in test results that the shear strength attributed to concrete in members without shear reinforcement does not increase in direct proportion with member depth. This factor is determined by ACI Equation (22.5.5.1.3) [ACI 22.5.5.1.3]:

$$\lambda_s = \sqrt{\frac{2}{1+(d/10)}} \leq 1.0 \tag{8.32}$$

It is evident from Equation (8.32) that λ_s is less than 1.0 for members with $d > 10.0$ in.

The term ρ_w is equal to the area of flexural reinforcement, A_s, at the section divided by $b_w d$ where b_w is commonly taken as one foot (see Figure 8.10). According to ACI R22.5.5.1, A_s may be taken as the sum of the areas of the longitudinal flexural reinforcement located more than two-thirds of the overall member depth away from the extreme compression fiber. For typical wall sections, A_s is the area of longitudinal tension reinforcement within the one-foot design strip (see Figure 8.10 for the case of a wall with two layers of longitudinal reinforcement).

According to the first Note in ACI Table 22.5.5.1, the axial force, N_u, which has the units of pounds, is to be taken as positive for compression forces acting on the gross area of the wall, A_g, and is to be taken as negative for tension forces. Also, $N_u / 6A_g \leq 0.05f_c'$ (ACI 22.5.5.1.2).

Values of $\sqrt{f_c'}$ used to calculate V_c are limited to 100 psi (the exception in ACI 22.5.3.2 is not applicable to walls; see ACI 22.5.3.1). This limitation on f_c' is primarily due to the fact that there is a lack of test data and practical experience with concrete having compressive strengths greater than 10,000 psi.

To minimize the likelihood of diagonal compression failure in the concrete and to limit the extent of cracking, the cross-sectional dimensions of a section must be selected to satisfy ACI Equation (22.5.1.2) [ACI 22.5.1.2]:

$$V_u \leq \phi(V_c + 8\sqrt{f_c'}b_w d) \tag{8.33}$$

Where out-of-plane shear strength requirements are not satisfied, the thickness of the wall must typically be increased. Increasing the compressive strength of the concrete is usually not as effective as increasing h.

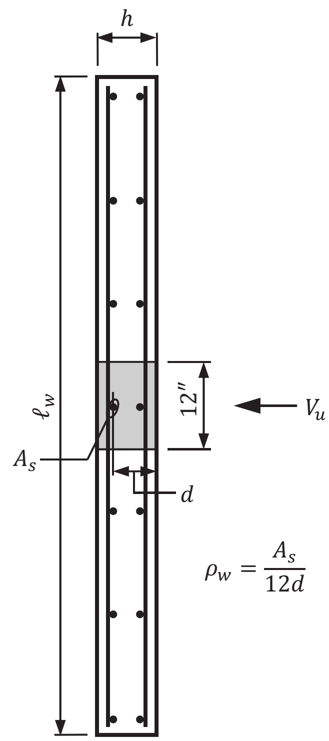


Figure 8.10 Out-of-plane shear force.

8.5 Reinforcement Limits

Both longitudinal (vertical) and transverse (horizontal) reinforcement must be provided in a reinforced concrete wall. Minimum longitudinal and transverse reinforcement requirements are given in ACI Table 11.6.1 and are summarized in Table 8.10 for deformed reinforcing bars with $f_y \geq 60{,}000$ psi and $V_u \leq 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$. The longitudinal reinforcement ratio, ρ_ℓ, corresponds to the vertical bars in the wall and is equal to the area of the vertical reinforcing bars divided by the gross area of the wall perpendicular to those bars, that is, the thickness of the wall times the length of the wall. Similarly, the transverse reinforcement ratio, ρ_t, corresponds to the horizontal bars in the wall and is equal to the area of the horizontal reinforcing bars divided by the gross area of the wall perpendicular to those bars, that is, the thickness of the wall times the height of the wall. For regularly spaced reinforcing bars, the reinforcement ratios can be determined on the basis of the applicable bar spacing instead of on the overall length or height of the wall.

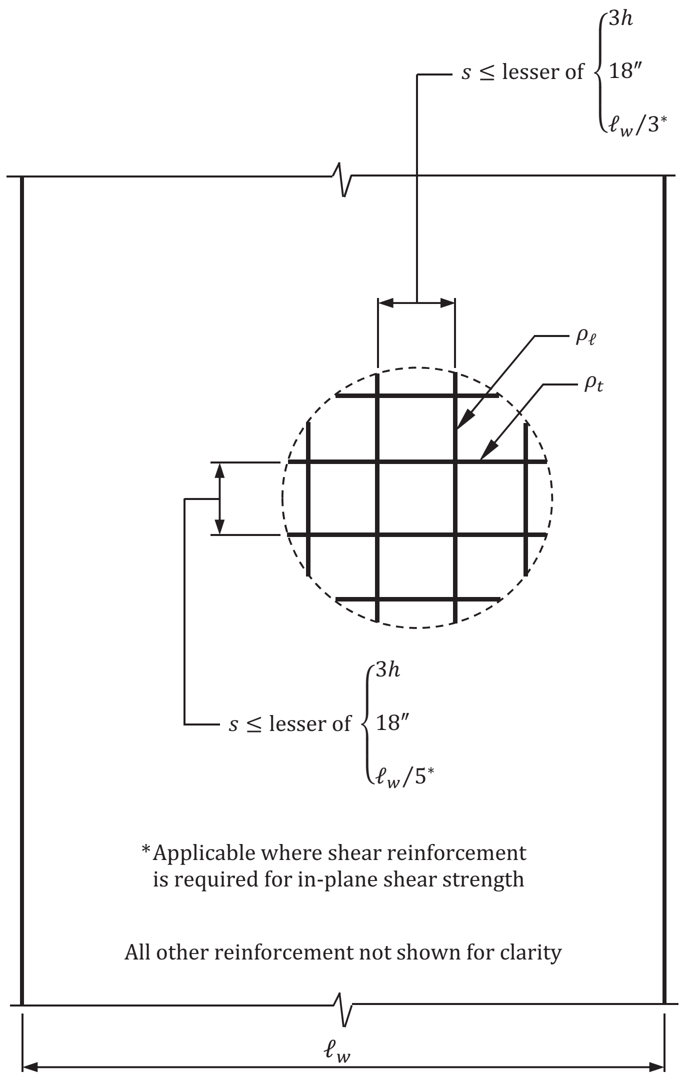
Table 8.10 Minimum Reinforcement for Walls in Accordance with ACI Table 11.6.1

Bar Size	Minimum longitudinal, ρ_ℓ	Minimum transverse, ρ_t
$\leq \#5$	0.0012	0.0020
$> \#5$	0.0015	0.0025

Where $V_u > 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$, the reinforcement requirements in ACI 11.6.2 must be satisfied. These requirements, along with the minimum reinforcement requirements of ACI 11.6.1, are given in Figure 8.11.

8.6 Determining the Wall Thickness

Because structural walls in buildings are typically located around elevator and stair openings, the length of a wall is often dictated by architectural requirements. The thickness of a wall, in conjunction with the specified length and material properties, needs to be determined so that strength (axial force, flexure, and shear) and serviceability requirements are satisfied.



Elevation

Longitudinal reinforcement, $\boldsymbol{\rho_\ell}$			Transverse reinforcement, $\boldsymbol{\rho_t}$		
$V_u \leq 0.5\phi\alpha_c\lambda\sqrt{f_c'}A_{cv}$		$V_u > 0.5\phi\alpha_c\lambda\sqrt{f_c'}A_{cv}$	$V_u \leq 0.5\phi\alpha_c\lambda\sqrt{f_c'}A_{cv}$		$V_u > 0.5\phi\alpha_c\lambda\sqrt{f_c'}A_{cv}$
≤ #5 bars	> #5 bars		≤ #5 bars	> #5 bars	
0.0012	0.0015	$0.0025 + 0.5(2.5 - h_w/\ell_w)(\rho_t - 0.0025) \geq 0.0025^{**}$	0.0020	0.0025	0.0025

**ρ_ℓ need not exceed ρ_t required by ACI 11.5.4.3

Figure 8.11 Minimum reinforcement requirements for cast-in-place reinforced concrete walls.

For walls where shear strength requirements govern, Equation (8.28) can be used to determine a minimum h based on a maximum in-plane shear force, V_u:

$$h \geq \frac{V_u}{\phi(\alpha_c\lambda\sqrt{f_c'} + \rho_t f_{yt})\ell_w} \tag{8.34}$$

where $\phi = 0.75$.

For walls satisfying the limitations of the simplified design method in ACI 11.5.3, the following equation can be used to determine minimum h [see Equation (8.27)]:

$$h = \frac{(P_u / \phi)}{1.1 f'_c \ell_w} + \sqrt{\left[\frac{(P_u / \phi)}{1.1 f'_c \ell_w}\right]^2 + \left(\frac{k\ell_c}{32}\right)^2} \tag{8.35}$$

where $\phi = 0.65$ for compression-controlled sections.

Relatively tall and/or slender walls can be governed by a combination of axial force and in-plane bending moments. A wall thickness and longitudinal wall reinforcement (size and spacing) must be selected so that an interaction diagram can be constructed. Because there is no closed-form solution for multiple factored load cases, iterations must be performed until all design strength criteria are satisfied, that is, $\phi P_n \geq P_u$ and $\phi M_n \geq M_u$ must be satisfied for all factored load combinations.

8.7 Determination of Required Reinforcement

8.7.1 Longitudinal Reinforcement

As noted in Section 8.6 of this publication, the size and spacing of the longitudinal reinforcement in a wall must be selected so that all design strength criteria are satisfied for combined flexure and axial force. No closed-form solution exists to determine the required ρ_ℓ for a given set of factored load combinations; it is usually determined by performing a number of iterations where at least the minimum reinforcement prescribed in ACI 11.6.1 and 11.6.2 are used in the first iteration.

Minimum longitudinal reinforcement per foot length of wall as a function of wall thickness for cases where $V_u \leq 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$ is given in Table 8.11 $(A_s = \rho_\ell bh = 0.0012 \times 12 \times h = 0.0144h$ for #5 bars and smaller; see Table 8.10 and Figure 8.11). Also included in the table is suggested reinforcing bar size and spacing. Two layers of reinforcement must be provided where the thickness of the wall is greater than 10 in., except for single-story basement walls and cantilever retaining walls (ACI 11.7.2.3). Each layer of reinforcement should be placed near each face.

Table 8.11 Minimum Required Longitudinal Reinforcement for Walls where $V_u \leq 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$

Wall Thickness, h (in.)	A_s (in.^{2}/ft)	Suggested Reinforcement
6	0.09	#3@14″
8	0.12	#3@10″
10	0.14	#4@16″
12*	0.17	#4@18″
14*	0.20	#4@18″
16*	0.23	#4@18″
18*	0.26	#4@18″

**Two layers of reinforcement are required.*

It is common for the walls in a low-rise building to have minimum longitudinal reinforcement over the entire height of the wall.

Where $V_u > 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$, the minimum reinforcement requirements of ACI 11.6.2 must be satisfied. Minimum longitudinal reinforcement per foot length of wall as a function of wall thickness where $V_u > 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$ assuming $\rho_t = 0.0025$ is given in Table 8.12 $(A_s = \rho_\ell bh = 0.0025 \times 12 \times h = 0.0300h;$ see Figure 8.11).

Table 8.12 Minimum Required Longitudinal Reinforcement for Walls where $V_u > 0.5\phi\,\alpha_c\lambda\sqrt{f'_c}A_{cv}$

Wall Thickness, h (in.)	A_s (in.^{2}/ft)	Suggested Reinforcement
6	0.18	#5@18″
8	0.24	#5@14″
10	0.30	#5@12″
12*	0.36	#5@18″
14*	0.42	#5@16″
16*	0.48	#5@14″
18*	0.54	#5@12″

**Two layers of reinforcement are required.*

8.7.2 Transverse Reinforcement

The required amount of transverse reinforcement, ρ_t, depends on the maximum factored in-plane shear force, V_u, which acts on the wall. Equation (8.28) can be solved for ρ_t :

$$\rho_t = \left(\frac{V_u}{\phi A_{cv}} - \alpha_c\lambda\sqrt{f'_c}\right)\frac{1}{f_{yt}} \qquad (8.36)$$

Where $V_u \le 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$, minimum ρ_t determined by ACI 11.6.1 must be provided. Minimum transverse reinforcement per foot length of wall as a function of wall thickness where $V_u \le 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$ is given in Table 8.13 ($A_s = \rho_t bh = 0.0020 \times 12 \times h = 0.0240h$ for #5 bars and smaller; see Table 8.10 and Figure 8.11).

Table 8.13 Minimum Required Transverse Reinforcement for Walls where $V_u \le 0.5\phi\,\alpha_c\lambda\sqrt{f'_c}A_{cv}$

Wall Thickness, h (in.)	A_s (in.^{2}/ft)	Suggested Reinforcement
6	0.14	#4@16″
8	0.19	#4@12″
10	0.24	#4@10″
12*	0.29	#4@16″
14*	0.34	#4@14″
16*	0.38	#4@12″
18*	0.43	#4@10″

**Two layers of reinforcement are required.*

Where $V_u > 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$, the minimum reinforcement requirements of ACI 11.6.2 must be satisfied. Minimum transverse reinforcement per foot length of wall as a function of wall thickness where $V_u > 0.5\phi\alpha_c\lambda\sqrt{f'_c}A_{cv}$ is given in Table 8.14 (that is, $A_s = \rho_\ell bh = 0.0025 \times 12 \times h = 0.0300h;$ see Figure 8.11).

Table 8.14 Minimum Required Transverse Reinforcement for Walls where $V_u > 0.5\phi \alpha_c \lambda \sqrt{f'_c} A_{cv}$

Wall Thickness, h (in.)	A_s (in.^{2}/ft)	Suggested Reinforcement
6	0.18	#5@18″
8	0.24	#5@14″
10	0.30	#5@12″
12*	0.36	#5@18″
14*	0.42	#5@16″
16*	0.48	#5@14″
18*	0.54	#5@12″

Two layers of reinforcement are required.

8.8 Reinforcement Detailing

8.8.1 Concrete Cover

Reinforcing bars are placed in walls with a minimum concrete cover to protect them from weather, fire, and other effects. Minimum cover requirements are given in ACI 20.5.1 (ACI 11.7.1.1). Concrete cover for walls is measured from the surface of the concrete to the outer edge of the layer of reinforcement closest to the wall surface (see Figure 8.12 for a wall where the transverse reinforcement is closest to the concrete surface). The minimum cover is equal to 0.75 in. for #11 and smaller reinforcing bars in walls not exposed to weather or in contact with the ground (ACI Table 20.5.1.3.1). For #14 and #18 bars, the minimum cover is equal 1.5 in.

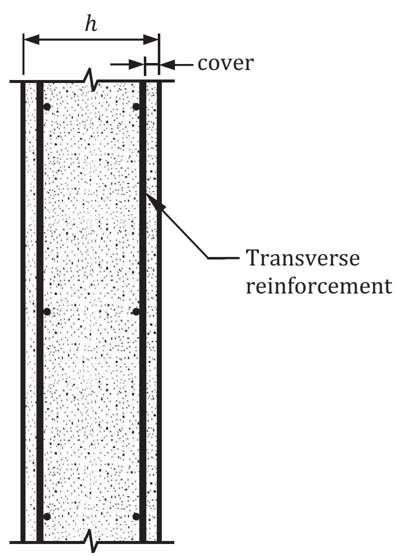


Figure 8.12 Concrete cover for walls.

8.8.2 Splices of Reinforcement

Overview

Splice lengths for longitudinal and transverse reinforcement in walls must be in accordance with the provisions in ACI 25.5 (ACI 11.7.1.3). Lap splices and mechanical splices are commonly used in walls. For the longitudinal bars, compression or tension splice lengths must be provided that satisfy the factored load combinations resisted by the wall. Tension lap and mechanical splices are used for the transverse reinforcement.

Lap Splices

Lap splices are the most popular and usually the most economical type of splices used in walls. The longitudinal reinforcement in a wall is permitted to be lap spliced immediately above the top of the slab (see Figure 8.13). This is the preferred location for ease of construction: The longitudinal bars from the wall below extend above the slab a distance that is greater than or equal to the required lap splice length, and the longitudinal bars in the wall above are tied to these bars after the floor below has been constructed.

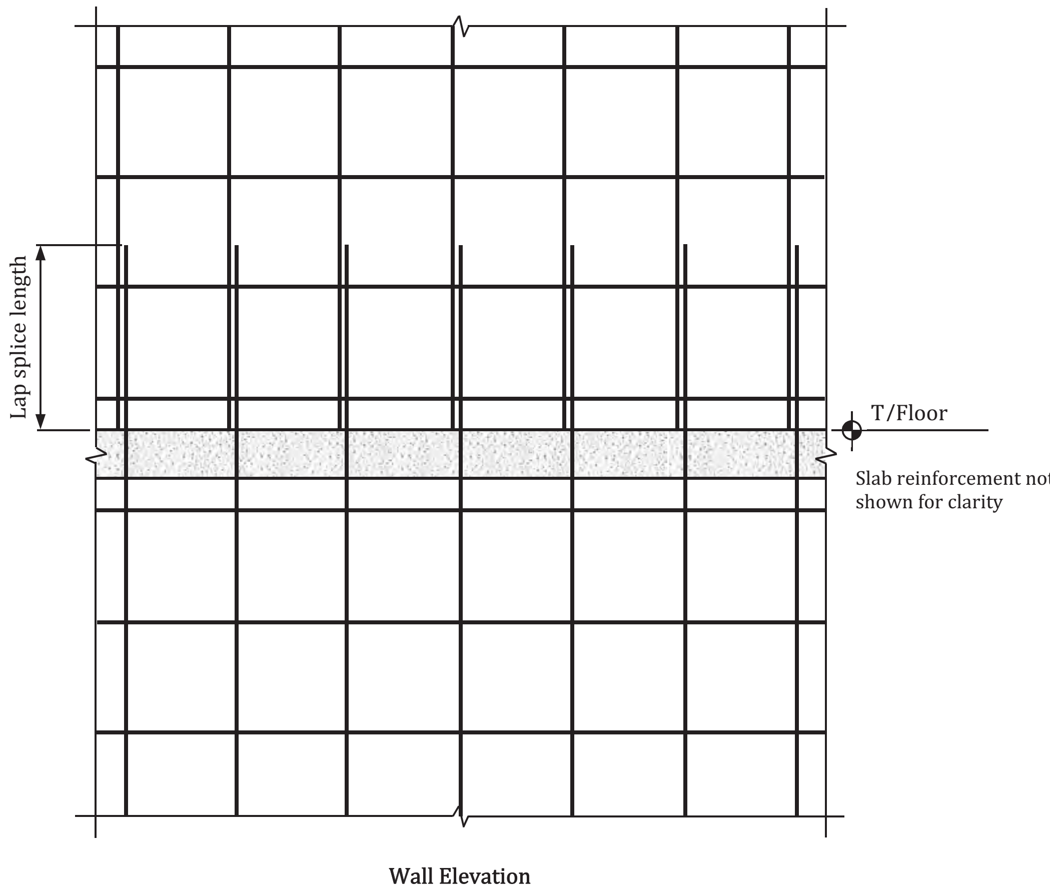


Figure 8.13 Location of lap splices for longitudinal bars in a wall.

Lap splices are not permitted for bars larger than #11 bars except in the case of compression lap splices where #14 or #18 bars are permitted to be spliced to #11 or smaller bars (ACI 25.5.1.1 and 25.5.5.3).

The type of lap splice that must be used (compressive or tensile) depends on the stress in the longitudinal bars due to the factored load combinations (see Figure 7.28 of this publication, which is also applicable to walls). Required lap splice lengths for the three zones indicated in Figure 7.28 are given in Table 8.15.

Table 8.15 Required Lap Splices Lengths

Zone	Stress in Longitudinal Bars	Minimum Lap Splice Length		Notes	ACI Section No.
1	All longitudinal bars are in compression	$f_y \leq 60{,}000$ psi	$\ell_{sc} = 0.0005 f_y d_b \geq 12$ in.	1–6	25.5.5.1
		$60{,}000 \text{ psi} < f_y \leq 80{,}000 \text{ psi}$	$\ell_{sc} = (0.0009 f_y - 24) d_b \geq 12$ in.		
		$f_y > 80{,}000$ psi	$\ell_{sc} = \text{greater of} \begin{cases} (0.0009 f_y - 24) d_b \\ \ell_{st} \text{ calculated by ACI 25.5.2.1} \end{cases}$		
2	Stress in the longitudinal bars on the tension face is tensile and $\leq 0.5 f_y$	Class A tension lap splice $\ell_{st} = \ell_d \geq 12$ in. Class B tension lap splice length $\ell_{st} = 1.3\ell_d \geq 12$ in.		6–9	25.5.2.1
3	Stress in the longitudinal bars on the tension face is tensile and $> 0.5 f_y$	Class B tension lap splice length $\ell_{st} = 1.3\ell_d \geq 12$ in.		6–9	25.5.2.1

Notes:

1. *For* $f'_c < 3{,}000$ psi, ℓ_{sc} *must be increased by (4/3) [ACI 25.5.5.1].*
2. *Compression lap splices must not be used for bars larger than #11 except as permitted in ACI 25.5.5.3 [ACI 25.5.5.2].*
3. *Compression lap splices of #14 and #18 bars to #11 or smaller bars are permitted in accordance with ACI 25.5.5.4 (see Note 4) [ACI 25.5.5.3].*
4. *Where bars of different size are lap spliced in compression,* ℓ_{sc} *is the longer of (a)* ℓ_{dc} *of the larger bar and (b)* ℓ_{sc} *of the smaller bar where* ℓ_{dc} *is calculated by ACI 25.4.9.1 [ACI 25.5.5.4]:*

$$\ell_{dc} = \text{greater of} \begin{cases} (f_y \psi_r / 50\lambda\sqrt{f'_c}) d_b \\ 0.0003 f_y \psi_r d_b \\ 8 \text{ in.} \end{cases}$$

where λ and ψ_r are given in ACI Table 25.4.9.3 *(see Table 8.16)*

5. ℓ_{st} is determined by ACI Table 25.5.2.1 where ℓ_d is determined by ACI 25.4.2.1(a) [ACI 25.5.2.1].
6. *Reduction of development length in accordance with ACI 25.4.10.1 is not permitted in calculating lap splice lengths (ACI 25.5.1.4).*
7. *Requirements for Class A and Class B tension lap splices are given in ACI Table 10.7.5.2.2 (see Table 8.17).*
8. *Tension development length* ℓ_d *is determined in accordance with ACI 25.4.2.1(a).*
9. *Where bars of different size are lap spliced together,* ℓ_{st} *is the greater of (a)* ℓ_d *of the larger bar and (b)* ℓ_{st} *of the smaller bar (ACI 25.5.2.2).*

Table 8.16 Modification Factors for Deformed Bars in Compression

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Confining reinforcement, Ψ_r	Longitudinal reinforcement enclosed in the following: 1. A spiral 2. A circular continuously wound tie with $d_b \geq 0.25$ in. and a pitch ≤ 4 in. 3. #4 ties in accordance with ACI 25.7.2 spaced ≤ 4 in. on center 4. Hoops in accordance with ACI 25.7.4 spaced ≤ 4 in. on center	0.75
	Other	1.0

Table 8.17 Requirements for Class A and Class B Tension Lap Splices

Stress in the longitudinal bars on the tension face	Splice Details	Splice Type	Tension Lap Splice Length, ℓ_{st}*
$\leq 0.5f_y$	There are ≤ 50 percent of the longitudinal bars spliced at any section and the lap splices on adjacent bars are staggered by at least ℓ_d	Class A	$\ell_{st} = \text{ longer of } \begin{cases} 1.0\ell_d \\ 12 \text{ in.} \end{cases}$
	Other	Class B	$\ell_{st} = \text{ longer of } \begin{cases} 1.3\ell_d \\ 12 \text{ in.} \end{cases}$
$> 0.5f_y$	All cases	Class B	$\ell_{st} = \text{ longer of } \begin{cases} 1.3\ell_d \\ 12 \text{ in.} \end{cases}$

**Tension development length ℓ_d is determined in accordance with ACI 25.4.2.1(a).*

Provisions for the development of deformed reinforcing bars in tension are given in ACI 25.4.2. The tension development length, ℓ_d, is determined using the provisions of ACI 25.4.2.3 or 25.4.2.4 in conjunction with the modification factors in ACI 25.4.2.5. The requirements of ACI 25.4.2.3 are based on those in ACI 25.4.2.4, so the latter requirements are covered first.

Method 1 – ACI 25.4.2.4

The development length in tension of a deformed reinforcing bar, ℓ_d, is determined by ACI Equation (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \tag{8.37}$$

The factors in Equation (8.37) are given in ACI Table 25.4.2.5; these factors can be found in Table 8.18 along with definitions for the other terms in Equation (8.37). When calculating ℓ_d by Equation (8.37), the confining term $(c_b + K_{tr}) / d_b$ must be taken less than or equal to 2.5 [ACI 25.4.2.4].

Table 8.18 Factors for Development of Deformed Bars in Tension

Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Reinforcement grade, ψ_g	Grade 40 or Grade 60	1.0
	Grade 80	1.15
	Grade 100	1.3
Epoxy, ψ_e^*	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover $< 3d_b$ or clear spacing $< 6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size, ψ_s	#7 and larger bars	1.0
	#6 and smaller bars	0.8
Casting position, ψ_t^*	More than 12 in. of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0
Spacing or cover dimension, c_b	All	Lesser of: 1. The distance from the center of a longitudinal bar to the nearest concrete surface. 2. One-half the center-to-center spacing of the longitudinal bars being developed (see Figure 8.14 for the case of the longitudinal bars in a wall being developed).
Transverse reinforcement index, K_{tr}	All	$K_{tr} = \frac{40A_{tr}}{sn}$ ** where: 1. A_{tr} = total cross-sectional area of all transverse reinforcement within a spacing s that crosses the potential plane of splitting through the longitudinal reinforcement being developed 2. s = center-to-center spacing of the transverse reinforcement 3. n = number of bars being developed along the plane of splitting

*The product $\psi_t \psi_e$ need not exceed 1.7.
**It is permitted to conservatively use $K_{tr} = 0$ if transverse (confining) reinforcement is present or required. For reinforcing bars with $f_y \geq 80{,}000$ psi spaced closer than 6.0 in. on center, transverse reinforcement must be provided along the development length such that $K_{tr} \geq 0.5d_b$ (ACI 25.4.2.2).

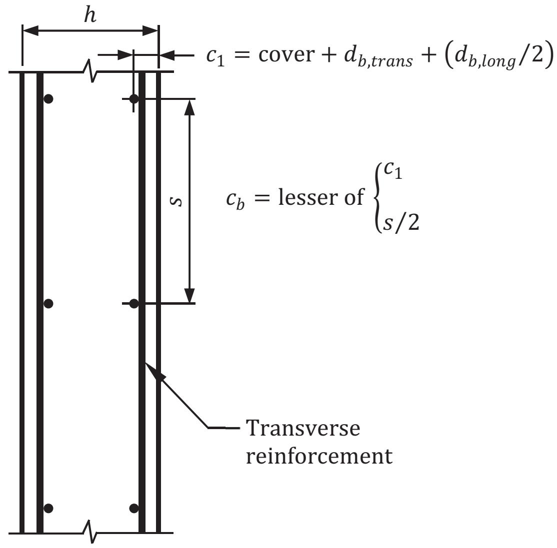


Figure 8.14 Spacing or cover dimension, c_b.

Method 2 – ACI 25.4.2.3

The method given in ACI 25.4.2.3 to determine ℓ_d is based on the requirements given in ACI 25.4.2.4 and pre-selected values of the confining term $(c_b + K_{tr})/d_b$. Two sets of spacing and cover cases are given in ACI Table 25.4.2.3 (see Table 8.19). The modifications factors in these equations are obtained from ACI Table 25.4.2.5 (see Table 8.18).

Table 8.19 Tension Development Length, ℓ_d, for Deformed Bars in Accordance with ACI 25.4.2.3

Spacing and Cover		#6 and Smaller Bars	#7 and Larger Bars
Case 1	Condition 1 1. Clear spacing of bars being developed or lap spliced $\geq d_b$ 2. Clear cover $\geq d_b$ 3. Stirrups or ties throughout ℓ_d not less than minimum required	$\left(\dfrac{f_y\psi_t\psi_e\psi_g}{25\lambda\sqrt{f'_c}}\right)d_b$	$\left(\dfrac{f_y\psi_t\psi_e\psi_g}{20\lambda\sqrt{f'_c}}\right)d_b$
	Condition 2 1. Clear spacing of bars being developed or lap spliced $\geq 2d_b$ 2. Clear cover $\geq d_b$		
Case 2	Other conditions	$\left(\dfrac{3f_y\psi_t\psi_e\psi_g}{50\lambda\sqrt{f'_c}}\right)d_b$	$\left(\dfrac{3f_y\psi_t\psi_e\psi_g}{40\lambda\sqrt{f'_c}}\right)d_b$

Mechanical and Welded Splices

Mechanical or welded splices are permitted in walls and must meet the requirements of ACI 25.5.7. These types of splices may be used in both tension and compression.

A variety of proprietary mechanical devices are available that can be used to splice longitudinal reinforcing bars in a wall (see Reference 9). These types of splices are commonly specified at locations where inordinately long lap splices would be required or where lap splices would cause congestion. Mechanical splices must be able to develop in tension or compression at least 125 percent of the yield strength of the longitudinal bars.

Welded splices must also be able to develop $1.25f_y$ of the longitudinal bars. These types of splices are primarily intended for #6 bars and larger. Welding of reinforcing bars must conform to the requirements of ACI 26.6.4.

8.8.3 Spacing of Longitudinal Reinforcement

Maximum spacing of longitudinal reinforcement in walls is given in ACI 11.7.2 (see Figure 8.11):

$$s \le \text{lesser of} \begin{cases} 3h \\ 18 \text{ in.} \\ \ell_w / 3 \end{cases} \tag{8.38}$$

The $\ell_w / 3$ limit in Equation (8.38) is applicable where shear reinforcement is required for in-plane strength.

8.8.4 Spacing of Transverse Reinforcement

Maximum spacing of transverse reinforcement in walls is given in ACI 11.7.3 (see Figure 8.11):

$$s \le \text{lesser of} \begin{cases} 3h \\ 18 \text{ in.} \\ \ell_w / 5 \end{cases} \tag{8.39}$$

The $\ell_w / 5$ limit in Equation (8.39) is applicable where shear reinforcement is required for in-plane strength.

8.8.5 Lateral Support of Longitudinal Reinforcement

Longitudinal reinforcement in a wall must be laterally supported with transverse ties where the following two conditions are satisfied (ACI 11.7.4.1):

1. The longitudinal reinforcement is required to resist compression (that is, the longitudinal reinforcement is required as compression reinforcement).
2. The area of longitudinal reinforcement, A_{st}, is greater than $0.01A_g$ where A_g is the gross cross-sectional area of the wall.

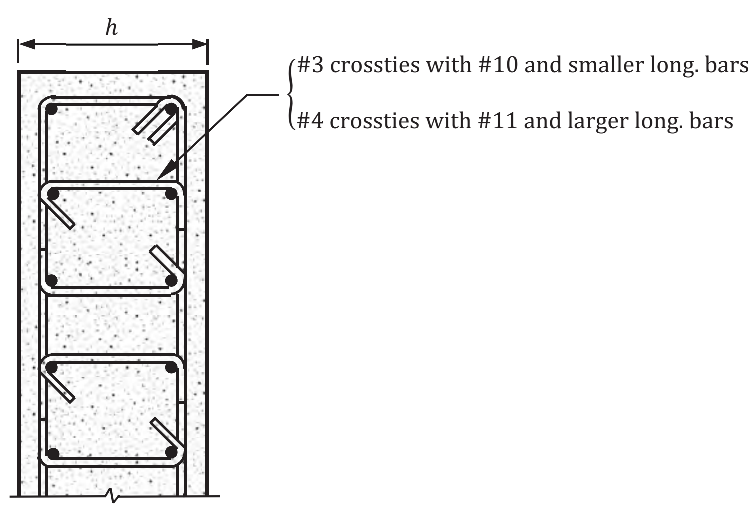
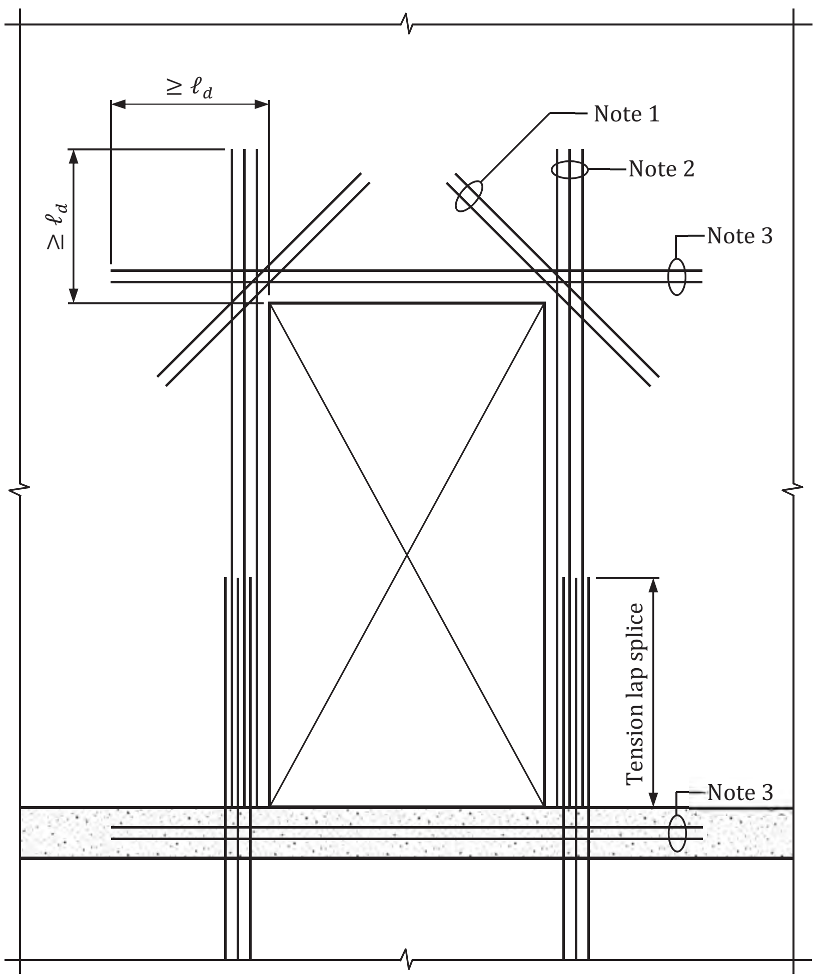


Figure 8.15 Transverse ties in a wall in accordance with ACI 11.7.4.1.

The size and extent of the required transverse reinforcement is not given in ACI 11.7.4.1. Similar to columns, it is recommended to specify #3 transverse ties for #10 and smaller longitudinal bars in the wall and #4 transverse ties for larger longitudinal bars. The transverse ties must engage all longitudinal bars subjected to compression considering all applicable factored load combinations. In order to develop #3 transverse ties in tension, a minimum 10-in. wall thickness is required. Similarly, a minimum 12-in.-thick wall is required in order to develop #4 transverse ties in tension.

An example of a wall where crossties are provided as the required transverse ties is given in Figure 8.15. This is the preferred type of transverse reinforcement for walls with 2 layers of longitudinal reinforcement compared to rectangular ties that enclose four of the longitudinal bars in a wall.



Slab and other wall reinforcement not shown for clarity

Notes

1. 2-#5×4′-0″ diagonal bars at each corner.
2. Provide at least one-half of the total area of interrupted longitudinal wall reinforcement or a minimum of 2-#5 bars at each side of the opening.
3. Provide at least one-half of the total area of interrupted transverse wall reinforcement or a minimum of 2-#5 bars at the top and bottom of the opening.

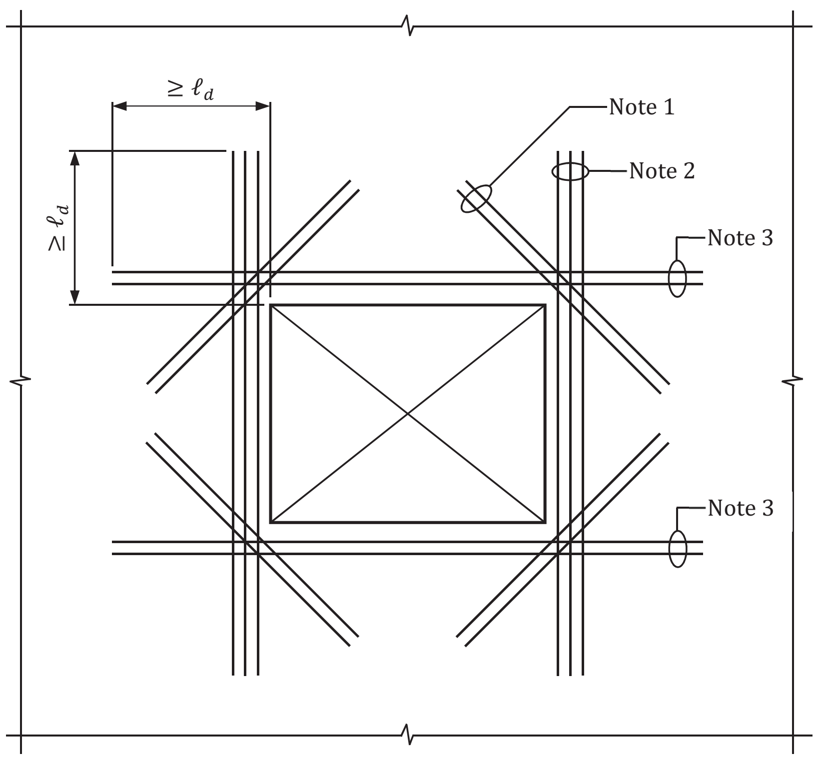
Figure 8.16 Recommended reinforcement details around a door opening in a reinforced concrete wall with two layers of reinforcement.

8.8.6 Reinforcement Around Openings

For walls with window, door, and similarly sized openings, the following additional reinforcement must be provided around the perimeter of the openings (ACI 11.7.5):

- A minimum of 1-#5 bar in walls with one layer of reinforcement
- A minimum of 2-#5 bars in walls with two layers of reinforcement

The additional reinforcement must be anchored to develop f_y in tension at the corners of the openings. These bars, which are commonly referred to as trim bars, essentially replace the reinforcement interrupted by the opening. More than the minimum amounts of trim bars may need to be provided based on an analysis of the wall with the opening(s). A recommended detail for the reinforcement around a typical door opening is given in Figure 8.16 for a wall with 2 layers of reinforcement. A similar detail for window and other wall openings is given in Figure 8.17.



Other wall reinforcement not shown for clarity

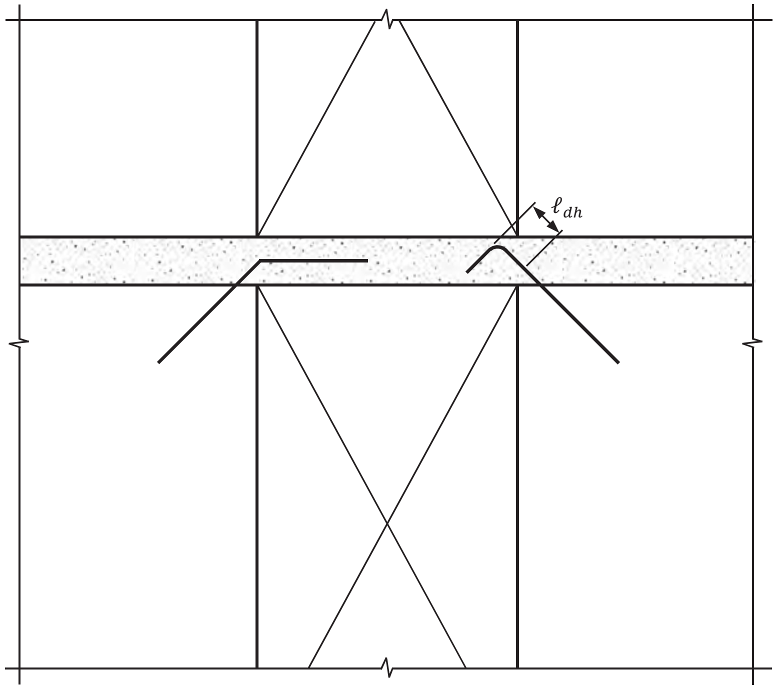
Notes

1. 2-#5×4′-0″ diagonal bars at each corner.
2. Provide at least one-half of the total area of interrupted longitudinal wall reinforcement or a minimum of 2-#5 bars at each side of the opening.
3. Provide at least one-half of the total area of interrupted transverse wall reinforcement or a minimum of 2-#5 bars at the top and bottom of the opening.

Figure 8.17 Recommended reinforcement details around an opening in a reinforced concrete wall with two layers of reinforcement.

The preferred reference point for measuring the tension development length, ℓ_d, of the longitudinal and transverse trim bars is the corner of the opening because it is a fixed point (see Figure 8.16 or Figure 8.17). Measuring ℓ_d from a longitudinal or transverse trim bar can be done, but the development length may end up being too short if the perpendicular trim bars adjacent to the opening are not at their intended locations.

For openings occurring close to the top of a wall or where openings are stacked on top of each other, the typical straight diagonal bar detail may not be possible. In order to properly develop the bars, either a standard hook needs to be provided at one end of the bar or the bar can be bent, as shown in Figure 8.18 for the case of stacked openings.



Other reinforcement not shown for clarity

Figure 8.18 Diagonal bar development at stacked wall openings.

8.9 Connections to Foundations

8.9.1 Overview

Vertical and horizontal forces must be transferred at the interface between a wall and a foundation in accordance with ACI 16.3 (ACI 11.2.2.2). Vertical compression forces are transferred by bearing on the concrete or by a combination of bearing and interface reinforcement. Tension forces must be transferred entirely by reinforcement, which may consist of extended longitudinal bars, dowels, anchor bolts, or mechanical connectors. Horizontal forces are transferred using the shear-friction provisions of ACI 22.9 or other appropriate methods.

8.9.2 Vertical Transfer

Compression

Where vertical compression forces are transferred to a foundation, bearing strength requirements of ACI 22.8 must be satisfied for both the wall and the foundation (ACI 16.3.3.4). For bearing on a wall, the factored bearing force, B_u, must be less than or equal to the design bearing strength, ϕB_n, where the nominal bearing strength, B_n, is given in ACI Table 22.8.3.2:

$$B_u \leq \phi B_n = \phi 0.85 f_c' A_1 \tag{8.40}$$

In this equation, ϕ is equal to 0.65 for bearing (ACI Table 21.2.1) and A_1 is the gross area of the wall.

For bearing on a foundation wider on all sides than the loaded area, the following equation must be satisfied:

$$B_u \leq \phi B_n = (\phi 0.85 f_c' A_1)\sqrt{A_2 / A_1} \leq 2(\phi 0.85 f_c' A_1) \tag{8.41}$$

The term A_2 is the area of the lower base of the largest frustrum of a pyramid, cone, or tapered wedge contained wholly within the foundation and having for its upper base the loaded area A_1 and having side slopes of 1 vertical to 2 horizontal. Areas A_1 and A_2 are depicted in Figure 8.19 for the case of a wall supported by a reinforced concrete foundation.

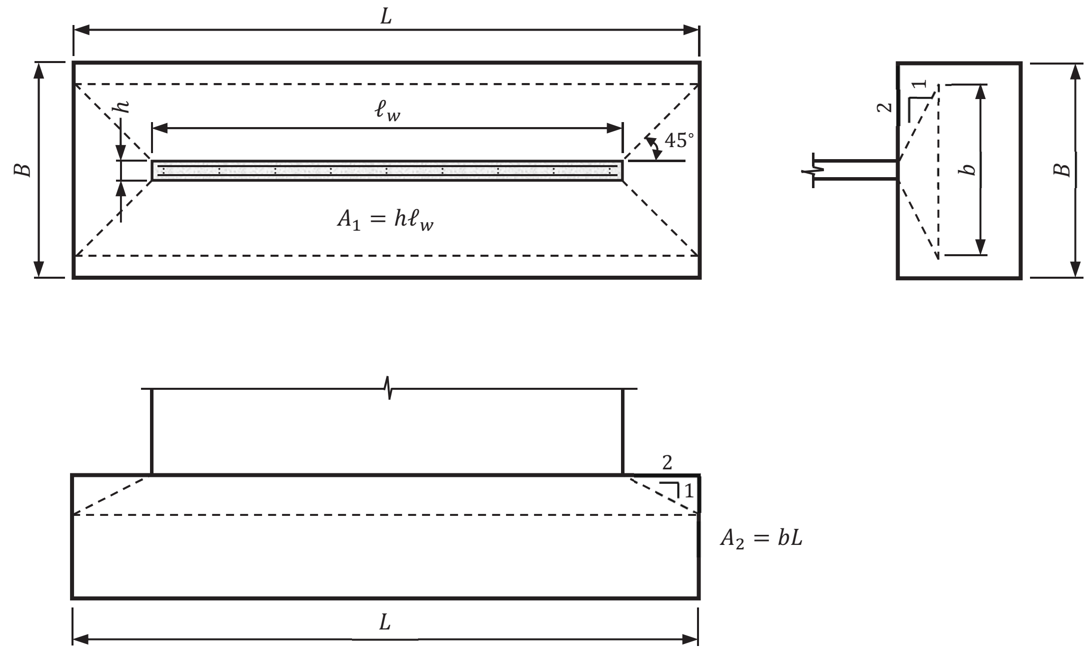


Figure 8.19 Determination of areas for bearing strength.

Where $B_u > \phi B_n$, the excess compression stress from the wall must be transferred by reinforcement to the foundation. The required area of interface reinforcement, A_s, is determined by the following equation:

$$A_s = \frac{B_u - \phi B_n}{\phi f_y} \geq A_{s,min} = \rho_{\ell,min} h \ell_w \quad (8.42)$$

In this equation, $\rho_{\ell,min}$ is the minimum longitudinal reinforcement ratio satisfying ACI 11.6.1 (ACI 16.3.4.2; see Table 8.10). The minimum amount of interface reinforcement, $A_{s,min}$, must be provided even where $B_u \leq \phi B_n$.

Dowel bars emanating from the foundation are the type of interface reinforcement commonly used due to ease of construction. The dowel bars, which are typically of the same size and spacing of the longitudinal reinforcement in the wall, are set in the foundation prior to placing the foundation concrete and are subsequently spliced to the longitudinal bars.

Illustrated in Figure 8.20 are dowel bars across the interface between a reinforced concrete wall and foundation where all the longitudinal bars in the wall are in compression for all factored load combinations. The dowel bars must extend into the foundation at least a compression development length, ℓ_{dc}, determined in accordance with ACI 25.4.9.2 (see Section 8.8.2 of this publication).

Standard hooks are typically provided at the ends of the dowel bars, which are tied to the flexural reinforcing bars in the foundation; this detail is far more economical than terminating the dowel bars above the flexural reinforcing bars. The hooked portion of the dowels cannot be considered effective for developing the dowels bars in compression (ACI 25.4.1.2). The following equation must be satisfied to ensure that the dowel bars are adequately developed into the foundation:

$$H \geq \ell_{dc} + r + (d_b)_d + 2(d_b)_f + \text{cover} \quad (8.43)$$

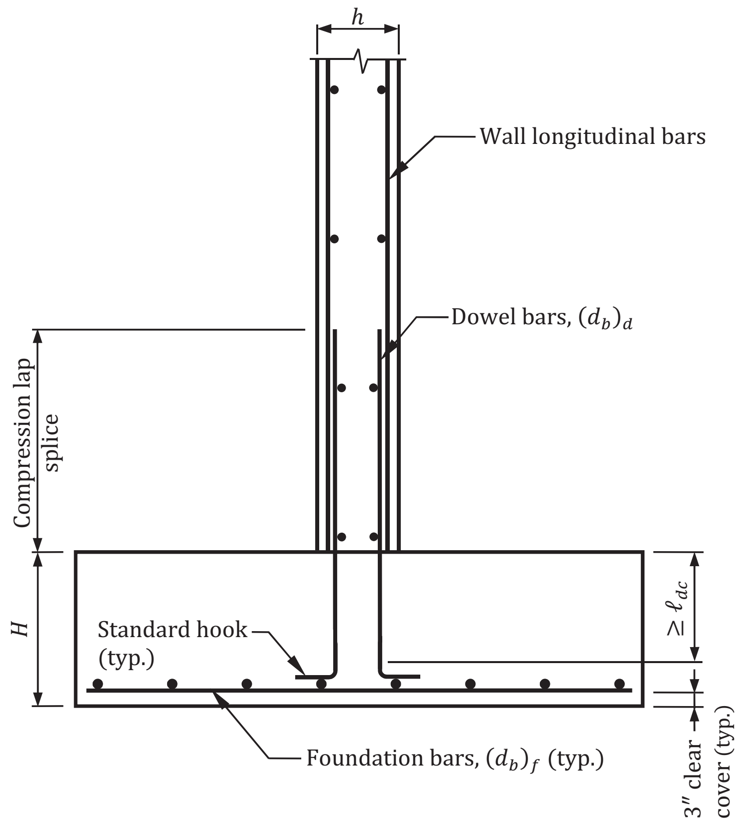


Figure 8.20 Dowel bars at the interface of a reinforced concrete wall and foundation where all the longitudinal bars in the wall are in compression.

In this equation, r is the radius of the dowel bar bend (see ACI Table 25.3.1, which contains minimum inside bend diameters for bars with standard 90-degree hooks). Where Equation (8.43) is not satisfied, one typical way to ensure adequate development is to increase the thickness of the foundation, H.

The dowel bars must also be fully developed in the wall; this is typically achieved by lap splicing the dowel bars to the longitudinal bars in the wall (see Figure 8.20). Where the dowel bars are the same size as the wall longitudinal bars, the minimum compression lap splice length is used (see Table 8.15). Where the dowel bars are larger in diameter than the wall longitudinal bars, the compression lap splice length must satisfy the requirements of ACI 25.5.5.4. Lap splices of #14 and #18 longitudinal bars in compression for all factored load combinations with #11 and smaller dowel bars are permitted provided the requirements of ACI 25.5.5.3 are satisfied (ACI 16.3.5.4).

Tension

Tension forces transferred from a wall to a foundation must be resisted entirely by reinforcement across the interface [ACI 16.3.1.2(b) and 16.3.5.2].

Tensile anchorage of the dowel bars into a foundation is typically accomplished by providing 90-degree standard hooks at the ends of the dowel bars with the development length of the hooked bar, ℓ_{dh}, determined in accordance with ACI 25.4.3 (see Figure 8.21):

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{8.44}$$

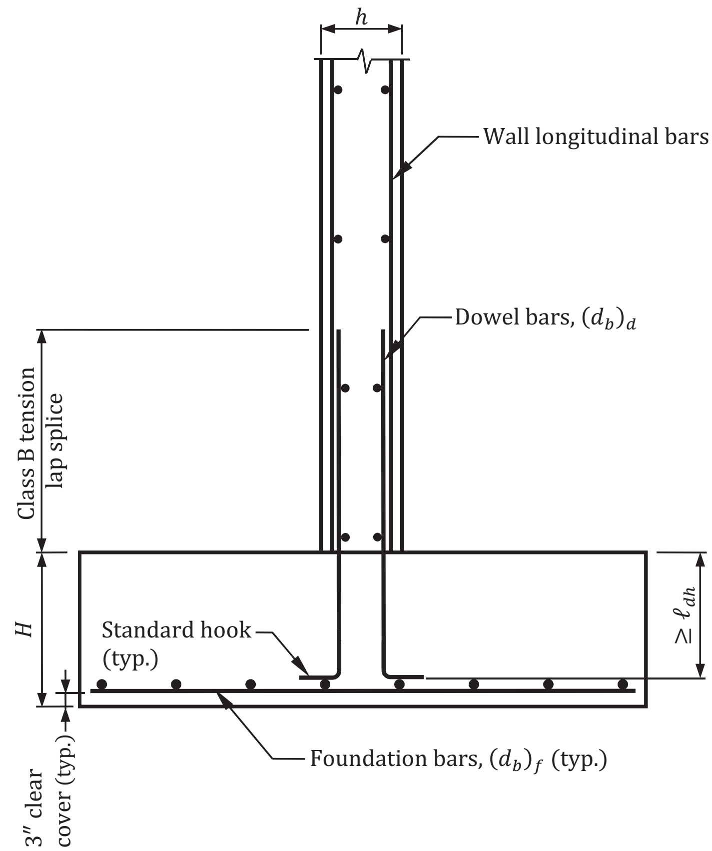


Figure 8.21 Dowel bars at the interface of a reinforced concrete wall and foundation where the longitudinal bars in the wall are in compression and tension.

This development length is measured from the critical section to the outside face of the hook (see Figure 4.10 of this publication). The modification factors in Equation (8.44) are given in ACI Table 25.4.3.2 (see Table 8.20).

Table 8.20 Modification Factors for Development of Hooked Bars in Tension

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Confining reinforcement, ψ_r	For #11 and smaller bars with $A_{th} \geq 0.4A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6

(table continued on next page)

Table 8.20 Modification Factors for Development of Hooked Bars in Tension (cont.)

Modification Factor	Condition	Value of Factor
Location, ψ_o	For #11 and smaller hooked bars 1. terminating inside a column core with side cover normal to the plane of the hook ≥ 2.5 in. or 2. with side cover normal to the plane of the hook $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c\,/\,15{,}000) + 0.6$
	$f'_c \geq 6{,}000$ psi	1.0

The confining reinforcement factor, ψ_r, is typically equal to 1.6 for hooked dowel bars in foundations because confining reinforcement is usually not provided in such cases.

The following equation must be satisfied to ensure that the dowel bars are adequately developed into the foundation (considering that the hooked portion of the dowel bars can be developed in tension):

$$H \geq \ell_{dh} + 2(d_b)_f + \text{cover} \tag{8.45}$$

Where Equation (8.45) is not satisfied, the depth of the foundation usually must be increased.

For development of the dowel bars into the wall, a tension lap splice or a mechanical connection in accordance with ACI 25.5.2 or 25.5.7, respectively, must be provided between the dowel bars and the longitudinal bars. In the case of lap splices, a Class B tension lap splice is required (see Table 8.15 and Figure 8.21).

8.9.3 Horizontal Transfer

The shear-friction method of ACI 22.9 is permitted to be used to determine the nominal shear strength, V_n, at the contact surface between the wall and the foundation (ACI 16.3.3.5). The required area of reinforcement A_{vf} across the interface is determined by the following equation, which is applicable to shear-friction reinforcement perpendicular to the interface (ACI 22.9.4.2):

$$A_{vf} \geq \frac{V_u}{\phi f_y \mu} \tag{8.46}$$

In this equation, V_u is the factored shear force due to the lateral force effects at the interface (in-plane or out-of-plane), the strength reduction factor ϕ is equal to 0.75, and μ is the coefficient of friction, which is obtained from ACI Table 22.9.4.2 (see Table 8.21).

Table 8.21 Coefficient of Friction, μ

Contact Surface Condition	μ
Concrete placed monolithically	1.4λ
Concrete placed against hardened concrete that is clean, free of laitance, and intentionally roughened to a full amplitude of approximately ¼ in.	1.0λ
Concrete placed against hardened concrete that is clean, free of laitance, and not intentionally roughened	0.6λ

Upper limits on shear-friction strength are given in ACI Table 22.9.4.4 (see Table 8.22). The term A_c is the area of concrete resisting V_u, which is equal to the gross cross-sectional area of the wall. Where the concrete strengths of the wall and the foundation are different, the smaller of the two must be used in these equations (ACI 22.9.4.4).

Table 8.22 Maximum V_n Across the Assumed Shear Plane

Contact Surface Condition	Maximum $V_n = V_u / \phi$*
Normalweight concrete placed monolithically or placed against hardened concrete intentionally roughened to a full amplitude of approximately ¼ in.	Least of $\begin{cases} 0.2f'_cA_c \\ (480+0.08f'_c)A_c \\ 1{,}600A_c \end{cases}$
Other cases	Least of $\begin{cases} 0.2f'_cA_c \\ 800A_c \end{cases}$

*A_c = *area of concrete resisting* V_u

Full tension anchorage of the shear-friction reinforcement must be provided into the foundation and into the wall. The lengths of these bars are determined in the same way as those for vertical transfer where tension forces are present.

The area of the dowel bars is usually determined initially based on the requirements for vertical transfer. That area is compared with the area required for horizontal transfer, and the larger of the two is provided at the interface.

8.10 Design Procedure

The design procedure in Figure 8.22 can be used in the design and detailing of reinforced concrete walls subjected to axial compression forces and uniaxial bending (in-plane or out-of-plane). Included in the figure are the section numbers, table numbers, and equation numbers where specific information in this chapter can be found.

8.11 Examples

8.11.1 Example 8.1 – Design of Reinforced Concrete Wall: Building #2, Interior Wall is Not Part of the SFRS, Simplified Design Method

Design a solid, interior reinforced concrete wall that is not part of the seismic force-resisting system (SFRS) in the first story of Building #2 subjected to the following axial forces, which are assumed to act at a 1.0-in. eccentricity with respect to the centroid of the wall (see Figure 8.23): $P_D = 2.0$ kips/ft and $P_L = 1.0$ kips/ft. Assume normal-weight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Step 1 – Determine the factored axial force Table 3.3

The following load combination governs:

$$P_u = 1.2P_D + 1.6P_L = (1.2 \times 2.0) + (1.6 \times 1.0) = 4.0 \text{ kips/ft}$$

Step 2 – Determine a trial wall thickness

Assume the Simplified Design Method can be used. This assumption is verified in Step 3 below.

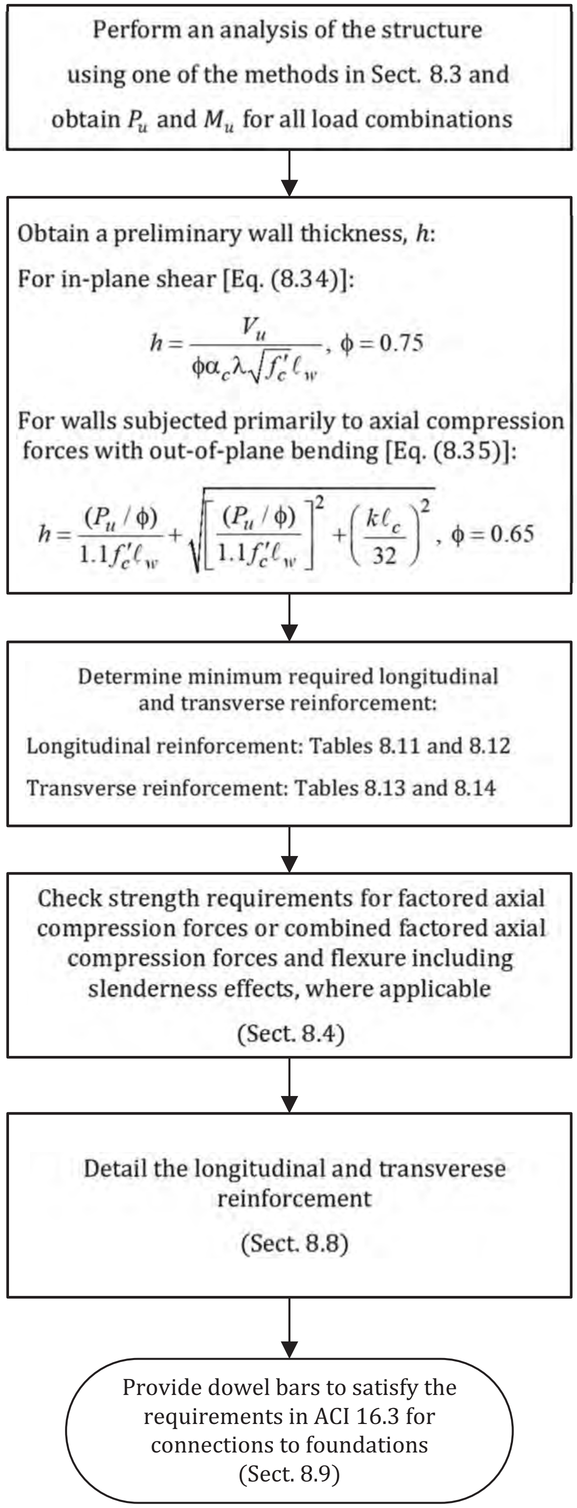


Figure 8.22 Design procedure for walls.

A minimum wall thickness can be determined by the following equation:

$$h = \frac{(P_u / \phi)}{1.1 f'_c \ell_w} + \sqrt{\left[\frac{(P_u / \phi)}{1.1 f'_c \ell_w}\right]^2 + \left(\frac{k \ell_c}{32}\right)^2} \qquad \text{Eq. (8.35)}$$

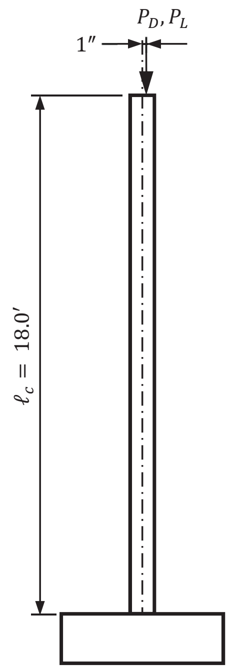


Figure 8.23 Interior bearing wall of Example 8.1.

Assuming the wall is braced at the top and bottom against lateral translation and unrestrained against rotation at both ends:

$k = 1.0$ Table 8.7

Therefore,

$$h = \frac{(4.0 / 0.65)}{1.1 \times 4 \times 12.0} + \sqrt{\left[\frac{(4.0 / 0.65)}{1.1 \times 4 \times 12.0}\right]^2 + \left(\frac{1.0 \times 18.0 \times 12}{32}\right)^2} = 6.9 \text{ in.}$$

For a bearing wall, the minimum thickness is equal to the following:

$$h = \frac{\ell_u}{25} = \frac{18.0 \times 12}{25} = 8.6 \text{ in.} > 4.0 \text{ in.}$$ Figure 8.1

Try $h = 9.0$ in.

Step 3 – Determine if the Simplified Design Method can be used ACI 11.5.3

Check the limitations of the Simplified Design Method:

1. The wall is solid and rectangular.
2. Eccentricity $e = 1.0$ in. $< h / 6 = 9.0 / 6 = 1.5$ in. Figure 8.8

Therefore, the Simplified Design Method can be used.

Step 4 – Determine the design strength of the wall

$$\phi P_n = \phi 0.55 f'_c A_g \left[1 - \left(\frac{k\ell_c}{32h}\right)^2\right]$$

Eq. (8.27)

$$= 0.65 \times 0.55 \times 4 \times (9.0 \times 12.0) \times \left[1 - \left(\frac{1.0 \times 18.0 \times 12}{32 \times 9.0}\right)^2\right] = 67.6 \text{ kips/ft} > P_u = 4.0 \text{ kips/ft}$$

Step 5 – Determine the minimum reinforcement in the wall

Assume #4 bars are used for the longitudinal and transverse reinforcement.

Minimum longitudinal reinforcement $= 0.0012 \times 9.0 \times 12.0 = 0.13 \text{ in.}^2$ Table 8.10

Provide a single layer of #4 bars spaced at 18 in. on center $(A_s = 0.13 \text{ in.}^2)$.

Minimum transverse reinforcement $= 0.0020 \times 9.0 \times 12.0 = 0.22 \text{ in.}^2$

Provide a single layer of #4 bars spaced at 10 in. on center $(A_s = 0.24 \text{ in.}^2)$.

Maximum spacing $= \text{lesser of} \begin{cases} 3h = 27 \text{ in.} \\ 18 \text{ in.} \end{cases} \geq \text{provided spacings}$ Figure 8.11

8.11.2 Example 8.2 – Design of Reinforced Concrete Wall: Building #2, Exterior Wall is Not Part of the SFRS, Moment Magnification Method, Out-of-Plane Forces

Design a solid, exterior reinforced concrete wall that is not part of the SFRS in the first story of Building #2 subjected to the following forces (see Figure 8.24):

Dead, $P_D = 1.7$ kips/ft

Live, $P_L = 0.8$ kips/ft

Wind, $w_W = 20 \text{ lb/ft}^2$ (it can be determined that the effects due to out-of-plane seismic effects in accordance with ASCE/SEI 13.3 are less than that due to wind load effects)

Assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement and the frame in the first story is nonsway. Also, the gravity loads are assumed to act concentrically on the wall.

Step 1 – Determine a trial wall thickness

There is no closed-form solution to determine h for a wall subjected to these types of loads.

A trial wall thickness of 8 in. is selected. The adequacy of this wall thickness will be checked in the following steps.

Step 2 – Determine the factored load combinations Table 3.3

Weight of the wall at mid-height $= (8.0 / 12) \times 0.15 \times (16.8 / 2) = 0.8$ kips/ft

Total dead load $= 1.7 + 0.8 = 2.5$ kips/ft

The gravity loads do not cause any bending moments in the wall because they act through the centroid of the wall.

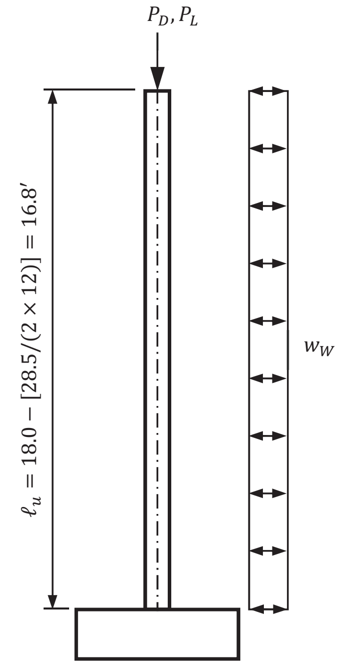


Figure 8.24 Exterior bearing wall of Example 8.2.

Assuming the wall is pinned at the top and bottom, the bending moment at the mid-height of the wall due to the uniformly distributed wind load is equal to the following:

$$M_w = 0.020 \times 16.8^2 \; / \; 8 = 0.7 \text{ ft-kips/ft} = 8.4 \text{ in.-kips/ft}$$

A summary of the factored load combinations is given in Table 8.23.

Table 8.23 Summary of Axial Forces and Bending Moments for the Wall in Example 8.2

Load Case		Axial Force (kips/ft)	Bending Moment (in.-kips/ft)
Dead (D)		2.5	0
Live (L)		0.8	0
Wind (W)		0	± 8.4
Load Combination			
ACI Eq. (5.3.1a)	$1.4D$	3.5	0
ACI Eq. (5.3.1b)	$1.2D + 1.6L$	4.3	0
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L$	3.4	8.4
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	2.3	8.4

Step 3 – Determine if slenderness effects need to be considered

Because the frame in the first story is a nonsway frame, slenderness effects need not be considered where ACI Eqs. (6.2.5.1b) and (6.2.5.1c) are satisfied (see Table 7.6 of this publication):

$$\frac{k\ell_u}{r} \leq \text{ lesser of } \begin{cases} 34 + 12(M_1 / M_2) \\ \\ 40 \end{cases}$$

$$\frac{k\ell_u}{r} = \frac{1.0 \times 16.8 \times 12}{0.3 \times 8.0} = 84 > 40$$

Therefore, slenderness effects must be considered in the design of this wall.

Step 4 – Determine the magnified moments in the wall using the moment magnification method ACI 6.6.4

The moment magnification method is covered in detail in Sect. 7.3.3 of this publication.

For nonsway frames, factored magnified moments, M_c, are determined by Eq. (7.2):

$$M_c = \delta M_2$$

where M_2 is the factored mid-height bending moment in the wall and the magnification factor, $\delta,$ is determined by Eq. (7.3):

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0$$

Because there are transverse loads between the supports, $C_m = 1.0.$ ACI 6.6.4.5.3(b)

The critical buckling load, P_c, is determined by Eq. (7.4):

$$P_c = \frac{\pi^2 (EI)_{eff}}{(k\ell_u)^2}$$

The effective stiffness, $(EI)_{eff}$, is determined by the following equation:

$$(EI)_{eff} = \frac{E_c I}{1 + \beta_{dns}}$$ Eq. (8.2)

$$I = \left(0.80 + \frac{25A_{st}}{A_g}\right)\left(1 - \frac{M_u}{P_u h} - \frac{0.5P_u}{P_o}\right)I_g$$

$$\geq 0.35I_g$$ Table 8.2

$$\leq 0.875I_g$$

It is evident that the area of longitudinal reinforcement in the wall, A_{st}, is needed to calculate I. For calculation purposes, the minimum amount of longitudinal reinforcement is assumed:

$$A_{st} = 0.0012 \times 8.0 \times 12.0 = 0.12 \text{ in.}^2/\text{ft}$$

Try a single layer of #4 bars spaced at 18 in. on center ($A_{st} = 0.13$ in.^{2}). The adequacy of this reinforcement will be checked in the following steps.

Calculations are provided for the load combination corresponding to ACI Eq. (5.3.1d) in Table 8.23.

For a 1-foot design strip:

$$P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st} = \{0.85 \times 4 \times [(8.0 \times 12.0) - 0.13]\} + (60 \times 0.13) = 333.8 \text{ kips}$$ ACI Eq. (22.4.2.2)

$$I_g = \frac{12.0 \times 8.0^3}{12} = 512.0 \text{ in.}^4$$

$$I = \left(0.80 + \frac{25 A_{st}}{A_g}\right)\left(1 - \frac{M_u}{P_u h} - \frac{0.5 P_u}{P_o}\right) I_g$$

$$= \left(0.80 + \frac{25 \times 0.13}{8.0 \times 12.0}\right) \times \left(1 - \frac{8.4}{3.4 \times 8.0} - \frac{0.5 \times 3.4}{333.8}\right) \times 512.0 = 292.9 \text{ in.}^4$$

$$> 0.35 \times 512.0 = 179.2 \text{ in.}^4$$

$$< 0.875 \times 512.0 = 448.0 \text{ in.}^4$$

$$E_c = w_c^{1.5} 33 \sqrt{f_c'} = 150.0^{1.5} \times 33 \times \sqrt{4{,}000} \,/\, 1{,}000 = 3{,}834 \text{ ksi}$$ ACI Eq. (19.2.2.1a)

$$\beta_{dns} = \frac{1.2 P_D}{1.2 P_D + 1.0 P_w + 0.5 P_L} = \frac{1.2 \times 2.5}{3.4} = 0.88$$

$$(EI)_{eff} = \frac{E_c I}{1 + \beta_{dns}} = \frac{3{,}834 \times 292.9}{1 + 0.88} = 597 \times 10^3 \text{ kip-in.}^2$$ Eq. (8.2)

$$P_c = \frac{\pi^2 (EI)_{eff}}{(k \ell_u)^2} = \frac{\pi^2 \times 597 \times 10^3}{(1.0 \times 16.8 \times 12)^2} = 145.0 \text{ kips}$$

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{0.75 P_c}} = \frac{1.0}{1 - \dfrac{3.4}{0.75 \times 145.0}} = 1.03 > 1.0$$

Therefore,

$$M_c = \delta M_2 = 1.03 \times 8.4 = 8.7 \text{ in.-kips}$$

Determine the minimum factored bending moment, $M_{2,min}$:

$$M_{2,min} = P_u(0.6 + 0.03h) = 3.4 \times [0.6 + (0.03 \times 8.0)] = 2.9 \text{ in.-kips} < M_c = 8.7 \text{ in.-kips}$$

Similar calculations can be performed for the load combination corresponding to ACI Eq. (5.3.1f).

A summary of the results from the moment magnification method for the load combinations that include wind load effects are given in Table 8.24.

Table 8.24 Summary of Results from the Moment Magnification Method for the Wall in Example 8.2

Load Combination	P_u (kips)	M_u (in.-kips)	β_{dns}	I (in.^{4})	$(EI)_{eff}$ x 10^3 (kip-in.^{2})	P_c (kips)	δ	M_c (in.-kips)
$1.2D + 1.0W + 0.5L$	3.4	8.4	0.88	292.9	597	145.0	1.03	8.7
$0.9D + 1.0W$	2.3	8.4	1.00	230.6	442	107.3	1.03	8.7

Step 5 – Check the adequacy of the section for combined flexure and axial forces ACI 22.4

The lower portion of the design strength interaction diagram for a 1-foot-wide section of the wall reinforced with #4 bars spaced at 18 in. on center is shown in Figure 8.25. Because all 4 load combinations fall within the design strength interaction diagram, the wall is adequate.

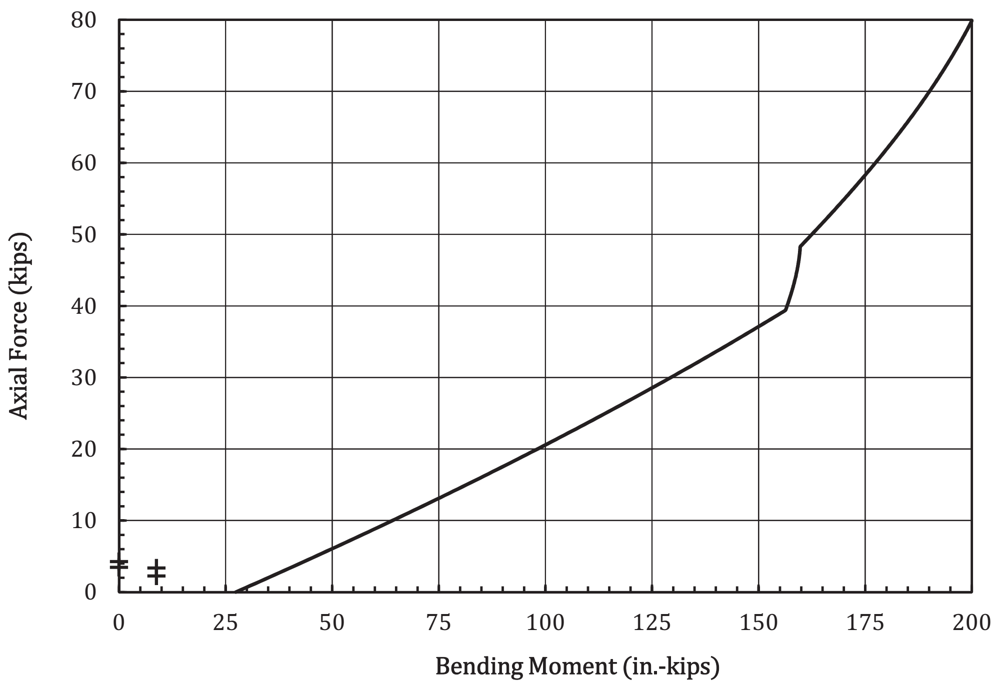


Figure 8.25 Design strength interaction diagram for the wall in Example 8.2.

Step 6 – Check the out-of-plane shear strength requirements ACI 22.5

The required shear strength, V_u, is determined based on the design wind force:

$V_u = 0.020 \times 16.8 / 2 = 0.2$ kips

The design shear strength, ϕV_n, is equal to the design shear strength of the concrete, ϕV_c :

$$\phi V_c = \phi\left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d \le \phi 5\lambda\sqrt{f_c'}b_w d \quad \text{Eq. (8.31)}$$

$$\lambda_s = \sqrt{\frac{2}{1+(d/10)}} = \sqrt{\frac{2}{1+(4.0/10)}} = 1.2 > 1.0\text{, use } 1.0 \quad \text{Eq. (8.32)}$$

$\lambda = 1.0$ for normalweight concrete Table 8.8

$$\rho_w = A_s / b_w d = 0.13 / (12.0 \times 4.0) = 0.0027$$

Therefore,

$$\phi V_c = 0.75 \times \left\{ \left[8 \times 1.0 \times 1.0 \times (0.0027)^{1/3} \times \sqrt{4{,}000} \right] + \frac{2{,}300}{6 \times 8.0 \times 12.0} \right\} \times 12.0 \times 4.0 / 1{,}000$$

$$= 2.7 \text{ kips} < \phi 5 \lambda \sqrt{f'_c} b_w d = 11.4 \text{ kips}$$

$$> V_u = 0.2 \text{ kips}$$

The axial force $N_u = 2{,}300$ lbs corresponding to ACI Eq. (5.3.1f) is used in determining ϕV_c because it results in a smaller value of ϕV_c than that determined using $N_u = 3{,}400$ lbs corresponding to ACI Eq. (5.3.1d).

The wall is adequate for out-of-plane shear.

8.11.3 Example 8.3 – Design of Reinforced Concrete Wall: Building #2, Exterior Wall is Not Part of the SFRS, Alternative Method for Out-of-Plane Forces

Design a solid, exterior reinforced concrete wall that is not part of the SFRS in the first story of Building #2 subjected to the following forces (see Figure 8.26):

Dead, $P_D = 2.0$ kips/ft

Live, $P_L = 0.8$ kips/ft

Wind, $w_W = 40$ lb/ft^2 (it can be determined that the effects due to out-of-plane seismic effects in accordance with ASCE/SEI 13.3 are less than that due to wind load effects)

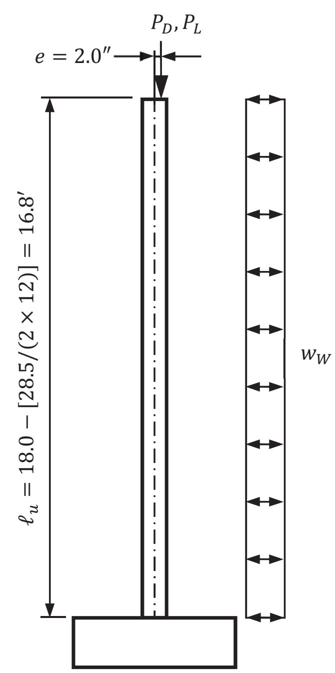


Figure 8.26 Exterior bearing wall in Example 8.3.

Assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement. Also assume an eccentricity of 2.0 in. between the dead and live load axial forces and the centroid of the wall.

Step 1 – Determine a trial wall thickness

There is no closed-form solution to determine h for a wall subjected to these types of loads.

A trial wall thickness of 8 in. is selected. The adequacy of this wall thickness will be checked in the following steps.

Step 2 – Determine the factored load combinations at the mid-height of the wall

Weight of the wall from top to mid-height $= \dfrac{8}{12} \times 0.150 \times \dfrac{16.8}{2} = 0.8 \text{ kips/ft}$

$$\text{Total dead load} = 0.8 + 2.0 = 2.8 \text{ kips/ft}$$

$$\text{Total live load} = 0.8 \text{ kips/ft}$$

Service dead load moment $= \dfrac{2.0 \times 2.0}{2} = 2.0 \text{ in.-kips/ft}$ (only the axial dead load causes a bending moment at mid-height)

Service live load moment $= \dfrac{0.8 \times 2.0}{2} = 0.8 \text{ in.-kips/ft}$

Wind load moment $= \dfrac{0.040 \times 16.8^2}{8} = 1.4 \text{ ft-kips/ft} = 16.9 \text{ in.-kips/ft}$

A summary of the factored load combinations is given in Table 8.25.

Table 8.25 Summary of Axial Forces and Bending Moments for the Wall in Example 8.3

Load Case		Axial Force (kips/ft)	Bending Moment (in.-kips/ft)
Dead (D)		2.8	2.0
Live (L)		0.8	0.8
Wind (W)		0	± 16.9
Load Combination			
ACI Eq. (5.3.1a)	$1.4D$	3.9	2.8
ACI Eq. (5.3.1b)	$1.2D + 1.6L$	4.6	3.7
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L$	3.8	19.7
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	2.5	18.7

Step 3 – Determine the factored load combinations at the mid-height of the wall, including slenderness effects

Using the direct calculation method of the alternative method [ACI 11.8.3.1(b)], the total factored moment, M_u, which includes slenderness effects, is determined by the following equation:

$$M_u = \frac{M_{ua}}{1 - \dfrac{5P_u\ell_c^2}{(0.75)48E_cI_{cr}}} \qquad \text{Eq. (8.10)}$$

where M_{ua} are the factored moments in the wall at mid-height (see Table 8.25).

A summary of the factored axial forces and bending moments, which include slenderness effects, is given in Table 8.26.

Table 8.26 Summary of Factored Axial Forces and Bending Moments, Including Slenderness Effects, for the Wall in Example 8.3

Load Combination	P_u (kips)	M_{ua} (in.-kips)	$A_{se,w}$ (in.^{2})	c (in.)	I_{cr} (in.^{4})	M_u (in.-kips)
$1.4D$	3.9	2.8	0.34	0.58	31.0	3.4
$1.2D + 1.6L$	4.6	3.7	0.35	0.60	31.6	4.7
$1.2D + 1.0W + 0.5L$	3.8	19.7	0.33	0.58	30.1	24.2
$0.9D + 1.0W$	2.5	18.7	0.31	0.54	28.8	21.4

Calculations are provided for the load combination corresponding to ACI Eq. (5.3.1d).

Maximum longitudinal bar spacing $= \text{lesser of } \begin{cases} 3h = 24 \text{ in.} \\ 18 \text{ in.} \end{cases}$ Figure 8.11

Assume the wall is reinforced with one layer of #5 bars spaced at 14 in. on center $(A_s = 0.27 \text{ in.}^2/\text{ft})$.

$$A_{se,w} = A_s + \frac{P_u h}{2f_y d} = 0.27 + \frac{3.8 \times 8.0}{2 \times 60 \times 4.0} = 0.33 \text{ in.}^2 \qquad \text{Eq. (8.7)}$$

$$c = \frac{A_s f_y + (P_u h / 2d)}{0.85 f_c' \ell_w \beta_1} = \frac{(0.27 \times 60) + [(3.8 \times 8.0) / (2 \times 4.0)]}{0.85 \times 4 \times 12.0 \times 0.85} = 0.58 \text{ in.} \qquad \text{Eq. (8.9)}$$

$E_s = 29{,}000$ ksi ACI 20.2.2.2

Check if the section is tension-controlled:

$$\varepsilon_t = 0.003\left(\frac{d_t}{c} - 1\right) = 0.003 \times \left(\frac{4.0}{0.58} - 1\right) = 0.0177$$

ACI Table 21.2.2

$$> \varepsilon_{ty} + 0.003 = (f_y / E_s) + 0.003 = (60 / 29{,}000) + 0.003 = 0.0051$$

Therefore, the section is tension-controlled. It can be determined that this wall section is tension-controlled for all load combinations. This satisfies the second limitation in ACI 11.8.1.1.

$$E_c = w_c^{1.5} 33\sqrt{f_c'} = 150.0^{1.5} \times 33 \times \sqrt{4{,}000} \,/\, 1{,}000 = 3{,}834 \text{ ksi}$$ ACI Eq. (19.2.2.1a)

$$E_s / E_c = 7.6 > 6$$ ACI 11.8.3.1

$$I_{cr} = \frac{E_s}{E_c}\left(A_s + \frac{P_u h}{2f_y d}\right)(d-c)^2 + \frac{\ell_w c^3}{3} = [7.6 \times 0.33 \times (4.0 - 0.58)^2] + \frac{12.0 \times 0.58^3}{3} = 30.1 \text{ in.}^4$$ Eq. (8.6)

Therefore,

$$M_u = \frac{M_{ua}}{1 - \dfrac{5P_u \ell_c^2}{(0.75)48E_c I_{cr}}} = \frac{19.7}{1 - \dfrac{5 \times 3.8 \times (16.8 \times 12)^2}{0.75 \times 48 \times 3{,}834 \times 30.1}} = 1.23 \times 19.7 = 24.2 \text{ in.-kips}$$

Similar calculations can be performed for the other load combinations.

Step 4 – Determine the cracking moment

$$f_r = 7.5\lambda\sqrt{f_c'} = 7.5 \times 1.0 \times \sqrt{4{,}000} = 474.3 \text{ psi}$$ ACI Eq. (19.2.3.1)

$$y_t = 8.0 / 2 = 4.0 \text{ in.}$$

$$I_g = \frac{12.0 \times 8.0^3}{12} = 512.0 \text{ in.}^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474.3 \times 512.0}{4.0 \times 1{,}000} = 60.7 \text{ in.-kips}$$ ACI Eq. (24.2.3.5)

Step 5 – Determine the design flexural strength of the wall and check the adequacy of the wall

The design flexural strength of the wall is determined by the following equation:

$$\phi M_n = \phi A_{se,w} f_y \left(d - \frac{a}{2}\right)$$ Eq. (8.11)

It is determined in Step 4 that the wall section is tension-controlled for all load combinations, so $\phi = 0.90$ (see Table 8.5).

The design flexural strengths are given in Table 8.27. For all load combinations, $\phi M_n > M_u$, so the wall is adequate for flexure. Also, $\phi M_n > M_{cr} = 60.7$ in.-kips for all load combinations, which satisfies the third limitation in ACI 11.8.1.1.

Table 8.27 Summary of Design Flexural Strengths for the Wall in Example 8.3

Load Combination	$A_{se,w}$ (in.^{2})	c (in.)	a (in.^{2})	ϕM_n (in.-kips)	M_u (in.-kips)
$1.4D$	0.34	0.58	0.49	68.9	3.4
$1.2D + 1.6L$	0.35	0.60	0.51	70.8	4.7
$1.2D + 1.0W + 0.5L$	0.33	0.58	0.49	66.9	24.0
$0.9D + 1.0W$	0.31	0.54	0.46	63.1	21.4

Step 6 – Check the maximum axial force at the mid-height of the wall section

According to the fourth limitation in ACI 11.8.1.1, P_u at the mid-height of the wall must be less than or equal to $0.06f'_c A_g = 0.06 \times 4 \times 8.0 \times 12.0 = 23.0$ kips.

It is evident that this limitation is satisfied for all load combinations (see Table 8.26).

Step 7 – Determine the maximum service-level at the mid-height of the wall section

Because there is no closed-form solution for Δ_s, assume $M_a < 2M_{cr} / 3$. Also assume the following value of Δ_s for the initial iteration:

$$\Delta_s = \left(\frac{M_{sa}}{M_{cr}}\right)\Delta_{cr}$$

The maximum service-level bending moment, M_{sa}, at the mid-height section of the wall is equal to the following:

$$M_{sa} = M_D + 0.5M_L + M_W = 2.0 + (0.5 \times 0.8) + (16.9 / 1.6) = 13.0 \text{ in.-kips} \qquad \text{Eq. (8.17)}$$

where the strength-level bending moment due to wind loads is divided by 1.6 to convert it to a service-level bending moment.

The deflection Δ_{cr} is determined by the following equation:

$$\Delta_{cr} = \frac{5M_{cr}\ell_c^2}{48E_cI_g} = \frac{5 \times 60.7 \times (16.8 \times 12)^2}{48 \times 3{,}834 \times 512.0} = 0.13 \text{ in.} \qquad \text{Eq. (8.15)}$$

Therefore,

$$\Delta_s = \left(\frac{M_{sa}}{M_{cr}}\right)\Delta_{cr} = \left(\frac{13.0}{60.7}\right) \times 0.13 = 0.03 \text{ in.}$$

Determine M_a from the following equation:

$$M_a = M_{sa} + P_s\Delta_s = 13.0 + \{[2.8 + (0.5 \times 0.8)] \times 0.03\} = 13.1 \text{ in.-kips} \qquad \text{Eq. (8.12)}$$

Because it has been assumed that $M_a < 2M_{cr} / 3$, determine Δ_s from the following equation:

$$\Delta_s = \left(\frac{M_a}{M_{cr}}\right)\Delta_{cr} = \left(\frac{13.1}{60.7}\right) \times 0.13 = 0.03 \text{ in.} \qquad \text{Eq. (8.13)}$$

This value of Δ_s is the same as the value initially assumed for Δ_s, so no additional iterations are required.

Check the initial assumption:

$M_a = 13.1$ in.-kips $< 2M_{cr} / 3 = 40.5$ in.-kips

Check that the fifth limitation of ACI 11.8.1.1 is satisfied:

$\Delta_s = 0.03$ in. $< \ell_c / 150 = (16.8 \times 12) / 150 = 1.3$ in.

Step 8 – Check the out-of-plane shear strength requirements ACI 22.5

The required shear strength, V_u, is determined based on the load combination corresponding to ACI Eq. (5.3.1d):

$V_u = [(1.2 \times 2.0) / 16.8] + [(0.5 \times 0.8) / 16.8] + [(0.040 \times 16.8) / 2] = 0.5$ kips

The design shear strength, ϕV_n, is equal to the design shear strength of the concrete, ϕV_c :

$$\phi V_c = \phi \left[8\lambda_s \lambda (\rho_w)^{1/3} \sqrt{f'_c} + \frac{N_u}{6A_g} \right] b_w d \le \phi 5 \lambda \sqrt{f'_c} b_w d \qquad \text{Eq. (8.31)}$$

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (4.0 / 10)}} = 1.2 > 1.0, \text{ use } 1.0 \qquad \text{Eq. (8.32)}$$

$\lambda = 1.0$ for normalweight concrete Table 8.8

$\rho_w = A_s / b_w d = 0.27 / (12.0 \times 4.0) = 0.0056$

Therefore,

$$\phi V_c = 0.75 \times \left\{ \left[8 \times 1.0 \times 1.0 \times (0.0056)^{1/3} \times \sqrt{4{,}000} \right] + \frac{3{,}800}{6 \times 8.0 \times 12.0} \right\} \times 12.0 \times 4.0 / 1{,}000$$

$$= 3.5 \text{ kips} < \phi 5 \lambda \sqrt{f'_c} b_w d = 11.4 \text{ kips}$$

$$> V_u = 0.5 \text{ kips}$$

The wall is adequate for out-of-plane shear.

8.11.4 Example 8.4 – Determination of Trial Wall Thickness of Reinforced Concrete Wall: Building #2, SDC C, Interior Wall is Part of the SFRS

Determine a trial wall thickness for the interior reinforced concrete wall along column line F that is part of the SFRS in the first story of Building #2 considering seismic forces in the east-west direction (see Figure 1.2). Assume normalweight concrete with $f'_c = 6{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2.

Solution

This interior wall is subjected to axial forces, bending moments, and shear forces. It is evident from Tables 3.15 and 3.27 of this publication that wind load effects are less than those from seismic effects.

There is no closed-form solution to determine *h* for a wall subjected to these combined load effects. However, a trial wall thickness can be obtained based on shear strength requirements:

$$h \geq \frac{V_u}{\phi(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_{yt})\ell_w} \qquad \text{Eq. (8.34)}$$

From Table 3.27, the base shear due to seismic effects is equal to 1,575.5 kips. The walls along column lines B and F each resist 50 percent of this base shear, so the factored shear force, V_u, in each wall in the first story is equal to 787.8 kips.

$$h_w / \ell_w = 139.0 / 36.5 = 3.8 > 2; \text{ therefore, } \alpha_c = 2 \qquad \text{Eq. (8.29)}$$

$\lambda = 1.0$ for normalweight concrete — Table 8.8

Assuming $\rho_t = 0.0025$ (see Figure 8.11):

$$h = \frac{V_u}{\phi(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_{yt})\ell_w} = \frac{787,800}{0.75 \times [(2 \times 1.0 \times \sqrt{6,000} + (0.0025 \times 60,000)] \times (36.5 \times 12)} = 7.9 \text{ in.}$$

A trial wall thickness of 12 in. is selected considering the axial forces and bending moments that this wall is subjected to (see Example 8.5).

8.11.5 Example 8.5 – Design of Reinforced Concrete Wall for Combined Flexure and Axal Forces: Building #2, SDC C, Interior Wall is Part of the SFRS

Design the interior reinforced concrete wall along column line F that is part of the SFRS in the first story of Building #2 for combined flexure and axial forces considering seismic forces in the east-west direction (see Figure 1.2). Assume a 12-in.-thick wall (see Example 8.4). Also assume normalweight concrete with $f'_c = 6,000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. $S_{DS} = 0.301$.

Step 1 – Determine the factored load combinations — Table 3.3

A three-dimensional, elastic analysis of the structure was performed using Ref. 14 with the walls subjected to the east-west seismic forces in Table 3.27 assuming the walls are fixed at the base (wind load effects are less than those from seismic effects and are not considered in this example; see Table 3.15). Rigid diaphragms were assigned at each level of the building and a moment of inertia equal to $0.35I_g$ was assigned to the walls in accordance with ACI Table 6.6.3.1.1(a) assuming the walls are cracked. The analysis showed that the lateral load effects are essentially resisted by the 36.5-ft wall webs in the direction of analysis, so no effective flange widths are considered in design; that is, all the lateral load effects are assigned to the webs.

A summary of the factored load combinations is given in Table 8.28 for the wall along column line F.

Table 8.28 Summary of Axial Forces and Bending Moments for the Wall in Example 8.4

Load Case	Axial Force (kips)	Bending Moment (ft-kips)
Dead (D)	6,306.6	0
Roof live (L_r)	59.3	—
Live (L)	1,304.6	0
Seismic (Q_E)	0	$\pm 77,586.8$

(table continued on next page)

Table 8.28 Summary of Axial Forces and Bending Moments for the Wall in Example 8.4 (cont.)

Load Case		Axial Force (kips)	Bending Moment (ft-kips)
Load Combination			
ACI Eq. (5.3.1a)	$1.4D$	8,829.2	0
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$	9,684.9	0
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$	8,315.1	0
ACI Eq. (5.3.1e)	$1.26D + Q_E + 0.5L$	8,598.6	$\pm 77{,}586.8$
ACI Eq. (5.3.1g)	$0.84D + Q_E$	5,297.5	$\pm 77{,}586.8$

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Eq. (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2 S_{DS} D$. According to ASCE/SEI 12.3.4.1, ρ can be taken as 1.0 for structures assigned to SDC C; therefore, $E = Q_E + (0.2 \times 0.301 \times D) = Q_E + 0.06D$. Therefore, the load combination becomes the following:

$$1.2D + 1.0E + 0.5L = 1.2D \pm Q_E + 0.06D + 0.5L = 1.26D + 0.5L \pm Q_E$$

In ACI Eq. (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E - 0.2 S_{DS} D = Q_E - 0.06D$, and the load combination becomes the following:

$$0.9D + 1.0E = 0.9D \pm Q_E - 0.06D = 0.84D \pm Q_E$$

Step 2 – Determine if the first story is nonsway or sway

The stability index, Q, is used to determine whether the first story is nonsway or sway:

$$Q = \frac{\Sigma P_u \Delta_o}{V_{us} \ell_c}$$

The following results are obtained from the analysis in the first story:

Total $P_D = 43{,}383$ kips

$P_{L_r} = 434$ kips

Total $P_L = 9{,}548$ kips

$P_{Q_E} = 0$

$\Delta_o = 0.03$ in.

$V_{us} = 1{,}575.5$ kips

The total factored load must correspond to the lateral loading case for which it is a maximum. In this example, ACI Eq. (5.3.1e) produces the largest load:

$$\Sigma P_u = 1.26 P_D + P_{Q_E} + 0.5 P_L = 59{,}437 \text{ kips}$$

Therefore,

$$Q = \frac{\Sigma P_u \Delta_o}{V_{us} \ell_c} = \frac{59,437 \times 0.03}{1,575.5 \times (18.0 \times 12)} = 0.01 < 0.05$$

Therefore, the frame in the first story is a nonsway frame.

Step 3 – Determine if slenderness effects must be considered

Because the first story frame is a nonsway frame, slenderness effects need not be considered where the following equation is satisfied:

$$\frac{k\ell_u}{r} \le \text{ lesser of } \begin{cases} 34 + 12(M_1 / M_2) \\ \\ 40 \end{cases}$$ Table 8.4

The moment at the top of the wall in the first story is equal to 60,814.6 ft-kips and the wall is bent in single curvature. Therefore, $M_1 / M_2 = -60,814.6 / 77,586.8 = -0.78$.

$r = 0.3 \times (36.5 \times 12) = 131.4$ in.

Therefore,

$$\frac{k\ell_u}{r} = \frac{1.0 \times 16.8 \times 12}{131.4} = 1.5 < 34 - (12 \times 0.78) = 24.6$$

Thus, slenderness effects need not be considered in the design of this wall in the direction of analysis.

Step 4 – Determine if the wall is adequate for combined flexure and axial forces

An estimate of the longitudinal reinforcement in the wall is needed in order to determine if the section is adequate or not for the factored load combinations in Table 8.28.

The minimum amount of longitudinal reinforcement depends on the magnitude of V_u with respect to the limit $0.5\phi\alpha_c\lambda\sqrt{f_c'}A_{cv}$ (see Figure 8.11).

From the analysis, $V_u = 1,575.5 / 2 = 787.8$ kips.

$h_w / \ell_w = 139.0 / 36.5 = 3.8 > 2$; therefore, $\alpha_c = 2$ Eq. (8.29)

$$V_u = 787.8 \text{ kips} > 0.5\phi\alpha_c\lambda\sqrt{f_c'}A_{cv} = 0.5 \times 0.75 \times 2 \times 1.0 \times \sqrt{6,000} \times (12.0 \times 438.0) / 1,000 = 305.4 \text{ kips}$$

Preliminary analysis indicates that minimum transverse reinforcement can be used (that is, $\rho_t = 0.0025$; see Figure 8.11 and Example 8.6).

Therefore,

Minimum $\rho_\ell = 0.0025 + 0.5(2.5 - h_w / \ell_w)(\rho_t - 0.0025) = 0.0025$ for $\rho_t = 0.0025$ Figure 8.11

Check the adequacy of the wall using minimum longitudinal reinforcement:

$$\text{Maximum longitudinal bar spacing} = \text{lesser of} \begin{cases} 3h = 36.0 \text{ in.} \\ 18 \text{ in.} \\ \ell_w / 3 = 438.0 / 3 = 146.0 \text{ in.} \end{cases}$$ Figure 8.11

Try 2 layers of #5 bars spaced at 18 in. on center $[\rho_\ell = (2 \times 0.31) / (12.0 \times 18.0) = 0.0029 > 0.0025)]$.

The design strength interaction diagram for this wall reinforced with 2 layers of #5 bars spaced at 18 in. on center is given in Figure 8.27. It was constructed based on equilibrium, strain compatibility, and the design assumptions in ACI 22.2 (see Section 7.4.3 and Figure 7.11 of this publication on how to construct interaction diagrams for rectangular sections). The load combinations from Table 8.28 are also given in the figure. Because all load combination points fall within the design strength interaction diagram, the wall is adequate.

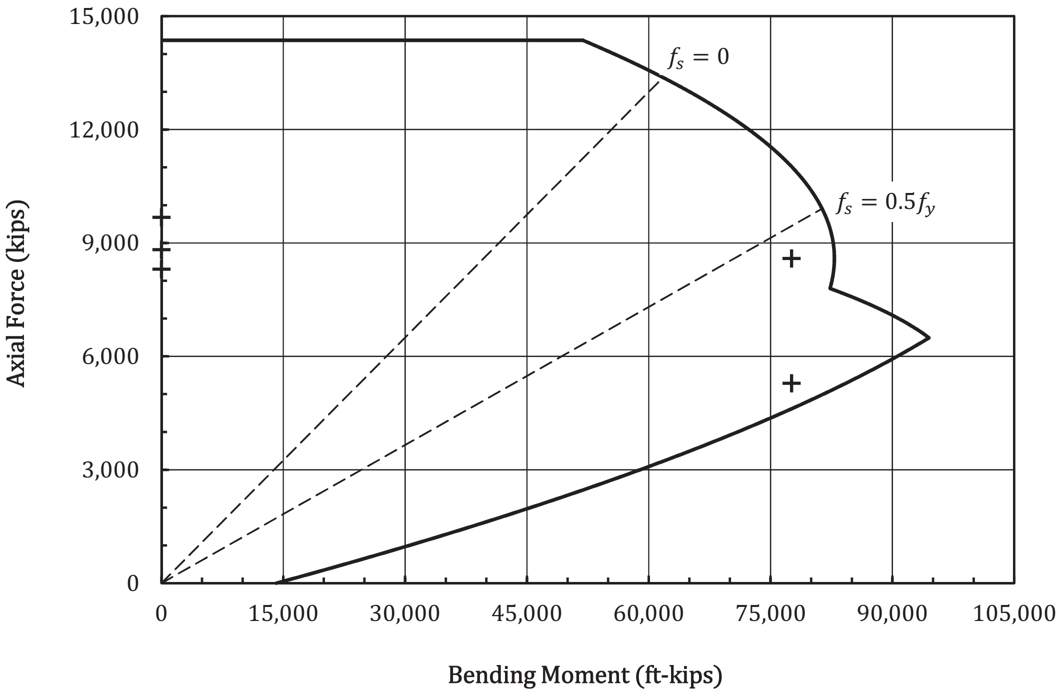


Figure 8.27 Design strength interaction diagram for the wall in Example 8.5

8.11.6 Example 8.6 – Design of Reinforced Concrete Wall for Shear Forces: Building #2, SDC C, Interior Wall is Part of the SFRS

Design the interior reinforced concrete wall along column line F that is part of the SFRS in the first story of Building #2 for shear forces considering seismic forces in the east-west direction (see Figure 1.2). Assume normalweight concrete with $f_c' = 6{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Example 8.5.

Step 1 – Determine the transverse reinforcement

An estimate of the transverse reinforcement in the wall is needed in order to determine if the section is adequate or not for the factored shear force $V_u = 787.8$ kips.

The minimum amount of transverse reinforcement depends on the magnitude of V_u with respect to the limit $0.5\phi\alpha_c\lambda\sqrt{f_c'}A_{cv}$ (see Figure 8.11).

$h_w / \ell_w = 139.0 / 36.5 = 3.8 > 2;$ therefore, $\alpha_c = 2$ Eq. (8.29)

$$V_u = 787.8 \text{ kips} > 0.5\phi\alpha_c\lambda\sqrt{f_c'}A_{cv} = 0.5 \times 0.75 \times 2 \times 1.0 \times \sqrt{6{,}000} \times (12.0 \times 438.0) / 1{,}000 = 305.4 \text{ kips}$$

Therefore, minimum $\rho_t = 0.0025$. Figure 8.11

Maximum transverse bar spacing $= \text{lesser of} \begin{cases} 3h = 36.0 \text{ in.} \\ 18 \text{ in.} \\ \ell_w / 5 = 438.0 / 5 = 87.6 \text{ in.} \end{cases}$ Figure 8.11

Try 2 layers of #5 bars spaced at 18 in. on center $[\rho_t = (2 \times 0.31) / (12.0 \times 18.0) = 0.0029 > 0.0025)]$.

Step 2 – Check the adequacy of the section

$$\begin{aligned} \phi V_n &= \phi(\alpha_c\lambda\sqrt{f_c'} + \rho_t f_{yt})A_{cv} \\ &= 0.75 \times [(2 \times 1.0 \times \sqrt{6{,}000}) + (0.0029 \times 60{,}000)] \times (12.0 \times 438.0) / 1{,}000 = 1{,}296.6 \text{ kips} \\ &< \phi 8\sqrt{f_c'}A_{cv} = 2{,}442.8 \text{ kips} \\ &> V_u = 787.8 \text{ kips} \end{aligned}$$

Eq. (8.28)

Therefore, the section is adequate for shear using transverse reinforcement consisting of 2 layers of #5 bars spaced at 18 in. on center.

8.11.7 Example 8.7 – Determination of Dowel Reinforcement at the Foundation of a Reinforced Concrete Wall: Building #2, SDC C, Interior Wall is Part of the SFRS

Determine the required dowel reinforcement for the interior reinforced concrete wall along column line F that is part of the SFRS in the first story of Building #2 considering seismic forces in the east-west direction assuming the wall is supported by a 3-ft-thick mat foundation (see Figure 1.2). Also assume normalweight concrete with $f_c' = 6{,}000$ psi and $f_c' = 4{,}000$ psi for the wall and foundation, respectively, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.2. See Examples 8.5 and 8.6.

Step 1 – Check the bearing stresses on the wall and mat foundation ACI 16.3.3.4

Bearing strength of the wall.

Factored bearing stress, b_u, is determined for compression and for compression and bending using the factored axial forces and bending moments determined by ACI Eqs. (5.3.1b) and ACI Eq. (5.3.1e), respectively (see Table 8.28).

- ACI Eq. (5.3.1b):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{9,684.9 \times 1,000}{12.0 \times 438.0} = 1,843 \text{ psi}$$

$$< \text{Design bearing strength} = \phi b_n = \phi 0.85 f_c' = 0.65 \times 0.85 \times 6,000 = 3,315 \text{ psi}$$

- ACI Eq. (5.3.1e):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{8,598.6 \times 1,000}{12.0 \times 438.0} = 1,636 \text{ psi}$$

$$\text{Bending stress (compression and tension)} = \frac{6M_u}{h\ell_w^2} = \frac{6 \times 77,586.8 \times 12,000}{12.0 \times 438.0^2} = 2,427 \text{ psi}$$

$$\text{Maximum compression stress} = 1,636 + 2,427 = 4,063 \text{ psi}$$

$$\text{Maximum tension stress} = 1,636 - 2,427 = -791 \text{ psi}$$

Tension forces cannot be transferred through bearing at the interface, so dowel bars will be provided that match the size and spacing of the longitudinal bars in the wall. This reinforcement ensures that both the compression and tension forces are adequately transferred from the wall into the mat foundation.

Bearing strength of the mat foundation.

There is no need to check the bearing strength of the mat foundation because dowel bars will be provided that match the size and spacing of the longitudinal bars in the wall.

Step 2 – Determine the required interface reinforcement ACI 16.3.4.1

From Example 8.5, the longitudinal reinforcement in the wall is 2 layers of #5 bars spaced at 18 in. on center; at this spacing, 25 rows of reinforcement are provided in the wall.

Check minimum area of reinforcement across the interface:

$$A_{s,min} = 0.0012A_g = 0.0012 \times 12.0 \times 438.0 = 6.31 \text{ in.}^2 < A_{s,provided} = 25 \times (2 \times 0.31) = 15.5 \text{ in.}^2$$ ACI 16.3.4.2

The longitudinal reinforcement ratio of 0.0012 is used to determine the minimum area of interface reinforcement in accordance with ACI 11.6.1 because #5 longitudinal bars (Grade 60) are used in the wall (see ACI Table 11.6.1).

Step 3 – Check horizontal force transfer

From Example 8.6, the factored shear force at the base of the wall is equal to 787.8 kips. This force is transferred horizontally between the wall and the foundation. The required area of shear-friction reinforcement is determined as follows assuming the surface between the wall and the foundation has not been intentionally roughened:

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{V_u}{\phi f_y (0.6\lambda)} = \frac{787.8}{0.75 \times 60 \times (0.6 \times 1.0)} = 29.2 \text{ in.}^2 \quad \text{Eq. (8.46)}$$

This area of reinforcement is larger than the area of reinforcement provided by the #5 dowel bars that match the size and spacing of the longitudinal reinforcement in the wall, which is equal to 15.5 in.^2

Therefore, try #7 dowel bars spaced at 18 in. on center $[A_{s,provided} = 25 \times (2 \times 0.60) = 30.0 \text{ in.}^2]$.

Check the upper shear limit:

$V_u = 787.7 \text{ kips} < \phi 0.2 f_c' A_c = 0.75 \times 0.2 \times 4 \times 12.0 \times 438.0 = 3{,}153.6 \text{ kips}$ Table 8.22

Use #7 dowel bars spaced at 18 in. on center.

Step 4 – Check the development of the dowel bars in the mat foundation

It is evident from Figure 8.27 that the stress in the longitudinal reinforcement farthest from the extreme compression face is tensile for the load combinations that include seismic effects. Therefore, the dowel bars must be developed for tension into the mat (and into the wall).

A standard 90-degree hook will be provided at the ends of the #7 dowel bars. Determine the tension development length, ℓ_{dh}, of the #7 dowel bars terminating in a hook:

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \quad \text{Eq. (8.44)}$$

$\psi_e = 1.0$ for uncoated bars Table 8.20

$\psi_r = 1.6$ (confining reinforcement not provided)

$\psi_o = 1.0$ (side cover normal to hook $> 6d_b$)

$\psi_c = (f_c' / 15{,}000) + 0.6 = (4{,}000 / 15{,}000) + 0.6 = 0.9$

$\lambda = 1.0$ for normalweight concrete

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} = \left(\dfrac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.9}{55 \times 1.0 \times \sqrt{4{,}000}}\right) \times 0.875^{1.5} = 20.3 \text{ in.} \\ 8d_b = 8 \times 0.875 = 7.0 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

Assuming two layers of #11 bars in the mat foundation with a clear cover of 3 in., determine the minimum mat thickness for the development of the #7 dowel bars:

$\text{Minimum } h = \ell_{dh} + 2(d_b)_f + \text{cover} = 20.3 + (2 \times 1.41) + 3.0 = 26.1 \text{ in.}$ Eq. (8.45)

The minimum $h = 26.1$ in. is less than the provided mat thickness of 36.0 in.

Step 5 – Determine the development length of the dowel bars in the wall

The #7 dowel bars must be lap spliced to the #5 longitudinal bars in the wall using a tension lap splice. Where bars of different size are lap spliced together, the required tension lap splice length is the greater of (a) ℓ_d of the larger bar and (b) ℓ_{st} of the smaller bar (see Note 9 in Table 8.15). Because all the dowel bars are spliced at the same location, a Class B tension lap splice is required (ACI Table 25.5.2.1).

Development length in tension, ℓ_d, of the #7 dowel bars:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ Eq. (8.37)

$\psi_t = 1.0$ Table 8.18

$\psi_e = 1.0$ for uncoated bars

$\psi_s = 1.0$ for #7 bars

$\psi_g = 1.0$ for Grade 60 reinforcement

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + (d_b)_{trans.} + 0.5(d_b)_{dowel} = 0.75 + 0.625 + (0.5 \times 0.875) = 1.8 \text{ in.} \\ \frac{s}{2} = \frac{18.0}{2} = 9.0 \text{ in.} \end{cases}$$ Figure 8.14

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) \,/\, d_b = (1.8 + 0) \,/\, 0.875 = 2.1 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0 \times \sqrt{6{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.1} \right) \times 0.875 = 24.2 \text{ in.} > 12.0 \text{ in.}$$

Tension lap splice length, ℓ_{st}, of the #5 longitudinal bars:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ Eq. (8.37)

$\psi_t = 1.0$ Table 8.18

$\psi_e = 1.0$ for uncoated bars

$\psi_s = 0.8$ for #5 bars

$\psi_g = 1.0$ for Grade 60 reinforcement

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + (d_b)_{trans.} + 0.5(d_b)_{long.} = 0.75 + 0.625 + (0.5 \times 0.625) = 1.7 \text{ in.} \\ \frac{s}{2} = \frac{18.0}{2} = 9.0 \text{ in.} \end{cases}$$ Figure 8.14

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) \,/\, d_b = (1.7 + 0) \,/\, 0.625 = 2.7 > 2.5, \text{ use } 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40}\frac{60{,}000}{1.0\times\sqrt{6{,}000}}\frac{1.0\times1.0\times0.8\times1.0}{2.5}\right)\times0.625 = 11.6 \text{ in.} < 12.0 \text{ in., use 12 in.}$$

$$\text{Class B lap splice length} = 1.3\ell_d = 1.3\times11.6 = 15.1 \text{ in.} < \ell_d = 24.2 \text{ in. of the \#7 dowel bars}$$

Provide a lap splice length of 2 ft-2 in.

Reinforcement details for this wall are given in Figure 8.28.

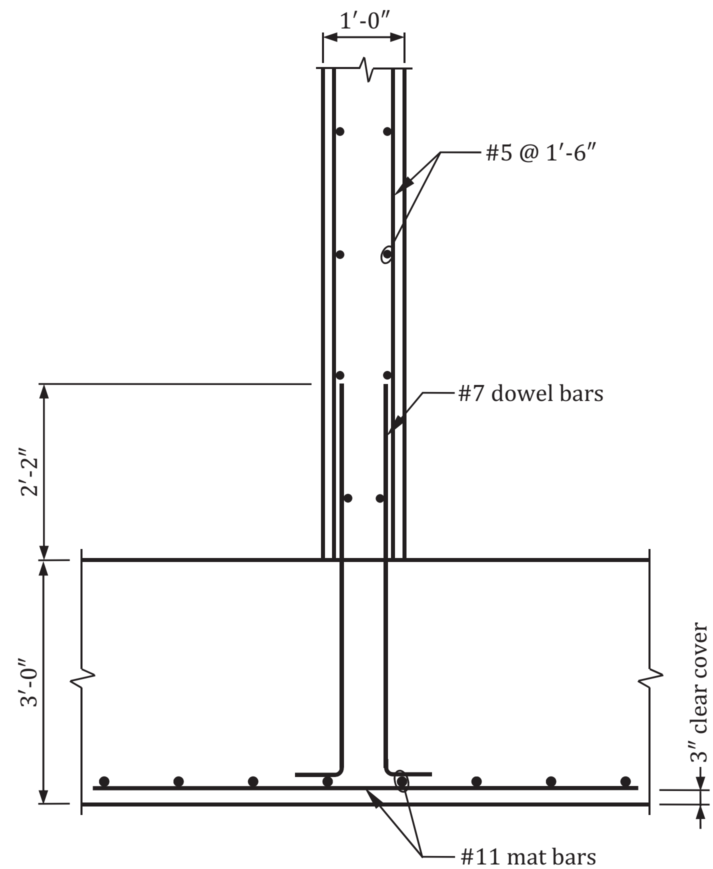


Figure 8.28 Reinforcement details for the wall in Examples 8.4 through 8.7.

Chapter 9
DIAPHRAGMS

9.1 Overview

Reinforced concrete diaphragms are roof and floor systems within a structure that resist and transfer gravity and lateral forces. Typical diaphragm actions in a reinforced concrete building are depicted in ACI Figure R12.1.1.

Diaphragms must support and transfer gravity loads to columns, walls, and other supporting elements in a building. The weight of the structure, superimposed dead loads, and live loads are common out-of-plane gravity loads applied to the surface of a diaphragm. Roof diaphragms must also be able to support effects due to rain, snow, and wind, to name a few.

Lateral forces from wind, earthquakes, and soil pressure are common types of forces transferred from a diaphragm to the lateral force-resisting system (LFRS) in a building. The load path from the application of the force on the diaphragm to the vertical elements of the LFRS depends on several factors, including the type of applied force and the rigidity (or, flexibility) of the diaphragm. The in-plane forces generate in-plane shear forces, bending moment, and axial forces in a diaphragm. The connections between the diaphragm and the vertical elements of the LFRS are very important; these connections must be properly designed and detailed otherwise there is no mechanism for proper load transfer to the LFRS.

Collectors, which are part of a diaphragm and are sometimes referred to as drag struts, are required where the LFRS does not extend the full depth of a diaphragm or where there is a discontinuity in the LFRS. These elements are parallel to the applied force and their main functions are to collect and transfer diaphragm shear forces to the vertical elements of the LFRS, distribute forces within the diaphragm, or both.

In general, diaphragms must be designed and detailed for the combined effects due to in-plane and out-of-plane load effects. Reinforcement is provided to resist the effects from in-plane and out-of-plane axial forces, bending moments, and shear forces.

The design and detailing of cast-in-place diaphragms with nonprestressed reinforcement are covered in this chapter. Provisions for diaphragms are given in ACI Chapter 12, which are applicable to diaphragms in buildings assigned to Seismic Design Category (SDC) A, B, and C.

9.2 Minimum Diaphragm Thickness

Diaphragms must have sufficient thickness so that all applicable strength and serviceability requirements are satisfied (ACI 12.3.1.1). The following load effects must be investigated for strength: (1) out-of-plane moments and shears due to gravity forces or combinations of gravity and lateral forces and (2) in-plane moments, in-plane shear forces, and axial forces due to wind, seismic, and other applicable lateral forces.

For buildings assigned to SDC A, B, or C, the minimum diaphragm thickness must be the largest of the following (ACI 12.3.1.2):

(1) Thickness based on the serviceability requirements of ACI 7.3.1.1 for one-way slabs or ACI 8.3.1 for two-way slabs.
(2) Thickness based on out-of-plane strength requirements.
(3) Thickness based on in-plane strength requirements.

Serviceability requirements for one-way slabs and two-way slabs are covered in Sections 4.2 and 5.2 of this publication, respectively. Methods are given in those sections that can be used to determine minimum slab thicknesses that satisfy deflection requirements.

Out-of-plane flexural requirements rarely have an impact on the required slab thickness of one-way and two-way slab systems. Reasonable amounts of flexural reinforcement can usually be provided at the critical sections using a slab thickness determined based on serviceability requirements. Two-way shear requirements may have an impact on the thickness of flat plate systems. Figure 5.3 in this publication can be used to determine a preliminary slab thickness for flat plates considering two-way shear stresses. A more refined analysis needs to be performed to ensure that two-way shear requirements are satisfied. In lieu of increasing the slab thickness or column size, two-way shear requirements can be satisfied by providing shear reinforcement (see Section 5.5.2 of this publication).

A slab thickness based on serviceability requirements is usually adequate to satisfy in-plane strength requirements. In-plane moments are resisted by chord reinforcement placed perpendicular to the direction of the lateral force. In most cases, this reinforcement is determined using a slab thickness sufficient for serviceability or other strength requirements; there is usually no need to increase the thickness of a slab based on in-plane flexural requirements. It is common for the design shear strength of the concrete alone to satisfy in-plane shear strength requirements of a diaphragm.

Fire-resistance requirements of the governing building code must also be considered when selecting a minimum slab thickness. For local jurisdictions that have adopted the IBC, minimum slab thickness for various fire-resistance ratings based on concrete type is given in IBC Table 722.2.2.1. It is common for fire-resistance requirements to be satisfied using a slab thickness that satisfies serviceability requirements, strength requirements, or both.

9.3 Required Strength

9.3.1 General

The factored load combinations in ACI Chapter 5 are to be used to calculate required strength of diaphragms, collectors, and their connections (see ACI 12.4.1.1). According to ACI 12.4.1.2, members must be designed for the simultaneous effects from out-of-plane and in-plane loading. For example, collectors are commonly designed for flexure and shear forces due to gravity loads in combination with axial compression and tension forces due to in-plane forces in the diaphragm.

9.3.2 Diaphragm Design Forces

Overview

In general, diaphragms must be designed to resist the effects from the following forces (see ACI 12.2.1 and ACI Figure R12.1.1):

(a) Diaphragm in-plane forces due to lateral loads acting on the building
(b) Diaphragm transfer forces
(c) Connection forces between the diaphragm and vertical framing or nonstructural elements
(d) Forces resulting from bracing vertical or sloped building elements
(e) Diaphragm out-of-plane forces due to gravity and other loads applied to the surface of the diaphragm

In-Plane Forces due to Lateral Loads

Methods to determine in-plane wind forces and seismic forces on diaphragms are given in Sections 3.6 and 3.7 of this publication. In the case of wind loads, the resultant wind force, W_x, at level x in a building is equal to the total wind pressure, p_x, at that level (which is sum of the windward pressure, p_z, and the leeward pressure, p_h) times the width of the building, B, times the tributary story height, h_x (see Figure 9.1). This force is applied at the centroid of the face of the building in the direction of analysis.

The seismic design force, F_{px}, is applied at the center of mass (CM) of the diaphragm at level x because it is an inertial force (see Figure 9.2). For rigid diaphragms (see Section 9.3.3 of this publication for classification of diaphragms), the diaphragm translates and rotates about the center of rigidity (CR).

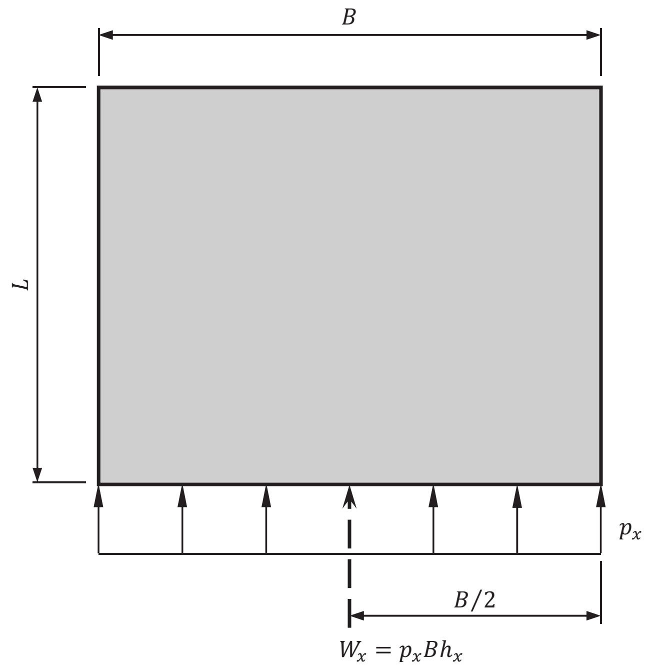


Figure 9.1 Resultant wind force acting on a diaphragm.

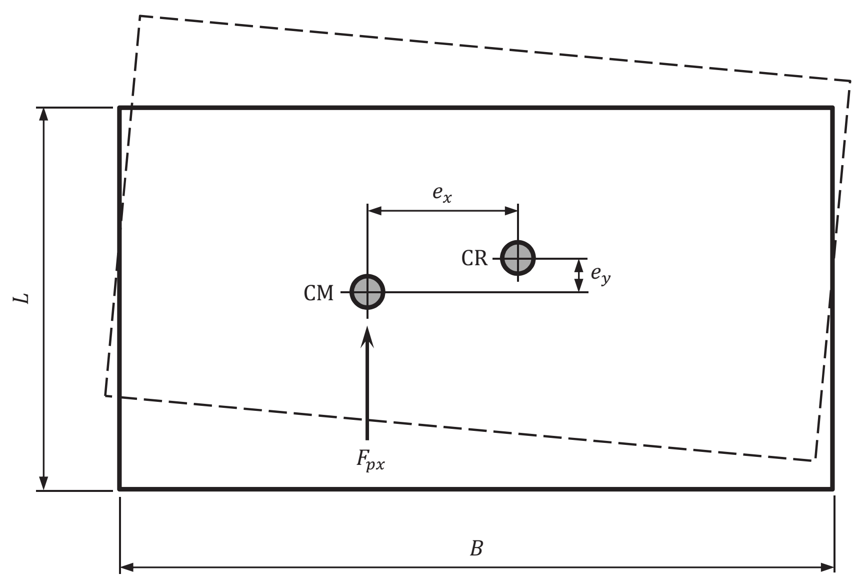


Figure 9.2 Seismic force acting on a rigid diaphragm.

For buildings with one or more basement levels, lateral forces are generated by soil pressure bearing against the basement walls (see Figure 9.3). The magnitude of the soil pressure is usually obtained from a geotechnical investigation. In cases where the results of such an investigation are not available, the lateral soil loads in IBC Table 1610.1 can be used (similar design lateral loads are provided in ASCE/SEI Table 3.2-1). It is evident from Figure 9.3 that a portion of the soil pressure is transferred to the foundation system and a portion is transferred to the reinforced concrete slab, which acts as a diaphragm. The diaphragm must be designed for the in-plane effects due to the soil pressure (plus any other applicable in-plane load effects) and must transfer the forces to the basement walls parallel to the direction of the soil pressure.

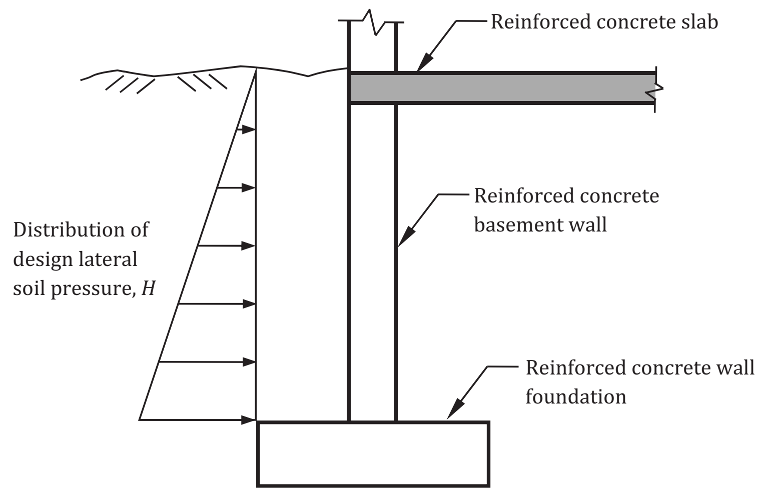


Figure 9.3 Distribution of at-rest soil pressure on a basement wall.

According to IBC 1803.5.12 and ASCE/SEI 11.8.3, basement walls and retaining walls supporting more than 6 ft of backfill height must be subjected to dynamic seismic lateral earth pressures due to design earthquake ground motions for structures assigned to SDC D, E, or F. However, no provisions or guidelines are provided on how to determine these dynamic pressures. A method to determine such forces is given in Reference 21.

In-Plane Forces due to Transfer Forces

A typical case where a diaphragm is subjected to transfer forces is illustrated in Figure 9.4. In addition to the effects from wind, seismic, and other applicable in-plane loads (which are collectively labelled as "Diaphragm force" in the figure), the podium slab must be designed for the effects due to the forces transferred from the shear walls above. The podium slab, in turn, transfers all the in-plane forces to the basement walls parallel to the direction of analysis.

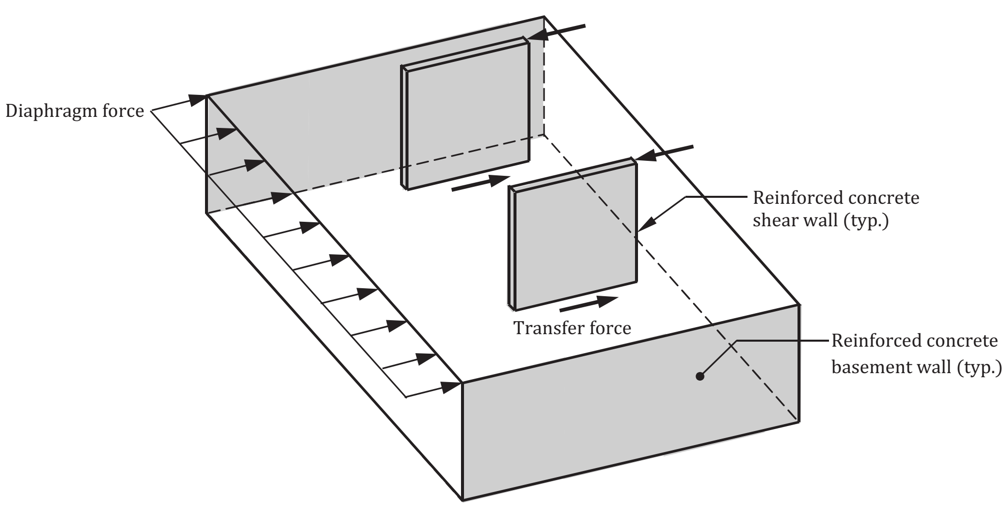


Figure 9.4 Transfer forces to a podium slab.

Connection Forces Between the Diaphragm and Vertical Framing or Nonstructural Elements

Nonstructural components, such as cladding, is commonly attached to reinforced concrete slabs over the height of a building. These components transfer wind pressures, inertial forces from earthquake shaking, or both, to the diaphragm through the connections between the two.

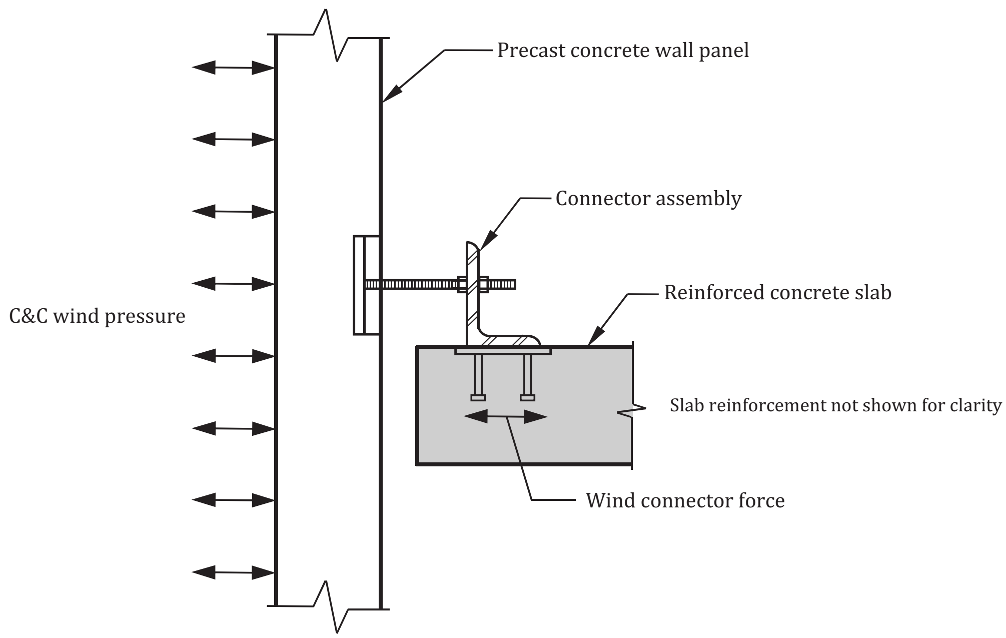


Figure 9.5 Wind forces from a nonstructural element transferred to a diaphragm.

Illustrated in Figure 9.5 is a generic tie-back connection between a precast concrete wall panel and a reinforced concrete slab, which is representative of the many types of connections between cladding and diaphragms. Component and cladding (C&C) wind pressure normal to the surface of the panel is transferred from the precast concrete wall panel to the diaphragm via the connector assembly. The diaphragm must be designed for this force in combination with all other applicable forces.

A similar force transfer to a diaphragm occurs for nonstructural wall panels subjected to out-of-plane seismic forces.

Forces Resulting from Bracing Vertical or Sloped Building Elements

Diaphragms must be designed for forces resulting from bracing vertical structural elements in a building. For example, in the case of a building frame system, it is assumed that all the lateral forces are resisted by the shear walls. When analyzed separately, shear walls and frames have different displacement profiles over the height of a building when subjected to lateral forces (see Figure 9.6). In an actual building, the diaphragms connect the columns and shear walls together, which means these elements must displace the same amount at each level. Thus, the resulting transfer forces caused by displacement compatibility between the columns that are not part of the LFRS and the shear walls must be resisted by the diaphragms.

Diaphragms must also be designed to resist forces from any sloped structural members in a building. An inclined column, such as the one illustrated in ACI Figure R12.1.1, transfers the horizontal components of the supported gravity and lateral forces, where applicable, into the reinforced concrete slab (diaphragm) in cases where there are no other local structural elements to counteract these forces.

Out-of-Plane Forces

One of the main functions of a diaphragm is to support and transfer gravity loads to the supporting structural members in a building. The weight of the structure, superimposed dead loads, and live loads are typical out-of-plane gravity loads applied to the surface of a diaphragm. Dead and live loads are determined in accordance with ASCE/SEI Chapters 3 and 4, respectively.

Snow and rain loads can occur on roof diaphragms, to name a few (ASCE/SEI Chapters 7 and 8). Wind uplift loads are also applicable, as are vertical accelerations due to seismic effects.

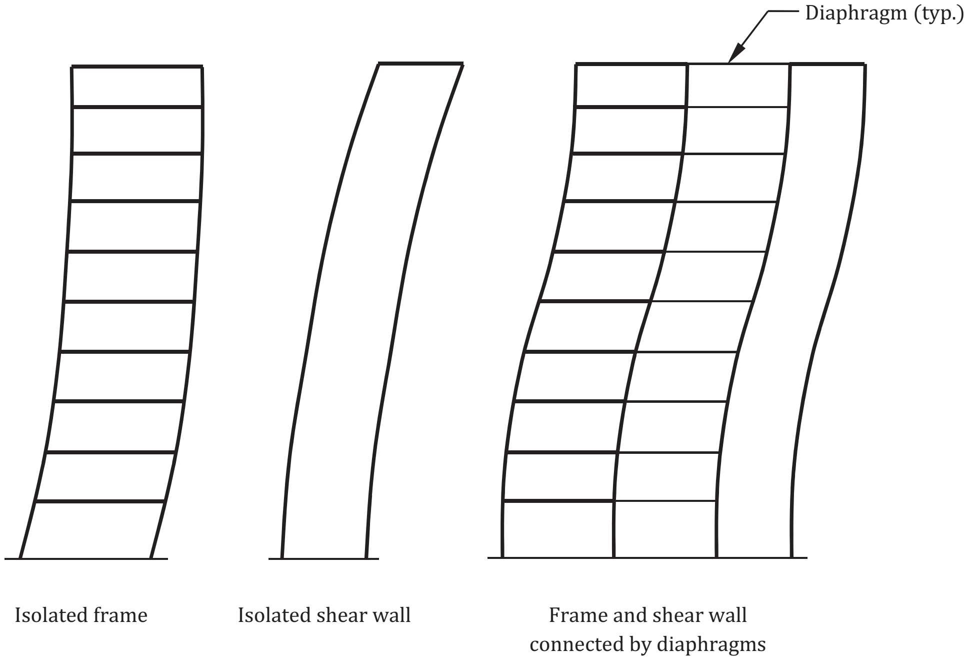


Figure 9.6 Diaphragm force transfer due to displacement compatibility.

Collector Design Forces

Where vertical elements of the LFRS do not extend the full depth of a diaphragm, collectors are needed to transfer forces between the diaphragm and the vertical elements of the LFRS (see ASCE/SEI Figure 12.10-1). Depending on several factors, a collector can be part of the slab or can be a beam.

In general, collectors must be designed for the combined effects due to gravity loads (flexure and shear) and lateral loads (axial tension and compression forces caused by in-plane shear transfer in the diaphragm due to wind forces, seismic forces, or both). The in-plane diaphragm forces are determined using the methods presented in Section 9.3.3 of this publication. Once the forces are determined, the diaphragm is modeled and analyzed using the methods in ACI 12.4.2, and the axial forces in a collector can be determined based on the selected method of analysis. Section 9.3.3 of this publication covers the analysis procedures in ACI 12.4.2. These procedures can be used to determine collector axial forces in diaphragms subjected to wind and seismic forces in buildings assigned to SDC A, B, or C.

9.3.3 Diaphragm Modeling and Analysis

Overview

General modeling and analysis requirements for diaphragms are given in ACI 12.4.2. The provisions in ACI 12.4.2.2 through 12.4.2.4 are to be used where the requirements of the general building code, such as the IBC or ASCE/SEI 7, are not applicable.

Some of the general analysis requirements in ACI Chapter 6 are applicable to diaphragms. For example, the provisions for elastic analysis in ACI 6.6.1 through 6.6.3 can be applied because diaphragms are designed to remain essentially elastic when subjected to in-plane wind and seismic forces.

In-plane stiffness modeling and analysis methods for diaphragms are covered below.

In-Plane Stiffness Modeling

It is permitted to use any set of reasonable and consistent assumptions for in-plane diaphragm stiffness (ACI 12.4.2.3). Distribution of forces in the diaphragm and displacements and forces in the vertical elements of the LFRS are contingent on the in-plane stiffness of the diaphragm.

A diaphragm is considered to be rigid where the in-plane deflection due to lateral forces is relatively small compared to that of the vertical elements of the LFRS. For purposes of analysis, rigid diaphragms displace and rotate as a rigid body when subjected to lateral forces, and the vertical elements of the LFRS move together accordingly (displacement compatibility). Also, lateral forces are distributed to the vertical elements of the LFRS in proportion to their relative rigidities (stiffnesses) and their location with respect to the center of rigidity (CR), which is the stiffness centroid within a diaphragm.

In contrast, the in-plane deflection of a flexible diaphragm is relatively large compared to that of the vertical elements of the LFRS, and the distribution of lateral forces to the vertical elements of the LFRS is independent of their relative rigidities. In such cases, lateral forces are distributed based on the tributary mass of the diaphragm to the vertical elements of the LFRS. For diaphragms of uniform material and weight, lateral forces can be distributed by tributary areas. Flexible diaphragms do not undergo rigid body rotation like rigid diaphragms.

A diaphragm is classified as semirigid where the in-plane deflection of the diaphragm and the deflection of the vertical elements of the LFRS are of the same order of magnitude.

A cast-in-place reinforced concrete diaphragm is permitted to be classified as rigid where the following criteria are satisfied:

- When subjected to lateral wind forces (ASCE/SEI 26.2): Span-to-depth ratio ≤ 2
- When subjected to lateral seismic forces (ASCE/SEI 12.3.1.2):
  (a) Span-to depth ratio ≤ 3 and
  (b) Structure has none of the horizontal irregularities in ASCE/SEI Table 12.3-1

When determining the span-to-depth ratio, the span is equal to the distance between lines of lateral resistance (such as walls and frames) in the direction of analysis (see Figure 9.7). The overall depth of the diaphragm in the direction of analysis is used to determine the span-to-depth ratio.

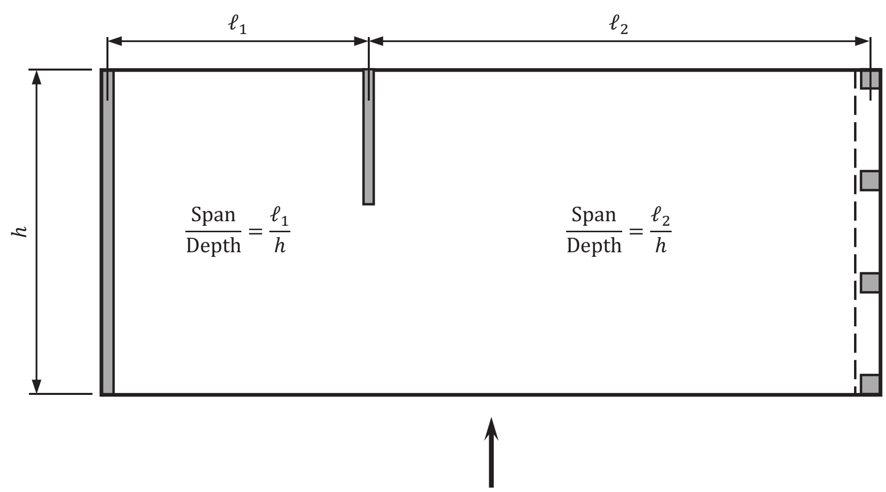


Figure 9.7 Definition of span-to-depth ratio for a diaphragm.

The reinforced concrete diaphragm illustrated in ACI Figure R12.4.2.3a may not be considered rigid in the direction of the lateral force because of its relatively large span-to-depth ratio, ℓ / h, where ℓ is the distance between the walls at each end, which have been designated to be part of the LFRS (the interior moment frames are not part of the LFRS), and h is the depth of the diaphragm in the direction of analysis. Similarly, ramps in parking structures may not be classified as rigid because of relatively long span-to-depth ratios, which are common in such structures.

According to ASCE/SEI 12.3.1.3, a diaphragm is permitted to be idealized as flexible where $\delta_{MDD} / \Delta_{ADVE} > 2$ [ASCE/SEI Equation (12.3-1)]. The term δ_{MDD} is the maximum in-plane deflection of the diaphragm and Δ_{ADVE} is the average drift of adjoining vertical elements of the LFRS over the story below the diaphragm under consideration when subjected to the in-plane force. A rigid diaphragm is defined in IBC 1604.4 based on deflections and drifts as well: A diaphragm is rigid for the purpose of distribution of story shear and torsional moment when the lateral deformation of the diaphragm, δ_{MDD}, is less than or equal to two times the average story drift, Δ_{ADVE}.

For diaphragms that cannot be idealized as either rigid or flexible, the in-plane stiffness of the diaphragm must be explicitly accounted for in the analysis. A three-dimensional analysis considering the stiffnesses of the diaphragms and the elements of the LFRS usually provides the most accurate distribution of forces in such cases.

To account for cracking in reinforced concrete diaphragms, a stiffness modifier should be applied to the gross in-plane stiffness of the diaphragm. In the case of wind forces, the structure is typically assumed to respond in the elastic range, so using 50 percent of the gross in-plane moment of inertia of the diaphragm would be appropriate (see ACI 6.6.3.1.2). For seismic forces, it has been shown that stiffness modifiers typically fall in the range of 0.15 to 0.50.

Analysis Methods

Once the diaphragm forces have been determined and the type of diaphragm has been established based on in-plane stiffness (rigid, semirigid, or flexible), the diaphragm and any collectors must be analyzed for the effects due to in-plane forces in combination with out-of-plane forces.

According to ACI 12.4.2.4, in-plane design moments, shear forces, and axial forces in diaphragms must be determined using any method that satisfies equilibrium and the design boundary conditions. Methods of analysis based on the following models are permitted to be used:

- A rigid diaphragm model where diaphragms can be idealized as rigid.
- A flexible diaphragm model where diaphragms can be idealized as flexible.
- A bounding analysis in which the design values are the envelope of values obtained by assuming upper bound and lower bound in-plane stiffnesses for the diaphragm in two or more separate analyses.
- A finite element model considering diaphragm flexibility.
- A strut-and-tie model in accordance with ACI 23.2.

The applicability of these models and other pertinent information are given in Table 9.1.

Table 9.1 Diaphragm Models

Model	Applicability	Notes
Rigid diaphragm	• For lateral wind forces, reinforced concrete slabs with a span-to-depth ratio of 2 or less • For lateral seismic forces, when the following two conditions are satisfied: (1) reinforced concrete slabs with a span-to-depth ratio of 3 or less and (2) structure possesses none of the horizontal irregularities in ASCE/SEI Table 12.3-1	Beam models are commonly used to determine diaphragm internal forces in buildings without major irregularities and/or transfer forces
Flexible diaphragm	A diaphragm is permitted to be idealized as flexible where the ratio of the maximum in-plane deflection of the diaphragm, δ_{MDD}, and the average story drift, Δ_{ADVE}, is greater than 2	Beam models are commonly used to determine diaphragm internal forces in buildings without major irregularities and/or transfer forces

(table continued on next page)

Table 9.1 Diaphragm Models (cont.)

Model	Applicability	Notes
Bounding analysis	Suitable for semirigid diaphragms	Diaphragm internal forces are taken as the envelope of values from the following two analyses: (1) rigid diaphragm on flexible supports and (2) flexible diaphragm on rigid supports
Finite element	Suitable for any diaphragm	This model is especially useful for irregularly shaped diaphragms and diaphragms with large transfer forces and/or openings
Strut-and-tie	Suitable for any diaphragm	Model should include consideration of force reversals that may occur under design load combinations

Beam models are widely used to determine internal forces in rigid and flexible diaphragms in buildings without major irregularities and/or transfer forces. Diaphragms are treated as beams that span between the vertical elements of the LFRS, which are idealized as either rigid or flexible (spring) supports. Beam models for rigid diaphragms are covered in detail below.

Equivalent Beam Model with Rigid Supports

This model is often used for low-rise buildings with regular geometries where two or more lines of lateral force-resisting systems are provided in a given direction that have approximately the same relative rigidity.

Diaphragm Forces

Illustrated in Figure 9.8 is a reinforced concrete diaphragm and a LFRS consisting of structural (shear) walls (it is assumed that the perimeter frames are not part of the LFRS). In the direction of analysis, the diaphragm is modeled as a beam whose depth is equal to the full diaphragm depth, L. The walls, which have the same in-plane stiffness, are assumed to be rigid supports in the direction of analysis. This method can also be used where walls are at the perimeter of the building or where more than two walls are in the direction of analysis (which means the equivalent beam is continuous). Because the walls do not extend the full depth of the diaphragm, collectors are needed to collect the shear from the diaphragm and to transfer it to the walls.

The equivalent in-plane load, w_u, in Figure 9.8 is uniformly distributed over the width of the diaphragm. The reactions in walls 1 and 2, R_1 and R_2, are the diaphragm forces transferred to the vertical elements of the LFRS, and are obtained from statics. It is permitted to assume that the diaphragm resists in-plane moment and axial force in accordance with ACI 22.3 and 22.4, which includes the assumption that strains vary linearly over the depth of the diaphragm. Shear and moment diagrams for the diaphragm are also given in Figure 9.8.

In this model, roof and floor systems act as the web of the equivalent beam, which resists the design shear forces. For diaphragms analyzed by this method, the shear flow, v_u, must be uniform over the depth L. For wall 2, $(v_u)_2 = R_2 / L$. For wall 1, the maximum uniform shear flow is equal to $(v_u)_1 = V_{u,max} / L$, which must be less than or equal to the design shear strength of the diaphragm. Shear flow is transferred from the diaphragm to the vertical elements of the LFRS by shear friction.

The diaphragm boundaries perpendicular to the lateral force act as the flanges of the equivalent beam (commonly referred to as chords) and resist the tension and compression forces induced in the diaphragm due to bending. These

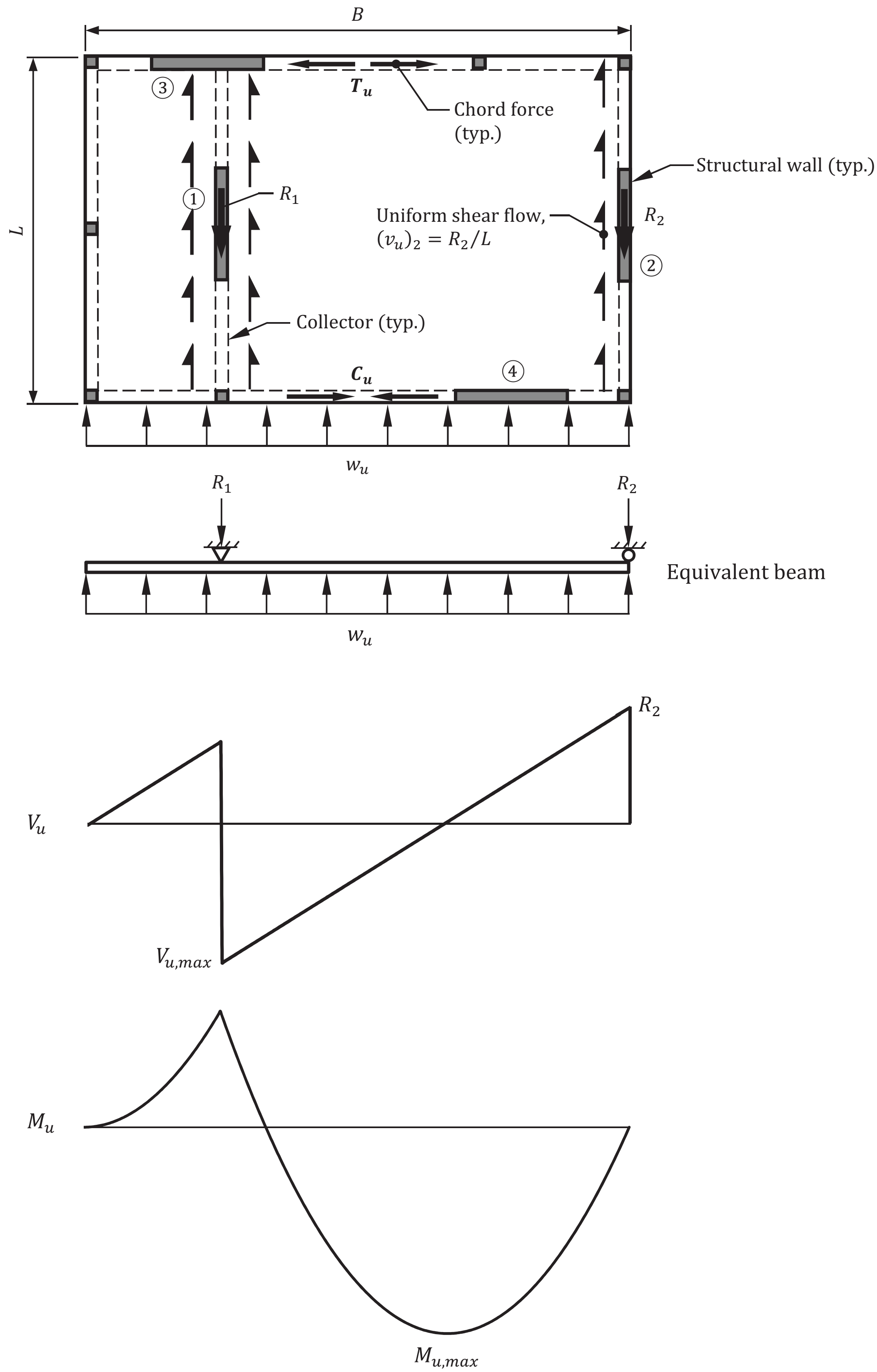


Figure 9.8 Equivalent beam model with rigid supports.

forces can be determined by dividing the maximum bending moment in the diaphragm by the distance between the forces (that is, the moment arm):

$$T_u = C_u = \frac{M_{u,max}}{d} \tag{9.1}$$

In this equation, d is the perpendicular distance between the chord forces, which is commonly taken as 95 percent of the total diaphragm depth in the direction of analysis (in this case, $d = 0.95L$). Reinforcement must be provided in the chords to resist the tension forces generated by this bending moment. Because wind and seismic forces can act in any direction, chord reinforcement is required at both boundaries of the diaphragm perpendicular to the direction of analysis.

Collector Forces

A reinforced concrete beam or a portion of a reinforced concrete slab in line with the LFRS can be used as a collector. In cases where the beam or slab has the same width as the vertical element of the LFRS it frames in to (for example, the width of the collector is equal to the thickness of the structural wall), all the collected forces are transferred directly into the member of the LFRS at its ends. This is illustrated in Figure 9.9 for wall 2, which is part of the LFRS of the building in Figure 9.8. The uniform factored shear force in the diaphragm at this location, $(v_u)_2$, is equal to the reaction in the wall, R_2, divided by the depth of the diaphragm in the direction of analysis, L. Similarly, the uniform factored shear force in the wall along its length is equal to $(v_{u,W})_2 = R_2 / \ell_2$. The net factored shear force at any point is equal to the difference between the unit factored shear forces in the wall and diaphragm at that point.

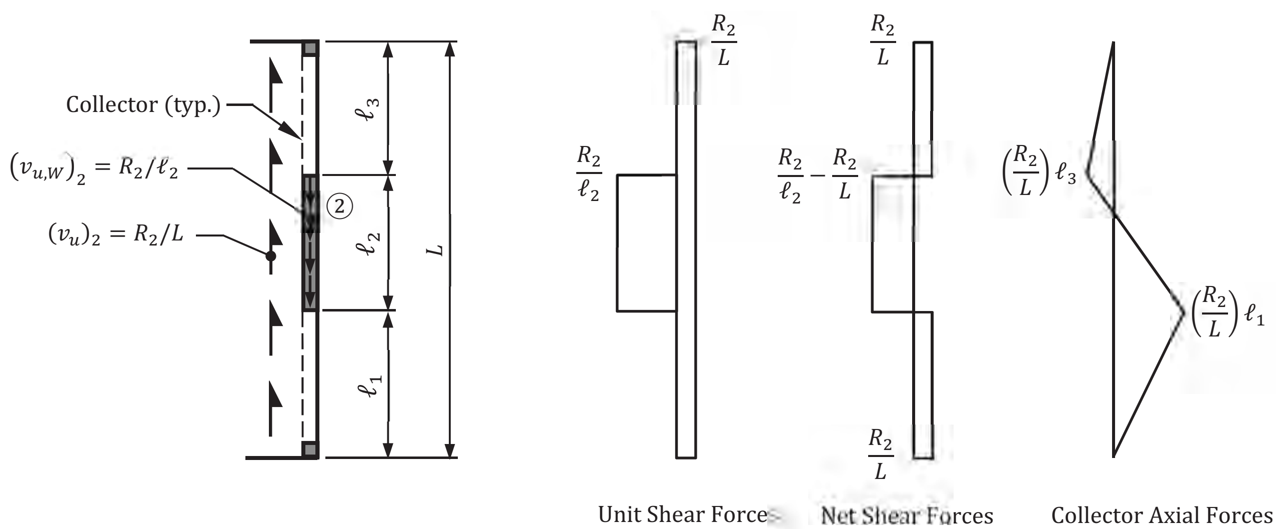


Figure 9.9 Unit shear forces, net shear forces, and collector forces in a diaphragm.

The axial forces in the collectors are determined by summing the areas in the net shear force diagram. At the edges of the diaphragm, the axial force in the collectors is equal to zero (Note: In cases where wind and/or seismic forces from perimeter elements, such as cladding, must be collected, the forces at the ends of a collector may not be zero). The axial force increases linearly as shear is transferred from the diaphragm to the collectors. At the end of the wall located a distance ℓ_3 from the edge of the diaphragm, the axial tension force in the collector is equal to $(R_2 / L)\ell_3$. At the other end of the wall, the axial compression force is equal to $(R_2 / L)\ell_1$ at a distance ℓ_1 from the edge of the diaphragm. Because wind and seismic forces can act in any direction, the axial forces in the collectors change from compression to tension and from tension to compression, so a collector must be designed for the most critical effects. Similar force diagrams can be obtained for wall 1.

Where a collector is assumed to be wider than the member of the LFRS element it frames in to, a part of the collector force is transferred directly into the member of the LFRS at its ends and a part is transferred through shear-friction along the length of the vertical element of the LFRS (see ACI Figure R12.5.4.1 for the case of a wall located at the edge of a diaphragm). Wider collectors are usually required in buildings where it is not practical or possible (from a design or constructability perspective) to provide collector elements concentric with the vertical elements of the LFRS. An example of where wider collectors may be necessary is in a building with relatively thin walls where only a narrow strip of slab is available to resist relatively large collector forces.

An effective slab width, b_{eff}, adjacent to the shear wall or frame is used to resist the collector forces. Requirements for b_{eff} are not covered in the code sections of ACI 318 or in ASCE/SEI 7. Reference 22 recommends using b_{eff} equal to the width of the vertical element of the LFRS plus a width on either side of the vertical element equal to one-half the contact length between the diaphragm and the vertical element (see Figure 9.10 for the effective collector width for wall 2 in Figure 9.8 where $\ell_1 = \ell_3$). ACI R12.5.4 recommends an effective slab width equal to basically the same as that in Reference 22.

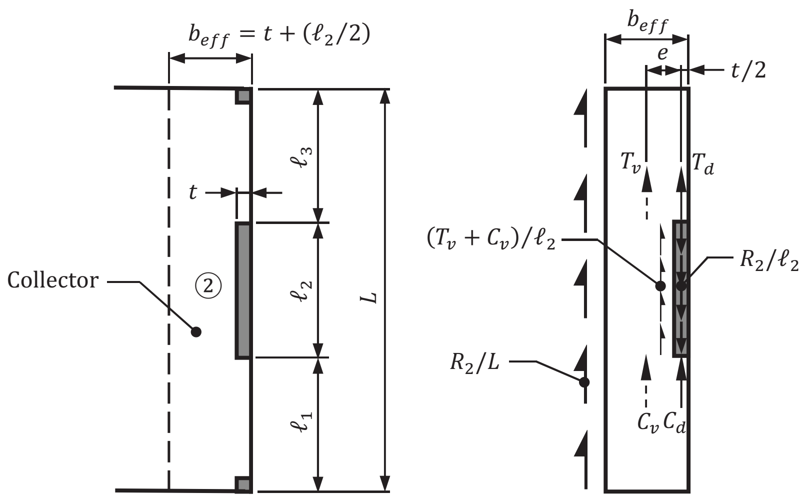


Figure 9.10 Effective slab width for collectors wider than the vertical elements of the LFRS.

Also illustrated in Figure 9.10 is the force transfer to wall 2. The tension and compression forces transferred directly into the ends of wall 2 are designated T_d and C_d, respectively. The tension force, T_d, is resisted by reinforcement in the wall parallel to the direction of analysis and the compression force, C_d, is resisted by the concrete. The remaining tension and compression forces T_v and C_v, respectively, are transferred through shear along ℓ_2, which is the length of wall 2. Shear-friction reinforcement is required to resist the total shear force $(R_2 + T_v + C_v)$; this reinforcement is placed perpendicular to the wall (see ACI Figure R12.5.4.1).

It is evident from Figure 9.10 that the forces T_v and C_v act at an eccentricity with respect to the centerline of wall 2. If it is assumed the reinforcement in the collector is uniformly distributed in the direction of analysis within b_{eff} (like that shown in ACI Figure R12.5.4.1), then the eccentricity, e, can be taken as $b_{eff} / 2$, and the in-plane bending moment resulting from this eccentricity and the portion of the collector force not transferred directly into the end of the wall must be considered in design (see Section 9.6.4 of this publication).

In general, the section of the collector concentric with the vertical element of the LFRS should be designed for a reasonable portion of the total collector force considering overall design and construction limitations. Reinforcing bar congestion at the ends of a shear wall, for example, is a practical limitation to consider when selecting the portion of the total tension force transferred directly into the end of a wall. In some cases, the total collector force will need to be resisted entirely by the shear in the slab adjacent to the vertical element of the LFRS.

Corrected Equivalent Beam Method with Spring Supports

This model is best suited for buildings with rigid diaphragms and lateral force-resisting systems with different stiffnesses. Effects of torsion are automatically accounted for in this method.

Diaphragm Forces and Center of Rigidity

The first step in determining the internal in-plane forces in a diaphragm based on the corrected equivalent beam model is to obtain the diaphragm forces transferred to the vertical elements of the LFRS. In multistory buildings, a three-dimensional model of the building is usually constructed with the appropriate lateral forces applied at each level over the height of the building. The forces in the vertical elements of the LFRS are obtained from an analysis of the structure using this model. In low-rise buildings with rigid diaphragms, the stiffnesses of the elements of the LFRS can be obtained using approximate analyses, which do not result in a significant loss of accuracy. Methods to determine approximate in-plane stiffnesses for walls and frames are given in Reference 21. Otherwise, stiffnesses can be obtained from a more refined analysis.

When utilizing an approximate method of analysis to determine the forces in the vertical elements of the LFRS, the location of the center of rigidity (CR) needs to be determined. The CR is the point on a floor/roof level where the equivalent story stiffness is assumed to be located. For buildings with rigid diaphragms, application of a lateral force through the CR produces only rigid body displacement of the story. Displacement and rotation occur where the lateral force is applied at any other point. The following equations can be used to locate the CR in the *x*-direction (x_{cr}) and *y*-direction (y_{cr}) from a selected origin (see Figure 9.11 where the origin is selected at the centroid of the column located at the bottom left of the framing system):

$$x_{cr} = \frac{\sum (k_i)_y x_i}{\sum (k_i)_y} \tag{9.2}$$

$$y_{cr} = \frac{\sum (k_i)_x y_i}{\sum (k_i)_x} \tag{9.3}$$

where $(k_i)_y$ = in-plane lateral stiffness of lateral-force-resisting element *i* in the *y*-direction

$(k_i)_x$ = in-plane lateral stiffness of lateral-force-resisting element *i* in the *x*-direction

x_i = distance in the *x*-direction from the origin to the centroid of lateral-force-resisting element *i*

y_i = distance in the *y*-direction from the origin to the centroid of lateral-force-resisting element *i*

All the elements of the LFRS parallel to the *y*- and *x*-directions of analyses are included in Equations (9.2) and (9.3), respectively; it is commonly assumed that out-of-plane resistance of a lateral-force-resisting element is negligible compared to its in-plane resistance. Thus, when determining the location of the CR in the *x*-direction by Equation (9.2), only the stiffnesses of walls 1 and 2 are considered. Note that a portion of wall 3 (that is, an effective flange width) may be included when determining the stiffness of wall 1. Similarly, when determining the location of the CR in the *y*-direction by Equation (9.3), only the stiffness of wall 3 (including an effective flange width of wall 1, if desired) and wall 4 are considered.

Once the location of the CR has been determined, the following equation can be used to determine $(R_i)_y$, which is the portion of the total story shear, V_y, resisted by element *i* considering both direct and torsional shear forces:

$$(R_i)_y = \frac{(k_i)_y}{\sum (k_i)_y} V_y \pm \frac{\overline{x}_i (k_i)_y}{\sum \overline{x}_i^2 (k_i)_y + \sum \overline{y}_i^2 (k_i)_x} V_y e_x \tag{9.4}$$

where $\overline{x}_i$ = perpendicular distance from the centroid of element *i* to the CR parallel to the *x*-axis

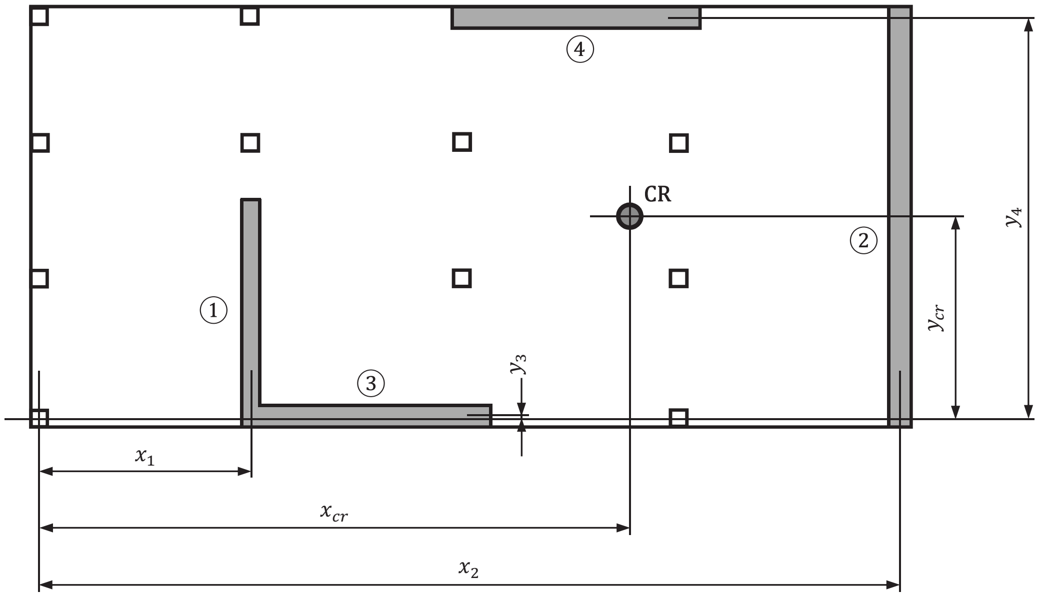


Figure 9.11 Location of center of rigidity (CR).

$\overline{y}_i$ = perpendicular distance from the centroid of element *i* to the CR parallel to the *y*-axis

e_x = perpendicular distance between V_y and the CR

Similarly, the following equation can be used to determine $(R_i)_x$, which is the portion of the total story shear, V_x, resisted by element *i* considering both direct and torsional shear forces:

$$(R_i)_x = \frac{(k_i)_x}{\sum (k_i)_x} V_x \pm \frac{\overline{y}_i (k_i)_x}{\sum \overline{x}_i^2 (k_i)_y + \sum \overline{y}_i^2 (k_i)_x} V_x e_y \qquad (9.5)$$

where e_y = perpendicular distance between V_x and the CR.

The first and second terms in these equations represent the direct shear forces and the torsional shear forces transferred to the vertical elements of the LFRS, respectively.

Once the diaphragm forces transferred to the vertical elements of the LFRS have been obtained using either an approximate or more refined analysis as outlined above, the second step is to determine an equivalent in-plane distributed load on the diaphragm that is in equilibrium with these forces (reactions). The distributed load is usually trapezoidal, which accounts for any torsional moments (see Figure 9.12). The loads w_1 and w_2 at each end of the equivalent beam can be obtained by using the equations for force equilibrium and moment equilibrium and then solving these two equations for the two unknowns w_1 and w_2.

For the rigid diaphragm in Figure 9.12, the force and moment equations of equilibrium in the direction of analysis are the following where moments are summed about the left edge of the diaphragm:

$$\frac{(w_1 + w_2)(b_1 + b_2 + b_3)}{2} = R_1 + R_2 + R_3 = V_y \qquad (9.6)$$

$$\frac{\left(\frac{w_1}{2} + w_2\right)(b_1 + b_2 + b_3)^2}{3} = R_1 b_1 + R_2(b_1 + b_2) + R_3(b_1 + b_2 + b_3) \qquad (9.7)$$

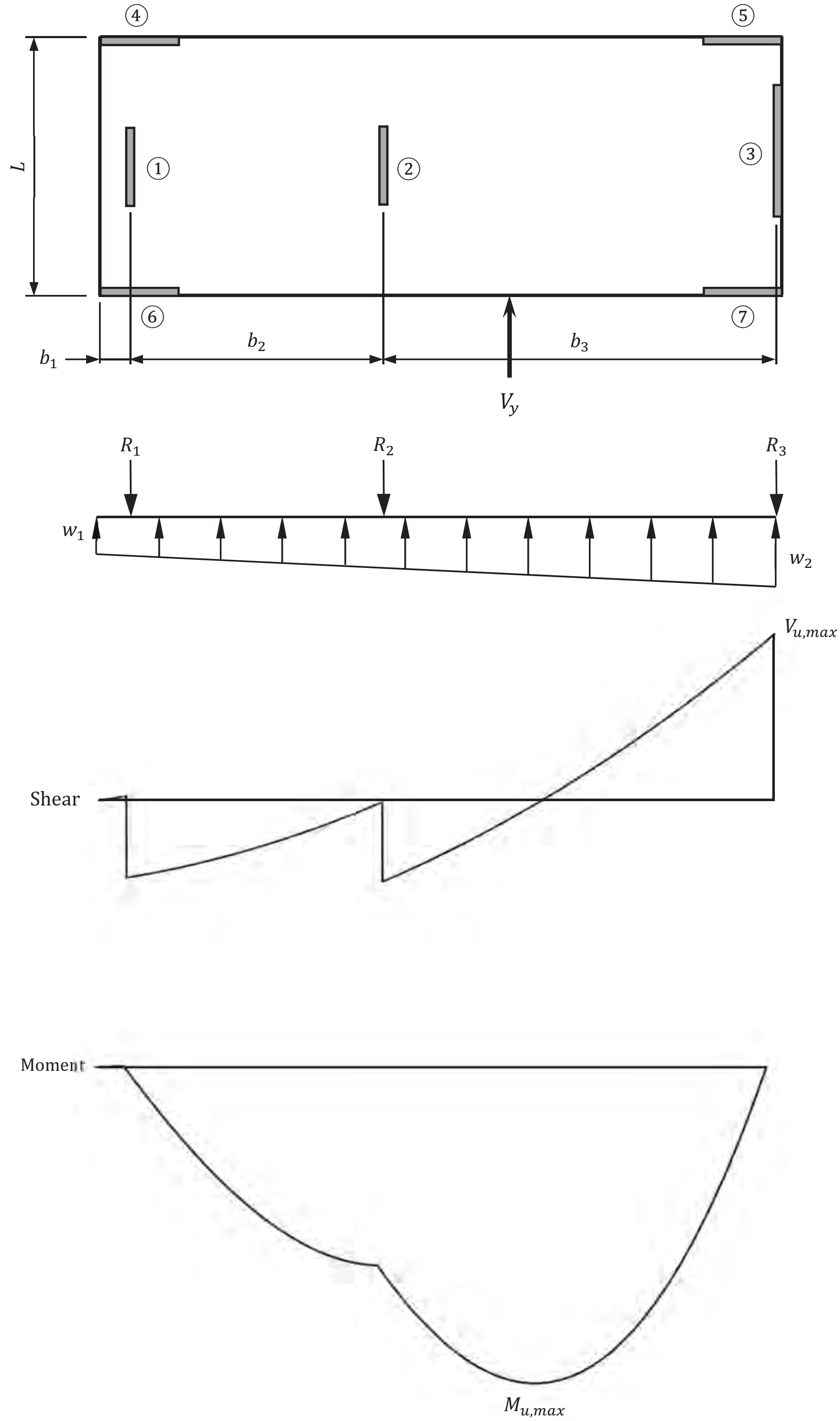


Figure 9.12 Equivalent distributed load, shear diagram, and moment diagram for a rigid diaphragm.

The resultant force of the trapezoidal distributed load is equal to the applied lateral force at that level obtained from analysis, which is reflected in Equation (9.6). The moment caused by the forces in the elements of the LFRS in the direction perpendicular to the in-plane force is often ignored in overall horizontal force distribution; it can be incorporated into the trapezoidal load, if desired.

Once w_1 and w_2 have been determined, the final step is to construct shear and moment diagrams for the diaphragm (see Figure 9.12). The shear diagram is used in (1) checking the design shear strength of the diaphragm, (2) designing the connections of the diaphragm to the vertical elements of the LFRS, and (3) determining the axial compression and tension forces in the collectors, if any. As noted previously, the moment diagram is used in determining the tension and compression forces in the chords, the former of which is needed to calculate the required area of chord reinforcement near the edges of the diaphragm.

Diaphragms with Openings

According to ACI 12.2.2, the effects of slab openings and slab voids must be considered in the analysis and design of diaphragms.

The corrected equivalent beam method can also be used for diaphragms with relatively large openings. For purposes of analysis, the diaphragm segments (commonly referred to as subdiaphragms) above and below the opening can be idealized as beams fixed at each end. It is assumed that the collector element on one side of the opening collects the uniform diaphragm shear on that side and transfers it to the subdiaphragms above and below the opening in proportion to their relative stiffness or mass. The collector on the other side of the opening then collects the shear from the subdiaphragms and transfers it to the portion of the diaphragm on that side of the opening. Thus, the loading on a subdiaphragm is based on the total applied force at that level and the relative stiffness or mass of the subdiaphragm. Secondary chord forces occur in each subdiaphragm due to local bending caused by this loading.

The diaphragm depicted in Figure 9.13 is the same as the one in Figure 9.12 but with a relatively large opening in it. The forces at each edge of the opening are designated w_L and w_R, which can be obtained from the overall trapezoidal force distribution.

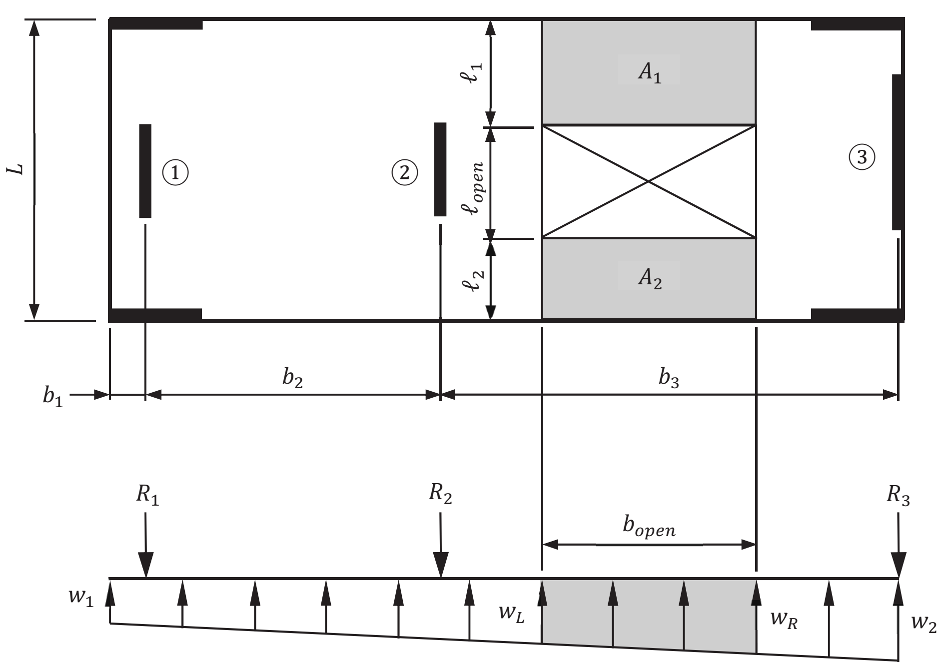


Figure 9.13 Diaphragm with a relatively large opening.

When a diaphragm is subjected to wind forces, the forces on the subdiaphragms above and below the opening can be approximately determined using the in-plane stiffness ratios of these subdiaphragms. The mass of the subdiaphragms is used to determine the forces when a diaphragm is subjected to seismic forces. The stiffness and mass ratios for the top and bottom subdiaphragms, f_{top} and f_{bot}, are given in Table 9.2. The mass ratios in Table 9.2 for use with seismic forces are based on the areas of the subdiaphragms, A_1 and A_2, where it is assumed that the diaphragm has the same thickness and material properties everywhere.

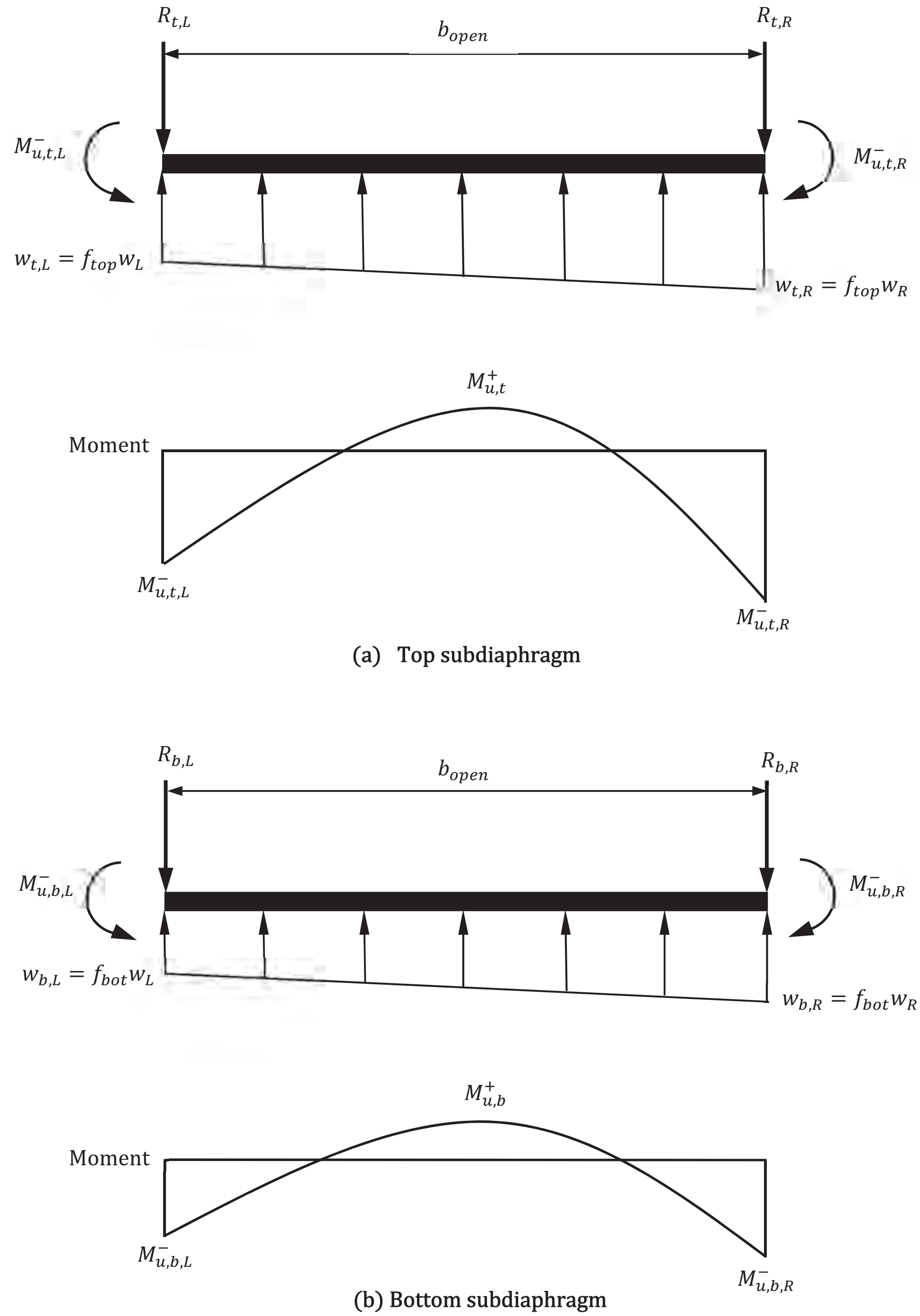


Figure 9.14 Free-body diagrams and moment diagrams for the subdiaphragms in Figure 9.13.

Table 9.2 Stiffness and Mass Ratios for Diaphragms with Openings

In-Plane Force	f_{top}	f_{bot}
Wind	$\frac{\ell_1^3}{\ell_1^3+\ell_2^3}$	$\frac{\ell_2^3}{\ell_1^3+\ell_2^3}$
Seismic	$\frac{A_1}{A_1+A_2}$	$\frac{A_2}{A_1+A_2}$

Free-body diagrams of the top and bottom subdiaphragms are given in Figure 9.14; as noted previously, the ends of these elements are assumed to be fixed. The moment diagrams in the figure can be obtained from statics where the appropriate f_{top} and f_{bot} determined from Table 9.2 are used to obtain the forces at each end of the subdiaphragm. The secondary tension (T_{u1} and T_{u2}) and compression (C_{u1} and C_{u2}) chord forces are determined using Equation (9.1) where $M_{u,max}$ is the maximum positive moment ($M_{u,t}^+$ and $M_{u,b}^+$) in each subdiaphragm.

The larger of the following two tension chord forces is used to determine the required area of chord reinforcement at the edges of the diaphragm where M_u is the bending moment in the diaphragm at the center of the opening (see Figure 9.15):

$$\text{Tension chord force} = \text{larger of} \begin{cases} T_u = \dfrac{M_{u,max}}{0.95L} \\ \\ T_{u,open} + T_{u,1} = \dfrac{M_u}{0.95L} + \dfrac{M_{u,t}^+}{0.95\ell_1} \end{cases} \tag{9.8}$$

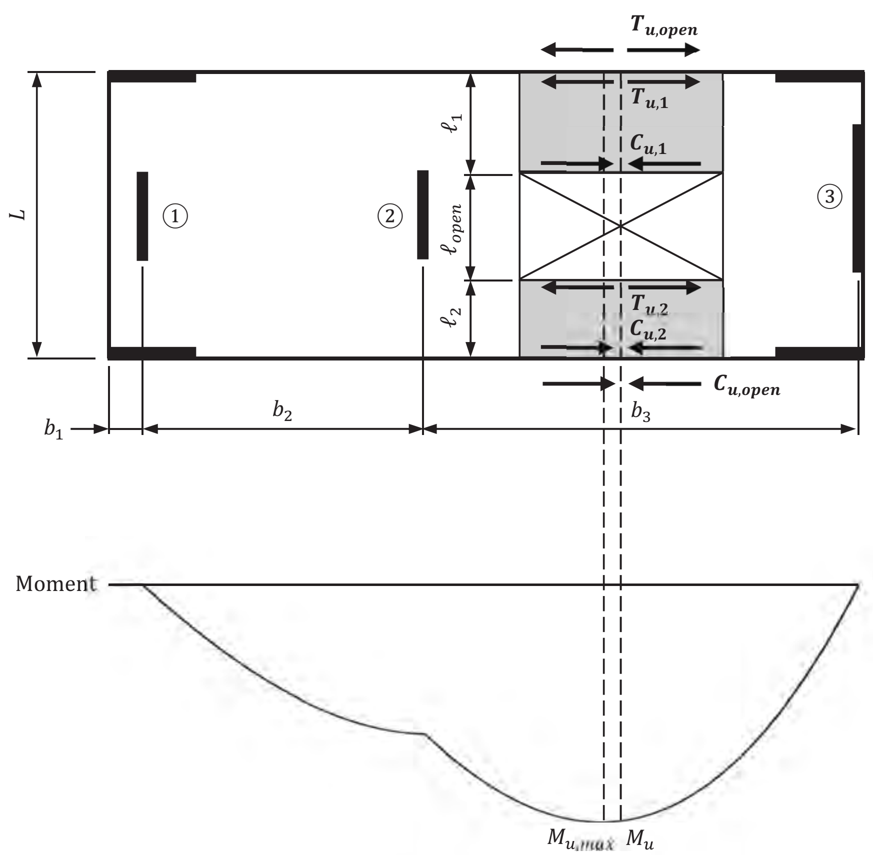


Figure 9.15 Determination of chord forces in a diaphragm with an opening.

Secondary chord forces also develop at the corners of the opening due to the negative moments at these locations. For the diaphragm in Figure 9.15, the tension chord force, $T_{u,2}$, along the bottom edge of the opening (that is, at the top of the bottom subdiaphragm) is equal to the larger of $M^{+}_{u,b,R} / (0.95\ell_2)$ and $M^{+}_{u,b,L} / (0.95\ell_2)$. Reinforcing bars must be provided along the entire top and bottom faces of the opening to resist the larger of the tension forces due to lateral forces acting in either direction.

9.4 Design Strength

9.4.1 General

For each applicable factored load combination in ACI Table 5.3.1, the applicable design strengths, ϕS_n, of diaphragms and connections must be greater than or equal to the applicable required strengths, U (ACI 12.5.1.1). As noted previously, interaction between load effects must be considered in design.

Strength reduction factors, ϕ, are determined in accordance with ACI 21.2. A summary of the strength reduction factors pertinent to the design of diaphragms and collectors subject to moment, axial force, or combined moment and axial force is given in Table 9.3 for members without spiral transverse reinforcement. The strain used to define a compression-controlled section, ε_{ty}, is equal to the specified yield strength of the reinforcement, f_y, divided by the modulus of elasticity of the reinforcing steel, E_s, which can be taken as 29,000,000 psi for any grade of reinforcement (ACI 20.2.2.2).

Table 9.3 Strength Reduction Factors, ϕ

Net Tensile Strain, ε_t	Classification	Strength Reduction Factor, ϕ*
$\varepsilon_t \leq \varepsilon_{ty}$	Compression-controlled	0.65
$\varepsilon_{ty} < \varepsilon_t \leq \varepsilon_{ty} + 0.003$	Transition**	$0.65 + \dfrac{0.25(\varepsilon_t - \varepsilon_{ty})}{0.003}$
$\varepsilon_t \geq \varepsilon_{ty} + 0.003$	Tension-controlled	0.90

**Applicable to transverse reinforcement other than spirals.*
***Sections classified as transition are permitted to use ϕ corresponding to compression-controlled sections.*

For diaphragms and collectors subjected to shear forces, $\phi = 0.75$ (ACI Table 21.2.1).

The design strength of a diaphragm depends on the type of model used to determine the internal force distribution (see Section 9.3.3 of this publication for permitted analysis models). Design strength requirements based on the model type are given in ACI 12.5.1.3 (see Table 9.4).

Table 9.4 Design Strength Requirements for Diaphragms

Model	Design Strength Requirements
Beam	ACI 12.5.2 through 12.5.4
Strut-and-tie	ACI 23.3
Finite element	ACI Chapter 22
Alternative	ACI 12.5.1.3(d)

9.4.2 Nominal Moment and Axial Force Strength

A diaphragm modeled as a beam is permitted to be designed for in-plane moment and axial force using the assumptions in ACI 22.3 (flexural strength) and 22.4 (axial strength or combined flexural and axial strength), which are the same assumptions used in the design of reinforced concrete beams, columns, and walls (ACI 12.5.2.1). This includes the assumption that strains vary linearly over the depth of the diaphragm when it is subjected to in-plane forces.

According to ACI 12.5.2.3, nonprestressed reinforcement and mechanical connectors used to resist tension chord forces in a diaphragm are permitted to be located within zones extending from the tension edge of the diaphragm a distance equal to 25 percent of the depth of the diaphragm in the direction of analysis (see ACI Figure R12.5.2.3). It is assumed that by placing the chord reinforcement within these zones, an essentially uniform shear flow over the depth of the diaphragm occurs, just like the case where the chord reinforcing bars are concentrated along the edges of the diaphragm. Where the depth of the diaphragm changes along its span, it is permitted to develop chord reinforcement into adjacent sections even where the reinforcement falls outside the 25 percent depth limit of the adjacent section.

9.4.3 Nominal Shear Strength

Nominal Shear Strength of Diaphragms

Nominal shear strength, V_n, for cast-in-place reinforced concrete diaphragms is determined by ACI Equation (12.5.3.3):

$$V_n = A_{cv}(2\lambda\sqrt{f_c'} + \rho_t f_y) \qquad (9.9)$$

where A_{cv} = gross area of the diaphragm (diaphragm thickness times width in the direction of analysis)

λ = modification factor accounting for the reduced mechanical properties of lightweight concrete

ρ_t = ratio of distributed slab transverse reinforcement to gross concrete area

The modification factor, λ, that reflects the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength is given in Table 9.5 based on equilibrium density and in Table 9.6 based on composition of aggregates in the concrete mix (see ACI 19.2.4).

Table 9.5 Values of λ Based on Equilibrium Density, w_c

Equilibrium Density, w_c	λ
$w_c \le 100$ lb/ft^3	0.75
100 lb/ft$^3 < w_c \le 135$ lb/ft^3	$0.0075w_c \le 1.0$
$w_c > 135$ lb/ft^3	1.0

Table 9.6 Values of λ Based on Composition of Aggregates

Concrete	Composition of Aggregates	λ
All-lightweight	Fine: ASTM C330 Coarse: ASTM C330	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330 and C33 Coarse: ASTM C330	0.75 to 0.85[(1)]

(table continued on next page)

Table 9.6 Values of λ Based on Composition of Aggregates (cont.)

Concrete	Composition of Aggregates	λ
Sand-lightweight	Fine: ASTM C33 Coarse: ASTM C330	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33 Coarse: Combination of ASTM C330 and ASTM C33	0.85 to 1.0(2)

(1) Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.
(2) Linear interpolation from 0.85 to 1.0 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of aggregate.

The transverse reinforcement ratio, ρ_t, is determined using the slab reinforcement parallel to the in-plane shear force. When calculating V_n by Equation (9.9), it is conservative to check the design shear strength requirements assuming ρ_t is equal to zero. Also, $\sqrt{f_c'}$ in this equation is limited to 100 psi (ACI 12.5.3.3).

The factored shear force, V_u, must not exceed the maximum design shear strength determined by ACI Equation (12.5.3.4):

$$V_u \leq \phi 8 A_{cv} \sqrt{f_c'} \tag{9.10}$$

Shear Transfer

Shear transfer between diaphragms, collectors, and vertical elements of the LFRS must be satisfied using the shear-friction provisions of ACI 22.9 or by mechanical connectors or dowels (ACI 12.5.3.7). In cast-in-place construction, shear transfer requirements must typically be satisfied at construction joints between the elements.

Where shear-friction provisions are used, the nominal shear strength across the assumed shear plane is determined by ACI Equation (22.9.4.2):

$$V_n = \mu A_{vf} f_y \tag{9.11}$$

In this equation, μ is the coefficient of friction, which is obtained from ACI Table 22.9.4.2 (see Table 9.7) and A_{vf} is the area of shear-friction reinforcement crossing the shear plane with the reinforcing bars oriented perpendicular to that plane.

Table 9.7 Coefficient of Friction, μ

Contact Surface Condition	μ
Concrete placed monolithically	1.4λ
Concrete placed against hardened concrete that is clean, free of laitance, and intentionally roughened to a full amplitude of approximately ¼ in.	1.0λ
Concrete placed against hardened concrete that is clean, free of laitance, and not intentionally roughened	0.6λ

Upper limits on shear-friction strength are given in ACI Table 22.9.4.4 (see Table 9.8). The term A_c is the area of concrete resisting V_u, which is equal to the gross cross-sectional area of the diaphragm in the direction of analysis.

Table 9.8 Maximum V_n Across the Assumed Shear Plane

Contact Surface Condition	Maximum $V_n = V_u / \phi$*
Normalweight concrete placed monolithically or placed against hardened concrete intentionally roughened to a full amplitude of approximately ¼ in.	Least of $\begin{cases} 0.2f'_c A_c \\ (480 + 0.08f'_c)A_c \\ 1{,}600A_c \end{cases}$
Other cases	Least of $\begin{cases} 0.2f'_c A_c \\ 800A_c \end{cases}$

*A_c = *area of concrete resisting* V_u

9.4.4 Collectors

Collectors are to be designed as tension members, compression members, or both in accordance with the design provisions for axial strength or combined flexural and axial strength in ACI 22.4 (ACI 12.5.4.2). In cases where collectors are subjected to relatively significant effects from gravity forces in addition to those from tension and compression axial forces due to in-plane lateral forces, design strength interaction diagrams are often constructed to check the adequacy of a collector for all applicable load combinations, just like for columns. Unlike typical columns, both the axial compression and tension parts of the interaction diagram are generally needed for the design of collectors.

In addition to the strength requirements noted above, the shear strength requirements in ACI 7.5.3 for one-way slabs and in ACI 8.5.3 for two-way slabs must be satisfied where a portion of a slab is used as a collector. Where beams are utilized as collectors, the strength requirements for shear or for combined shear and torsion in ACI 9.5.3 and 9.5.4 must also be satisfied.

9.5 Reinforcement Limits

The area of reinforcement in diaphragms must be at least equal to the area corresponding to shrinkage and temperature reinforcement given in ACI 24.4, which is equal to $0.0018A_g$ (ACI 12.6.1); this is also the minimum area of flexural reinforcement for one-way slabs (see ACI 7.6.1.1 and 12.6.2). For two-way slabs, the minimum flexural reinforcement is also equal to $0.0018A_g$, except in the effective slab width specified in ACI 8.4.2.2.3 where the minimum area of reinforcement is equal to the larger of $0.0018A_g$ and the area determined by ACI Equation (8.6.1.2) in cases where the factored two-way shear stress exceeds the value given in ACI 8.6.1.2.

Reinforcement required to resist in-plane forces in diaphragms must be in addition to the reinforcement required to resist other load effects, such as those from gravity (ACI 12.6.3). Shrinkage and temperature reinforcement, however, is permitted to also resist diaphragm in-plane forces.

9.6 Determination of Required Reinforcement

9.6.1 Chord Reinforcement

The maximum tension chord force, T_u, which is determined by Equation (9.1), must be resisted by reinforcement perpendicular to the direction of the applied in-plane force. At any location within the diaphragm, T_u must be less than or equal to the design tension strength of the chord reinforcement:

$$T_u \le \phi T_n = \frac{\phi A_{s(\text{chord})}}{f_y} \tag{9.12}$$

In this equation, $\phi = 0.90$ for reinforcing bars in tension and $A_{s(\text{chord})}$ is the required area of chord reinforcement. For a given T_u, Equation (9.12) can be used to determine $A_{s(\text{chord})}$:

$$A_{s(\text{chord})} \geq \frac{T_u}{\phi f_y} \tag{9.13}$$

For diaphragms with openings, the maximum tension chord force is determined by Equation (9.8) and that force is used in Equation (9.13) to determine $A_{s(\text{chord})}$ at the edges of the diaphragm (see Figure 9.15). The required area of chord reinforcement along the edges of the opening perpendicular to the direction of analysis can also be determined by Equation (9.13) using the subdiaphragm tension chord forces $T_{u,1}$ and $T_{u,2}$ (see Figure 9.15 and Section 9.3.3 of this publication).

Because wind and seismic forces can act in any direction, chord reinforcement is required along all edges of diaphragms and openings.

For diaphragms in buildings assigned to Seismic Design Category (SDC) A, B, or C, diaphragm chords subjected to compression forces need not be designed and detailed as columns (ACI R12.5.4.2). However, in cases where diaphragm boundaries (edges of a diaphragm or edges of an opening in a diaphragm) are subjected to relatively large compression forces compared to the axial compression strength of the boundary element, transverse reinforcement, such as hoops, should be used to confine the concrete. Such transverse reinforcement is usually feasible where there are beams along the diaphragm boundaries; without beams, the hoops must be developed in the slab, which is not always possible, especially for relatively thin slabs. In such cases, a thicker slab may be required if beams are not an option.

9.6.2 Diaphragm Shear Reinforcement

The following requirements must be satisfied for design shear strength of diaphragms (see ACI 12.5.3 and Section 9.4.3 of this publication):

$$V_u \leq \text{ lesser of } \begin{cases} \phi A_{cv}(2\lambda\sqrt{f'_c} + \rho_t f_y) \\ \\ \phi 8 A_{cv}\sqrt{f'_c} \end{cases} \tag{9.14}$$

Required shear strength, V_u, due to in-plane forces is determined using one of the models in Section 9.3.3 of this publication. For diaphragms modeled as beams with chord reinforcement concentrated near it edges, the factored shear flow, v_u, is uniformly distributed over the depth of the diaphragm (see Figure 9.8).

The shear reinforcement ratio, ρ_t, is equal to the area of the uniformly distributed slab reinforcement oriented parallel to the shear force divided by the gross area of the slab perpendicular to that reinforcement. Equation (9.9) can be solved for the required ρ_t :

$$\rho_t \geq \frac{(V_u \,/\, \phi A_{cv}) - 2\lambda\sqrt{f'_c}}{f_y} \tag{9.15}$$

It is evident from Equation (9.15) that shear reinforcement is not required where $2\lambda\sqrt{f'_c} \geq (V_u \,/\, \phi A_{cv})$. Providing a slab thickness adequate for serviceability or, where applicable, for two-way shear is usually adequate for in-plane shear strength as well.

Where in-plane shear reinforcement is required, it is usually combined with the flexural reinforcement in the slab in the direction of analysis (where two layers of flexural reinforcement are provided in a slab, the combination usually occurs with the bottom reinforcement). Thus, the total area of reinforcement in such cases is equal to the area of shear reinforcement determined by Equation (9.15) plus the area of flexural reinforcement. Combining the reinforcement in this fashion generally results in simpler detailing and more efficient placement of the reinforcing bars in the field.

9.6.3 Shear Transfer Reinforcement

Shear transfer reinforcement must be provided between diaphragms, collectors, and vertical elements of the LFRS. In cast-in-place construction, shear transfer reinforcement is provided mainly at construction joints between the elements. The required area of shear-friction reinforcement, A_{vf}, which depends on the overall layout of the elements, the magnitude of the factored shear force at the location of interest, the width of the collector with respect to the width of the vertical element of the LFRS it frames into, and the construction sequence, can be determined by Equation (9.11):

$$A_{vf} \geq \frac{V_u / \ell}{\phi \mu f_y} \tag{9.16}$$

In this equation, V_u is the factored shear force at the interface of interest, ℓ is the length of the interface of interest, and μ is determined by Table 9.7 considering the contact surface condition at the interface. Note that A_{vf} has the units of square inches per unit length in Equation (9.16).

Shear transfer reinforcement is typically required at the interface between the diaphragm and the vertical elements of the LFRS and at the interface between the diaphragm and the collectors (if any). The equations in Table 9.9, which are based on Equation (9.16), can be used to determine the required area of shear-friction reinforcement at these locations.

Table 9.9 Required Shear Transfer Reinforcement in Diaphragms

Case	Shear Transfer Interface	Required A_{vf}
1	Between the diaphragm and the vertical elements of the LFRS	$A_{vf} \geq \frac{V_{u1} / \ell_{LFRS}}{\phi \mu f_y}$
2	Between the diaphragm and the collectors	$A_{vf} \geq \frac{R_i / L}{1.4 \phi \lambda f_y}$

In Case 1, V_{u1} is equal to the reaction R_i in the vertical element of the LFRS where there are no collectors or where a collector is required and the collector has the same width as the vertical element of the LFRS it frames in to. Where a collector is wider than the vertical element, $V_{u1} = R_i + T_v + C_v$ (see Section 9.3.3 of this publication). Also, ℓ_{LFRS} is the length of the vertical element of the LFRS in the direction of analysis (for example, the length of the structural wall or the total length of the moment-resisting frames minus the length of any openings in the diaphragm adjacent to these elements). For buildings assigned to SDC A, B, or C, μ is equal to 1.0λ or 0.6λ depending on the contact surface condition provided at the interface (see Table 9.7).

In Case 2, L is the overall depth of the diaphragm in the direction of analysis and μ is equal to 1.4λ for concrete placed monolithically because the collector is either part of the slab or, where beams are used as collectors, the beams and slab are placed at the same time (that is, there is no cold joint between these two elements).

The construction method has an impact on the determination of shear transfer reinforcement. The first type of construction method is illustrated in Figure 9.16 where structural walls are utilized as the vertical elements of the LFRS. In this method, the wall below is constructed first to the underside of the slab; the slab and then the wall above are subsequently constructed in that order. Cold joints typically occur at the top and bottom surfaces of the slab (diaphragm) along the length of the wall, ℓ_w. The shear-friction reinforcement in Figure 9.16 consists of dowel bars connecting the slab to the wall below. In this case, the required area of the dowel bars, A_{vf}, is calculated by the equation in Table 9.9 for Case 1: $A_{vf} = (V_{u1} / \ell_w) / \phi \mu f_y$.

The second type of construction method is shown in Figure 9.17. In this case, the wall is constructed ahead of the slab (for example, where slip forms are used to construct the wall). A cold joint occurs at the face of the wall and shear transfer reinforcement must be provided at that joint. Proprietary form saver systems are commonly used in this type of construction mainly for economic reasons. The shear-friction reinforcement in Figure 9.17 consists of

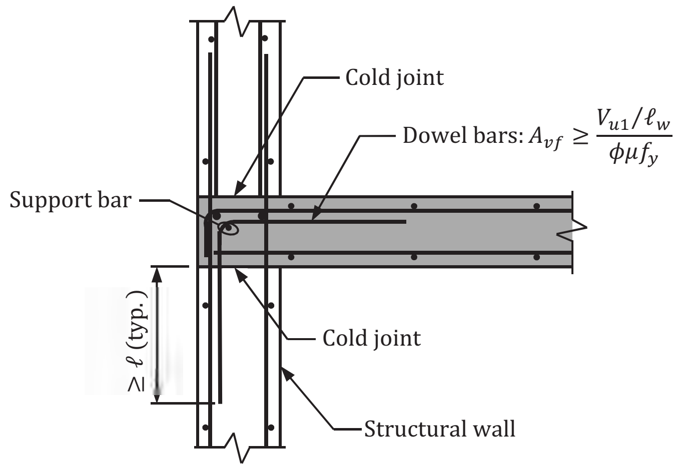


Figure 9.16 Shear transfer between a diaphragm and a structural wall utilizing dowel bars – Construction Method 1.

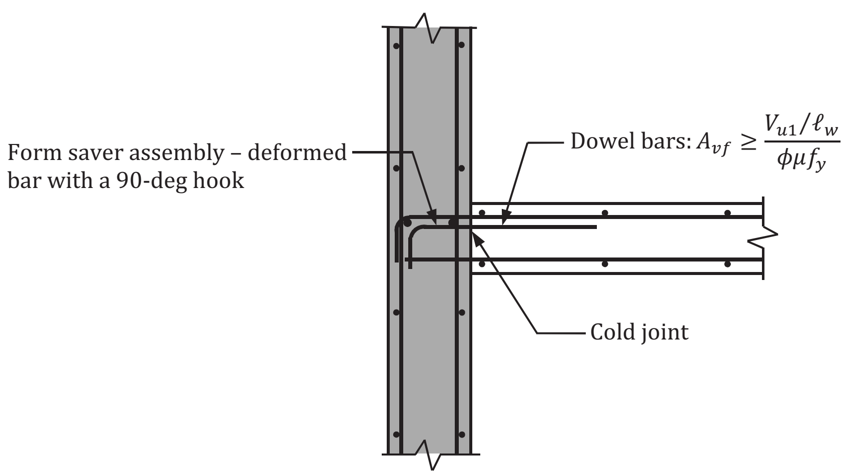


Figure 9.17 Shear transfer between a diaphragm and a structural wall utilizing dowel bars – Construction Method 2.

dowel bars, each of which requires its own form saver. The required area of the dowel bars, A_{vf}, is calculated by the equation in Table 9.9 for Case 1: $A_{vf} = (V_{u1} / \ell_w) / \phi\mu f_y$.

Methods utilizing shrinkage and temperature reinforcement or the bottom reinforcement in a slab as shear-friction reinforcement are given in Reference 21.

The above discussion is equally valid for moment-resisting frames that are utilized as part of the LFRS instead of, or in combination with, structural walls.

9.6.4 Reinforcement Due to Eccentricity of Collector Forces

For collectors wider than the width of the vertical elements of the LFRS that they frame in to, an in-plane bending moment is generated in a diaphragm adjacent to the vertical elements due in part to the forces T_v and C_v, which act at an eccentricity from the centerline of the vertical elements (see Figure 9.10). A method to determine the required reinforcement to resist this bending moment is given below, which is based on the method given in Reference 22.

Illustrated in Figure 9.18 is a portion of a diaphragm with a structural wall that is part of the LFRS located at its edge and a collector wider than the thickness of the wall. The internal forces acting on the free-body diagram of the diaphragm adjacent to the wall are indicated in the figure based on some simplifying, conservative assumptions. The eccentric bending moment, M_e, due to these internal forces can be approximated by the following equation:

$$M_e = (T_v + C_v)e - V_s\ell_w \tag{9.17}$$

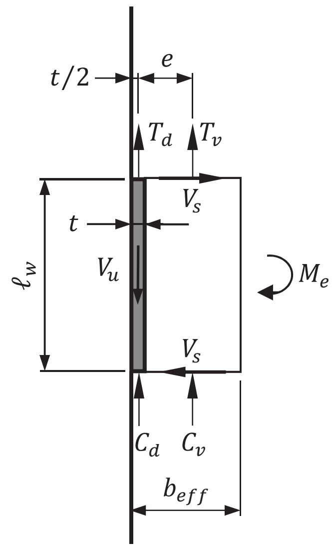


Figure 9.18 Internal forces in a diaphragm where the collector is wider than the vertical element of the LFRS.

In this equation, V_s is the shear strength of the diaphragm based on only the reinforcing bars in the slab (that is, $V_s = A_{cv}\rho_t f_y$). The nominal strength of the concrete, V_c, is not included because tension forces are present, which makes $V_c = 0$.

The area of tension reinforcement, $A_{s(\text{ecc})}$, required to resist M_e can be determined by the following equation:

$$A_{s(\text{ecc})} = \frac{M_e \,/\, 0.95\ell_w}{\phi f_y} \tag{9.18}$$

where $\phi = 0.90$ for reinforcing bars in tension. This reinforcement is placed perpendicular to the face of the wall at both ends and must be developed into the slab and into the wall (see Figure 9.19).

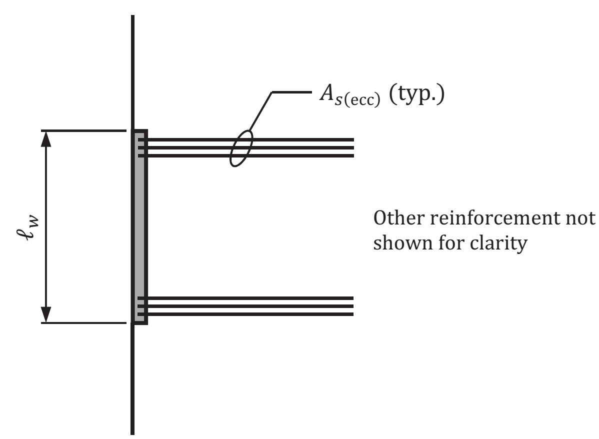


Figure 9.19 Required tension reinforcement where the collector is wider than the vertical element of the LFRS.

9.6.5 Collector Reinforcement

Overview

In general, collectors must be designed as tension members, compression members, or both in accordance with the provisions of ACI 22.4 for members subjected to axial strength or combined flexural and axial strength (ACI 12.5.4.2).

The types and amount of reinforcement and the detailing requirements that must be satisfied depend mostly on (1) the type of collector (that is, a collector that is a portion of the slab or is a beam) and (2) the width of the collector with respect to the width of the vertical element of the LFRS that it frames in to.

Slabs

Where slabs are utilized as collectors, the following equation can be used to determine the required area of longitudinal reinforcement, $A_{s(\text{collector})}$, in a collector to resist the factored axal tension force, T_u :

$$A_{s(\text{collector})} = \frac{T_u}{\phi f_y} \tag{9.19}$$

where $\phi = 0.90$ for reinforcing bars in tension. This reinforcement is provided in addition to the flexural reinforcement in the slab required for gravity loads. A typical detail where the collector is the same width as the vertical element of the LFRS is given in Figure 9.20.

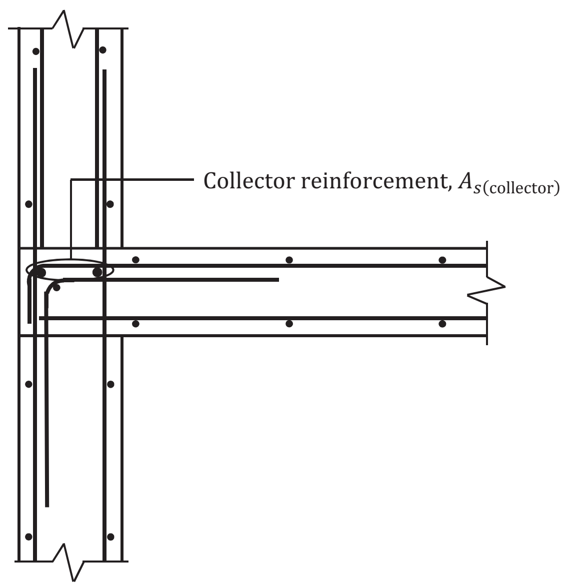


Figure 9.20 Longitudinal reinforcement in a collector that has the same width as the vertical element of the LFRS.

For the case of axial compression forces, C_u must be less than or equal to the design axial compression strength of the section at zero eccentricity, ϕP_o :

$$C_u \le \phi P_o = \phi[0.85 f'_c(A_g - A_s) + f_y A_s] \tag{9.20}$$

In this equation, $\phi = 0.65$ for compression-controlled sections (ACI Table 21.2.2), A_g is equal to the thickness of the slab times the width of the collector, and A_s is the area of longitudinal reinforcement in A_g. The design strength requirements for axial compression rarely govern.

For slabs wider than the vertical element of the LFRS, the total area of longitudinal tension reinforcement, $A_{s(\text{collector})}$, required to resist T_u can also be determined by Equation (9.19). The axial tension force T_d and the corresponding required area of tension reinforcement, $A_{s(d)}$, in the width of the vertical element of the LFRS is selected considering design and construction limitations. Once the size and number of reinforcing bars are chosen based on $A_{s(d)}$, the area of reinforcement, $A_{s(v)}$, required in the effective slab width outside the width of the vertical elements is equal to $A_{s(\text{collector})}$ minus the area of longitudinal reinforcement provided in the width of the vertical elements of the LFRS. The reinforcing bars corresponding to $A_{s(v)}$ are usually uniformly distributed over the effective width of the collector (see Figure 9.21).

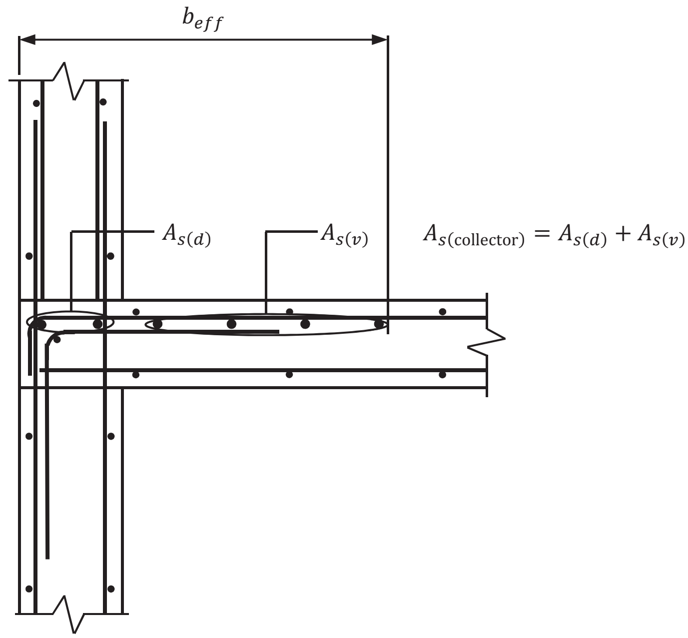


Figure 9.21 Longitudinal reinforcement in a collector wider than the vertical element of the LFRS.

For collectors wider than the vertical element of the LFRS, the design axial strength defined in Equation (9.20) must be checked for C_d and C_v using the appropriate A_g and A_s for each segment.

Beams

Beams designated as collector elements must be designed for the combined effects from flexure, shear, torsion, axial compression forces, and axial tension forces due to gravity and lateral loads. Applicable design and detailing provisions in ACI Chapter 9 must be satisfied along with the applicable detailing requirements.

Longitudinal reinforcement must be determined for the combined effects due to flexure, torsion, and axial compression and tension forces, and transverse reinforcement must be determined for combined effects due to shear and torsion, where applicable. Generally, a design strength interaction diagram is constructed, which includes both the axial compression and tension portions, to determine whether the collector is adequate for all the combined factored flexure and axial load effects. The information provided above for slabs wider than the vertical elements of the LFRS is also applicable to beams.

9.7 Reinforcement Detailing

Requirements for concrete cover, development lengths, splices, bar spacing, and reinforcement detailing for diaphragms and collectors are given in ACI 12.7. A summary of these requirements is given in Table 9.10.

Table 9.10 Reinforcement Detailing Requirements for Diaphragms and Collectors

Requirement	ACI Section No.	
Concrete cover	20.5.1	
Development lengths	25.4	
Splices	25.5	
Spacing	Minimum	25.2
	Maximum	12.7.2.2
Reinforcement detailing	One-way slabs	7.7
	Two-way slabs	8.7

At critical sections in diaphragms and collectors, tension and compression forces in the reinforcement must be adequately developed on each side of those sections (ACI 12.7.3.2). Tension reinforcement must extend beyond the point at which it is no longer required a distance equal to at least the tension development length, ℓ_d, determined in accordance with ACI 25.4.2 except at diaphragm edges and at expansion joints (ACI 12.7.3.3).

Longitudinal reinforcement in collectors must extend into the vertical elements of the LFRS a length equal to at least the greater of the two lengths given in ACI 12.5.4.3. This requirement is illustrated in ACI Figure R12.5.4.3. Additional information on detailing requirements for diaphragms and collectors is given in Reference 21.

9.8 Design Procedure

The design procedure in Figure 9.22 can be used in the design and detailing of reinforced concrete diaphragms. Included in the figure are the section numbers and equation numbers where specific information on that topic in this chapter can be found. Comprehensive flowcharts associated with each of the steps in Figure 9.22 are given in Reference 21.

9.9 Examples

9.9.1 Example 9.1 – Determination of Diaphragm In-Plane Forces: Building #1 (Framing Option B), SDC A, Collectors Not Required

Determine the diaphragm forces in the slab at the second-floor level of Building #1, Framing Option B for wind forces in the north-south direction assuming all the frames along column lines 1 and 7 are part of the LFRS (see Figure 1.1). Also assume an 8.5-in. thick slab, 28.0 by 24.0 in. beams, and 24.0 by 24.0 in. columns.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the diaphragm design in-plane forces

From Table 3.10 in Example 3.1, the wind force in the north-south direction at the second-floor level is equal to 51.3 kips. This force is applied at the centroid of the building face at this level.

Step 2 – Determine the classification of the diaphragm Sect. 9.3.3

For forces in the north-south direction, the span-to-depth ratio is equal to $150.0 / 94.0 = 1.6$.

Because the span-to-depth ratio is less than 2.0, the diaphragm can be classified as rigid. ASCE/SEI 26.2

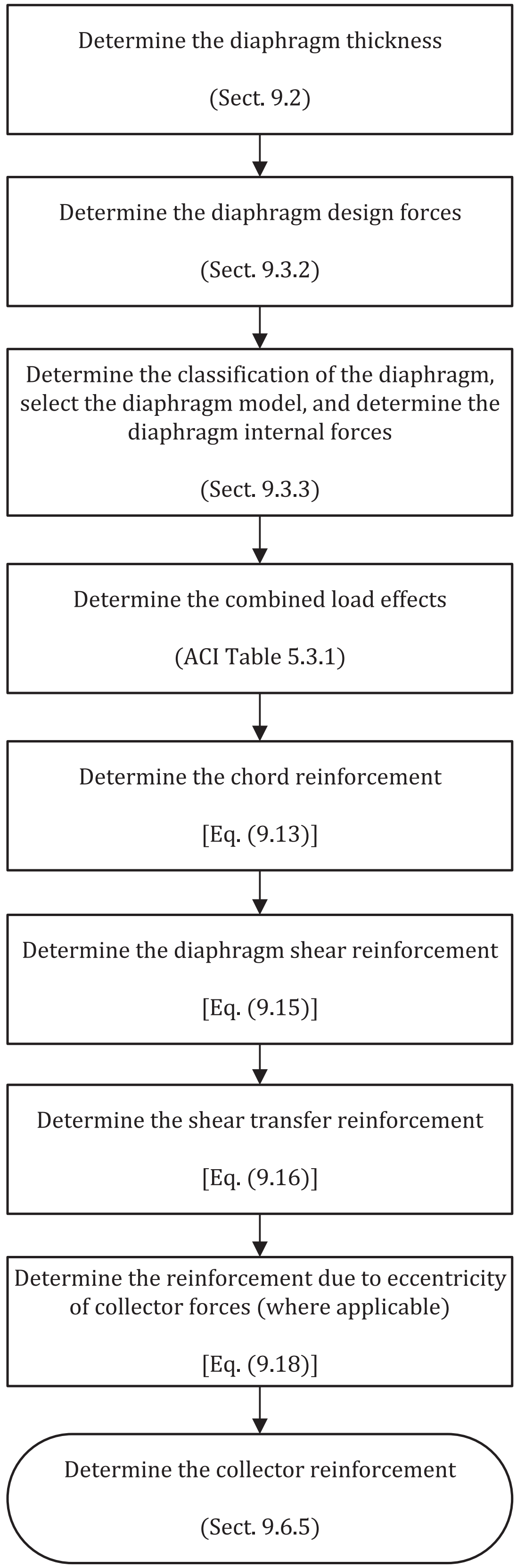


Figure 9.22 Design procedure for diaphragms.

Step 3 – Select the diaphragm model Table 9.1

Because this building has no irregularities and is not subjected to any transfer forces, and because the frames along column lines 1 and 7 have the same lateral stiffness (that is, the sizes of the beams and columns are the same in both frames), an equivalent beam model with rigid supports is selected to determine the internal forces for this diaphragm.

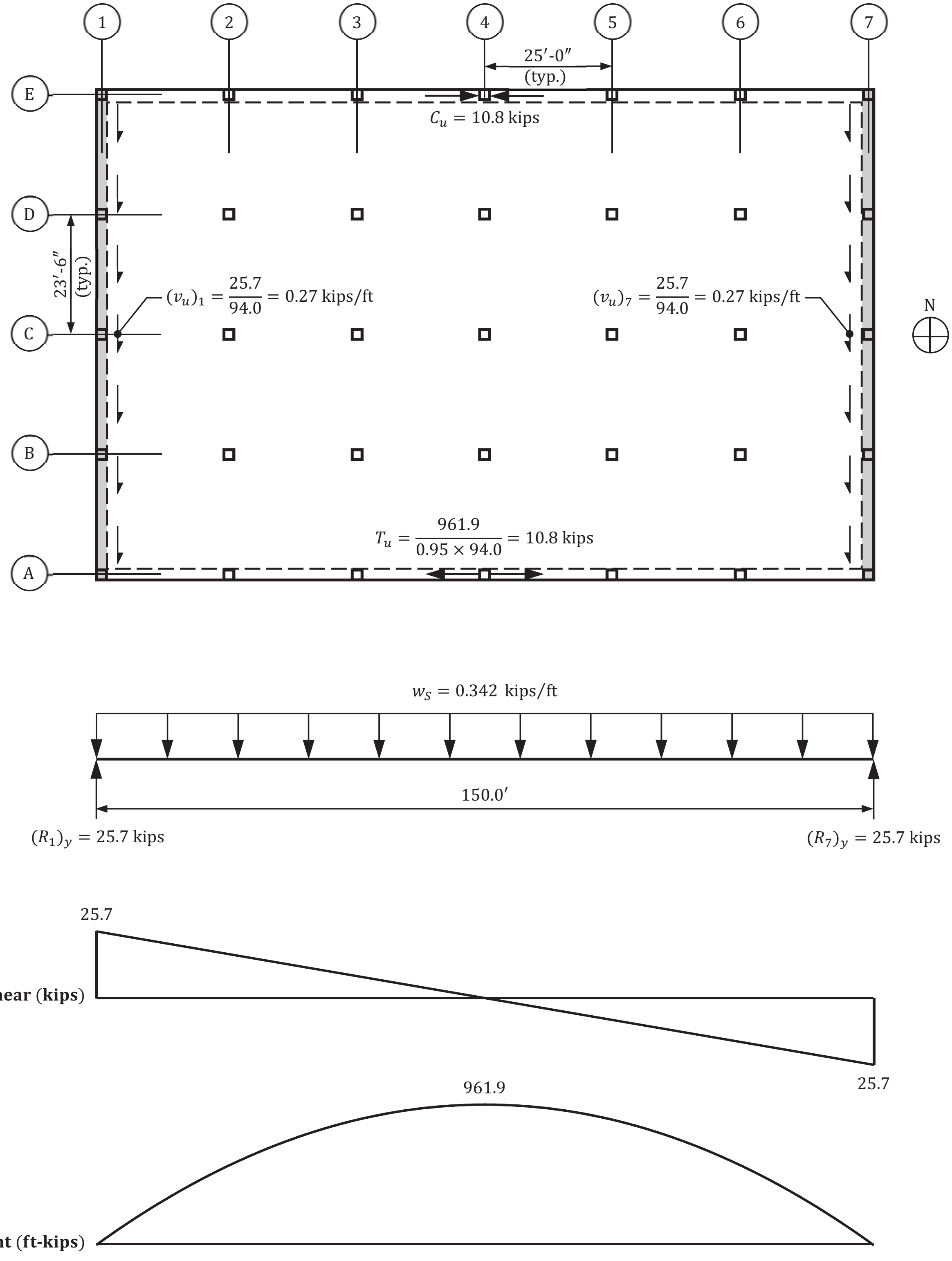


Figure 9.23 Equivalent beam model for the diaphragm in Example 9.1 for wind in the south direction.

Step 4 – Determine the diaphragm internal forces Sect. 9.3.3

The equivalent beam model for the diaphragm at the second-floor level is given in Figure 9.23 for wind in the south direction. Because the moment-resisting frames on column lines 1 and 7 are identical, the CR is located $150.0 / 2 = 75.0$ ft from either end of the diaphragm, which coincides with the location where the wind force is applied at this level. Thus, there is no inherent torsional moment, which means the diaphragm is subjected to an equivalent uniformly distributed wind load equal to $w_S = 51.3 / 150.0 = 0.342$ kips/ft.

- Reactions in each of the moment-resisting frames along column lines 1 and 7

  $(R_1)_y = (R_7)_y = 0.342 \times 150.0 / 2 = 25.7$ kips

  The shear and moment diagrams are also given in Figure 9.23.

- Chord forces at the edges of the diaphragm perpendicular to the direction of analysis

$$T_u = C_u = \frac{M_{u,max}}{d} = \frac{961.9}{0.95 \times 94.0} = 10.8 \text{ kips} \qquad \text{Eq. (9.1)}$$

- Maximum unit shear forces in the diaphragm

$$(v_u)_1 = (v_u)_7 = \frac{25.7}{94.0} = 0.27 \text{ kips/ft}$$

Collectors are not required because the moment-resisting frames extend the entire depth of the diaphragm in the direction of analysis.

9.9.2 Example 9.2 – Determination of Diaphragm Reinforcement: Building #1 (Framing Option B), SDC A, Collectors Not Required

Determine the required diaphragm reinforcement in the slab at the second-floor level of Building #1, Framing Option B for wind forces in the north-south direction assuming all the frames along column lines 1 and 7 are part of the LFRS (see Figure 1.1). Also assume an 8.5-in. thick slab, normalweight concrete with $f'_c = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 9.1.

Step 1 – Determine the chord reinforcement

From Example 9.1, $T_u = 10.8$ kips.

$$A_{s(\text{chord})} = \frac{T_u}{\phi f_y} = \frac{10.8}{0.9 \times 60} = 0.20 \text{ in.}^2 \qquad \text{Eq. (9.13)}$$

At column lines A and E, provide 1-#4 bar located just outside of the cross-section of the beams in the moment frames (see Figure 9.24).

Step 2 – Determine the diaphragm shear reinforcement

The largest factored unit shear forces occur along column lines 1 and 7 and are equal to 0.27 kips/ft (see Example 9.1).

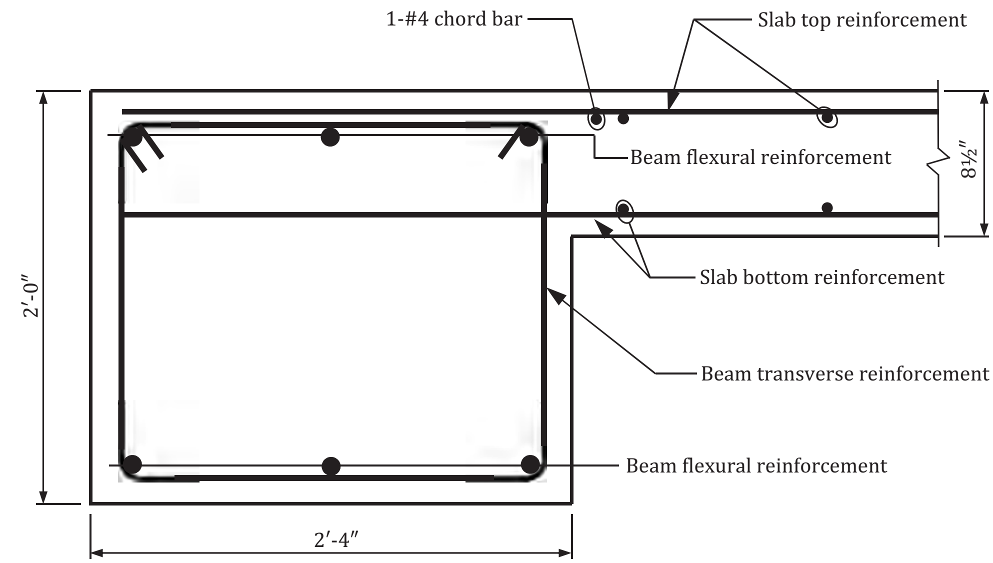


Figure 9.24 Location of chord reinforcement in the diaphragm in Example 9.2 for wind forces in the north-south direction.

Check shear strength requirements assuming $\rho_t = 0$: Eq. (9.14)

$$V_u = 0.27 \text{ kips/ft} < \text{lesser of } \begin{cases} \phi A_{cv}(2\lambda\sqrt{f_c'} + \rho_t f_y) = (0.75 \times 8.5 \times 12.0) \times [(2 \times 1.0 \times \sqrt{4{,}000}) + 0] / 1{,}000 = 9.7 \text{ kips/ft} \\ \\ \phi 8 A_{cv}\sqrt{f_c'} = 0.75 \times 8 \times 8.5 \times 12.0 \times \sqrt{4{,}000} / 1{,}000 = 38.7 \text{ kips/ft} \end{cases}$$

Therefore, no shear reinforcement is required to satisfy shear strength requirements.

Step 3 – Determine the shear transfer reinforcement

Only shear transfer reinforcement between the diaphragm and the moment-resisting frames is required:

$$A_{vf} = \frac{(v_u)_1}{\phi \mu f_y} = \frac{0.27}{0.75 \times 1.4 \times 1.0 \times 60} = 0.004 \text{ in.}^2/\text{ft}$$ Eq. (9.16)

A value of $\mu = 1.4\lambda$ for monolithic concrete is used because the concrete for the slab and beams in the moment-resisting frames are placed at the same time (see Table 9.7).

Because the required area of shear-friction reinforcement is very small, it is safe to assume that the bottom flexural reinforcement in the slab can be used as the shear transfer reinforcement between the diaphragm and the moment-resisting frames along column lines 1 and 7.

9.9.3 Example 9.3 – Determination of Diaphragm In-Plane Forces: Building #1 (Framing Option B), SDC A, Collectors Required, Collector Width the Same as the Width of the Vertical Elements of the LFRS

Determine the diaphragm forces in the slab at the second-floor level of Building #1, Framing Option B for wind forces in the north-south direction assuming only the frames between column lines B and D along column lines 1 and 7 are part of the LFRS (see Figure 1.1). It is assumed that the width of the collectors (beams) are equal to the widths of the beams in the moment-resisting frames. Also assume an 8.5-in. thick slab, 28.0 by 24.0 in. beams, and 24.0 by 24.0 in. columns.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the diaphragm design in-plane forces

From Table 3.10 in Example 3.1, the wind force in the north-south direction at the second-floor level is equal to 51.3 kips. This force is applied at the centroid of the building face at this level.

Step 2 – Determine the classification of the diaphragm — Sect. 9.3.3

For forces in the north-south direction, the span-to-depth ratio is equal to $150.0 / 94.0 = 1.6$.

Because the span-to-depth ratio is less than 2.0, the diaphragm can be classified as rigid. — ASCE/SEI 26.2

Step 3 – Select the diaphragm model — Table 9.1

Because this building has no irregularities and is not subjected to any transfer forces, and because the frames between column lines B and D along column lines 1 and 7 have the same lateral stiffness (that is, the sizes of the beams and columns are the same in both frames), an equivalent beam model with rigid supports is selected to determine the internal forces for this diaphragm.

Step 4 – Determine the diaphragm internal forces — Sect. 9.3.3

The equivalent beam model for the diaphragm at the second-floor level is given in Figure 9.25 for wind in the south direction. Because the moment-resisting frames between column lines B and D along column lines 1 and 7 are identical, the CR is located $150.0 / 2 = 75.0\ \text{ft}$ from either end of the diaphragm, which coincides with the location where the wind force is applied at this level. Thus, there is no inherent torsional moment, which means the diaphragm is subjected to an equivalent uniformly distributed wind load equal to $w_S = 51.3 / 150.0 = 0.342\ \text{kips/ft}$.

- Reactions in each of the moment-resisting frames between column lines B and D along column lines 1 and 7

  $(R_1)_y = (R_7)_y = 0.342 \times 150.0 / 2 = 25.7\ \text{kips}$

  The shear and moment diagrams are also given in Figure 9.25.

- Chord forces at the edges of the diaphragm perpendicular to the direction of analysis

  $$T_u = C_u = \frac{M_{u,max}}{d} = \frac{961.9}{0.95 \times 94.0} = 10.8\ \text{kips} \qquad \text{Eq. (9.1)}$$

- Maximum unit shear forces in the moment-resisting frames between column lines B and D

  $$(v_{u,F})_1 = (v_{u,F})_7 = \frac{25.7}{47.0} = 0.55\ \text{kips/ft}$$

- Maximum unit shear force in the diaphragm

  $$(v_u)_1 = (v_u)_7 = \frac{25.7}{94.0} = 0.27\ \text{kips/ft}$$

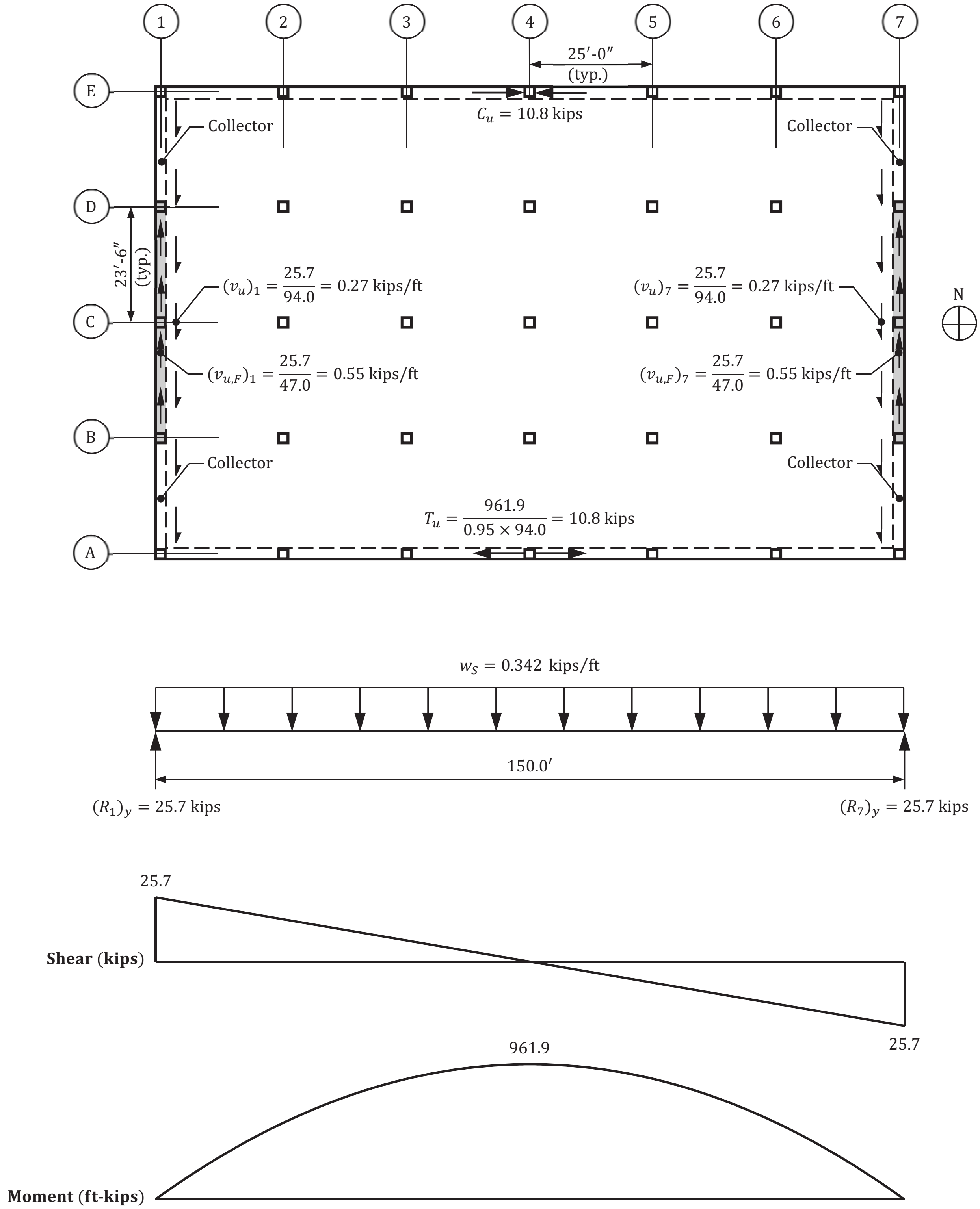


Figure 9.25 Equivalent beam model for the diaphragm in Example 9.3 for wind in the south direction.

- Collector forces

  Collectors are required because the moment-resisting frames do not extend the entire depth of the diaphragm in the direction of analysis.

The unit shear forces, net shear forces, and collector forces along column line 1 are given in Figure 9.26 where the collectors are the beams between column lines A and B and between column lines D and E. Note that in Figure 9.26, the forces are shown with more significant figures after the decimal place than are used in the above calculations. The main reason for this is to demonstrate that the maximum axial force in the collector is equal to the same value regardless of which end of the net shear force diagram is used to calculate it. Using a smaller number of significant figures results in two slightly different values of the maximum collector force, which is due to due roundoff only.

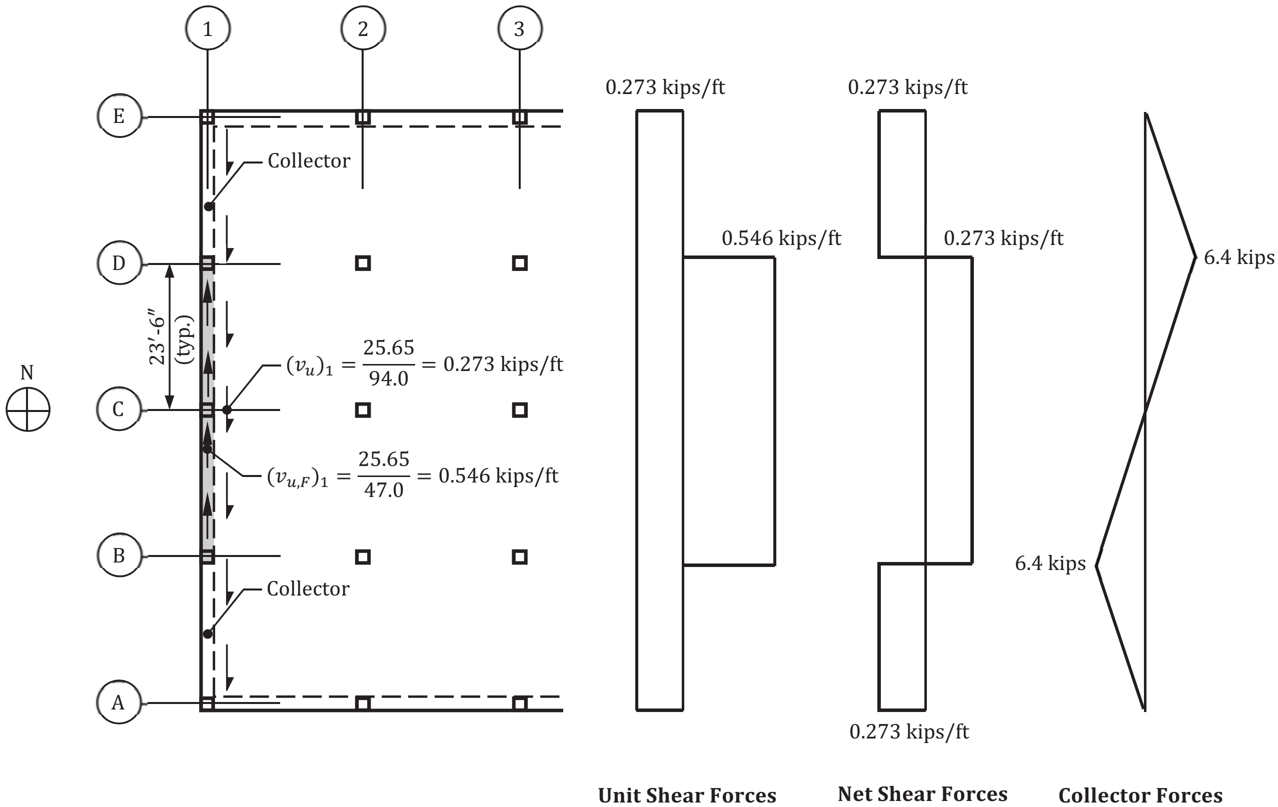


Figure 9.26 Unit shear forces, net shear forces, and collector forces in the diaphragm in Example 9.3.

9.9.4 Example 9.4 – Determination of Diaphragm Reinforcement: Building #1 (Framing Option B), SDC A, Collectors Required, Collector Width the Same as the Width of the Vertical Elements of the LFRS

Determine the required diaphragm reinforcement in the slab at the second-floor level of Building #1, Framing Option B for wind forces in the north-south direction assuming only the frames between column lines B and D along column lines 1 and 7 are part of the LFRS (see Figure 1.1). It is assumed that the width of the collectors (beams) are equal to the widths of the beams in the moment-resisting frames. Also assume an 8.5-in. thick slab, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 9.3.

Step 1 – Determine the chord reinforcement

From Example 9.3, $T_u = 10.8$ kips.

$$A_{s(\text{chord})} = \frac{T_u}{\phi f_y} = \frac{10.8}{0.9 \times 60} = 0.20 \text{ in.}^2 \qquad \text{Eq. (9.13)}$$

At column lines A and E, provide 1-#4 bar located just outside of the cross-section of the beams in the moment frames (see Figure 9.24).

Step 2 – Determine the diaphragm shear reinforcement

The largest factored unit shear forces occur along column lines 1 and 7 and are equal to 0.27 kips/ft (see Example 9.3).

Check shear strength requirements assuming $\rho_t = 0$: Eq. (9.14)

$$V_u = 0.27 \text{ kips/ft} < \text{lesser of} \begin{cases} \phi A_{cv}(2\lambda\sqrt{f_c'} + \rho_t f_y) = (0.75 \times 8.5 \times 12.0) \times [(2 \times 1.0 \times \sqrt{4{,}000}) + 0] \,/\, 1{,}000 = 9.7 \text{ kips/ft} \\ \phi 8 A_{cv}\sqrt{f_c'} = 0.75 \times 8 \times 8.5 \times 12.0 \times \sqrt{4{,}000} \,/\, 1{,}000 = 38.7 \text{ kips/ft} \end{cases}$$

Therefore, no shear reinforcement is required to satisfy shear strength requirements.

Step 3 – Determine the shear transfer reinforcement

Shear transfer reinforcement between the diaphragm and the moment-resisting frames between column lines B and D:

$$A_{vf} = \frac{(v_{u,F})_1}{\phi \mu f_y} = \frac{0.55}{0.75 \times 1.4 \times 1.0 \times 60} = 0.009 \text{ in.}^2/\text{ft} \qquad \text{Eq. (9.16)}$$

A value of $\mu = 1.4\lambda$ for monolithic concrete is used because the concrete for the slab and beams in the moment-resisting frames are placed at the same time (see Table 9.7).

Because the required area of shear-friction reinforcement is very small, it is safe to assume that the bottom flexural reinforcement in the slab can be used as the shear transfer reinforcement between the diaphragm and the moment-resisting frames between column lines B and D along column lines 1 and 7.

Shear transfer reinforcement between the diaphragm and the collectors:

$$A_{vf} = \frac{(v_u)_1}{\phi \mu f_y} = \frac{0.27}{0.75 \times 1.4 \times 1.0 \times 60} = 0.004 \text{ in.}^2/\text{ft} \qquad \text{Eq. (9.16)}$$

A value of $\mu = 1.4\lambda$ for monolithic concrete is used because the concrete for the slab and beams in the moment-resisting frames are placed at the same time (see Table 9.7).

Because the required area of shear-friction reinforcement is very small, it is safe to assume that the bottom flexural reinforcement in the slab can be used as the shear transfer reinforcement between the diaphragm and the collectors.

Step 4 – Determine the collector reinforcement

The collector beams must be designed for the combined effects due to gravity loads and the axial force due to wind loads, which is obtained from Figure 9.26.

A summary of the axial forces, bending moments, and shear forces for a typical collector beam is given in Table 9.11.

Table 9.11 Summary of Axial Forces, Bending Moments, and Shear Forces for a Typical Collector Beam in Example 9.4

Load Case		Axial Force (kips)	Bending Moment (ft-kips)			Shear (kips)
			Exterior Negative	Positive	First Interior Negative	
Dead		0	−16.1	63.8	−95.1	25.9
Live (L)		0	−9.4	29.7	−41.7	10.6
Wind (W)		±6.4	—	—	—	—
Load Combination						
ACI Eq. (5.3.1a)	$1.4D$	0	−22.5	89.3	−133.1	36.3
ACI Eq. (5.3.1b)	$1.2D + 1.6L$	0	−34.4	124.1	−180.8	48.0
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L$	±6.4	−24.0	91.4	−135.0	36.4
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	±6.4	−14.5	57.4	−85.6	23.3

The collector beams are also subjected to a uniform torsional load from the slab. Because the torsional moments can be redistributed, the beam can be designed for the compatibility torsional moment (see Sect. 6.3.1 of this publication).

The longitudinal reinforcement in the beam is determined based on the factored axial forces, bending moments, and torsional moments (calculations not given here; see Chapter 6 of this publication and Figure 9.27). The design strength interaction diagram (including the tension portion) is given in Figure 9.28. Included in the figure are the factored load combinations in Table 9.11. It is evident that the longitudinal reinforcement is adequate because all the factored load combination points fall within the diagram.

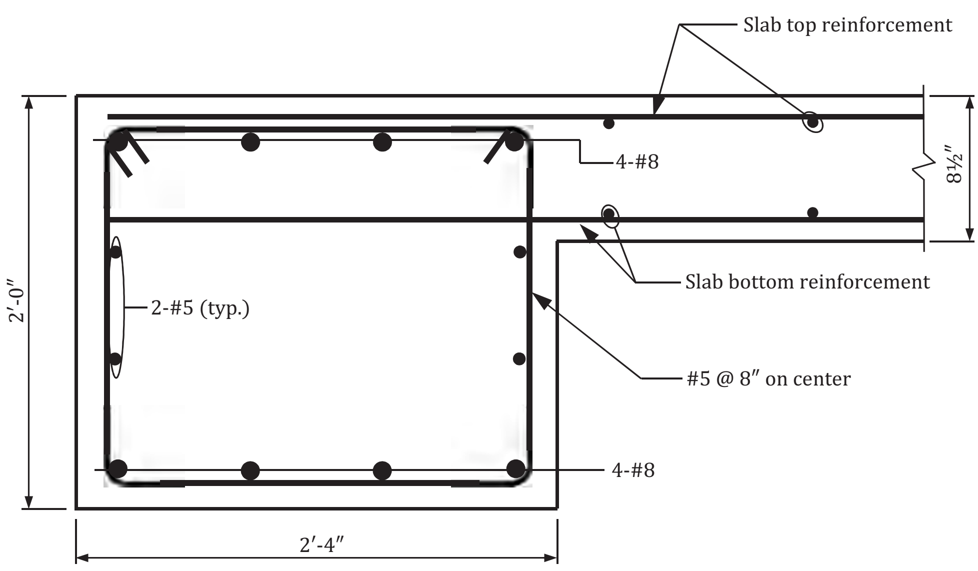


Figure 9.27 Required reinforcement for the collector beam in Example 9.4.

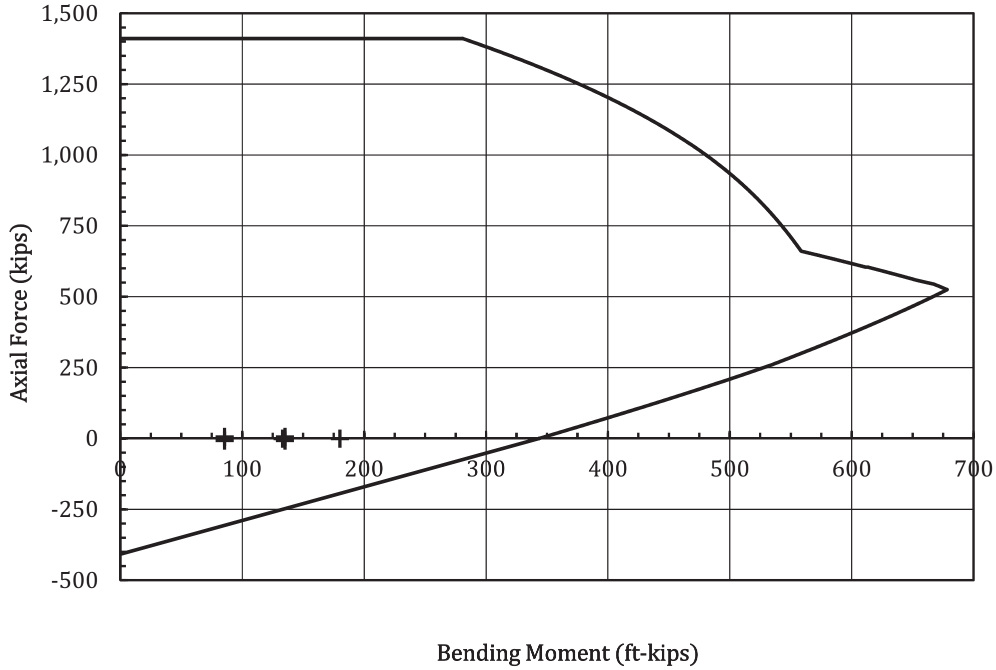


Figure 9.28 Design strength interaction diagram for the collector beam in Example 9.4.

9.9.5 Example 9.5 – Determination of Diaphragm In-Plane Forces: Building #1 (Framing Option C), SDC A, Collectors Required, Collector Width the Same as the Width of the Vertical Elements of the LFRS

Determine the diaphragm forces in the slab at the second-floor level of Building #1, Framing Option C for wind forces in the north-south direction assuming the lateral forces are resisted by the following moment-resisting frames (see Figure 1.1):

- Frames on column lines 1 and 7 between column lines A and E
- Frames on column line 3 between column lines B and D
- Frames on column lines A and E between column lines 1 and 7

It is assumed that the width of the collectors (beams) are equal to the widths of the beams in the moment-resisting frames. Also assume a 7.0-in. thick slab, 28.0 by 24.0 in. beams, and 24.0 by 24.0 in. columns.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the diaphragm design in-plane forces

From Table 3.10 in Example 3.1, the wind force in the north-south direction at the second-floor level is equal to 51.3 kips. This force is applied at the centroid of the building face at this level.

Step 2 – Determine the classification of the diaphragm Sect. 9.3.3

For forces in the north-south direction, the span-to-depth ratios are equal to $50.0 / 94.0 = 0.53$ and $100.0 / 94.0 = 1.1$.

Because the span-to-depth ratios are less than 2.0, the diaphragm can be classified as rigid. ASCE/SEI 26.2

Step 3 – Select the diaphragm model Table 9.1

This building has no irregularities and is not subjected to any transfer forces. Because the frame on column line 3 has a different lateral stiffness than the frames on column lines 1 and 7, the corrected equivalent beam model with spring supports is selected to determine the internal forces in this diaphragm.

Step 4 – Determine the diaphragm internal forces Sect. 9.3.3

The corrected equivalent beam model for the diaphragm at the second-floor level is depicted in Figure 9.29 for wind in the south direction.

- Frame stiffnesses

  It can be determined that the stiffnesses of the frames along column lines 1, 3, and 7 are equal to the following (Ref. 21):

  $$(k_1)_y = (k_7)_y = 0.5445E_c$$

  $$(k_3)_y = 0.3072E_c$$

  Assume the frames along column lines A and E are designated as the LFRS for wind forces in the east-west direction. The stiffnesses of these frames are equal to the following:

  $$(k_A)_x = (k_E)_x = 0.7642E_c$$

- Location of the CR

  The location of the CR in the *x*-direction is determined by the following equation where the origin is taken at column line 1:

  $$x_{cr} = \frac{\sum (k_i)_y x_i}{\sum (k_i)_y} = \frac{(0.3072E_c \times 50.0) + (0.5445E_c \times 150.0)}{(2 \times 0.5445E_c) + 0.3072E_c} = 69.5 \text{ ft} \qquad \text{Eq. (9.2)}$$

  The eccentricity between the location of the load application and the CR is equal to $e_x = 75.0 - 69.5 = 5.5$ ft.

  From symmetry, $y_{cr} = 94.0 / 2 = 47.0$ ft from column line A or E.

- Reactions in the frames

  The reactions in the frames in the north-south direction of analysis are obtained by the following equation:

  $$(R_i)_y = \frac{(k_i)_y}{\sum (k_i)_y} V_y \pm \frac{\overline{x}_i (k_i)_y}{\sum \overline{x}_i^2 (k_i)_y + \sum \overline{y}_i^2 (k_i)_x} V_y e_x \qquad \text{Eq. (9.4)}$$

  The reactions in the frames perpendicular to the direction of analysis are obtained by the following equation:

  $$(R_i)_x = \pm \frac{\overline{y}_i (k_i)_x}{\sum \overline{x}_i^2 (k_i)_y + \sum \overline{y}_i^2 (k_i)_x} V_y e_x$$

  Reactions in the frames due to the 51.3-kip wind force in the south direction are given in Table 9.12.

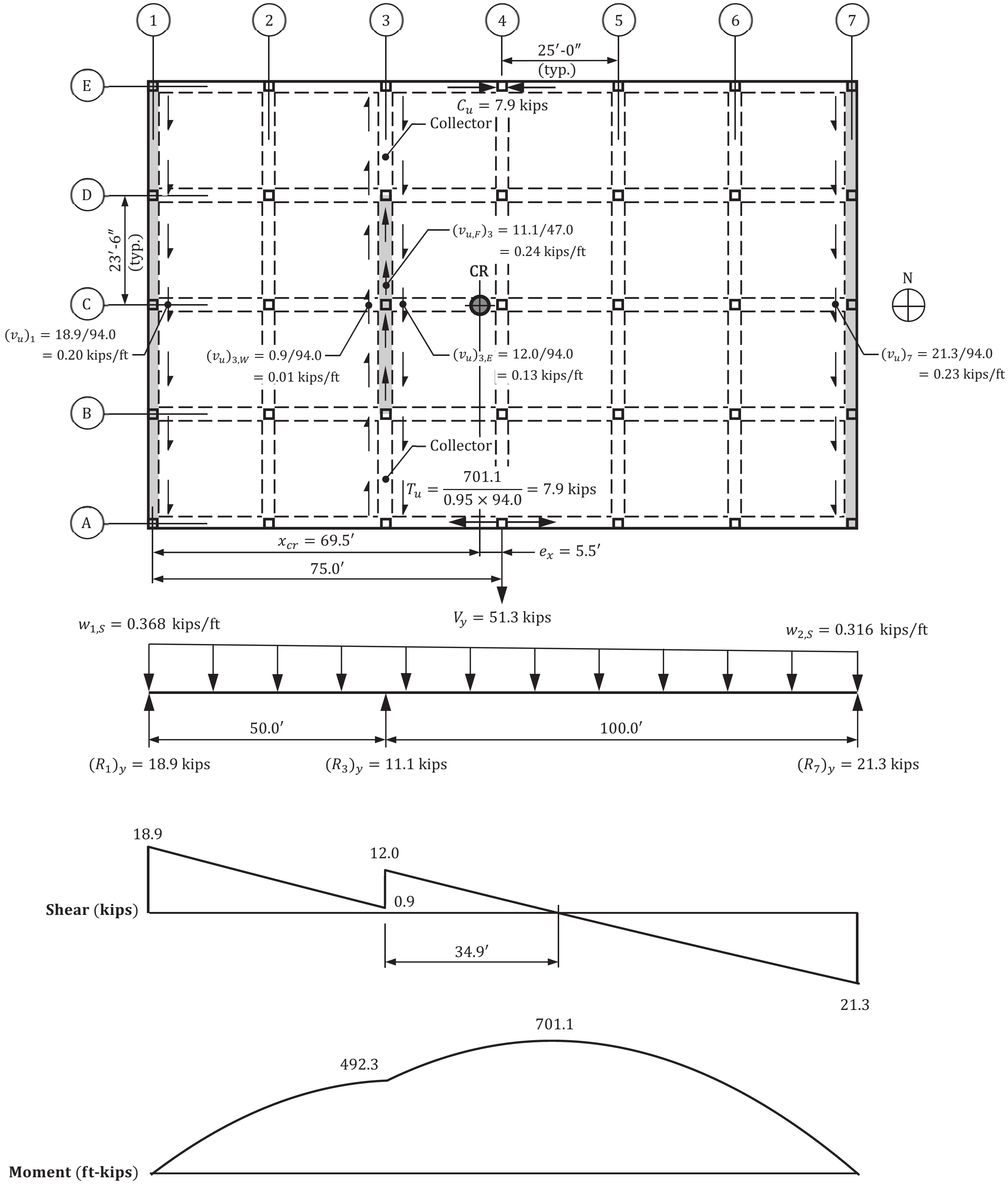


Figure 9.29 Equivalent beam model for the diaphragm in Example 9.5 for wind in the south direction

Table 9.12 Reactions in the Frames of Example 9.5 for the Wind Force in the South Direction

Frame	x_i (ft)	y_i (ft)	$\frac{(k_i)_y}{E_c}$	$\frac{(k_i)_x}{E_c}$	$\bar{x}_i$ (ft)	$\bar{y}_i$ (ft)	$\frac{\bar{x}_i^2(k_i)_y}{E_c}$	$\frac{\bar{y}_i^2(k_i)_x}{E_c}$	$\frac{(k_i)_y V_y}{\Sigma x_i(k_i)_y}$ (kips)*	$\left(\frac{\bar{x}_i(k_i)_y V_y e_x}{\Sigma\bar{x}_i(k_i)_y+\Sigma\bar{y}_i^2(k_i)_x}\right)^*$ (kips)	R_i (kips)
1	0.0	—	0.5445	—	69.5	—	2,630	—	20.0	–1.1	18.9
3	50.0	—	0.3072	—	19.5	—	117	—	11.3	–0.2	11.1
7	150.0	—	0.5445	—	80.5	—	3,529	—	20.0	1.3	21.3
A	—	0.0	—	0.7642	—	47.0	—	1,688	—	1.1	1.1
E	—	94.0	—	0.7642	—	47.0	—	1,688	—	–1.1	–1.1
		Σ	1.396	1.528			6,276**	3,376**	51.3	0.0	51.3

*For frames along column lines A and E, replace $\bar{x}_i(k_i)_y$ with $\bar{y}_i(k_i)_x$ in this equation.
**$\sum\bar{x}_i^2(k_i)_y+\bar{y}_i^2(k_i)_x=(6{,}276+3{,}376)E_c=9{,}652E_c$

- Equivalent in-plane distributed loads

  The equivalent in-plane load for wind in the south direction is trapezoidal, which accounts for the eccentricity between the location of the load application and the CR. The equations for force and moment equilibrium are solved simultaneously for the two unknowns $w_{1,S}$ and $w_{2,S}$ where moments are summed about column line 1:

$$\frac{(w_{1,S}+w_{2,S})B}{2}=\frac{(w_{1,S}+w_{2,S})\times 150.0}{2}=V_y=51.3 \qquad \text{Eq. (9.6)}$$

$$\frac{\left(\frac{w_{1,S}}{2}+w_{2,S}\right)B^2}{3}=\frac{\left(\frac{w_{1,S}}{2}+w_{2,S}\right)\times 150.0^2}{3}=(R_3)_y x_3+(R_7)_y x_7=(11.1\times 50.0)+(21.3\times 150.0) \qquad \text{Eq. (9.7)}$$

  Therefore, $w_{1,S}=0.368$ kips/ft and $w_{2,S}=0.316$ kips/ft.

  The shear and moment diagrams are also given in Figure 9.29.

- Chord forces at the edges of the diaphragm perpendicular to the direction of analysis:

$$T_u=C_u=\frac{M_{u,max}}{d}=\frac{701.1}{0.95\times 94.0}=7.9 \text{ kips} \qquad \text{Eq. (9.1)}$$

- Maximum unit shear forces in the moment-resisting frames occurs along column line 3:

$$(v_{u,F})_3=\frac{11.1}{47.0}=0.24 \text{ kips/ft}$$

- Maximum unit shear force in the diaphragm occurs along column line 7:

$$(v_u)_7=\frac{21.3}{94.0}=0.23 \text{ kips/ft}$$

- Collector forces

  Collectors are required on column line 3 because the moment-resisting frames do not extend the entire depth of the diaphragm in the direction of analysis.

  The unit shear forces, net shear forces, and collector forces along column line 3 are given in Figure 9.30 where the collectors are the beams between column lines A and B and between column lines D and E.

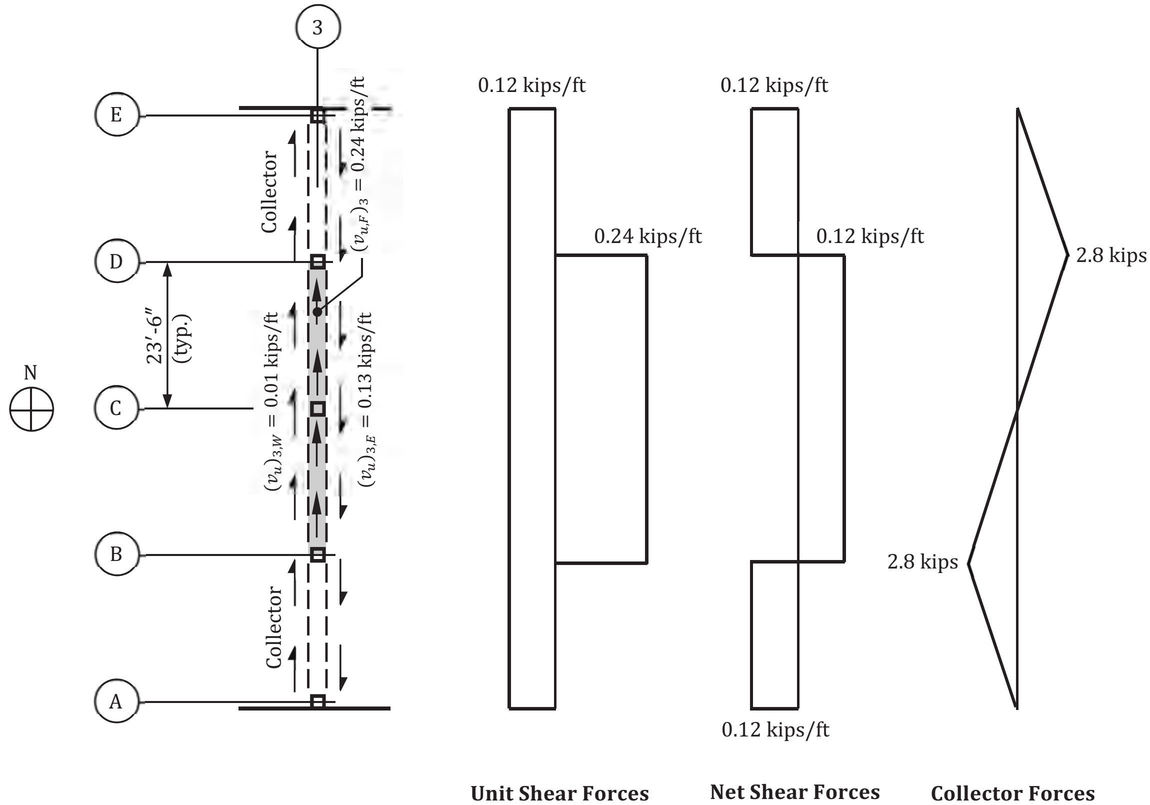


Figure 9.30 Unit shear forces, net shear forces, and collector forces in the diaphragm in Example 9.5.

9.9.6 Example 9.6 – Determination of Diaphragm Reinforcement: Building #1 (Framing Option C), SDC A, Collectors Required, Collector Width the Same as the Width of the Vertical Elements of the LFRS

Determine the required diaphragm reinforcement in the slab at the second-floor level of Building #1, Framing Option C for wind forces in the north-south direction assuming the lateral forces are resisted by the following moment-resisting frames (see Figure 1.1):

- Frames on column lines 1 and 7 between column lines A and E
- Frames on column line 3 between column lines B and D
- Frames on column lines A and E between column lines 1 and 7

It is assumed that the width of the collectors (beams) are equal to the widths of the beams in the moment-resisting frames. Also assume a 7.0-in. thick slab, 28.0 by 24.0 in. beams, 24.0 by 24.0 in. columns, normalweight concrete with $f_c' = 4{,}000$ psi, and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 9.5.

Step 1 – Determine the chord reinforcement

From Example 9.5, $T_u = 7.9$ kips.

$$A_{s(\text{chord})} = \frac{T_u}{\phi f_y} = \frac{7.9}{0.9 \times 60} = 0.15 \text{ in.}^2 \qquad \text{Eq. (9.13)}$$

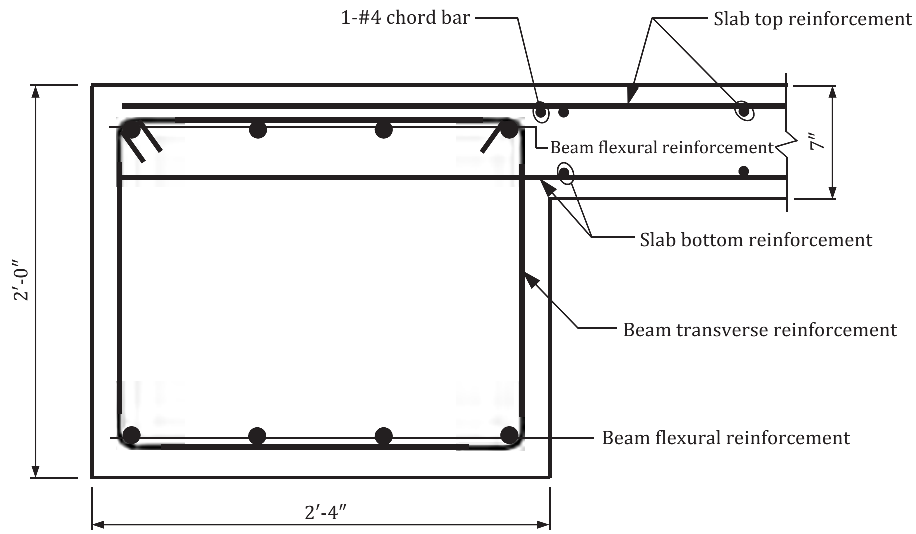


Figure 9.31 Location of chord reinforcement in the diaphragm in Example 9.6 for wind forces in the north-south direction.

At column lines A and E, provide 1-#4 bar located just outside of the cross-section of the beams in the moment frames (see Figure 9.31).

Step 2 – Determine the diaphragm shear reinforcement

The largest factored unit shear force occurs along column line 7 and is equal to 0.23 kips/ft (see Example 9.5).

Check shear strength requirements assuming $\rho_t = 0$: Eq. (9.14)

$$V_u = 0.23 \text{ kips/ft} < \text{lesser of } \begin{cases} \phi A_{cv}(2\lambda\sqrt{f_c'} + \rho_t f_y) = (0.75 \times 7.0 \times 12.0) \times [(2 \times 1.0 \times \sqrt{4{,}000}) + 0] \,/\, 1{,}000 = 8.0 \text{ kips/ft} \\ \\ \phi 8 A_{cv}\sqrt{f_c'} = 0.75 \times 8 \times 7.0 \times 12.0 \times \sqrt{4{,}000} \,/\, 1{,}000 = 31.9 \text{ kips/ft} \end{cases}$$

Therefore, no shear reinforcement is required to satisfy shear strength requirements.

Step 3 – Determine the shear transfer reinforcement

Shear transfer reinforcement between the diaphragm and the moment-resisting frames on column line 3 between column lines B and D:

$$A_{vf} = \frac{(v_{u,F})_3}{\phi \mu f_y} = \frac{0.24}{0.75 \times 1.4 \times 1.0 \times 60} = 0.004 \text{ in.}^2/\text{ft} \qquad \text{Eq. (9.16)}$$

A value of $\mu = 1.4\lambda$ for monolithic concrete is used because the concrete for the slab and beams in the moment-resisting frames are placed at the same time (see Table 9.7).

Because the required area of shear-friction reinforcement is very small, it is safe to assume that the bottom flexural reinforcement in the slab can be used as the shear transfer reinforcement between the diaphragm and the moment-resisting frames on column line 3 between column lines B and D.

Shear transfer reinforcement between the diaphragm and the collectors:

$$A_{vf} = \frac{(v_u)_{3,E} - (v_u)_{3,W}}{\phi \mu f_y} = \frac{0.12}{0.75 \times 1.4 \times 1.0 \times 60} = 0.002 \text{ in.}^2/\text{ft} \qquad \text{Eq. (9.16)}$$

A value of $\mu = 1.4\lambda$ for monolithic concrete is used because the concrete for the slab and beams in the moment-resisting frames are placed at the same time (see Table 9.7).

Because the required area of shear-friction reinforcement is very small, it is safe to assume that the bottom flexural reinforcement in the slab can be used as the shear transfer reinforcement between the diaphragm and the collectors.

Step 4 – Determine the collector reinforcement

The collector beams must be designed for the combined effects due to gravity loads and the axial force due to wind loads, which is obtained from Figure 9.30.

A summary of the axial forces, bending moments, and shear forces for a typical collector beam is given in Table 9.13.

Table 9.13 Summary of Axial Forces, Bending Moments, and Shear Forces for a Typical Collector Beam in Example 9.6

Load Case		Axial Force (kips)	Bending Moment (ft-kips)			Shear (kips)
			Exterior Negative	Positive	First Interior Negative	
Dead		0	−61.6	55.4	−77.7	21.2
Live (L)		0	−27.4	26.3	−36.2	9.2
Wind (W)		± 2.8	—	—	—	—
Load Combination						
ACI Eq. (5.3.1a)	$1.4D$	0	−86.2	77.6	−108.8	29.7
ACI Eq. (5.3.1b)	$1.2D + 1.6L$	0	−117.8	108.6	−151.2	40.2
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L$	± 2.8	−87.6	79.6	−111.3	30.0
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	± 2.8	−55.4	49.9	−69.9	19.1

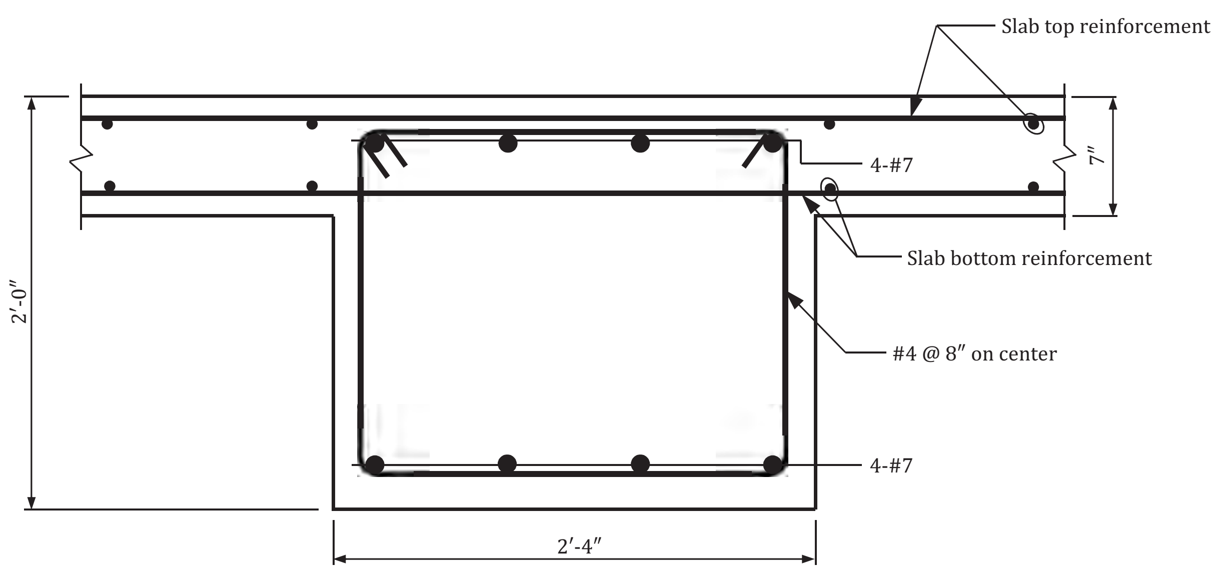


Figure 9.32 Required reinforcement for the collector beam in Example 9.6.

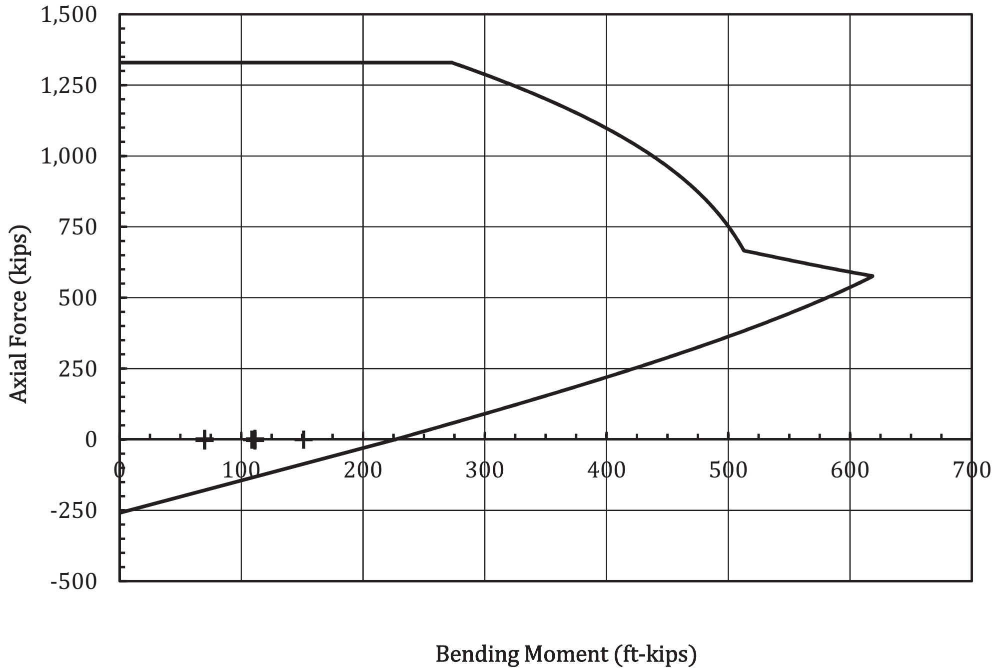


Figure 9.33 Design strength interaction diagram for the collector beam in Example 9.6.

The longitudinal reinforcement in the beam is determined based on the factored axial forces and bending moments (calculations not given here; see Chapter 6 of this publication and Figure 9.32). The design strength interaction diagram (including the tension portion) is shown in Figure 9.33. Included in the figure are the factored load combinations in Table 9.13. It is evident that the longitudinal reinforcement is adequate because all the factored load combination points fall within the diagram.

9.9.7 Example 9.7 – Determination of Diaphragm In-Plane Forces: Building #2, SDC C, Collectors Required, Collector Width the Same as the Width of the Vertical Elements of the SFRS

Determine the diaphragm forces in the slab at the second-floor level of Building #2 for seismic forces in the east-west direction assuming only the walls on column lines B and F are part of the SFRS (see Figure 1.2). Also assume a 4.5-in. thick slab and 12-in.-thick walls. It is assumed that the portions of the slab in line with the walls are the collectors and the widths of the collectors are equal to the thicknesses of the walls.

Design data are given in Sect. 1.2.2.

Step 1 – Determine the diaphragm design in-plane forces

From Table 3.31 in Example 3.14, the seismic diaphragm design force in the east-west direction at the second-floor level is equal to 345.6 kips. This force is applied at the CM, which is located at the centroid of the floor plate at this level due to symmetry.

Step 2 – Determine the classification of the diaphragm Sect. 9.3.3

For forces in the east-west direction, the span-to-depth ratio is equal to $120.0 / 120.5 = 1.0 < 3.0$.

Also, the structure does not have any of the horizontal irregularities in ASCE/SEI Table 12.3-1.

Therefore, the diaphragm can be classified as rigid. ASCE/SEI 12.3.1.2

Step 3 – Select the diaphragm model Table 9.1

Because this building has no irregularities and is not subjected to any transfer forces, and because the walls on column lines B and F have the same lateral stiffness (that is, the thicknesses and the lengths of both walls are the same), an equivalent beam model with rigid supports is selected to determine the internal forces in this diaphragm.

Step 4 – Determine the diaphragm internal forces Sect. 9.3.3

The equivalent beam model for the diaphragm at the second-floor level is given in Figure 9.34 for seismic forces in the west direction. Because the walls on column lines B and F have the same thickness and length (that is, the same in-plane stiffness), the CR is located $180.0 / 2 = 90.0$ ft from either end of the diaphragm, which coincides with the location of the CM at this level. Thus, there is no inherent torsional moment, which means the diaphragm is subjected to an equivalent uniformly distributed seismic load equal to $345.6 / 180.0 = 1.92$ kips/ft.

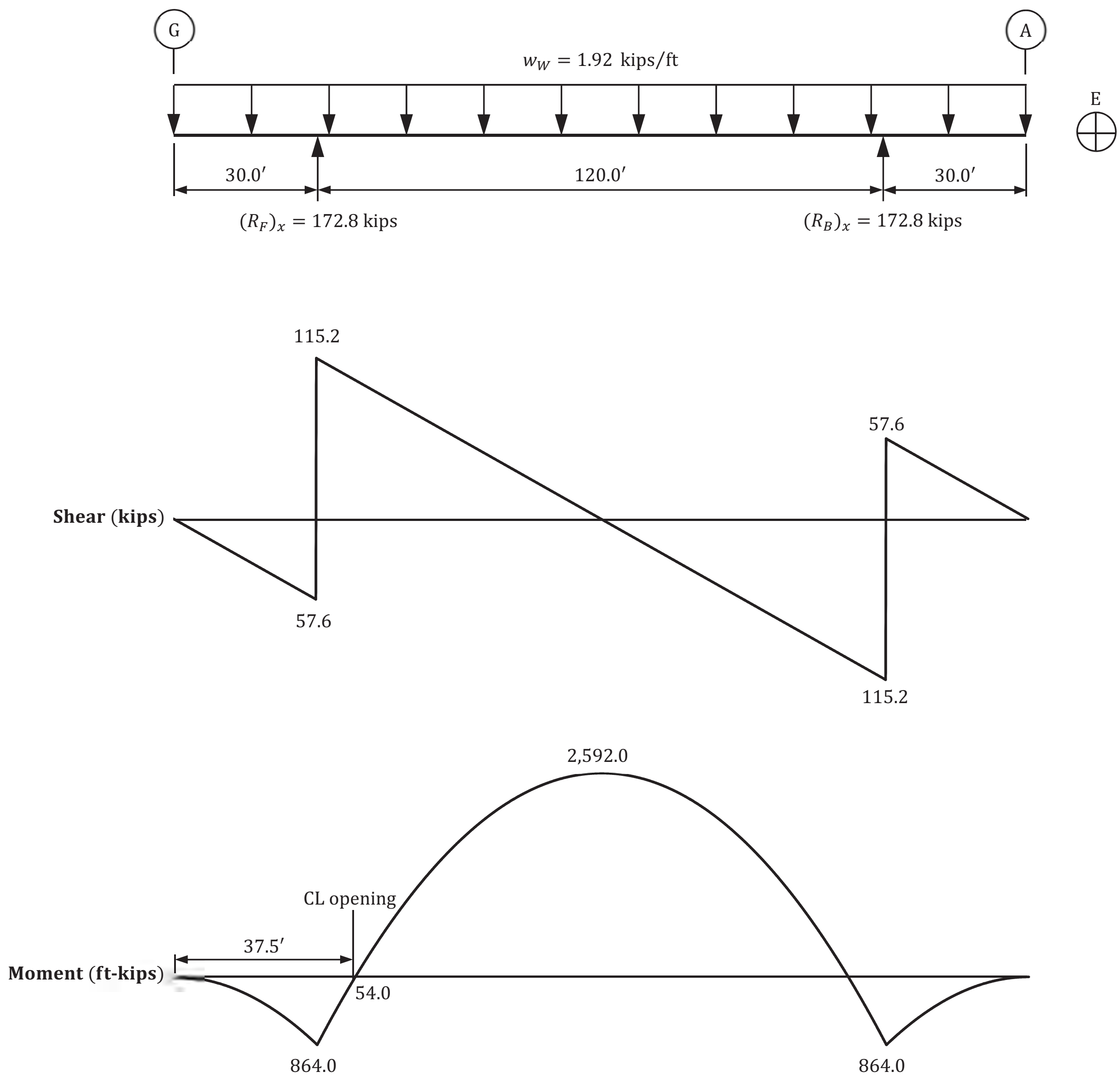


Figure 9.34 Equivalent beam model for the diaphragm in Example 9.7 for seismic forces in the west direction.

- Reactions in each of the walls on column lines B and F

  $(R_B)_x = (R_F)_x = 1.92 \times 180.0 / 2 = 172.8 \text{ kips}$

  The shear and moment diagrams are also given in Figure 9.34.

- Chord forces

  The maximum tension chord force based on the maximum bending moment in the diaphragm is equal to the following:

$$T_u = C_u = \frac{M_{u,max}}{d} = \frac{2,592.0}{0.95 \times 120.5} = 22.6 \text{ kips} \qquad \text{Eq. (9.1)}$$

The primary tension chord force at the center of the opening, which located 37.5 ft from column line A or G, is calculated using the moment in the diaphragm at this location (see Figure 9.35):

$$T_{u,open} = \frac{M_{u,open}}{d} = \frac{54.0}{0.95 \times 120.5} = 0.5 \text{ kips} \qquad \text{Eq. (9.8)}$$

where $M_{u,open}$ is determined from the moment diagram in Figure 9.34.

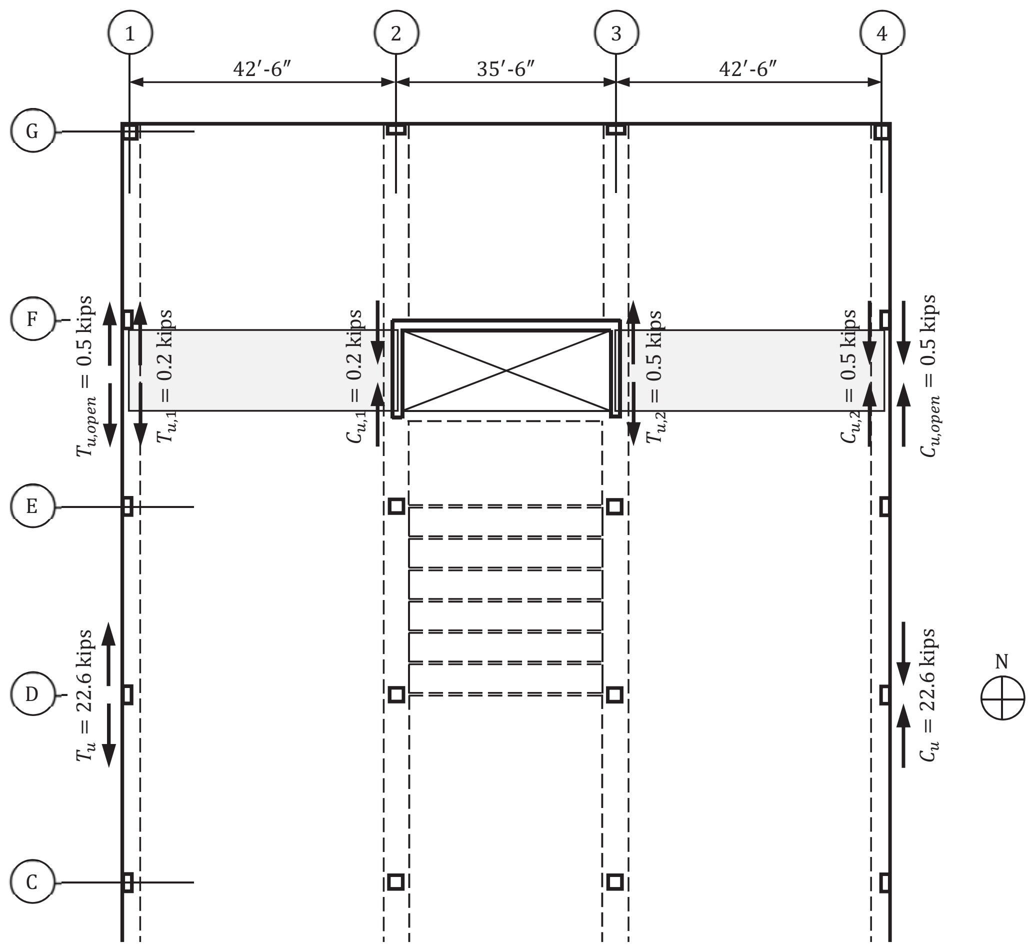


Figure 9.35 Chord forces in the diaphragm of Example 9.7 for the seismic force in the west direction.

The secondary tension chord force at the center of the opening, $T_{u,1}$, is determined by the following equation:

$$T_{u,1} = \frac{M_u^+}{0.95\ell_1} \tag{Eq. (9.8)}$$

In this equation, M_u^+ is the positive bending moment in the subdiaphragm to the west of the opening.

It is assumed that the subdiaphragms are fixed at both ends and are subjected to a portion of the total uniform diaphragm load, w_W, based on the mass of the segment. Because each segment has the same area, each segment resists 50 percent of w_W. From statics, M_u^+ is equal to the following:

$$M_u^+ = \frac{(w_W / 2)b_{open}^2}{24} = \frac{(1.92 / 2) \times 15.0^2}{24} = 9.0 \text{ ft-kips}$$

Therefore, the secondary tension chord force at the center of the opening is equal to the following:

$$T_{u,1} = \frac{M_u^+}{0.95\ell_1} = \frac{9.0}{0.95 \times 42.5} = 0.2 \text{ kips}$$

The total tension chord force at the center of the opening is equal to $0.5 + 0.2 = 0.7$ kips, which is less than the 22.6-kip tension chord force determined for the overall diaphragm.

Secondary tension chord forces develop at the corners of the openings due to the negative bending moments in the subdiaphragm to the east of the opening. The tension chord force occurring along column line 3 is equal to the following:

$$T_{u,2} = \frac{M_u^-}{0.95\ell_2} = \frac{(w_W / 2)b_{open}^2 / 12}{0.95\ell_2} = \frac{(1.92 / 2) \times 15.0^2 / 12}{0.95 \times 42.5} = 0.5 \text{ kips}$$

- Unit shear forces

  Along column lines B and F, the shear force in the slab is equal to 57.6 kips (see Figure 9.34), which is distributed over a length of 120.5 ft. Therefore, the unit shear force is equal to the following:

$$(v_u)_B = (v_u)_F = \frac{57.6}{120.5} = 0.48 \text{ kips/ft}$$

  Just to the north of column line B and to the south of column line F, the shear force is equal to 115.2 kips, which is distributed over a length of $120.5 - 35.5 = 85.0$ ft due to the openings. Thus, the unit shear forces at these locations are equal to the following:

$$(v_u)_B = (v_u)_F = \frac{115.2}{85.0} = 1.36 \text{ kips/ft}$$

- Collector forces

  Collectors are required because the walls along column lines B and F do not extend the entire depth of the diaphragm in the direction of analysis. In this example, the portions of the slab in line with the walls are the collectors and the widths of the collectors are equal to the thicknesses of the walls.

  The unit shear forces, net shear forces, and collector forces along column line F are given in Figure 9.36.

  In buildings assigned to SDC C, collectors and their connections must be designed for the maximum of the three forces given in ASCE/SEI 12.10.2.1:

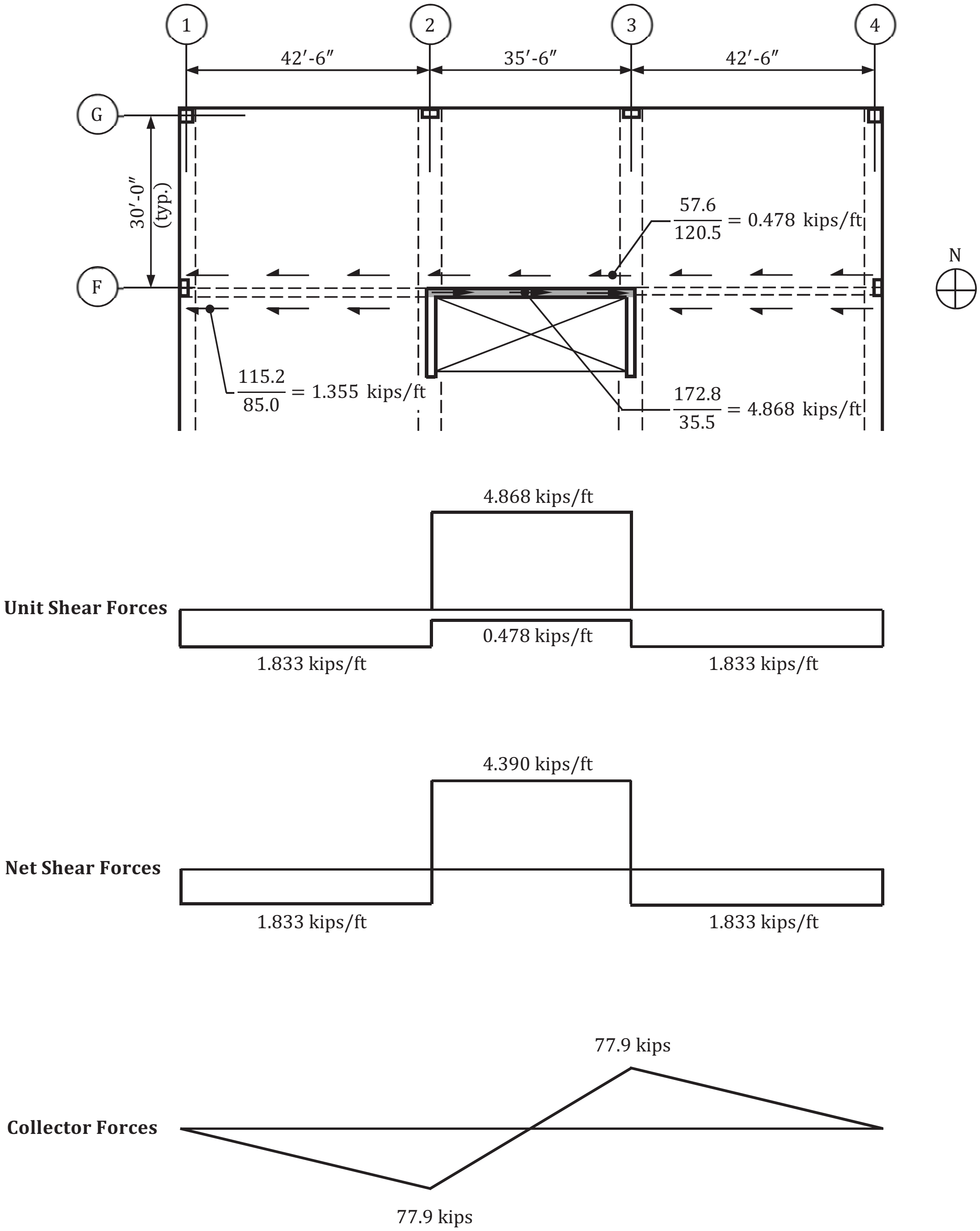


Figure 9.36 Unit shear forces, net shear forces, and collector forces in the diaphragm in Example 9.7.

1. Forces calculated using the seismic load effects including overstrength of ASCE/SEI 12.4.3 with the seismic forces determined by the Equivalent Lateral Force Procedure of ASCE/SEI 12.8 or the modal response spectrum analysis procedure of ASCE/SEI 12.9.1.

   For the ordinary building frame system with ordinary reinforced concrete shear walls in the east-west direction, the overstrength factor, Ω_o, is equal to 2.5 (see ASCE/SEI Table 12.2-1). Therefore, the required in-plane diaphragm force at the second-floor level is equal to $2.5 \times 24.8 = 62.0$ kips (see Table 3.31 in Example 3.14).

2. Forces calculated using the seismic load effects including overstrength of ASCE/SEI 12.4.3 with the seismic forces determined by ASCE/SEI Eq. (12.10-1).

   Using the information in Table 3.31, the diaphragm force, F_{px}, at the second-floor level is equal to the following:

$$F_{p2} = \left(\sum_{i=2}^{R} F_i \,/ \sum_{i=2}^{R} w_i\right) w_{p2} = (1{,}575.5 \,/\, 43{,}763) \times 3{,}840 = 0.036 \times 3{,}840 = 138.2 \text{ kips}$$

   The required in-plane force including overstrength is equal to $2.5 \times 138.2 = 345.6$ kips.

3. Forces calculated using the load combinations of ASCE/SEI 2.3.6 with the seismic forces determined by ASCE/SEI Eq. (12.10-2).

   The diaphragm force, F_{px}, at the second-floor level based on ASCE/SEI Eq. (12.10-2) is equal to 345.6 kips (see Table 3.31).

   Therefore, the collectors and their connections to the vertical elements of the SFRS must be designed for the effects due to the 345.6-kip in-plane force stipulated in the second and third requirements. Thus, the required axial force in the collector is equal to 77.9 kips (see Figure 9.36).

9.9.8 Example 9.8 – Determination of Diaphragm Reinforcement: Building #2, SDC C, Collectors Required, Collector Width the Same as the Width of the Vertical Elements of the SFRS

Determine the required diaphragm reinforcement in the slab at the second-floor level of Building #2 for seismic forces in the east-west direction assuming only the walls on column lines B and F are part of the SFRS (see Figure 1.2). Also assume a 4.5-in. thick slab, 12-in.-thick walls, normalweight concrete with $f'_c = 4{,}000$ psi, and Grade 60 reinforcement. It is assumed that the portions of the slab in line with the walls are the collectors and the widths of the collectors are equal to the thicknesses of the walls.

Design data are given in Sect. 1.2.2. See Example 9.7.

Step 1 – Determine the chord reinforcement

From Example 9.7, $T_u = 22.6$ kips at column lines 1 and 4.

$$A_{s(\text{chord})} = \frac{T_u}{\phi f_y} = \frac{22.6}{0.9 \times 60} = 0.42 \text{ in.}^2 \qquad \text{Eq. (9.13)}$$

Provide 1-#6 bar ($A_{s,provided} = 0.44$ in.^{2}) located near the edges of the diaphragm at mid-depth of the 4.5-in.-thick slab.

At the edges of the openings:

$$A_{s(\text{chord})} = \frac{T_{u,2}}{\phi f_y} = \frac{0.5}{0.9 \times 60} = 0.01 \text{ in.}^2$$

The required amount of secondary chord reinforcement along the slab edges adjacent to the openings at column lines 2 and 3 is nominal. The collector reinforcement for seismic force transfer in the north-south direction will be provided along these edges of the opening, which is greater than 0.01 in.^{2}

Step 2 – Determine the diaphragm shear reinforcement

The largest factored unit shear forces occur along column lines B and F and are equal to 1.36 kips/ft (see Figure 9.36).

Check shear strength requirements assuming $\rho_t = 0$: Eq. (9.14)

$$V_u = 1.36 \text{ kips/ft} < \text{lesser of} \begin{cases} \phi A_{cv}(2\lambda\sqrt{f_c'} + \rho_t f_y) = (0.75 \times 4.5 \times 12.0) \times [(2 \times 1.0 \times \sqrt{4{,}000}) + 0] \,/\, 1{,}000 = 5.1 \text{ kips/ft} \\ \phi 8 A_{cv}\sqrt{f_c'} = 0.75 \times 8 \times 4.5 \times 12.0 \times \sqrt{4{,}000} \,/\, 1{,}000 = 20.5 \text{ kips/ft} \end{cases}$$

Therefore, no shear reinforcement is required to satisfy shear strength requirements.

Step 3 – Determine the shear transfer reinforcement

- Shear transfer reinforcement between the diaphragm and walls on column lines B and F assuming the slab and walls are not placed at the same time and the interface is not intentionally roughened (see Figure 9.36 and Table 9.7):

  $$A_{vf} = \frac{(v_{u,W})_B}{\phi \mu f_y} = \frac{(v_{u,W})_F}{\phi \mu f_y} = \frac{4.87}{0.75 \times 0.6 \times 1.0 \times 60} = 0.18 \text{ in.}^2/\text{ft} \qquad \text{Eq. (9.16)}$$

  Provide #4 dowel bars spaced at 12 in. on center $(A_{s,provided} = 0.20 \text{ in.}^2/\text{ft})$ over the lengths of the walls.

- Shear transfer reinforcement between the diaphragm and the collectors (see Figure 9.36):

  $$A_{vf} = \frac{(v_u)_B}{\phi \mu f_y} = \frac{(v_u)_F}{\phi \mu f_y} = \frac{1.36 + 0.48}{0.75 \times 1.4 \times 1.0 \times 60} = 0.03 \text{ in.}^2/\text{ft}$$

  A value of $\mu = 1.4\lambda$ for monolithic concrete is used because the collectors are part of the slab and have the same widths as the thickness of the walls (see Table 9.7).

  Because the area of shear-friction is very small, it is safe to assume that the bottom flexural reinforcement in the slab oriented in the north-south direction can be used as the shear transfer reinforcement between the diaphragm and the collectors.

Step 4 – Determine the collector reinforcement

The maximum axial force in the collectors along column lines B and F is equal to 77.9 kips (see Figure 9.36). The required area of longitudinal collector reinforcement is equal to the following:

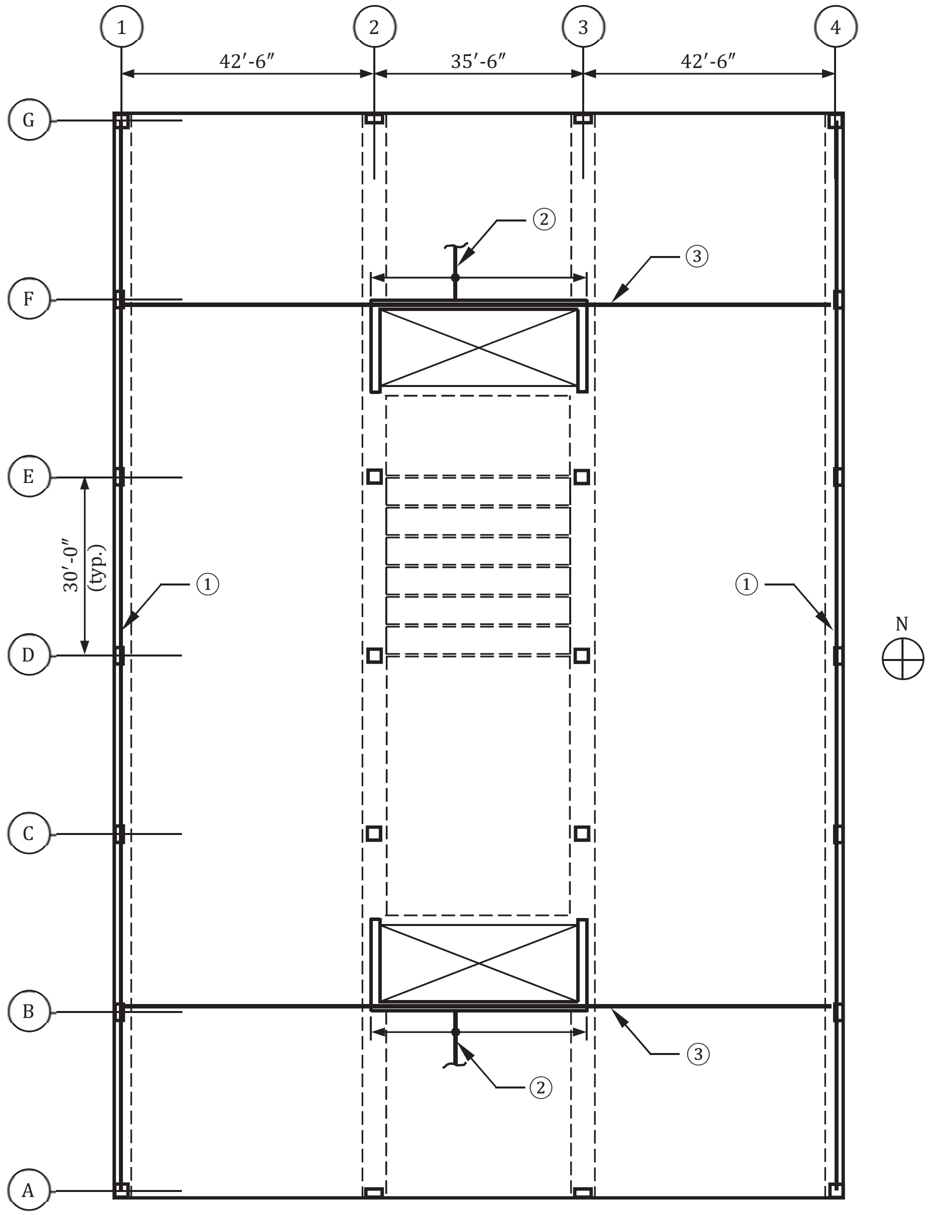
$$A_{s(\text{collector})} = \frac{T_u}{\phi f_y} = \frac{77.9}{0.9 \times 60} = 1.44 \text{ in.}^2 \qquad \text{Eq. (9.19)}$$

Provide 3-#7 bars $(A_{s,provided} = 1.80 \text{ in.}^2)$ along column lines B and F. These reinforcing bars are in addition to any other reinforcement in the slab and must be placed within the 12.0-in. thickness of the walls. Collector reinforcement must extend along the length of the walls in accordance with ACI 12.5.4.3. Extending all the collector bars the full length of the walls ensures uniform shear flow across the wall segment below the slab, which must resist the shear force from the wall above plus the tension force transferred by the collector.

Check the axial compression force in the collectors assuming the sections are reinforced with 3-#7 longitudinal bars:

$$C_u = 77.9 \text{ kips} < \phi[0.85 f_c'(A_g - A_s) + f_y A_s] = 0.65 \times \{[0.85 \times 4 \times (54.0 - 1.80)] + (60 \times 1.80)\} = 185.6 \text{ kips} \qquad \text{Eq. (9.20)}$$

Reinforcement details for the second-floor diaphragm are given in Figure 9.37 for seismic forces in the east-west direction. For simpler detailing, the 3-#7 collector bars on column lines B and F are extended the full depth of the diaphragm. Similar details can be obtained for seismic forces in the north-south direction.



1. ① — Chord reinforcement: 1-#6 bar
2. ② — Shear transfer reinforcement: #4 @ 12″ dowel bars
3. ③ — Collector reinforcement: 3-#7 bars
4. Provide standard 90-deg hooks at the ends of all bars
5. Provide Class B lap splices or Type 2 mechanical connectors where required
6. Other reinforcement not shown for clarity

Figure 9.37 Reinforcement details for the diaphragm at the second-floor level in Examples 9.7 and 9.8.

Comments. It is important to ensure that the collector reinforcement does not cause any congestion issues within the wall. If congestion issues or any other problems occur, the collectors can be made wider than the thickness of the walls (see Examples 9.9 and 9.10 below).

9.9.9 Example 9.9 – Determination of Diaphragm In-Plane Forces: Building #2, SDC C, Collectors Required, Collector Width Wider than the Width of the Vertical Elements of the SFRS

Determine the diaphragm forces in the slab at the second-floor level of Building #2 for seismic forces in the east-west direction assuming only the walls on column lines B and F are part of the SFRS (see Figure 1.2). Also assume a 4.5-in. thick slab and 12-in.-thick walls. It is assumed that the portions of the slab in line with the walls are the collectors and the widths of the collectors are wider than the thicknesses of the walls.

Design data are given in Sect. 1.2.2.

Step 1 – Determine the diaphragm design in-plane forces

From Table 3.31 in Example 3.14, the seismic force in the east-west direction at the second-floor level is equal to 345.6 kips. This force is applied at the CM, which is located at the centroid of the floor plate at this level due to symmetry.

Step 2 – Determine the classification of the diaphragm Sect. 9.3.3

For forces in the east-west direction, the span-to-depth ratio is equal to $120.0 / 120.5 = 1.0 < 3.0$.

Also, the structure does not have any of the horizontal irregularities in ASCE/SEI Table 12.3-1.

Therefore, the diaphragm can be classified as rigid. ASCE/SEI 12.3.1.2

Step 3 – Select the diaphragm model Table 9.1

Because this building has no irregularities and is not subjected to any transfer forces, and because the walls on column lines B and F have the same lateral stiffness (that is, the thicknesses and the lengths of both walls are the same), an equivalent beam model with rigid supports is selected to determine the internal forces in this diaphragm.

Step 4 – Determine the diaphragm internal forces Sect. 9.3.3

The equivalent beam model for the diaphragm at the second-floor level is depicted in Figure 9.34 for seismic forces in the west direction. Because the walls on column lines B and F have the same thickness and length (that is, in-plane stiffness), the CR is located $180.0 / 2 = 90.0$ ft from either end of the diaphragm, which coincides with the location of the CM at this level. Thus, there is no inherent torsional moment, which means the diaphragm is subjected to an equivalent uniformly distributed wind load equal to $345.6 / 180.0 = 1.92$ kips/ft.

- Reactions in each of the walls on column lines B and F

  $$(R_B)_x = (R_F)_x = 1.92 \times 180.0 / 2 = 172.8 \text{ kips}$$

  The shear and moment diagrams are also given in Figure 9.34.

- Chord forces

  The maximum tension chord force based on the maximum bending moment in the diaphragm is equal to the following:

  $$T_u = C_u = \frac{M_{u,max}}{d} = \frac{2{,}592.0}{0.95 \times 120.5} = 22.6 \text{ kips} \quad \text{Eq. (9.1)}$$

The primary tension chord force at the center of the opening, which located 37.5 ft from column line A or G, is equal to the following (see Figure 9.35):

$$T_{u,open} = \frac{M_{u,open}}{d} = \frac{54.0}{0.95 \times 120.5} = 0.5 \text{ kips} \qquad \text{Eq. (9.8)}$$

where $M_{u,open}$ is determined from the moment diagram in Figure 9.34.

The secondary tension chord force at the center of the opening, $T_{u,1}$, is determined by the following equation:

$$T_{u,1} = \frac{M_u^+}{0.95\ell_1} \qquad \text{Eq. (9.8)}$$

In this equation, M_u^+ is the positive bending moment in the subdiaphragm to the west of the opening.

It is assumed that the subdiaphragms are fixed at both ends and are subjected to a portion of the total uniform diaphragm load, w_W, based on the mass of the segment. Because each segment has the same area, each segment resists 50 percent of w_W. From statics, M_u^+ is equal to the following:

$$M_u^+ = \frac{(w_W / 2)b_{open}^2}{24} = \frac{(1.92 / 2) \times 15.0^2}{24} = 9.0 \text{ ft-kips}$$

Therefore, the secondary tension chord force at the center of the opening is equal to the following:

$$T_{u,1} = \frac{M_u^+}{0.95\ell_1} = \frac{9.0}{0.95 \times 42.5} = 0.2 \text{ kips}$$

The total tension chord force at the center of the opening is equal to $0.5 + 0.2 = 0.7$ kips, which is less than the 22.6-kip tension chord force determined for the overall diaphragm.

Secondary tension chord forces develop at the corners of the openings due to the negative bending moments in the subdiaphragm to the east of the opening. The tension chord force occurring along column line 3 is equal to the following:

$$T_{u,2} = \frac{M_u^-}{0.95\ell_2} = \frac{(w_W / 2)b_{open}^2 / 12}{0.95\ell_2} = \frac{(1.92 / 2) \times 15.0^2 / 12}{0.95 \times 42.5} = 0.5 \text{ kips}$$

- Unit shear forces

  Along column lines B and F, the shear force in the slab is equal to 57.6 kips (see Figure 9.34), which is distributed over a length of 120.5 ft. Therefore, the unit shear force is equal to the following:

$$(v_u)_B = (v_u)_F = \frac{57.6}{120.5} = 0.48 \text{ kips/ft}$$

Just to the north of column line B and to the south of column line F, the shear force is equal to 115.2 kips, which is distributed over a length of $120.5 - 35.5 = 85.0$ ft because of the openings. Thus, the unit shear forces at these locations are equal to the following:

$$(v_u)_B = (v_u)_F = \frac{115.2}{85.0} = 1.36 \text{ kips/ft}$$

- Collector forces

  Collectors are required because the walls along column lines B and F do not extend the entire depth of the diaphragm in the direction of analysis. In this example, the portions of the slab in line with the walls are the collectors and the widths of the collectors are taken greater than the thicknesses of the walls. The effective slab width, b_{eff}, is equal to the following:

  $$b_{eff} = t + (\ell_w / 2) = (12.0 / 12) + (35.5 / 2) = 18.75 \text{ ft}$$ Sect. 9.3.3

  In buildings assigned to SDC C, collectors and their connections must be designed for the maximum of the three forces given in ASCE/SEI 12.10.2.1:

1. Forces calculated using the seismic load effects including overstrength of ASCE/SEI 12.4.3 with the seismic forces determined by the Equivalent Lateral Force Procedure of ASCE/SEI 12.8 or the modal response spectrum analysis procedure of ASCE/SEI 12.9.1.

   For the ordinary building frame system with ordinary reinforced concrete shear walls in the east-west direction, the overstrength factor, Ω_o, is equal to 2.5 (see ASCE/SEI Table 12.2-1). Therefore, the required in-plane diaphragm force at the second-floor level is equal to $2.5 \times 24.8 = 62.0$ kips (see Table 3.31 in Example 3.14).

2. Forces calculated using the seismic load effects including overstrength of ASCE/SEI 12.4.3 with the seismic forces determined by ASCE/SEI Eq. (12.10-1).

   Using the information in Table 3.31, the diaphragm force, F_{px}, at the second-floor level is equal to the following:

   $$F_{p2} = \left(\sum_{i=2}^{R} F_i / \sum_{i=2}^{R} w_i \right) w_{p2} = (1{,}575.5 / 43{,}763) \times 3{,}840 = 0.036 \times 3{,}840 = 138.2 \text{ kips}$$

   The required in-plane force including overstrength is equal to $2.5 \times 138.2 = 345.6$ kips.

3. Forces calculated using the load combinations of ASCE/SEI 2.3.6 with the seismic forces determined by ASCE/SEI Eq. (12.10-2).

   The diaphragm force, F_{px}, at the second-floor level based on ASCE/SEI Eq. (12.10-2) is equal to 345.6 kips (see Table 3.31).

   Therefore, the collectors and their connections to the vertical elements of the SFRS must be designed for the effects due to the 345.6-kip in-plane force stipulated in the second and third requirements. Thus, the total axial force in the collector is equal to 77.9 kips (see Figure 9.36).

For purposes of analysis, it is assumed that 20 percent of the total axial force is transferred directly into the wall $(T_d = C_d = 0.20 \times 77.9 = 15.6 \text{ kips})$ and 80 percent is transferred by shear-friction adjacent to the wall $(T_v = C_v = 77.9 - 15.6 = 62.3 \text{ kips})$.

9.9.10 Example 9.10 – Determination of Diaphragm Reinforcement: Building #2, SDC C, Collectors Required, Collector Width Wider than the Width of the Vertical Elements of the SFRS

Determine the required diaphragm reinforcement in the slab at the second-floor level of Building #2 for seismic forces in the east-west direction assuming only the walls on column lines B and F are part of the SFRS (see Figure 1.2). Also assume a 4.5-in. thick slab, 12-in.-thick walls, normalweight concrete with $f'_c = 4{,}000$ psi, and Grade 60 reinforcement. It is assumed that the portions of the slab in line with the walls are the collectors and the widths of the collectors are wider than the thicknesses of the walls.

Design data are given in Sect. 1.2.2. See Example 9.9.

Step 1 – Determine the chord reinforcement

From Example 9.9, $T_u = 22.6 \text{ kips}$ at column lines 1 and 4.

$$A_{s(\text{chord})} = \frac{T_u}{\phi f_y} = \frac{22.6}{0.9 \times 60} = 0.42 \text{ in.}^2 \qquad \text{Eq. (9.13)}$$

Provide 1-#6 bar $(A_{s,provided} = 0.44 \text{ in.}^2)$ located near the edges of the diaphragm at mid-depth of the 4.5-in.-thick slab.

At the edges of the openings:

$$A_{s(\text{chord})} = \frac{T_{u,2}}{\phi f_y} = \frac{0.5}{0.9 \times 60} = 0.01 \text{ in.}^2$$

The required amount of secondary chord reinforcement along the slab edges adjacent to the openings at column lines 2 and 3 is nominal. The collector reinforcement for seismic force transfer in the north-south direction will be provided along these edges of the opening, which is greater than 0.01 in.^2

Step 2 – Determine the diaphragm shear reinforcement

The largest factored unit shear forces occur along column lines B and F and are equal to 1.36 kips/ft (see Figure 9.36).

Check shear strength requirements assuming $\rho_t = 0$: Eq. (9.14)

$$V_u = 1.36 \text{ kips/ft} < \text{lesser of } \begin{cases} \phi A_{cv}(2\lambda\sqrt{f_c'} + \rho_t f_y) = (0.75 \times 4.5 \times 12.0) \times [(2 \times 1.0 \times \sqrt{4{,}000}) + 0] \,/\, 1{,}000 = 5.1 \text{ kips/ft} \\ \phi 8 A_{cv}\sqrt{f_c'} = 0.75 \times 8 \times 4.5 \times 12.0 \times \sqrt{4{,}000} \,/\, 1{,}000 = 20.5 \text{ kips/ft} \end{cases}$$

Therefore, no shear reinforcement is required to satisfy shear strength requirements.

Step 3 – Determine the shear transfer reinforcement

Shear transfer reinforcement between the diaphragm and walls on column lines B and F assuming the slab and walls are not placed at the same time and the interface is not intentionally roughened.

The total shear force transferred between the diaphragms and the walls considering 80 percent of the collector axial force is transferred by shear is equal to the following (see Figure 9.36, Table 9.7, and Step 4 in Example 9.9):

$$(v_{u,W})_B = (v_{u,W})_F = \frac{R_F}{\ell_w} + \frac{(T_v + C_v)}{\ell_w} = \frac{172.8}{35.5} + \frac{2 \times 62.3}{35.5} = 4.87 + 3.51 = 8.38 \text{ kips/ft}$$

The required area of shear-friction reinforcement is equal to the following:

$$A_{vf} = \frac{(v_{u,W})_B}{\phi \mu f_y} = \frac{(v_{u,W})_F}{\phi \mu f_y} = \frac{8.38}{0.75 \times 0.6 \times 1.0 \times 60} = 0.31 \text{ in.}^2/\text{ft} \qquad \text{Eq. (9.16)}$$

Provide #5 dowel bars spaced at 12 in. on center $(A_{s,provided} = 0.31 \text{ in.}^2/\text{ft})$ over the lengths of the walls.

Shear transfer reinforcement between the diaphragm and the collectors (see Figure 9.36):

$$A_{vf} = \frac{(v_u)_B}{\phi \mu f_y} = \frac{(v_u)_F}{\phi \mu f_y} = \frac{1.36 + 0.48}{0.75 \times 1.4 \times 1.0 \times 60} = 0.03 \text{ in.}^2/\text{ft}$$

A value of $\mu = 1.4\lambda$ for monolithic concrete is used because the collectors are part of the slab (see Table 9.7).

Because the area of shear-friction is very small, it is safe to assume that the bottom flexural reinforcement in the slab oriented in the north-south direction can be used as the shear transfer reinforcement between the diaphragm and the collectors.

Step 4 – Determine the collector reinforcement

The total area of collector reinforcement is equal to the following:

$$A_{s(\text{collector})} = \frac{T_u}{\phi f_y} = \frac{77.9}{0.9 \times 60} = 1.44 \text{ in.}^2 \qquad \text{Eq. (9.19)}$$

Assuming 20 percent of the maximum axial force in the collectors along column lines B and F is transferred directly into the ends of the walls, the required area of longitudinal collector reinforcement, $A_{s(d)}$, is equal to the following:

$$A_{s(d)} = \frac{T_d}{\phi f_y} = \frac{0.2T_u}{\phi f_y} = \frac{0.2 \times 77.9}{0.9 \times 60} = 0.29 \text{ in.}^2 \qquad \text{Eq. (9.19)}$$

Provide 1-#5 bar $(A_{s,provided} = 0.31 \text{ in.}^2)$ along column lines B and F. These reinforcing bars are in addition to any other reinforcement in the slab and must be placed within the 12.0-in. thickness of the walls.

The remaining required collector reinforcement is uniformly distributed within b_{eff} :

$$A_{s(v)} = \frac{0.8T_u}{\phi f_y} = \frac{0.8 \times 77.9}{0.9 \times 60} = 1.15 \text{ in.}^2$$

Provide 4-#5 bars $(A_{s,provided} = 1.24 \text{ in.}^2)$ uniformly spaced within $b_{eff} = 18.75$ ft.

Check the axial compression force in the collectors.

For the 12.0-in.-wide section reinforced with 1-#5 bar:

$$C_d = 0.2 \times 77.9 = 15.6 \text{ kips}$$

$$< \phi[0.85 f_c'(A_g - A_s) + f_y A_s] = 0.65 \times \{[0.85 \times 4 \times (54.0 - 0.31)] + (60 \times 0.31)\} = 130.7 \text{ kips} \qquad \text{Eq. (9.20)}$$

For the $(18.75 \times 12) - 12.0 = 213.0\text{-in.-wide}$ section reinforced with 4-#5 bars:

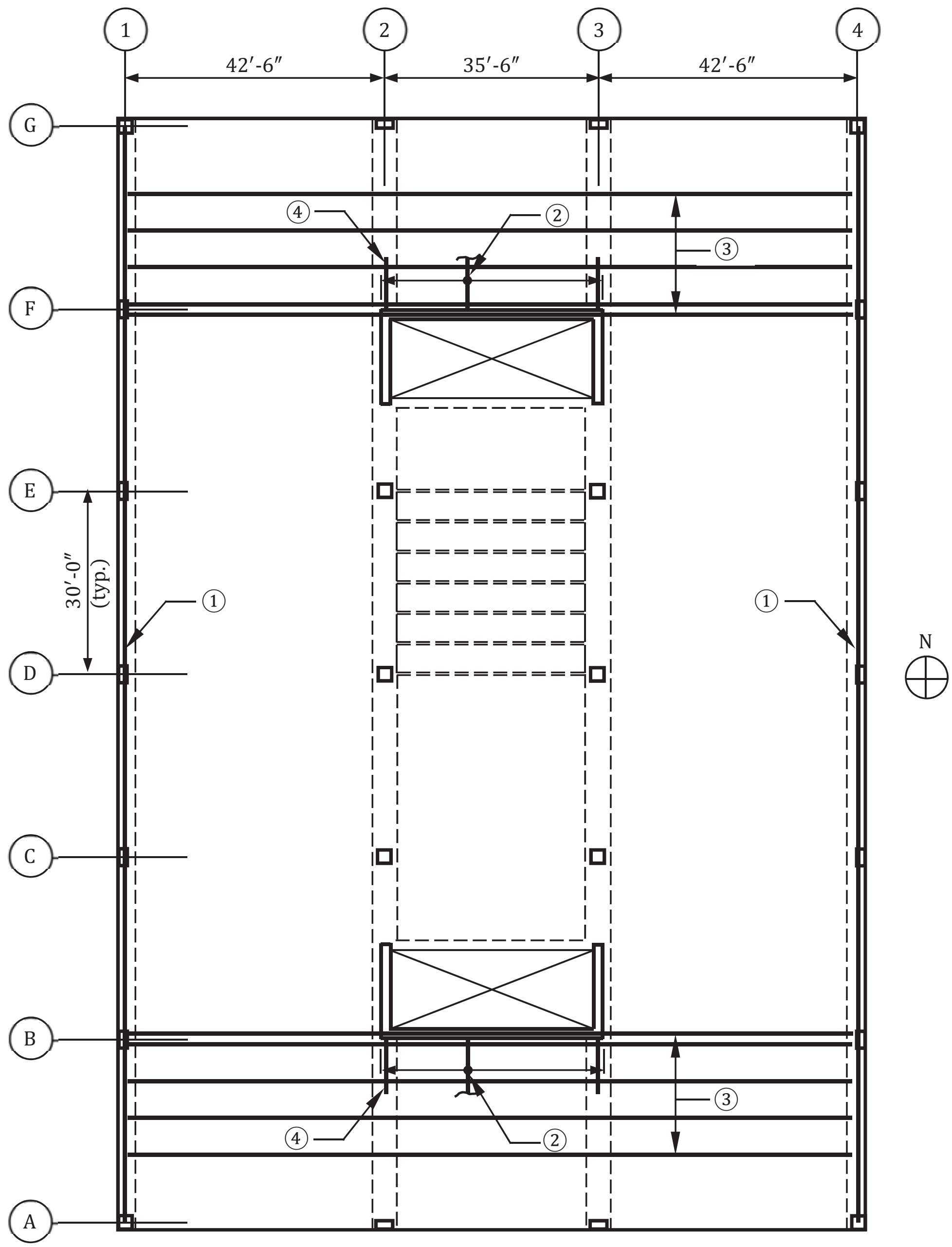
$$C_v = 0.8 \times 77.9 = 62.3 \text{ kips}$$

$$< \phi[0.85 f_c'(A_g - A_s) + f_y A_s] = 0.65 \times \{[0.85 \times 4 \times (958.5 - 1.24)] + (60 \times 1.24)\} = 2{,}163.9 \text{ kips}$$

Step 5 – Determine the reinforcement due to eccentricity of collector forces

The in-plane bending moment, M_e, due to the eccentricity, e, between the portion of the collector force not transferred directly in to the ends of the walls and the centerline of the walls can be determined by the following equation:

$$M_e = (T_v + C_v)e - V_s \ell_w \quad \text{Eq. (9.17)}$$



1. ① — Chord reinforcement: 1-#6 bar
2. ② — Shear transfer reinforcement: #5 @ 12″ dowel bars
3. ③ — Collector reinforcement: 5-#5 bars
4. ④ — Reinforcement due to eccentricity of collector forces: 2-#6 bars
5. Provide standard 90-deg hooks at the ends of all bars
6. Provide Class B lap splices or Type 2 mechanical connectors where required
7. Other reinforcement not shown for clarity

Figure 9.38 Reinforcement details for the diaphragm at the second-floor level in Examples 9.9 and 9.10.

In this equation, $e = b_{eff} / 2 = 18.75 / 2 = 9.4$ ft. Conservatively taking the shear strength of the diaphragm due to the reinforcement, V_s, equal to zero, M_e is equal to the following:

$$M_e = (2 \times 62.3) \times 9.4 = 1{,}171.2 \text{ ft-kips}$$

The required reinforcement, $A_{s(\mathrm{ecc})}$, is equal to the following:

$$A_{s(\mathrm{ecc})} = \frac{M_e / 0.95\ell_w}{\phi f_y} = \frac{1{,}171.2 / (0.95 \times 35.5)}{0.9 \times 60} = 0.64 \text{ in.}^2 \qquad \text{Eq. (9.18)}$$

Provide 2-#6 bars $(A_{s,provided} = 0.88 \text{ in.}^2)$ place perpendicular to the face of the wall at both ends; these bars must be developed for tension into the slab and into the wall.

Reinforcement details for the second-floor diaphragm are given in Figure 9.38 for seismic forces in the east-west direction. For simpler detailing, all the collector bars on column lines B and F are extended the full depth of the diaphragm. Similar details can be obtained for seismic forces in the north-south direction.

Chapter 10
FOUNDATIONS

10.1 Overview

The main function of a foundation is to transmit the loads from the structure above to the soil below. In building structures, the loads are transmitted directly or indirectly by columns and walls. Foundations must be located on a soil or rock stratum with adequate strength to support the loads; in other words, the loads must be spread out over a sufficient area so the resulting pressure is not greater than the allowable bearing capacity of the soil or rock. In addition to strength, the total settlement and the differential settlement between adjoining foundations must be limited to tolerable amounts in order to prevent possible damage to the structure. Overturning, sliding, and rotation must also be limited to tolerable values.

Numerous types of shallow and deep foundations are available to support the loads from the superstructure of a building. The design and detailing of the following foundation systems are covered in this chapter:

- Isolated spread footings
- Walls footings
- Combined spread footings
- Drilled piers (caissons)

Provisions for foundations are given in ACI Chapter 13, which are applicable to foundations supporting buildings assigned to Seismic Design Category (SDC) A and B.

Detailed information on the design of pile caps is given in Reference 23. Design and detailing methods for cantilevered retaining walls are given in Reference 20.

10.2 Design Criteria

Foundations must be proportioned such that the requirements in the governing building code for bearing effects, stability against overturning, and sliding at the soil-foundation interface are satisfied (ACI 13.2.6.1). Typically, the base area of a spread footing or the number and arrangement of deep foundation members are determined using allowable strengths of the soil or rock beneath the foundation (which have usually been established by a geotechnical investigation) and allowable stress load combinations. However, nominal geotechnical strengths with resistance factors and strength design load combinations may also be used. When determining the base dimensions of footings, the minimum moment requirement for slenderness considerations given in ACI 6.6.4.5 need not be considered for transfer of forces and moments to a footing; only the calculated end moment at the base of a column needs to be transferred.

Factored reactions are used to design foundation members except as permitted by ACI 13.4.2 for deep foundation members (ACI 13.2.6.3). For example, the thickness of a footing and the required area of flexural reinforcement are determined using factored bending moments. Similarly, shear strength requirements must be satisfied using factored shear forces. As note above, it is permitted to design deep foundation members using allowable stress load combinations and allowable strengths (ACI 13.4.2).

Foundation design is permitted to be based on any procedure that satisfies equilibrium and geometric compatibility (ACI 13.2.6.4). The strut-and-tie method in ACI Chapter 23 may be used to design foundations (ACI 13.2.6.5).

10.3 Footings

10.3.1 Overview

In general, footings are shallow foundations that transfer the load from the superstructure to a soil stratum beneath the structure relatively close to the ground surface. A soil stratum must be identified with adequate bearing capacity to support the loads. To avoid possible frost heave, shallow foundations must be placed below the frost line at the site (where applicable). A geotechnical report provides guidance on the soil bearing capacity and the appropriate depth for particular site conditions.

Spread footings typically support one or more vertical elements. The isolated spread footing in Figure 10.1 supports a single column. Its function is to spread the column load to the soil so that the maximum pressure beneath the footing is less than or equal to the permissible bearing capacity of the soil. Flexural reinforcement is provided in two orthogonal directions at the bottom of the footing.

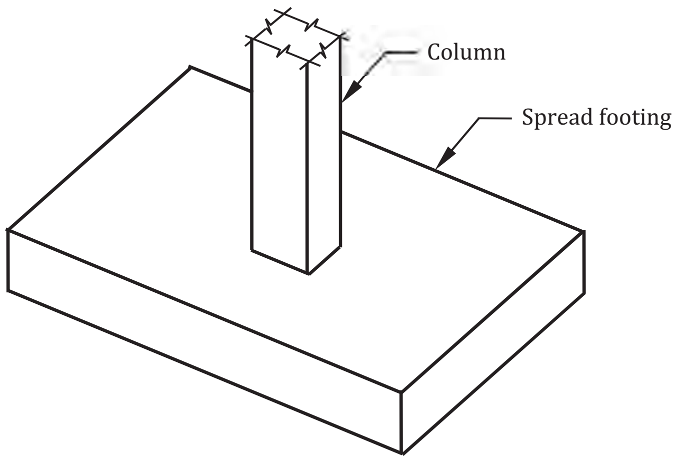


Figure 10.1 An isolated spread footing.

Wall footings are usually continuous under the length of a wall and are similar to spread footings (see Figure 10.2). Flexural reinforcement is placed at the bottom of the footing perpendicular to the face of the wall and shrinkage and temperature reinforcement is provided parallel to the length of the wall.

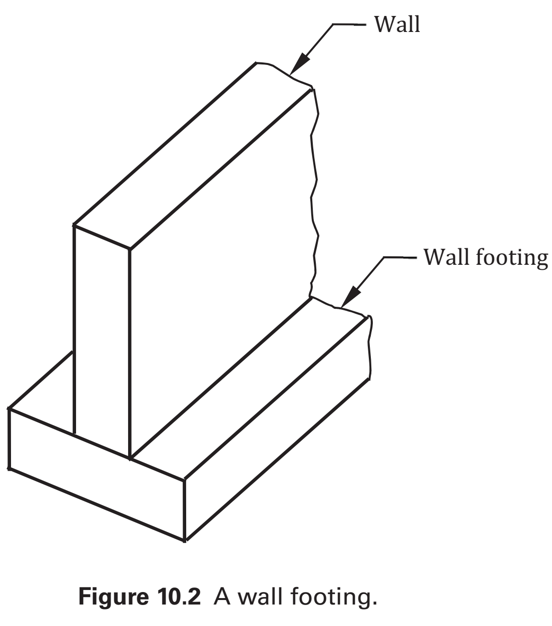


Figure 10.2 A wall footing.

A combined footing is a spread footing that supports multiple columns or walls on the same footing (see Figure 10.3). This type of footing is commonly used where the space between adjoining footings is small or where a building is close to a property line. If the load at the interior column is greater than the load at the edge column for the arrangement shown in the figure, the footing can be sized so that the pressure at the base of the footing is uniform. Where an exterior column has a greater load than an adjacent interior column, a trapezoidal footing can be used to achieve a uniform distribution of pressure beneath the footing (see Figure 10.4).

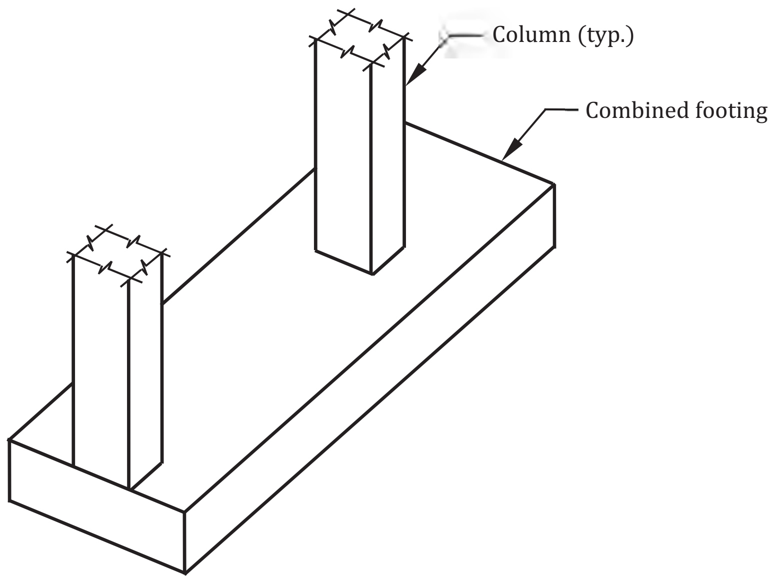


Figure 10.3 A rectangular combined footing.

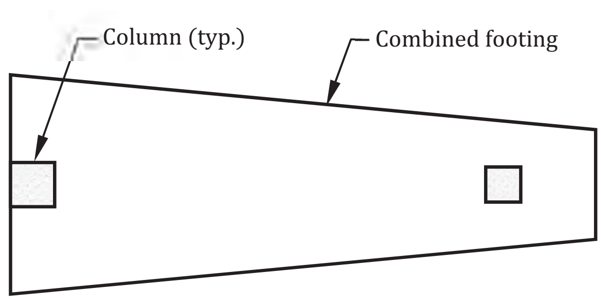


Figure 10.4 A trapezoidal spread footing.

A strap footing (or, cantilever footing) consists of an eccentrically loaded footing connected to an adjacent footing by a strap beam, as shown in Figure 10.5. The purpose of the strap beam is to transmit the moment from the eccentrically loaded footing to the adjacent footing so that a uniform soil pressure is achieved beneath both footings. The strap beam must be rigid to avoid rotation of the exterior footing; providing a strap beam with a moment of inertia of at least two times that of the footing will usually accomplish this.

The design and detailing of reinforced concrete footings generally involves determining the following:

1. Base area of the footing
2. Thickness of the footing
3. Required amount of flexural and interface reinforcement
4. Reinforcement details

Methods on how to determine these items are given in the following sections.

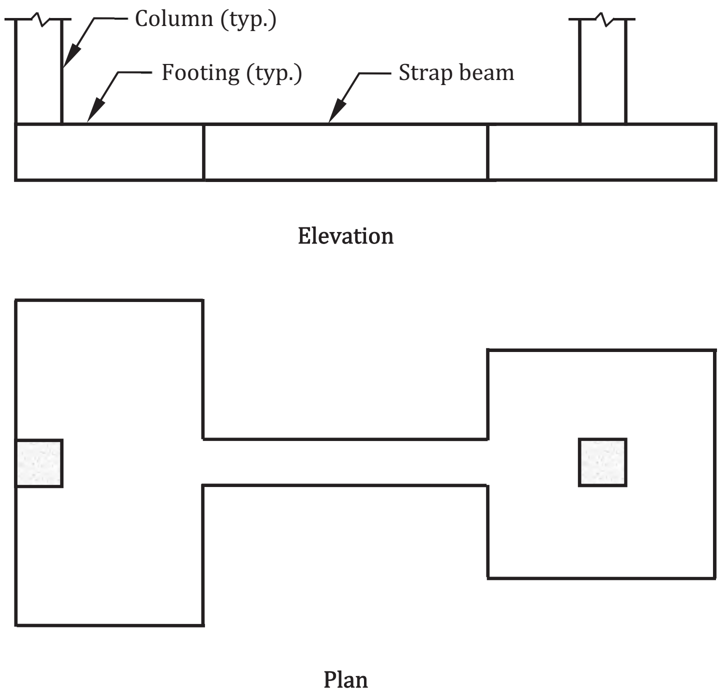


Figure 10.5 A strap footing.

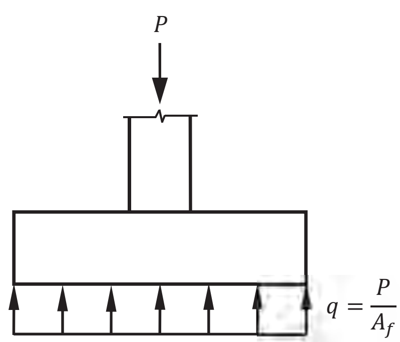


Figure 10.6 Soil pressure distribution beneath a concentrically loaded footing.

10.3.2 Determining the Base Area of a Footing

Overview

The base area of a footing subjected to sustained gravity loads is usually determined using allowable stress load combinations and an allowable soil bearing capacity determined from a geotechnical investigation (the presumptive load-bearing values in IBC Table 1806.2 may also be used where permitted). The allowable soil stress provided in the geotechnical report is either the stress associated with soil bearing failure (including a factor of safety) or an equivalent stress meant to control settlement of the footing based on the anticipated sustained loads. It is common for footings to be designed in the elastic range for the case of sustained gravity loads without any uplift.

Bearing failure is the primary design consideration when footings are subjected to wind and/or seismic effects. For these types of transient loads, it is common practice to use allowable stress load combinations to size the footing with an allowable bearing capacity equal to the static bearing capacity multiplied by a factor to account for the transient nature of the loads. Footings are typically designed in the elastic range when subjected to the effects from wind loads, and uplift is commonly permitted provided the maximum bearing stress is less than or equal to the allowable value.

Isolated Spread Footing

Consider the footing in Figure 10.6, which is subjected to a service axial load, P, acting through the centroid of the footing base area, A_f. For purposes of design, the footing is assumed to be rigid and the resulting soil pressure, q, at the base of the footing is assumed to be uniform. The required area of the footing is equal to the total service load divided by the net permissible soil pressure, q_p, which is equal to the allowable bearing capacity of the soil, q_a, minus the weight of the surcharge above the base of the footing (the surcharge consists of the weight of the soil and concrete above the base plus any additional service surcharge applied at the surface):

$$A_f = BL = \frac{P}{q_p} \tag{10.1}$$

In this equation, B and L are the base dimensions of the footing.

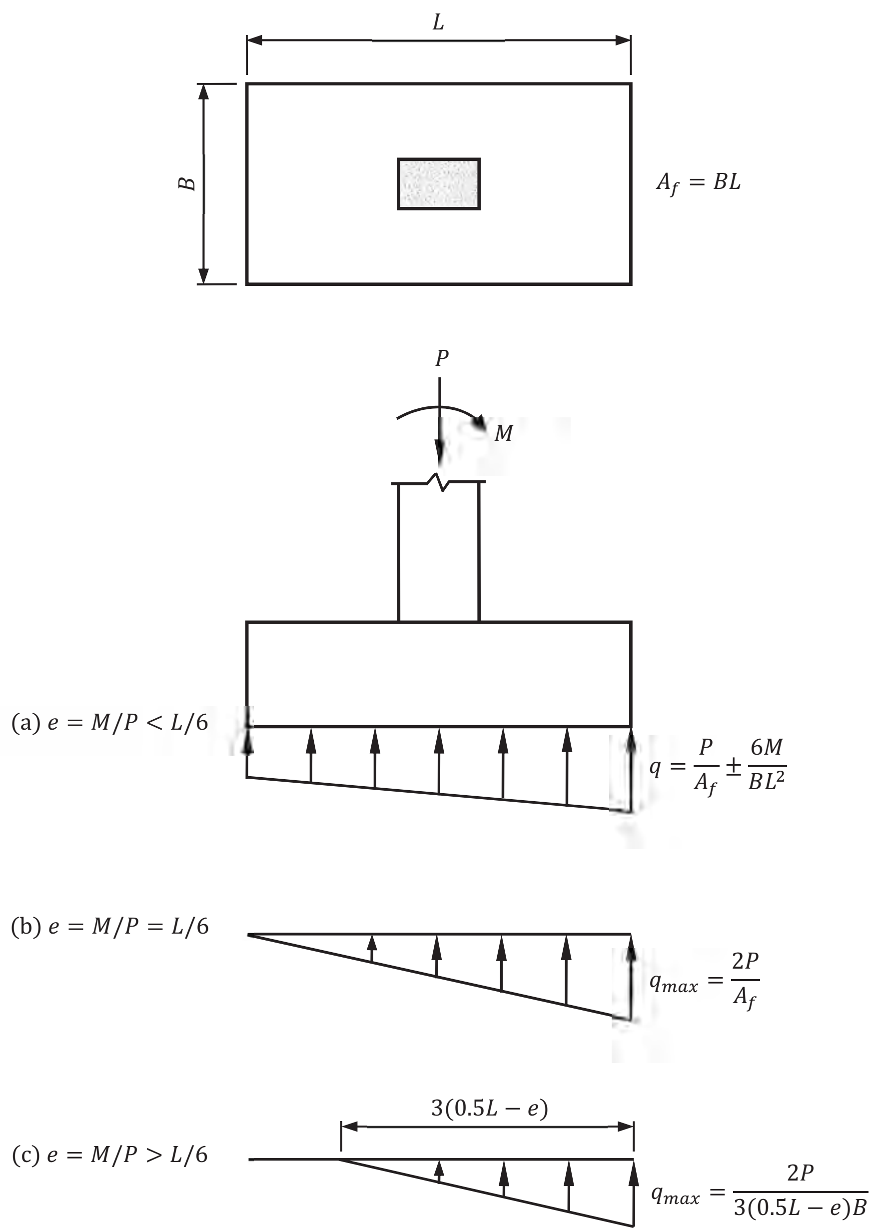


Figure 10.7 Soil pressure distribution beneath a footing subjected to axial force and moment. (a) $e < L/6$. (b) $e = L/6$. (c) $e > L/6$.

For footings subjected to an axial force and a moment, or to an axial force at an eccentricity, e, from the centroid of the footing, the total stress at the base of the footing is the sum of the stresses due to the axial force, P (stress $=$ axial force/footing area) and the bending moment, M (stress $=$ moment/section modulus of footing). The total pressure is assumed to vary linearly, as shown in Figure 10.7(a) for the case where the axial force falls within the kern of the footing area, that is, where $e = M / P < L / 6$. The minimum and maximum pressures at the edges of the footing in this case can be determined from the following equation:

$$q = \frac{P}{A_f} \pm \frac{6M}{BL^2} \tag{10.2}$$

Where $e = L / 6$, the pressure at one edge of the footing is equal to zero and the maximum pressure at the other edge is equal to $2P / A_f$ [see Figure 10.7(b)].

The base dimensions of the footing, B and L, can be obtained by the following equation where $e \leq L / 6$:

$$q_{max} = \frac{P}{BL} + \frac{6M}{BL^2} \leq q_p \tag{10.3}$$

It is evident that bearing capacity requirements are satisfied for any combination of B and L that satisfies this equation.

Where $e > L / 6$, the combined minimum stress determined by Equation (10.2) is less than zero, which means the stress is tensile. Because tension cannot be transmitted between the footing and the soil, Equation (10.2) is no longer valid, and the pressure distribution shown in Figure 10.7(c) must be used. In such cases, B and L can be obtained by the following equation:

$$q_{max} = \frac{2P}{3(0.5L - e)B} \leq q_p \tag{10.4}$$

In the case of square footings, Equation (10.4) can be solved for $B = L$ directly:

$$B = L \geq e + \frac{\sqrt{3}}{3}\sqrt{3e^2 + 4(P / q_p)} \tag{10.5}$$

Combined Footings

It is desirable to design combined footings so that the pressure is uniform over the entire area of the footing; among other things, this helps prevent the footing from rotating. For the combined rectangular footing in Figure 10.8, the columns are supporting axial forces P_1 and P_2 at the locations x_1 and y_1 measured from the edge of the column on the left.

In order for the pressure beneath the footing to be uniform, the resultant force $P = P_1 + P_2$ must act through the centroid of the footing. The distance x from the edge of the footing to P is obtained by summing moments about this point and solving for x :

$$x = \frac{P_1 x_1 + P_2 x_2}{P} \tag{10.6}$$

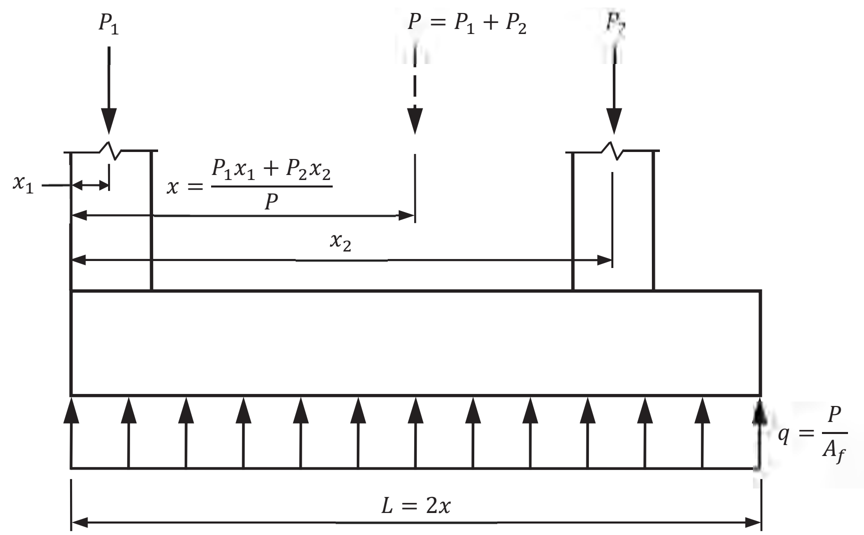


Figure 10.8 Uniform soil pressure distribution for a combined rectangular footing.

A uniform pressure is obtained by setting the footing dimension L equal to $2x$. Once L has been determined, the width of the footing, B, can be calculated by Equation (10.1):

$$B = \frac{P}{Lq_p} \tag{10.7}$$

The dimensions of a combined trapezoidal footing can be determined using similar methods. Consider the trapezoidal footing in Figure 10.9. Assuming the length of the footing, L, is given based on the size and spacing of the columns, the dimensions B_1 and B_2 can be determined so that a uniform pressure occurs beneath the footing.

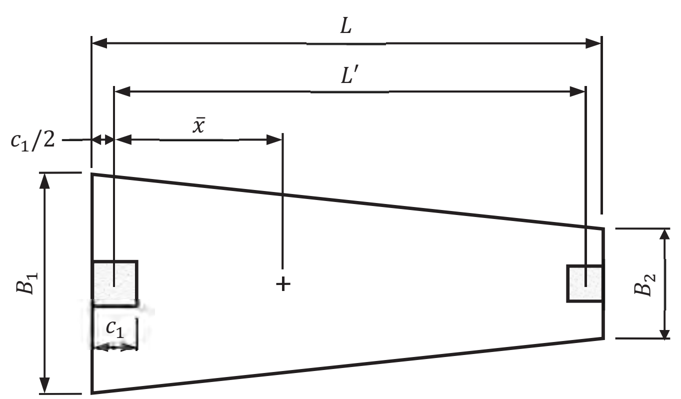


Figure 10.9 Dimensions of a combined trapezoidal footing.

The term $\bar{x}$ is the distance from the center of the heavier-loaded column on the left to the location where the centroid of the footing coincides with the resultant of the column loads, $P = P_1 + P_2$. It is evident that a trapezoidal footing is not possible if $\bar{x} + 0.5c_1 < L\,/\,3$; in such cases, the result is a triangular footing with the column on the right not fully supported on the foundation. A combined rectangular footing occurs where $\bar{x} + 0.5c_1 = L\,/\,2$. Therefore, a combined trapezoidal footing is possible where $L\,/\,3 < \bar{x} + 0.5c_1 < L\,/\,2$. The following equation can be used to determine the location of the centroid of the footing with respect to the left edge where $x = \bar{x} + 0.5c_1$:

$$x = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \tag{10.8}$$

The distance $\bar{x}$ from the center of the left column to the resultant axial force, P, is obtained by summing moments about this point and solving for $\bar{x}$:

$$\bar{x} = \frac{P_2 L'}{P} = x - 0.5c_1 = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) - 0.5c_1 \tag{10.9}$$

where L' is the distance between P_1 and P_2 (that is, the distance between the centerlines of the columns).

The pressure at the base of the footing must be less than or equal to the permissible soil pressure:

$$q_p = \frac{P}{A_f} = \frac{2P}{L(B_1 + B_2)} \tag{10.10}$$

Equations (10.9) and (10.10) can be solved for the two unknowns B_1 and B_2.

10.3.3 Determining the Thickness of a Footing

Overview

Once the required area of a footing has been established on the basis of the service loads and the permissible bearing capacity of the soil, the thickness, h, can be determined considering both flexure and shear.

Isolated spread footings and wall footings must be designed for bending moments caused by the pressure developed at the base of the footing due to factored loads. Requirements for one-way and two-way shear must also be satisfied (two-way shear is not applicable to wall footings). Methods on how to determine the thickness of a footing based on these three criteria are given below.

A minimum depth of 6 in. is required above the bottom reinforcement of footings (ACI 13.3.1.2). According to ACI Table 20.5.1.3.1, the minimum concrete cover to the reinforcement is equal to 3 in. for concrete cast against and permanently exposed to earth. Thus, the minimum thickness is equal to approximately 10 in.

It is also important to consider the minimum footing thickness based on the development of the dowel bars emanating from the footing, which are spliced to the longitudinal reinforcement in the columns or walls. Information on the required development lengths is covered in Section 10.3.6 of this publication.

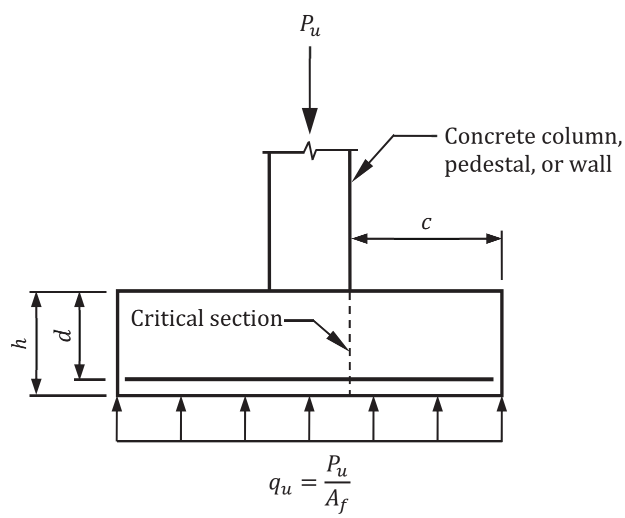


Figure 10.10 Factored pressure at the base of a concentrically loaded, isolated spread footing.

Minimum Thickness Based on Flexural Requirements

A footing must be designed for bending moments induced by the pressure developed at the base of the footing from the factored loads. A concentrically loaded, isolated spread footing is illustrated in Figure 10.10. The factored pressure, q_u, is equal to the factored axial force, P_u, divided by the area of the footing, A_f.

The critical section for flexure for an isolated footing supporting a concrete column, pedestal, or wall is located at the face of the supported member (ACI Table 13.2.7.1). The maximum factored bending moment, M_u, at this critical section can be calculated by the following equation:

$$M_u = \frac{q_u c^2}{2} \tag{10.11}$$

where c is the distance from the critical section to the edge of the footing (that is, the length of the cantilever).

In cases where the factored pressure at the base of the footing is nonuniform, M_u can be determined from statics, just like in the case of factored uniform pressure, or it can be conservatively determined by Equation (10.11) assuming the maximum pressure at the edge of the footing is uniformly distributed across the entire width of the footing.

Critical section locations for two other cases are given in ACI Table 13.2.7.1: (1) footings supporting masonry walls and (2) footings supporting columns with a steel base plate. In the first case, the critical section is located halfway between the center and the face of the wall, and for the second case, it is located halfway between the face of the column and the edge of the base plate.

Once the maximum bending moment, M_u, at the critical section has been determined, the required area of flexural reinforcement, A_s, is determined in the same way as any other flexural member. The nominal strength coefficient of resistance, R_n, is calculated by the following equation:

$$R_n = \frac{M_u}{\phi b_w d^2} = \rho f_y \left(1 - \frac{0.59 \rho f_y}{f_c'}\right) \tag{10.12}$$

where $\rho = A_s / b_w d$.

The required thickness of the footing based on flexural requirements can be obtained by assuming the minimum area of reinforcing steel prescribed in ACI 13.3.2.1 and 13.3.3.1 is used in the footing, which is equal to 0.0018 times the gross area of the footing. Because the design strength equation for flexure is based on the effective depth, d, and not the overall thickness of the member, 0.0018 must be modified to account for this. Assuming $d / h \cong 0.9$, then $\rho = 0.0018 / 0.9 = 0.0020$.

Assuming $\rho = 0.0020$, Grade 60 reinforcement, a tension-controlled section $(\phi = 0.90)$, and a one-foot-wide design strip, Equation (10.12) reduces to the following:

$$R_n = \frac{M_u}{0.9 d^2} = (0.0020 \times 60) \times \left(1 - \frac{0.59 \times 0.0020 \times 60}{f_c'}\right) = 0.12 \times \left(1 - \frac{0.0708}{f_c'}\right) \tag{10.13}$$

where f_c' has the units of ksi.

Thus, for a factored bending moment M_u at the critical section of the footing, the minimum effective depth of the footing, d, that satisfies flexural requirements can be determined by the following equation:

$$d \geq \sqrt{\frac{M_u}{0.108\left(1 - \frac{0.0708}{f_c'}\right)}} \tag{10.14}$$

In this equation, d has the units of inches, f_c' has the units of ksi, and M_u has the units of ft-kips.

Substituting Equation (10.11) into Equation (10.14) results in the following equation for concentrically loaded footings:

$$d \geq c\sqrt{\frac{P_u / A_f}{0.216\left(1 - \dfrac{0.0708}{f_c'}\right)}} \tag{10.15}$$

where d is in inches, c is in feet, P_u is in kips, A_f is in square feet, and f_c' is in ksi.

For $f_c' = 4{,}000$ psi, Equation (10.15) reduces to the following:

$$d \geq 2.2c\sqrt{\frac{P_u}{A_f}} \tag{10.16}$$

Minimum Thickness Based on Shear Requirements

Overview

The one-way and two-way shear requirements in ACI 8.5.3.1.1 and 8.5.3.1.2, respectively, must be satisfied for two-way footings with the critical sections for factored shear forces measured from the locations of the critical sections for M_u in ACI Table 13.2.7.1 (ACI 13.2.7.2). Note that the size effect modification factor, λ_s, defined in ACI 22.5 for one-way strength and in ACI 22.6 for two-way strength need not be considered when determining the design shear strength of one-way shallow footings, two-way isolated footings, or two-way combined footings (ACI 13.2.6.2).

One-way Shear

For one-way shear, the factored shear force, V_u, at the critical section, determined using the factored pressure at the base of the footing q_u, must be less than or equal to the design shear strength, ϕV_c, determined in accordance with ACI 22.5.5.1.

The tributary area used in the calculation of V_u for a concentrically loaded, isolated spread footing supporting a column is given in Figure 10.11. The critical section is located a distance d from the face of the column, and the following equation must be satisfied at this location:

$$V_u = q_u B(c - d) \leq \phi V_c = \phi 8\lambda(\rho_w)^{1/3}\sqrt{f_c'}Bd \tag{10.17}$$

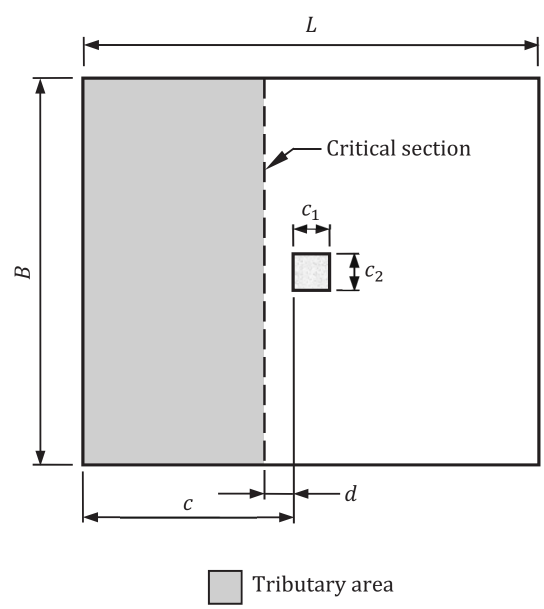


Figure 10.11 Critical section for one-way shear in an isolated spread footing.

In this equation, the design shear strength ϕV_c corresponds to members with less than the required minimum shear reinforcement (see ACI Table 22.5.5.1). The modification factor, λ, that reflects the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength is given in Table 10.1 based on equilibrium density and in Table 10.2 based on composition of aggregates in the concrete mix (see ACI 19.2.4).

Table 10.1 Values of λ Based on Equilibrium Density, w_c

Equilibrium Density, w_c	λ
$w_c \leq 100 \text{ lb/ft}^3$	0.75
$100 \text{ lb/ft}^3 < w_c \leq 135 \text{ lb/ft}^3$	$0.0075 w_c \leq 1.0$
$w_c > 135 \text{ lb/ft}^3$	1.0

Table 10.2 Values of λ Based on Composition of Aggregates

Concrete	Composition of Aggregates	λ
All-lightweight	Fine: ASTM C330 Coarse: ASTM C330	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330 and C33 Coarse: ASTM C330	0.75 to 0.85(1)
Sand-lightweight	Fine: ASTM C33 Coarse: ASTM C330	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33 Coarse: Combination of ASTM C330 and ASTM C33	0.85 to 1.0(2)

(1) Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.

(2) Linear interpolation from 0.85 to 1.0 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of aggregate.

The term ρ_w is the ratio of the flexural reinforcement to the gross area of the concrete area. Assuming minimum flexural reinforcement $(\text{that is, } \rho_w = 0.0020; \text{ see above})$, Equation (10.17) can be solved for the minimum d required to satisfy one-way shear requirements:

$$d \geq \frac{q_u c}{q_u + \phi\lambda\sqrt{f'_c}} \tag{10.18}$$

Two-way Shear

For two-way shear, the factored shear stress, v_u, at the critical section, determined using the factored pressure at the base of the footing, q_u, must be less than or equal to the design shear strength, ϕv_c, determined in accordance with ACI 22.6.5. For a concentrically loaded footing, the following equation must be satisfied at the critical section, which is located $d / 2$ from the face of the column (see Figure 10.12):

$$v_u = \frac{q_u[BL - (c_1 + d)(c_2 + d)]}{b_o d} \leq \phi v_c \tag{10.19}$$

where b_o is the perimeter of the critical section and v_c is the least of the values defined in ACI Table 22.6.5.2.

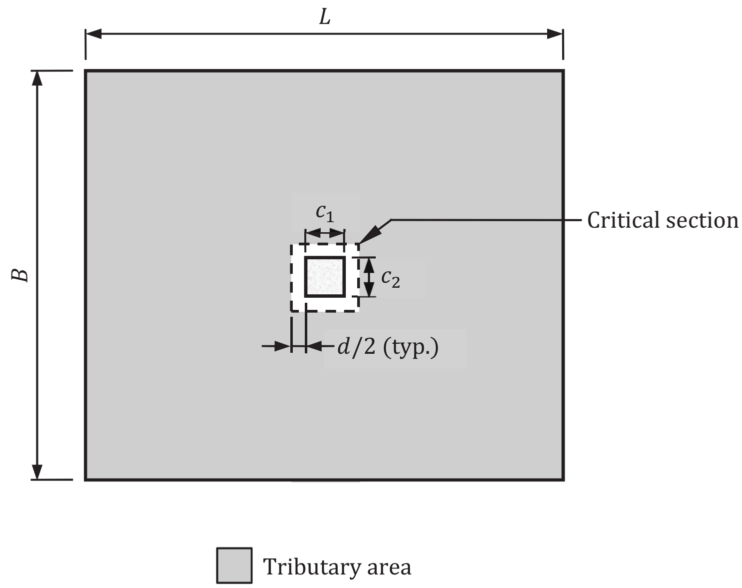


Figure 10.12 Critical section for two-way shear in an isolated spread footing.

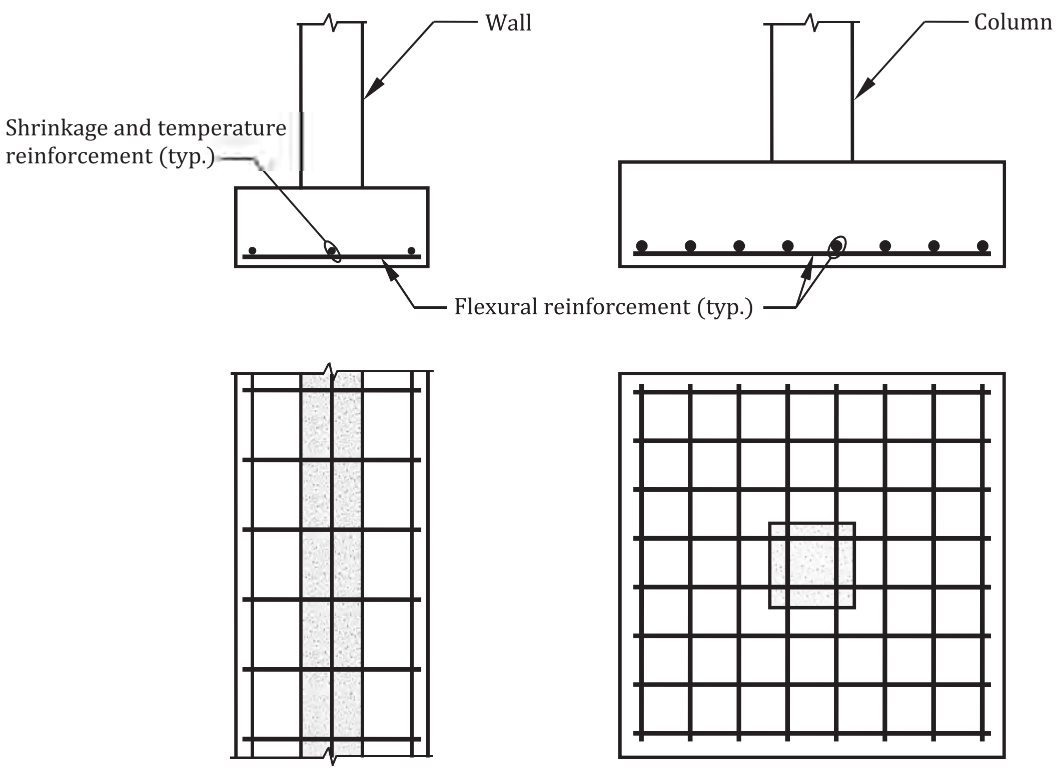


Figure 10.13 Distribution of flexural reinforcement in one-way footings and square two-way footings.

For footings supporting square columns, $\phi v_c = \phi 4\lambda\sqrt{f_c'}$ typically governs. Therefore, the following equation can be used to determine the minimum d required to satisfy two-way shear requirements for a square column supported by an isolated footing:

$$d \geq c_1\left[\frac{-a_1 + \sqrt{a_1^2 + q_u a_2 a_3}}{2a_2}\right] \tag{10.20}$$

where $a_1 = 0.5q_u + \phi v_c$

$a_2 = 0.25q_u + \phi v_c$

$a_3 = (A_f / c_1^2) - 1$

The largest d calculated by Equations (10.15), (10.18), and (10.20) is used in determining the overall thickness of a footing. Providing an effective depth equal to at least the largest of these three calculated values automatically satisfies all requirements for flexure, one-way shear, and two-way shear. The overall depth of the footing, h, can be obtained by adding 4 in. (3-in. cover plus the diameter of a flexural reinforcing bar) to the largest calculated effective depth, d.

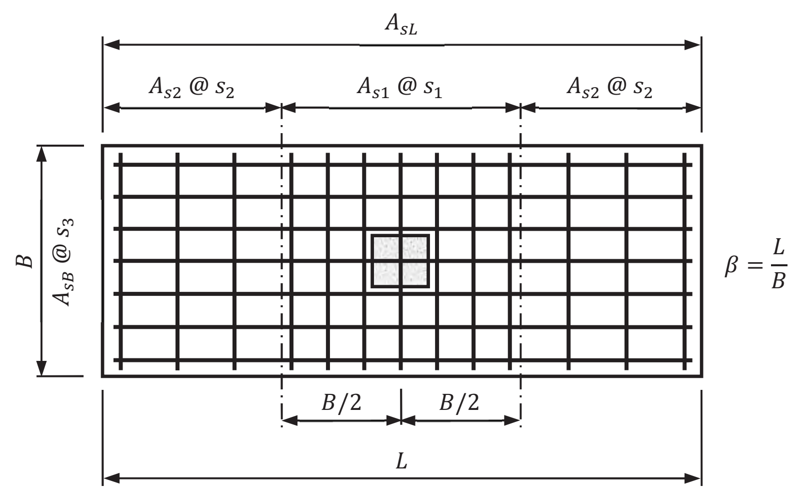
10.3.4 Determining the Flexural Reinforcement

By determining the effective depth d using the methods outlined in Section 10.3.3 of this publication, minimum (or, close to minimum) flexural reinforcement will be provided in the footing. It is evident that minimum reinforcement is applicable if d determined by Equation (10.15) or (10.16) is larger than those determined by Equations (10.18) and (10.20). Also, if d determined by Equation (10.18) or (10.20) is the largest, then minimum reinforcement is also applicable because d determined by Equation (10.15) or (10.16), which is smaller, is based on minimum reinforcement. In general, minimum reinforcement must be calculated using a footing thickness based on the largest effective depth determined by Equations (10.15), (10.18), and (10.20).

10.3.5 Detailing the Flexural Reinforcement

Requirements for the distribution of flexural reinforcement for one-way shallow footings are given in ACI 13.3.2.2 and for two-way isolated footings in ACI 13.3.3.2 and 13.3.3.3. For wall footings and square two-way footings, reinforcement is to be uniformly distributed across the entire width of the footing (see Figure 10.13). The same reinforcement is used in both orthogonal directions in square two-way footings because the factored bending moments at the critical sections are the same.

Requirements for distribution of flexural reinforcement in rectangular two-way footings are given in ACI 13.3.3.3. Reinforcement in the long direction is uniformly distributed across the entire width of the footing. In the short direction, a portion of the total reinforcement, $\gamma_s A_s$, must be uniformly distributed over a band width centered on the



$$A_{s1} = \frac{2}{\beta + 1} A_{sL}$$

$$A_{s2} = \frac{A_{sL} - A_{s1}}{2}$$

Figure 10.14 Distribution of flexural reinforcement in rectangular footings.

column or pedestal equal to the length of the short side of the footing where $\gamma_s = 2/(\beta+1)$ and β is the ratio of the long side to the short side of the footing (see Figure 10.14). The remainder of the reinforcement outside of the center band must be uniformly distributed.

For easier placement in the field, it is common practice to increase the required amount of reinforcement in the short direction by $2\beta/(\beta+1)$ and to space it uniformly across the long dimension of the footing instead of distributing the reinforcing bars as shown in Figure 10.14.

10.3.6 Development of Flexural Reinforcement

Development of reinforcement in footings must be in accordance with ACI Chapter 25 (ACI 13.2.8.1). The critical sections for development of reinforcement are the same as those given in ACI 13.2.7.1 for the factored bending moment and at all other vertical planes where changes of section or reinforcement occur (ACI 13.2.8.3). In other words, reinforcing bars must extend at least a tension development length, ℓ_d, beyond the critical section for flexure.

For a concrete column supported by an isolated footing, ℓ_d must be less than or equal to the available development length:

$$\ell_d \leq \frac{L - c_1}{2} - \text{cover} \tag{10.21}$$

where L and c_1 are the lengths of the footing and the column in the direction of analysis, respectively, and the cover is no less than 3 in. (see Figure 10.15).

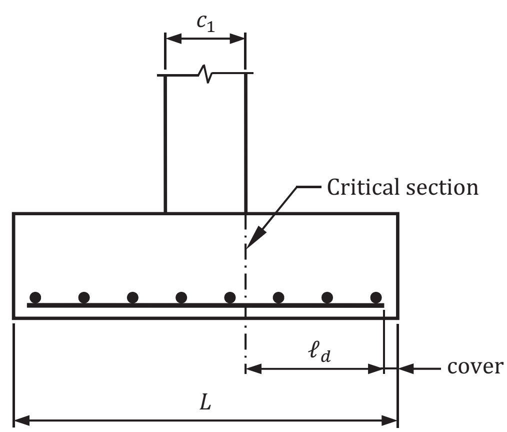


Figure 10.15 Development length of flexural reinforcement in footings.

Equation (10.21) can also be used for wall footings where the thickness of the wall, t_{wall}, is substituted for c_1.

Requirements for the development of deformed reinforcing bars in tension are given in ACI 25.4.2. The tension development length, ℓ_d, is determined using the provisions of ACI 25.4.2.3 or 25.4.2.4 in conjunction with the modification factors in ACI 25.4.2.5. The requirements of ACI 25.4.2.3 are based on those in ACI 25.4.2.4, so the latter requirements are covered first.

Method 1 – ACI 25.4.2.4

The development length in tension of a deformed reinforcing bar, ℓ_d, is determined by ACI Equation (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \tag{10.22}$$

The terms in Equation (10.22) are as follows (ACI Table 25.4.2.5):

- *Modification factor for lightweight concrete,* λ. This factor reflects the lower tensile strength of lightweight concrete:

$$\lambda = \begin{cases} 0.75 \text{ for lightweight concrete} \\ \\ 1.0 \text{ for normalweight concrete} \end{cases} \tag{10.23}$$

- *Reinforcement grade factor,* ψ_g. This factor accounts for the effect of reinforcement yield strength on the required development length:

$$\psi_g = \begin{cases} 1.0 \text{ for Grade 40 or Grade 60} \\ \\ 1.15 \text{ for Grade 80} \\ \\ 1.3 \text{ for Grade 100} \end{cases} \tag{10.24}$$

- *Reinforcement coating factor,* ψ_e. This factor accounts for the reduced bond strength between the concrete and epoxy-coated or zinc and epoxy dual-coated reinforcing bars:

$$\psi_e = \begin{cases} 1.5 \text{ for epoxy-coated or zinc and epoxy dual-coated bars with clear} \\ \text{cover} < 3d_b \text{ or clearing spacing} < 6d_b \\ \\ 1.2 \text{ for epoxy-coated or zinc and epoxy dual-coated bars for all other conditions} \\ \\ 1.0 \text{ for uncoated or zinc-coated (galvanized) bars} \end{cases} \tag{10.25}$$

- *Reinforcement size factor,* ψ_s. This factor reflects the more favorable performance of smaller diameter reinforcing bars:

$$\psi_s = \begin{cases} 1.0 \text{ for \#7 and larger bars} \\ \\ 0.8 \text{ for \#6 and smaller bars} \end{cases} \tag{10.26}$$

- *Casting position factor,* ψ_t. This factor reflects the adverse effects that can occur to the top horizontal reinforcement in a member due to vertical migration of water and mortar, which collect on the underside of the bars during placement of the concrete:

$$\psi_t = \begin{cases} 1.3 \text{ where more than 12 in. of fresh concrete is placed below the horizontal reinforcement} \\ \\ 1.0 \text{ in all other cases} \end{cases} \tag{10.27}$$

  According to the footnote in ACI Table 25.4.2.5, $\psi_t \psi_e$ need not exceed 1.7.

- *Spacing or cover dimension*, c_b. This term is defined as follows:

$$c_b = \text{lesser of} \begin{cases} \text{the distance from the center of a bar to the nearest concrete surface} \\ \\ \text{one-half the center-to-center spacing of the bars being developed} \end{cases} \tag{10.28}$$

- *Transverse reinforcement index,* K_{tr}. The transverse reinforcement index represents the role of confining reinforcement across the potential splitting planes. This index is determined by ACI Equation (25.4.2.3b):

$$K_{tr} = \frac{40A_{tr}}{sn} \tag{10.29}$$

where $A_{tr} =$ total cross-sectional area of all transverse reinforcement within a spacing s crossing the potential plane of splitting through the reinforcement being developed

$s =$ center-to-center spacing of the transverse reinforcement

$n =$ number of bars being developed across the plane of splitting

It is permitted to conservatively use $K_{tr} = 0$ if transverse reinforcement is present or required. For footings, K_{tr} is typically zero because transverse reinforcement is not used in such members.

The confining term $(c_b + K_{tr})/d_b$ must be taken less than or equal to 2.5 in Equation (10.22) [ACI 25.4.2.4]. When this term is less than 2.5, splitting failures are likely to occur. A pullout failure of the reinforcement is more likely when this term is greater than 2.5, so an increase in the anchorage capacity due to an increase in cover or amount of confining reinforcement is not likely.

The tension development length is permitted to be reduced in cases where the flexural reinforcement is greater than that required from analysis, except for the six cases in ACI 25.4.10.2 (ACI 25.4.10.1). The reduction factor applied to ℓ_d is equal to the required area of flexural reinforcement divided by the provided area of reinforcement.

Method 2 – ACI 25.4.2.3

The method given in ACI 25.4.2.3 to determine ℓ_d is based on the requirements given in ACI 25.4.2.4 and preselected values of the confining term $(c_b + K_{tr})/d_b$. Two sets of spacing and cover cases are given in ACI Table 25.4.2.3 (see Table 10.3).

Table 10.3 Tension Development Length, ℓ_d, in Accordance with ACI 25.4.2.3

Spacing and Cover		#6 and Smaller Bars	#7 and Larger Bars
Case 1	Condition 1 1. Clear spacing of bars being developed or lap spliced $\geq d_b$ 2. Clear cover $\geq d_b$ 3. Stirrups or ties throughout ℓ_d not less than ACI 318 minimum	$\left(\frac{f_y\psi_t\psi_e\psi_g}{25\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e\psi_g}{20\lambda\sqrt{f_c'}}\right)d_b$
	Condition 2 1. Clear spacing of bars being developed or lap spliced $\geq 2d_b$ 2. Clear cover $\geq d_b$		
Case 2	Other conditions	$\left(\frac{3f_y\psi_t\psi_e\psi_g}{50\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{3f_y\psi_t\psi_e\psi_g}{40\lambda\sqrt{f_c'}}\right)d_b$

In Case 1, the spacing, cover, and confinement of the bars being developed meet the requirements under Condition 1 or Condition 2. When the requirements of either Condition 1 or Condition 2 are met, it is assumed $(c_b + K_{tr}) / d_b = 1.5$; that value is substituted into Equation (10.22), which results in the equations in the first row of Table 10.3.

Case 2 is applicable where the requirements in either Condition 1 or Condition 2 in Case 1 are not satisfied; in this case, it is assumed $(c_b + K_{tr}) / d_b = 1.0$. Tension development lengths, ℓ_d, must be the greater of the values determined by the equations in Table 10.3 and 12 in.

In typical footings, the clear spacing between the flexural reinforcing bars is usually greater than or equal to $2d_b$ and the clear cover is typically greater than or equal to d_b. Therefore, ℓ_d can be calculated by the equations for Case 1 in Table 10.3. The development lengths calculated by these equations are usually longer than those calculated by Equation (10.22).

10.3.7 Force Transfer at the Base of Supported Members

Overview

Vertical and horizontal forces must be transferred at the interface between the supported member and a footing (ACI 13.2.2). Requirements for force transfer from a column, wall, or pedestal to a footing are given in ACI 16.3.

Vertical compressive forces are transferred by bearing on the concrete or by a combination of bearing and interface reinforcement. Tensile forces must be transferred entirely by reinforcement, which may consist of extended longitudinal bars, dowels, anchor bolts, or mechanical connectors.

Lateral forces are transferred using the shear-friction provisions of ACI 22.9 or any other appropriate methods.

Vertical Transfer – Compression

Where vertical compressive forces are transferred to a footing, bearing strength requirements of ACI 22.8 must be satisfied for both the supported member and the footing (ACI 16.3.3.4). For bearing on the supported member, the factored bearing force must be less than or equal to the design bearing strength, ϕB_n (ACI Table 22.8.3.2):

$$B_u \le \phi B_n = \phi 0.85 f_c' A_1 \quad (10.30)$$

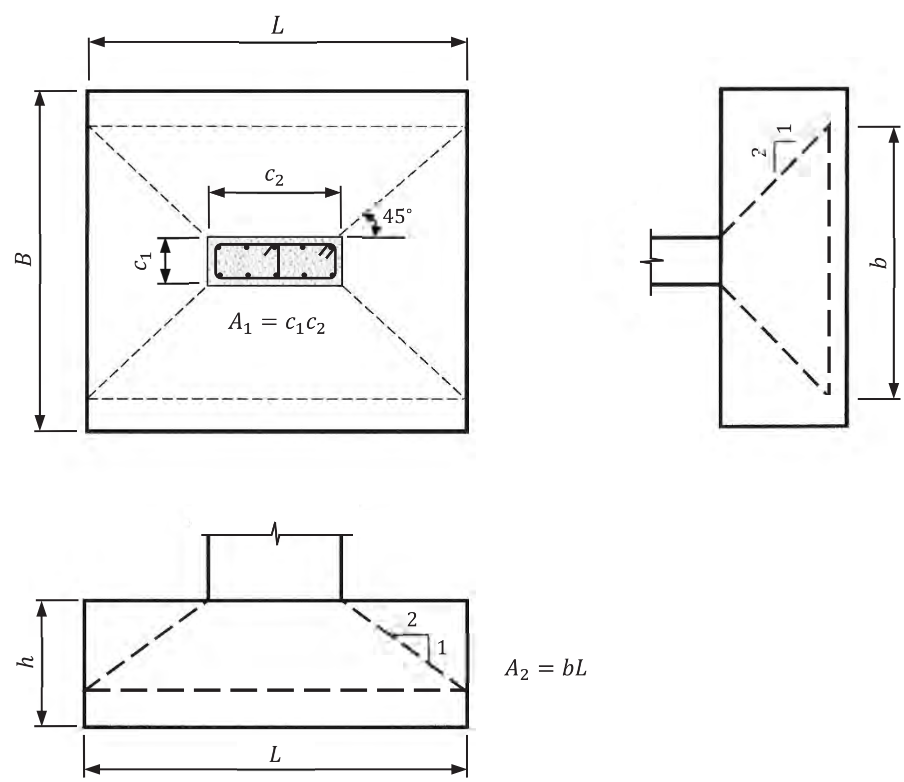


Figure 10.16 Determination of areas A_1 and A_2.

In this equation, the strength reduction factor ϕ for bearing is equal to 0.65 (ACI Table 21.2.1) and A_1 is the area of the column, wall, or pedestal supported by the footing.

For bearing on a footing, the following equation must be satisfied:

$$B_u \leq \phi B_n = (\phi 0.85 f'_c A_1)\sqrt{A_2 / A_1} \leq 2(\phi 0.85 f'_c A_1) \tag{10.31}$$

The term A_2 is defined as the area of the lower base of the largest frustrum of a pyramid, cone, or tapered wedge contained wholly within the footing and having for its upper base the loaded area A_1 and having side slopes of 1 vertical to 2 horizontal (see Figure 10.16).

In cases where $B_u > \phi B_n$, the excess compressive stress from the supported member must be transferred by reinforcement to the footing. The required area of interface reinforcement can be calculated by the following equation:

$$A_s = \frac{B_u - \phi B_n}{\phi f_y} \geq A_{s,min} \tag{10.32}$$

A minimum area of reinforcement, $A_{s,min}$, must be provided across the interface even where $B_u \leq \phi B_n$. Requirements for $A_{s,min}$ are determined on the basis of the type of member supported by the footing (ACI 16.3.4):

Cast-in-place columns or pedestal supported on footings. The minimum area of interface reinforcement in this case is equal to 0.5 percent of the gross area of the supported member (ACI 16.3.4.1). The interface reinforcement can consist of (a) extended longitudinal reinforcing bars from the column or wall into the footing or (b) dowels emanating from the footing. For ease of construction, dowel bars are the type of interface reinforcement commonly used.

Cast-in-place walls supported on footings. The minimum area of interface reinforcement is equal to the minimum longitudinal wall reinforcement given in ACI Table 11.6.1 (ACI 16.3.4.2). For $f_y \geq 60{,}000$ psi:

$$A_{s,min} = \begin{cases} 0.0012 A_g & \text{for \#5 and smaller bars} \\ 0.0015 A_g & \text{for other deformed bars} \end{cases} \tag{10.33}$$

Illustrated in Figure 7.38 of this publication are dowel bars across the interface between a reinforced concrete column and footing for the case where all of the longitudinal bars in the column are in compression. The dowel bars are set in the footing prior to casting the footing concrete and are subsequently spliced to the column or wall longitudinal bars. Dowels should be positioned so as not to interfere with the longitudinal bars or the tie hooks from the column.

Where all the longitudinal bars of the supported member are in compression for all the factored load combinations, the dowel bars must extend into the footing at least a compression development length, ℓ_{dc}, determined in accordance with ACI 25.4.9.2:

$$\ell_{dc} = \text{greater of} \begin{cases} (f_y \psi_r / 50\lambda\sqrt{f'_c}) d_b \\ 0.0003 f_y \psi_r d_b \\ 8 \text{ in.} \end{cases} \tag{10.34}$$

The terms λ and ψ_r are given in ACI Table 25.4.9.3 (see Table 10.4).

Table 10.4 Modification Factors for Deformed Bars in Compression

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Confining reinforcement, ψ_r	Longitudinal reinforcement enclosed in the following: 1. A spiral 2. A circular continuously wound tie with $d_b \geq 0.25$ in. and a pitch ≤ 4 in. 3. #4 ties in accordance with ACI 25.7.2 spaced ≤ 4 in. on center 4. Hoops in accordance with ACI 25.7.4 spaced ≤ 4 in. on center	0.75
	Other	1.0

Standard hooks are typically provided at the ends of the dowel bars, which are tied to the reinforcing bars in the footing; this detail is far more economical than terminating the dowel bars above the footing reinforcing bars. The hooked portion of the dowels cannot be considered effective for developing the dowels bars in compression (ACI 25.4.1.2). The following equation must be satisfied to ensure that the dowel bars are adequately developed into the footing:

$$h \geq \ell_{dc} + r + (d_b)_d + 2(d_b)_f + \text{cover} \tag{10.35}$$

In this equation, r is the radius of the dowel bar bend; minimum inside bend diameters for bars with standard 90-degree hooks are given in ACI Table 25.3.1. Also, $(d_b)_d$ and $(d_b)_f$ are the diameters of the dowel bars and footing bars, respectively. In cases where Equation (10.35) is not satisfied, either a greater number of smaller dowel bars can be used or the depth of the footing must be increased.

For wall footings, the following equation must be satisfied to ensure that the dowel bars are adequately developed into the footing:

$$h \geq \ell_{dc} + r + (d_b)_d + \text{cover} \tag{10.36}$$

The dowel bars must also be fully developed in the supported member and are typically lap spliced to the longitudinal bars. Where the dowel bars are the same size as the longitudinal bars, the minimum compression lap splice length is equal to the following [ACI 25.5.5.1(a)]:

$$\ell_{sc} = \begin{cases} \text{For } f_y \leq 60{,}000 \text{ psi: larger of } 0.0005 f_y d_b \text{ and } 12 \text{ in.} \\ \text{For } 60{,}000 \text{ psi} < f_y \leq 80{,}000 \text{ psi: larger of } (0.0009 f_y - 24) d_b \text{ and } 12 \text{ in.} \\ \text{For } f_y > 80{,}000 \text{ psi: larger of } (0.0009 f_y - 24) d_b \text{ and } \ell_{st} \text{ calculated by ACI 25.5.2.1} \end{cases} \tag{10.37}$$

The lap splice length must be increased by one-third where $f'_c < 3{,}000$ psi (ACI 25.5.5.1).

Where the dowel bars are different in size than the longitudinal bars, the compression lap splice length, ℓ_{sc}, must be greater than or equal to the longer of the following (ACI 25.5.5.4):

1. The development length in compression, ℓ_{dc}, of the larger bar determined in accordance with ACI 25.4.9.1.
2. The compression lap splice length, ℓ_{sc}, of the smaller bar determined in accordance with ACI 25.5.5.1.

Requirements that permit #14 and #18 column bars in compression to be lap spliced to #11 and smaller dowel bars are given in ACI 16.3.5.4.

Vertical Transfer – Tension

Tensile forces transferred from a supported member to a footing must be resisted entirely by reinforcement across the interface [ACI 16.3.1.2(b) and 16.3.5.2], and dowel bars must be provided for all the bars in the supported member.

Tensile anchorage of the dowel bars into the footing is typically accomplished by providing 90-degree standard hooks at the ends of the dowel bars with the development length of the hooked bar, ℓ_{dh}, determined in accordance with ACI 25.4.3 (see Figure 7.39 of this publication for the case of a column supported by a footing):

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}} \right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{10.38}$$

This development length is measured from the critical section to the outside face of the hook (see Figure 4.10 of this publication). The modification factors in Equation (10.38) are given in ACI Table 25.4.3.2 (see Table 10.5).

Table 10.5 Modification Factors for Development of Hooked Bars in Tension

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Confining reinforcement, ψ_r	For #11 and smaller bars with $A_{th} \geq 0.4A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For #11 and smaller hooked bars 1. terminating inside a column core with side cover normal to the plane of the hook ≥ 2.5 in. or 2. with side cover normal to the plane of the hook $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f_c' < 6{,}000$ psi	$(f_c' / 15{,}000) + 0.6$
	$f_c' \geq 6{,}000$ psi	1.0

The confining reinforcement factor, ψ_r, is typically equal to 1.6 for hooked dowel bars in footings because confining reinforcement is usually not provided in such members.

The following equation must be satisfied to ensure that the dowel bars are adequately developed into the footing (considering that the hooked portion of the dowel bars can be developed in tension):

$$h \geq \ell_{dh} + 2(d_b)_f + \text{cover} \tag{10.39}$$

Where Equation (10.39) is not satisfied, the depth of the footing usually must be increased.

For development of the dowel bars into the supported member, a tension lap splice or a mechanical connection in accordance with ACI 25.5.2 or 25.5.7, respectively, must be provided between the dowel bars and the longitudinal bars. In the case of lap splices, a Class B tension lap splice is required.

Horizontal Transfer

The shear-friction method of ACI 22.9 is permitted to be used to determine the nominal shear strength, V_n, at the contact surface between the supported member and the foundation (ACI 16.3.3.5). The required area of reinforcement, A_{vf}, across the interface between the supported member and the footing is determined by the following equation, which is applicable to shear-friction reinforcement perpendicular to the interface (ACI 22.9.4.2):

$$A_{vf} \geq \frac{V_u}{\phi f_y \mu} \tag{10.40}$$

In this equation, V_u is the factored shear force due to the lateral force effects at the interface, the strength reduction factor ϕ is equal to 0.75, and μ is the coefficient of friction, which is obtained from ACI Table 22.9.4.2 (see Table 10.6).

Table 10.6 Coefficient of Friction, μ

Contact Surface Condition	μ
Concrete placed monolithically	1.4λ
Concrete placed against hardened concrete that is clean, free of laitance, and intentionally roughened to a full amplitude of approximately ¼ in.	1.0λ
Concrete placed against hardened concrete that is clean, free of laitance, and not intentionally roughened	0.6λ

Upper limits on shear-friction strength are given in ACI Table 22.9.4.4 (see Table 10.7). The term A_c is the area of concrete resisting V_u. For example, where a column is supported by a footing, A_c is equal to the gross area of the column. Where the concrete strengths of the supported member and the footing are different, the smaller of the two must be used in these equations (ACI 22.9.4.4).

Table 10.7 Maximum V_n Across the Assumed Shear Plane

Contact Surface Condition	Maximum $V_n = V_u / \phi$*
Normalweight concrete placed monolithically or placed against hardened concrete intentionally roughened to a full amplitude of approximately ¼ in.	Least of $\begin{cases} 0.2f'_cA_c \\ (480 + 0.08f'_c)A_c \\ 1{,}600A_c \end{cases}$
Other cases	Least of $\begin{cases} 0.2f'_cA_c \\ 800A_c \end{cases}$

*A_c = *area of concrete resisting* V_u

The required amount of shear friction reinforcement, A_{vf}, is permitted to be determined by Equation (10.40) based on a net force equal to $(V_u / \phi) - C_{net}$ where C_{net} is a permanent net compression force transmitted across the assumed shear plane (ACI 22.9.4.5). According to ACI 22.9.4.6, the area of reinforcement required to resist a net factored tension force across the assumed shear plane is to be added to the area of reinforcement required for shear friction that crosses the assumed shear plane. For supported members transmitting a bending moment across the shear plane, the flexural compression and tension forces are in equilibrium and do not change the resultant compression force, $A_{vf}f_y$, acting across the shear plane or the shear-friction resistance. Thus, it is not necessary to provide additional reinforcement to resist the flexural tension stresses, unless the required flexural tension reinforcement exceeds the amount of shear-transfer reinforcement provided in the flexural tension zone.

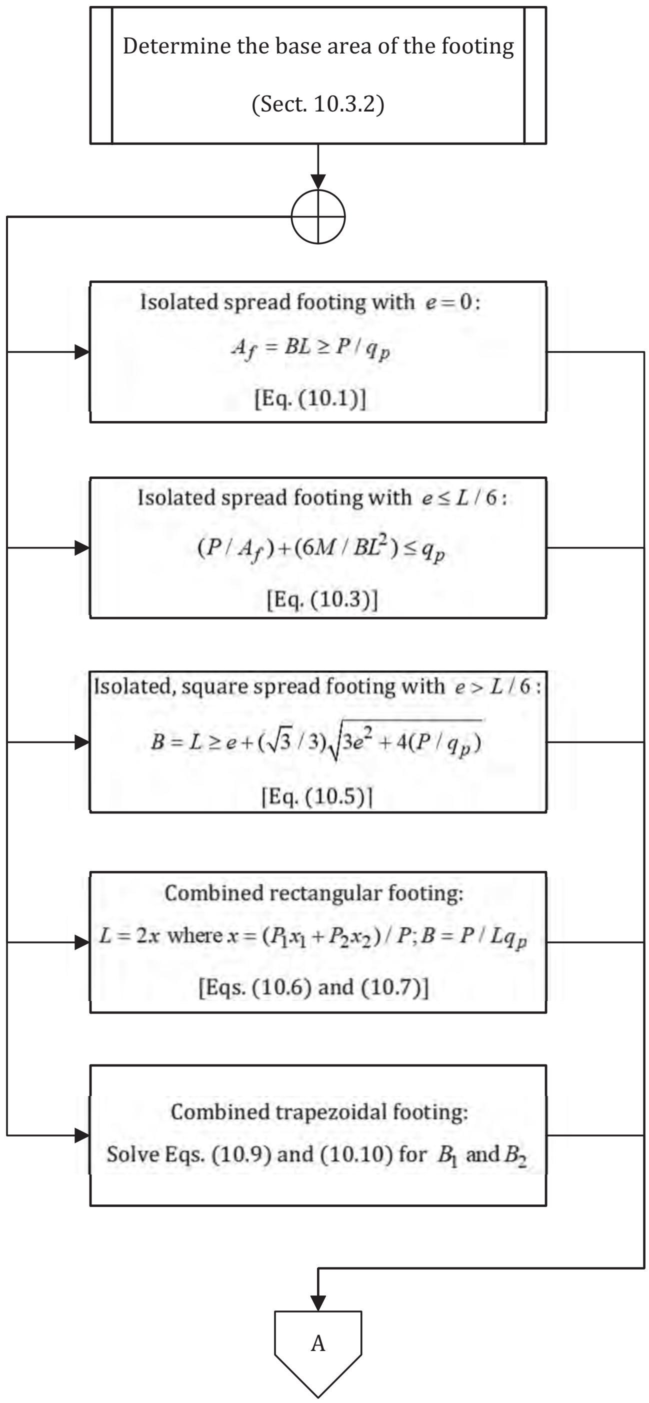


Figure 10.17 Design procedure for footings.

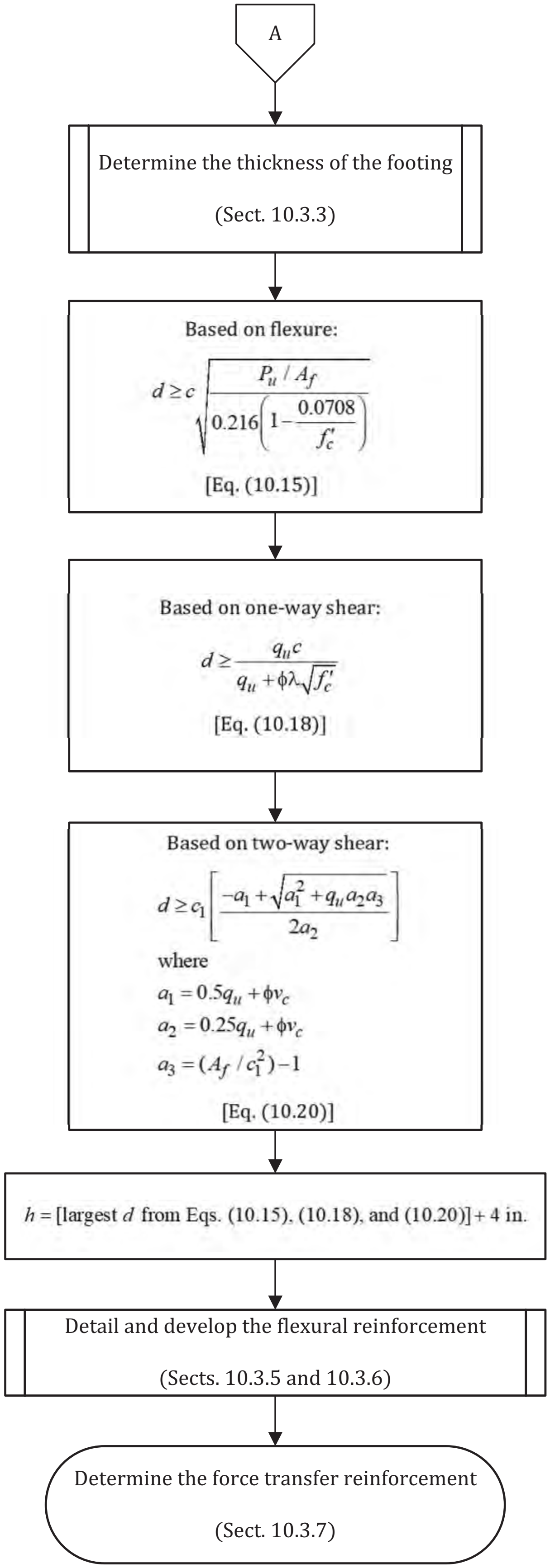


Figure 10.17 Design procedure for footings (cont.).

Full tension anchorage of the shear-friction reinforcement must be provided into the footing and into the supported member. The lengths of these bars are determined in the same way as those for vertical transfer where tension forces are present.

The area of the dowel bars is usually determined initially based on the requirements for vertical transfer. That area is compared with the area required for horizontal transfer, and the larger of the two is provided at the interface.

10.3.8 Design Procedure

The design procedure in Figure 10.17 can be used in the design and detailing of reinforced concrete footings. Included in the figure are the section numbers and equation numbers where specific information in this chapter can be found.

10.4 Drilled Piers

10.4.1 Overview

A drilled pier, which is also referred to as a pier or caisson, is a type of deep foundations that transfers the loads from the superstructure to a soil or rock stratum usually well below the ground surface. The bottom of the pier may be belled to provide a larger end-bearing area (see Figure 10.18).

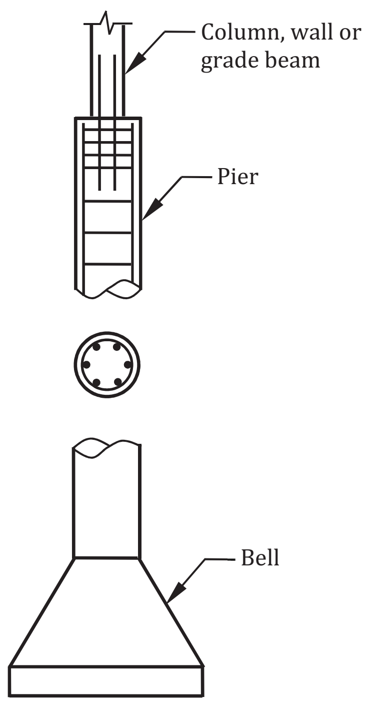


Figure 10.18 Typical drilled pier with a bell.

The loads from the supported members are transferred to the pier by bearing. Skin resistance (or, friction), point bearing, or a combination of the two are ways in which the load is transferred to the soil surrounding and below the pier or bell. Drilled piers are categorized based on the manner in which the loads are transferred to the soil or rock. The discussion below focuses on drilled piers subjected primarily to axial forces that transfer the forces to the soil/ rock by end bearing.

10.4.2 Design Methods

Overview

The design of drilled piers is permitted to be in accordance with the allowable axial strength method in ACI 13.4.2 for members subjected primarily to axial forces or the strength design method in ACI 13.4.3 (ACI 13.4.1.2). For drilled piers without bells, the diameter of the pier is determined based on (1) ACI 13.4.2 or 13.4.3 or (2) the allowable bearing capacity of the soil/rock. For drilled piers with bells, the diameter of the pier is determined based on ACI 13.4.2 or 13.4.3 while the diameter of the bell is based on the allowable bearing capacity of the soil/rock.

Allowable Axial Strength

Drilled piers may be designed using the load combinations for allowable stress design in ASCE/SEI 2.4 and the allowable compressive strengths given in ACI Table 13.4.2.1 (see Table 10.8) provided the following conditions are satisfied:

- The drilled pier is laterally supported its entire length.
- The applied forces cause bending moments in the drilled pier less than the moment due to an accidental eccentricity of 5 percent of the member diameter (that is, $e = M / P < 0.05d_{pier}$ where d_{pier} is the diameter of the pier).

Drilled piers not meeting one or both of these conditions must be designed using the strength design method in ACI 13.4.3 (ACI 13.4.2.2).

Table 10.8 Maximum Allowable Compressive Strength for Drilled Piers

Type of Drilled Pier	Maximum Allowable Compressive Strength, P_a
Uncased cast-in-place concrete drilled pier	$0.3f'_cA_g + 0.4f_yA_s$
Cast-in-place concrete drilled pier in rock or within a pipe, tube, or other permanent metal casing that does not satisfy the conditions in ACI 13.4.2.3	$0.33f'_cA_g + 0.4f_yA_s$
Metal-cased cast-in-place concrete drilled pier confined in accordance with ACI 13.4.2.3	$0.4f'_cA_g$

In Table 10.8, A_g is the gross cross-sectional area of the concrete in the pier. Where temporary or permanent casing is used, the inside face of the casing is considered to be the concrete surface. Also, A_s is the area of the longitudinal reinforcement in the drilled pier; it must not include the area of any casing, pipes, or tubes present.

It is evident from Table 10.8 that the allowable compressive strength depends on whether the drilled pier has metal casing or not and if metal casing is provided, whether the pier can be considered to be confined or not by the casing in accordance with ACI 13.4.2.3.

Strength Design

The strength design provisions in ACI 13.4.3 can be used to design any drilled pier (ACI 13.4.3.1). For drilled piers subjected to axial compression loads only, the pier is designed using the nominal strengths for columns, which are given in ACI 10.5, and the strength reduction factors in ACI Table 13.4.3.2 (see Table 10.9). For drilled piers subjected to combined axial force and moment, tension, and shear, the strength reduction factors in ACI Table 21.2.1 must be used instead. The design strengths must be less than or equal to the required strengths determined by the appropriate factored load combinations in ACI 5.3.

Table 10.9 Compressive Strength Reduction Factors ϕ for Drilled Piers

Type of Drilled Pier	Compressive Strength Reduction Factor, ϕ
Uncased cast-in-place concrete drilled pier	0.55
Cast-in-place concrete drilled pier in rock or within a pipe, tube, or other permanent metal casing that does not satisfy the conditions in ACI 13.4.2.3	0.60
Metal-cased cast-in-place concrete drilled pier confined in accordance with ACI 13.4.2.3	0.65

According to ACI 13.4.3.2, the provisions of ACI 22.4.2.4 and 22.4.2.5 pertaining to tie reinforcement and spiral reinforcement for lateral support of longitudinal reinforcement in compression members, respectively, need not apply to the lateral reinforcement in drilled piers.

Longitudinal reinforcement must be provided in a drilled pier subjected to uplift or where the factored maximum bending moment, M_u, is greater than $0.4M_{cr}$ where M_{cr} is the cracking moment determined by ACI Equation (24.2.3.5) [ACI 13.4.4.1]:

$$M_{cr} = \frac{f_r I_g}{y_t} \tag{10.41}$$

The term y_t is the distance from the centroidal axis of the gross section, neglecting longitudinal reinforcement, to the tension face of the member and the modulus of rupture, f_r, is determined by ACI Equation (19.2.3.1):

$$f_r = 7.5\lambda\sqrt{f'_c} \tag{10.42}$$

The requirement for longitudinal reinforcement need not be satisfied if the drilled pier is enclosed by a structural steel pipe or tube.

Where soil cannot provide lateral resistance or where a pier extends above the soil surface or through subsurface layers of air or water, the pier must be designed as a column using the applicable provisions in ACI Chapter 10 (ACI 13.4.4.2). This includes checking whether the effects of slenderness must be considered or not.

10.4.3 Determining the Pier Size

Allowable Axial Strength

For drilled piers designed using the allowable axial strength method, the diameter of the pier, d_{pier}, can be determined by setting the maximum governing axial load, P, from the applicable load combinations for allowable stress design in ASCE/SEI 2.4 equal to the appropriate allowable compressive strength, P_a, in Table 10.8 and solving for d_{pier}, which is equal to $\sqrt{4A_g / \pi}$.

The equations in Table 10.10 can be used to determine d_{pier} where it is assumed $A_s = 0.005A_g$ for the drilled piers in the first two rows of the table.

Table 10.10 Required Pier Diameter Based on Allowable Axial Strength

Type of Drilled Pier	Required Pier Diameter, d_{pier}
Uncased cast-in-place concrete drilled pier*	$\left[\frac{4P}{\pi(0.3f'_c + 0.002f_y)}\right]^{1/2}$
Cast-in-place concrete drilled pier in rock or within a pipe, tube, or other permanent metal casing that does not satisfy the conditions in ACI 13.4.2.3*	$\left[\frac{4P}{\pi(0.33f'_c + 0.002f_y)}\right]^{1/2}$
Metal-cased cast-in-place concrete drilled pier confined in accordance with ACI 13.4.2.3	$\left[\frac{4P}{\pi(0.4f'_c)}\right]^{1/2}$

**It is assumed* $A_s = 0.005A_g$ *for this type of drilled pier.*

Strength Design

For drilled piers subjected to axial compression loads only, d_{pier} can be determined by setting the maximum governing axial load, P_u, from the applicable load combinations in ACI 5.3 equal to the design axial load strength, $\phi P_{n,max}$, given in ACI Table 22.4.2.1 using the appropriate strength reduction factor in Table 10.9.

The following equation can be used to determine d_{pier} where $\rho_g = A_s / A_g$ is the assumed percentage of longitudinal reinforcement in the pier:

$$d_{pier} = \left\{\frac{4P_u}{\phi\pi 0.80[0.85f'_c(1-\rho_g) + f_y\rho_g]}\right\}^{1/2} \tag{10.43}$$

Initially, ρ_g can be taken as 0.005 and can be subsequently adjusted, if required.

Where significant bending moments and shear forces are transferred in combination with axial loads, the pier is typically designed considering the lateral restraint provided by the soil (which is usually modeled as spring supports). Approximate methods of analysis are given in Reference 24; included is a procedure to calculate the lateral deflection at the top of the pier.

10.4.4 Determining the Bell Diameter

For the case of end-bearing drilled piers, the diameter of the bell, d_{bell}, can be determined using the service axial load, P, and the allowable bearing capacity of the soil or rock, q_a :

$$d_{bell} = \left(\frac{4P}{\pi q_a}\right)^{1/2} \tag{10.44}$$

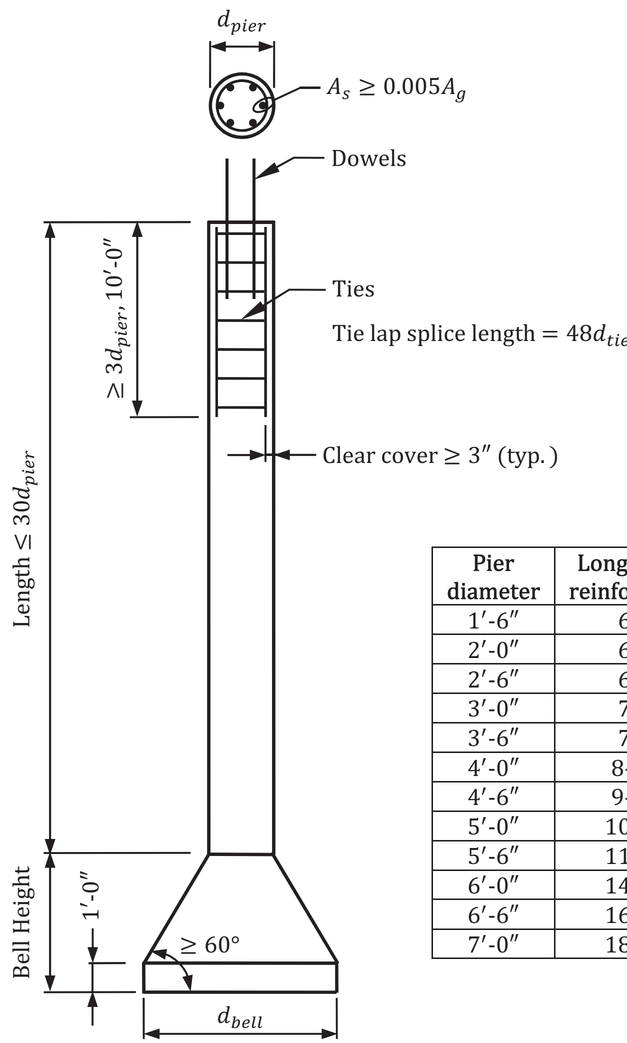
The height of the bell depends on the diameter of the pier and the slope of the bell. The thickness of the lower portion of the bell that is not sloped is typically 1 foot. The angle of the sloped portion is usually 60 degrees or more so the effects of vertical shear do not need to be considered in the design.

In lieu of using service-level loads, factored axial forces and nominal soil/rock bearing strengths and strength reduction factors can be used to determine the diameter of a bell.

Tables that facilitate the selection of pier and bell diameters for drilled piers subjected to axial compression and combined axial compression and flexure are given in Reference 25.

10.4.5 Reinforcement Details

Recommended reinforcement details for drilled piers subjected to primarily axial compression loads are given in Figure 10.19. The recommended longitudinal reinforcement in the figure is based on a minimum longitudinal reinforcement ratio of 0.005, which corresponds to columns with cross-sections larger than required for the applied loads (ACI 10.3.1.2). The minimum embedment length of the longitudinal reinforcement is the longer of $3d_{pier}$ or 10 ft.



Pier diameter	Longitudinal reinforcement	Ties
1′-6″	6-#5	#3 @ 10″
2′-0″	6-#6	#3 @ 12″
2′-6″	6-#7	#3 @ 14″
3′-0″	7-#8	#3 @ 16″
3′-6″	7-#9	#3 @ 18″
4′-0″	8-#10	#4 @ 18″
4′-6″	9-#10	#4 @ 18″
5′-0″	10-#11	#4 @ 18″
5′-6″	11-#11	#4 @ 18″
6′-0″	14-#11	#4 @ 18″
6′-6″	16-#11	#4 @ 18″
7′-0″	18-#11	#4 @ 18″

Figure 10.19 Recommended reinforcement details for drilled piers subjected to axial compression loads.

As noted in Section 10.4.2, longitudinal reinforcement must be provided to resist the tensile forces in drilled piers subjected to uplift or to a factored bending moment exceeding 40 percent of the cracking moment, M_{cr}, unless the drilled pier is enclosed by a structural steel pipe or tube. The longitudinal bars must extend into the pier a sufficient distance to fully develop the bars in tension.

Vertical and horizontal loads must be transferred from the supported member to the top of the pier. The force transfer requirements given in Section 10.3.7 of this publication are applicable to drilled piers.

10.5 Examples

10.5.1 Example 10.1 – Design of a Wall Footing Subjected to Axial Compression: Building #2

Design a footing for the 9-in.-thick interior reinforced concrete wall of Building #2 in Example 8.1 (see Figure 1.2). The wall is reinforced with one layer of #4 longitudinal bars spaced at 18 in. on center. Assume Grade 60 reinforcement and normalweight concrete with $f_c' = 3{,}000$ psi for the footing and $f_c' = 4{,}000$ psi for the wall. Also assume a permissible soil pressure of 2,000 lb/ft^2.

Design data are given in Sect. 1.2.2.

Step 1 – Determine the width of the footing

From Example 8.1, the axial service dead and live loads are equal to the following:

$$P_D = 2.0 \text{ kips/ft}$$

$$P_L = 1.0 \text{ kips/ft}$$

Use the basic load combinations for allowable stress design in ASCE/SEI 2.4.1 to determine the maximum axial force transferred to the footing:

1. $P_D = 2.0$ kips/ft
2. $P_D + P_L = 2.0 + 1.0 = 3.0$ kips

Assuming the axial forces act at the centroid of the wall, the required area of the footing is equal to the following:

$$A_f = \frac{P_D + P_L}{q_p} = \frac{3.0 \times 1{,}000}{2{,}000} = 1.5 \text{ ft}^2/\text{ft} \qquad \text{Eq. (10.1)}$$

A 1 ft-6 in.-wide wall footing is adequate for soil bearing capacity. However, it is unlikely that width is adequate to develop the flexural reinforcement. Therefore, try a 4-ft wide footing.

Step 2 – Determine the thickness of the footing

Use the load combinations in ACI Table 5.3.1 to determine the thickness of the footing:

$$1.4P_D = 1.4 \times 2.0 = 2.8 \text{ kips/ft} \qquad \text{ACI Eq. (5.3.1a)}$$

$$1.2P_D + 1.6P_L = (1.2 \times 2.0) + (1.6 \times 1.0) = 4.0 \text{ kips/ft} \qquad \text{ACI Eq. (5.3.1b)}$$

The maximum factored pressure at the base of the footing is equal to the following:

$$q_u = \frac{P_u}{A_f} = \frac{4.0}{4.0 \times 1.0} = 1.0 \text{ kips/ft}^2$$

Minimum effective depth for flexure:

$$d \geq c\sqrt{\frac{P_u / A_f}{0.216\left(1 - \dfrac{0.0708}{f_c'}\right)}} = \left(\frac{4.0}{2} - \frac{9.0}{2 \times 12}\right) \times \sqrt{\frac{1.0}{0.216 \times \left(1 - \dfrac{0.0708}{3.0}\right)}} = 3.5 \text{ in.} \qquad \text{Eq. (10.15)}$$

Minimum effective depth for one-way shear:

$$d \geq \frac{q_u c}{q_u + \phi\lambda\sqrt{f_c'}} = \frac{\left(\frac{1.0 \times 1,000}{144}\right) \times \left(\frac{4.0 \times 12}{2} - \frac{9.0}{2}\right)}{\left(\frac{1.0 \times 1,000}{144}\right) + (0.75 \times 1.0 \times \sqrt{3,000})} = 2.8 \text{ in.} \qquad \text{Eq. (10.18)}$$

The minimum effective depth for flexure governs in this case. However, a minimum depth of 6 in. is required above the bottom reinforcement of the footing (ACI 13.3.1.2). Therefore, at least a 10-in.-thick footing is required. It is unlikely a 10-in. footing thickness is adequate to properly develop the dowel bars from the footing to the wall.

Try a 16-in.-thick footing $(d = 12 \text{ in.})$.

Step 3 – Determine the required flexural reinforcement

Because the provided effective depth of 12 in. is greater than the 3.5-in. minimum effective depth calculated for flexure, minimum flexural reinforcement is adequate:

$$A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 16.0 = 0.35 \text{ in.}^2/\text{ft} \qquad \text{ACI 7.6.1.1}$$

Try #5 bars spaced at 10 in. on center $(A_{s,provided} = 0.37 \text{ in.}^2/\text{ft})$. The 10-in. spacing is less than the maximum spacing of $3h = 48$ in. or 18 in. (ACI 7.7.2.3).

Step 4 – Check if the flexural reinforcement can be developed for tension

The tension development length, ℓ_d, of the #5 bars must be less than or equal to the available development length:

$$\ell_d \leq \frac{L - t_{wall}}{2} - \text{cover} \qquad \text{Eq. (10.21)}$$

Determine ℓ_d from ACI Eq. (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s\psi_g}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b \geq 12.0 \text{ in.} \qquad \text{Eq. (10.22)}$$

For normalweight concrete, $\lambda = 1.0$. Eq. (10.23)

For Grade 60 reinforcement, $\psi_g = 1.0$. Eq. (10.24)

For uncoated reinforcing bars, $\psi_e = 1.0$. Eq. (10.25)

For #5 bars, $\psi_s = 0.8$. Eq. (10.26)

For less than 12 in. of concrete placed below the flexural reinforcement, $\psi_t = 1.0$. Eq. (10.27)

$$c_b = \text{lesser of} \begin{cases} 3.0 + (0.625 / 2) = 3.3 \text{ in.} \\ \\ 10.0 / 2 = 5.0 \text{ in.} \end{cases} \qquad \text{Eq. (10.28)}$$

Because there is no transverse reinforcement, $K_{tr} = 0$. Eq. (10.29)

$$(c_b + K_{tr}) / d_b = (3.3 + 0) / 0.625 = 5.3 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0 \times \sqrt{3{,}000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{2.5} \right) \times 0.625 = 16.4 \text{ in.} > 12.0 \text{ in.}$$

$$< \frac{L - t_{wall}}{2} - \text{cover} = \frac{(4.0 \times 12) - 9.0}{2} - 3.0 = 16.5 \text{ in.}$$

Because the available development length is greater than the tension development length, the #5 bars can be fully developed for flexure.

Step 5 – Determine the shrinkage and temperature reinforcement ACI 24.4.3.2

$$A_{s,min} = 0.0018bh = 0.0018 \times (4.0 \times 12) \times 16.0 = 1.38 \text{ in.}^2$$

Provide 5-#5 bars ($A_{s,provided} = 1.55 \text{ in.}^2$) uniformly spaced within the 48-in. width; this reinforcement is perpendicular to the main flexural reinforcement.

Step 6 – Determine the required dowel reinforcement

Only vertical forces need to be considered for force transfer between the wall and footing.

- Check the bearing force on the wall:

  $$\phi B_n = \phi 0.85 f_c' A_1 = 0.65 \times 0.85 \times 4.0 \times (9.0 \times 12.0) = 238.7 \text{ kips} > B_u = 4.0 \text{ kips} \quad \text{Eq. (10.30)}$$

- Check the bearing force on the footing:

  $$B_u \le \phi B_n = (\phi 0.85 f_c' A_1)\sqrt{A_2 / A_1} \le 2(\phi 0.85 f_c' A_1) \quad \text{Eq. (10.31)}$$

  Using Figure 10.16, A_2 is determined as follows:

  Thickness of footing $= 16.0$ in.

  Horizontal projection for a 1:2 slope $= 2 \times 16.0 = 32.0$ in.

  Projected length $b = 32.0 + 9.0 + 32.0 = 73.0 \text{ in.} > 48.0 \text{ in.}$

  Therefore, $A_2 = 48.0 \times 12.0 = 576.0 \text{ in.}^2$

  $$\sqrt{A_2 / A_1} = \sqrt{576.0 / (9.0 \times 12.0)} = 2.3 > 2.0; \text{ use } 2.0$$

  $$\phi B_n = 2(\phi 0.85 f_c' A_1) = 2 \times 0.65 \times 0.85 \times 3.0 \times (9.0 \times 12.0) = 358.0 \text{ kips} > B_u = 4.0 \text{ kips}$$

- Determine the required interface reinforcement

  Because the design bearing strength is adequate, provide the minimum area of reinforcement across the interface:

  $$A_{s,min} = 0.0012 A_g \text{ for \#5 and smaller deformed bars} = 0.0012 \times 9.0 \times 12.0 = 0.13 \text{ in.}^2/\text{ft} \quad \text{Eq. (10.33)}$$

  Try #4 bars spaced at 18 in. on center ($A_s = 0.13 \text{ in.}^2/\text{ft}$), which matches the size and spacing of the longitudinal bars in the wall.

- Development of the dowel bars into the footing

  The dowel bars must extend into the footing at least a compression development length, ℓ_{dc}:

$$\ell_{dc} = \text{ greater of } \begin{cases} (f_y \psi_r \,/\, 50\lambda\sqrt{f'_c})d_b = [(60{,}000 \times 1.0) \,/\, (50 \times 1.0 \times \sqrt{3{,}000})] \times 0.5 = 11.0 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 0.5 = 9.0 \text{ in.} \\ 8 \text{ in.} \end{cases} \qquad \text{Eq. (10.34)}$$

where $\psi_r = 1.0$ for reinforcing bars without confinement. Table 10.4

The minimum footing thickness for the development of the dowel bars in compression is the following:

$$h \geq \ell_{dc} + r + (d_b)_d + \text{cover} = 11.0 + (3 \times 0.50) + 0.50 + 3.0 = 16.0 \text{ in.} \qquad \text{Eq. (10.36)}$$

The provided footing thickness is equal to the required footing thickness, so the dowel bars can be fully developed for compression into the footing.

- Development of the dowel bars into the wall

  The dowel bars are lap spliced to the longitudinal bars in the wall. The dowel bars and the longitudinal bars are the same size, so the compression lap splice length, ℓ_{sc}, is equal to the following for Grade 60 reinforcement:

$$\ell_{sc} = \text{greater of} \begin{cases} 0.0005 f_y d_b = 0.0005 \times 60{,}000 \times 0.5 = 15.0 \text{ in.} \\ 12.0 \text{ in.} \end{cases} \qquad \text{Eq. (10.37)}$$

  Provide a lap splice length equal to 1 ft-3 in.

Reinforcement details for the wall footing are given in Figure 10.20.

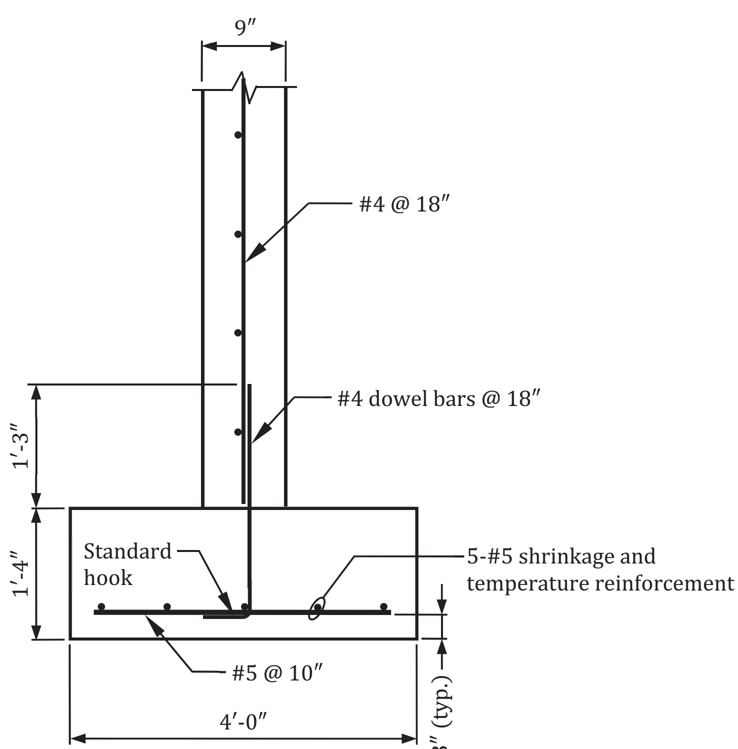


Figure 10.20 Reinforcement details for the wall footing in Example 10.1.

10.5.2 Example 10.2 – Design of a Square Isolated Spread Footing Subjected to Axial Compression: Building #1 (Framing Option B), SDC A

Design a square isolated footing for the 18-in. square interior column of Building #1, Framing Option B in Example 7.1 (see Figure 1.1). The column is reinforced with 4-#9 bars. Assume Grade 60 reinforcement and normalweight concrete with $f'_c = 4{,}000$ psi for the column and the footing. Also assume a permissible soil pressure of 4,000 lb/ft^2.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the base dimensions of the footing

From Example 7.1, the service axial dead, roof live, and live loads are equal to the following:

$$P_D = 342.7 \text{ kips}$$

$$P_{L_r} = 11.8 \text{ kips}$$

$$P_L = 152.8 \text{ kips}$$

Use the basic load combinations for allowable stress design in ASCE/SEI 2.4.1 to determine the maximum axial force transferred to the footing:

1. $P_D = 342.7$ kips
2. $P_D + P_L = 342.7 + 152.8 = 495.5$ kips
3. $P_D + P_{L_r} = 342.7 + 11.8 = 354.5$ kips
4. $P_D + 0.75P_L + 0.75P_{L_r} = 342.7 + (0.75 \times 152.8) + (0.75 \times 11.8) = 466.2$ kips

Using ASCE/SEI Eq. 2, the required area of the footing is equal to the following:

$$A_f = \frac{P_D + P_L}{q_p} = \frac{495.5 \times 1{,}000}{4{,}000} = 123.9 \text{ ft}^2 \qquad \text{Eq. (10.1)}$$

$$B = L = \sqrt{A_f} = 11.1 \text{ ft}$$

An 11 ft-6 in. square isolated footing is adequate for soil bearing capacity.

Step 2 – Determine the thickness of the footing

Use the load combinations in ACI Table 5.3.1 to determine the thickness of the footing:

$$1.4P_D = 1.4 \times 342.7 = 479.8 \text{ kips} \qquad \text{ACI Eq. (5.3.1a)}$$

$$1.2P_D + 1.6P_L + 0.5P_{L_r} = (1.2 \times 342.7) + (1.6 \times 152.8) + (0.5 \times 11.8) = 661.6 \text{ kips} \qquad \text{ACI Eq. (5.3.1b)}$$

$$1.2P_D + 1.6P_{L_r} + 0.5P_L = (1.2 \times 342.7) + (1.6 \times 11.8) + (0.5 \times 152.8) = 506.5 \text{ kips} \qquad \text{ACI Eq. (5.3.1c)}$$

The maximum factored pressure at the base of the footing is equal to the following:

$$q_u = \frac{P_u}{A_f} = \frac{661.6}{11.5^2} = 5.0 \text{ kips/ft}^2$$

- Minimum effective depth for flexure:

$$d \geq c\sqrt{\frac{P_u / A_f}{0.216\left(1 - \frac{0.0708}{f_c'}\right)}} = \left(\frac{11.5}{2} - \frac{18.0}{2 \times 12}\right) \times \sqrt{\frac{5.0}{0.216 \times \left(1 - \frac{0.0708}{4.0}\right)}} = 24.3 \text{ in.} \qquad \text{Eq. (10.15)}$$

- Minimum effective depth for one-way shear:

$$d \geq \frac{q_u c}{q_u + \phi\lambda\sqrt{f_c'}} = \frac{\left(\frac{5.0 \times 1,000}{144}\right) \times \left(\frac{11.5 \times 12}{2} - \frac{18.0}{2}\right)}{\left(\frac{5.0 \times 1,000}{144}\right) + (0.75 \times 1.0 \times \sqrt{4,000})} = 25.4 \text{ in.} \qquad \text{Eq. (10.18)}$$

- Minimum effective depth for two-way shear:

$$a_1 = 0.5q_u + \phi v_c = \left(\frac{0.5 \times 5.0 \times 1,000}{144}\right) + (0.75 \times 4 \times \sqrt{4,000}) = 207.1 \text{ psi}$$

$$a_2 = 0.25q_u + \phi v_c = \left(\frac{0.25 \times 5.0 \times 1,000}{144}\right) + (0.75 \times 4 \times \sqrt{4,000}) = 198.4 \text{ psi}$$

$$a_3 = (A_f / c_1^2) - 1 = \frac{(11.5 \times 12)^2}{18.0^2} - 1 = 57.8$$

$$d \geq c_1\left[\frac{-a_1 + \sqrt{a_1^2 + q_u a_2 a_3}}{2a_2}\right] = 18.0 \times \left\{\frac{-207.1 + \sqrt{207.1^2 + \left[\left(\frac{5.0 \times 1,000}{144}\right) \times 198.4 \times 57.8\right]}}{2 \times 198.4}\right\} = 20.7 \text{ in.} \qquad \text{Eq. (10.20)}$$

The minimum effective depth for one-way shear governs in this case.

Try a 30-in.-thick footing $(d = 26.0 \text{ in.})$.

Step 3 – Determine the required flexural reinforcement

Because the provided effective depth of 26.0 in. is greater than the 24.3-in. minimum effective depth calculated for flexure, minimum flexural reinforcement is adequate:

$$A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 30.0 = 0.65 \text{ in.}^2/\text{ft} \qquad \text{ACI 7.6.1.1}$$

Try #7 bars spaced at 10 in. on center $(A_{s,provided} = 0.72 \text{ in.}^2/\text{ft})$. The 10-in. spacing is less than the maximum spacing of $3h = 90$ in. or 18 in.

Step 4 – Check if the flexural reinforcement can be developed for tension

The tension development length, ℓ_d, of the #7 bars must be less than or equal to the available development length:

$$\ell_d \leq \frac{L - c_1}{2} - \text{cover} \qquad \text{Eq. (10.21)}$$

Determine ℓ_d from ACI Eq. (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s\psi_g}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b \geq 12.0 \text{ in.} \qquad \text{Eq. (10.22)}$$

For normalweight concrete, $\lambda = 1.0$. Eq. (10.23)

For Grade 60 reinforcement, $\psi_g = 1.0$. Eq. (10.24)

For uncoated reinforcing bars, $\psi_e = 1.0$. Eq. (10.25)

For #7 bars, $\psi_s = 1.0$. Eq. (10.26)

For less than 12 in. of concrete placed below the flexural reinforcement, $\psi_t = 1.0$. Eq. (10.27)

$$c_b = \text{lesser of} \begin{cases} 3.0 + (0.875 / 2) = 3.4 \text{ in.} \\ \\ 10.0 / 2 = 5.0 \text{ in.} \end{cases} \quad \text{Eq. (10.28)}$$

Because there is no transverse reinforcement, $K_{tr} = 0$. Eq. (10.29)

$(c_b + K_{tr}) / d_b = (3.4 + 0) / 0.875 = 3.9 > 2.5,$ use 2.5

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0 \times \sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} \right) \times 0.875 = 24.9 \text{ in.} > 12.0 \text{ in.}$$

$$< \frac{L - c_1}{2} - \text{cover} = \frac{(11.5 \times 12) - 18.0}{2} - 3.0 = 57.0 \text{ in.}$$

Because the available development length is greater than the tension development length, the #7 bars can be fully developed for flexure.

Step 5 – Determine the required dowel reinforcement

Only vertical forces need to be considered for force transfer between the column and footing.

- Check the bearing force on the column:
  $\phi B_n = \phi 0.85 f_c' A_1 = 0.65 \times 0.85 \times 4.0 \times 18.0^2 = 716.0 \text{ kips} > B_u = 661.6 \text{ kips}$ Eq. (10.30)
- Check the bearing force on the footing:
  $B_u \le \phi B_n = (\phi 0.85 f_c' A_1)\sqrt{A_2 / A_1} \le 2(\phi 0.85 f_c' A_1)$ Eq. (10.31)

  Using Figure 10.16, A_2 is determined as follows:

  Thickness of footing $= 30.0$ in.

  Horizontal projection for a 1:2 slope $= 2 \times 30.0 = 60.0$ in.

  Projected length $b = 60.0 + 18.0 + 60.0 = 138.0 \text{ in.} = 11.5 \times 12 = 138.0 \text{ in.}$

  Therefore, $A_2 = b^2 = 138.0^2 = 19{,}044.0 \text{ in.}^2$

  $\sqrt{A_2 / A_1} = \sqrt{19{,}044 / 18.0^2} = 7.7 > 2.0;$ use 2.0

  $\phi B_n = 2(\phi 0.85 f_c' A_1) = 2 \times 0.65 \times 0.85 \times 4.0 \times 18.0^2 = 1{,}432.1 \text{ kips} > B_u = 661.6 \text{ kips}$
- Determine the required interface reinforcement
  Because the design bearing strength is adequate, provide the minimum area of reinforcement across the interface:

$A_{s,min} = 0.005A_g = 0.005 \times 18.0^2 = 1.62 \text{ in.}^2$ Sect. 10.3.7

Try 4-#6 dowel bars $(A_{s,provided} = 1.76 \text{ in.}^2)$.

- Development of the dowel bars into the footing

  The dowel bars must extend into the footing at least a compression development length, ℓ_{dc} :

$$\ell_{dc} = \text{ greater of } \begin{cases} (f_y\psi_r \,/\, 50\lambda\sqrt{f'_c})d_b = [(60{,}000 \times 1.0) \,/\, (50 \times 1.0 \times \sqrt{4{,}000})] \times 0.75 = 14.2 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 0.75 = 13.5 \text{ in.} \\ 8 \text{ in.} \end{cases} \quad \text{Eq. (10.34)}$$

  where $\psi_r = 1.0$ for reinforcing bars without confinement. Table 10.4

  The minimum footing thickness for the development of the dowel bars in compression is the following:

$$h \geq \ell_{dc} + r + (d_b)_d + 2(d_b)_f + \text{cover} = 14.2 + (3 \times 0.75) + (2 \times 0.875) + 3.0 = 21.2 \text{ in.} \quad \text{Eq. (10.35)}$$

  The provided footing thickness is greater than the required footing thickness, so the dowel bars can be fully developed for compression into the footing.

- Development of the dowel bars into the column

  The dowel bars are lap spliced to the longitudinal bars in the column. Because the dowel bars are smaller in diameter than the longitudinal bars, the compression lap splice length, ℓ_{sc}, must be greater than or equal to the larger of the following:

  Development length in compression, ℓ_{dc}, of the #9 longitudinal bars:

$$\ell_{dc} = \text{ greater of } \begin{cases} (f_y\psi_r \,/\, 50\lambda\sqrt{f'_c})d_b = [(60{,}000 \times 1.0) \,/\, (50 \times 1.0 \times \sqrt{4{,}000})] \times 1.128 = 21.4 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 1.128 = 20.3 \text{ in.} \\ 8 \text{ in.} \end{cases} \quad \text{Eq. (10.34)}$$

  Compression lap splice length, ℓ_{sc}, of the #6 dowel bars:

$$\ell_{sc} = \text{greater of } \begin{cases} 0.0005 f_y d_b = 0.0005 \times 60{,}000 \times 0.75 = 22.5 \text{ in.} \\ 12.0 \text{ in.} \end{cases} \quad \text{Eq. (10.37)}$$

  Provide a lap splice length equal to 2 ft-0 in.

Reinforcement details for the footing are given in Figure 10.21.

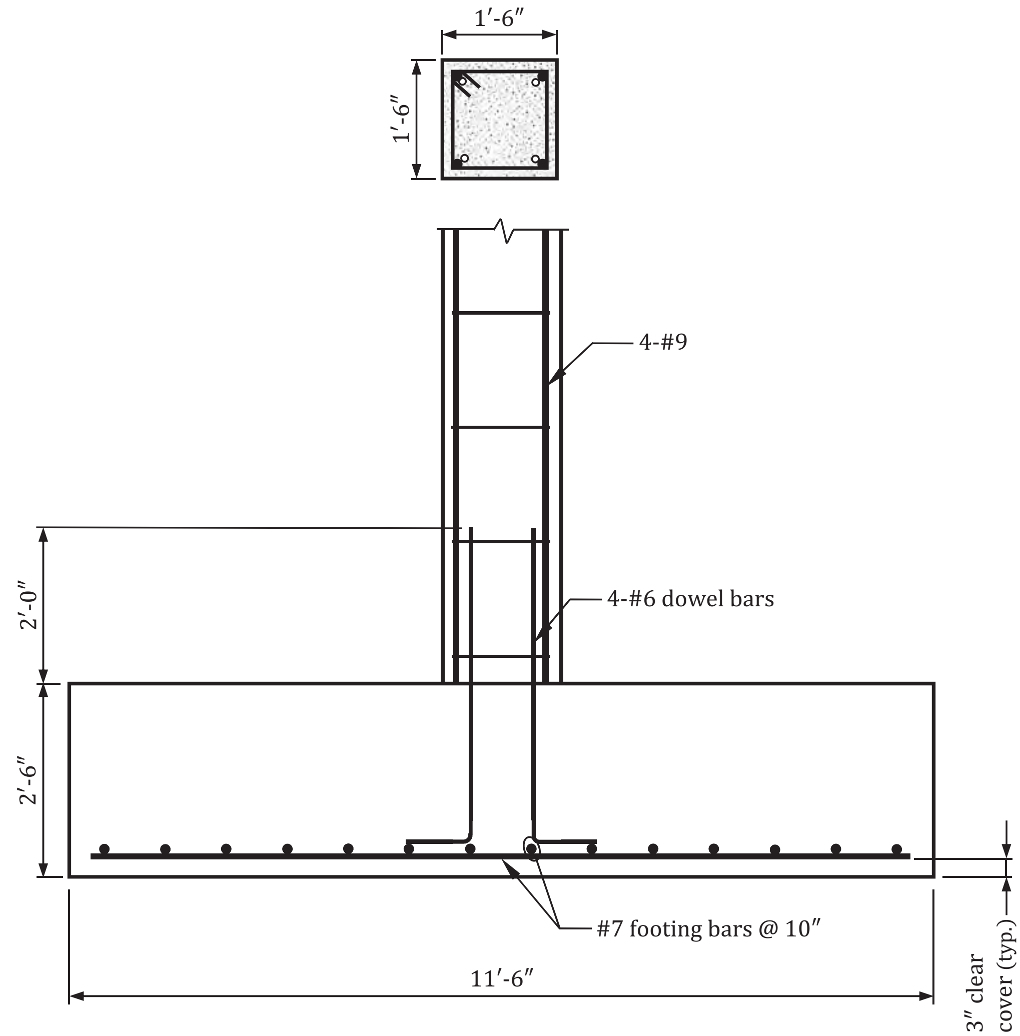


Figure 10.21 Reinforcement details for the footing in Example 10.2.

10.5.3 Example 10.3 – Design of a Rectangular Isolated Spread Footing Subjected to Axial Compression: Building #1 (Framing Option B), SDC A

Design a rectangular isolated footing for the 18-in. square interior column of Building #1, Framing Option B in Example 7.1 (see Figure 1.1). Because of property line restrictions, one dimension of the footing is limited to 8 ft. The column is reinforced with 4-#9 bars. Assume Grade 60 reinforcement and normalweight concrete with $f_c' = 4{,}000 \text{ psi}$ for the column and the footing. Also assume a permissible soil pressure of 4,000 lb/ft^2.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the base dimensions of the footing

From Example 7.1, the service axial dead, roof live, and live loads are equal to the following:

$P_D = 342.7 \text{ kips}$

$P_{L_r} = 11.8 \text{ kips}$

$P_L = 152.8 \text{ kips}$

Use the basic load combinations for allowable stress design in ASCE/SEI 2.4.1 to determine the maximum axial force transferred to the footing:

1. $P_D = 342.7$ kips
2. $P_D + P_L = 342.7 + 152.8 = 495.5$ kips
3. $P_D + P_{L_r} = 342.7 + 11.8 = 354.5$ kips
4. $P_D + 0.75P_L + 0.75P_{L_r} = 342.7 + (0.75 \times 152.8) + (0.75 \times 11.8) = 466.2$ kips

Using ASCE/SEI Eq. 2, the required area of the footing is equal to the following:

$$A_f = \frac{P_D + P_L}{q_p} = \frac{495.5 \times 1{,}000}{4{,}000} = 123.9 \text{ ft}^2 \qquad \text{Eq. (10.1)}$$

Given $B = 8.0$ ft, $L = 123.9 / 8.0 = 15.5$ ft

An 8 ft-0 in. by 15 ft-6 in. isolated footing is adequate for soil bearing capacity.

Step 2 – Determine the thickness of the footing

Use the load combinations in ACI Table 5.3.1 to determine the thickness of the footing:

$$1.4P_D = 1.4 \times 342.7 = 479.8 \text{ kips} \qquad \text{ACI Eq. (5.3.1a)}$$

$$1.2P_D + 1.6P_L + 0.5P_{L_r} = (1.2 \times 342.7) + (1.6 \times 152.8) + (0.5 \times 11.8) = 661.6 \text{ kips} \qquad \text{ACI Eq. (5.3.1b)}$$

$$1.2P_D + 1.6P_{L_r} + 0.5P_L = (1.2 \times 342.7) + (1.6 \times 11.8) + (0.5 \times 152.8) = 506.5 \text{ kips} \qquad \text{ACI Eq. (5.3.1c)}$$

The maximum factored pressure at the base of the footing is equal to the following:

$$q_u = \frac{P_u}{A_f} = \frac{661.6}{8.0 \times 15.5} = 5.3 \text{ kips/ft}^2$$

The minimum effective depth of the footing is determined using the longest cantilever length, c.

Minimum effective depth for flexure:

$$d \geq c\sqrt{\frac{P_u / A_f}{0.216\left(1 - \dfrac{0.0708}{f'_c}\right)}} = \left(\frac{15.5}{2} - \frac{18.0}{2 \times 12}\right) \times \sqrt{\frac{5.3}{0.216 \times \left(1 - \dfrac{0.0708}{4.0}\right)}} = 35.0 \text{ in.} \qquad \text{Eq. (10.15)}$$

Minimum effective depth for one-way shear:

$$d \geq \frac{q_u c}{q_u + \phi\lambda\sqrt{f'_c}} = \frac{\left(\dfrac{5.3 \times 1{,}000}{144}\right) \times \left(\dfrac{15.5 \times 12}{2} - \dfrac{18.0}{2}\right)}{\left(\dfrac{5.3 \times 1{,}000}{144}\right) + (0.75 \times 1.0 \times \sqrt{4{,}000})} = 36.7 \text{ in.} \qquad \text{Eq. 10.18)}$$

Minimum effective depth for two-way shear:

$$a_1 = 0.5q_u + \phi v_c = \left(\frac{0.5 \times 5.3 \times 1{,}000}{144}\right) + (0.75 \times 4 \times \sqrt{4{,}000}) = 208.1 \text{ psi}$$

$$a_2 = 0.25q_u + \phi v_c = \left(\frac{0.25 \times 5.3 \times 1{,}000}{144}\right) + (0.75 \times 4 \times \sqrt{4{,}000}) = 198.9 \text{ psi}$$

$$a_3 = (A_f / c_1^2) - 1 = \frac{8.0 \times 15.5 \times 144}{18.0^2} - 1 = 54.1$$

$$d \geq c_1 \left[\frac{-a_1 + \sqrt{a_1^2 + q_u a_2 a_3}}{2a_2} \right] = 18.0 \times \left\{ \frac{-208.1 + \sqrt{208.1^2 + \left[\left(\frac{5.3 \times 1{,}000}{144} \right) \times 198.9 \times 54.1 \right]}}{2 \times 198.9} \right\} = 20.6 \text{ in.} \qquad \text{Eq. (10.20)}$$

The minimum effective depth for one-way shear governs in this case.

Try a 42-in.-thick footing $(d = 38.0 \text{ in.})$.

Step 3 – Determine the required flexural reinforcement

Because the provided effective depth of 38.0 in. is greater than the 35.0-in. minimum effective depth calculated for flexure, minimum flexural reinforcement is adequate:

$$A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 42.0 = 0.91 \text{ in.}^2/\text{ft} \qquad \text{ACI 7.6.1.1}$$

Reinforcement in the long direction is uniformly distributed across the 8.0-ft width. Thus, try #9 bars spaced at 12 in. on center $(A_{s,provided} = 1.00 \text{ in.}^2/\text{ft})$. The 12-in. spacing is less than the maximum spacing of $3h = 126$ in. or 18.0 in.

In the short direction, the portion of the total reinforcement, $\gamma_s A_s$, is uniformly distributed over a band width equal to $B = 8.0$ ft centered on the column (see Figure 10.14).

$$\beta = 15.5 / 8.0 = 1.94$$

$$\gamma_s = 2 / (\beta + 1) = 2 / (1.94 + 1) = 0.68$$

Therefore, $0.68 \times 0.91 = 0.62 \text{ in.}^2/\text{ft}$ must be distributed within an 8.0 ft width.

Within the $15.5 - 8.0 = 7.5$ ft width, provide $0.91 - 0.62 = 0.29 \text{ in.}^2/\text{ft}$.

Alternatively, provide the following amount of reinforcement, which can be uniformly distributed across the 15.5-ft width:

$$A_s = \left(\frac{2\beta}{\beta + 1} \right) A_{s,min} = \left(\frac{2 \times 1.94}{1.94 + 1} \right) \times 0.91 = 1.20 \text{ in.}^2/\text{ft} \qquad \text{Sect. 10.3.5}$$

Try #9 bars spaced at 10 in. on center $(A_{s,provided} = 1.20 \text{ in.}^2/\text{ft})$.

Step 4 – Check if the flexural reinforcement can be developed for tension

The tension development length, ℓ_d, of the #9 bars must be less than or equal to the available development length:

$$\ell_d \leq \frac{B - c_1}{2} - \text{cover} \qquad \text{Eq. (10.21)}$$

Determine ℓ_d from ACI Eq. (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \qquad \text{Eq. (10.22)}$$

For normalweight concrete, $\lambda = 1.0$. Eq. (10.23)

For Grade 60 reinforcement, $\psi_g = 1.0$. Eq. (10.24)

For uncoated reinforcing bars, $\psi_e = 1.0$. Eq. (10.25)

For #9 bars, $\psi_s = 1.0$. Eq. (10.26)

For less than 12 in. of concrete placed below the flexural reinforcement, $\psi_t = 1.0$. Eq. (10.27)

$$c_b = \text{lesser of} \begin{cases} 3.0 + (1.128 / 2) = 3.6 \text{ in.} \\ \\ 10.0 / 2 = 5.0 \text{ in.} \end{cases} \qquad \text{Eq. (10.28)}$$

Because there is no transverse reinforcement, $K_{tr} = 0$. Eq. (10.29)

$(c_b + K_{tr}) / d_b = (3.6 + 0) / 1.128 = 3.2 > 2.5, \text{ use } 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0 \times \sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} \right) \times 1.128 = 32.1 \text{ in.} > 12.0 \text{ in.}$$

$$< \frac{B - c_1}{2} - \text{cover} = \frac{(8.0 \times 12) - 18.0}{2} - 3.0 = 36.0 \text{ in.}$$

Because the available development length is greater than the tension development length, the #9 bars can be fully developed for flexure. It is evident that the #9 bars can be fully developed for tension in the perpendicular direction as well.

Step 5 – Determine the required dowel reinforcement

Only vertical forces need to be considered for force transfer between the column and footing.

- Check the bearing force on the column:

  $\phi B_n = \phi 0.85 f'_c A_1 = 0.65 \times 0.85 \times 4.0 \times 18.0^2 = 716.0 \text{ kips} > B_u = 661.6 \text{ kips}$ Eq. (10.30)

- Check the bearing force on the footing:

  $B_u \le \phi B_n = (\phi 0.85 f'_c A_1)\sqrt{A_2 / A_1} \le 2(\phi 0.85 f'_c A_1)$ Eq. (10.31)

  Using Figure 10.16, A_2 is determined as follows:

  Thickness of footing $= 42.0$ in.

  Horizontal projection for a 1:2 slope $= 2 \times 42.0 = 84.0$ in.

  Projected length $b = 84.0 + 18.0 + 84.0 = 186.0$ in.

  Therefore, $A_2 = 186.0 \times 96.0 = 17{,}856 \text{ in.}^2$

  $\sqrt{A_2 / A_1} = \sqrt{17{,}856 / 18.0^2} = 7.4 > 2.0; \text{ use } 2.0$

  $\phi B_n = 2(\phi 0.85 f'_c A_1) = 2 \times 0.65 \times 0.85 \times 4.0 \times 18.0^2 = 1{,}432.1 \text{ kips} > B_u = 661.6 \text{ kips}$ s

- Determine the required interface reinforcement

  Because the design bearing strength is adequate, provide the minimum area of reinforcement across the interface:

$A_{s,min} = 0.005A_g = 0.005 \times 18.0^2 = 1.62 \text{ in.}^2$ Sect. 10.3.7

Try 4-#6 dowel bars ($A_{s,provided} = 1.76 \text{ in.}^2$).

- Development of the dowel bars into the footing

  The dowel bars must extend into the footing at least a compression development length, ℓ_{dc} :

$$\ell_{dc} = \text{ greater of } \begin{cases} (f_y\psi_r / 50\lambda\sqrt{f_c'})d_b = [(60{,}000 \times 1.0) / (50 \times 1.0 \times \sqrt{4{,}000})] \times 0.75 = 14.2 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 0.75 = 13.5 \text{ in.} \\ 8 \text{ in.} \end{cases}$$ Eq. (10.34)

  where $\psi_r = 1.0$ for reinforcing bars without confinement. Table 10.4

  The minimum footing thickness for the development of the dowel bars in compression is the following:

$$h \geq \ell_{dc} + r + (d_b)_d + 2(d_b)_f + \text{cover} = 14.2 + (3 \times 0.75) + (2 \times 1.128) + 3.0 = 21.7 \text{ in.}$$ Eq. (10.35)

  The provided footing thickness is greater than the required footing thickness, so the dowel bars can be fully developed for compression into the footing.

- Development of the dowel bars into the column

  The dowel bars are lap spliced to the longitudinal bars in the column. Because the dowel bars are smaller in diameter than the longitudinal bars, the compression lap splice length, ℓ_{sc}, must be greater than or equal to the larger of the following:

  Development length in compression, ℓ_{dc}, of the #9 longitudinal bars:

$$\ell_{dc} = \text{ greater of } \begin{cases} (f_y\psi_r / 50\lambda\sqrt{f_c'})d_b = [(60{,}000 \times 1.0) / (50 \times 1.0 \times \sqrt{4{,}000})] \times 1.128 = 21.4 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 1.128 = 20.3 \text{ in.} \\ 8 \text{ in.} \end{cases}$$ Eq. (10.34)

  Compression lap splice length, ℓ_{sc}, of the #6 dowel bars:

$$\ell_{sc} = \text{greater of } \begin{cases} 0.0005 f_y d_b = 0.0005 \times 60{,}000 \times 0.75 = 22.5 \text{ in.} \\ 12.0 \text{ in.} \end{cases}$$ Eq. (10.37)

  Provide a lap splice length equal to 2 ft-0 in.

Reinforcement details for this footing are similar to those in Figure 10.21.

10.5.4 Example 10.4 – Design of a Square Isolated Spread Footing Subjected to Axial Compression and Flexure: Building #1 (Framing Option C), SDC A

Design a square isolated footing for the 20-in. square edge column of Building #1, Framing Option C in Example 7.14 (see Figure 1.1). The column is reinforced with 4-#9 bars. Assume Grade 60 reinforcement and normalweight concrete with $f_c' = 4{,}000$ psi for the column and the footing. Also assume a permissible soil pressure of 3,000 lb/ft^2.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the base dimensions of the footing

The allowable stress and strength design load combinations are given in Table 10.11 (see Example 7.14). The allowable stress design load combinations are from ASCE/SEI 2.4.1. The strength design load combinations are from ACI Table 5.3.1. The bending moment in Table 10.11 due to wind is at the bottom of the column.

Table 10.11 Summary of Axial Forces and Bending Moments for Column C1

Load Case		Axial Force (kips)	Bending Moment (ft-kips)
Dead (D)		233.3	0
Roof live (L_r)		5.9	—
Live (L)		76.4	0
Wind (W)		0	± 63.9
Allowable Stress Load Combinations			
ASCE/SEI Eq. 1	D	233.3	0
ASCE/SEI Eq. 2	$D + L$	309.7	0
ASCE/SEI Eq. 3	$D + L_r$	239.2	0
ASCE/SEI Eq. 4	$D + 0.75L + 0.75L_r$	295.0	0
ASCE/SEI Eq. 5	$D + 0.6W$	233.3	± 38.3
ASCE/SEI Eq. 6	$D + 0.75L + 0.75(0.6W) + 0.75L_r$	295.0	± 28.8
ASCE/SEI Eq. 7	$0.6D + 0.6W$	140.0	± 38.3
Strength Design Load Combinations			
ACI Eq. (5.3.1a)	$1.4D$	326.6	0
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$	405.2	0
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$	327.6	0
	$1.2D + 0.5W + 1.6L_r$	289.4	± 32.0
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L + 0.5L_r$	321.1	± 63.9
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	210.0	± 63.9

Determine the required base dimensions of the footing based on axial force only (ASCE/SEI Eq. 2) and then check if the maximum combined pressure due to axial force and bending moment is less than the permissible soil pressure.

$$A_f = \frac{P_D + P_L}{q_p} = \frac{309.7 \times 1{,}000}{3{,}000} = 103.2 \text{ ft}^2 \qquad \text{Eq. (10.1)}$$

$$B = L = \sqrt{A_f} = 10.2 \text{ ft}$$

Try a 10 ft-6 in. square isolated footing.

Check the maximum combined pressure based on ASCE/SEI Eq. 5:

Pressure due to axial force $= \dfrac{P}{A_f} = \dfrac{233{,}300}{10.5^2} = 2{,}116 \text{ lb/ft}^2$

Pressure due to moment $= \dfrac{6M}{BL^2} = \dfrac{6 \times 38.3 \times 1{,}000}{10.5^3} = 199 \text{ lb/ft}^2$

Total pressure $= 2{,}116 + 199 = 2{,}315 \text{ lb/ft}^2 < q_p = 3{,}000 \text{ lb/ft}^2$

Check the maximum combined pressure based on ASCE/SEI Eq. 6:

Pressure due to axial force $= \dfrac{P}{A_f} = \dfrac{295{,}000}{10.5^2} = 2{,}676 \text{ lb/ft}^2$

Pressure due to moment $= \dfrac{6M}{BL^2} = \dfrac{6 \times 28.8 \times 1{,}000}{10.5^3} = 149 \text{ lb/ft}^2$

Total pressure $= 2{,}676 + 149 = 2{,}825 \text{ lb/ft}^2 < q_p = 3{,}000 \text{ lb/ft}^2$

A 10 ft-6 in. isolated square footing is adequate for soil bearing capacity.

Note that there is no net uplift (tension) at the base of the footing for any of the allowable stress load combinations in Table 10.11.

Step 2 – Determine the thickness of the footing

Based on the strength design load combinations in Table 10.11, the maximum factored pressure at the base of the footing based on only axial force is obtained from ACI Eq. (5.3.1b), which is a uniform stress over the base area of the footing:

$$q_u = \frac{P_u}{A_f} = \frac{405.2}{10.5^2} = 3.7 \text{ kips/ft}^2$$

Minimum effective depth for flexure:

$$d \geq c\sqrt{\frac{P_u / A_f}{0.216\left(1 - \dfrac{0.0708}{f_c'}\right)}} = \left(\frac{10.5}{2} - \frac{20.0}{2 \times 12}\right) \times \sqrt{\frac{3.7}{0.216 \times \left(1 - \dfrac{0.0708}{4.0}\right)}} = 18.4 \text{ in.} \qquad \text{Eq. (10.15)}$$

Minimum effective depth for one-way shear:

$$d \geq \frac{q_u c}{q_u + \phi\lambda\sqrt{f_c'}} = \frac{\left(\dfrac{3.7 \times 1{,}000}{144}\right) \times \left(\dfrac{10.5 \times 12}{2} - \dfrac{20.0}{2}\right)}{\left(\dfrac{3.7 \times 1{,}000}{144}\right) + (0.75 \times 1.0 \times \sqrt{4{,}000})} = 18.6 \text{ in.} \qquad \text{Eq. (10.18)}$$

Minimum effective depth for two-way shear:

$$a_1 = 0.5q_u + \phi v_c = \left(\frac{0.5 \times 3.7 \times 1{,}000}{144}\right) + (0.75 \times 4 \times \sqrt{4{,}000}) = 202.6 \text{ psi}$$

$$a_2 = 0.25q_u + \phi v_c = \left(\frac{0.25 \times 3.7 \times 1{,}000}{144}\right) + (0.75 \times 4 \times \sqrt{4{,}000}) = 196.2 \text{ psi}$$

$$a_3 = (A_f / c_1^2) - 1 = \frac{(10.5 \times 12)^2}{20.0^2} - 1 = 38.7$$

$$d \geq c_1\left[\frac{-a_1 + \sqrt{a_1^2 + q_u a_2 a_3}}{2a_2}\right] = 20.0 \times \left\{\frac{-202.6 + \sqrt{202.6^2 + \left[\left(\frac{3.7 \times 1{,}000}{144}\right) \times 196.2 \times 38.7\right]}}{2 \times 196.2}\right\} = 14.4 \text{ in.} \qquad \text{Eq. (10.20)}$$

The minimum effective depths determined above are based on a uniform factored pressure at the base of the footing due to axial force only. Check two-way shear strength requirements based on ACI Eq. (5.3.1d) for combined axial force and bending assuming $d = 20.0$ in.

$$q_{u,max} = \frac{P_u}{BL} + \frac{6M_u}{BL^2} = \frac{321.1}{10.5^2} + \frac{6 \times 63.9}{10.5^3} = 2.91 + 0.33 = 3.24 \text{ kips/ft}^2$$

$$q_{u,min} = \frac{P_u}{BL} - \frac{6M_u}{BL^2} = \frac{321.1}{10.5^2} - \frac{6 \times 63.9}{10.5^3} = 2.91 - 0.33 = 2.58 \text{ kips/ft}^2$$

The critical section around the 20-in. column is at a distance of $d / 2 = 10.0$ in. from the face of the column (see Figure 10.12). The total shear stress is equal to the shear stress due to the factored axial force plus the shear stress due to the transferred bending moment.

The magnitude of the factored pressure along the length of the footing is equal to the following:

$$q_u = \frac{(3.24 - 2.58)x}{10.5} + 2.58 = 0.063x + 2.58$$

where x is measured from the edge of the footing with minimum factored pressure.

The boundaries of the critical sections are at the following locations from the edge of the footing with minimum factored pressure:

$$x_1 = \frac{B}{2} - \frac{c_1}{2} - \frac{d}{2} = \frac{10.5}{2} - \frac{20.0}{2 \times 12} - \frac{20.0}{2 \times 12} = 3.6 \text{ ft}$$

$$x_2 = \frac{B}{2} + \frac{c_1}{2} + \frac{d}{2} = \frac{10.5}{2} + \frac{20.0}{2 \times 12} + \frac{20.0}{2 \times 12} = 6.9 \text{ ft}$$

The factored pressures at these locations are equal to the following:

$$q_u \ @ \ x_1 = (0.063 \times 3.6) + 2.58 = 2.81 \text{ kips/ft}^2$$

$$q_u \ @ \ x_2 = (0.063 \times 6.9) + 2.58 = 3.02 \text{ kips/ft}^2$$

The factored shear force at the critical section, V_u, is equal to the factored axial force from the column minus the factored soil pressure bounded by the critical section (see Figure 10.22):

$$V_u = 321.1 - \left[2.81 \times \left(\frac{40.0}{12}\right)^2\right] - \left[\frac{1}{2} \times (3.02 - 2.81) \times \left(\frac{40.0}{12}\right)^2\right] = 321.1 - 32.4 = 288.7 \text{ kips}$$

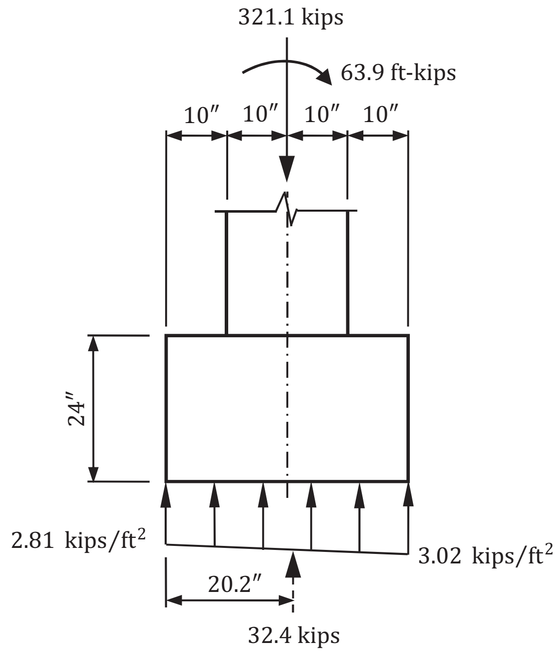


Figure 10.22 Free-body diagram of the critical section for two-way shear in Example 10.4.

The moment transferred between the column and the footing, M_{sc}, is obtained by summing moments about the centroid of the footing:

$$M_{sc} = 63.9 - \left(32.4 \times \frac{20.2 - 20.0}{12}\right) = 63.4 \text{ ft-kips}$$

The factor γ_v is determined by ACI Eq. (8.4.4.2.2):

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 1 - \frac{1}{1 + (2/3)\sqrt{40.0 / 40.0}} = 0.4$$

The section properties of the critical section are determined using Case 1 in Table 5.11 of this publication for an interior column:

$$A_c = 2(b_1 + b_2)d = 2 \times (40.0 + 40.0) \times 20.0 = 3{,}200.0 \text{ in.}^2$$

$$\frac{J}{c_{AB}} = \frac{b_1 d(b_1 + 3b_2) + d^3}{3} = \frac{[40.0 \times 20.0 \times (4 \times 40.0)] + 20.0^3}{3} = 45{,}333 \text{ in.}^3$$

The total factored shear stress at the critical section is equal to the following:

$$v_{u(AB)} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{288{,}700}{3{,}200.0} + \frac{0.4 \times 63.4 \times 12{,}000}{45{,}333} = 90.2 + 6.7 = 96.9 \text{ psi} < \phi 4\sqrt{f_c'} = 189.7 \text{ psi}$$

Therefore, a minimum effective depth of 20.0 in. is also adequate for two-way shear based on combined axial force and bending.

The minimum effective depth for flexure and for one-way shear are not determined using ACI Eq. (5.3.1d) because the maximum factored pressure at the base of the footing based on ACI Eq. (5.3.1b) is greater than that based on ACI Eq. (5.3.1d).

Try a 24-in.-thick footing $(d = 20.0 \text{ in.})$.

Step 3 – Determine the required flexural reinforcement

Because the provided effective depth of 20.0 in. is greater than the 18.4-in. minimum effective depth calculated for flexure, minimum flexural reinforcement is adequate:

$$A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 24.0 = 0.52 \text{ in.}^2/\text{ft} \qquad \text{ACI 7.6.1.1}$$

Try #6 bars spaced at 10 in. on center $(A_{s,provided} = 0.53 \text{ in.}^2/\text{ft})$. The 10-in. spacing is less than the maximum spacing of $3h = 72$ in. or 18 in.

Step 4 – Check if the flexural reinforcement can be developed for tension

The tension development length, ℓ_d, of the #6 bars must be less than or equal to the available development length:

$$\ell_d \leq \frac{L - c_1}{2} - \text{cover} \qquad \text{Eq. (10.21)}$$

Determine ℓ_d from ACI Eq. (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \qquad \text{Eq. (10.22)}$$

For normalweight concrete, $\lambda = 1.0$. Eq. (10.23)

For Grade 60 reinforcement, $\psi_g = 1.0$. Eq. (10.24)

For uncoated reinforcing bars, $\psi_e = 1.0$. Eq. (10.25)

For #6 bars, $\psi_s = 0.8$. Eq. (10.26)

For less than 12 in. of concrete placed below the flexural reinforcement, $\psi_t = 1.0$. Eq. (10.27)

$$c_b = \text{lesser of} \begin{cases} 3.0 + (0.75 / 2) = 3.4 \text{ in.} \\ \\ 10.0 / 2 = 5.0 \text{ in.} \end{cases} \qquad \text{Eq. (10.28)}$$

Because there is no transverse reinforcement, $K_{tr} = 0$. Eq. (10.29)

$$(c_b + K_{tr}) / d_b = (3.4 + 0) / 0.75 = 4.5 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0 \times \sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{2.5} \right) \times 0.75 = 17.1 \text{ in.} > 12.0 \text{ in.}$$

$$< \frac{L - c_1}{2} - \text{cover} = \frac{(10.5 \times 12) - 20.0}{2} - 3.0 = 50.0 \text{ in.}$$

Because the available development length is greater than the tension development length, the #6 bars can be fully developed for flexure.

Step 5 – Determine the required dowel reinforcement

- Bearing strength of the column

  Factored bearing stress, b_u, is determined for compression using the factored axial force determined by ACI Eqs. (5.3.1b) and for compression plus bending using the factored axial force and bending moment determined by ACI Eqs. (5.3.1d) and (5.3.1f) [see Table 10.11].

  ACI Eq. (5.3.1b):

$$\text{Axial compression stress} = b_u = \frac{P_u}{A_1} = \frac{405.2 \times 1{,}000}{20.0^2} = 1{,}013 \text{ psi}$$

  ACI Eq. (5.3.1d):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{321.1 \times 1{,}000}{20.0^2} = 803 \text{ psi}$$

$$\text{Bending stress (compression and tension)} = \frac{6M_u}{c_1^3} = \frac{6 \times 63.9 \times 12{,}000}{20.0^3} = 575 \text{ psi}$$

$$\text{Maximum compression stress} = 803 + 575 = 1{,}378 \text{ psi}$$

$$\text{Minimum compression stress} = 803 - 575 = 228 \text{ psi}$$

  ACI Eq. (5.3.1f):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{210.0 \times 1{,}000}{20.0^2} = 525 \text{ psi}$$

$$\text{Bending stress (compression and tension)} = \frac{6M_u}{c_1^3} = \frac{6 \times 63.9 \times 12{,}000}{20.0^3} = 575 \text{ psi}$$

$$\text{Maximum compression stress} = 525 + 575 = 1{,}100 \text{ psi}$$

$$\text{Maximum tension stress} = 525 - 575 = -50 \text{ psi}$$

  Design bearing strength:

$$\phi b_n = \phi 0.85 f_c' = 0.65 \times 0.85 \times 4{,}000 = 2{,}210 \text{ psi}$$

  Tension forces cannot be transferred through bearing at the interface, so dowel bars will be provided that match the size of the longitudinal bars in the column. This reinforcement ensures that both the compression and tension forces are adequately transferred from the column into the footing.

- Bearing strength of the footing

  There is no need to check the bearing strength of the footing because interface reinforcement must be provided due to the net tension stress from the factored axial force and bending moment determined by ACI Eq. (5.3.1f).

- Determine the required interface reinforcement

  Try 4-#9 dowel bars. This reinforcement matches the longitudinal reinforcement in the column and ensures that both the compression and tension forces are adequately transferred through the interface.

  Check minimum area of reinforcement across the interface:

$$A_{s,min} = 0.005A_g = 0.005 \times 20.0^2 = 2.00 \text{ in.}^2 < A_{s,provided} = 4.00 \text{ in.}^2$$

- Development of the dowel bars into the footing

  A standard 90-degree hook will be provided at the ends of the #9 dowel bars. Determine the tension development length, ℓ_{dh}, of the #9 dowel bars terminating in a hook:

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \qquad \text{Eq. (10.38)}$$

$\psi_e = 1.0$ for uncoated bars — Table 10.5

$\psi_r = 1.6$ (confining reinforcement not provided)

$\psi_o = 1.0$ (side cover normal to hook $> 6d_b$)

$\psi_c = (f_c' / 15{,}000) + 0.6 = (4{,}000 / 15{,}000) + 0.6 = 0.9$

$\lambda = 1.0$ for normalweight concrete

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} = \left(\dfrac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.9}{55 \times 1.0 \times \sqrt{4{,}000}}\right) \times 1.128^{1.5} = 29.8 \text{ in.} \\ 8d_b = 8 \times 1.128 = 9.0 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

$$\text{Minimum } h = \ell_{dh} + 2(d_b)_f + \text{cover} = 29.8 + (2 \times 0.75) + 3.0 = 34.3 \text{ in.} \qquad \text{Eq. (10.39)}$$

The minimum $h = 34.3$ in. is greater than the provided footing thickness of 24.0 in. Thus, increase the footing thickness to 36.0 in. to accommodate development of the #9 dowel bars.

Determine minimum flexure reinforcement in the footing based on the increased thickness:

$$A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 36.0 = 0.78 \text{ in.}^2/\text{ft} \qquad \text{ACI 7.6.1.1}$$

Use #8 bars spaced at 12 in. on center ($A_{s,provided} = 0.79$ in.^{2}/ft). The 12-in. spacing is less than the maximum spacing of $3h = 108$ in. or 18 in. Note that the #9 dowel bars can be fully developed in the 36-in.-thick footing with the #8 bars for flexure. Also, the #8 bars can be fully developed for flexure based on the width of the footing.

- Development of the dowel bars into the column

  The dowel bars must be lap spliced to the longitudinal reinforcement in the column using a tension lap splice. Because all the dowel bars are spliced at the same location, a Class B tension lap splice is required (ACI Table 25.5.2.1).

  Development length in tension, ℓ_d, of the #9 bars:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\right) d_b \geq 12.0 \text{ in.} \qquad \text{Eq. (10.22)}$$

For normalweight concrete, $\lambda = 1.0$. Eq. (10.23)

For Grade 60 reinforcement, $\psi_g = 1.0$. Eq. (10.24)

For uncoated reinforcing bars, $\psi_e = 1.0$. Eq. (10.25)

For #9 bars, $\psi_s = 1.0$. Eq. (10.26)

$\psi_t = 1.0$. Eq. (10.27)

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + (d_b)_{tie} + 0.5(d_b)_{long.} = 1.5 + 0.375 + (0.5 \times 1.128) = 2.4 \text{ in.} \\ \dfrac{s}{2} = \dfrac{20.0 - (2 \times 1.5) - (2 \times 0.375) - 1.128}{2} = 7.6 \text{ in.} \end{cases}$$ Eq. (10.28)

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.4 + 0) / 1.128 = 2.1 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60,000}{1.0 \times \sqrt{4,000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.1} \right) \times 1.128 = 38.2 \text{ in.} > 12.0 \text{ in.}$$

$\text{Class B lap splice length} = 1.3\ell_d = 1.3 \times 38.2 = 49.7 \text{ in.}$

Provide a lap splice length of 4 ft-2 in.

- Check horizontal force transfer

From Example 7.18, $V_u = 6.5$ kips is transferred horizontally between the column and footing. The required area of shear-friction reinforcement is determined as follows assuming the surface between the column and footing has not been intentionally roughened:

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{V_u}{\phi f_y (0.6\lambda)} = \frac{6.5}{0.75 \times 60 \times (0.6 \times 1.0)} = 0.24 \text{ in.}^2$$ Table 10.6, Eq. (10.40)

The 4-#9 dowel bars provide 4.00 in.^2 across the interface, which is greater than the required amount of 0.24 in.^2

Check the upper shear limit:

$$V_u = 6.5 \text{ kips} < \text{ least of } \begin{cases} \phi 0.2 f'_c A_c = 0.75 \times 0.2 \times 4 \times 20.0^2 = 240.0 \text{ kips} \\ \phi 800 A_c = 0.75 \times 800 \times 20.0^2 / 1,000 = 240.0 \text{ kips} \end{cases}$$ Table 10.7

Reinforcement details for this footing are similar to those in Figure 10.21.

10.5.5 Example 10.5 – Design of a Square Isolated Spread Footing Subjected to Axial Compression and Flexure: Building #1 (Framing Option B), SDC A

Design a square isolated footing for the 24-in. square edge column of Building #1, Framing Option B in Example 7.25 (see Figure 1.1). The column is reinforced with 4-#11 bars. Assume Grade 60 reinforcement and normalweight concrete with $f'_c = 4,000$ psi for the column and the footing. Also assume a permissible soil pressure of 3,500 lb/ft^2.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the base dimensions of the footing

The allowable stress and strength design load combinations are given in Table 10.12 (see Example 7.25). The allowable stress design load combinations are from ASCE/SEI 2.4.1. The strength design load combinations are from ACI 5.3.1 and ASCE/SEI 2.3.1, which are the same. The bending moments in Table 10.12 due to gravity and wind are at the bottom of the column, the latter of which are magnified to account for the effects of slenderness (see Tables 7.44 and 7.46 of this publication).

Table 10.12 Summary of Axial Forces and Bending Moments for Column D1

Load Case			Axial Force (kips)	Bending Moment (ft-kips)
Dead (D)			224.4	2.8
Roof live (L_r)			5.9	0
Live (L)			76.4	2.4
Wind (W)			± 1.9	± 232.1
Allowable Stress Load Combinations				
ASCE/SEI Eq. 1	D		224.4	2.8
ASCE/SEI Eq. 2	$D + L$		300.8	5.2
ASCE/SEI Eq. 3	$D + L_r$		230.3	2.8
ASCE/SEI Eq. 4	$D + 0.75L + 0.75L_r$		286.1	4.6
ASCE/SEI Eq. 5	$D + 0.6W$	SSR	223.3	-136.5
		SSL	225.3	142.1
ASCE/SEI Eq. 6	$D + 0.75L + 0.75(0.6W) + 0.75L_r$	SSR	285.3	-99.9
		SSL	287.0	109.1
ASCE/SEI Eq. 7	$0.6D + 0.6W$	SSR	133.5	-137.6
		SSL	135.8	140.9
Strength Design Load Combinations				
ACI Eq. (5.3.1a)	$1.4D$		314.2	3.9
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		394.5	7.2
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		316.9	4.6
	$1.2D + 0.5W + 1.6L_r$	SSR	277.8	-125.5
		SSL	279.7	132.3
ACI Eq. (5.3.1d)	$1.2D + 1.0W + 0.5L + 0.5L_r$	SSR	308.5	-255.4
		SSL	312.3	264.6
ACI Eq. (5.3.1f)	$0.9D + 1.0W$	SSR	200.1	-248.2
		SSL	203.9	253.2

Determine the required base dimensions of the footing based on axial force only (ASCE/SEI Eq. 2) and then check if the maximum combined pressure due to axial force and bending moment is less than the permissible soil pressure.

$$A_f = \frac{P_D + P_L}{q_p} = \frac{300.8 \times 1{,}000}{3{,}500} = 85.9 \text{ ft}^2 \qquad \text{Eq. (10.1)}$$

$$B = L = \sqrt{A_f} = 9.3 \text{ ft}$$

Try a 9 ft-6 in. square isolated footing.

Check the maximum combined pressure based on ASCE/SEI Eq. 5:

$$\text{Pressure due to axial force} = \frac{P}{A_f} = \frac{225{,}300}{9.5^2} = 2{,}496 \text{ lb/ft}^2$$

$$\text{Pressure due to moment} = \frac{6M}{BL^2} = \frac{6 \times 142.1 \times 1{,}000}{9.5^3} = 994 \text{ lb/ft}^2$$

$$\text{Total pressure} = 2{,}496 + 994 = 3{,}490 \text{ lb/ft}^2 < q_p = 3{,}500 \text{ lb/ft}^2$$

Check the maximum combined pressure based on ASCE/SEI Eq. 6:

$$\text{Pressure due to axial force} = \frac{P}{A_f} = \frac{287{,}000}{9.5^2} = 3{,}180 \text{ lb/ft}^2$$

$$\text{Pressure due to moment} = \frac{6M}{BL^2} = \frac{6 \times 109.1 \times 1{,}000}{9.5^3} = 764 \text{ lb/ft}^2$$

$$\text{Total pressure} = 3{,}180 + 764 = 3{,}944 \text{ lb/ft}^2 > q_p = 3{,}500 \text{ lb/ft}^2$$

Therefore, a 9 ft-6 in. square isolated footing is not adequate for soil bearing capacity based on combined axial force and bending.

It can be determined that a 10 ft-6 in. isolated square footing is adequate.

Note that there is no net uplift (tension) at the base of the footing for any of the allowable stress load combinations in Table 10.12. based on a 10 ft-6 in. footing.

Step 2 – Determine the thickness of the footing

The factored pressure determined by ACI Eq. (5.3.1b) is trapezoidal over the base area of the footing, and thw maximum pressure is equal to the following:

$$q_u = \frac{P_u}{BL} + \frac{6M_u}{BL^2} = \frac{394.5}{10.5^2} + \frac{6 \times 7.2}{10.5^3} = 3.58 + 0.04 = 3.62 \text{ kips/ft}^2$$

The factored pressure determined by ACI Eq. (5.3.1d) is trapezoidal over the base area of the footing, and the maximum pressure is equal to the following:

$$\text{Maximum } q_u = \frac{P_u}{BL} + \frac{6M_u}{BL^2} = \frac{312.3}{10.5^2} + \frac{6 \times 264.6}{10.5^3} = 2.83 + 1.37 = 4.20 \text{ kips/ft}^2$$

Conservatively assume this pressure is uniformly distributed across the entire width of the footing and determine the minimum effective depths based on this assumption.

Minimum effective depth for flexure:

$$d \geq c\sqrt{\frac{P_u / A_f}{0.216\left(1-\dfrac{0.0708}{f_c'}\right)}} = \left(\frac{10.5}{2}-\frac{24.0}{2\times 12}\right)\times\sqrt{\frac{4.20}{0.216\times\left(1-\dfrac{0.0708}{4.0}\right)}} = 18.9 \text{ in.} \qquad \text{Eq. (10.15)}$$

Minimum effective depth for one-way shear:

$$d \geq \frac{q_u c}{q_u + \phi\lambda\sqrt{f_c'}} = \frac{\left(\dfrac{4.20\times 1{,}000}{144}\right)\times\left(\dfrac{10.5\times 12}{2}-\dfrac{24.0}{2}\right)}{\left(\dfrac{4.20\times 1{,}000}{144}\right)+(0.75\times 1.0\times\sqrt{4{,}000})} = 19.4 \text{ in.} \qquad \text{Eq. (10.18)}$$

Minimum effective depth for two-way shear:

$$a_1 = 0.5q_u + \phi v_c = \left(\frac{0.5\times 4.20\times 1{,}000}{144}\right)+(0.75\times 4\times\sqrt{4{,}000}) = 204.3 \text{ psi}$$

$$a_2 = 0.25q_u + \phi v_c = \left(\frac{0.25\times 4.20\times 1{,}000}{144}\right)+(0.75\times 4\times\sqrt{4{,}000}) = 197.0 \text{ psi}$$

$$a_3 = (A_f / c_1^2) - 1 = \frac{(10.5\times 12)^2}{24.0^2} - 1 = 26.6$$

$$d \geq c_1\left[\frac{-a_1+\sqrt{a_1^2+q_u a_2 a_3}}{2a_2}\right] = 24.0\times\left\{\frac{-204.3+\sqrt{204.3^2+\left[\left(\dfrac{4.20\times 1{,}000}{144}\right)\times 197.0\times 26.6\right]}}{2\times 197.0}\right\} = 14.4 \text{ in.} \qquad \text{Eq. (10.20)}$$

The minimum effective depth for one-way shear governs in this case.

Try a 24-in.-thick footing $(d = 20.0 \text{ in.})$.

Step 3 – Determine the required flexural reinforcement

Because the provided effective depth of 20.0 in. is greater than the 18.9-in. minimum effective depth calculated for flexure, minimum flexural reinforcement is adequate:

$$A_{s,min} = 0.0018bh = 0.0018\times 12.0\times 24.0 = 0.52 \text{ in.}^2/\text{ft} \qquad \text{ACI 7.6.1.1}$$

Try #6 bars spaced at 10 in. on center $(A_{s,provided} = 0.53 \text{ in.}^2/\text{ft})$. The 10-in. spacing is less than the maximum spacing of $3h = 72$ in. or 18 in.

Step 4 – Check if the flexural reinforcement can be developed for tension

The tension development length, ℓ_d, of the #6 bars must be less than or equal to the available development length:

$$\ell_d \leq \frac{L-c_1}{2} - \text{cover} \qquad \text{Eq. (10.21)}$$

Determine ℓ_d from ACI Eq. (25.4.2.4a):

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \quad \text{Eq. (10.22)}$$

For normalweight concrete, $\lambda = 1.0$. Eq. (10.23)

For Grade 60 reinforcement, $\psi_g = 1.0$. Eq. (10.24)

For uncoated reinforcing bars, $\psi_e = 1.0$. Eq. (10.25)

For #6 bars, $\psi_s = 0.8$. Eq. (10.26)

For less than 12 in. of concrete placed below the flexural reinforcement, $\psi_t = 1.0$. Eq. (10.27)

$$c_b = \text{lesser of} \begin{cases} 3.0 + (0.75 / 2) = 3.4 \text{ in.} \\ \\ 10.0 / 2 = 5.0 \text{ in.} \end{cases} \quad \text{Eq. (10.28)}$$

Because there is no transverse reinforcement, $K_{tr} = 0$. Eq. (10.29)

$(c_b + K_{tr}) / d_b = (3.4 + 0) / 0.75 = 4.5 > 2.5,$ use 2.5

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0 \times \sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{2.5} \right) \times 0.75 = 17.1 \text{ in.} > 12.0 \text{ in.}$$

$$< \frac{L - c_1}{2} - \text{cover} = \frac{(10.5 \times 12) - 24.0}{2} - 3.0 = 48.0 \text{ in.}$$

Because the available development length is greater than the tension development length, the #6 bars can be fully developed for flexure.

Step 5 – Determine the required dowel reinforcement

- Bearing strength of the column

  Factored bearing stress, b_u, is determined for compression plus bending using the factored axial forces and bending moments determined by ACI Eqs. (5.3.1b) and (5.3.1d) [see Table 10.12].

  ACI Eq. (5.3.1b):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{394.5 \times 1{,}000}{24.0^2} = 685 \text{ psi}$$

$$\text{Bending stress (compression and tension)} = \frac{6M_u}{c_1^3} = \frac{6 \times 7.2 \times 12{,}000}{24.0^3} = 38 \text{ psi}$$

$$\text{Maximum compression stress} = 685 + 38 = 723 \text{ psi}$$

$$\text{Minimum compression stress} = 685 - 38 = 647 \text{ psi}$$

ACI Eq. (5.3.1d):

$$\text{Axial compression stress} = \frac{P_u}{A_1} = \frac{312.3 \times 1,000}{24.0^2} = 542 \text{ psi}$$

$$\text{Bending stress (compression and tension)} = \frac{6M_u}{c_1^3} = \frac{6 \times 264.6 \times 12,000}{24.0^3} = 1,378 \text{ psi}$$

$$\text{Maximum compression stress} = 542 + 1,378 = 1,920 \text{ psi}$$

$$\text{Maximum tension stress} = 542 - 1,378 = -836 \text{ psi}$$

Design bearing strength:

$$\phi b_n = \phi 0.85 f_c' = 0.65 \times 0.85 \times 4,000 = 2,210 \text{ psi}$$

Tension forces cannot be transferred through bearing at the interface, so dowel bars will be provided that match the size of the longitudinal bars in the column. This reinforcement ensures that both the compression and tension forces are adequately transferred from the column into the footing.

- Bearing strength of the footing

  There is no need to check the bearing strength of the footing because interface reinforcement must be provided due to the net tension stress from the factored axial force and bending moment determined by ACI Eq. (5.3.1d).

- Determine the required interface reinforcement

  Try 4-#11 dowel bars. This reinforcement matches the longitudinal reinforcement in the column and ensures that both the compression and tension forces are adequately transferred through the interface.

  Check minimum area of reinforcement across the interface:

$$A_{s,min} = 0.005A_g = 0.005 \times 24.0^2 = 2.88 \text{ in.}^2 < A_{s,provided} = 6.24 \text{ in.}^2$$

- Development of the dowel bars into the footing

  A standard 90-degree hook will be provided at the ends of the #11 dowel bars. Determine the tension development length, ℓ_{dh}, of the #11 dowel bars terminating in a hook:

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases}$$ Eq. (10.38)

$\psi_e = 1.0$ for uncoated bars — Table 10.5

$\psi_r = 1.6$ (confining reinforcement not provided)

$\psi_o = 1.0$ (side cover normal to hook $> 6d_b$)

$\psi_c = (f_c' / 15,000) + 0.6 = (4,000 / 15,000) + 0.6 = 0.9$

$\lambda = 1.0$ for normalweight concrete

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} = \left(\dfrac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.9}{55 \times 1.0 \times \sqrt{4{,}000}}\right) \times 1.41^{1.5} = 41.6 \text{ in.} \\ 8d_b = 8 \times 1.41 = 11.3 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

$$\text{Minimum } h = \ell_{dh} + 2(d_b)_f + \text{cover} = 41.6 + (2 \times 0.75) + 3.0 = 46.1 \text{ in.} \qquad \text{Eq. (10.39)}$$

The minimum $h = 46.1$ in. is greater than the provided footing thickness of 24.0 in. Thus, increase the footing thickness to 48.0 in. to accommodate development of the #11 dowel bars.

Determine minimum flexure reinforcement in the footing based on the increased thickness:

$$A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 48.0 = 1.04 \text{ in.}^2/\text{ft} \qquad \text{ACI 7.6.1.1}$$

Use #10 bars spaced at 14 in. on center $(A_{s,provided} = 1.09 \text{ in.}^2/\text{ft})$. The 14-in. spacing is less than the maximum spacing of $3h = 144$ in. or 18 in. Note that the #11 dowel bars can be fully developed in the 48-in.-thick footing with the #10 bars for flexure.

- Development of the dowel bars into the column

Development length in tension, ℓ_d, of the #11 bars:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\right) d_b \geq 12.0 \text{ in.} \qquad \text{Eq. (10.22)}$$

For normalweight concrete, $\lambda = 1.0$. Eq. (10.23)

For Grade 60 reinforcement, $\psi_g = 1.0$. Eq. (10.24)

For uncoated reinforcing bars, $\psi_e = 1.0$. Eq. (10.25)

For #11 bars, $\psi_s = 1.0$. Eq. (10.26)

$\psi_t = 1.0$. Eq. (10.27)

$$c_b = \text{lesser of} \begin{cases} \text{cover} + (d_b)_{tie} + 0.5(d_b)_{long.} = 1.5 + 0.5 + (0.5 \times 1.41) = 2.7 \text{ in.} \\ \dfrac{s}{2} = \dfrac{24.0 - (2 \times 1.5) - (2 \times 0.5) - 1.41}{2} = 9.3 \text{ in.} \end{cases} \qquad \text{Eq. (10.28)}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.7 + 0) / 1.41 = 1.9 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0 \times \sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{1.9}\right) \times 1.41 = 52.8 \text{ in.} > 12.0 \text{ in.}$$

$$\text{Class B lap splice length} = 1.3\ell_d = 1.3 \times 52.8 = 68.6 \text{ in.}$$

Provide a lap splice length of 5 ft-9 in.

- Check horizontal force transfer

  From Example 7.26, $V_u = 26.2 \text{ kips}$ is transferred horizontally between the column and the footing. The required area of shear-friction reinforcement is determined as follows assuming the surface between the column and the footing has not been intentionally roughened:

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{V_u}{\phi f_y (0.6\lambda)} = \frac{26.2}{0.75 \times 60 \times (0.6 \times 1.0)} = 0.97 \text{ in.}^2$$ Table 10.6, Eq. (10.40)

  The 4-#11 dowel bars provides 6.24 in.^2 across the interface, which is greater than the required amount of 0.97 in.^2

  Check the upper shear limit:

$$V_u = 26.2 \text{ kips} < \begin{cases} \phi 0.2 f_c' A_c = 0.75 \times 0.2 \times 4 \times 24.0^2 = 345.6 \text{ kips} \\ \\ \phi 800 A_c = 0.75 \times 800 \times 24.0^2 \, / \, 1{,}000 = 345.6 \text{ kips} \end{cases}$$ Table 10.7

Reinforcement details for this footing are similar to those in Figure 10.21.

Comments. The thickness of the footing and the amount of flexural reinforcement in the footing can be reduced by using smaller longitudinal bars in the column and smaller dowel bars in the footing. It can be determined that 8-#8 longitudinal bars are adequate for the 24-in.-square column. Eight #8 dowel bars with a standard 90-degree hook can be fully developed in a 30-in.-thick footing. The corresponding minimum flexural reinforcement in the footing is #8 bars spaced at 14 in. on center.

10.5.6 Example 10.6 – Design of a Combined Rectangular Spread Footing Subjected to Axial Compression: Building #1 (Framing Option B), SDC A

Design a combined rectangular spread footing supporting columns A2 and B2 in Building #1, Framing Option B (see Figure 1.1). The columns, which are 18 in. by 18 in. and are reinforced with 4-#9 bars, are not part of the LFRS. Assume Grade 60 reinforcement and normalweight concrete with $f_c' = 4{,}000 \text{ psi}$ for the columns and the footing. Also assume a permissible soil pressure of 3,000 lb/ft^2.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the base dimensions of the footing

The service axial dead, roof live, and live loads are equal to the following:

- Column A2

  $P_D = 182.3 \text{ kips}$

  $P_{L_r} = 6.3 \text{ kips}$

  $P_L = 81.3 \text{ kips}$

- Column B2

  $P_D = 342.7 \text{ kips}$

  $P_{L_r} = 11.8 \text{ kips}$

  $P_L = 152.8 \text{ kips}$

Use the basic load combinations for allowable stress design in ASCE/SEI 2.4.1 to determine the maximum axial force transferred to the footing:

- Column A2
  1. $P_D = 182.3$ kips
  2. $P_D + P_L = 182.3 + 81.3 = 263.6$ kips
  3. $P_D + P_{L_r} = 182.3 + 6.3 = 188.6$ kips
  4. $P_D + 0.75P_L + 0.75P_{L_r} = 182.3 + (0.75 \times 81.3) + (0.75 \times 6.3) = 248.0$ kips
- Column B2
  1. $P_D = 342.7$ kips
  2. $P_D + P_L = 342.7 + 152.8 = 495.5$ kips
  3. $P_D + P_{L_r} = 342.7 + 11.8 = 354.5$ kips
  4. $P_D + 0.75P_L + 0.75P_{L_r} = 342.7 + (0.75 \times 152.8) + (0.75 \times 11.8) = 466.2$ kips

ASCE/SEI Eq. 2 governs for both columns.

The base dimension of the footing is determined so that the pressure at the base of the footing is uniform (see Figure 10.23):

$$x = \frac{P_1 x_1 + P_2 x_2}{P} = \frac{\left(263.6 \times \frac{9.0}{12}\right) + \left[495.5 \times \left(23.5 + \frac{9.0}{12}\right)\right]}{263.6 + 495.5} = 16.1 \text{ ft} \qquad \text{Eq. (10.6)}$$

$L = 2x = 2 \times 16.1 = 32.2$ ft

Use $L = 32$ ft-3 in.

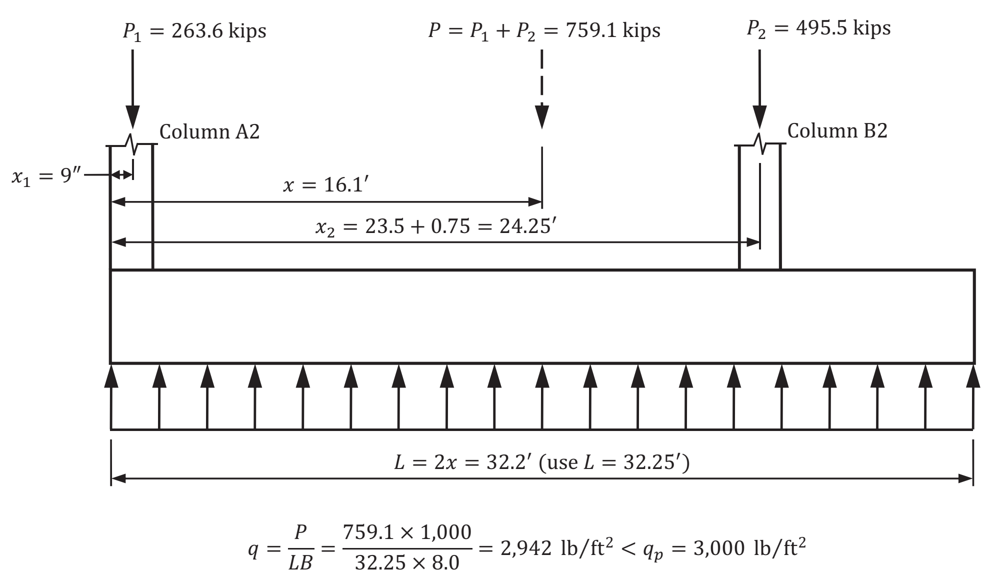


Figure 10.23 Combined footing in Example 10.6

$$B = \frac{P}{Lq_p} = \frac{263.6 + 495.5}{32.25 \times 3.0} = 7.9 \text{ ft} \qquad \text{Eq. (10.7)}$$

Use $B = 8$ ft-0 in.

Step 2 – Determine the thickness of the footing

The factored axial forces on each column based on the factored load combinations in ACI Table 5.3.1 are the following:

- Column A2

  ACI Eq. (5.3.1a): $1.4P_D = 255.2$ kips

  ACI Eq. (5.3.1b): $1.2P_D + 1.6P_L + 0.5P_{L_r} = 352.0$ kips

  ACI Eq. (5.3.1c): $1.2P_D + 1.6P_{L_r} + 0.5P_L = 269.5$ kips

- Column B2

  ACI Eq. (5.3.1a): $1.4P_D = 479.8$ kips

  ACI Eq. (5.3.1b): $1.2P_D + 1.6P_L + 0.5P_{L_r} = 661.6$ kips

  ACI Eq. (5.3.1c): $1.2P_D + 1.6P_{L_r} + 0.5P_L = 506.5$ kips

The largest axial forces and pressure at the base of the footing are based on ACI Eq. (5.3.1b).

The factored pressure, q_u, at the base of the footing is equal to the following:

$$q_u = \frac{352.0 + 661.6}{32.25 \times 8.0} = 3.93 \text{ kips/ft}^2$$

The factored uniformly distributed load, w_u, along the length of the footing is obtained by multiplying q_u by the width of the footing, B :

$$w_u = 3.93 \times 8.0 = 31.44 \text{ kips/ft}$$

The thickness of the footing is determined using the factored distributed load considering both flexure and shear. The shear and moment diagrams for the footing are given in Figure 10.24. Note that in order for the moment diagram to close, the footing length is taken as 32.2 ft instead of 32.25 ft.

- One-way shear

  The maximum factored shear force, V_u, occurs at the interior column and is equal to 411.9 kips. The factored shear force at the critical section located a distance d from the face of the column is equal to the following (see Figure 10.25):

  $$V_u = 411.9 - \left[31.44 \times \left(\frac{1.5}{2} + d\right)\right] = 388.3 - 31.44d$$

  Assuming a reinforcement ratio $\rho_w = 0.0060$ in the footing at this section, the design shear strength is equal to the following for members without shear reinforcement:

  $$\phi V_c = \phi 8\lambda(\rho_w)^{1/3}\sqrt{f_c'}Bd = 0.75 \times 8 \times 1.0 \times (0.0060)^{1/3} \times \sqrt{4{,}000} \times 8.0 \times 12 \times d \,/\, 1{,}000 = 6.62d \qquad \text{ACI Table 22.5.5.1}$$

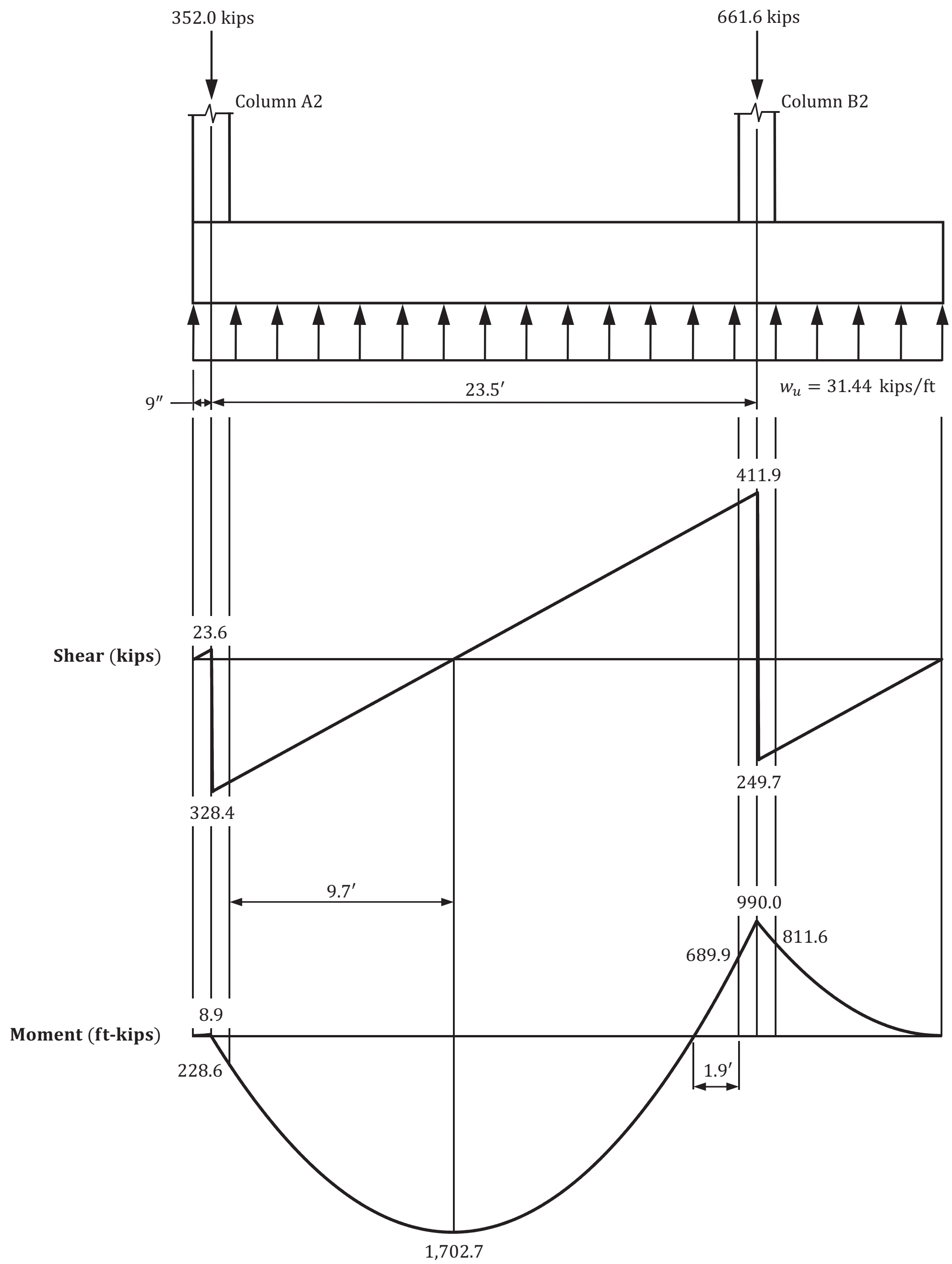


Figure 10.24 Shear and moment diagrams for the combined footing in Example 10.6.

Set the required shear strength equal to the design shear strength and solve for d:

$388.3 - (31.44\ /\ 12)d = 6.62d \rightarrow d = 42.0$ in.

Thus, an effective depth equal to 42.0 in. is adequate for one-way shear.

- Two-way shear
  Two-way shear requirements are checked at each column assuming $d = 42.0$ in.

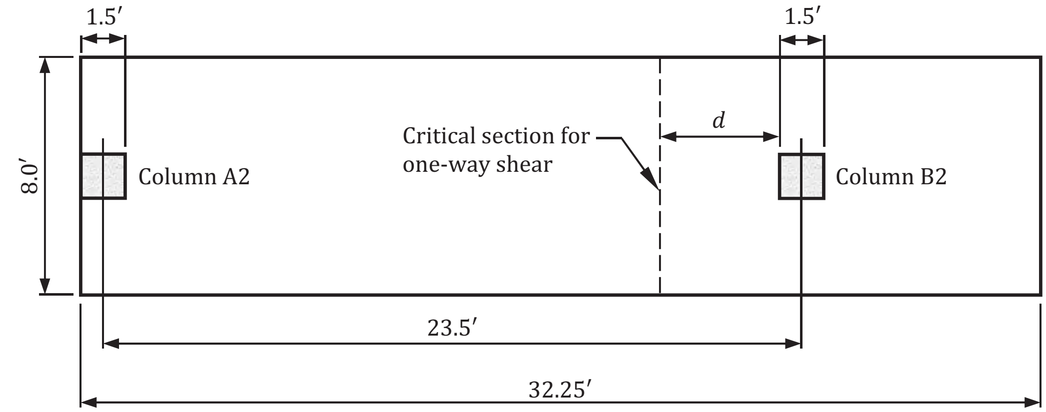


Figure 10.25 Critical section for one-way shear.

Edge column – Column A2

The edge column has a three-sided critical section located $d / 2 = 21.0$ in. from the face of the column in both directions (see Figure 10.26).

The maximum factored shear force at the face of the critical section is equal to the factored column load minus the factored soil pressure in the area bounded by the critical section:

$$V_u = 352.0 - \left[\frac{3.93}{144} \times (39.0 \times 60.0)\right] = 352.0 - 63.9 = 288.1 \text{ kips}$$

Determine the section properties of the critical section using Case 3 in Table 5.11 of this publication for an edge column bending perpendicular to the edge:

$$c_{AB} = \frac{b_1^2}{2b_1 + b_2} = \frac{39.0^2}{(2 \times 39.0) + 60} = 11.0 \text{ in.}$$

$$c_{CD} = b_1 - c_{AB} = 39.0 - 11.0 = 28.0 \text{ in.}$$

$$A_c = (2b_1 + b_2)d = [(2 \times 39.0) + 60.0] \times 42.0 = 5{,}796.0 \text{ in.}^2$$

$$\frac{J_c}{c_{AB}} = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} = 130{,}507 \text{ in.}^3$$

The load from the column and the load from the factored soil pressure in the area bounded by the critical section do not act through the same point; thus, a transfer moment, M_{sc}, occurs at the critical section. Determine M_{sc} at the centroid of the critical section (see Figure 10.26):

$$M_{sc} = \left(352.0 \times \frac{28.0 - 9.0}{12}\right) - \left(63.9 \times \frac{28.0 - 19.5}{12}\right) = 557.3 - 45.3 = 512.0 \text{ ft-kips}$$

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.35$$

The total factored shear stress at the face of critical section AB is determined by Eq. (5.14) of this publication:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{288{,}100}{5{,}796.0} + \frac{0.35 \times 512.0 \times 12{,}000}{130{,}507} = 49.7 + 16.5 = 66.2 \text{ psi}$$

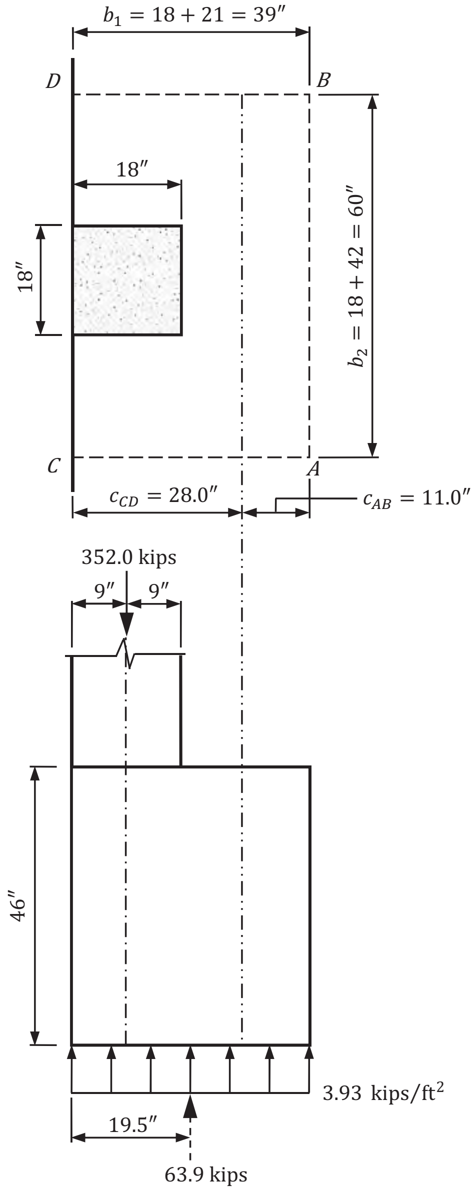


Figure 10.26 Critical section for two-way shear at the edge column.

The two-way shear design strength for members without shear reinforcement, ϕv_c, is equal to the least of the values determined by the three equations in ACI Table 22.6.5.2. For square columns, the first of these equations governs:

$$\phi v_c = \phi 4\sqrt{f_c'} = 0.75 \times 4 \times \sqrt{4{,}000} = 189.7 \text{ psi} > v_{u|AB} = 66.2 \text{ psi}$$

Interior column – Column B2

The interior column has a four-sided critical section located $d / 2 = 21.0$ in. from the face of the column in both directions.

The maximum factored shear force at the face of the critical section is equal to the factored column load minus the factored soil pressure in the area bounded by the critical section:

$$V_u = 661.6 - \left[\frac{3.93}{144} \times (60.0 \times 60.0)\right] = 661.6 - 98.3 = 563.3 \text{ kips}$$

The load from the column and the load from the factored soil pressure in the area bounded by the critical section act through the centroid of the critical section; thus, no transfer moment occurs at this location.

Determine the section properties of the critical section using Case 1 in Table 5.11 of this publication for an interior column:

$$c_{AB} = c_{CD} = 60.0 / 2 = 30.0 \text{ in.}$$

$$A_c = 2(b_1 + b_2)d = 2 \times (60.0 + 60.0) \times 42.0 = 10{,}080.0 \text{ in.}^2$$

The total factored shear stress at the critical section is equal to the following:

$$v_{u|AB} = \frac{V_u}{A_c} = \frac{563{,}300}{10{,}080} = 55.9 \text{ psi} < \phi v_c = 189.7 \text{ psi}$$

Therefore, an effective depth equal to 42.0 in. is adequate for two-way shear.

Use a 46.0-in.-thick footing.

Step 3 – Determine the required flexural reinforcement

- Moment in the longitudinal direction near midspan

  From Figure 10.24, the moment near midspan is equal to 1,702.7 ft-kips. The required flexural reinforcement to resist this moment must be placed at the top of the footing section; it is analogous to positive reinforcement.

$$R_n = \frac{M_u}{\phi B d^2} = \frac{1{,}702.7 \times 12{,}000}{0.9 \times (8.0 \times 12) \times 42.0^2} = 134.1 \text{ psi}$$

$$\rho = \frac{0.85 f_c'}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4.0}{60.0} \times \left[1 - \sqrt{1 - \frac{2 \times 134.1}{0.85 \times 4{,}000}}\right] = 0.0023$$

  At this section, the required shear strength is zero, so a minimum ρ_w is not needed to satisfy one-way shear strength requirements. Thus,

$$A_s = 0.0023 \times 12.0 \times 42.0 = 1.16 \text{ in.}^2/\text{ft} > A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 46.0 = 0.99 \text{ in.}^2/\text{ft}$$

  Use #9 bars spaced at 10 in. on center $(A_s = 1.20 \text{ in.}^2/\text{ft})$.

- Moment in the longitudinal direction at the interior column

  From Figure 10.24, the maximum moment occurs at the right face of the interior column and is equal to 811.6 ft-kips. The required flexural reinforcement to resist this moment must be placed at the bottom of the footing section; it is analogous to negative reinforcement.

$$R_n = \frac{M_u}{\phi B d^2} = \frac{811.6 \times 12{,}000}{0.9 \times (8.0 \times 12) \times 42.0^2} = 63.9 \text{ psi}$$

$$\rho = \frac{0.85 f_c'}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4.0}{60.0} \times \left[1 - \sqrt{1 - \frac{2 \times 63.9}{0.85 \times 4{,}000}}\right] = 0.0011 < \rho_{min} = 0.0020$$

At the left face of the interior column, the moment is equal to 689.9 ft-kips, which would require minimum flexural reinforcement. However, the factored shear force at this location is maximum and the design one-way shear strength was determined in Step 2 using $\rho_w = 0.0060$. Therefore, provide the following flexural reinforcement at the interior column:

$$A_s = 0.0060 \times 12.0 \times 42.0 = 3.02 \text{ in.}^2$$

Use #11 bars spaced at 6 in. on center $(A_s = 3.12 \text{ in.}^2/\text{ft})$.

- Moment in the longitudinal direction at the edge column

From Figure 10.24, the moment at the interior face of the edge column is equal to 228.6 ft-kips, which requires minimum reinforcement:

$$A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 46.0 = 0.99 \text{ in.}^2/\text{ft}$$

Note that this moment is positive, which causes tension on the top of the footing. Therefore, reinforcement must be provided at the top of the footing at this location.

Check that the minimum flexural reinforcement is adequate to satisfy the moment transfer requirements in ACI 8.4.2.2.

The transfer moment M_{sc} was determined in Step 2 and is equal to 512.0 ft-kips. A fraction of this moment $\gamma_f M_{sc} = 0.65 \times 512.0 = 332.8 \text{ ft-kips}$ is transferred over the following effective width:

$$b_{eff} = c_2 + 3h = 18.0 + (3 \times 46.0) = 156.0 \text{ in.} = 13.0 \text{ ft} > B = 8.0 \text{ ft}$$

Therefore, the entire width of the footing is available to resist this transfer moment.

Determine the required flexural reinforcement:

$$R_n = \frac{M_u}{\phi B d^2} = \frac{332.8 \times 12{,}000}{0.9 \times (8.0 \times 12) \times 42.0^2} = 26.2 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] = \frac{0.85 \times 4.0}{60.0} \times \left[1 - \sqrt{1 - \frac{2 \times 26.2}{0.85 \times 4{,}000}}\right] = 0.0004 < \rho_{min} = 0.0020$$

Check the minimum reinforcement required by ACI 8.6.1.2:

$$v_{uv} = \frac{V_u}{A_c} = \frac{288{,}100}{5{,}796.0} = 49.7 \text{ psi} < \phi 2\lambda\sqrt{f'_c} = 0.75 \times 2 \times 1.0 \times \sqrt{4{,}000} = 94.9 \text{ psi}$$

Therefore, the minimum reinforcement requirements of ACI 8.6.1.2 need not be satisfied.

Extending the #9 top bars spaced at 10 in. (which were determined for the moment at midspan) the full length of the footing satisfies the flexural requirements at the face of the edge column.

Provide minimum flexural reinforcement at the bottom of the footing at this location. Thus, use #9 bars spaced at 12 in. on center $(A_s = 1.00 \text{ in.}^2/\text{ft})$.

- Moment in the transverse direction at the interior column

The bending moment in the transverse direction is determined assuming the factored load from the column is distributed over a width on each side of the column. The actual width is not important at this stage because the moments are independent of it.

The factored distributed load and factored bending moment at the interior column in the transverse direction are equal to the following (see Figure 10.27):

$$w_u = \frac{661.6}{8.0} = 82.7 \text{ kips/ft}$$

$$M_u = \frac{82.7 \times 3.25^2}{2} = 436.8 \text{ ft-kips}$$

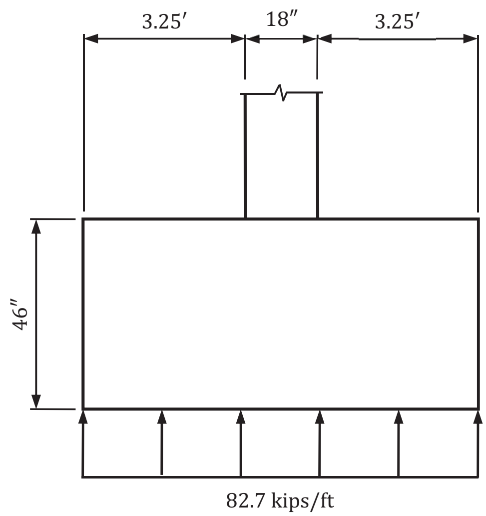


Figure 10.27 Factored distributed load in the transverse direction at the interior column.

Assuming the effective width of the transverse member is equal to the width of the column plus the effective depth, d (that is, the effective width extends $d/2$ on each side of the column), the required flexural reinforcement to resist this moment is equal to the following:

$$R_n = \frac{M_u}{\phi b d^2} = \frac{436.8 \times 12{,}000}{0.9 \times (18.0 + 42.0) \times 42.0^2} = 55.0 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] = \frac{0.85 \times 4.0}{60.0} \times \left[1 - \sqrt{1 - \frac{2 \times 55.0}{0.85 \times 4{,}000}}\right] = 0.0009 < \rho_{min} = 0.0020$$

$$A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 46.0 = 0.99 \text{ in.}^2/\text{ft}$$

Use #9 bars spaced at 12 in. on center $(A_s = 1.00 \text{ in.}^2/\text{ft})$.

- Moment in the transverse direction at the edge column

  The factored distributed load and factored bending moment at the interior column in the transverse direction are equal to the following:

$$w_u = \frac{352.0}{8.0} = 44.0 \text{ kips/ft}$$

$$M_u = \frac{44.0 \times 3.25^2}{2} = 232.4 \text{ ft-kips}$$

Assuming the effective width of the transverse member is equal to the width of the column plus $d/2$, the required flexural reinforcement to resist this moment is equal to the following:

$$R_n = \frac{M_u}{\phi b d^2} = \frac{232.4 \times 12{,}000}{0.9 \times (18.0 + 21.0) \times 42.0^2} = 45.0 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] = \frac{0.85 \times 4.0}{60.0} \times \left[1 - \sqrt{1 - \frac{2 \times 45.0}{0.85 \times 4{,}000}}\right] = 0.0008 < \rho_{min} = 0.0020$$

$$A_{s,min} = 0.0018bh = 0.0018 \times 12.0 \times 46.0 = 0.99 \text{ in.}^2/\text{ft}$$

Use #9 bars spaced at 12 in. on center $(A_s = 1.00 \text{ in.}^2/\text{ft})$.

At all critical sections for flexure in both directions, the provided spacing is less than the maximum spacing of $3h = 138.0$ in. and 18.0 in.

Step 4 – Check if the flexural reinforcement can be developed for tension

- Reinforcement in the longitudinal direction near midspan

  The #9 bars at 10 in. on center must be developed on both sides of the critical section, which is located 9.7 ft from the interior face of the edge column (see Figure 10.24). It is evident from the moment diagram that the points of inflection occur within and very close to the edge and interior columns, respectively. Therefore, these top bars are extended over the entire footing length.

- Reinforcement in the longitudinal direction at the columns

  The bottom bars at the edge column are extended to the left face of the interior column. The bottom bars at the interior column must be developed passed the point of inflection (which is located 1.9 ft from the left face of the interior column; see Figure 10.24) in accordance with ACI 9.7.3.8.4.

- Reinforcement in the transverse direction

  It can be determined that the #9 bottom bars can be fully developed for tension using straight bars without hooks.

Step 5 – Determine the required dowel reinforcement

Only vertical forces need to be considered for force transfer between the columns and the footing. Calculations are given below for the interior column. Similar calculations can be performed for the edge column.

- Check the bearing force on the column:

  $\phi B_n = \phi 0.85 f'_c A_1 = 0.65 \times 0.85 \times 4.0 \times 18.0^2 = 716.0 \text{ kips} > B_u = 661.6 \text{ kips}$ Eq. (10.30)

- Check the bearing force on the footing:

  $B_u \le \phi B_n = (\phi 0.85 f'_c A_1)\sqrt{A_2 / A_1} \le 2(\phi 0.85 f'_c A_1)$ Eq. (10.31)

  Using Figure 10.16, A_2 is determined as follows:

  Thickness of footing $= 42.0$ in.

  Horizontal projection for a 1:2 slope $= 2 \times 42.0 = 84.0$ in.

  Projected length $b = 84.0 + 18.0 + 84.0 = 186.0$ in.

  Therefore, $A_2 = 186.0 \times 96.0 = 17{,}856 \text{ in.}^2$

$$\sqrt{A_2 / A_1} = \sqrt{17{,}856 / 18.0^2} = 7.4 > 2.0;\ \text{use } 2.0$$

$$\phi B_n = 2(\phi 0.85 f_c' A_1) = 2 \times 0.65 \times 0.85 \times 4.0 \times 18.0^2 = 1{,}432.1 \text{ kips} > B_u = 661.6 \text{ kips}$$

- Determine the required interface reinforcement

  Because the design bearing strength is adequate, provide the minimum area of reinforcement across the interface:

  $$A_{s,min} = 0.005A_g = 0.005 \times 18.0^2 = 1.62 \text{ in.}^2$$ Sect. 10.3.7

  Try 4-#6 dowel bars ($A_{s,provided} = 1.76 \text{ in.}^2$).

- Development of the dowel bars into the footing

  The dowel bars must extend into the footing at least a compression development length, ℓ_{dc} :

  $$\ell_{dc} = \text{ greater of } \begin{cases} (f_y \psi_r / 50\lambda\sqrt{f_c'})d_b = [(60{,}000 \times 1.0) / (50 \times 1.0 \times \sqrt{4{,}000})] \times 0.75 = 14.2 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 0.75 = 13.5 \text{ in.} \\ 8 \text{ in.} \end{cases}$$ Eq. (10.34)

  where $\psi_r = 1.0$ for reinforcing bars without confinement. Table 10.4

  The minimum footing thickness for the development of the dowel bars in compression is the following:

  $$h \geq \ell_{dc} + r + (d_b)_d + 2(d_b)_f + \text{cover} = 14.2 + (3 \times 0.75) + (2 \times 1.41) + 3.0 = 22.3 \text{ in.}$$ Eq. (10.35)

  The provided footing thickness is greater than the required footing thickness, so the dowel bars can be fully developed for compression into the footing.

- Development of the dowel bars into the column

  The dowel bars are lap spliced to the longitudinal bars in the column. Because the dowel bars are smaller in diameter than the longitudinal bars, the compression lap splice length, ℓ_{sc}, must be greater than or equal to the larger of the following:

  1. Development length in compression, ℓ_{dc}, of the #9 longitudinal bars:

     $$\ell_{dc} = \text{ greater of } \begin{cases} (f_y \psi_r / 50\lambda\sqrt{f_c'})d_b = [(60{,}000 \times 1.0) / (50 \times 1.0 \times \sqrt{4{,}000})] \times 1.128 = 21.4 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 1.128 = 20.3 \text{ in.} \\ 8 \text{ in.} \end{cases}$$ Eq. (10.34)

  2. Compression lap splice length, ℓ_{sc}, of the #6 dowel bars:

     $$\ell_{sc} = \text{greater of } \begin{cases} 0.0005 f_y d_b = 0.0005 \times 60{,}000 \times 0.75 = 22.5 \text{ in.} \\ 12.0 \text{ in.} \end{cases}$$ Eq. (10.37)

  Provide a lap splice length equal to 2 ft-0 in.

Reinforcement details for this footing are given in Figure 10.28.

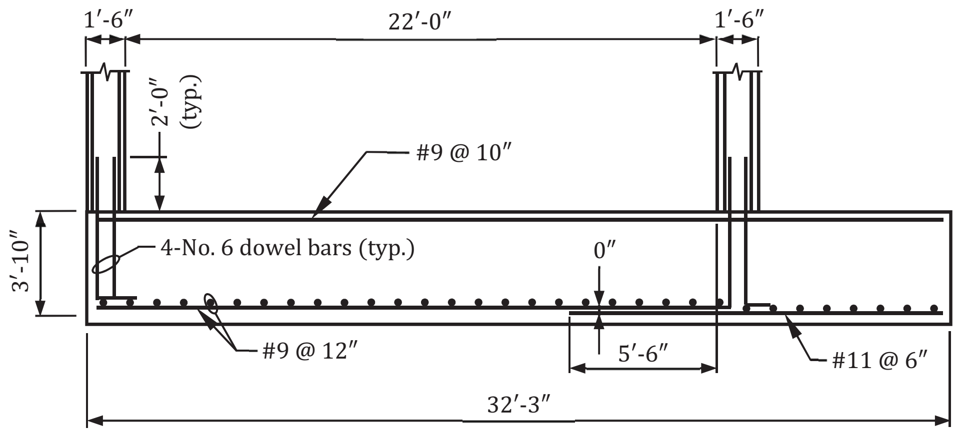


Figure 10.28 Reinforcement details for the combined rectangular footing in Example 10.6.

10.5.7 Example 10.7 – Design of a Drilled Pier Subjected to Axial Compression: Building #1 (Framing Option B), SDC A

Design a drilled pier supporting a typical interior column in Building #1, Framing Option B (see Figure 1.1). The column, which is 18 in. by 18 in. and is reinforced with 4-#9 bars, is not part of the LFRS. Assume Grade 60 reinforcement and normalweight concrete with $f_c' = 4{,}000$ psi for the columns and the drilled pier. Also assume the following for the drilled pier: (1) lateral support is provided over the entire height, (2) bending moments due to the effects of gravity loads are negligible, (3) no casing is provided, and (4) the allowable bearing capacity of the rock is 10,000 lb/ft^2.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the pier size

Because the two conditions in ACI 13.4.2.1 are satisfied, it is permitted to design the drilled pier using allowable stress design.

The service axial dead, roof live, and live loads on a typical interior column are equal to the following:

$P_D = 342.7 \text{ kips}$

$P_{L_r} = 11.8 \text{ kips}$

$P_L = 152.8 \text{ kips}$

Use the basic load combinations for allowable stress design in ASCE/SEI 2.4.1 to determine the maximum axial force transferred to the drilled pier:

1. $P_D = 342.7 \text{ kips}$
2. $P_D + P_L = 342.7 + 152.8 = 495.5 \text{ kips}$
3. $P_D + P_{L_r} = 342.7 + 11.8 = 354.5 \text{ kips}$
4. $P_D + 0.75P_L + 0.75P_{L_r} = 342.7 + (0.75 \times 152.8) + (0.75 \times 11.8) = 466.2 \text{ kips}$

ASCE/SEI Eq. 2 governs.

The diameter of the pier can be determined by the following equation, which does not consider the contribution of the longitudinal reinforcement:

$$d_{pier} = \left[\frac{4P}{\pi(0.3f_c')}\right]^{1/2} = \left[\frac{4 \times 495.5}{\pi \times 0.3 \times 4.0}\right]^{1/2} = 22.9 \text{ in.}$$ Table 10.10

A pier with a diameter of 2 ft-0 in. is adequate; however, a 3 ft-0 in. diameter is used based on the size of the 1.5-ft square column.

Step 2 – Determine the bell size

The diameter of the bell can be determined by the following equation:

$$d_{bell} = \left(\frac{4P}{\pi q_a}\right)^{1/2} = \left(\frac{4 \times 495.5}{\pi \times 10.0}\right)^{1/2} = 7.9 \text{ ft}$$ Eq. (10.19)

Use a bell with a diameter of 8 ft-0 in.

Step 3 – Determine the longitudinal and transverse reinforcement

Because the drilled pier is subjected to primarily axial compression loads, provide a minimum longitudinal reinforcement ratio of 0.005:

$$A_s = 0.005 \times (\pi \times 36.0^2 / 4) = 5.09 \text{ in.}^2$$

Use 7-#8 longitudinal bars ($A_{s,provided} = 5.53 \text{ in.}^2$). Figure 10.19

Length of longitudinal bars $= 3d_{pier} = 3 \times 3.0 = 9.0 \text{ ft} < 10.0 \text{ ft}, \text{ use } 10.0 \text{ ft}$

Also, use #3 ties spaced at 16 in. on center.

Step 4 – Determine the required dowel reinforcement

Only vertical forces need to be considered for force transfer between the column and the pier.

The maximum factored axial force transferred from the column to the pier is equal to the following:

ACI Eq. (5.3.1b): $1.2P_D + 1.6P_L + 0.5P_{L_r} = 661.6 \text{ kips}$

- Check the bearing force on the column:

  $$\phi B_n = \phi 0.85 f_c' A_1 = 0.65 \times 0.85 \times 4.0 \times 18.0^2 = 716.0 \text{ kips} > B_u = 661.6 \text{ kips}$$ Eq. (10.30)

- Check the bearing force on the pier:

  $$B_u \le \phi B_n = (\phi 0.85 f_c' A_1)\sqrt{A_2 / A_1} \le 2(\phi 0.85 f_c' A_1)$$ Eq. (10.31)

  The largest square that can be inscribed in the 3-ft-diameter pier has sides equal to $3.0 / \sqrt{2} = 2.1$ ft.

  Therefore, $A_2 = (2.1 \times 12)^2 = 635.0 \text{ in.}^2$

  $$\sqrt{A_2 / A_1} = \sqrt{635.0 / 18.0^2} = 1.4 < 2.0$$

  $$\phi B_n = 0.65 \times 0.85 \times 4.0 \times 18.0^2 \times 1.4 = 1{,}002.5 \text{ kips} > B_u = 661.6 \text{ kips}$$

- Determine the required interface reinforcement

  Because the design bearing strength is adequate, provide the minimum area of reinforcement across the interface:

  $A_{s,min} = 0.005A_g = 0.005 \times 18.0^2 = 1.62 \text{ in.}^2$ Sect. 10.3.7

  Use 4-#6 dowel bars ($A_{s,provided} = 1.76 \text{ in.}^2$).

- Development of the dowel bars into the pier

  The dowel bars must extend into the footing at least a compression development length, ℓ_{dc} :

$$\ell_{dc} = \text{ greater of } \begin{cases} (f_y\psi_r \,/\, 50\lambda\sqrt{f_c'})d_b = [(60{,}000 \times 1.0) \,/\, (50 \times 1.0 \times \sqrt{4{,}000})] \times 0.75 = 14.2 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 0.75 = 13.5 \text{ in.} \\ 8 \text{ in.} \end{cases}$$ Eq. (10.34)

  where $\psi_r = 1.0$ for reinforcing bars without confinement. Table 10.4

  Use #6 dowel bars 1 ft-4 in. in length.

- Development of the dowel bars into the column

  The dowel bars are lap spliced to the longitudinal bars in the column. Because the dowel bars are smaller in diameter than the longitudinal bars, the compression lap splice length, ℓ_{sc}, must be greater than or equal to the larger of the following:

  (1) Development length in compression, ℓ_{dc}, of the #9 longitudinal bars:

$$\ell_{dc} = \text{ greater of } \begin{cases} (f_y\psi_r \,/\, 50\lambda\sqrt{f_c'})d_b = [(60{,}000 \times 1.0) \,/\, (50 \times 1.0 \times \sqrt{4{,}000})] \times 1.128 = 21.4 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 1.128 = 20.3 \text{ in.} \\ 8 \text{ in.} \end{cases}$$ Eq. (10.34)

  (2) Compression lap splice length, ℓ_{sc}, of the #6 dowel bars:

$$\ell_{sc} = \text{greater of } \begin{cases} 0.0005 f_y d_b = 0.0005 \times 60{,}000 \times 0.75 = 22.5 \text{ in.} \\ 12.0 \text{ in.} \end{cases}$$ Eq. (10.37)

  Provide a lap splice length equal to 2 ft-0 in.

Reinforcement details for this pier are given in Figure 10.29.

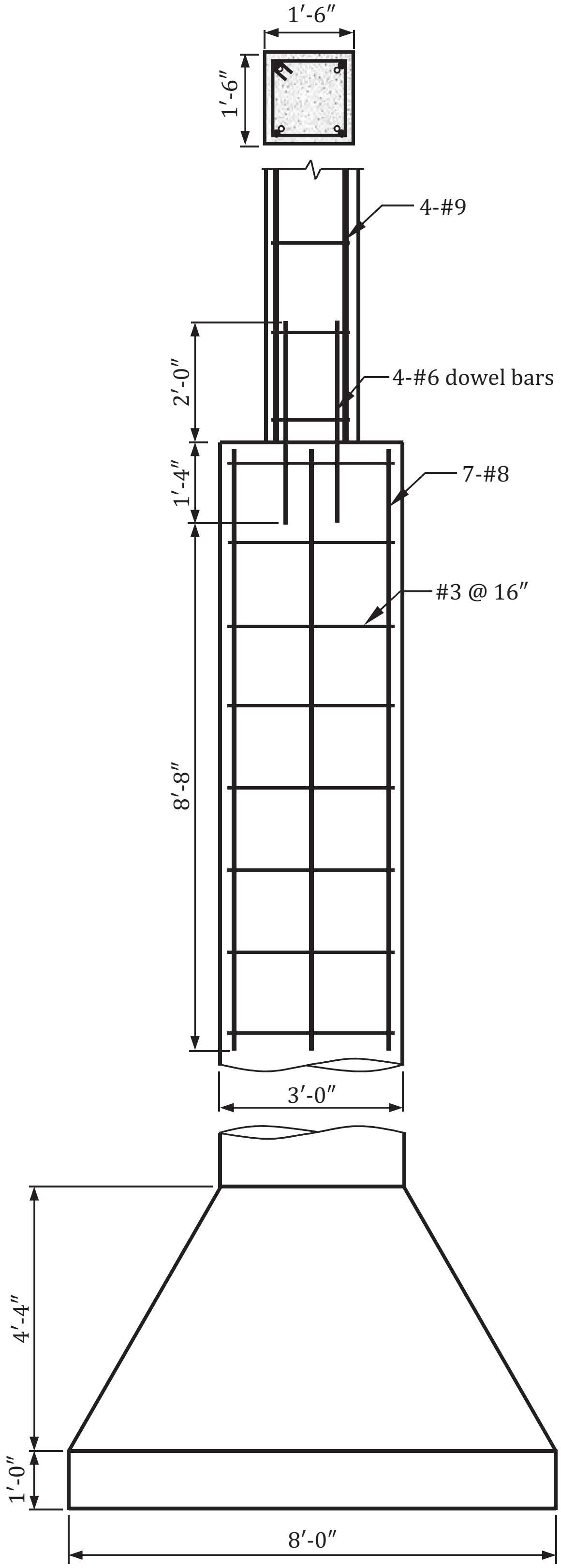


Figure 10.29 Reinforcement details for the drilled pier in Example 10.7.

Chapter 11
BEAM-COLUMN AND SLAB-COLUMN JOINTS

11.1 Overview

Joints in a reinforced concrete structure transfer axial forces from the column above and must resist the horizontal shear forces from the connected beams and/or slabs framing into the joint. According to ACI R15.1, a joint is the portion of a structure that is common to intersecting members whereas a connection consists of a joint and portions of adjoining members. In order for a joint to adequately transfer the applicable forces, it must be confined by structural members (beams and/or slabs), by transverse reinforcement, or by both.

Provisions for beam-column and slab-column joints are given in ACI Chapter 15, which are applicable to buildings assigned to Seismic Design Category (SDC) A and B.

11.2 Design Criteria

The general requirements in ACI 15.2 for the design and detailing of beam-column and slab-column joints in cast-in-place reinforced concrete buildings are summarized in Table 11.1.

Table 11.1 General Requirements for Beam-Column and Slab-Column Joints in Accordance with ACI 15.2

Requirement	ACI Section No.
Beam-column and slab-column joints must satisfy the detailing provisions of ACI 15.3 and the strength requirements of ACI 15.4.	15.2.1
Beam-column and slab-column joints must satisfy the requirements in ACI 15.5 for transfer of axial force through the floor system.	15.2.2
Beam-column joints must be designed for the shear forces resulting from moment transfer at the joint due to gravity, wind, earthquake, and any other lateral force.	15.2.3
At corner joints between two members (such as a roof-level exterior joint in a building), the effects of moment reversals within the joint must be considered in the design of the joint.	15.2.4
The strut-and-tie method in ACI Chapter 23 must be used to analyze and design a joint where the depth of the beam framing into the joint that generates the joint shear is more than twice the depth of the column in the direction of analysis. In addition to the requirements of ACI Chapter 23, the joint shear strength requirements of ACI 15.4.2 and the joint detailing requirements of ACI 15.3 must be satisfied.	15.2.5
Column and beam extensions are assumed to provide continuity through a beam-column joint provided the requirements in ACI 15.2.6 and 15.2.7 are satisfied, respectively (see Figures 11.1 and 11.2). Extensions do not contribute to joint shear force if they do not support externally applied loads.	15.2.6, 15.2.7
For a given direction of analysis, a joint is considered to be laterally confined where two transverse beams (that is, two beams framing into the column in the direction perpendicular to the direction of analysis) satisfy the three requirements in ACI 15.2.8 (see Figure 11.3).	15.2.8
Strength and detailing requirements in accordance with the applicable provisions in ACI Chapter 8, ACI 15.3.2, and ACI 22.6 must be satisfied for slab-column connections that transfer moment.	15.2.9

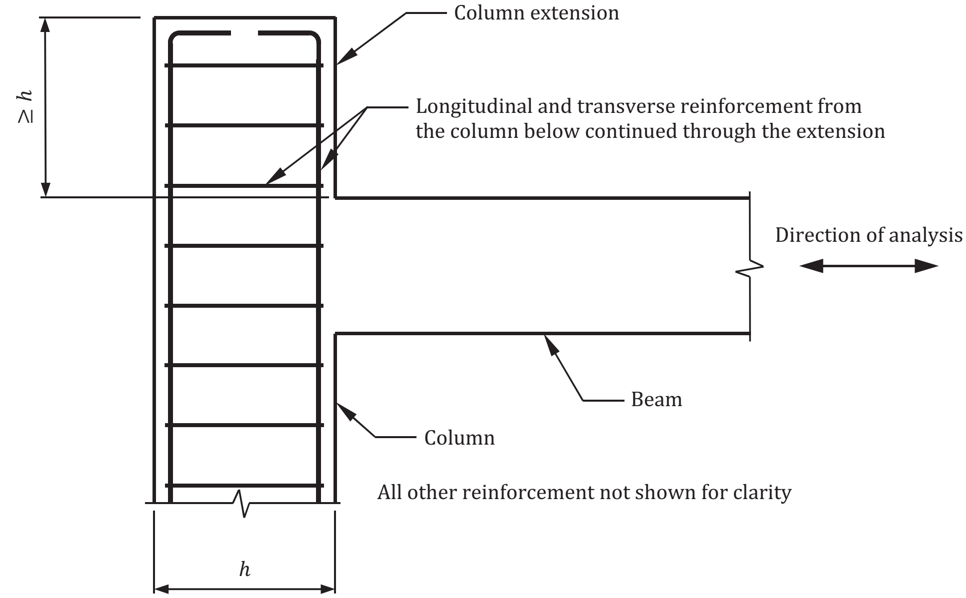


Figure 11.1 A column extension that provides continuity through a beam-column joint in accordance with ACI 15.2.6.

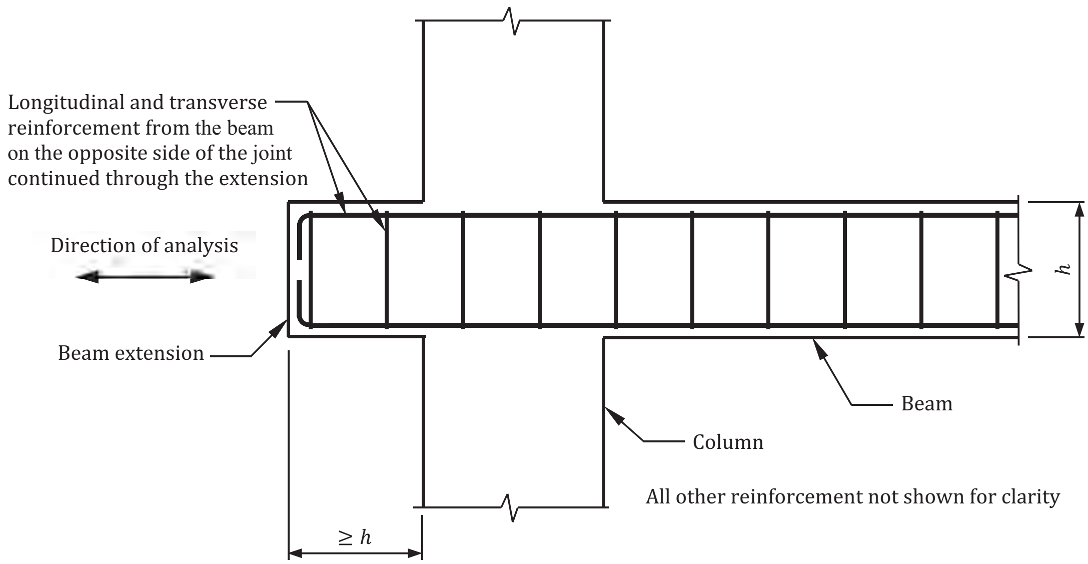


Figure 11.2 A beam extension that provides continuity through a beam-column joint in accordance with ACI 15.2.7.

11.3 Detailing of Joints

11.3.1 Beam-Column Joint Transverse Reinforcement

Beam-column joints must be detailed in accordance with ACI 15.3.1.2 through 15.3.1.4 unless all the following are satisfied (ACI 15.3.1.1):

(a) The joint is confined by transverse beams in accordance with ACI 15.2.8 for all shear directions considered (see Figure 11.3).

(b) The joint is not part of a designated seismic-force-resisting system (SFRS).

(c) The joint is not part of a structure assigned to SDC D, E, or F.

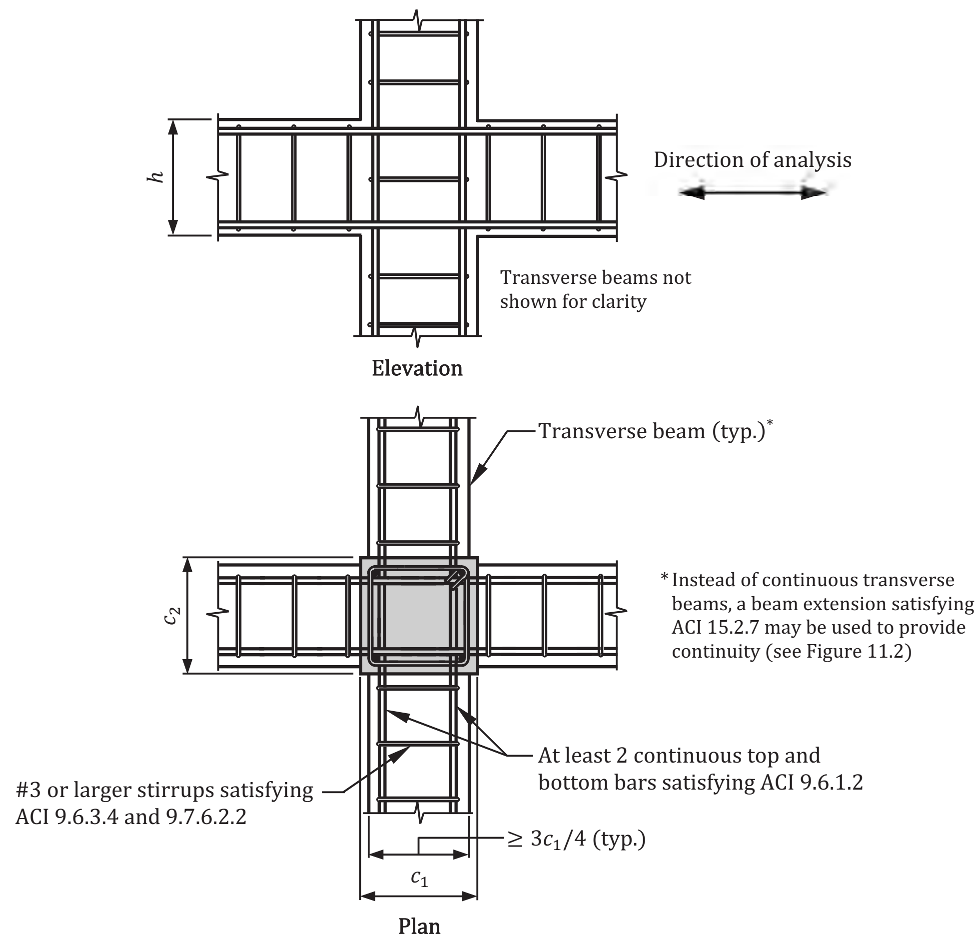


Figure 11.3 Requirements for joint confinement by transverse beams in accordance with ACI 15.2.8.

The detailing requirements of ACI 15.3.1.2 through 15.3.1.4 are applicable to beam-column joints that are not confined (such as at a joint for an edge or a corner column). These detailing requirements are illustrated in Figure 11.4.

11.3.2 Slab-Column Joint Transverse Reinforcement

According to ACI 15.3.2.1, column transverse reinforcement must be continued through any slab-column joint that is not laterally supported on four sides by the slab (that is, it must be continued through the joints at edge and corner columns). Such reinforcement must also be provided through column capitals, drop panels, and shear caps that are present.

11.3.3 Longitudinal Reinforcement

Longitudinal reinforcement in a column or beam that is terminated in the joint or within a column or beam extension must satisfy the applicable development requirements in ACI 25.4 (ACI 15.3.3.1). Where standard hooks are used to develop the longitudinal reinforcement, the hook must be turned toward the mid-depth of the beam or column (ACI 15.3.3.2).

An example of an edge or corner column where the longitudinal reinforcement from the beam is terminated in the joint with a standard 90-degree hook is illustrated in Figure 11.5. In accordance with ACI 15.3.3.2, the hooks are turned toward the mid-depth of the beam. The critical section for development of the hooked bars is taken at the face of the column.

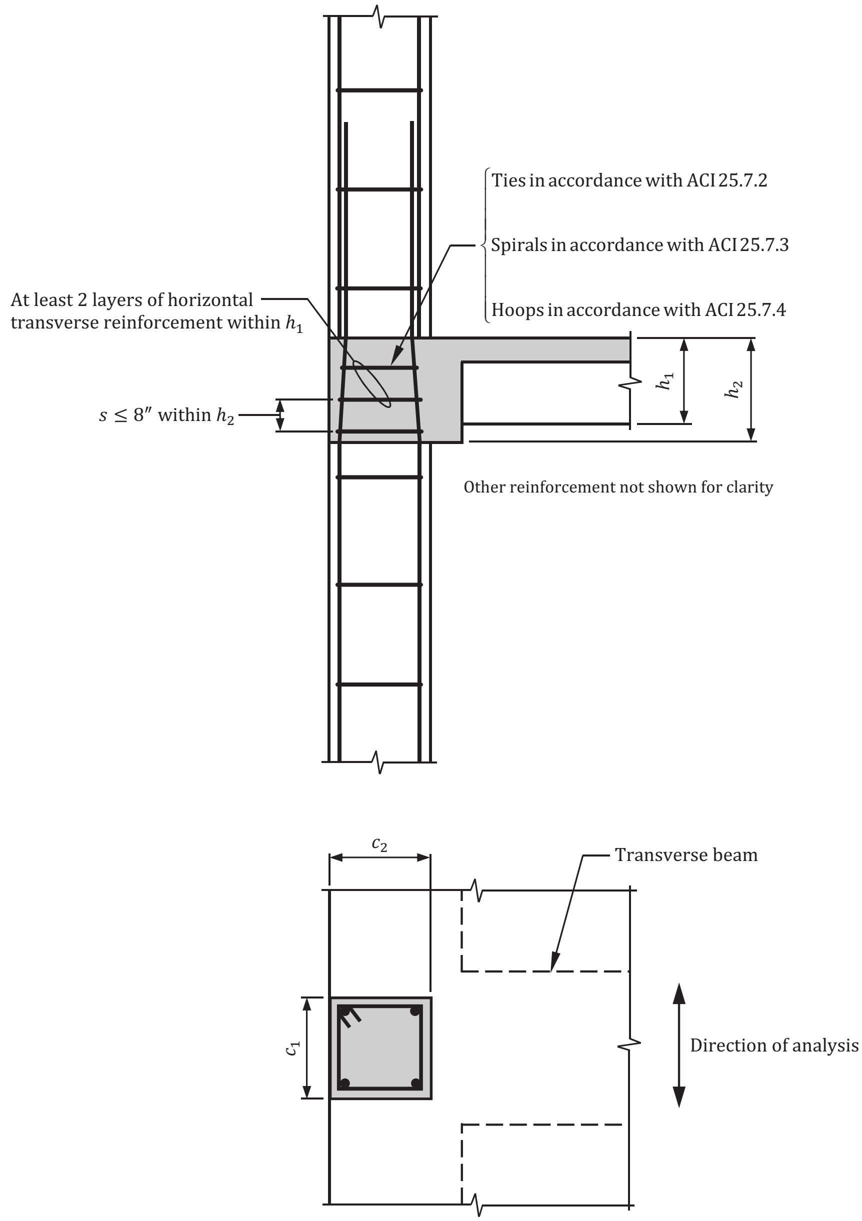


Figure 11.4 Detailing requirements for beam-column joints in accordance with ACI 15.3.1.2 through 15.3.1.4.

The development length of a deformed reinforcing bar in tension with a standard hook, ℓ_{dh}, is given in ACI 25.4.3.1:

$$\ell_{dh} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f'_c}} \right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{11.1}$$

This development length is measured from the critical section (which is taken at the face of the column) to the outside face of the hook. The modification factors in Equation (11.1) are given in Table 11.2 (see ACI Table 25.4.3.2).

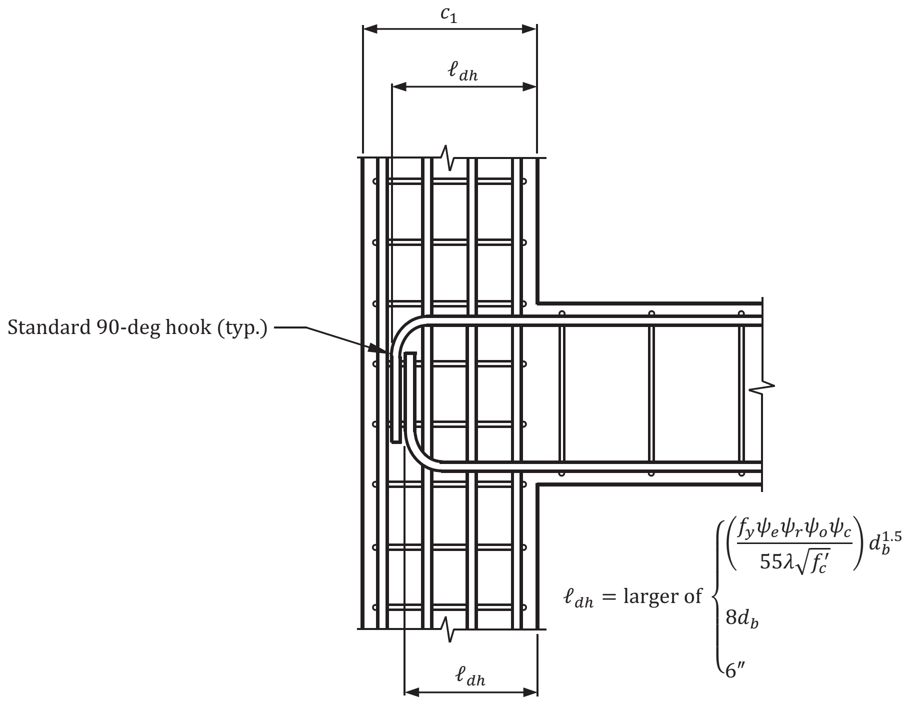


Figure 11.5 Longitudinal beam reinforcement terminated in a beam-column joint with a standard 90-degree hook.

Table 11.2 Modification Factors for Development of Hooked Bars in Tension

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Confining reinforcement, ψ_r	For #11 and smaller bars with $A_{th} \geq 0.4A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For #11 and smaller hooked bars 1. terminating inside a column core with side cover normal to the plane of the hook ≥ 2.5 in. or 2. with side cover normal to the plane of the hook $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c\,/\,15{,}000) + 0.6$
	$f'_c \geq 6{,}000$ psi	1.0

The term A_{th} is the total cross-sectional area of ties or stirrups confining the hooked bars and A_{hs} is the total cross-sectional area of the hooked bars being developed at the same critical section. The term s is the center-to-center spacing of the hooked bars, which have a nominal diameter d_b. Detailing requirements for A_{th} are given in ACI 25.4.3.3, which are illustrated in ACI Figures R25.4.3.3a and R25.4.3.3b for confining reinforcement placed parallel and perpendicular to hooked bars being developed, respectively.

An example of an edge or corner column where the longitudinal reinforcement from the beam is terminated in the joint with a standard head is illustrated in Figure 11.6. Headed deformed bars are permitted to be used only when the conditions of ACI 25.4.4.1 are satisfied:

(a) Bar must conform to ACI 20.2.1.6
(b) Bar size must be #11 or smaller
(c) Net bearing area of head, A_{brg}, must be at least $4A_b$ where A_b is the area of the bar
(d) Concrete must be normalweight
(e) Clear cover to the bar must be greater than or equal to $2d_b$ where d_b is the nominal diameter of the bar
(f) Center-to-center spacing of the bars must be greater than or equal to $3d_b$

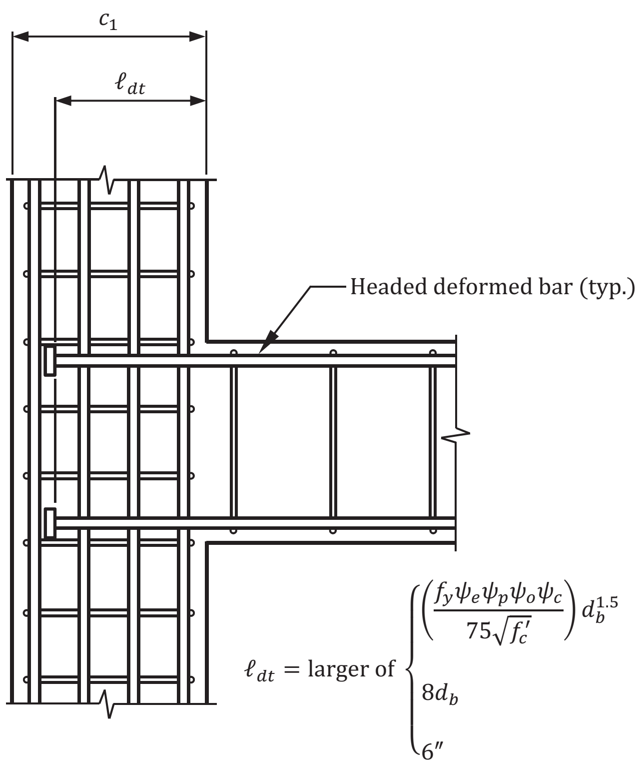


Figure 11.6 Longitudinal beam reinforcement terminated in a beam-column joint with a standard head.

The development length of a headed deformed reinforcing bar in tension, ℓ_{dt}, is given in ACI 25.4.4.2:

$$\ell_{dt} = \text{greater of} \begin{cases} \left(\dfrac{f_y\psi_e\psi_p\psi_o\psi_c}{75\sqrt{f_c'}}\right)d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{11.2}$$

This development length is measured from the critical section (which is taken at the face of the column) to the bearing face of the head (see ACI Figures R25.4.4.2a and R25.4.4.2b). The modification factors in Equation (11.2) are given in Table 11.3 (see ACI Table 25.4.4.3).

Where beam negative moment reinforcement is provided by headed deformed bars that terminate in a joint, the column must extend above the top of the joint a distance of at least the horizontal depth of the joint in the direction of analysis (ACI 25.4.4.6). If such an extension is not possible or feasible, the beam reinforcement must be enclosed by additional vertical joint reinforcement that provides equivalent confinement to the top face of the joint. This situation is likely to occur for the joints in the top story of the structure; for joints below the top story where the columns are continuous above the joint, this requirement is not applicable.

Table 11.3 Modification Factors for Development of Headed Bars in Tension

Modification Factor	Condition	Value of Factor
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Parallel tie reinforcement, ψ_p	For #11 and smaller bars with $A_{tt} \geq 0.3A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For headed bars (1) terminating inside a column core with side cover to bar ≥ 2.5 in. or (2) with side cover to bar $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c / 15{,}000) + 0.6$
	$f'_c \geq 6{,}000$ psi	1.0

The term ψ_p accounts for confining effects provided by stirrups or ties that are oriented parallel to the development length of headed bars. At beam-column joints, $\psi_p = 1.0$ where tie reinforcement with a total cross-sectional area A_{tt} is provided that satisfies the requirements in ACI 25.4.4.4 (see ACI Figure R25.4.4.4). Where such reinforcement is not provided in beam-column joints, $\psi_p = 1.6$.

11.4 Strength Requirements for Beam-Column Joints

11.4.1 Required Shear Strength

Overview

The required factored horizontal joint shear, V_u, is determined at the mid-height of a joint in the direction of analysis from statics. According to ACI 15.4.1.1, V_u is calculated using flexural tensile and compressive beam forces and column shear consistent with (a) or (b):

(a) The maximum moment transferred between the beam and the column based on a factored load analysis for beam-column joints with continuous beams in the direction of analysis.

(b) The negative and positive nominal flexural strengths of the beam at the face of the joint, M_n^- and M_n^+.

Joints in Moment Frames Subjected to Gravity Loads Only

A free-body diagram of an interior column or an edge column with the direction of analysis parallel to the edge in a moment frame that is subjected to the effects from gravity loads only is given in Figure 11.7. The bending moments M_1^- and M_2^- and the shear forces V_1 and V_2 that are transferred from the beam to the joint are shown in the figure. It is assumed for purposes of discussion that $M_1^- > M_2^-$. It is also assumed that the point of inflection is at the mid-height of the column, which is a reasonable assumption for columns above the first story and below the top story.

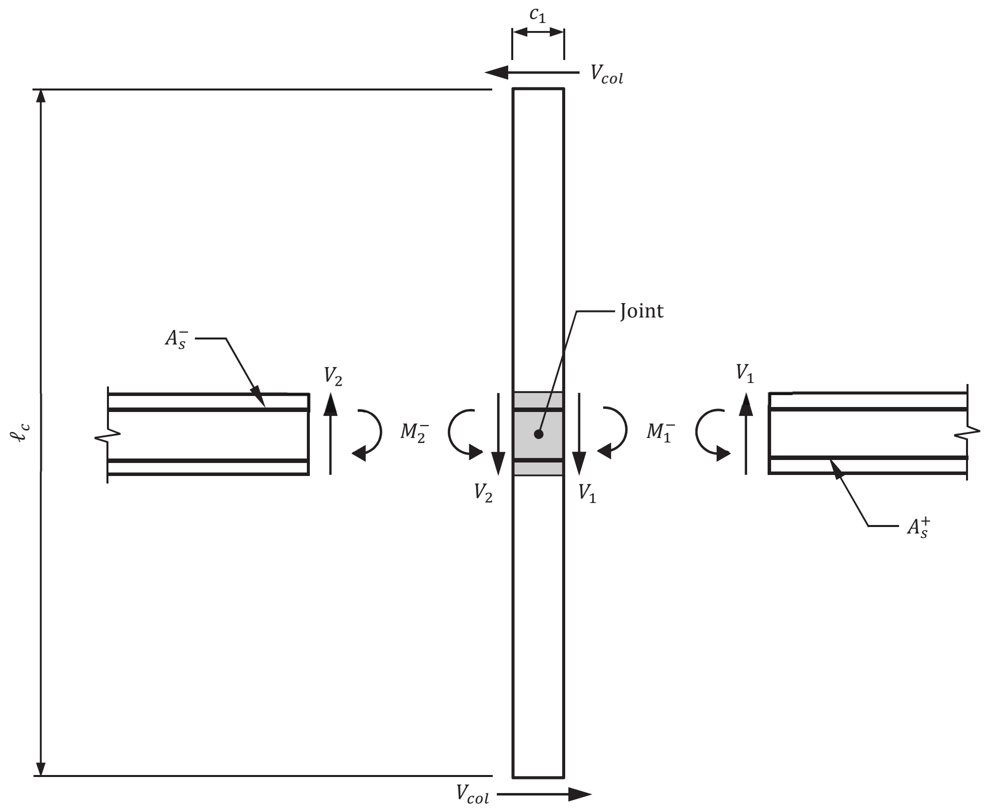


Figure 11.7 Free-body diagram of an interior column or an edge column with the direction of analysis parallel to the edge in a moment frame subjected to gravity loads.

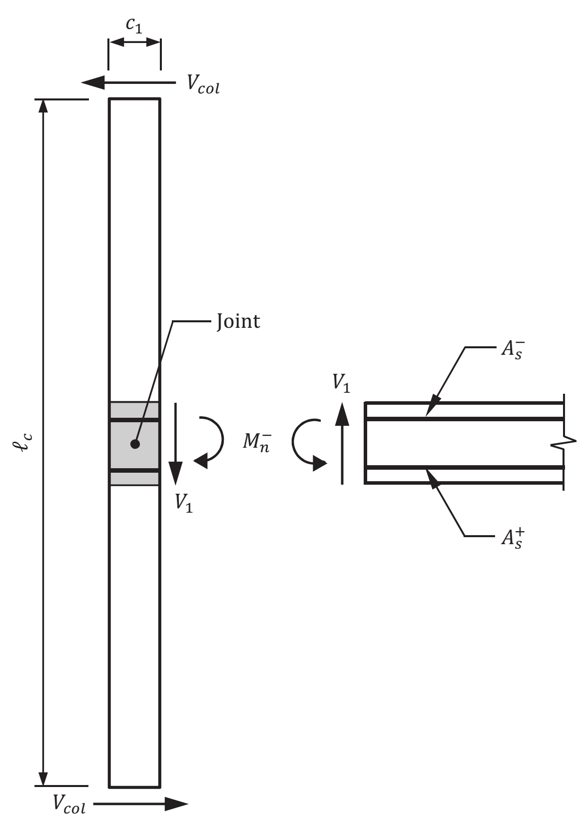


Figure 11.8 Free-body diagram of an edge column with the direction of analysis perpendicular to the edge or a corner column in a moment frame subjected to gravity loads.

The length ℓ_c is equal to the depths of the beams plus one-half the clear story height above and below the joint.

Where the negative and positive reinforcing bars in the beams framing into an interior joint are continuous through the joint, the forces are transmitted through the joint by the reinforcing bars, which means there is no need to check the shear strength of the joint.

The free-body diagram of an edge column with the direction of analysis perpendicular to the edge or a corner column in a moment frame that is subjected to the effects from gravity loads only is given in Figure 11.8.

The shear force in the column is determined from equilibrium by summing moments about the center of the joint:

$$V_{col} = \frac{M_1^-}{\ell_c} + \frac{V_1(c_1 / 2)}{\ell_c} \tag{11.3}$$

The loads that are transferred from the beam and column to the joint are shown on the free-body diagram of the joint in Figure 11.9(a) where P_2 and P_3 are the axial forces and M_2 and M_3 are the bending moments in the column. It is assumed that the axial force in the beam is negligible. The internal forces in the joint due to these loads are given in Figure 11.9(b). Tension and compression forces are denoted "T" and "C", respectively. In order to satisfy horizontal equilibrium, the tension force in the negative reinforcement, T_1, must be equal to the compression force resisted by the concrete, C_1.

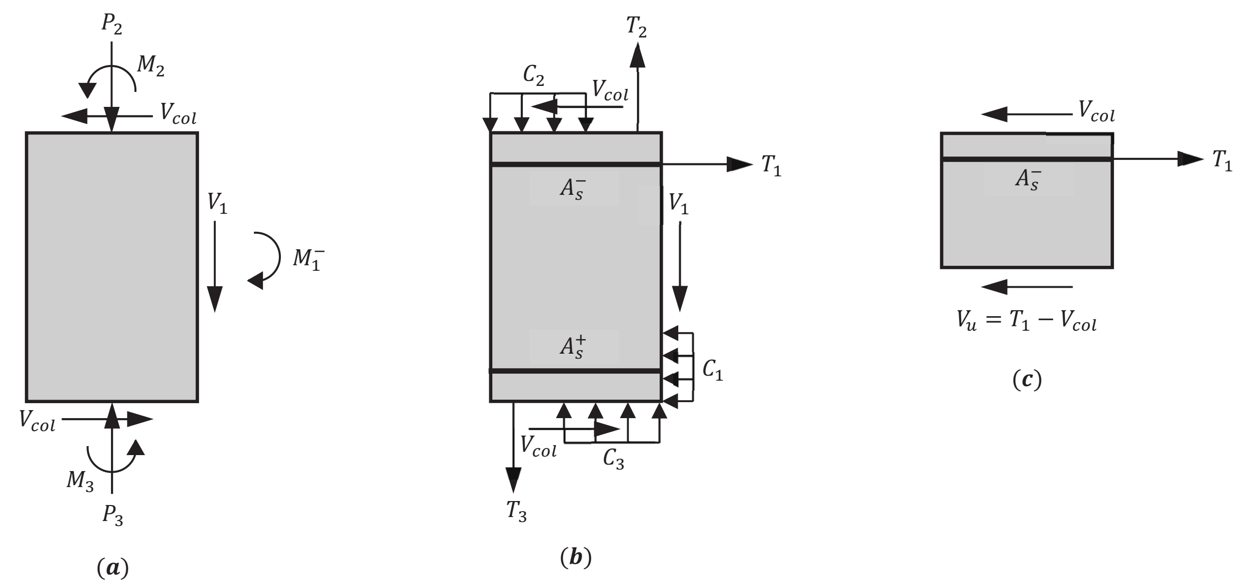


Figure 11.9 Free-body diagram of a joint in a moment frame subjected to gravity loads.
(a) Load transfer to the joint. (b) Internal forces in the joint. (c) Horizontal shear force in the joint.

A free-body diagram of the upper half of the joint with only horizontal forces is given in Figure 11.9(c). The horizontal shear force in the joint, V_u, can be determined from equilibrium by summing forces in the horizontal direction:

$$V_u = T_1 - V_{col} \tag{11.4}$$

Where a factored load analysis is used to determine V_u (which is the first option in ACI 15.4.1.1), the tension force in the negative reinforcement, T_1, is determined based on (1) $M_1^- = M_{u1}^-$, which is the factored bending moment at the face of the joint determined from the factored load analysis and (2) f_{s1}, which is the corresponding stress in the negative reinforcement; that is, $T_1 = A_s^- f_{s1}$.

In lieu of using a factored load analysis, V_u can be determined based on the negative nominal flexural strength of the beam, M_n^- (which is the second option in ACI 15.4.1.1). In such cases, $T_1 = A_s^- f_y$, and the shear force in the joint is equal to the following:

$$V_u = A_s^- f_y - V_{col} = A_s^- f_y - \left[\frac{M_n^-}{\ell_c} + \frac{V_1(c_1 / 2)}{\ell_c}\right] \tag{11.5}$$

where M_n^- is determined by the following equation:

$$M_n^- = A_s^- f_y\left(d - \frac{a}{2}\right) = A_s^- f_y\left(d - \frac{A_s^- f_y}{1.7 f_c' b}\right) \tag{11.6}$$

The shear force $[V_1(c_1 / 2) / \ell_c]$ is usually much smaller than the shear force (M_n^- / ℓ_c), and, thus, is often not included when determining V_u by Equation (11.5). This simplification is conservative because the calculated value of V_u based on $[V_1(c_1 / 2) / \ell_c] = 0$ is greater than the calculated value where this shear force is included.

Joints in Moment Frames Subjected to Gravity and Lateral Loads

The free-body diagram of an interior column or an edge column with the direction of analysis parallel to the edge in a moment frame subjected to gravity and lateral forces is given in Figure 11.10 for sidesway to the left where nominal flexural strengths are used to determine V_u.

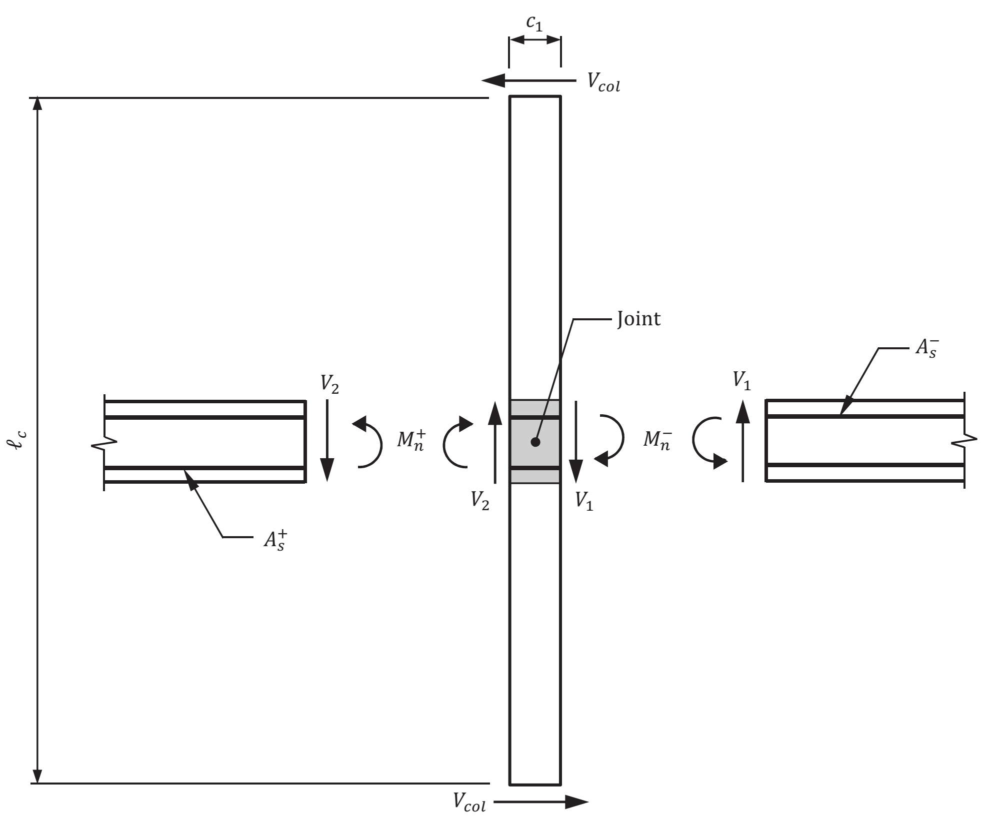


Figure 11.10 Free-body diagram of an interior column or an edge column with the direction of analysis parallel to the edge in a moment frame subjected to gravity and lateral loads.

The shear force in the column, V_{col}, can be obtained from equilibrium by summing moments about the center of the joint:

$$V_{col} = \frac{M_n^- + M_n^+}{\ell_c} + \frac{(V_1 + V_2) \times (c_1 / 2)}{\ell_c} \tag{11.7}$$

where M_n^- is determined by Equation (11.6) and M_n^+ is determined by the following equation:

$$M_n^+ = A_s^+ f_y \left(d - \frac{a}{2} \right) = A_s^+ f_y \left(d - \frac{A_s^+ f_y}{1.7 f_c' b} \right) \tag{11.8}$$

The loads that are transferred from the beam and column to the joint are shown on the free-body diagram of the joint in Figure 11.11(a) and the internal forces in the joint due to these loads are given in Figure 11.11(b). In order to satisfy horizontal equilibrium, the tension forces in the reinforcement must be equal to the compression forces resisted by the concrete, that is, $T_1 = C_1$ and $T_2 = C_2$.

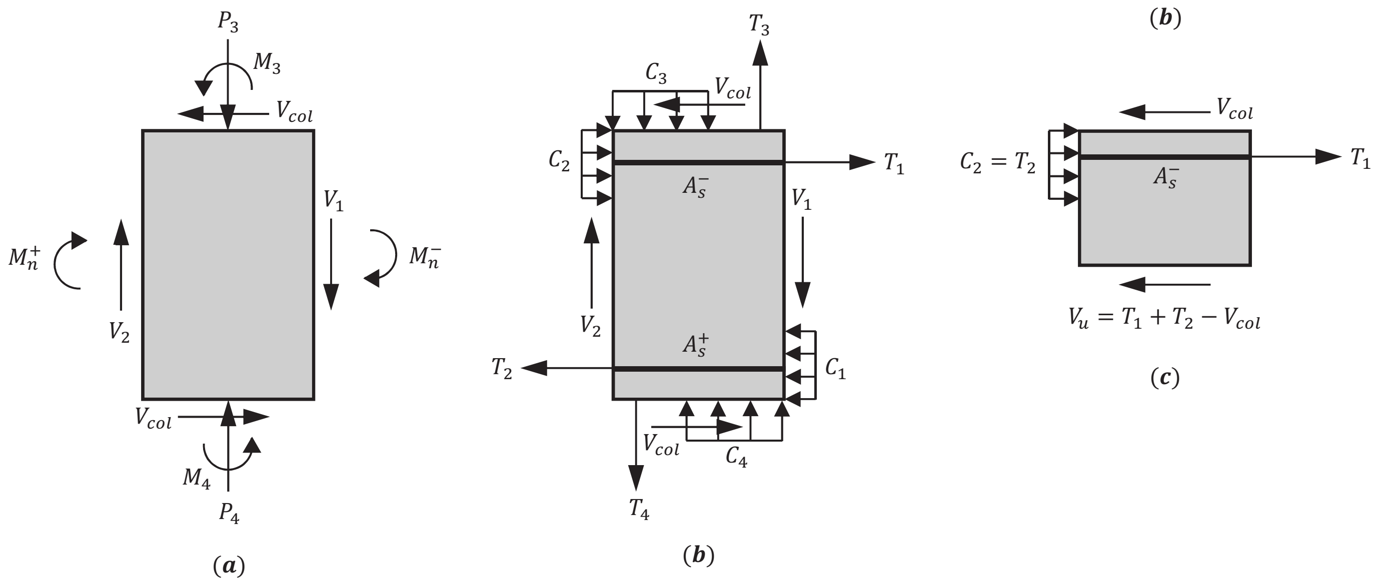


Figure 11.11 Free-body diagram of a joint in a moment frame subjected to gravity and lateral loads. (a) Load transfer to the joint. (b) Internal forces in the joint. (c) Horizontal shear force in a joint.

A free-body diagram of the upper half of the joint with only horizontal forces is shown in Figure 11.11(c) where it is assumed that the axial forces on the beams are negligible. The horizontal shear force in the joint, V_u, is obtained from equilibrium by summing forces in the horizontal direction:

$$V_u = T_1 + T_2 - V_{col} \tag{11.9}$$

where $T_1 = A_s^- f_y$ and $T_2 = A_s^+ f_y$.

Equation (11.9) can be rewritten in the following form:

$$V_u = (A_s^- + A_s^+) f_y - V_{col} \tag{11.10}$$

The horizontal shear force in the joint for sidesway to the right is the same as that for sidesway to the left where the beams framing into the joint have the same negative and positive nominal flexural strengths at both ends of the beams. Otherwise, V_u must be determined by Equation (11.10) for sidesway to the right, and the larger of the two factored shear forces must be used to check shear strength requirements.

For edge columns with the direction of analysis perpendicular to the edge and for corner columns in a moment frame that is subjected to both gravity and lateral loads, the horizontal shear force must be determined for both the positive and negative nominal flexural strengths (that is, sidesway to the right and to the left), and the larger of the two must be used to check shear strength requirements.

For sidesway to the right:

$$V_{col} = \frac{M_n^+}{\ell_c} + \frac{V_1 \times (c_1 / 2)}{\ell_c} \tag{11.11}$$

$$V_u = A_s^+ f_y - V_{col} \tag{11.12}$$

For sidesway to the left:

$$V_{col} = \frac{M_n^-}{\ell_c} + \frac{V_1 \times (c_1 / 2)}{\ell_c} \tag{11.13}$$

$$V_u = A_s^- f_y - V_{col} \tag{11.14}$$

It is assumed in all the above methods for determining V_u that the beams in the direction of analysis are as wide as or narrower than the columns that they frame in to. For beams that are significantly wider than the columns, longitudinal reinforcement in the beams passes outside of the confined core of the column, and the force generated by that reinforcement cannot be equilibrated by the compression strut in the joint. Guidelines on how to address this situation are given in Reference 26. It is proposed that joint shear strength for beams that are at most 4 in. wider than the columns can be determined using the methods presented above.

For columns in the first or top story of a moment frame or for moment frames where the above assumption regarding the location of the inflection point is not suitable, similar analyses can be performed to determine V_u using appropriate assumptions for the case at hand. The results from the overall analysis of the moment frame can be used as a guide to locate points of inflection in the columns. Alternatively, V_u can be conservatively determined by setting V_{col} equal to zero.

It is important to note that joint shear strength is evaluated in each direction of analysis separately; it is not required to consider interaction in two principal directions simultaneously.

11.4.2 Design Shear Strength

The design shear strength, ϕV_n, of a cast-in-place beam-column joint must be greater than or equal to the required shear strength, V_u (ACI 15.4.2.1). The strength reduction factor, ϕ, is equal to 0.75 for shear in accordance with ACI Table 21.2.1.

The nominal shear strength of the joint, V_n, is determined using ACI Table 15.4.2.3 and depends on the continuity of the columns and beams framing into the joint and whether the joint is confined or not by transverse beams (see Table 11.4).

Table 11.4 Nominal Joint Shear Strength, V_n

Column	Beam in Direction of Analysis	Confinement by Transverse Beams*	V_n (lbs)**
Continuous or a column extension is provided that satisfies ACI 15.2.6	Continuous or a beam extension is provided that satisfies ACI 15.2.7	Confined	$24\lambda\sqrt{f'_c}A_j$
		Not confined	$20\lambda\sqrt{f'_c}A_j$
	Other	Confined	$20\lambda\sqrt{f'_c}A_j$
		Not confined	$15\lambda\sqrt{f'_c}A_j$
Other	Continuous or a beam extension is provided that satisfies ACI 15.2.7	Confined	$20\lambda\sqrt{f'_c}A_j$
		Not confined	$15\lambda\sqrt{f'_c}A_j$
	Other	Confined	$15\lambda\sqrt{f'_c}A_j$
		Not confined	$12\lambda\sqrt{f'_c}A_j$

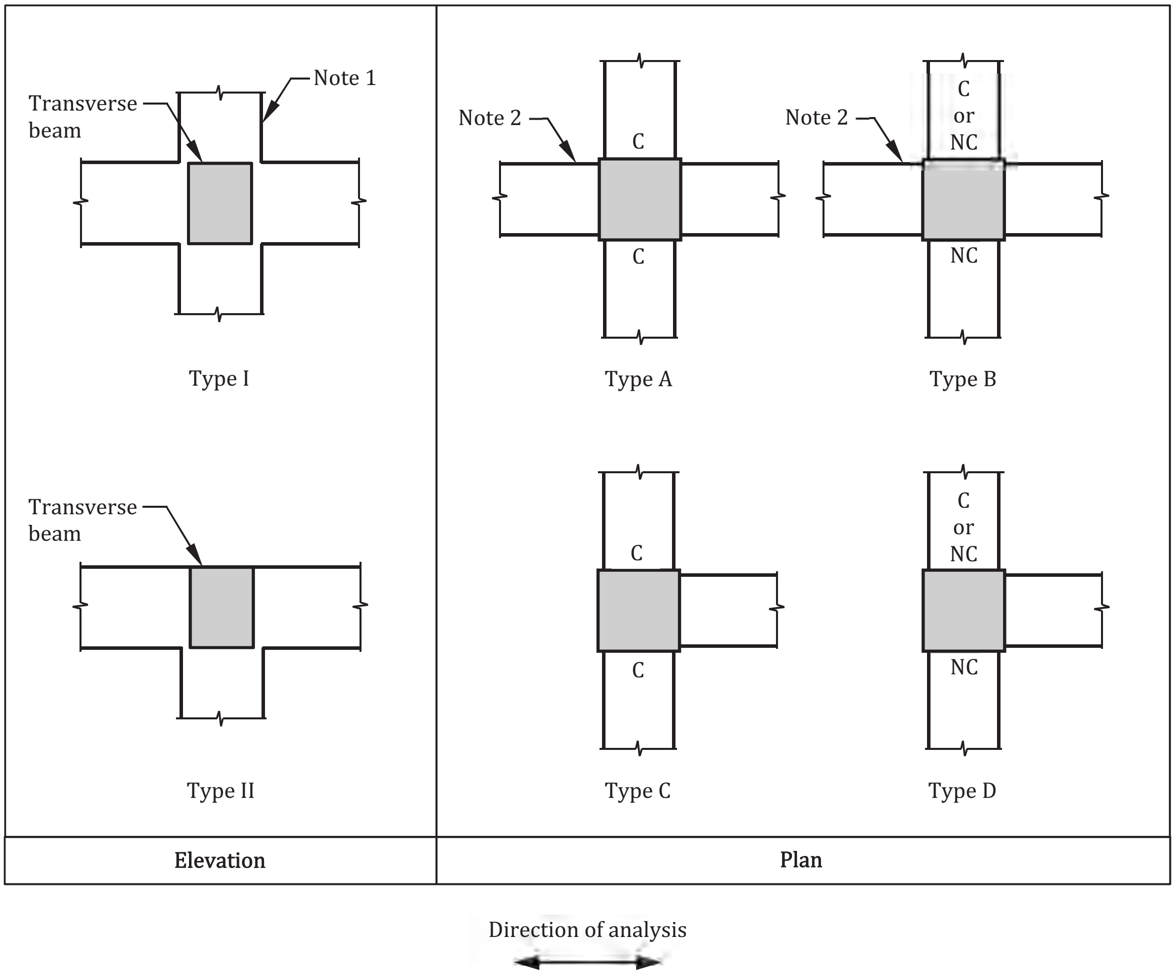
** Transverse beams that satisfy the requirements of ACI 15.2.8 are considered to provide confinement (see Figure 11.3). Examples of the various joint types in this table are given in Figure 11.12.*

*** The modification factor, λ, that reflects the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength is equal to 0.75 for lightweight concrete and 1.0 for normalweight concrete.*

Examples of the various joint types in Table 11.4 and the corresponding V_n are given in Figure 11.12. Joints with continuous columns (that is, columns frame into the top and bottom of the joint) or with a column extension that satisfies the requirements of ACI 15.2.6 are designated Type I. Joints with noncontinuous columns (that is, joints where columns frame into the bottom of the joints only, such as those in the top story of a building), are designated Type II. Type A and Type B designations in Figure 11.12 correspond to joints with continuous beams in the direction of analysis or with beam extensions that satisfy the requirements in ACI 15.2.7. Type C and Type D designations correspond to joints with noncontinuous beams in the direction of analysis. A designation of "C" at the face of a joint means a transverse beam provides confinement at that face in accordance with ACI 15.2.8. An "NC" designation means that no transverse beam is present at that face or a transverse beam is present but does not satisfy the confinement requirements in ACI 15.2.8.

The nominal shear strengths, V_n, can be obtained from Figure 11.12 for the various combinations of joint types. For example, a Type I-A joint consists of a continuous column and continuous beams in the direction of analysis with confinement provided by transverse beams on both faces; in this case, $V_n = 24\lambda\sqrt{f'_c}A_j$. This joint type corresponds to an interior column located in the structure other than the top story. Similarly, a corner column in the top story of a structure corresponds to a Type II-D joint, and $V_n = 12\lambda\sqrt{f'_c}A_j$.

The term A_j is the effective cross-sectional area within a joint, which is given in ACI 15.4.2.4. By definition, A_j is equal to the product of the joint depth and the effective joint width. The depth of the joint is always equal to the depth of the column parallel to the direction of analysis, c_1 (see Figure 11.13). The effective joint width, w, is equal to the width of the column perpendicular to the direction of analysis where the beams in the direction of analysis are as wide as or wider than the column. In such cases, $A_j = c_1 \times c_2$. For beams not as wide as the column, w is equal to the lesser of $b + c_1$ and $b + 2x$ where b is the beam width and x is the perpendicular distance from the edge of the beam to the nearest side face of the column.



Joint Type	V_n
I-A	$24\lambda\sqrt{f_c'}A_j$
I-B	$20\lambda\sqrt{f_c'}A_j$
I-C	$20\lambda\sqrt{f_c'}A_j$
I-D	$15\lambda\sqrt{f_c'}A_j$
II-A	$20\lambda\sqrt{f_c'}A_j$
II-B	$15\lambda\sqrt{f_c'}A_j$
II-C	$15\lambda\sqrt{f_c'}A_j$
II-D	$12\lambda\sqrt{f_c'}A_j$

Notes

1. Column is continuous or a column extension satisfying ACI 15.2.6 is provided (see Figure 11.1).
2. Beam in direction of analysis is continuous or a beam extension satisfying ACI 15.2.7 is provided (see Figure 11.2).
3. C – Transverse beam provides confinement in accordance with ACI 15.2.8 (see Figure 11.3).
4. NC – Transverse beam is not provided or transverse beam does not provide confinement in accordance with ACI 15.2.8 (see Figure 11.3).

Figure 11.12 Examples of joint types and the corresponding nominal joint shear strength, V_n.

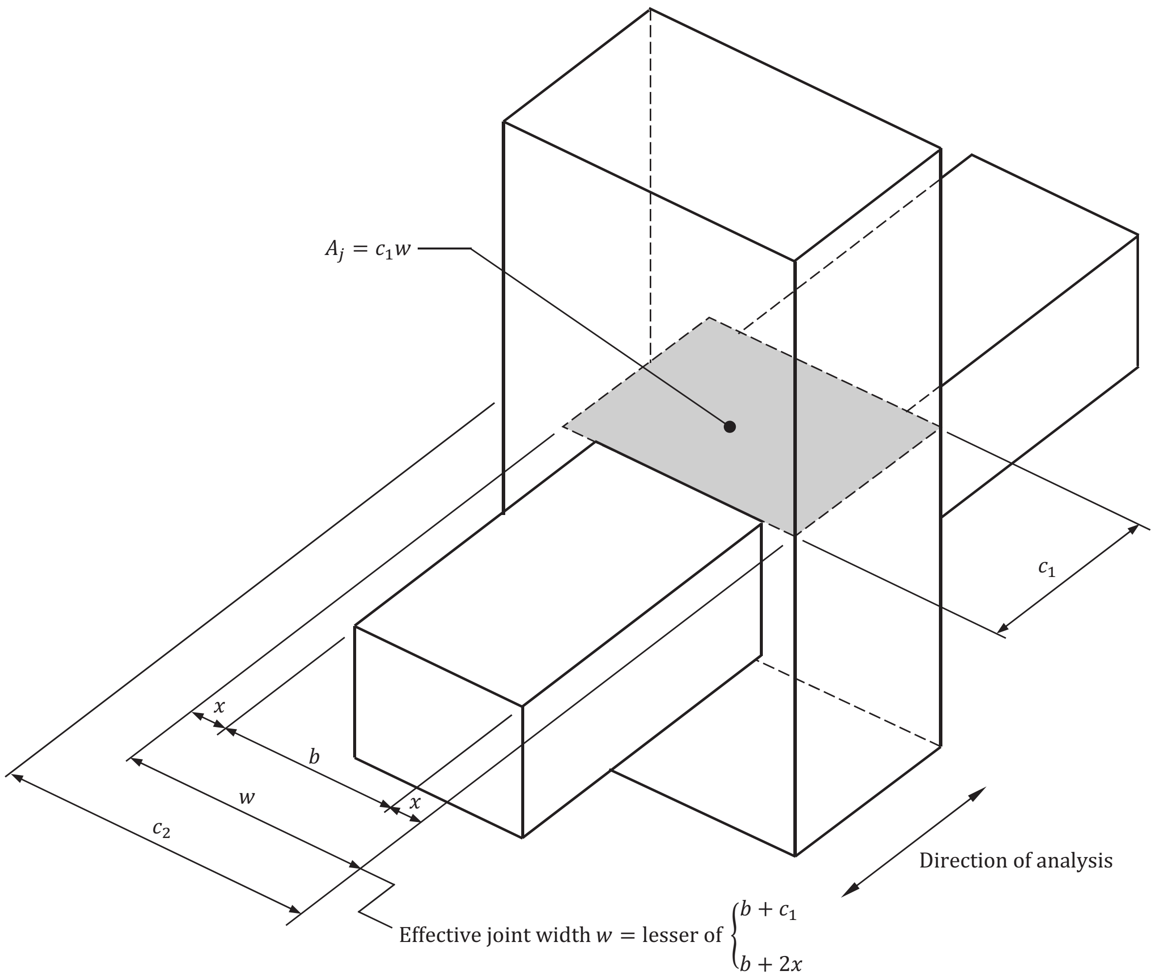


Figure 11.13 Effective cross-sectional area within a joint, A_j.

11.5 Transfer of Column Axial Force Through the Floor System

The requirements in ACI 15.5 consider the effect of the strength of the concrete in a floor system on the axial strength of a column. If the strength of the concrete in the floor system is significantly less than that in the column, confinement of the joint by the floor system may not be adequate, which may have an impact on load transfer through the floor system.

The provisions in ACI 15.5.1 must be satisfied where the compressive strength of the floor system is less than 70 percent of the compressive strength of a column. For edge and corner columns, one of the following requirements must be satisfied:

1. Concrete specified for the column must be placed in the floor system at the column location and must extend at least 2 ft into the floor system from the faces of the column for the full depth of the floor system. The column concrete within this zone must be fully integrated with the concrete for the floor system.
2. The design strength of a column through a floor system must be calculated using the specified concrete compressive strength of the floor system. Vertical dowels and transverse reinforcement must be provided to achieve the required design strength, if required.

In the first option (which is sometimes referred to as "puddling"), two different concrete mixtures must be provided in the floor system. The column and floor system concrete mixtures must remain plastic long enough so that the two can be vibrated and well-integrated within the designated area.

In the second option, the design strength of the column is checked using the concrete compressive strength of the floor system. In cases where the strength requirements of the column are not satisfied, dowel bars and transverse reinforcement can be used to increase the design strength of the column.

In addition to the two options described above, one additional option is available for interior columns: For interior columns laterally supported by beams of approximately equal depth that satisfy the continuity requirements in ACI 15.2.7 and for slab-column joints supported on four sides by the slab, it is permitted to calculate the design strength of the column at the joint using a concrete compressive strength equal to 75 percent of the specified compressive strength of the column concrete plus 35 percent of the specified compressive strength of the floor system concrete. When utilizing this option, the ratio of the column concrete strength to the floor system concrete strength must not exceed 2.5 in the design calculations. Therefore, the design strength of the column in an interior joint must be determined using the following concrete strength, $(f_c')_{design}$:

$$(f_c')_{design} = 0.75(f_c')_{column} + 0.35(f_c')_{floor} \leq 2.225(f_c')_{floor} \tag{11.15}$$

11.6 Examples

11.6.1 Example 11.1 – Check of Joint Shear Strength, Edge Column is Not Part of the LFRS: Building #1 (Framing Option C), SDC A

Check the joint shear strength in the north-south direction for edge column E2 of Building #1, Framing Option C at the second-floor level (see Figure 1.1). Assume the following: (1) the column is not part of the LFRS, (2) the column is 24-in. square, (3) the column is reinforced with 8-#8 longitudinal bars and #4 ties, (4) beams frame into the joint on three faces, (5) the beams are 24-in. wide and 24-in. deep, and (6) the beams are reinforced with 5-#7 bars at the top of the section and 5-#6 bars at the bottom of the section. Also assume Grade 60 reinforcement and normalweight concrete with $f_c' = 4{,}000$ psi.

Design data are given in Sect. 1.2.1.

Step 1 – Check if the beam longitudinal reinforcement can be developed for tension in the edge column

Determine the tension development length of the #7 bars assuming standard 90-degree hooks are provided at the ends of the bars:

$$\ell_{dh} = \left(\frac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5}$$ Eq. (11.1)

For uncoated reinforcing bars, $\psi_e = 1.0$. Table 11.2

For hooked bars without confinement in accordance with ACI 25.4.3.3, $\psi_r = 1.6$.

For #7 hooked bars terminating within the column core with side cover normal to the plane of the hook greater than 2.5 in., $\psi_o = 1.0$.

For $f_c' = 4{,}000$ psi, $\psi_c = (f_c' / 15{,}000) + 0.6 = (4{,}000 / 15{,}000) + 0.6 = 0.87$.

For normalweight concrete, $\lambda = 1.0$.

Therefore,

$$\ell_{dh} = \left(\frac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.87}{55 \times 1.0 \times \sqrt{4{,}000}}\right) \times 0.875^{1.5} = 19.7 \text{ in.} > 8d_b = 7.0 \text{ in. and } 6.0 \text{ in.}$$

Available development length $= c_1 - \text{cover} - (d_b)_{tie} - (d_b)_{long.} = 24.0 - 1.5 - 0.5 - 1.0 = 21.0 \text{ in.} > 19.7 \text{ in.}$

Therefore, the #7 bars can be fully developed within the column with standard 90-degree hooks at the ends of the bars.

Determine the tension development length of the #7 bars assuming standard heads are provided at the ends of the bars:

$$\ell_{dt} = \left(\frac{f_y \psi_e \psi_p \psi_o \psi_c}{75\sqrt{f'_c}}\right) d_b^{1.5}$$ Eq. (11.2)

For uncoated reinforcing bars, $\psi_e = 1.0$. Table 11.3

For headed bars without parallel tie reinforcement satisfying the conditions in ACI Table 25.4.4.3, $\psi_p = 1.6$.

For #7 headed bars terminating within the column core with side cover to the bar greater than 2.5 in., $\psi_o = 1.0$.

For $f'_c = 4{,}000$ psi, $\psi_c = (f'_c / 15{,}000) + 0.6 = (4{,}000 / 15{,}000) + 0.6 = 0.87$.

Therefore,

$$\ell_{dt} = \left(\frac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.87}{75 \times \sqrt{4{,}000}}\right) \times 0.875^{1.5} = 14.4 \text{ in.} > 8d_b = 7.0 \text{ in. and } 6.0 \text{ in.}$$

Available development length $= c_1 - \text{cover} - (d_b)_{tie} - (d_b)_{long.} = 24.0 - 1.5 - 0.5 - 1.0 = 21.0 \text{ in.} > 14.4 \text{ in.}$

Therefore, the #7 bars can be fully developed within the column with standard heads at the ends of the bars.

Step 2 – Determine the factored horizontal joint shear force ACI 15.4.1.1

Because the joint is not part of the LFRS (that is, it is subjected to the effects from gravity loads only), and because the area of the negative flexural reinforcement in the beam is greater than the area of the positive reinforcement, the following equation for sidesway to the left can be used to determine V_u conservatively assuming $V_{col} = 0$:

$$V_u = A_s^- f_y = (5 \times 0.60) \times 60.0 = 180.0 \text{ kips}$$ Eq. (11.14)

Step 3 – Determine the design shear strength of the joint

Columns frame into the bottom and top of the joint (that is, the column is continuous). The beam in the direction of analysis is not continuous. The transverse beams provide confinement in accordance with ACI 15.2.8 because the 24-in. widths are greater than $0.75c_1 = 0.75 \times 24.0 = 18.0$ in. (see Figure 11.3). Therefore, the joint type is I-C (see Figure 11.12).

Because the beam in the direction of analysis is the same width as the column, the effective cross-sectional area within the joint is equal to the area of the column:

$$A_j = c_1 \times c_2 = 24.0 \times 24.0 = 576.0 \text{ in.}^2$$ Figure 11.13

The design shear strength of the joint is equal to the following for a Type I-C joint:

$$\phi V_n = \phi 20\lambda\sqrt{f'_c} A_j = 0.75 \times 20 \times 1.0 \times \sqrt{4{,}000} \times 576.0 / 1{,}000 = 546.4 \text{ kips} > V_u = 180.0 \text{ kips}$$ Figure 11.12

11.6.2 Example 11.2 – Check of Joint Shear Strength, Edge Column is Part of the LFRS: Building #1 (Framing Option C), SDC A

Check the joint shear strength in the north-south direction for edge column C1 of Building #1, Framing Option C at the second-floor level (see Figure 1.1). Assume the following: (1) the column is part of the LFRS, (2) the column is 20-in. square, (3) the column is reinforced with 4-#9 longitudinal bars and #4 ties, (4) beams frame into the joint on three faces, (5) the beams are 18-in. wide and 24-in. deep and are centered on the column, and (6) the beams are reinforced with 3-#9 bars at the top of the section and 3-#8 bars at the bottom of the section. Also assume Grade 60 reinforcement and normalweight concrete with $f'_c = 4{,}000$ psi.

Design data are given in Sect. 1.2.1.

Step 1 – Check if the beam longitudinal reinforcement can be developed for tension in the edge column

Determine the tension development length of the #9 bars assuming standard 90-degree hooks are provided at the ends of the bars:

$$\ell_{dh} = \left(\frac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f'_c}} \right) d_b^{1.5}$$ Eq. (11.1)

For uncoated reinforcing bars, $\psi_e = 1.0$. Table 11.2

For hooked bars without confinement in accordance with ACI 25.4.3.3, $\psi_r = 1.6$.

For #9 hooked bars terminating within the column core with side cover normal to the plane of the hook greater than 2.5 in., $\psi_o = 1.0$.

For $f'_c = 4{,}000$ psi, $\psi_c = (f'_c / 15{,}000) + 0.6 = (4{,}000 / 15{,}000) + 0.6 = 0.87$.

For normalweight concrete, $\lambda = 1.0$.

Therefore,

$$\ell_{dh} = \left(\frac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.87}{55 \times 1.0 \times \sqrt{4{,}000}} \right) \times 1.128^{1.5} = 28.8 \text{ in.} > 8d_b = 9.0 \text{ in. and } 6.0 \text{ in.}$$

Available development length $= c_1 - \text{cover} - (d_b)_{tie} - (d_b)_{long.} = 20.0 - 1.5 - 0.5 - 1.128 = 16.9 \text{ in.} < 28.8 \text{ in.}$

Therefore, the #9 bars cannot be fully developed within the column with standard 90-degree hooks at the ends of the bars.

Assuming the size of the column cannot be increased and no confining reinforcement is provided, decrease the size of the longitudinal bars. It can be determined that 7-#6 top bars and 6-#6 bottom bars can be fully developed in tension within the column using standard 90-degree hooks at the ends of the bars.

Determine the tension development length of the #9 bars assuming standard heads are provided at the ends of the bars:

$$\ell_{dt} = \left(\frac{f_y \psi_e \psi_p \psi_o \psi_c}{75\sqrt{f'_c}} \right) d_b^{1.5}$$ Eq. (11.2)

For uncoated reinforcing bars, $\psi_e = 1.0$. Table 11.3

For headed bars without parallel tie reinforcement satisfying the conditions in ACI Table 25.4.4.3, $\psi_p = 1.6$.

For #9 headed bars terminating within the column core with side cover to the bar greater than 2.5 in., $\psi_o = 1.0$.

For $f'_c = 4{,}000$ psi, $\psi_c = (f'_c / 15{,}000) + 0.6 = (4{,}000 / 15{,}000) + 0.6 = 0.87$.

Therefore,

$$\ell_{dt} = \left(\frac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.87}{75 \times \sqrt{4{,}000}} \right) \times 1.128^{1.5} = 21.1 \text{ in.} > 8d_b = 9.0 \text{ in. and } 6.0 \text{ in.}$$

Available development length $= c_1 - \text{cover} - (d_b)_{tie} - (d_b)_{long.} = 20.0 - 1.5 - 0.5 - 1.128 = 16.9 \text{ in.} < 21.1 \text{ in.}$

Therefore, the #9 bars cannot be fully developed within the column with standard heads at the ends of the bars.

It can be determined that 5-#7 top bars and 4-#7 bottom bars can be fully developed in tension within the column using standard heads at the ends of the bars.

The headed bars are used in the remainder of this example.

Step 2 – Determine the factored horizontal joint shear force ACI 15.4.1.1

The following equation can be used to determine V_u conservatively assuming $V_{col} = 0$:

$$V_u = (A_s^- + A_s^+)f_y = [(5 \times 0.60) + (4 \times 0.60)] \times 60.0 = 324.0 \text{ kips} \quad \text{Eq. (11.10)}$$

Step 3 – Determine the design shear strength of the joint

Columns frame into the bottom and top of the joint (that is, the column is continuous). The beam in the direction of analysis is continuous. The transverse beam provides confinement in accordance with ACI 15.2.8 because the 18-in. width is greater than $0.75c_1 = 0.75 \times 20.0 = 15.0$ in. (see Figure 11.3). Therefore, the joint type is I-B (see Figure 11.12).

Because the beams in the direction of analysis are narrower than the column, the effective joint width, w, is equal to the following:

$$w = \text{lesser of} \begin{cases} b + c_1 = 18.0 + 20.0 = 38.0 \text{ in.} \\ \\ b + 2x = 18.0 + (2 \times 1.0) = 20.0 \text{ in.} \end{cases}$$

$$A_j = c_1 \times w = 20.0 \times 20.0 = 400.0 \text{ in.}^2 \quad \text{Figure 11.13}$$

The design shear strength of the joint is equal to the following for a Type I-B joint:

$$\phi V_n = \phi 20\lambda\sqrt{f_c'}A_j = 0.75 \times 20 \times 1.0 \times \sqrt{4{,}000} \times 400.0 / 1{,}000 = 379.5 \text{ kips} > V_u = 324.0 \text{ kips} \quad \text{Figure 11.12}$$

11.6.3 Example 11.3 – Check of Joint Shear Strength, Corner Column is Part of the LFRS: Building #1 (Framing Option B), SDC A

Check the joint shear strength in the north-south direction for corner column E1 of Building #1, Framing Option B at the second-floor level (see Figure 1.1). Assume the following: (1) the column is part of the LFRS, (2) the column is 20-in. square, (3) the column is reinforced with 4-#9 longitudinal bars and #4 ties, (4) beams frame into the joint on two faces, (5) the beams are 18-in. wide and 24-in. deep and are flush with the outside edge of the column, and (6) the beams are reinforced with 7-#6 bars at the top of the section and 6-#6 bars at the bottom of the section. Assume Grade 60 reinforcement and normalweight concrete with $f_c' = 4{,}000$ psi.

Design data are given in Sect. 1.2.1.

Step 1 – Check if the beam longitudinal reinforcement can be developed for tension in the corner column

Determine the tension development length of the #6 bars assuming standard 90-degree hooks are provided at the ends of the bars:

$$\ell_{dh} = \left(\frac{f_y \psi_e \psi_r \psi_o \psi_c}{55\lambda\sqrt{f_c'}}\right) d_b^{1.5} \quad \text{Eq. (11.1)}$$

For uncoated reinforcing bars, $\psi_e = 1.0$. Table 11.2

For hooked bars without confinement in accordance with ACI 25.4.3.3, $\psi_r = 1.6$.

For #6 hooked bars terminating within the column core with side cover normal to the plane of the hook greater than 2.5 in., $\psi_o = 1.0$.

For $f'_c = 4{,}000$ psi, $\psi_c = (f'_c / 15{,}000) + 0.6 = (4{,}000 / 15{,}000) + 0.6 = 0.87$.

For normalweight concrete, $\lambda = 1.0$.

Therefore,

$$\ell_{dh} = \left(\frac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.87}{55 \times 1.0 \times \sqrt{4{,}000}} \right) \times 0.75^{1.5} = 15.6 \text{ in.} > 8d_b = 6.0 \text{ in. and } 6.0 \text{ in.}$$

Available development length $= c_1 - \text{cover} - (d_b)_{tie} - (d_b)_{long.} = 20.0 - 1.5 - 0.5 - 1.128 = 16.9 \text{ in.} > 15.6 \text{ in.}$

Therefore, the #6 bars can be fully developed within the column with standard 90-degree hooks at the ends of the bars.

Determine the tension development length of the #6 bars assuming standard heads are provided at the ends of the bars:

$$\ell_{dt} = \left(\frac{f_y \psi_e \psi_p \psi_o \psi_c}{75\sqrt{f'_c}} \right) d_b^{1.5}$$ Eq. (11.2)

For uncoated reinforcing bars, $\psi_e = 1.0$. Table 11.3

For headed bars without parallel tie reinforcement satisfying the conditions in ACI Table 25.4.4.3, $\psi_p = 1.6$.

For #6 headed bars terminating within the column core with side cover to the bar greater than 2.5 in., $\psi_o = 1.0$.

For $f'_c = 4{,}000$ psi, $\psi_c = (f'_c / 15{,}000) + 0.6 = (4{,}000 / 15{,}000) + 0.6 = 0.87$.

Therefore,

$$\ell_{dt} = \left(\frac{60{,}000 \times 1.0 \times 1.6 \times 1.0 \times 0.87}{75 \times \sqrt{4{,}000}} \right) \times 0.75^{1.5} = 11.4 \text{ in.} > 8d_b = 6.0 \text{ in. and } 6.0 \text{ in.}$$

Available development length $= c_1 - \text{cover} - (d_b)_{tie} - (d_b)_{long.} = 20.0 - 1.5 - 0.5 - 1.128 = 16.9 \text{ in.} > 11.4 \text{ in.}$

Therefore, the #6 bars can be fully developed within the column with standard heads at the ends of the bars.

Step 2 – Determine the factored horizontal joint shear force ACI 15.4.1.1

Because the area of the negative flexural reinforcement in the beam is greater than the area of the positive reinforcement, the following equation for sidesway to the left can be used to determine V_u conservatively assuming $V_{col} = 0$:

$$V_u = A_s^- f_y = (7 \times 0.44) \times 60.0 = 184.8 \text{ kips}$$ Eq. (11.14)

Step 3 – Determine the design shear strength of the joint

Columns frame into the bottom and top of the joint (that is, the column is continuous). The beam in the direction of analysis is not continuous. The transverse beam provides confinement in accordance with ACI 15.2.8 because the 18-in. width is greater than $0.75c_1 = 0.75 \times 20.0 = 15.0$ in. (see Figure 11.3). Therefore, the joint type is I-D (see Figure 11.12).

Because the beam in the direction of analysis is narrower than the column, the effective joint width, w, is equal to the following:

$$w = \text{lesser of} \begin{cases} b + c_1 = 18.0 + 20.0 = 38.0 \text{ in.} \\ b + 2x = 18.0 + (2 \times 0) = 18.0 \text{ in.} \end{cases}$$

$$A_j = c_1 \times w = 20.0 \times 18.0 = 360.0 \text{ in.}^2$$ Figure 11.13

The design shear strength of the joint is equal to the following for a Type I-D joint:

$$\phi V_n = \phi 15 \lambda \sqrt{f_c'} A_j = 0.75 \times 15 \times 1.0 \times \sqrt{4{,}000} \times 360.0 / 1{,}000 = 256.2 \text{ kips} > V_u = 184.8 \text{ kips}$$ Figure 11.12

11.6.4 Example 11.4 – Check of Joint Shear Strength, Edge Column is Part of the SFRS: Building #1 (Framing Option B), SDC B

Check the joint shear strength in the north-south direction for edge column D1 of Building #1, Framing Option B at the second-floor level (see Figure 1.1). Assume the following: (1) the column is part of the LFRS, (2) the column is 24-in. square, (3) the column is reinforced with 8-#8 longitudinal bars and #4 ties, (4) beams frame into the joint on two faces, (5) the beams are 24-in. wide and 24-in. deep, and (6) the beams are reinforced with 7-#7 bars at the top of the section and 3-#7 bars at the bottom of the section. Also assume Grade 60 reinforcement and normalweight concrete with $f_c' = 4{,}000$ psi.

Design data are given in Sect. 1.2.1.

Step 1 – Determine the factored horizontal joint shear force ACI 15.4.1.1

Determine the shear force in the column, V_{col}, conservatively neglecting V_1 and V_2 :

$$V_{col} = \frac{M_n^- + M_n^+}{\ell_c}$$ Eq. (11.7)

$$M_n^- = A_s^- f_y \left(d - \frac{A_s^- f_y}{1.7 f_c' b} \right) = (7 \times 0.60) \times 60 \times \left[21.5 - \frac{(7 \times 0.60) \times 60}{1.7 \times 4.0 \times 24.0} \right] / 12 = 419.1 \text{ ft-kips}$$ Eq. (11.6)

$$M_n^+ = A_s^+ f_y \left(d - \frac{A_s^+ f_y}{1.7 f_c' b} \right) = (3 \times 0.60) \times 60 \times \left[21.5 - \frac{(3 \times 0.60) \times 60}{1.7 \times 4.0 \times 24.0} \right] / 12 = 187.5 \text{ ft-kips}$$ Eq. (11.8)

From analysis of the structure, the point of inflection in the column above the joint occurs approximately 6.1 ft from the bottom of the column and the point of inflection in the column below the joint occurs approximately 9.4 ft from the top of the column. Therefore, $\ell_c = 6.1 + 9.4 = 15.5$ ft.

$$V_{col} = \frac{M_n^- + M_n^+}{\ell_c} = \frac{419.1 + 187.5}{15.5} = 39.1 \text{ kips}$$

The factored shear force in the joint is equal to the following:

$$V_u = (A_s^- + A_s^+) f_y - V_{col} = \{[(7 \times 0.60) + (3 \times 0.60)] \times 60.0\} - 39.1 = 360.0 - 39.1 = 320.9 \text{ kips}$$ Eq. (11.10)

Step 2 – Determine the design shear strength of the joint

Columns frame into the bottom and top of the joint (that is, the column is continuous). The beam in the direction of analysis is continuous. There are no beams in the transverse direction. Therefore, the joint type is I-B (see Figure 11.12).

Because the beam in the direction of analysis is the same width as the column, the effective cross-sectional area within the joint is equal to the area of the column:

$A_j = c_1 \times c_2 = 24.0 \times 24.0 = 576.0 \text{ in.}^2$ Figure 11.13

The design shear strength of the joint is equal to the following for a Type I-B joint:

$\phi V_n = \phi 20 \lambda \sqrt{f_c'} A_j = 0.75 \times 20 \times 1.0 \times \sqrt{4{,}000} \times 576.0 / 1{,}000 = 546.4 \text{ kips} > V_u = 320.9 \text{ kips}$ Figure 11.12

Step 3 – Determine the detailing for the joint

Because the joint is part of the SFRS, the detailing requirements in ACI 15.3.1.2 through 15.3.1.4 must be satisfied.

The details for this joint are given in Figure 11.14 (see Figure 11.4).

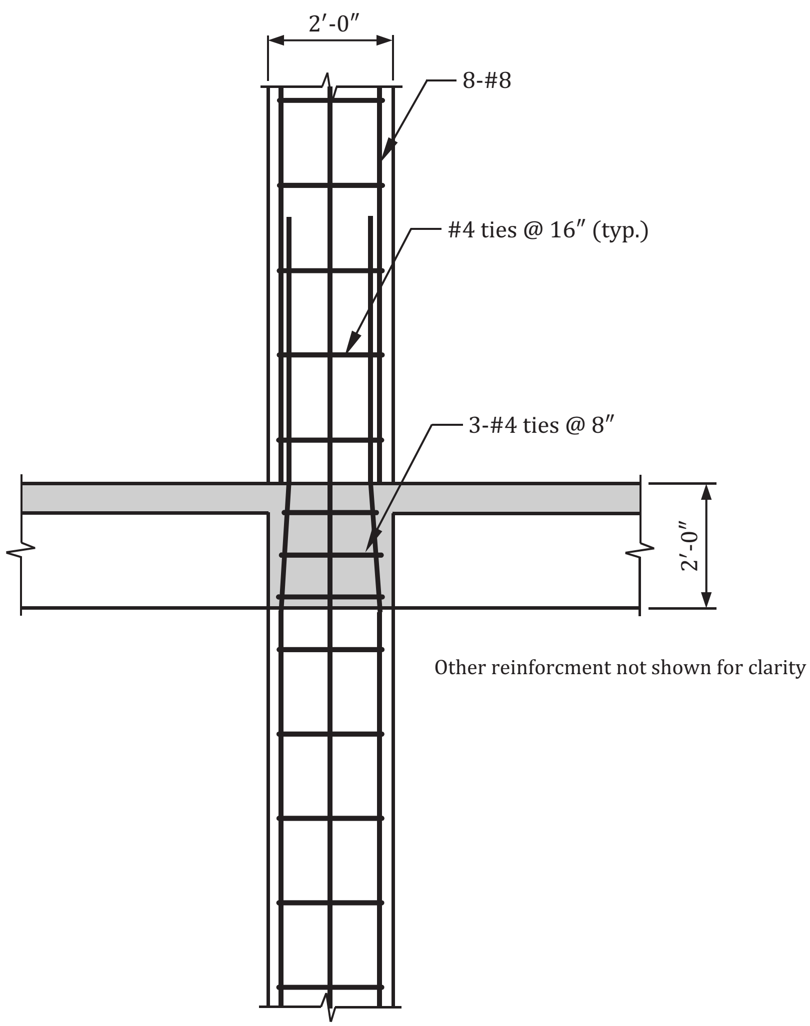


Figure 11.14 Reinforcement details for the joint in Example 11.4.

11.6.5 Example 11.5 – Adequacy of Transfer of Column Axial Force, Interior Column: Building #1 (Framing Option B), SDC A

Determine the adequacy of the transfer of column axial force for the interior column C3 of Building #1, Framing Option B (see Figure 1.1). Assume the column is 24-in. square. Also assume the concrete for the column is normalweight with $f_c' = 6{,}000$ psi and the concrete for the floor system is normalweight concrete with $f_c' = 4{,}000$ psi.

Design data are given in Sect. 1.2.1.

Solution

The provisions of ACI 15.5.1 must be satisfied where the compressive strength of the floor system is less than 70 percent of the compressive strength of the column.

In this example, $0.7 \times 6{,}000 = 4{,}200 \text{ psi} > 4{,}000 \text{ psi}$, so one of the requirements in ACI 15.5.1 must be satisfied.

Because the column is an interior column that is laterally supported on all four faces, any one of three requirements in ACI 15.5.1 can be used to satisfy transfer of column axial force through the joint.

In the first requirement, the 6,000-psi column concrete must be fully integrated with the 4,000-psi floor system concrete within a zone extending into the floor system at least 2 ft from the four faces of the column for the full depth of the floor system.

In the second requirement, the 24-in. column must be designed at the joint using the 4,000-psi floor system concrete. Vertical dowels and transverse reinforcement must be provided to achieve the design strength, if required.

In the third requirement, the 24-in. column must be designed at the joint using the following concrete compressive strength:

$$(f_c')_{design} = 0.75(f_c')_{column} + 0.35(f_c')_{floor} = (0.75 \times 6{,}000) + (0.35 \times 4{,}000)$$

Eq. (11.15)

$$= 5{,}900 \text{ psi} < 2.225(f_c')_{floor} = 8{,}900 \text{ psi}$$

Chapter 12
EARTHQUAKE-RESISTANT STRUCTURES — OVERVIEW

12.1 Overview

Design and detailing requirements for structures assigned to Seismic Design Category (SDC) B through F are given in ACI Chapter 18, including those for members designated as part of the seismic-force-resisting system (SFRS) in buildings assigned to SDC B and C. For buildings assigned to SDC D through F, requirements are given for members designated as part of the SFRS and for members not designated as part of the SFRS.

The provisions in ACI Chapter 18 are considered to be the minimum required in order for cast-in-place concrete structures to respond in the nonlinear range when subjected to design-level ground motions without critical strength decay. To achieve an inelastic response, particular detailing requirements must be satisfied, which are related to the SDC. The purpose of the detailing is to dissipate energy caused by the ground motion; even though a decrease in structure stiffness is likely to occur during a design earthquake event, the design and detailing requirements ensure that the strength of the structure is not compromised.

This chapter provides an overview of the design and detailing requirements in ACI Chapter 18. Included is information on the various structural systems available to resist the effects from earthquakes based on SDC.

12.2 Seismic Design Category

In order to apply the applicable design and detailing requirements in ACI Chapter 18, all structures must be assigned to a SDC in accordance with ACI 4.4.6.1 (ACI 18.2.1).

The SDC is a trigger mechanism for many seismic requirements, including the following:

- Permissible SFRS
- Limitations on building height
- Consideration of structural irregularities
- The need for additional special inspections, structural testing, and structural observation for seismic resistance

A method to determine the SDC in accordance with ASCE/SEI 11.6 is given in Section 3.3 and Figure 3.1 of this publication.

12.3 Design and Detailing Requirements

Structures assigned to SDC A need not satisfy the requirements in ACI Chapter 18. The requirements that must be satisfied for structures assigned to SDC B through F are given in ACI 18.2.1.3 through 18.2.1.5 (see Table 12.1).

Table 12.1 Requirements to be Satisfied Based on SDC

SDC	ACI Section No.
B	18.2.2
C	18.2.2, 18.2.3, and 18.13
D, E, or F	18.2.2 through 18.2.8 18.12 through 18.14

The sections of ACI Chapter 18 that must be satisfied in typical applications based on SDC are given in Table 12.2 (see ACI Table R18.2). The applicable design and detailing requirements in ACI Chapters 1 through 17 and 19 through 26 must also be satisfied; a dash in Table 12.2 signifies the application is governed by the requirements of those chapters for a given SDC (that is, no additional requirements are given in ACI Chapter 18).

Table 12.2 Sections of ACI Chapter 18 That Must be Satisfied in Typical Applications Based on SDC*

Application	ACI Chapter 18 Section No.		
	SDC B	SDC C	SDC D, E, or F
Analysis and design requirements	18.2.2	18.2.2	18.2.2 18.2.4
Material requirements	—	—	18.2.5 through 18.2.8
Design and detailing of frame members	18.3	18.4	18.6 through 18.9
Design and detailing of structural walls and coupling beams	—	—	18.10
Design and detailing of diaphragms and trusses	—	18.12**	18.12
Design and detailing of foundations	—	18.13	18.13
Design and detailing of frames members not designated as part of the SFRS	—	—	18.14

** The applicable design and detailing requirements in ACI Chapters 1 through 17 and 19 through 26 must also be satisfied. Where the requirements in ACI Chapter 18 conflict with other chapters, the requirements of ACI Chapter 18 govern (ACI 18.2.1.2).*
*** The requirements of ACI 18.12 must be satisfied for buildings assigned to SDC C only where precast concrete diaphragms are used as part of the SFRS (ACI 18.12.1.2).*

12.4 Structural Systems

12.4.1 Overview

For jurisdictions that have adopted the IBC, structural systems designated to part of the SFRS are restricted to those given in ASCE/SEI Table 12.2-1; other systems are permitted provided the conditions in ACI 18.2.1.7 are satisfied (ACI 18.2.1.6). The cast-in-place reinforced concrete systems in ASCE/SEI Table 12.2-1 are given in Table 12.3. Note that the number in Table 12.3 preceding each system is the number from ASCE/SEI Table 12.2-1. Seismic design coefficient and factors R, Ω_o, and C_d, structural system limitations, and building height limits are given in the table based on SDC. Descriptions of each SFRS are given in Table 12.4 and additional information is provided in the following sections (Note: Cantilevered column systems are not covered in this publication). Information on the design coefficient and factors in Table 12.3 is given in Table 12.5.

Table 12.3 Design Coefficient and Factors for Seismic-Force-Resisting Systems of Reinforced Concrete

Seismic-Force-Resisting System	R	Ω_o	C_d	Structural System Limitations Including Structural Height Limits (ft)				
				Seismic Design Category				
				B	C	D	E	F
A. Bearing Wall Systems								
1. Special reinforced concrete shear walls	5	$2^1/_2$	5	NL	NL	160	160	100
2. Ordinary reinforced concrete shear walls	4	$2^1/_2$	4	NL	NL	NP	NP	NP
B. Building Frame Systems								
4. Special reinforced concrete shear walls	6	$2^1/_2$	5	NL	NL	160	160	100
5. Ordinary reinforced concrete shear walls	5	$2^1/_2$	$4^1/_2$	NL	NL	NP	NP	NP

(table continued on next page)

Table 12.3 Design Coefficient and Factors for Seismic-Force-Resisting Systems of Reinforced Concrete (cont.)

Seismic-Force-Resisting System	R	Ω_o	C_d	Structural System Limitations Including Structural Height Limits (ft) — Seismic Design Category				
				B	C	D	E	F
C. Moment-Resisting Frame Systems								
3. Special reinforced concrete moment frames	8	3	$5^1/_2$	NL	NL	NL	NL	NL
4. Intermediate reinforced concrete moment frames	5	3	$4^1/_2$	NL	NL	NP	NP	NP
5. Ordinary reinforced concrete moment frames	3	3	$2^1/_2$	NL	NP	NP	NP	NP
D. Dual Systems with Special Moment Frames Capable of Resisting at Least 25% of Prescribed Seismic Forces								
3. Special reinforced concrete shear walls	7	$2^1/_2$	$5^1/_2$	NL	NL	NL	NL	NL
4. Ordinary reinforced concrete shear walls	6	$2^1/_2$	5	NL	NL	NP	NP	NP
E. Dual Systems with Intermediate Moment Frames Capable of Resisting at Least 25% of Prescribed Seismic Forces								
2. Special reinforced concrete shear walls	$6^1/_2$	$2^1/_2$	5	NL	NL	160	100	100
8. Ordinary reinforced concrete shear walls	$5^1/_2$	$2^1/_2$	$4^1/_2$	NL	NL	NP	NP	NP
F. Shear Wall-Frame Interactive System with Ordinary Reinforced Concrete Moment Frames and Ordinary Reinforced Concrete Shear Walls								
—	$4^1/_2$	$2^1/_2$	4	NL	NP	NP	NP	NP

NL = No height limit
NP = Not permitted
A "shear wall" is defined in ACI 2.3 as a "structural wall".

The terms "ordinary," "intermediate," and "special" in Table 12.3 are associated with the levels of inelastic response assumed in the determination of the design earthquake forces. A higher level of inelastic response means more stringent requirements for member proportioning and detailing, which results in an accompanying increase in deformation capacity.

Table 12.4 Seismic-Force-Resisting Systems of Reinforced Concrete

System	Description
Bearing wall	Bearing walls provide support for most or all the gravity loads, and resistance to seismic forces is provided by the same bearing walls acting as shear walls. These systems do not have an essentially complete space frame that provides support for gravity loads.
Building frame	A structural system with an essentially complete space frame that supports the gravity loads and shear walls that resist the seismic forces. It is assumed that the seismic forces are allocated to the shear walls only; no interaction is considered between the shear walls and the frames.

(table continued on next page)

Table 12.4 Seismic-Force-Resisting Systems of Reinforced Concrete (cont.)

System	Description
Moment-resisting frame	Gravity loads are supported by an essentially complete space frame and seismic forces are resisted primarily by flexural action of designated frame members (the entire space frame or selected portions of the space frame may be designated as the SFRS).
Dual	An essentially complete space frame provides support for gravity loads and resistance to seismic forces is provided by moment-resisting frames and shear walls. The frames and shear walls are designed to resist seismic forces in proportion to their relative rigidities. An additional requirement is that the moment frames alone must be capable of resisting at least 25 percent of the seismic forces (i.e., the frames act as a backup to the shear walls).
Shear wall-frame interactive	An essentially complete space frame provides support for gravity loads and resistance to seismic forces is provided by moment-resisting frames and shear walls, which are designed to resist seismic forces in proportion to their relative rigidities. The shear strength of the shear walls must be at least 75 percent of the design story shear at each story, and the moment-resisting frames must be capable of resisting at least 25 percent of the design story shear in every story.

Table 12.5 Design Coefficient and Factors Related to Seismic-Force-Resisting Systems

Design Coefficient and Factors	Description
Response modification coefficient, R	This coefficient accounts for the ability of a SFRS to respond to ground shaking in a ductile manner without loss of load-carrying capacity. It basically represents the ratio of the forces that would develop under the ground motion specified in ASCE/SEI 7 if the structure had responded to the ground motion in a linear-elastic manner. A system with no ability to respond in a ductile manner has an R-value equal to 1, while systems capable of a highly ductile response have an R-value equal to 8. The seismic force effects on a building are essentially reduced by this coefficient. Although the required design strength decreases as R increases, the design and detailing requirements substantially increase with increasing R.
Overstrength factor, Ω_o	The purpose of this factor is to ensure that critical elements in the SFRS that are not within anticipated hinge regions remain essentially elastic during the design-level earthquake. In the design and detailing of certain members, such as collector elements, the reactions obtained from an analysis based on the code-prescribed forces must be multiplied by Ω_o to ensure that such elements remain elastic for the duration of ground shaking.
Deflection amplification factor, C_d	This factor is used to adjust the lateral displacements determined for a structure using the code-prescribed seismic forces to the actual anticipated lateral displacements during the design-level earthquake. Values of C_d are equal to or slightly less than the corresponding R-values. The more ductile a system is (that is, the greater the R-value), the greater the difference is between the values of R and C_d.

12.4.2 Bearing Wall Systems

Overview

In a bearing wall system, bearing walls provide support for most or all of the gravity loads, and resistance to seismic forces is provided by the same bearing walls acting as shear walls (see Figure 12.1).

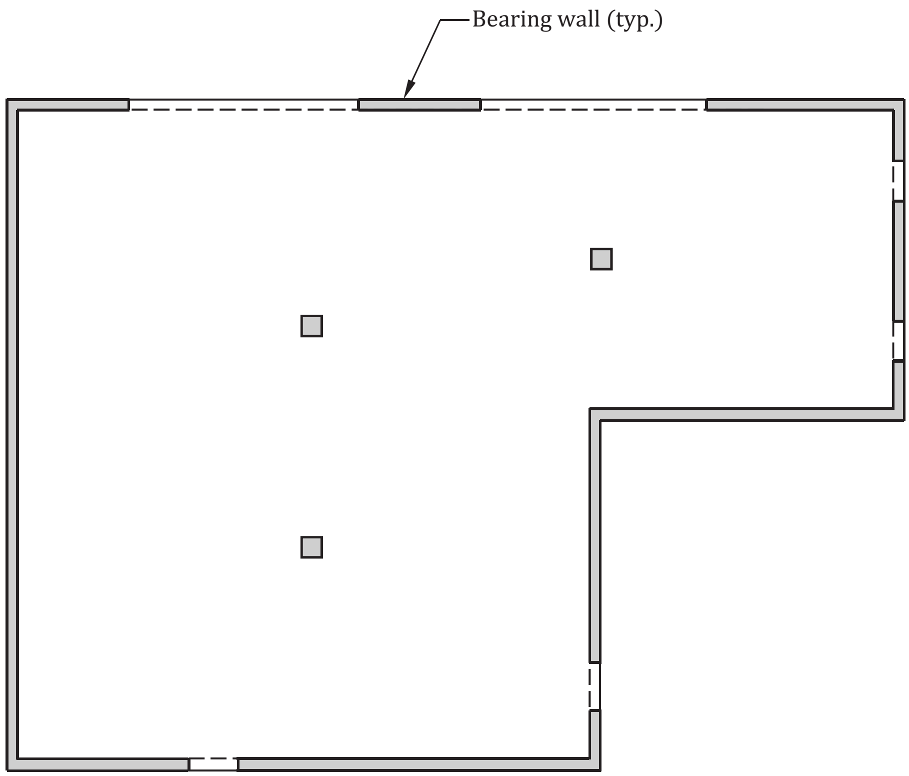


Figure 12.1 Example of a building with a bearing wall system.

SDC B

Ordinary reinforced concrete shear walls are permitted to be used in structures assigned to SDC B without any limitations. Such walls must satisfy the applicable requirements of ACI Chapters 1 to 17 and 19 to 26; the provisions of ACI Chapter 18 need not be satisfied. An ordinary reinforced concrete shear wall system is the minimum system type that needs to be provided for SDC B; a system listed for a higher SDC can always be provided if desired (which in this case would be a system with special reinforced concrete shear walls), as long as all the applicable design and detailing requirements for that system are satisfied (this is true for any type of system covered in the following sections).

SDC C

Ordinary reinforced concrete shear walls are permitted to be used in structures assigned to SDC C without any limitations. It is assumed the design and detailing requirements in ACI Chapter 11 are compatible with the anticipated level of inelastic response when the walls are subjected to the effects from moderately strong ground motion.

SDC D, E, or F

Special reinforced concrete shear walls are required in buildings assigned to SDC D, E, or F. The building height is limited to 160 ft for structures assigned to SDC D and E and to 100 ft for structures assigned to SDC F. The design and detailing requirements of ACI 18.2.2 through 18.2.8 and 18.10 must be satisfied for walls in structures assigned to SDC D, E, or F.

12.4.3 Building Frame Systems

Overview

A building frame system is a structural system with an essentially complete space frame that supports the gravity loads and shear walls that resist the seismic forces (see, for example, Building #3 in Figure 1.3 of this publication). It is assumed the seismic forces are allocated to the shear walls only; no interaction is considered between the shear walls and the frames.

SDC B

Ordinary reinforced concrete shear walls are permitted to be used in buildings assigned to SDC B with no limitations. The applicable design and detailing requirements of ACI Chapters 1 to 17 and 19 to 26 must be satisfied.

SDC C

Structures assigned to SDC C are permitted to utilize ordinary reinforced concrete shear walls with no limitations. Like in the case of bearing wall systems, it is assumed the design and detailing requirements in ACI Chapter 11 are compatible with the anticipated level of inelastic response.

SDC D, E, or F

Special reinforced concrete shear walls are required to be used in a building frame system (along with the applicable building height limits) in structures assigned to SDC D and higher. The deformational compatibility requirements of ACI 18.14 must also be satisfied: The beam-column frames must be designed to resist the effects caused by the lateral deflections due to the seismic forces because they are connected to the walls by the diaphragm at each level. In other words, the frame members, which are not designated as part of the SFRS, must be able to support the tributary gravity loads when subjected to the design displacements caused by the seismic forces.

12.4.4 Moment-Resisting Frame Systems

Overview

In a moment-resisting frame system, gravity loads are supported by an essentially complete space frame and seismic forces are resisted primarily by flexural action of designated frame members (see, for example, Building #1, Framing Option C in Figure 1.1 of this publication). The entire space frame or selected portions of the space frame may be designated as the SFRS.

SDC B

An ordinary reinforced concrete moment is permitted to be used in buildings assigned to SDC B with no limitations. In addition to the requirements of ACI Chapters 1 to 17 and 19 to 26, the requirements of ACI 18.3 must be satisfied.

SDC C

Structures assigned to SDC C are permitted to utilize intermediate reinforced concrete moment frames with no limitations. The design and detailing requirements in ACI 18.4 must be satisfied.

SDC D, E, or F

Special reinforced concrete moment frames are required in structures assigned to SDC D, E, or F. This system can be used without any limitations and must be designed and detailed in accordance with the provisions in ACI 18.2.2 through 18.2.8 and ACI 18.6 through 18.8.

12.4.5 Dual Systems

Overview

In a dual system, an essentially complete space frame provides support for gravity loads and resistance to seismic forces is provided by moment-resisting frames and by shear walls (see, for example, Building #4 in Figure 1.4 of this publication). The frames and shear walls are designed to resist seismic forces in proportion to their relative rigidities. An additional requirement is that the moment frames alone must be capable of resisting at least 25 percent of the seismic forces (i.e., the frames act as a backup to the shear walls).

SDC B

Any of the dual systems listed in ASCE/SEI Table 12.2-1 are permitted to be used in structures assigned to SDC B. It is common for shear wall-frame interactive systems to be specified in such cases.

SDC C

A dual system with intermediate moment frames and ordinary reinforced concrete shear walls is permitted as a minimum for structures assigned to SDC C. The moment frames, which must be designed to independently resist 25 percent of the code-prescribed seismic forces, must satisfy the requirements of ACI 18.4. The requirements of ACI Chapters 1 to 17 and 19 to 26 must be satisfied for the walls.

SDC D, E, or F

Dual systems with special moment frames and special reinforced concrete shear walls are required for structures assigned to SDC D, E, or F without limitations. Each component must be designed and detailed in accordance with the applicable provisions of ACI 18.2.1.6. A dual system with intermediate moment frames and special reinforced concrete shear walls is also permitted in ASCE/SEI Table 12.2-1 with the following limitations: (1) for SDC D, the building height is limited to 160 ft and (2) for SDC E and F, the building height is limited to 100 ft. The use of intermediate moment frames as part of a dual system in SDC D, E, or F is not recommended (see ACI R18.2).

12.4.6 Shear Wall-Frame Interactive Systems

Shear wall-frame interactive systems are similar to dual systems in that an essentially complete space frame provides support for gravity loads and resistance to seismic forces is provided by moment-resisting frames and by shear walls, which are designed to resist lateral forces in proportion to their relative rigidities.

Shear wall-frame interactive systems with ordinary reinforced concrete moment frames and ordinary reinforced concrete shear walls are permitted to be used in structures assigned to SDC B with no limitations; dual systems are required in structures assigned to SDC C and above. The shear strength of the shear walls must be at least 75 percent of the design story shear at each story, and the moment-resisting frames must be capable of resisting at least 25 percent of the design story shear in every story (ASCE/SEI 12.2.5.8). Other than the requirements in ACI 18.3 for ordinary moment frames, no special seismic design and detailing requirements are needed for this system.

Chapter 13
EARTHQUAKE-RESISTANT STRUCTURES — SDC B and C

13.1 Overview

This chapter covers the design and detailing requirements in ACI Chapter 18 for the following:

- Frame members (ACI 18.3 and 18.4)
- Foundations (ACI 18.13)

Ordinary moment frames are permitted to be used as a minimum for structures assigned to Seismic Design Category (SDC) B with no limitations (see Table 12.3 of this publication). Requirements for ordinary moment frames are given in ACI 18.3. For structures assigned to SDC C, intermediate moment frames are permitted to be used with no limitations. Requirements for intermediate moment frames are given in ACI 18.4.

The requirements in ACI Chapter 13 must be satisfied for foundations supporting structures assigned to SDC B. The applicable design and detailing requirements for this case are covered in Chapter 10 of this publication. Requirements for foundations supporting structures assigned to SDC C are given in Section 13.4 below.

No requirements are given in ACI Chapter 18 for the design and detailing of cast-in-place reinforced concrete diaphragms in buildings assigned to SDC B or C. The requirements in ACI Chapter 12 are applicable in such cases (see Chapter 9 of this publication). Collector elements must be designed for the applicable forces determined in accordance with ASCE/SEI 12.10.2.1.

Ordinary reinforced concrete shear walls are permitted to be used in bearing wall, building frame, and shear wall-frame interactive seismic-force-resisting systems (SFRS) in structures assigned to SDC B or C (see Table 12.3 of this publication). As noted in Section 12.4 of this publication, it is assumed the design and detailing requirements given in ACI Chapter 11 for ordinary shear walls are compatible with the anticipated level of inelastic response when the walls are subjected to the effects from moderately strong ground motion associated with SDC C. Design and detailing requirements for ordinary walls are given in Chapter 8 of this publication.

It is important to note that the applicable seismic design and detailing requirements in ACI Chapter 18 must be satisfied even in cases where strength design load combinations including earthquake effects do not govern the design of the structural member. For example, if the combined effects due to gravity and wind forces govern with respect to the required area of flexural reinforcement in a beam that is part of a moment-resisting frame in SDC B, the seismic flexural detailing requirements for SDC B in ACI 18.3.2 must be satisfied even though wind load effects govern the design.

13.2 Ordinary Moment Frames (SDC B)

13.2.1 Overview

Design and detailing requirements for beams, columns, and beam-column joints in ordinary moment frames are given in ACI 18.3. The provisions in ACI 18.3 are not applicable to moment frames consisting of slabs and columns; the requirements in ACI Chapter 8 must be used to design and detail two-way slab-column moment frames in structures assigned to SDC B (see Chapter 5 of this publication).

Gravity and lateral load effects on members in an ordinary moment frame are obtained from an analysis method in ACI 6.2. Design strength load combinations for structures assigned to SDC B are given in Table 13.1 for gravity, wind, and seismic load effects (see Section 3.5 of this publication).

Table 13.1 Strength Design Load Combinations for Structures Assigned to SDC B

ACI Equation Number	ASCE/SEI 7 Load Combination	Load Combination
5.3.1a	1	$U = 1.4D$
5.3.1b	2	$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
5.3.1c	3	$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$
5.3.1d	4	$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
5.3.1e	6	$U = 1.2D + Q_E + 1.0L + 0.2S$
5.3.1f	5	$U = 0.9D + 1.0W$
5.3.1g	7	$U = 0.9D + Q_E$

13.2.2 Beams

The required beam size for strength and serviceability and the required flexural, shear, and torsion reinforcement are determined using the provisions in ACI Chapter 9 for the combined effects due to gravity and lateral forces (see Chapter 6 of this publication).

In addition to the applicable detailing requirements in ACI Chapter 9, the detailing requirements in ACI 18.3.2 must be satisfied for beams in ordinary moment frames that are part of the SFRS (see Figure 13.1):

- The flexural reinforcement at the top and bottom faces of the section must include two bars continuous along the span.
- The area of the continuous bottom bars in the section must have an area of at least 25 percent of the maximum area of the bottom flexural reinforcement along the span.
- The flexural reinforcement must be anchored to develop f_y in tension at the face of the support.

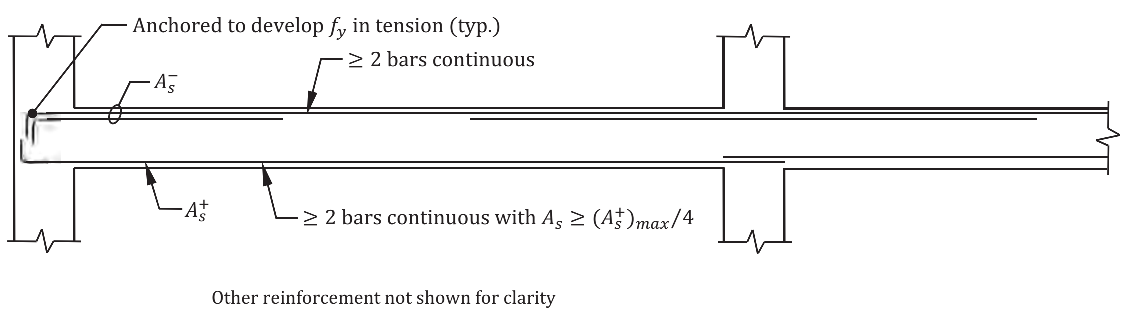


Figure 13.1 Requirements for beams in ordinary moment frames in structures assigned to SDC B.

The requirements for flexural reinforcement are intended to improve seismic force resistance and structural integrity for structures not expected to be subjected to strong ground motion.

13.2.3 Columns

The required column size and the required longitudinal and transverse reinforcement for columns in ordinary moment frames that are part of the SFRS in structures assigned to SDC B are determined using the provisions in ACI Chapter 10 for the combined effects due to gravity and lateral forces (see Chapter 7 of this publication).

Shear strength requirements for certain columns in ordinary moment frames that are part of the SFRS are given in ACI 18.3.3. Where the unsupported length of a column, ℓ_u, is less than or equal to five times the plan dimension of the column, c_1, in the direction of analysis, the design shear strength of the column must be taken as the lesser of the following:

- The shear force, V_u, associated with the development of nominal flexural strengths, M_n, of the column at each restrained end of the unsupported length due to reverse curvature bending (see Figure 13.2). In the figure, M_{nt} and M_{nb} are the nominal flexural strengths at the top and bottom of the column, respectively, in the direction of analysis. These flexural strengths must be calculated for the factored axial force, P_u, consistent with the direction of analysis resulting in the highest flexural strength. Sidesway to the right and sidesway to the left must both be considered.
- The maximum shear force obtained from the design load combinations of ACI Chapter 5 that include the earthquake effect, E, with $\Omega_o E$ substituted for E in the load combinations. The term Ω_o is the overstrength factor for ordinary moment frames of reinforced concrete, which is equal to 3 (see ASCE/SEI Table 12.2-1 or Table 12.3 in this publication).

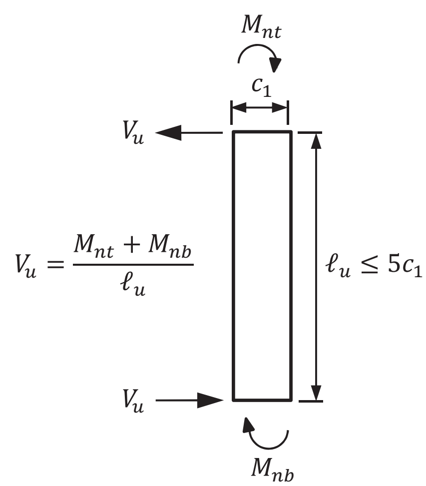


Figure 13.2 Design shear strength of columns in ordinary moment frames that are part of the SFRS in accordance with ACI 18.3.3.

The provisions for the design shear strength are intended to provide additional shear capacity in relatively squat columns, which are vulnerable to shear failure under earthquake loading.

13.2.4 Beam-Column Joints

Beam-column joints in ordinary moment frames that are part of the SFRS in structures assigned to SDC B must be designed and detailed in accordance with the provisions in ACI Chapter 15 (ACI 18.3.4; see Chapter 11 of this publication).

According to ACI 18.3.4, the required factored horizontal joint shear, V_u, is determined at the mid-height of the joint in the direction of analysis from statics using tensile and compressive beam forces and column shear consistent with the negative and positive nominal flexural strengths of the beam at the face of the joint, M_n^- and M_n^+.

The information in Section 11.4.1 of this publication can be used to calculate V_u for interior and edge/corner joints subjected to gravity and lateral forces. The design shear strength of joints, ϕV_n, can be determined using Figure 11.12 in Section 11.4.2 of this publication. For joints in buildings assigned to SDC B, the strength reduction factor, ϕ, is equal to 0.75.

13.3 Intermediate Moment Frames (SDC C)

13.3.1 Overview

Design and detailing requirements for beams, columns, beam-column joints, slab-column joints, and two-way slabs without beams designated part of the SFRS in intermediate moment frames are given in ACI 18.4.

Gravity and lateral load effects on members in an intermediate moment frame are obtained from an analysis method in ACI 6.2. Design strength load combinations for structures assigned to SDC C are given in Table 13.2 for gravity, wind, and seismic load effects (see Section 3.5 of this publication).

Table 13.2 Strength Design Load Combinations for Structures Assigned to SDC C

ACI Equation Number	ASCE/SEI 7 Load Combination	Load Combination
5.3.1a	1	$U = 1.4D$
5.3.1b	2	$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
5.3.1c	3	$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$
5.3.1d	4	$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
5.3.1e	6	$U = (1.2 + 0.2S_{DS})D + Q_E + 1.0L + 0.2S$
5.3.1f	5	$U = 0.9D + 1.0W$
5.3.1g	7	$U = (0.9 - 0.2S_{DS})D + Q_E$

13.3.2 Beams

Overview

The required beam size for strength and serviceability and the required flexural reinforcement are determined using the provisions in ACI Chapter 9 for the combined effects due to gravity and lateral forces (see Chapter 6 of this publication). The shear strength requirements in ACI 18.4.2.3 and the detailing requirements in ACI 18.4.2.4 through 18.4.2.6 must be satisfied in addition to any relevant provisions in ACI Chapter 9.

Flexural Strength Requirements

Requirements having a possible impact on the amount of positive flexural reinforcement in beams that are part of an intermediate moment frame are given in ACI 18.4.2.2. At the faces of the joints, the positive moment strength, M_n^+, must be at least equal to 33 percent of the corresponding negative moment strength, M_n^-, at that joint. The area of the positive flexural reinforcement in the beam at a joint may need to be increased to satisfy this requirement. Also, at any section along the length of a beam, the negative and positive moment strength must be equal to at least 20 percent of the maximum moment strength provided at the face of either joint.

A summary of the flexural requirements in ACI Chapter 9 and ACI 18.4.2.2 for beams in an intermediate moment frame is given in Figure 13.3.

There are no restrictions where lap splices of the flexural reinforcement can be located along the span. The top reinforcement is usually spliced near midspan and the bottom reinforcement is spliced near the supports. Because plastic hinges have the potential to form at the ends of a beam, it would be appropriate to locate the splices of the bottom reinforcement somewhere between the end of the anticipated plastic hinge region and midspan. The assumed length of the anticipated plastic hinge region is equal to $2h$ from each end of a beam where h is the overall depth of the beam (ACI 18.4.2.4).

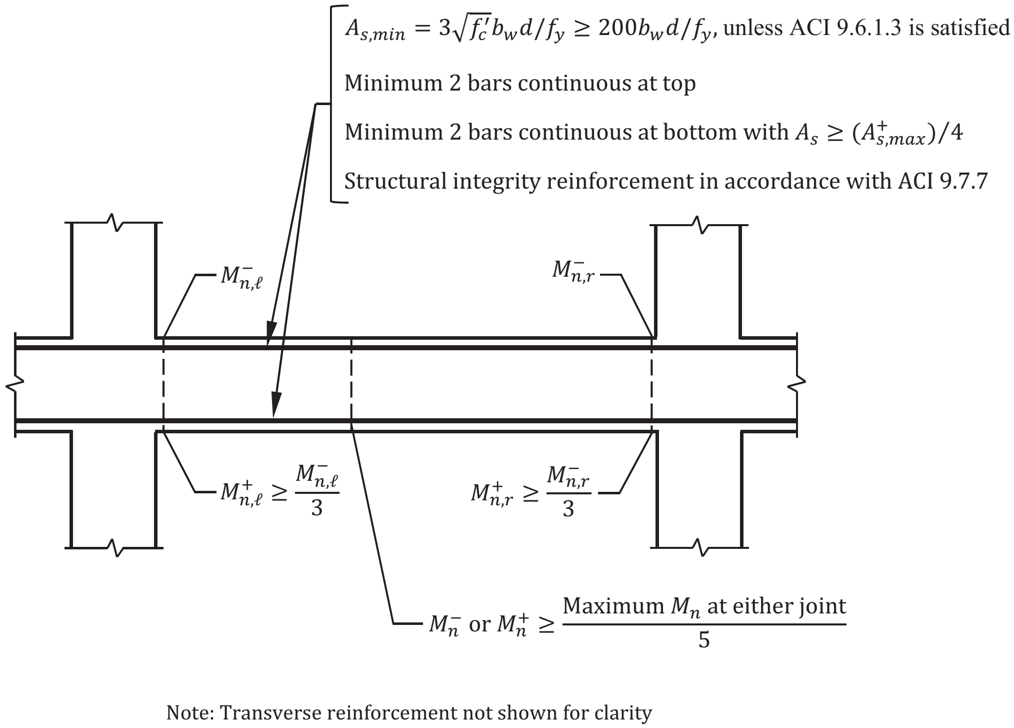


Figure 13.3 Flexural requirements for beams in intermediate moment frames.

Shear Strength Requirements

Required shear reinforcement in beams of intermediate moment frames is calculated differently than in beams of ordinary moment frames, the latter of which is determined solely on the effects from factored load combinations (see Section 13.2.2 of this publication).

According to ACI 18.4.2.3, the design shear strength of a beam, ϕV_n, which must be greater than or equal to the required shear strength, V_u, is equal to the lesser of the following:

- The sum of the shear forces associated with development of the negative and positive nominal flexural strengths, M_n^- and M_n^+, at each end of the beam and the factored gravity and vertical earthquake loads acting along the span [ACI 18.4.2.3(a)].
- The maximum shear force obtained from the factored load combinations that include earthquake load effects, E, where E is taken to be twice that prescribed by the governing building code [ACI 18.4.2.3(b)].

The requirement in ACI 18.4.2.3(a) is illustrated in Figure 13.4 for a beam with a factored uniform gravity load w_u along its span. Because earthquake effects can act in either direction, both sidesway to the right and sidesway to the left must considered. ACI Equation (5.3.1e) is applicable when determining w_u, which must include the vertical earthquake load effect $0.2S_{DS}D$ (see Table 13.2). Thus, $0.2S_{DS}D$ is combined with $1.2D$, $0.5L$, and $0.2S$, where it is assumed the load factor on L can be taken as 0.5 in accordance with ACI 5.3.3. The total factored shear force at each end of a beam, V_u, is determined from statics. In cases where the top reinforcement is the same at both ends of a beam $(A^-_{s,\ell} = A^-_{s,r})$ and the bottom reinforcement is continuous over the entire span, maximum V_u is the same for both sidesway to the right and to the left.

According to ACI 18.4.2.3(b), the maximum V_u is determined from analysis of the structure using the factored load combination $1.2D + 1.0E + 0.5L + 0.2S$ where $E = 2(Q_E + 0.2S_{DS}D)$. Thus, the following equation can be used to determine V_u :

$$V_u = (1.2 + 0.4S_{DS})V_D + 2.0V_{Q_E} + 0.5V_L + 0.2V_S \tag{13.1}$$

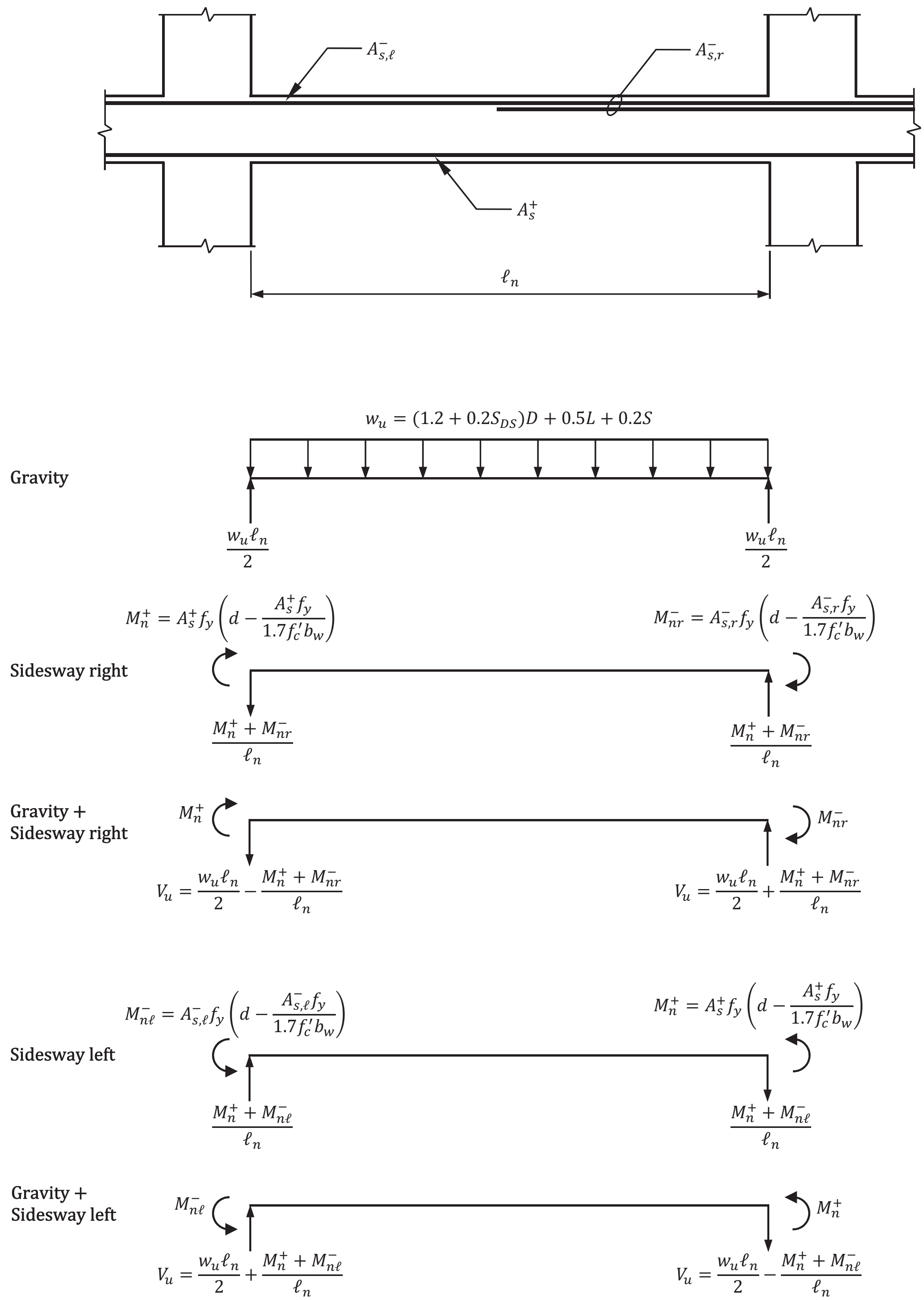


Figure 13.4 Required shear forces for beams in intermediate moment frames in accordance with ACI 18.4.2.3(a).

The lesser of the factored shear forces determined by ACI 18.4.2.3(a) and 18.4.2.3(b) is used in determining the required size and spacing of the shear reinforcement.

As noted previously, plastic hinges can form at the ends of a beam during a design-level seismic event and the concrete must be properly confined at these locations. Hoops must be provided within a distance of at least $2h$ from each end of a beam, which is the assumed length of the anticipated plastic hinge region. Hoops are defined as a closed tie or continuously wound tie made up of one or several reinforcement elements, each having seismic hooks at each end conforming to ACI 25.3.4 (ACI 25.7.4; see Figure 13.5). The ends of the reinforcement elements must engage a longitudinal bar in the beam and the extensions must project into the interior of the hoop. Hoops formed by stirrups with seismic hooks and crossties (Details B and C in Figure 13.5) are preferred over those formed by closed stirrups with seismic hooks (Detail A) because the longitudinal bars in the beam can be placed more easily and efficiently. Crossties must conform to ACI 25.3.5. Note that interlocking headed deformed bars are not permitted be a closed tie (ACI 25.7.4.2).

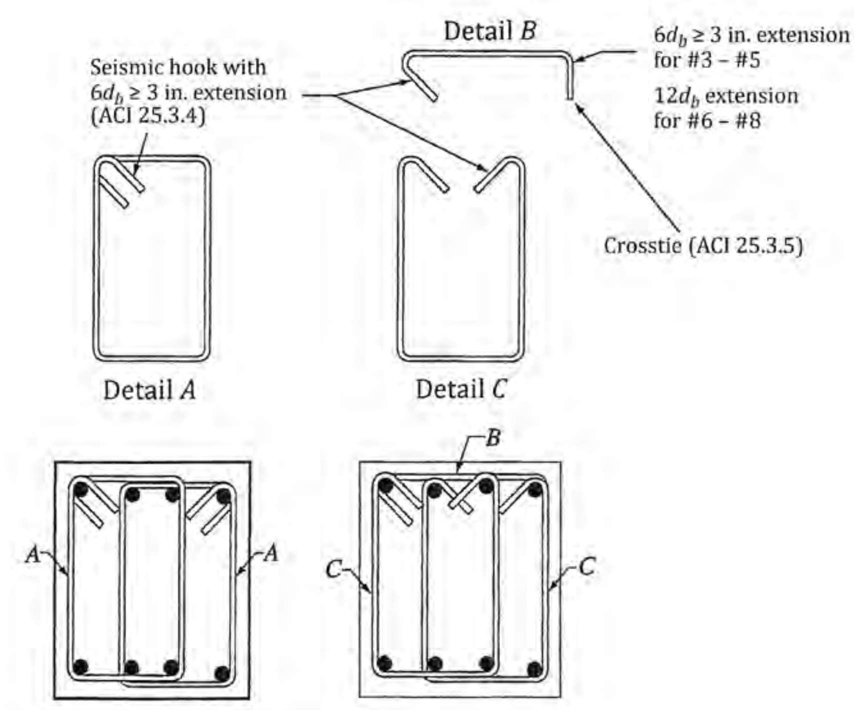


Figure 13.5 Examples of hoops and overlapping hoops.

The size and spacing of the hoops within $2h$ can be determined by the following equation (ACI 22.5.8.5.1):

$$\frac{A_v}{s} \geq \frac{(V_u - \phi V_c)}{\phi f_{yt} d} \tag{13.2}$$

where $\phi = 0.75$ for shear (see ACI Table 21.2.1) and V_c is determined by ACI 22.5.5.1. Where $A_v \geq A_{v,min}$, the following equation from ACI Table 22.5.5.1 can be used to determine V_c for beams with negligible axial force, N_u :

$$V_c = 2\lambda\sqrt{f'_c} b_w d \tag{13.3}$$

The modification factor that reflects the reduced mechanical properties of lightweight concrete to normalweight concrete of the same compressive strength, λ, is determined in accordance with ACI 19.2.4.

Once the size and spacing of the hoops are determined based on the governing V_u, the spacing must be checked against the maximum spacing requirements in ACI 18.4.2.4. The calculated hoop spacing, s, within $2h$ must be less than or equal to the smallest of the following (see Figure 13.6):

$$s \leq \text{smallest of} \begin{cases} d / 4 \\ 8 \times \text{diameter of the smallest longitudinal bar enclosed} \\ 24 \times \text{diameter of the hoop bar} \\ 12 \text{ in.} \end{cases} \tag{13.4}$$

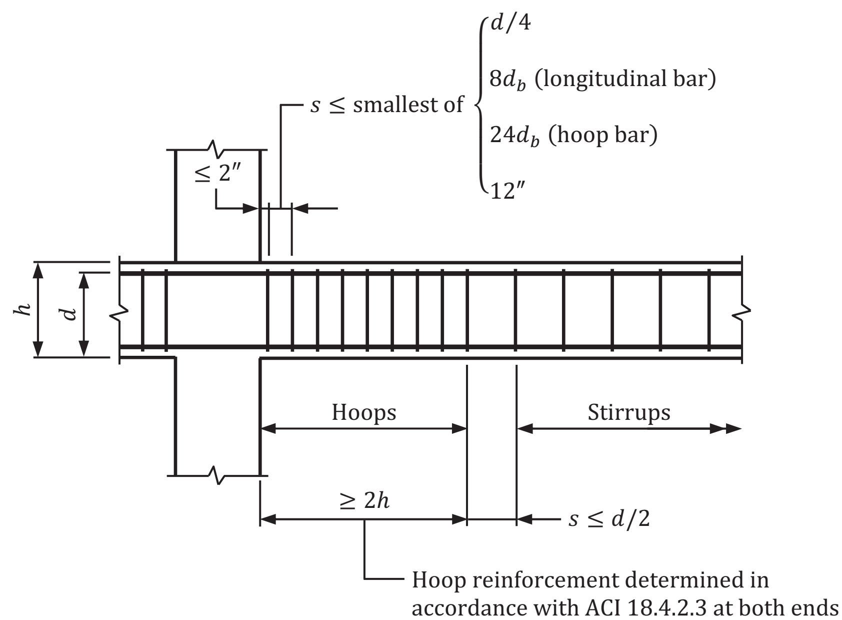


Figure 13.6 Transverse reinforcement requirements for beams in intermediate moment frames.

Outside of the anticipated plastic hinge regions, it is permitted to use stirrups spaced no greater than $d / 2$ on center (ACI 18.4.2.5). The factored shear force, V_u, is calculated at the location where the hoops are terminated and the size and spacing of the stirrups are determined using Equation (13.2).

For beams subjected to a factored axial compressive force greater than $A_g f'_c / 10$, transverse reinforcement conforming to the tie requirements in ACI 25.7.2.2 and either ACI 25.7.2.3 or 25.7.2.4 must be provided in the region of the beam outside of the anticipated plastic hinge regions (ACI 18.4.2.6). These additional requirements are intended to provide lateral support for the longitudinal reinforcement subjected to factored axial compressive forces greater than the prescribed limit.

13.3.3 Columns

Overview

The required column size and the required longitudinal reinforcement for columns in intermediate moment frames that are part of the SFRS in structures assigned to SDC C are determined using the provisions in ACI Chapter 10 for the combined effects due to gravity and lateral forces (see Chapter 7 of this publication). The shear strength requirements in ACI 18.4.3.1 and the detailing requirements in ACI 18.4.3.2 through 18.4.3.6 must be satisfied in addition to any relevant provisions in ACI Chapter 10.

Shear Strength Requirements

Plastic hinges can form at the ends of a column during a design-level seismic event and the concrete must be properly confined within these regions. Columns must be spirally reinforced in accordance with ACI Chapter 10 or transverse reinforcement in the form of hoops must be provided over a distance of ℓ_o from each end of a column where ℓ_o is the assumed length of the anticipated plastic hinge region (ACI 18.4.3.3):

$$\ell_o \geq \text{longest of} \begin{cases} \text{Clear span of the column}/6 \\ \\ \text{Maximum cross-sectional dimension of the column} \\ \\ \text{18 in.} \end{cases} \tag{13.5}$$

Shear strength requirements for columns in intermediate moment frames are given in ACI 18.4.3.1; these requirements are similar to those for beams in intermediate moment frames.

The design shear strength of a column, ϕV_n, must be greater than or equal to the required shear strength, V_u, which is equal to the lesser of the following:

- The shear forces associated with development of the nominal flexural strengths, M_{nt} and M_{nb}, at the top and bottom of the column due to reverse curvature bending. The nominal flexural strength is calculated for the factored axial force, consistent with the direction of analysis, resulting in the largest nominal flexural strength.
- The maximum shear force obtained from the factored load combinations that include earthquake effects, E, where $\Omega_o E$ is substituted for E.

The requirement in ACI 18.4.3.1(a) is illustrated in Figure 13.7. The nominal flexural strengths at the top and bottom of the column, M_{nt} and M_{nb}, are applied in reverse curvature bending and the factored shear force, V_u, is obtained from statics:

$$V_u = \frac{M_{nt} + M_{nb}}{\ell_u} \tag{13.6}$$

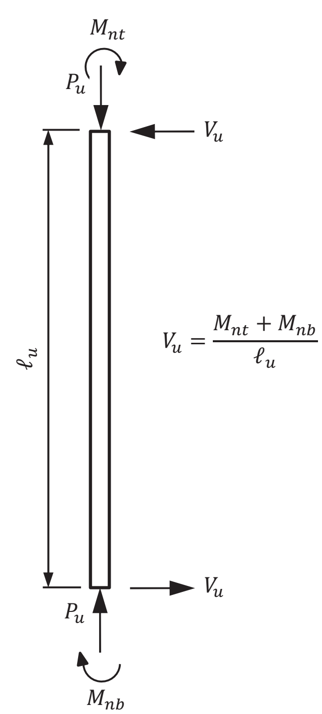


Figure 13.7 Required shear forces for columns in intermediate moment frames in accordance with ACI 18.4.3.1(a).

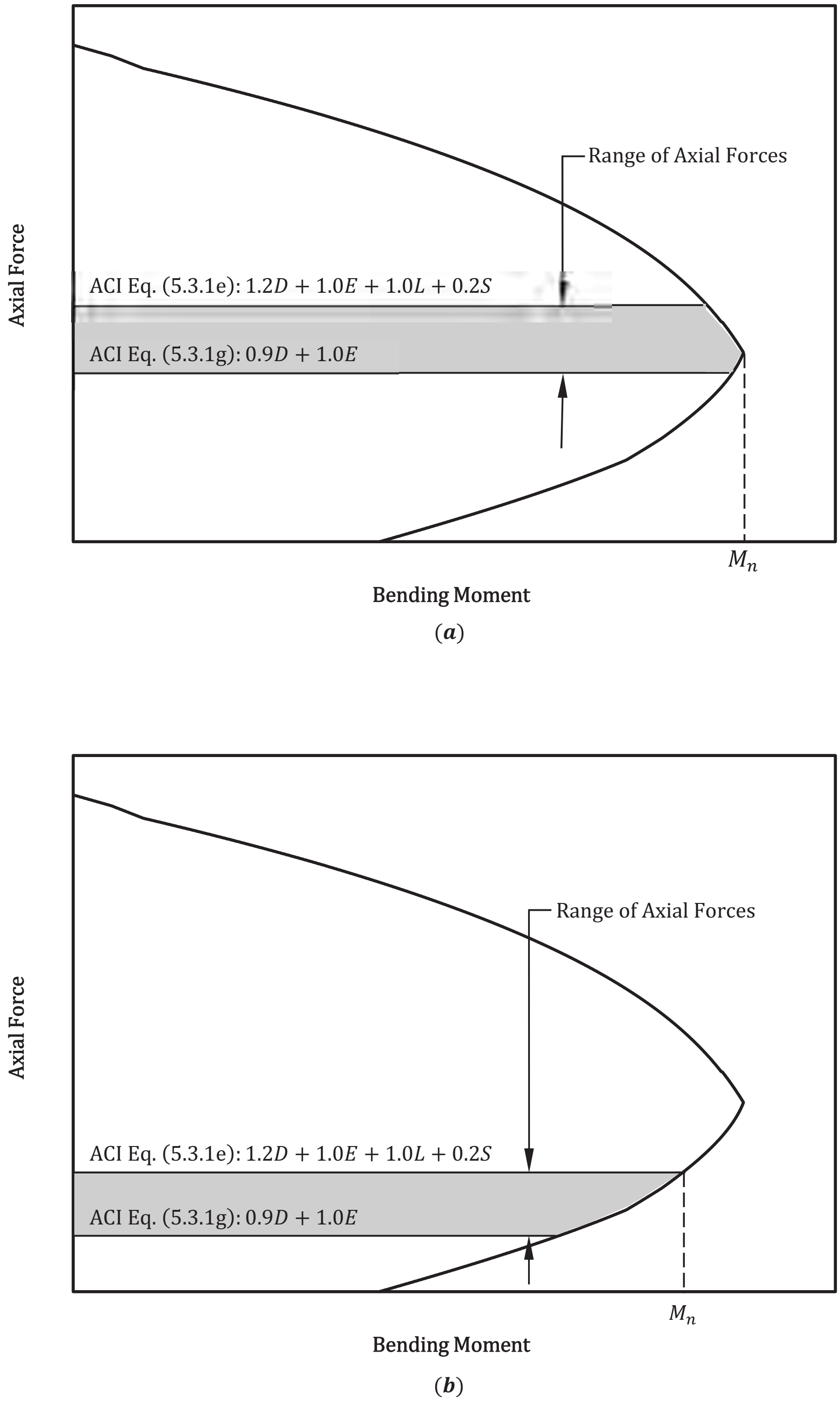


Figure 13.8 Nominal strength interactions diagrams for columns in intermediate moment frames.

When determining the nominal flexural strengths, the factored axial force, P_u, within the range of axial forces including earthquake effects must be selected that results in the largest nominal flexural strength. Nominal strength interaction diagrams for a column are given in Figure 13.8. For the range of axial forces shown in Figure 13.8(a), the largest M_n occurs at the balanced point even though the factored axial force corresponding to this nominal flexural strength is not obtained from either of the two applicable load combinations, which are ACI Equations (5.3.1e) and (5.3.1g). In this case, M_n at the balanced point must be used in the calculation of V_u. For the range of axial forces in Figure 13.8(b), the largest M_n is equal to the nominal flexural strength associated with the factored axial force from ACI Equation (5.3.1e) because that nominal flexural strength is greater than the nominal flexural strengths associated with any of the other factored axial forces in that range.

The requirement in ACI 18.4.3.1(b) is similar to that for beams in intermediate moment frames except instead of increasing E by a factor of 2, it is increased by the overstrength factor, Ω_o, which is equal to 3 for intermediate moment frames (see ASCE/SEI Table 12.2-1 or Table 12.3 in this publication). This higher factor recognizes the importance of columns compared to beams in regard to shear design.

The lesser of the factored shear forces from these two requirements is used in determining the required size and spacing of the shear reinforcement. In general, shear strength in the regions of flexural yielding is provided by both concrete (V_c) and transverse reinforcement (V_s). The size and spacing of the transverse reinforcement can be determined by the following equation (ACI 22.5.8.5.1):

$$\frac{A_v}{s} \geq \frac{(V_u - \phi V_c)}{\phi f_{yt} d} \tag{13.7}$$

where $\phi = 0.75$ for shear (see ACI Table 21.2.1) and V_c is determined by ACI 22.5.5.1. For rectilinear hoops, A_v is equal to the area of all hoop legs or all hoop plus crosstie legs in the direction of analysis within the longitudinal center-to-center spacing, s. For circular ties, A_v is equal to two times the area of the tie bar within s (ACI 22.5.8.5.6). Similarly, for spirals, A_v is equal to two times the area of the spiral bar within the spiral pitch, s.

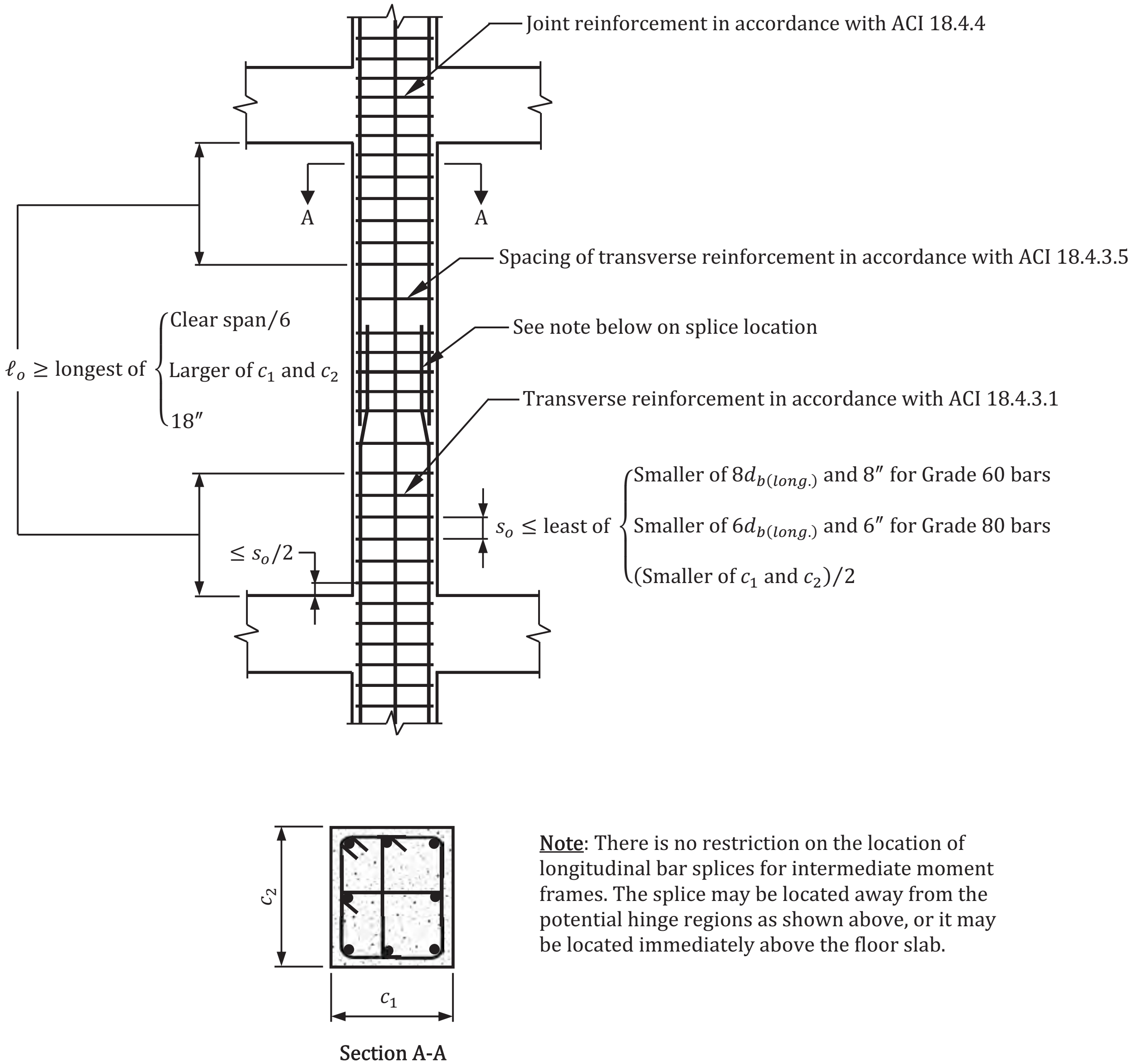
Once the size and spacing of the transverse reinforcement has been determined based on the governing V_u, the spacing must be checked against the maximum spacing requirements in ACI 18.4.3.3 for hoops and ACI 25.7.3.1 for spirals.

Within ℓ_o, the calculated hoop spacing based on V_u must be less than or equal to s_o (see Figure 13.9):

$$s_o \leq \text{least of} \begin{cases} \text{Smaller of } 8d_b \text{ of the smallest longitudinal bar enclosed and 8 in. for Grade 60 bars} \\ \text{Smaller of } 6d_b \text{ of the smallest longitudinal bar enclosed and 6 in. for Grade 80 bars} \\ \text{One-half the smallest cross-sectional dimension of the column} \end{cases} \tag{13.8}$$

For columns with spiral reinforcement, the calculated spiral pitch based on V_u must be less than or equal to $(d_{b,spiral} + 3.0 \text{ in.})$ where $d_{b,spiral}$ is the diameter of the spiral bar (ACI 25.7.3.1). The spiral pitch must be greater than or equal to the greater of $(d_{b,spiral} + 1.0 \text{ in.})$ and $[d_{b,spiral} + (4d_{agg} / 3)]$ where d_{agg} is the nominal maximum size of coarse aggregate in the concrete mix.

Outside of the anticipated plastic hinge regions, the spacing of the transverse reinforcement must conform to the lateral reinforcement provisions in ACI 25.7.2 for ties and ACI 25.7.3 for spirals, and to the maximum spacing requirements for shear reinforcement in ACI 10.7.6.5.2 (ACI 18.4.3.5). The least spacing obtained from these requirements must be used in the center region of a column.



Note: There is no restriction on the location of longitudinal bar splices for intermediate moment frames. The splice may be located away from the potential hinge regions as shown above, or it may be located immediately above the floor slab.

Figure 13.9 Transverse reinforcement requirements for columns with hoops in intermediate moment frames.

Columns Supporting Reactions from Discontinuous Stiff Members

Where a stiff member, such as a wall, is discontinued and supported on columns instead of on a foundation, the columns must contain transverse reinforcement at a spacing of s_o in accordance with ACI 18.4.3.3 for columns in intermediate moment frames over the full height at all levels beneath the level at which the discontinuity occurs where the factored axial compressive force related to earthquake effects in the columns exceeds $A_g f_c' / 4$ (ACI 18.4.3.6). The limit of $A_g f_c' / 4$ pertains to columns designed using load combinations that include the overstrength factor Ω_o, which are given in ASCE/SEI 2.3.6 (see Table 3.5 of this publication). Otherwise, the limit is $A_g f_c' / 10$. The transverse reinforcement must extend above and below the column in accordance with ACI 18.7.5.6(b), which is the requirement applicable to columns supporting discontinued stiff members in special moment frames (see Figure 13.10).

As noted previously, the required reinforcement details presented above must be provided even where the effects from wind loads govern the design of a column.

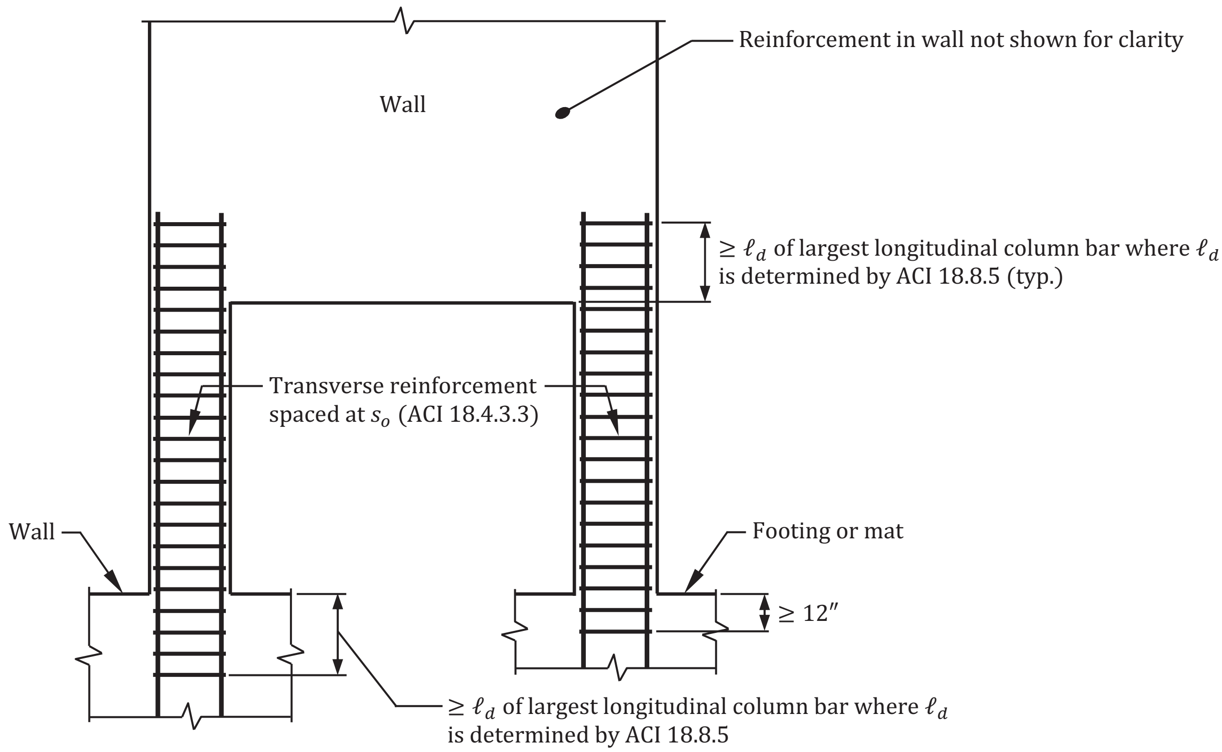


Figure 13.10 Transverse reinforcement requirements for columns supporting discontinuous stiff members in intermediate moment frames.

13.3.4 Joints

Beam-Column Joints

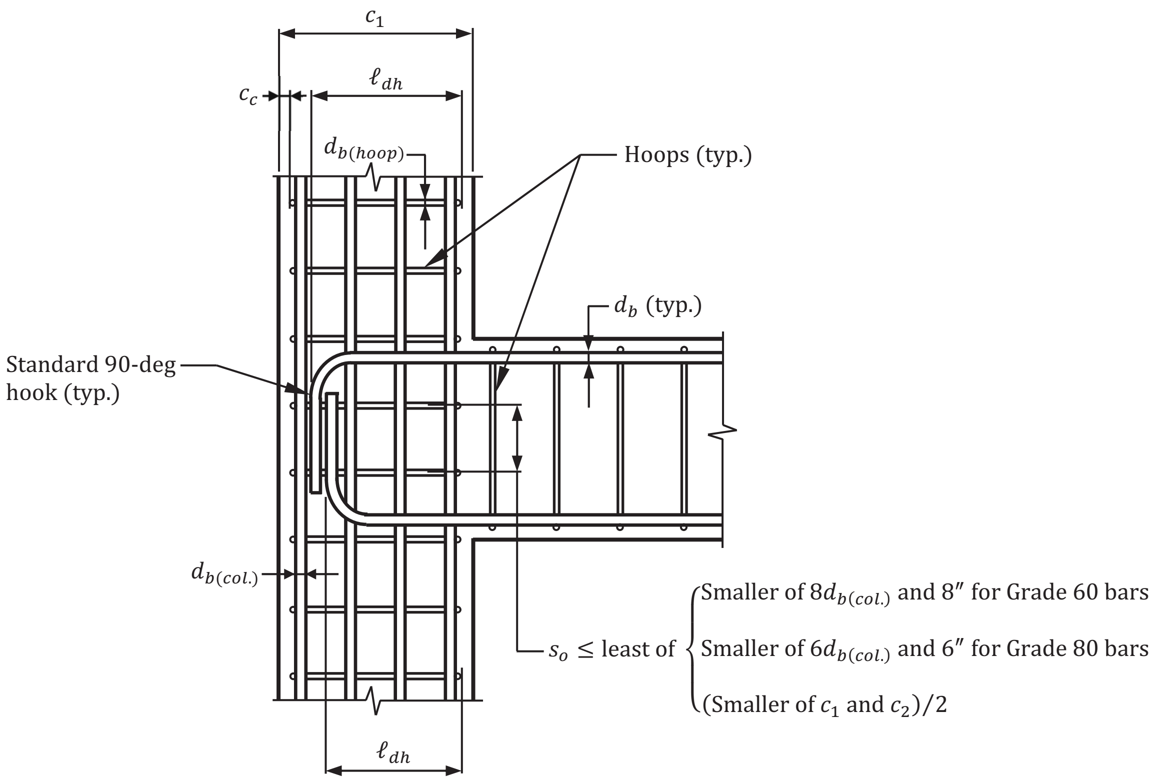
Beam-column joints in intermediate moment frames must satisfy the requirements in ACI 15.3.1.2, 15.3.1.3, and 18.4.4.2 through 18.4.4.5 (ACI 18.4.4.1). Shear strength requirements for beam-column joints are given in ACI 18.4.4.7.

The strut-and-tie method in ACI Chapter 23 must be used to analyze and design a joint where the depth of the beam framing into the joint generating the joint shear is more than twice the depth of the column in the direction of analysis (ACI 18.4.4.2). In addition to the requirements of ACI Chapter 23, the joint shear strength requirements of ACI 15.4.2 and the joint detailing requirements of ACI 18.4.4.3 through 18.4.4.5 must be satisfied.

Beam longitudinal reinforcement terminated in a joint must extend to the far face of the joint core and must be developed in tension in accordance with the tension development length requirements of ACI 18.8.5 for joints in special moment frames and in compression in accordance with ACI 25.4.9 (ACI 18.4.4.3).

An example of an edge or corner column where the longitudinal reinforcement from the beam is terminated in a standard 90-degree hook is given in Figure 13.11. In accordance with ACI 18.8.5.1, the hook must be located within the confined core of the column and it must be bent into the joint. The critical section for development of the hooked bars is taken at the outside edge of the joint transverse reinforcement and not at the face of column because the concrete outside of the confined core may spall during a design-level seismic event.

The development length of a deformed #3 through #11 reinforcing bar with a standard hook in tension, ℓ_{dh}, is determined by ACI Equation (18.8.5.1). The modification factor, λ, is equal to 0.75 for concrete with lightweight aggregate and 1.0 otherwise. The following equations can be used to determine ℓ_{dh} based on the requirements in ACI 18.8.5.1:



Normalweight concrete	$\ell_{dh} = \text{greater of} \begin{cases} f_y d_b / 65\sqrt{f'_c} \\ 8d_b \\ 6'' \end{cases}$
Lightweight concrete	$\ell_{dh} = \text{greater of} \begin{cases} f_y d_b / (65 \times 0.75)\sqrt{f'_c} \\ 10d_b \\ 7.5'' \end{cases}$
For epoxy-coated or zinc and epoxy dual-coated bars, multiply ℓ_{dh} by 1.2.	

Figure 13.11 Development of flexural reinforcement with standard hooks in beams of intermediate moment frames.

For normalweight concrete:

$$\ell_{dh} = \text{greater of} \begin{cases} f_y d_b \,/\, 65\sqrt{f'_c} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{13.9}$$

For lightweight concrete:

$$\ell_{dh} = \text{greater of} \begin{cases} f_y d_b / (65 \times 0.75)\sqrt{f_c'} \\ 10d_b \\ 7.5 \text{ in.} \end{cases} \tag{13.10}$$

These development lengths are measured from the outside edge of the joint transverse reinforcement to the outside face of the hook (see Figure 13.11).

In order for the hooked longitudinal bars to be fully developed in the joint for tension, the available development length must be greater than or equal to ℓ_{dh} :

$$\text{Available development length} = c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} \geq \ell_{dh} \tag{13.11}$$

In this equation, c_c is the clear cover to the hoop reinforcement in the joint, $d_{b(hoop)}$ is the diameter of the hoop reinforcement in the joint, and $d_{b(col.)}$ is the diameter of the longitudinal reinforcement in the column.

In lieu of hooked bars, #3 through #11 straight bars may be used provided the bars are properly developed in accordance with ACI 18.8.5.3 and 18.8.5.4. The required development lengths of #3 through #11 straight bars are multiples of the hooked bar development lengths in ACI 18.8.5.1: for top bars, the factor is 3.25 and for bottom bars, it is 2.5. Note that #14 and #18 bars are not included because of the lack of information pertaining to anchorage of these bar sizes when subjected to load reversals.

An example of an edge or corner column with headed deformed bars is given in Figure 13.12. Such bars are permitted to be used when the conditions of ACI 25.4.4.1 are satisfied:

(a) Bar must conform to ACI 20.2.1.6
(b) Bar size must be #11 or smaller
(c) Net bearing area of head, A_{brg}, must be at least $4A_b$ where A_b is the area of the bar
(d) Concrete must be normalweight
(e) Clear cover to the bar must be greater than or equal to $2d_b$ where d_b is the nominal diameter of the bar
(f) Center-to-center spacing of the bars must be greater than or equal to $3d_b$

According to ACI 18.8.5.2, the development length of a headed deformed reinforcing bar in tension, ℓ_{dt}, is given in ACI 25.4.4.2 where $1.25f_y$ is substituted for f_y :

$$\ell_{dt} = \text{greater of} \begin{cases} \left(\dfrac{1.25 f_y \psi_e \psi_p \psi_o \psi_c}{75\sqrt{f_c'}}\right) d_b^{1.5} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{13.12}$$

[Note: The development lengths of headed bars determined in accordance with ACI 18.8.5.2 are greater than the development lengths of hooked bars determined in accordance with ACI 18.8.5.1, which is not correct. ACI Committee 318 is aware of this and is working to modify the provisions in ACI 18.8.5.2 accordingly.]

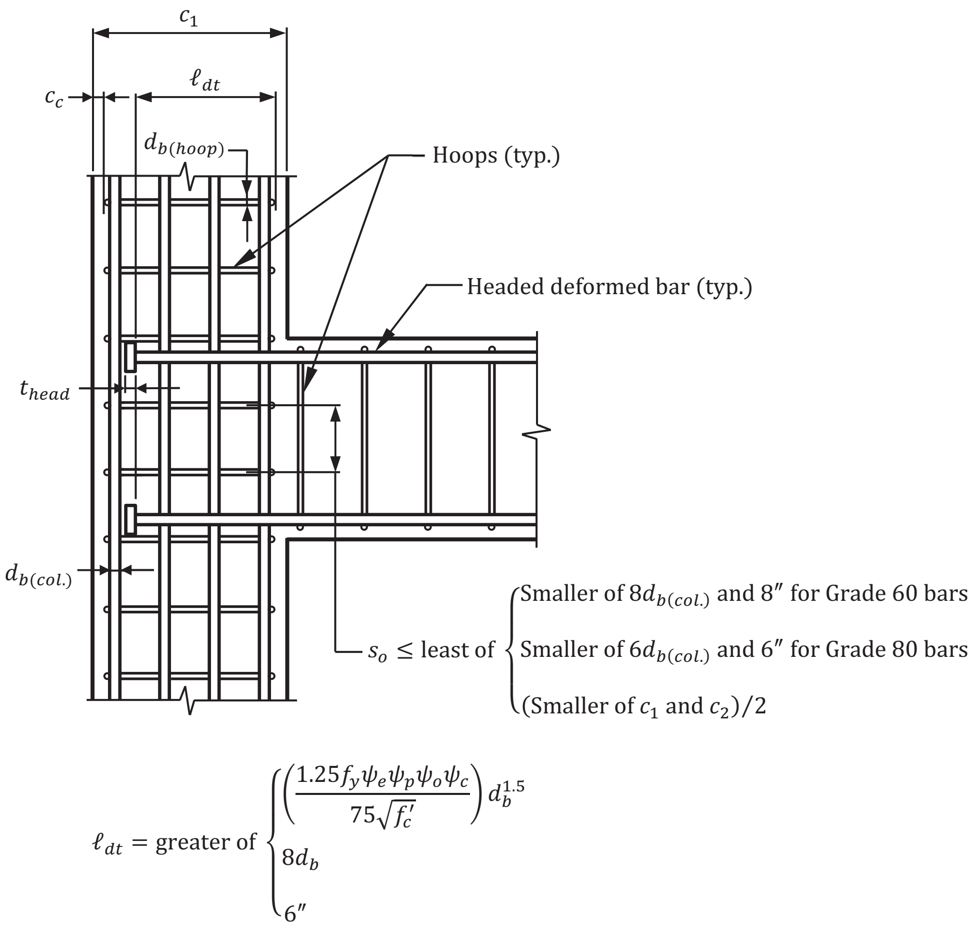


Figure 13.12 Development of flexural reinforcement with headed bars in beams of intermediate moment frames.

The modification factors in Equation (13.12) are given in Table 13.3 (see ACI Table 25.4.4.3).

Table 13.3 Modification Factors for Development of Headed Bars in Tension

Modification Factor	Condition	Value of Factor
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Parallel tie reinforcement, ψ_p	For #11 and smaller bars with $A_{tt} \geq 0.3A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For headed bars (1) terminating inside a column core with side cover to bar ≥ 2.5 in. or (2) with side cover to bar $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c / 15{,}000) + 0.6$
	$f'_c \geq 6{,}000$ psi	1.0

The term ψ_p accounts for confining effects provided by transverse reinforcement oriented parallel to the development length of headed bars. At beam-column joints, the total cross-sectional area of transverse reinforcement, A_{tt}, must be located within $8d_b$ of the centerline of the headed bar toward the middle of the joint where d_b is the nominal diameter of the headed bar (see ACI Figure R25.4.4.4). Where A_{tt} is greater than or equal to $0.3A_{hs}$ or where the center-to center spacing of the headed bars is greater than or equal to $6d_b$, $\psi_p = 1.0$. The term A_{hs} is the total cross-sectional area of the headed bars being developed.

In order for the headed longitudinal bars to be fully developed in the joint for tension, the available development length must be greater than or equal to ℓ_{dt}:

$$\text{Available development length} = c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} - t_{head} \geq \ell_{dt} \tag{13.13}$$

where t_{head} is the thickness of the head.

The development length, ℓ_{dc}, for deformed bars in compression is determined in accordance with ACI 25.4.9:

$$\ell_{dc} = \text{ greater of } \begin{cases} \left(\dfrac{f_y \psi_r}{50\lambda\sqrt{f_c'}}\right) d_b \\ 0.0003 f_y \psi_r d_b \\ 8 \text{ in.} \end{cases} \tag{13.14}$$

The modification factors in Equation (13.14) are given in Table 13.4 (see ACI Table 25.4.9.3).

Table 13.4 Modification Factors for Development of Deformed Bars in Compression

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Confining reinforcement, ψ_r	Reinforcement enclosed within the following: • A spiral • A circular continuously wound tie with $d_b \geq 1/4$ in. and pitch of 4 in. • #4 bar ties in accordance with ACI 25.7.2 spaced ≤ 4 in. on center • Hoops in accordance with ACI 25.7.4 spaced ≤ 4 in. on center	0.75
	Other	1.0

This development length corresponds to the straight portion of a hooked or headed bar measured from the critical section (in this case, the outside edge of the joint transverse reinforcement) to the onset of the bend for hooked bars and from the critical section to the head for headed bars. Therefore, in order for the hooked longitudinal bars to be fully developed in the joint for compression, the available development length must be greater than or equal to ℓ_{dc}:

$$\text{Available development length} = c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} - d_b - r \geq \ell_{dc} \tag{13.15}$$

In this equation, c_c is the clear cover to the hoop reinforcement in the column, $d_{b(hoop)}$ is the diameter of the hoop reinforcement in the column, $d_{b(col.)}$ is the diameter of the longitudinal reinforcement in the column, d_b is the diameter of the longitudinal reinforcement in the beam, and r is the bend radius of the beam longitudinal bar determined in accordance with ACI Table 25.3.1 for standard 90-degree hooks.

For headed bars to be fully developed in the joint for compression, the following equation must be satisfied:

$$\text{Available development length} = c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} - t_{head} \geq \ell_{dc} \tag{13.16}$$

According to ACI 18.4.4.4, the size and spacing of the required transverse reinforcement at the ends of a column in an intermediate moment frame must be carried through a joint within the height of the deepest beam framing into the joint (see Figures 13.11 and 13.12).

Headed bars used as negative reinforcement in beams terminated in a joint where there is no column above (such as, in the top story of a building) require confinement along the top face of the joint. Two confinement options are provided in ACI 18.4.4.5: (a) the column below the joint must be extended above the top of the joint a distance equal

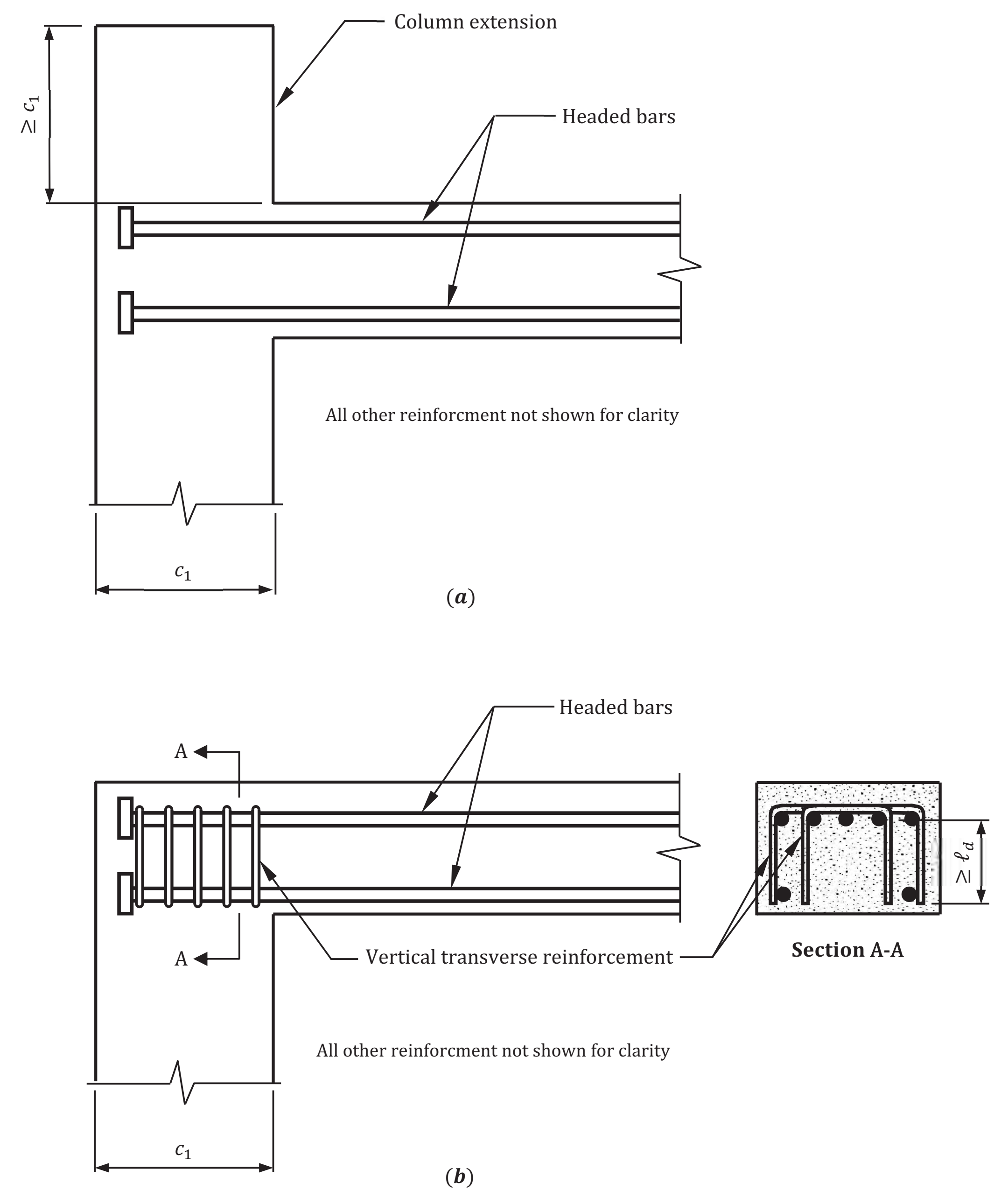


Figure 13.13 Confinement for headed bars in joints of intermediate moment frames. (a) Column extension. (b) Vertical transverse reinforcement.

to at least the depth of the joint in the direction of analysis [see Figure 13.13(a)] or (b) vertical joint reinforcement must be provided that hooks around the headed bars and extends downward into the joint in addition to the column longitudinal reinforcement [see Figure 13.13(b)]. Design recommendations for the vertical joint reinforcement are given in Reference 26.

Slab-Column Joints

According to ACI 18.4.4.6, slab-column joints in intermediate moment frames must satisfy the transverse reinforcement requirements of ACI 15.3.2. Column transverse reinforcement must be continued through any slab-column joint not laterally supported on four sides by the slab (that is, it must be continued through the joints at edge and corner columns). Such reinforcement must also be provided through column capitals, drop panels, and shear caps. Additionally, at least one layer of joint transverse reinforcement must be provided between the top and bottom layers of flexural reinforcement in the slab.

Shear Strength Requirements for Beam-Column Joints

The design shear strength of cast-in-place beam-column joints in intermediate moment frames must satisfy the following equation (ACI 18.4.4.7.1):

$$\phi V_n \geq V_u \tag{13.17}$$

The horizontal joint shear force, V_u, is determined using the requirements in ACI 18.3.4 for joints in ordinary moment frames (ACI 18.4.4.7.2), that is, V_u is determined at the mid-height of the joint in the direction of analysis from statics using tensile and compressive beam forces and column shear consistent with the negative and positive nominal flexural strengths of the beam at the face of the joint, M_n^- and M_n^+ (see Section 13.2.4 of this publication). The information in Section 11.4.1 of this publication can be used to calculate V_u for interior and edge/corner joints subjected to gravity and lateral forces. For convenience, equations for the shear force in the column, V_{col}, and V_u are given in Table 13.5 for sidesway to the left assuming the point of inflection is at the mid-height of the column, which is a reasonable assumption for columns above the first story and below the top story (see Figure 13.14). When calculating V_{col}, it is conservative to disregard the terms associated with the beam shear forces, V_1 and V_2. Similarly, it is conservative to disregard V_{col} when calculating V_u.

Table 13.5 Horizontal Joint Shear Forces for Interior, Edge, and Corner Columns

Shear Force	Column Location	
	Interior Columns and Edge Columns with the Direction of Analysis Parallel to the Edge	**Edge Columns with the Direction of Analysis Perpendicular to the Edge and Corner Columns**
V_{col}	$V_{col} = \dfrac{M_n^- + M_n^+}{\ell_c} + \dfrac{(V_1 + V_2) \times (c_1 / 2)}{\ell_c}$	$V_{col} = \dfrac{M_n^-}{\ell_c} + \dfrac{V_1 \times (c_1 / 2)}{\ell_c}$
V_u	$V_u = (A_s^- + A_s^+)f_y - V_{col}$	$V_u = A_s^- f_y - V_{col}$

In accordance with ACI Table 21.2.1, the strength reduction factor, ϕ, is equal to 0.75 for shear (ACI 18.4.4.7.3).

The nominal joint shear strength, V_n, in an intermediate moment frame is determined using the requirements of ACI 18.8.4.3 for joints in special moment frames (ACI 18.4.4.7.4). The continuity of the columns and beams framing into the joint and whether the joint is confined or not by transverse beams have an influence on V_n. Continuous columns and beams refer to members present on both sides of a joint. Alternatively, column and beam extensions are assumed to provide continuity through a beam-column joint provided the requirements in ACI 15.2.6 and 15.2.7 are satisfied, respectively (see Figures 11.1 and 11.2 of this publication). A joint is considered to be laterally confined where two transverse beams (that is, two beams framing into the column in the direction perpendicular to the direction of analysis) satisfy the three requirements in ACI 15.2.8 (see Figure 11.3 of this publication). The nominal joint shear strengths in ACI Table 18.8.4.3 are given in Table 13.6.

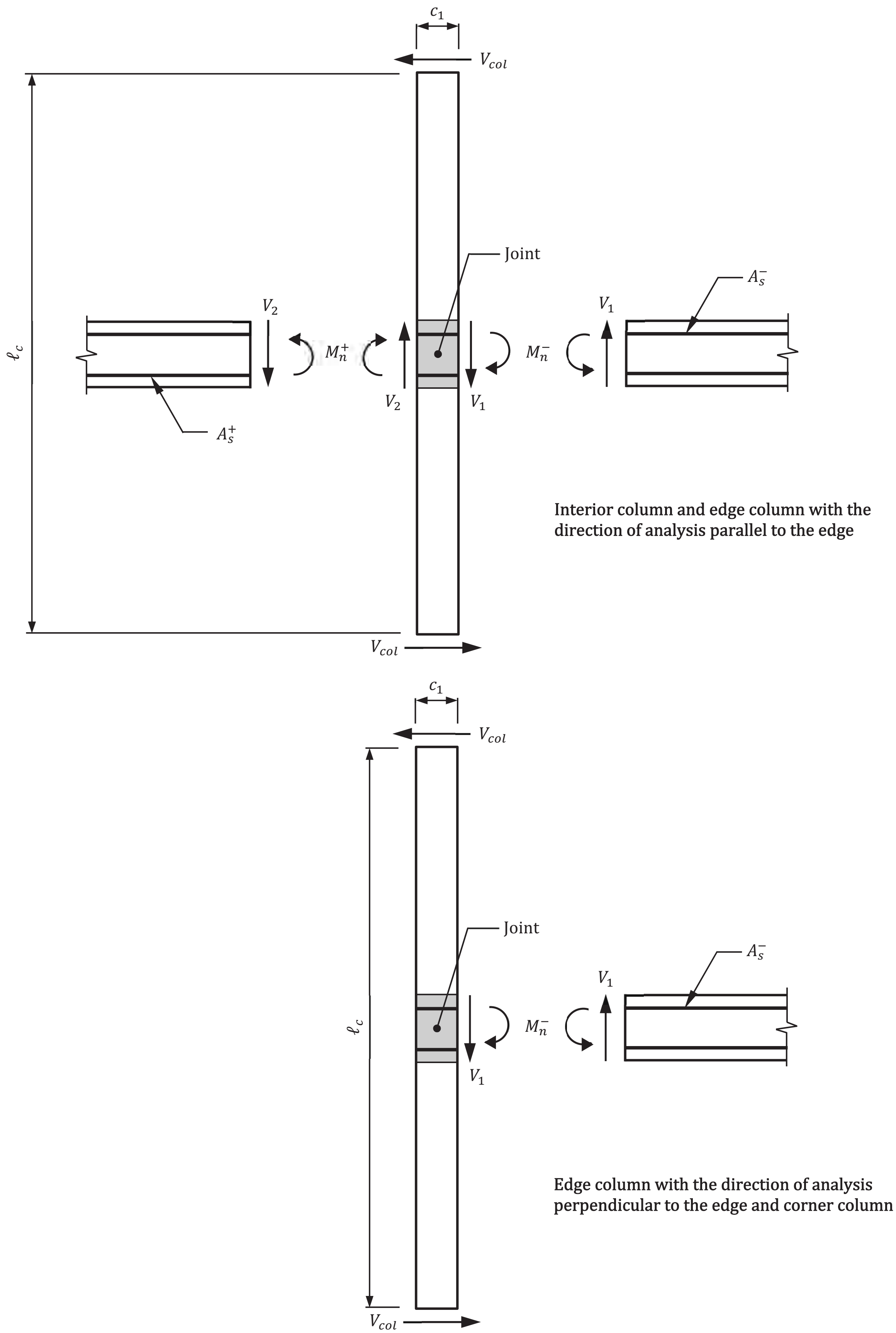


Figure 13.14 Free-body diagrams of columns in an intermediate moment frame subjected to gravity and lateral loads.

Table 13.6 Nominal Joint Shear Strength, V_n

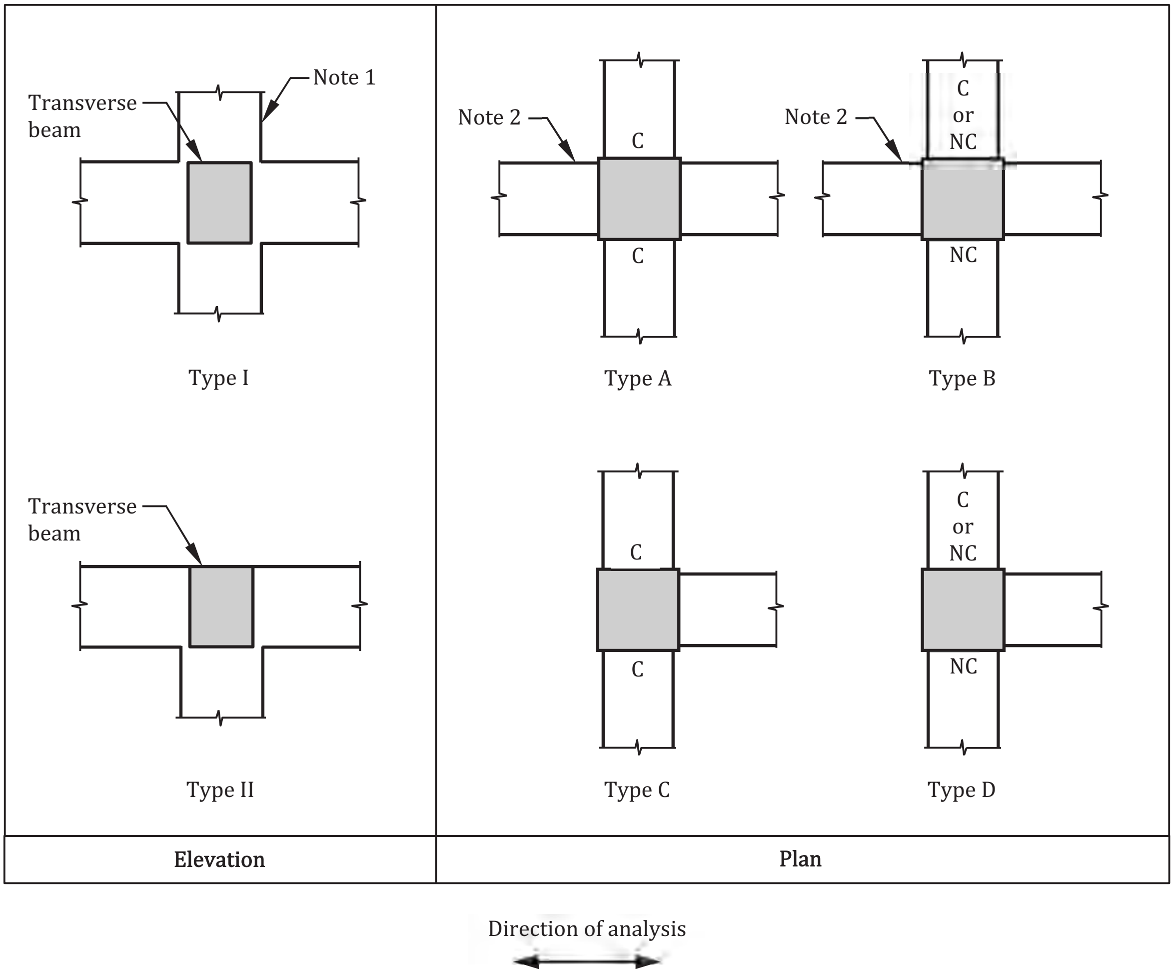
Column	Beam in Direction of Analysis	Confinement by Transverse Beams*	V_n (lbs)**
Continuous or a column extension is provided that satisfies ACI 15.2.6	Continuous or a beam extension is provided that satisfies ACI 15.2.7	Confined	$20\lambda\sqrt{f'_c}A_j$
		Not confined	$15\lambda\sqrt{f'_c}A_j$
	Other	Confined	$15\lambda\sqrt{f'_c}A_j$
		Not confined	$12\lambda\sqrt{f'_c}A_j$
Other	Continuous or a beam extension is provided that satisfies ACI 15.2.7	Confined	$15\lambda\sqrt{f'_c}A_j$
		Not confined	$12\lambda\sqrt{f'_c}A_j$
	Other	Confined	$12\lambda\sqrt{f'_c}A_j$
		Not confined	$8\lambda\sqrt{f'_c}A_j$

**Transverse beams that satisfy the requirements of ACI 15.2.8 are considered to provide confinement (see Figure 11.3 of this publication). Examples of the various joint types in this table are given in Figure 13.15.*
***The modification factor, λ, that reflects the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength is equal to 0.75 for lightweight concrete and 1.0 for normalweight concrete.*

Examples of the various joint types in Table 13.6 and the corresponding V_n are given in Figure 13.15. Joints with continuous columns (that is, columns frame into the top and bottom of the joint) or with a column extension that satisfies the requirements of ACI 15.2.6 are designated Type I. Joints with noncontinuous columns (that is, joints where columns frame into the bottom of the joints only, such as those in the top story of a building), are designated Type II. Type A and Type B designations in Figure 13.15 correspond to joints with continuous beams in the direction of analysis or with beam extensions that satisfy the requirements in ACI 15.2.7. Type C and Type D designations correspond to joints with noncontinuous beams in the direction of analysis. A designation of "C" at the face of a joint means a transverse beam provides confinement at that face in accordance with ACI 15.2.8. An "NC" designation means that no transverse beam is present at that face or a transverse beam is present but does not satisfy the confinement requirements in ACI 15.2.8.

The nominal shear strengths, V_n, can be obtained from Figure 13.15 for the various combinations of joint types. For example, a Type I-A joint consists of a continuous column and continuous beams in the direction of analysis with confinement provided by transverse beams on both faces; in this case, $V_n = 20\lambda\sqrt{f'_c}A_j$. This joint type corresponds to an interior column located in the structure other than the top story. Similarly, a corner column in the top story of a structure corresponds to a Type II-D joint, and $V_n = 8\lambda\sqrt{f'_c}A_j$.

The term A_j is the effective cross-sectional area within a joint, which is given in ACI 15.4.2.4. By definition, A_j is equal to the product of the joint depth and the effective joint width. The depth of the joint is always equal to the depth of the column parallel to the direction of analysis, c_1 (see Figure 11.13 of this publication). The effective joint width, w, is equal to the width of the column perpendicular to the direction of analysis where the beams in the direction of analysis are as wide as or wider than the column. In such cases, $A_j = c_1 \times c_2$. For beams not as wide as the column, w is equal to the lesser of $b + c_1$ and $b + 2x$ where b is the beam width and x is the perpendicular distance from the edge of the beam to the nearest side face of the column.



Joint Type	V_n
I-A	$20\lambda\sqrt{f_c'}A_j$
I-B	$15\lambda\sqrt{f_c'}A_j$
I-C	$15\lambda\sqrt{f_c'}A_j$
I-D	$12\lambda\sqrt{f_c'}A_j$
II-A	$15\lambda\sqrt{f_c'}A_j$
II-B	$12\lambda\sqrt{f_c'}A_j$
II-C	$12\lambda\sqrt{f_c'}A_j$
II-D	$8\lambda\sqrt{f_c'}A_j$

Notes

1. Column is continuous or a column extension satisfying ACI 15.2.6 is provided (see Figure 11.1 of this publication).
2. Beam in direction of analysis is continuous or a beam extension satisfying ACI 15.2.7 is provided (see Figure 11.2 of this publication).
3. C – Transverse beam provides confinement in accordance with ACI 15.2.8 (see Figure 11.3 of this publication).
4. NC – Transverse beam is not provided or transverse beam does not provide confinement in accordance with ACI 15.2.8 (see Figure 11.3 of this publication).

Figure 13.15 Examples of joint types and the corresponding nominal joint shear strength, V_n.

13.3.5 Two-way Slabs Without Beams

Overview

Flat plate systems, which are two-way slabs without beams, are permitted to be part of the SFRS in intermediate moment frames. As noted in Section 5.2.2 of this publication, the thickness of a flat plate system is usually determined first on the basis of the appropriate deflection requirements. Two-way shear requirements may control the thickness of the slab instead of serviceability requirements. Headed shear stud reinforcement provides an economical way to increase the two-way shear resistance of a slab.

Analysis Methods

Flat plate floor and roof systems can be modeled for lateral load resistance using any method satisfying both equilibrium and compatibility. It is common to assume the intermediate moment frames consist of column strips and columns. This essentially means all the lateral load effects get assigned to the column strips. In lieu of that method, an effective beam width can be determined that is a fraction of the transverse width of the slab based on a model that produces results in reasonable agreement with test data.

The combined effects due to gravity and lateral load effects are obtained using the strength design load combinations in Table 13.2. It is permitted to combine the results of the gravity load analysis with the results from the lateral load analysis (ACI 8.4.1.9). In other words, the slab can be analyzed for gravity loads using any rational method, including the Direct Design Method in Section 5.3.4 of this publication (where applicable), and the results from that method can be combined with the results from the lateral load analysis assuming, for example, the column strips alone resist the lateral load effects.

Required Flexural Reinforcement

Once the thickness of the slab has been determined on the basis of deflection and two-way shear criteria and the slab has been analyzed for the effects due to gravity and lateral loads, the required flexural reinforcement can be determined in the column strips and middle strips using $\phi M_n = M_u$ (see Section 5.4.2 of this publication).

As in the case for slab-column joints in buildings assigned to SDC A and B, special flexural requirements must be satisfied at slab-column joints in buildings assigned to SDC C where intermediate moment frames consisting of two-way slabs without beams are utilized. It is assumed the slab moment resisted by the column at a slab-column joint, M_{sc}, due to gravity and earthquake effects is transferred by a combination of flexure and eccentricity of shear (ACI 18.4.5.1). The portion of M_{sc} transferred by flexure is equal to $\gamma_f M_{sc}$ where γ_f is determined by ACI Equation (8.4.2.2.2):

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} \tag{13.18}$$

In this equation, b_1 and b_2 are the dimensions of the critical section for shear measured parallel and perpendicular to the direction of analysis, respectively.

The requirements for the effective slab width resisting $\gamma_f M_{sc}$ at edge and corner columns for intermediate moment frames are given in ACI 18.4.5.2 and are illustrated in ACI Figure R18.4.5.1 (see Figure 13.16). Flexural reinforcement perpendicular to the edge is not considered fully effective to resist $\gamma_f M_{sc}$ unless it is placed within the effective slab widths indicated in Figure 13.16. The portion of the effective slab width extending beyond the face of the column is the lesser of $1.5h$ or c_t where c_t is the distance from the interior face of the column to the slab edge in the direction of analysis. The definition of the effective slab width is based on yield lines that extend from the interior corners or corner of the column to the edge of the slab at a 45-degree angle. The effective slab widths for interior columns and columns bending parallel to the edge are the same as those for ordinary moment frames, which are given in ACI Table 8.4.2.2.3. The amount of flexural reinforcement required to resist $\gamma_f M_{sc}$ must be placed within the effective width defined in Figure 13.16 and must be greater than or equal to one-half of the reinforcement in the column strip at the support (ACI 18.4.5.3). Additional flexural reinforcement may be required in the slab to satisfy this strength requirement.

It is possible for a reversal of moments to occur at the joints. In such cases, bottom reinforcement is required in addition to the top reinforcement within the effective slab width.

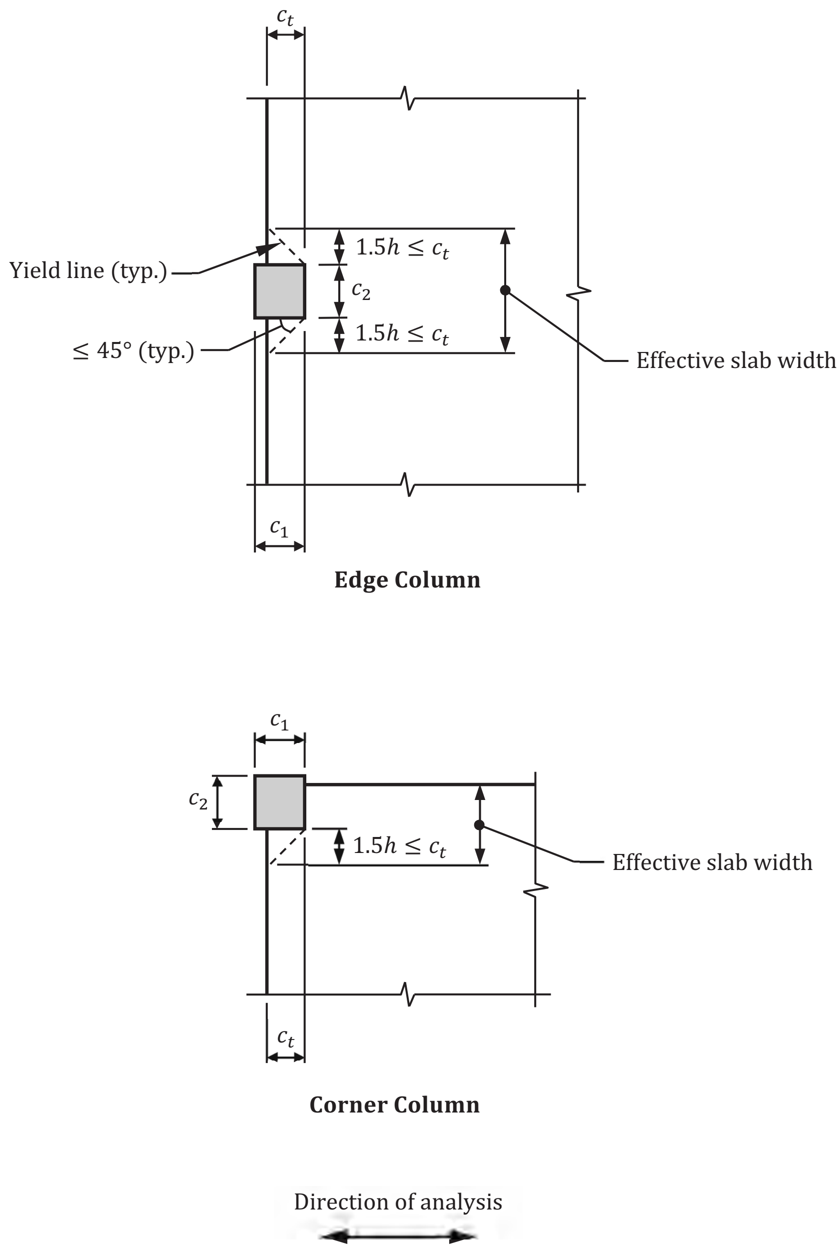


Figure 13.16 Effective slab width in edge and corner connections of intermediate moment frames consisting of two-way slabs without beams.

Detailing the Flexural Reinforcement

The detailing requirements in ACI 8.7.1 through 8.7.4 for flexural reinforcement must be satisfied for all two-way slab systems, including those in buildings assigned to SDC C (see Section 5.6 of this publication). For two-way slabs that are part of an intermediate moment frame, the detailing provisions of ACI 18.4.5.3 through 18.4.5.7 must also be satisfied.

As noted above, at least one-half of the reinforcement provided in the column strip at the support must be placed within the effective slab width given in ACI Table 8.4.2.2.3 (ACI 18.4.5.3). This requirement is illustrated in Figure 13.17.

The detailing requirements in ACI 18.4.5.4 through 18.4.5.7 are given in Figure 13.18.

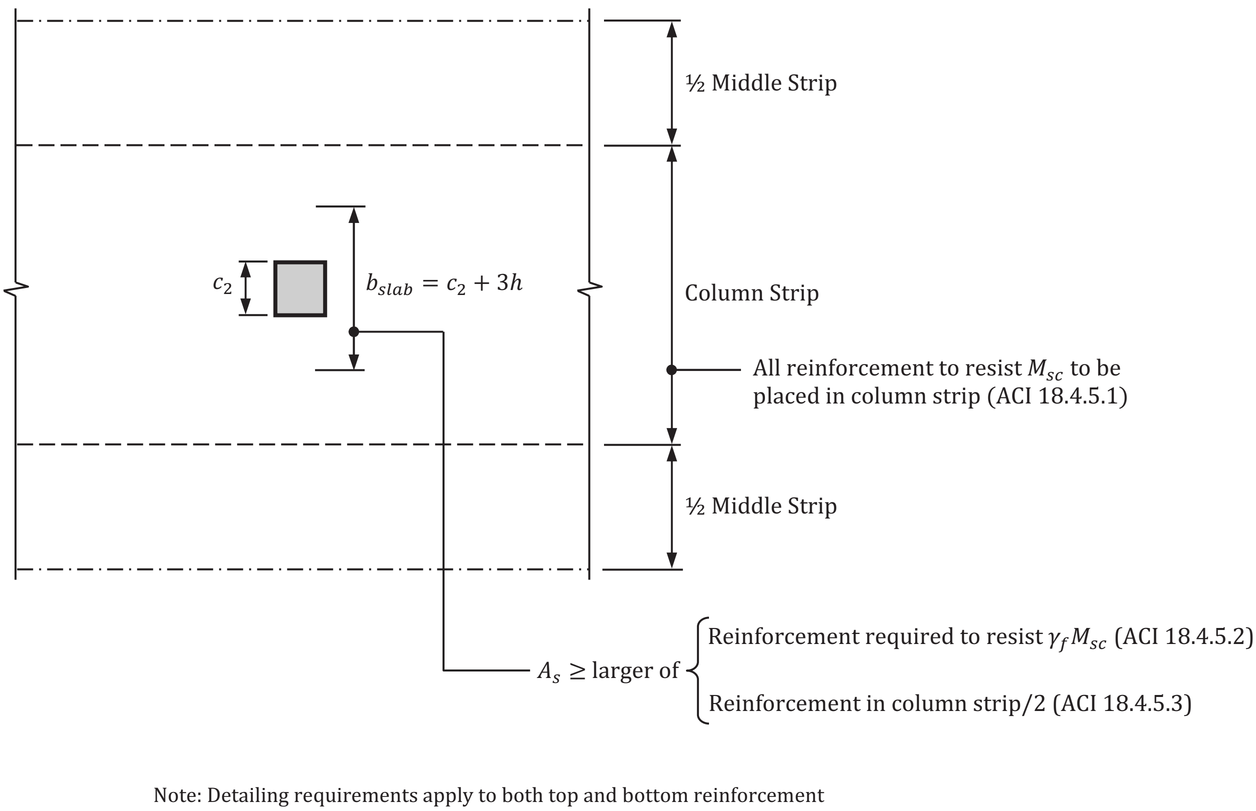


Figure 13.17 Reinforcement details at supports of two-way slabs without beams.

Shear Strength Requirements

The one-way and two-way shear requirements in ACI 8.4.3 and 8.4.4, respectively, must be satisfied for shear design in two-way slabs (see Chapter 5 of this publication).

For two-way systems without column-line beams, it has been shown that slab-column connections are susceptible to punching shear failure and reduced lateral displacement ductility during seismic events where shear stresses due to gravity loads are relatively large. A limit of $0.4\phi v_c$ on shear caused by factored gravity loads without moment transfer is prescribed in ACI 18.4.5.8 where the nominal two-way shear strength of the concrete, v_c, is determined in accordance with ACI 22.6.5. This requirement need not be satisfied where the provisions of ACI 18.14.5 are satisfied, which are applicable to slab-column connections in frames that are not part of the SFRS.

13.4 Foundations

No design and detailing requirements other than those in ACI Chapter 13 are needed for foundations supporting structures assigned to SDC B. The information in Chapter 10 of this publication can be used in such cases.

Design and detailing requirements for piles, pile caps, and deep foundations (including drilled piers) supporting structures assigned to SDC C are given in ACI 18.13 (there are no requirements in ACI 18.13 for spread footings supporting buildings assigned to SDC C). Requirements for slabs-on-ground resisting in-plane earthquake forces from walls or columns that are part of the SFRS are given in ACI 18.13.3.2. The requirements that must be satisfied for drilled piers (caissons) are given in Table 13.7.

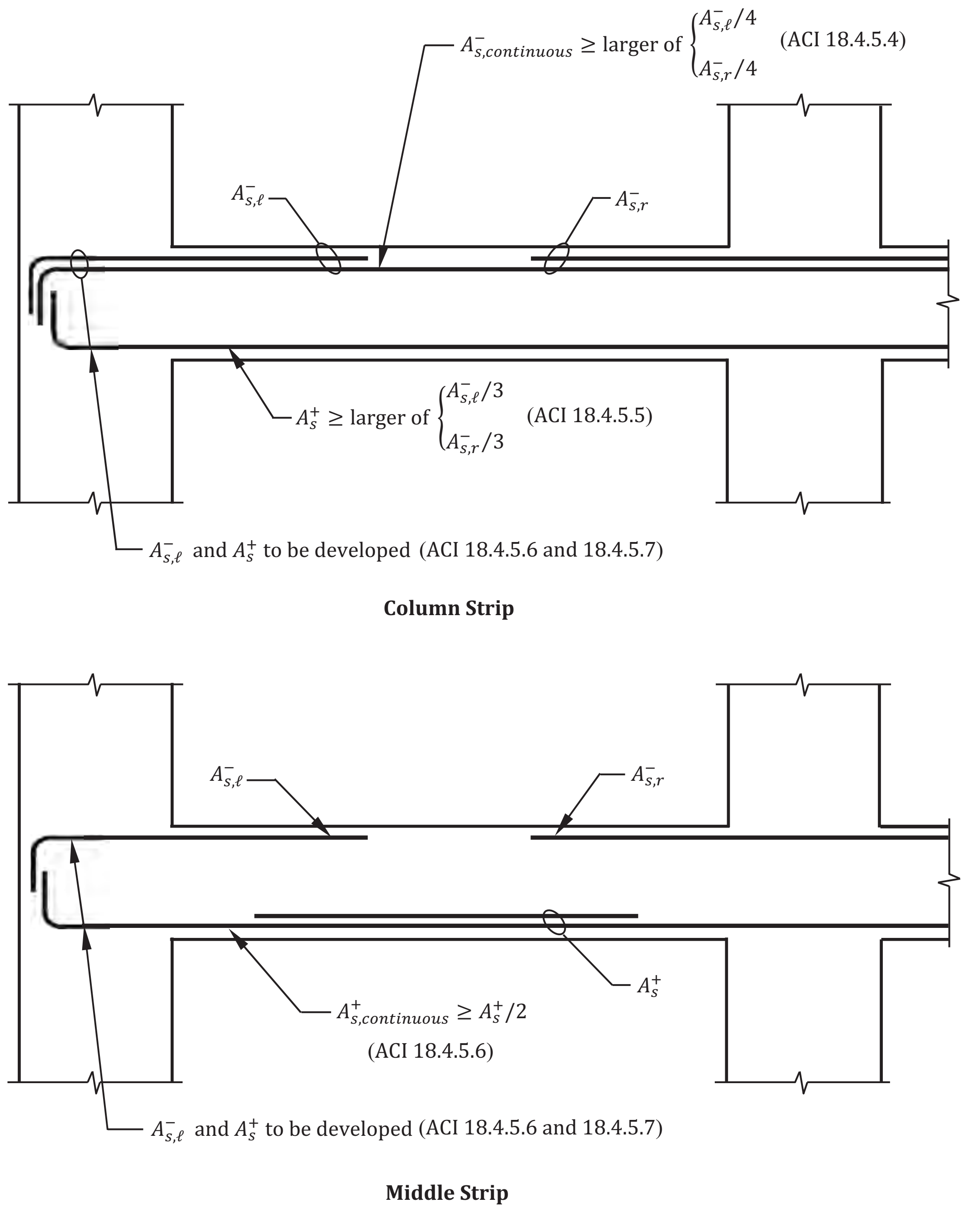


Figure 13.18 Reinforcement details for two-way slabs without beams in intermediate moment frames.

Table 13.7 Requirements for Drilled Piers Supporting Structures Assigned to SDC C

Requirement	ACI Section No.
Caissons must be interconnected by foundation seismic ties in orthogonal directions, unless it can be demonstrated that equivalent restraint is provided by other means.	18.13.4.1
Foundation seismic ties must have a design strength in tension and compression at least equal to $0.1S_{DS}$ times the greater of (1) the factored dead load plus factored live load supported by the pile cap or (2) the factored dead load plus factored live load supported by the column. This requirement is permitted to be waived if it can be demonstrated that another means of equivalent restraint will be provided.	18.13.4.3
Caissons resisting design tension forces must have continuous longitudinal reinforcement over their length.	18.13.5.2
Minimum longitudinal and transverse reinforcement required in ACI 18.13.5.7 must be extended over the entire unsupported length of the portion of a caisson in air or water, or in soil not capable of providing adequate lateral restraint to prevent buckling throughout this length.	18.13.5.3
Hoops, spirals, and ties in caissons must be terminated with seismic hooks.	18.13.5.4
Longitudinal and transverse reinforcement in uncased cast-in-place caissons must be provided in accordance with ACI Table 18.13.5.7.1 and with ACI 18.13.5.7.2 and 18.13.5.7.3.	18.13.5.7
Longitudinal reinforcement in caissons resisting tension forces must be detailed to transfer the tension forces within the pile cap to the supported structural members.	18.13.6.1
Caisson longitudinal reinforcement must be embedded in the pile cap a distance equal to the development length in compression or tension. Alternatively, it may be embedded by the use of field-placed dowels anchored in the caisson.	18.13.6.2

The purpose of the foundation seismic ties specified in ACI 18.13.4.1 is to minimize the relative movement of the supported member relative to the movement of the caisson. A grade beam can be used as a seismic tie (see Figure 13.19).

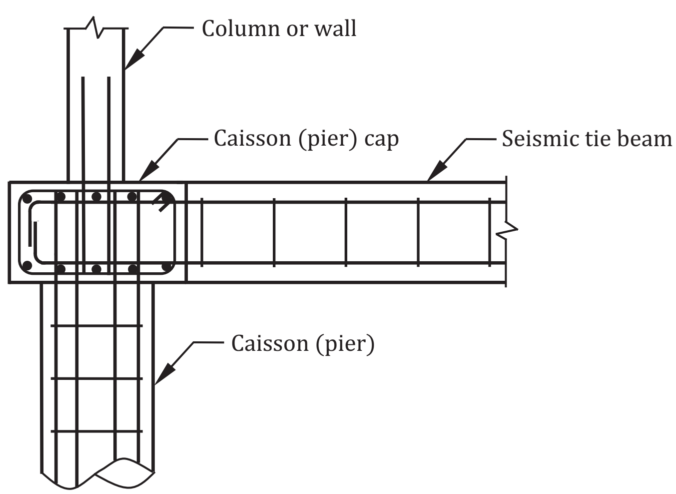


Figure 13.19 Foundation tie for caissons supporting structures in SDC C.

The foundation seismic tie design strength specified in ACI 18.13.4.3 is meant to provide a minimum connection between adjacent caissons. The required design strength for the tie beam must be at least $0.1S_{DS}$ times the larger force from the column or pile cap, whichever applies, on either end of the tie beam. Longitudinal reinforcement in the tie beam, which is designed for the applicable factored tension and compression forces, must be anchored in the caisson (see Figure 13.19).

Requirements for uncased cast-in-place or augered concrete piles or piers are illustrated in Figure 13.20 for a circular pile or pier with spiral reinforcement where it is assumed a hard or stiff layer of soil is present over the entire length of the pile or pier.

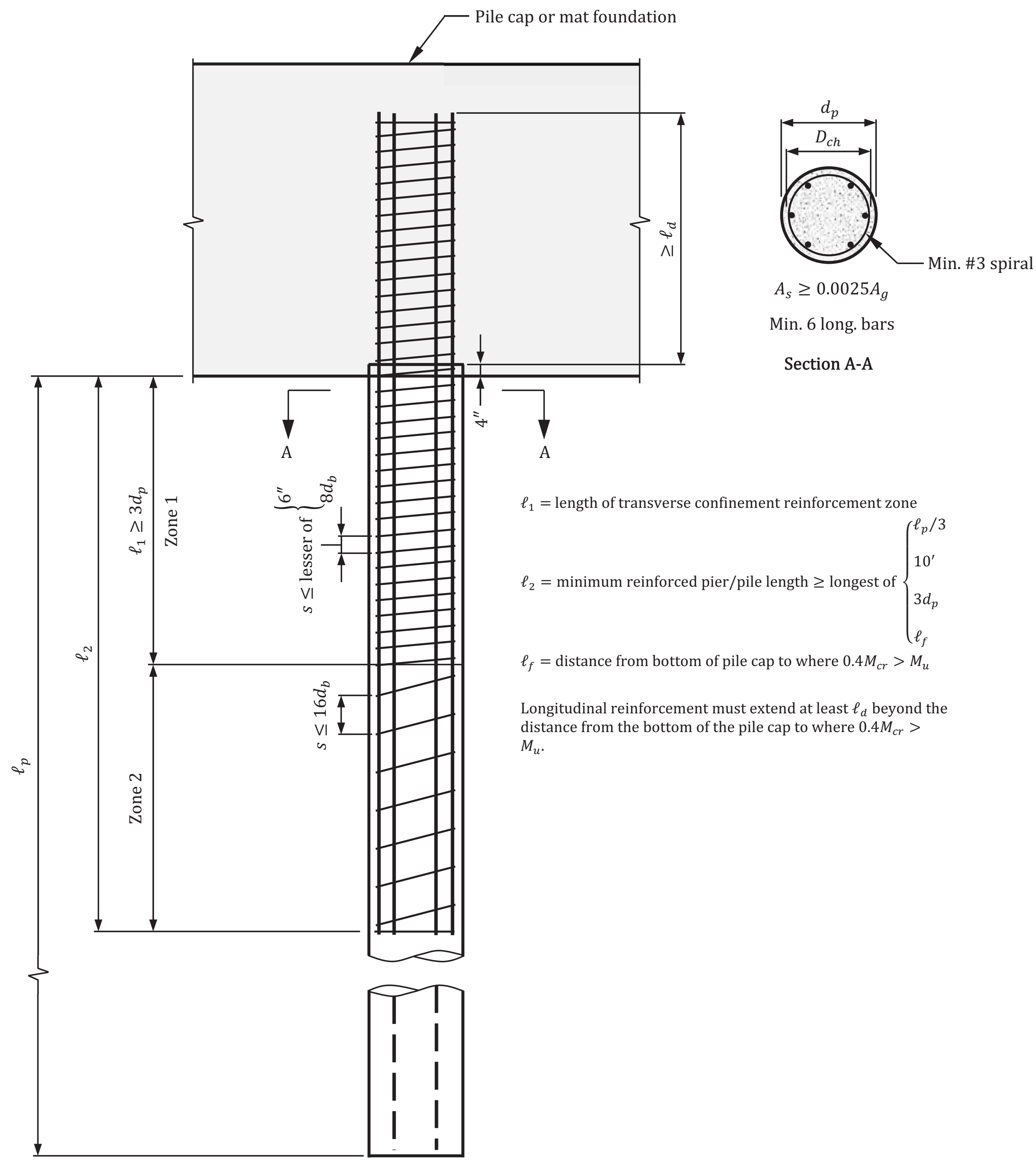


Figure 13.20 Requirements for uncased cast-in-place or augered concrete piles or piers supporting structures assigned to SDC C – Hard or stiff soil over entire length.

In the figure, ℓ_1 is the length of the required transverse confinement reinforcement zone and ℓ_2 is the minimum length where reinforcement is required. These lengths are defined in ACI Table 18.13.5.7.1 and are the same for any Site Class. Zone 1 in Figure 13.20 corresponds to the segment where minimum transverse confinement reinforcement is required over the length ℓ_1. Zone 2 corresponds to the segment where minimum transverse reinforcement is required in the remainder of the required reinforcement length (that is, in the region outside of Zone 1 where the length is equal to $\ell_2 - \ell_1$). The dashed longitudinal reinforcement corresponds to the case where the pile/pier must resist design tension forces (ACI 18.13.5.2).

Where a portion of a pile or pier is in air or water or is in soil not capable of providing adequate lateral restrain to prevent buckling, Zone 1 longitudinal and transverse reinforcement must be provided over the entire unsupported length (ACI 18.13.5.3).

The purpose of the requirements in ACI 18.13.6.1 and 18.13.6.2 is to ensure that a complete load path is present that transfers the force from the longitudinal reinforcement in the supported member through the pile (caisson) cap to the longitudinal reinforcement in the caisson.

13.5 Examples

13.5.1 Example 13.1 – Determination of Flexural Reinforcement: Beam in Building #1 (Framing Option B), Beam is Part of the SFRS (Intermediate Moment Frame), SDC C

Determine the required flexural reinforcement for the beam along column line 1 between column lines C and D in Building #1, Framing Option B, at the second-floor level assuming the beam is part of the SFRS and the dimensions of the beam are 28 in. by 24 in. (see Figure 1.1). The slab is 8.5 in. thick and the columns are 24.0 in. by 24.0 in. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. It is determined in Example 3.5, Part (c) that the building is assigned to SDC C where $S_{DS} = 0.192$.

Step 1 – Determine the factored bending moments along the span

A summary of the service bending moments due to the dead and live loads at the critical sections of the beam is given in Table 13.8.

Table 13.8 Bending Moments (ft-kips) due to Service Dead and Live Loads at the Second-Floor Level for the Beam in Example 13.1

Negative – Line C	Positive	Negative – Line D
$M_D^- = -81.3$	$M_D^+ = 55.8$	$M_D^- = -84.0$
$M_L^- = -31.9$	$M_L^+ = 22.3$	$M_L^- = -33.3$

The bending moments in the beam due to earthquake loads are given in Table 13.9. The "plus-minus" sign preceding the tabulated values signifies the earthquake loads can act in both the north direction and the south direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the east elevation of the building). The bending moments due to wind loads are smaller than those due to the earthquake loads and are not considered in this example.

Table 13.9 Bending Moments (ft-kips) due to Earthquake Loads at the Second-Floor Level for the Beam in Example 13.1

Negative – Line C	Positive	Negative – Line D
± 220.8	—	± 220.0

The design bending moments from the governing load combinations are given in Table 13.10 (see Table 13.2).

Table 13.10 Design Bending Moments (ft-kips) at the Second-Floor Level for the Beam in Example 13.1

Load Combination			Negative – Line C	Positive	Negative – Line D
ACI Eq. (5.3.1a)	$1.4D$		–113.8	78.1	–117.6
ACI Eq. (5.3.1b)	$1.2D + 1.6L$		–148.6	102.6	–154.1
ACI Eq. (5.3.1e)	$1.24D + Q_E + 0.5L$	SSR	104.0	80.3	–340.8
		SSL	–337.6	80.3	99.2
ACI Eq. (5.3.1g)	$0.86D + Q_E$	SSR	150.9	48.0	–292.2
		SSL	–290.7	48.0	147.8

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Equation (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = Q_E + (0.2 \times 0.192)D = Q_E + 0.04D$ where, as noted in Section 3.5 of this publication, the redundancy factor, ρ, is permitted to be taken as 1.0 for buildings assigned to SDC C (ASCE/SEI 12.3.4.1). Therefore, this load combination becomes the following:

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + (Q_E + 0.04D) = 1.24D + Q_E + 0.5L$$

Similarly, in ACI Equation (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E + 0.2S_{DS}D = Q_E - (0.2 \times 0.192)D = Q_E - 0.04D$, and this load combination becomes the following:

$$0.9D + 1.0E = 0.9D + (Q_E - 0.04D) = 0.86D + Q_E$$

Step 2 – Determine the required flexural reinforcement at the critical sections

The required area of flexural reinforcement, A_s, at the critical sections is determined by the following equations, which are given in Section 6.5.1 of this publication for rectangular sections with a single layer of tension reinforcement where $d = 24.0 - 2.5 = 21.5$ in. (an effective slab width at positive moment critical sections is not considered in this example):

$$R_n = \frac{M_u}{\phi b d^2}$$

$$A_s = \frac{0.85 f'_c b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right]$$

A summary of the required flexural reinforcement is given in Table 13.11. It is evident that all sections are tension-controlled because $A_s < A_{s,t}$.

Table 13.11 Required Flexural Reinforcement for the Beam in Example 13.1

Location		M_u (ft-kips)	R_n (psi)	A_s (in.^{2})*
Negative – Line C	SSR	150.9	155	2.01
	SSL	–337.6	348	3.69

(table continued on next page)

Table 13.11 Required Flexural Reinforcement for the Beam in Example 13.1 (cont.)

Location		M_u (ft-kips)	R_n (psi)	A_s (in.^{2})*
Positive		102.6	106	2.01
Negative – Line D	SSR	–340.8	351	3.73
	SSL	147.8	152	2.01

*Min. $A_s = 200 b_w d / f_y = 200 \times 28.0 \times 21.5 / 60{,}000 = 2.01$ in.^{2}
Max. $A_{s,t} = 0.018 b_w d = 0.018 \times 28.0 \times 21.5 = 10.84$ in.^{2}

Step 3 – Select the flexural reinforcement

Because the beam is subjected to torsional loading from the dead and live loads, additional longitudinal reinforcement may be required on all four faces of the beam in accordance with ACI 9.7.5.1 (see Example 13.3). In order to determine the required shear reinforcement, the nominal flexural strengths of the beam must be known, which depend on the area of flexural reinforcement. Therefore, at this stage in the design, more reinforcement than required for flexure alone is provided because of torsional loading. It will be checked in Example 13.4 whether all the applicable requirements are satisfied once the total area of longitudinal reinforcement is determined based on flexure and torsion.

Select the size and number of reinforcing bars based on the maximum and minimum spacing requirements in ACI 24.3 and 25.2, respectively.

Negative reinforcement – Line C:

Use 10-#6 bars ($A_{s,provided} = 4.40$ in.$^2 > 3.69$ in.^{2}; maximum and minimum number of longitudinal bars in a single layer for a 28.0-in.-wide beam are equal to 12 and 4 from Tables 6.8 and 6.9 of this publication, respectively).

Positive reinforcement:

Use 6-#6 bars ($A_{s,provided} = 2.64$ in.$^2 > 2.01$ in.^{2}; maximum and minimum number of longitudinal bars in a single layer for a 28.0-in.-wide beam are equal to 12 and 4 from Tables 6.8 and 6.9 of this publication, respectively).

The 6-#6 bars ($\phi M_n = 245.5$ ft-kips) are also adequate for the 150.9 ft-kip and 147.8 ft-kip positive moments at the faces of the supports at column lines C and D, respectively (see Table 13.11).

Negative reinforcement – Line D:

Use 10-#6 bars ($A_{s,provided} = 4.40$ in.$^2 > 3.73$ in.^{2}; maximum and minimum number of longitudinal bars in a single layer for a 28.0-in.-wide beam are equal to 12 and 4 from Tables 6.8 and 6.9 of this publication, respectively).

Step 4 – Check the nominal strength requirements of ACI 18.4.2.2

The positive moment strength at the face of a joint must be greater than or equal to one-third the negative moment strength provided at that location (ACI 18.4.2.2). This requirement is satisfied at both joints assuming the 6-#6 bottom bars are continuous over the span:

$$M_n^+ \text{ (6-\#6)} = A_s^+ f_y \left(d - \frac{A_s^+ f_y}{1.7 f_c' b_w} \right) = (2.64 \times 60) \times \left(21.5 - \frac{2.64 \times 60}{1.7 \times 4.0 \times 28.0} \right) / 12 = 272.8 \text{ ft-kips}$$

$$> \frac{M_n^- \text{ (10-\#6)}}{3} = \frac{A_s^- f_y}{3} \left(d - \frac{A_s^- f_y}{1.7 f_c' b_w} \right) = \frac{(4.40 \times 60)}{3} \times \left(21.5 - \frac{4.40 \times 60}{1.7 \times 4.0 \times 28.0} \right) / 12 = \frac{442.5}{3} = 147.5 \text{ ft-kips}$$

Also, the negative or positive moment strength at any section of the beam must be greater than or equal to one-fifth the maximum moment strength provided at the face of either joint, which in this example is equal to $442.5 / 5 = 88.8$ ft-kips. Providing 6-#6 continuous bottom bars and 10-#6 continuous top bars satisfies this requirement.

13.5.2 Example 13.2 – Determination of Shear Reinforcement: Beam in Building #1 (Framing Option B), Beam is Part of the SFRS (Intermediate Moment Frame), SDC C

Determine the required shear reinforcement for the beam along column line 1 between column lines C and D in Building #1, Framing Option B, at the second-floor level assuming the beam is part of the SFRS and the dimensions of the beam are 28 in. by 24 in. (see Figure 1.1). The slab is 8.5 in. thick and the columns are 24.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. It is determined in Example 3.5, Part (c) that the building is assigned to SDC C where $S_{DS} = 0.192$. See Example 13.1.

Step 1 – Determine the factored shear forces

Design shear strength must be greater than or equal to the lesser of the two values calculated by ACI 18.4.2.3(a) and 18.4.2.3(b).

According to ACI 18.4.2.3(a), maximum V_u is obtained by adding the shear forces associated with the development of the nominal flexural strengths at each end of the beam and the shear forces from the factored gravity and vertical earthquake loads. Sidesway to the right and to the left must both be investigated to obtain the maximum V_u.

The largest shear force associated with seismic effects is obtained from ACI Equation (5.3.1e). The factored load w_u that is uniformly distributed over the length of the beam is obtained by substituting $E = Q_E + 0.2S_{DS}D = 0.04D$ into ACI Equation (5.3.1e):

$$w_u = 1.2w_D + 0.04w_D + 0.5w_L = (1.24 \times 2.2) + (0.5 \times 0.8) = 3.1 \text{ kips/ft}$$

Shear forces due to the combination of gravity loads and nominal flexural strength for sidesway to the right are given in Figure 13.21 assuming the 6-#6 bottom bars are continuous over the span (see Figure 13.4). The maximum V_u is equal to 66.6 kips. Because the negative flexural reinforcement is the same at both supports, the maximum V_u is also equal to 66.6 kips for sidesway to the left.

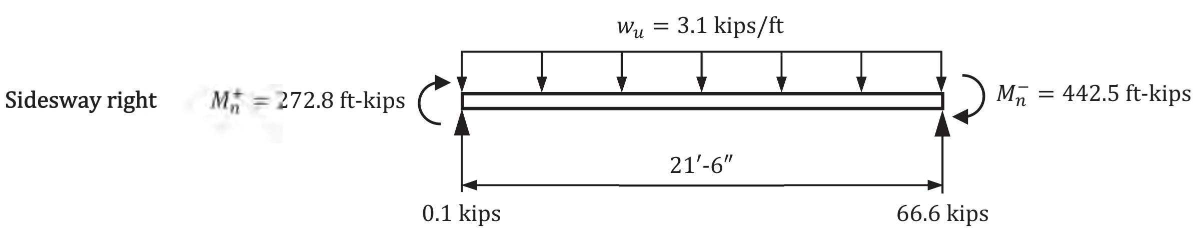


Figure 13.21 Factored shear forces in accordance with ACI 18.4.2.3(a).

According to ACI 18.4.2.3(b), maximum V_u is determined from analysis of the structure using the factored load combination $1.2D + 1.0E + 0.5L$ where $E = 2(Q_E + 0.2S_{DS}D)$. Thus, the following equation can be used to determine maximum V_u:

$$V_u = (1.2 + 0.4S_{DS})V_D + 2.0V_{Q_E} + 0.5V_L = 1.28V_D + 2.0V_{Q_E} + 0.5V_L \quad \text{Eq. (13.1)}$$

From analysis, $V_D = 23.9$ kips, $V_{Q_E} = 20.5$ kips, and $V_L = 9.1$ kips.

$$V_u = (1.28 \times 23.9) + (2.0 \times 20.5) + (0.5 \times 9.1) = 76.1 \text{ kips}$$

Therefore, use $V_u = 66.6$ kips. ACI 18.4.2.3

Step 2 – Determine the required shear reinforcement

Assuming $A_v \geq A_{v,min}$:

$$\phi V_c = \phi 2\lambda\sqrt{f_c'}b_w d = 0.75 \times 2 \times 1.0 \times \sqrt{4{,}000} \times 28.0 \times 21.5 / 1{,}000 = 57.1 \text{ kips}$$ ACI Table 22.5.5.1

Transverse reinforcement required for shear:

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi f_{yt} d} = \frac{66.6 - 57.1}{0.75 \times 60 \times 21.5} = 0.0098 \text{ in.}^2/\text{in.}$$ ACI 22.5.8.5.3

Because the beam is subjected to torsional loading, additional transverse reinforcement may be required (see Example 13.3).

13.5.3 Example 13.3 – Determination of Torsion Reinforcement: Beam in Building #1 (Framing Option B), Beam is Part of the SFRS (Intermediate Moment Frame), SDC C

Determine the required torsion reinforcement for the beam along column line 1 between column lines C and D in Building #1, Framing Option B, at the second-floor level assuming the beam is part of the SFRS and the dimensions of the beam are 28 in. by 24 in. (see Figure 1.1). The slab is 8.5 in. thick and the columns are 24.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. It is determined in Example 3.5, Part (c) that the building is assigned to SDC C where $S_{DS} = 0.192$. See Examples 13.1 and 13.2.

Step 1 – Determine the maximum factored torsional moment

From analysis, the maximum factored torsional moment, T_u, is equal to 140.4 ft-kips.

Step 2 – Determine if torsional effects must be considered

Torsion can be neglected where the factored torsional moment from analysis is less than the threshold torsion:

$$T_u < \phi T_{th} = \phi\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$ ACI 22.7.1.1

Determine the overhanging flange width, b_e, permitted to be used in the calculation of A_{cp} and p_{cp}:

$$b_e = \text{least of} \begin{cases} h_b = 24.0 - 8.5 = 15.5 \text{ in.} \\ 4h = 4 \times 8.5 = 34.0 \text{ in.} \end{cases}$$ ACI 9.2.4.4

Torsional section properties including the overhanging flange:

$$A_{cp} = b_w(h_b + h) + hb_e = [28.0 \times (15.5 + 8.5)] + (8.5 \times 15.5) = 804 \text{ in.}^2$$ Figure 6.3

$$p_{cp} = 2(h_b + h + b_w + b_e) = 2 \times (15.5 + 8.5 + 28.0 + 15.5) = 135 \text{ in.}$$

$A_{cp}^2 \,/\, p_{cp} = 4{,}788$ in.

Torsional section properties without the overhanging flange:

$A_{cp} = b_w(h_b + h) = 28.0 \times (15.5 + 8.5) = 672$ in.^{2}

$p_{cp} = 2(h_b + h + b_w) = 2 \times (15.5 + 8.5 + 28.0) = 104$ in.

$A_{cp}^2 \,/\, p_{cp} = 4{,}342$ in.

Because $A_{cp}^2 \,/\, p_{cp}$ for the beam with the overhanging flange is greater than that for the beam without the overhanging flange, use $A_{cp}^2 \,/\, p_{cp} = 4{,}788$ in. [ACI 9.2.4.4(b)].

$T_u = 140.4$ ft-kips $> \phi T_{th} = 0.75 \times 1.0 \times \sqrt{4{,}000} \times 4{,}788 \,/\, 12{,}000 = 18.9$ ft-kips

Therefore, torsional effects must be considered.

Because the edge beam is part of an indeterminate system in which redistribution of internal forces can occur following torsional cracking, the maximum factored torsional moment need not exceed the cracking torsional moment:

$$T_u = \phi T_{cr} = \phi 4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right) = 0.75 \times 1.0 \times 4 \times \sqrt{4{,}000} \times 4{,}788 \,/\, 12{,}000 = 75.7 \text{ ft-kips}$$ ACI 22.7.3.2

This is equivalent to a factored uniformly distributed torsional loading, t_u, equal to the following:

$$t_u = \frac{2 \times 75.7}{21.5} = 7.0 \text{ ft-kips/ft}$$

Step 3 – Check the adequacy of the cross-sectional dimensions

With 1.5-in. clear cover to #3 hoops:

$x_1 = b_w - 2c - d_s = 28.0 - (2 \times 1.5) - 0.375 = 24.6$ in. Figure 6.17

$y_1 = h_b + h - 2c - d_s = 15.5 + 8.5 - (2 \times 1.5) - 0.375 = 20.6$ in.

$A_{oh} = x_1 y_1 = 24.6 \times 20.6 = 506.8$ in.^{2}

$p_h = 2(x_1 + y_1) = 2 \times (24.6 + 20.6) = 90.4$ in.

From Step 2, $T_u = 75.7$ ft-kips.

From Step 1 in Example 13.2, $V_u = 66.6$ kips.

From Step 2 in Example 13.2, $V_c = 57.1 \,/\, 0.75 = 76.1$ kips.

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} = \sqrt{\left(\frac{66.6 \times 1{,}000}{28.0 \times 21.5}\right)^2 + \left(\frac{75.7 \times 12{,}000 \times 90.4}{1.7 \times 506.8^2}\right)^2} = 218.2 \text{ psi}$$

ACI 22.7.7.1

$$< \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) = 0.75 \times \left(\frac{76.1 \times 1{,}000}{28.0 \times 21.5} + 8\sqrt{4{,}000}\right) = 474.3 \text{ psi}$$

Therefore, the cross-sectional dimensions are adequate.

Step 4 – Determine the required transverse reinforcement for torsion

$$\frac{A_t}{s} = \frac{T_u}{2\phi \cot\theta A_o f_{yt}} = \frac{75.7 \times 12{,}000}{2 \times 0.75 \times \cot 45 \times (0.85 \times 506.8) \times 60{,}000} = 0.0234 \text{ in.}^2/\text{in.}$$ ACI 22.7.6.1

Step 5 – Determine the required longitudinal reinforcement for torsion

$$A_\ell = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right) \cot^2\theta = 0.0234 \times 90.4 \times \left(\frac{60}{60}\right) \times \cot^2 45 = 2.12 \text{ in.}^2$$ ACI 22.7.6.1

$$A_{\ell,min} = \text{lesser of} \begin{cases} \dfrac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\dfrac{A_t}{s}\right)p_h\left(\dfrac{f_{yt}}{f_y}\right) = \dfrac{5\sqrt{4{,}000} \times 804}{60{,}000} - (0.0234 \times 90.4) = 2.12 \text{ in.}^2 \\[2ex] \dfrac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\dfrac{25b_w}{f_{yt}}\right)p_h\left(\dfrac{f_{yt}}{f_y}\right) = \dfrac{5\sqrt{4{,}000} \times 804}{60{,}000} - \left[\left(\dfrac{25 \times 28.0}{60{,}000}\right) \times 90.4\right] = 3.18 \text{ in.}^2 \end{cases}$$ ACI 9.6.4.3

Use $A_\ell = 2.12 \text{ in.}^2$

The transverse and longitudinal reinforcement for torsion are combined with the transverse reinforcement required for shear and the longitudinal reinforcement required for flexure (see Example 13.4).

13.5.4 Example 13.4 – Design for Combined Flexure, Shear, and Torsion: Beam in Building #1 (Framing Option B), Beam is Part of the SFRS (Intermediate Moment Frame), SDC C

Design the beam along column line 1 between column lines C and D in Building #1, Framing Option B, at the second-floor level for the combined effects due to flexure, shear, and torsion assuming the beam is part of the SFRS and the dimensions of the beam are 28 in. by 24 in. (see Figure 1.1). The slab is 8.5 in. thick and the columns are 24.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. It is determined in Example 3.5, Part (c) that the building is assigned to SDC C where $S_{DS} = 0.192$. See Examples 13.1, 13.2, and 13.3.

Step 1 – Determine the total transverse reinforcement

The total required area of transverse reinforcement per stirrup leg is equal to that required for shear (Example 13.2) plus that required for torsion (Example 13.3):

$$\frac{A_v}{2s} + \frac{A_t}{s} = \frac{0.0098}{2} + 0.0234 = 0.0283 \text{ in.}^2/\text{in.}$$

ACI 9.6.4.2

$$> \text{greater of} \begin{cases} 0.375\sqrt{f_c'}b_w / f_{yt} = 0.375 \times \sqrt{4{,}000} \times 28.0 / 60{,}000 = 0.0111 \text{ in.}^2/\text{in.} \\[2ex] 25b_w / f_{yt} = 25 \times 28.0 / 60{,}000 = 0.0117 \text{ in.}^2/\text{in.} \end{cases}$$

The maximum spacing of the hoops over the length $2h = 2 \times 24.0 = 48.0$ in. from the face of the support at each end of the beam is the smallest of the following:

- $d / 4 = 21.5 / 4 = 5.4$ in. ACI 18.4.2.4, Figure 13.6
- $8d_b$ of smallest longtudinal bar $= 8 \times 0.75 = 6.0$ in.
- $24d_b$ of the hoop bar $= 24 \times 0.375 = 9.0$ in.
- 12.0 in.

The maximum spacing of the legs of the transverse reinforcement across the width of the member is determined in accordance with ACI Table 9.7.6.2.2 where $V_s = (V_u / \phi) - V_c = (66.6 / 0.75) - 76.1$ kips $= 12.7$ kips $< 4\sqrt{f_c'}b_w d$ $= 152.3$ kips :

Maximum spacing across width $= d = 21.5$ in. < 24.0 in.

Assuming #3 transverse bars with 2 legs, center-to-center spacing $= 28.0 - (2 \times 1.5) - 0.375 = 24.6$ in. > 21.5 in.

Thus, 3 legs must be provided.

Try 11-#3 hoops (3 legs) at each end of the beam spaced at 5 in. on center with the first hoop located 2 in. from the face of the support.

Provided transverse reinforcement (per leg) $= 0.11 / 5.0 = 0.0220$ in.^{2}/in. < 0.0283 in.^{2}/in.

Therefore, use 11-#4 hoops (3 legs) at each end of the beam spaced at 5 in. on center (provided transverse reinforcement per leg $= 0.20 / 5.0 = 0.0400$ in.^{2}/in.).

The provided hoop spacing of 5 in. is less than the lesser of the following, which is the maximum permitted spacing for the transverse torsional reinforcement:

$$s_{max} = \text{lesser of} \begin{cases} d / 2 = 21.5 / 2 = 10.8 \text{ in.} \\ p_h / 8 = 90.4 / 8 = 11.3 \text{ in.} \\ 12.0 \text{ in.} \end{cases}$$ ACI 9.7.6.3.3

For the remainder of the beam, the maximum closed stirrup spacing is $d / 2 = 21.5 / 2 = 10.8$ in. ACI 18.4.2.5

Try #4 closed stirrups (3 legs) at 10 in. on center for the remainder of the beam.

At 52 in. from the face of the support:

$$V_u = 66.6 - [3.1 \times (52.0 / 12)] = 53.2 \text{ kips}$$

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi f_{yt} d} = \frac{53.2 - 57.1}{0.75 \times 60 \times 21.5} < 0$$ ACI 22.5.8.5.3

$$T_u = 72.7 - [7.0 \times (52.0 / 12)] = 42.4 \text{ ft-kips}$$

$$\frac{A_t}{s} = \frac{T_u}{2\phi \cot\theta A_o f_{yt}} = \frac{42.4 \times 12{,}000}{2 \times 0.75 \times \cot 45 \times (0.85 \times 506.8) \times 60{,}000} = 0.0131 \text{ in.}^2/\text{in.}$$ ACI 22.7.6.1

Provided transverse reinforcement (per leg) $= 0.20 / 10.0 = 0.0200 \text{ in.}^2/\text{in.} > 0.0131 \text{ in.}^2/\text{in.}$

Therefore, use #4 closed stirrups spaced at 10 in. for the remainder of the beam.

Step 2 – Determine the total longitudinal reinforcement

From Step 5 in Example 13.3, $A_\ell = 2.12 \text{ in.}^2$ This reinforcement must be distributed around the perimeter of the beam with a maximum spacing of 12.0 in. and must be combined with that required for flexure (ACI 9.7.5.1 and 9.5.4.3). Thus, assign $2.12 / 4 = 0.53 \text{ in.}^2$ to each face of the beam.

Use 2-#5 bars on each side face of the beam $(A_{s,provided} = 0.62 \text{ in.}^2 > 0.53 \text{ in.}^2)$.

Check spacing of side bars:

With #6 top and bottom longitudinal bars, #4 hoops and closed stirrups, and 1.5-in. clear cover, the clear spacing between the bars on each side face $= \{24.0 - [2 \times (1.5 + 0.5 + 0.75 + 0.625)]\} / 3 = 5.8 \text{ in.}$

Center-to-center spacing of the #5 bars $= 5.8 + 0.625 = 6.4 \text{ in.} < 12.0 \text{ in.}$

Center-to-center spacing of the #5 and #6 bars $= 5.8 + (0.625 / 2) + (0.75 / 2) = 6.5 \text{ in.} < 12.0 \text{ in.}$

Check requirement of ACI 9.7.5.2:

$d_b = 0.625 \text{ in.} > 0.042s = 0.042 \times 10.0 = 0.42 \text{ in. and } 3/8 \text{ in.}$

where the largest spacing of the transverse reinforcement, s, is used to check this requirement.

The remaining longitudinal reinforcement for torsion is distributed equally between the top and bottom of the section: $\{2.12 - [(2 \times (2 \times 0.31)]\} / 2 = 0.44 \text{ in.}^2$

- Negative – Line C

  Total longitudinal reinforcement $= 3.69 + 0.44 = 4.13 \text{ in.}^2$ Table 13.11

  The 10-#6 bars selected in Step 3 of Example 13.1 are adequate $(A_{s,provided} = 4.40 \text{ in.}^2 > 4.13 \text{ in.}^2)$.

- Positive

  Total longitudinal reinforcement $= 2.01 + 0.44 = 2.45 \text{ in.}^2$

  The 6-#6 bars selected in Step 3 of Example 13.1 are adequate $(A_{s,provided} = 2.64 \text{ in.}^2 > 2.45 \text{ in.}^2)$.

- Negative – Line D

  Total longitudinal reinforcement $= 3.73 + 0.44 = 4.17 \text{ in.}^2$

The 10-#6 bars selected in Step 3 of Example 13.1 are adequate $(A_{s,provided} = 4.40 \text{ in.}^2 > 4.17 \text{ in.}^2)$.

Because the negative and positive flexural reinforcement that was selected in Example 13.1 are adequate for the combined effects of flexure and torsion, the negative and positive nominal flexural strengths are the same as determined previously, which means the nominal flexural strength requirements of ACI 18.4.2.2 are automatically satisfied (see Step 4 in Example 13.1) and the shear forces based on the nominal flexural strengths do not need to be revised (see Step 1 of Example 13.2).

Step 3 – Detail the longitudinal reinforcement

Assume the 10-#6 top bars and 6-#6 bottom bars are also required in an end span. These bars must extend to the far end of the joint core (that is, the core of the edge column) and must be fully developed in tension in accordance with ACI 18.8.5 and in compression in accordance with ACI 25.4.9 (ACI 18.4.4.3).

Assuming standard 90-degree hooks are provided at the ends of the #6 bars, the tension development length of a deformed hooked bar, ℓ_{dh}, is the greater of the following for normalweight concrete:

$$\ell_{dh} = \text{greater of} \begin{cases} f_y d_b / 65\sqrt{f_c'} = (60{,}000 \times 0.75) / (65 \times \sqrt{4{,}000}) = 11.0 \text{ in.} \\ 8d_b = 8 \times 0.75 = 6.0 \text{ in.} \\ 6 \text{ in.} \end{cases} \qquad \text{Eq. (13.9)}$$

Assuming #8 longitudinal bars and #3 hoops in the edge column, the available development length is equal to the following (see Figure 13.11):

$$c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} = 24.0 - (2 \times 1.5) - 0.375 - 1.0 = 19.6 \text{ in.} > \ell_{dh} = 11.0 \text{ in.} \qquad \text{Eq. (13.11)}$$

Therefore, the #6 longitudinal bars in the beam can be fully developed in tension in an edge column using standard 90-degree hooks.

The development length of a deformed bar in compression, ℓ_{dc}, is the greater of the following:

$$\ell_{dc} = \text{greater of} \begin{cases} \left(\dfrac{f_y \psi_r}{50\lambda\sqrt{f_c'}}\right) d_b = \left(\dfrac{60{,}000 \times 1.0}{50 \times 1.0 \times \sqrt{4{,}000}}\right) \times 0.75 = 14.2 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 0.75 = 13.5 \text{ in.} \\ 8 \text{ in.} \end{cases} \qquad \text{Eq. (13.14)}$$

Assuming #8 longitudinal bars and #3 hoops in the edge column, the available development length is equal to the following for the straight portion of the hooked bar [Eq. (13.15)]:

$$\begin{aligned} c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} - d_b - r &= 24.0 - (2 \times 1.5) - 0.375 - 1.0 - 0.75 - (3 \times 0.75) \\ &= 16.6 \text{ in.} > \ell_{dc} = 14.2 \text{ in.} \end{aligned} \qquad \text{Eq. (13.15)}$$

where r is the bend radius of the beam longitudinal bar, which is equal to $6d_b / 2 = 3d_b$ for #6 bars (see ACI Table 25.3.1).

Therefore, the #6 longitudinal bars in the beam can be fully developed in compression in an edge column.

Reinforcement details for this beam at an interior span are given in Figure 13.22.

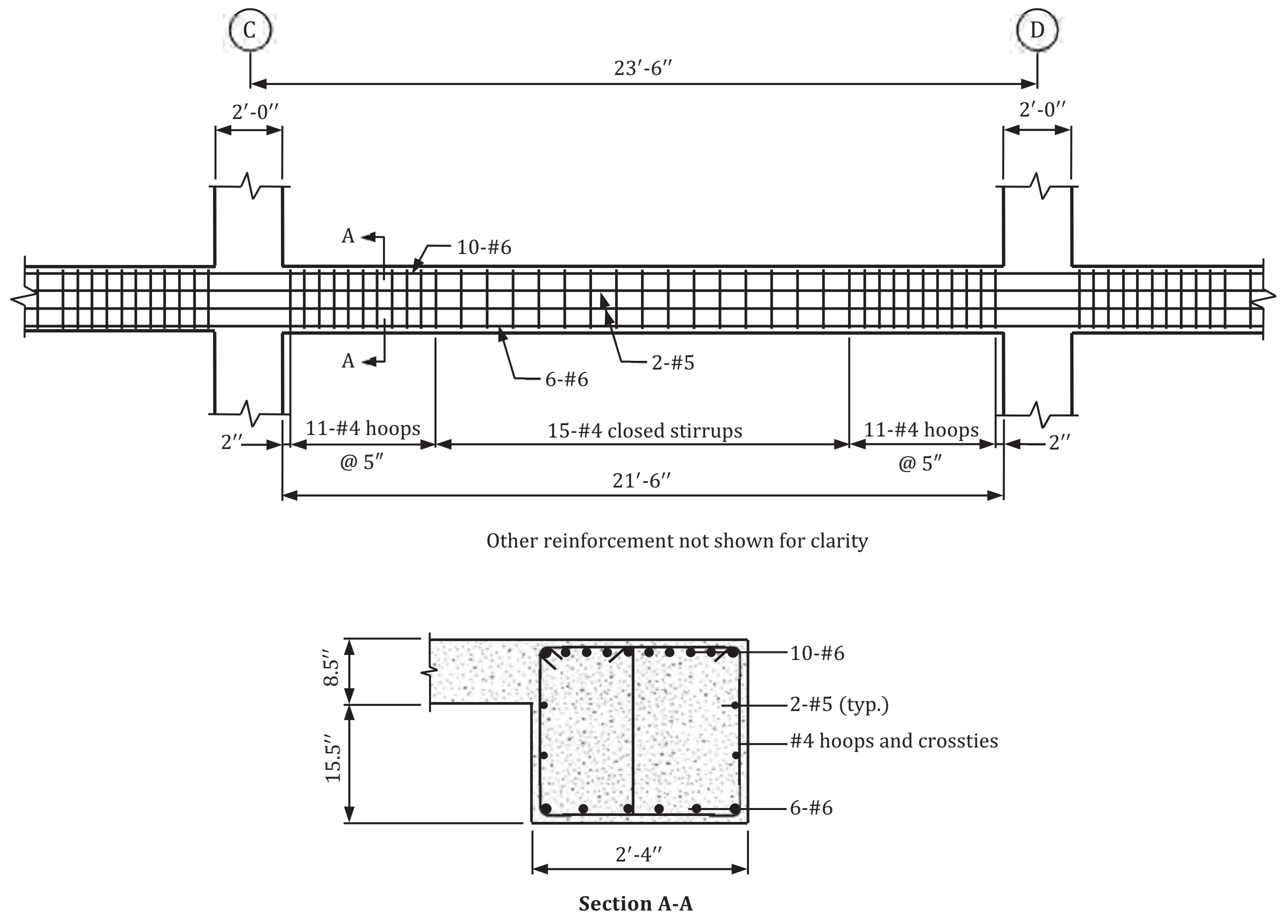


Figure 13.22 Reinforcement details for the beam in Examples 13.1 through 13.4.

13.5.5 Example 13.5 – Determination of Longitudinal Reinforcement: Column in Building #1 (Framing Option B), Column is Part of the SFRS (Intermediate Moment Frame), SDC C

Determine the required longitudinal reinforcement for column D1 in the first story of Building #1, Framing Option B assuming the column is part of the SFRS and the dimensions of the column are 24 in. by 24 in. (see Figure 1.1). The slab is 8.5 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. It is determined in Example 3.5, Part (c) that the building is assigned to SDC C where $S_{DS} = 0.192$.

Step 1 – Determine the factored axial forces and bending moments Table 13.2

A summary of the axial forces and bending moments at the bottom of column D1 in the first story is given in Table 13.12 for sidesway to the right (SSR) and sidesway to the left (SSL). Factored wind load effects are smaller than those due to factored seismic load effects and are not considered in this example. It can be determined that the first story is nonsway and slenderness effects need not be considered (see Sect. 7.3.3 of this publication).

Table 13.12 Summary of Axial Forces and Bending Moments for Column D1

Load Case			Axial Force (kips)	Bending Moment (ft-kips)
Dead (D)			306.7	−2.8
Roof live (L_r)			7.3	—
Live (L)			88.5	−1.4
Seismic (Q_E)			±4.7	±441.4
Load Combination				
ACI Eq. (5.3.1a)	$1.4D$		429.4	−3.9
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		513.3	−5.6
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		424.0	−4.1
ACI Eq. (5.3.1e)	$1.24D + Q_E + 0.5L$	SSR	429.3	437.2
		SSL	419.9	−445.6
ACI Eq. (5.3.1g)	$0.86D + Q_E$	SSR	268.5	439.0
		SSL	259.1	−443.8

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Equation (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = Q_E + (0.2 \times 0.192)D = Q_E + 0.04D$ where, as noted in Section 3.5 of this publication, the redundancy factor, ρ, is permitted to be taken as 1.0 for buildings assigned to SDC C (ASCE/SEI 12.3.4.1). Therefore, this load combination becomes the following:

$$1.2D + 1.0E + 0.5L = 1.2D + (Q_E + 0.04D) + 0.5L = 1.24D + Q_E + 0.5L$$

Similarly, in ACI Equation (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E + 0.2S_{DS}D = Q_E - (0.2 \times 0.192)D = Q_E - 0.04D$, and this load combination becomes the following:

$$0.9D + 1.0E = 0.9D + (Q_E - 0.04D) = 0.86D + Q_E$$

Step 2 – Determine the required area of longitudinal reinforcement

Based on a 24-in. square tied column, the required reinforcement ratio, ρ_g, is equal to the following for the largest factored axial force in Table 13.12, which occurs without any appreciable bending moments:

$$\rho_g = \frac{(P_u/\phi\phi_c A_g) - 0.85f'_c}{f_y - 0.85f'_c} = \frac{[513.3\ /\ (0.65 \times 0.80 \times 24.0^2)] - (0.85 \times 4)}{60 - (0.85 \times 4)} < 0 \qquad \text{Eq. (7.32)}$$

Therefore, use the minimum required $\rho_g = 0.01$:

$$A_{st} = 0.01 \times 24.0^2 = 5.76 \text{ in.}^2$$

Select 8-#8 bars $(A_{st,provided} = 6.32 \text{ in.}^2)$.

Step 2 – Check if the selected longitudinal reinforcement is adequate for all load combinations

The design strength interaction diagram for this column reinforced with 8-#8 bars is given in Figure 13.23. Also shown in the figure are load combination points for the factored axial forces and bending moments in Table 13.12.

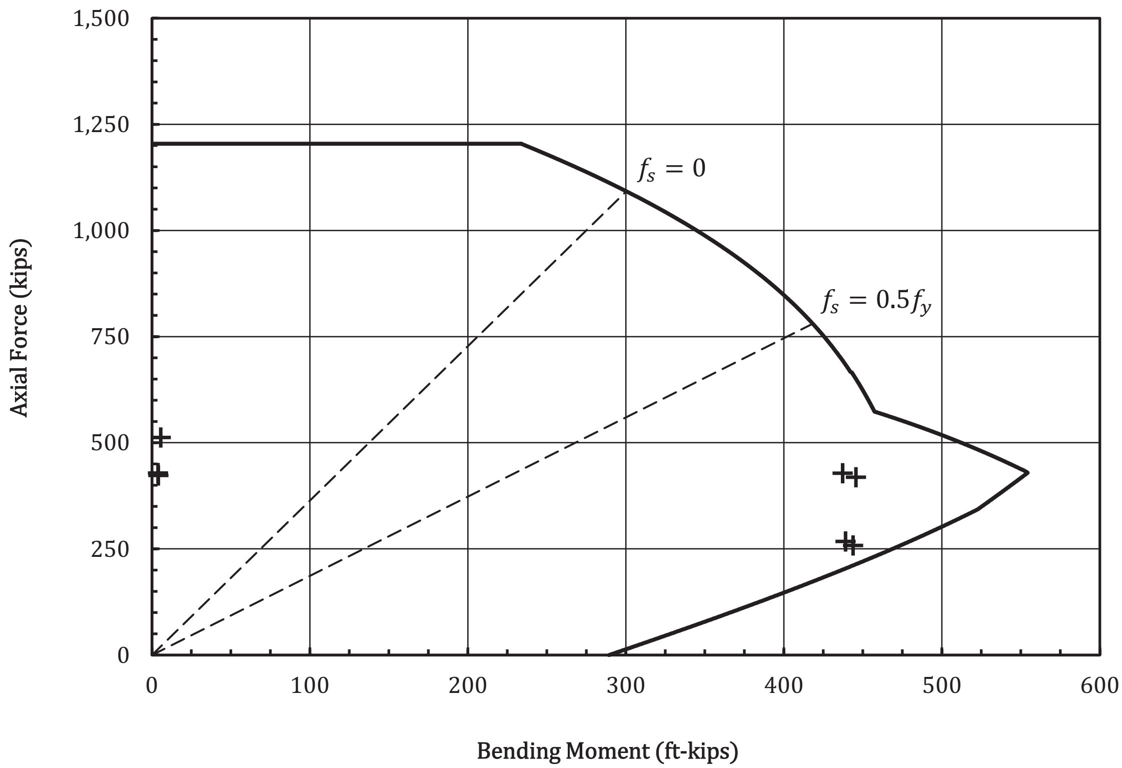


Figure 13.23 Design strength interaction diagram for column D1.

It is evident from the figure that the column is adequate for all factored load combinations.

Step 3 – Check the minimum number of longitudinal bars and the minimum face dimension of the column

For rectangular, tied columns:

Minimum number of longitudinal bars with rectangular ties $= 4$ — Table 7.16

Minimum face dimension $= 12$ in. for 3-#8 bars per face and normal lap splices < 24 in. — Table 7.17

Use 8-#8 bars.

13.5.6 Example 13.6 – Determination of Transverse Reinforcement: Column in Building #1 (Framing Option B), Column is Part of the SFRS (Intermediate Moment Frame), SDC C

Determine the required transverse reinforcement for column D1 in the first story of Building #1, Framing Option B assuming the column is part of the SFRS and the dimensions of the column are 24 in. by 24 in. (see Figure 1.1). The slab is 8.5 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. It is determined in Example 3.5, Part (c) that the building is assigned to SDC C where $S_{DS} = 0.192$. See Example 13.5.

Step 1 – Determine the factored shear forces

Design shear strength must be greater than or equal to the lesser of the two values determined by ACI 18.4.3.1(a) and 18.4.3.1(b).

According to ACI 18.4.3.1(a), maximum V_u is obtained from the development of the nominal flexural strengths at each end of the column for the factored axial force consistent with the direction of analysis resulting in the highest moment strength. Sidesway to the right and to the left must both be investigated to obtain the maximum V_u.

The nominal strength diagram and the factored load combinations that include seismic load effects are given in Figure 13.24 for the load effects at the bottom of the column. In this example, the axial force and the corresponding nominal flexural strength at the top of the column is smaller than the axial force and the corresponding nominal flexural strength at the bottom of the column. Conservatively use the nominal flexural strength at the bottom of the column at both the top and bottom of the column to determine V_u.

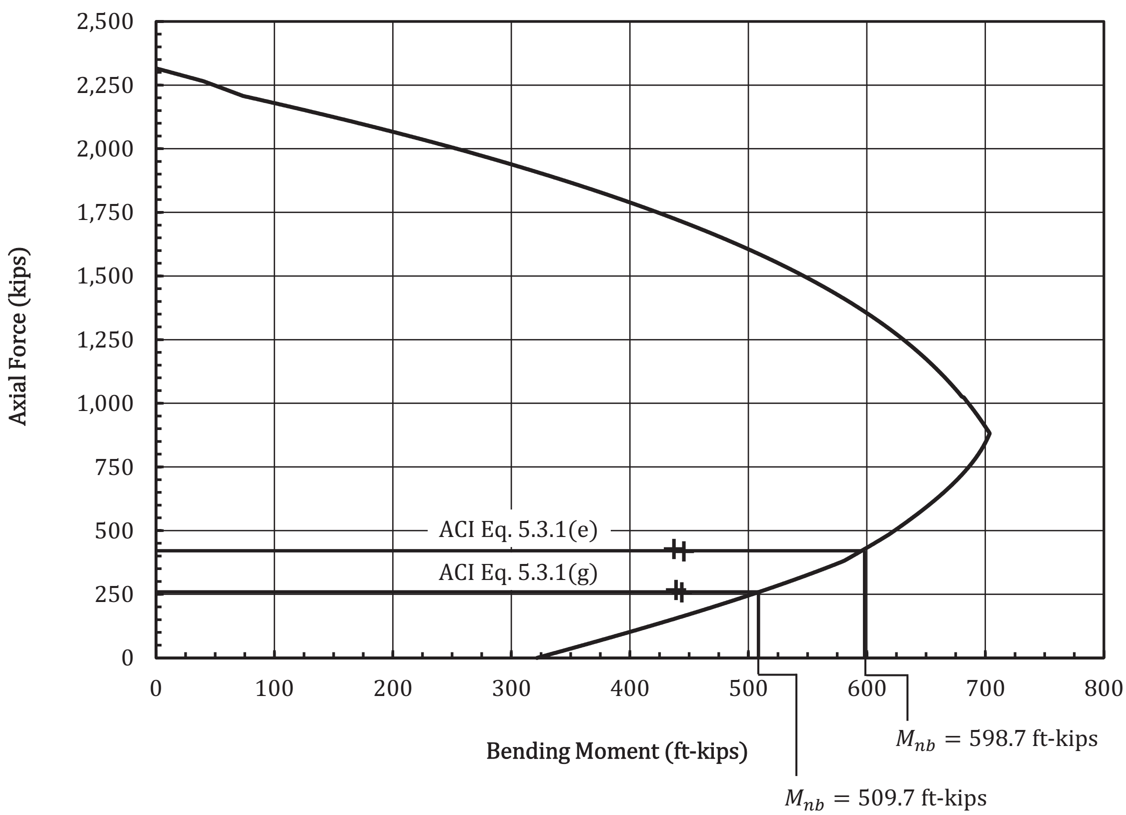


Figure 13.24 Nominal strength interaction diagram for column D1.

It is evident from Figure 13.24 that the largest M_{nb} within the range of factored axial forces is obtained from ACI Eq. 5.3.1(e) for sidesway to the right.

For $P_n = P_u / \phi = 429.3 / 0.9 = 477.0$ kips, $M_{nb} = 598.7$ ft-kips

$$V_u = \frac{2M_{nb}}{\ell_u} = \frac{2 \times 598.7}{14.0 - \dfrac{24.0}{2 \times 12}} = 92.1 \text{ kips}$$

According to ACI 18.4.3.1(b), maximum V_u is determined from analysis of the structure using the factored load combination $1.2D + 1.0E + 0.5L$ where $E = \Omega_o(Q_E + 0.2S_{DS}D)$. Thus, the following equation can be used to determine maximum V_u where $\Omega_o = 3$ for intermediate moment frames (see Table 12.3 of this publication):

$$V_u = (1.2 + 0.6S_{DS})V_D + 3.0V_{Q_E} + 0.5V_L = 1.32V_D + 3.0V_{Q_E} + 0.5V_L$$

From analysis, $V_D = 0.6$ kips, $V_{Q_E} = 46.5$ kips, and $V_L = 0.3$ kips.

$$V_u = (1.32 \times 0.6) + (3.0 \times 46.5) + (0.5 \times 0.3) = 140.4 \text{ kips}$$

Therefore, use $V_u = 92.1$ kips. ACI 18.4.3.1

Step 2 – Determine the required shear reinforcement

Assuming $A_v \geq A_{v,min}$:

$$\phi V_c = \phi\left(2\lambda\sqrt{f_c'} + \frac{N_u}{6A_g}\right)b_w d$$

ACI Table 22.5.5.1

$$= 0.75 \times \left[(2 \times 1.0 \times \sqrt{4{,}000}) + \frac{429{,}300}{6 \times 24.0^2}\right] \times 24.0 \times 17.8 \,/\, 1{,}000 = 80.3 \text{ kips}$$

where $N_u = 429.3$ kips, which is the axial force determined by ACI Eq. (5.3.1e) corresponding to the nominal flexural strength used to determine the maximum shear force (see Table 13.12), and $d = 17.8$ in. is determined from a strain compatibility analysis of the section. Neutral axis depth $c = 8.0$ in. for this case, which means two layers of longitudinal reinforcement are in tension.

$$\text{Center-to-center spacing of the longitudinal bars} = \frac{24.0 - [2 \times (1.5 + 0.375)] - 1.0}{2} = 9.6 \text{ in.}$$

Therefore, d is equal to the following:

$$d = 24.0 - \left[1.5 + 0.375 + \frac{1.0}{2} + \frac{(2 \times 0.79) \times 9.6}{5 \times 0.79}\right] = 17.8 \text{ in.}$$

Provide at least the minimum shear reinforcement in ACI 10.6.2.2 considering the requirements in ACI 18.4.3.3.

According to ACI 18.4.3.3, hoops must be provided at each end of the column over the length ℓ_o :

$$\ell_o \geq \text{longest of} \begin{cases} \ell_u/6 = (13.0 \times 12) \,/\, 6 = 26.0 \text{ in.} \\ c_1 = c_2 = 24 \text{ in.} \\ 18 \text{ in.} \end{cases} \qquad \text{Eq. (13.5)}$$

The maximum spacing of the hoops is the least of the following for Grade 60 reinforcement:

$$s_o \leq \text{least of} \begin{cases} 8d_b = 8 \times 1.0 = 8.0 \text{ in.} \\ 8.0 \text{ in.} \\ 0.5c_1 = 0.5c_2 = 12.0 \text{ in.} \end{cases} \qquad \text{Eq. (13.8)}$$

Assume #3 hoops, which are permitted to be used with #8 longitudinal bars (see ACI 25.7.2.2 and Figure 7.32 of this publication). Check the spacing requirements of ACI 25.7.2.3(b):

$$\text{Clear space between longitudinal bars} = \frac{24.0 - [2 \times (1.5 + 0.375)] - 1.0}{2} - 1.0 = 8.6 \text{ in.} > 6.0 \text{ in.}$$

Therefore, #3 crossties must be provided around the interior longitudinal bars.

Determine the required spacing of the #3 hoops based on V_u :

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (3 \times 0.11) \times 60 \times 17.8}{92.1 - 80.3} = 22.4 \text{ in.} > s_o = 8.0 \text{ in.} \qquad \text{ACI 22.5.8.5.3}$$

Check the minimum shear reinforcement requirements using an 8.0 in. hoop spacing:

$$A_{v,min} = \text{greater of} \begin{cases} 0.75\sqrt{f_c'} b_w s / f_{yt} = 0.75 \times \sqrt{4{,}000} \times 24.0 \times 8.0 / 60{,}000 = 0.15 \text{ in.}^2 \\ 50 b_w s / f_{yt} = 50 \times 24.0 \times 8.0 / 60{,}000 = 0.16 \text{ in.}^2 \end{cases} \qquad \text{ACI 10.6.2.2}$$

Provided $A_v = 3 \times 0.11 = 0.33 \text{ in.}^2 > 0.16 \text{ in.}^2$

Use #3 hoops with crossties spaced at 8.0 in. over a length of at least 26.0 in. from each end of the column. This transverse reinforcement is also provided within the joint, which satisfies the requirements of ACI 18.4.4.4.

Locate the first hoop no more than $s_o / 2 = 8.0 / 2 = 4.0$ in. from the face of the joint. ACI 18.4.3.4

For the remainder of the column, it is permitted to use #3 ties and crossties at a spacing of no more than the lesser of $d / 2 = 17.7 / 2 = 8.9$ in. and 24.0 in. because $V_s = A_v f_{yt} d / s = 43.8 \text{ kips} < 4\sqrt{f_c'} b_w d = 107.5 \text{ kips}$ (ACI 18.4.3.5 and 10.7.6.5.2). Use #3 ties and crossties at a spacing of 8.0 in., which matches the spacing of the hoops in the plastic hinge region.

13.5.7 Example 13.7 – Determination of Lap Splice Length: Column in Building #1 (Framing Option B), Column is Part of the SFRS (Intermediate Moment Frame), SDC C

Determine the required lap splice length for column D1 in the first story of Building #1, Framing Option B assuming the column is part of the SFRS and the dimensions of the column are 24 in. by 24 in. (see Figure 1.1). The slab is 8.5 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. It is determined in Example 3.5, Part (c) that the building is assigned to SDC C where $S_{DS} = 0.192$. See Examples 13.5 and 13.6.

Step 1 – Determine the tension development length

The tension development length, ℓ_d, of the #8 longitudinal bars for column D1 is determined using the requirements in ACI 25.4.2.4:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

$\psi_t = 1.0$ ACI Table 25.4.2.5

$\psi_e = 1.0$ for uncoated bars

$\psi_s = 1.0$ for #8 bars

$\psi_g = 1.0$ for Grade 60 reinforcement

$$c_b = \text{ lesser of } \begin{cases} \text{cover} + (d_b)_{tie} + 0.5(d_b)_{long.} = 1.5 + 0.375 + (0.5 \times 1.0) = 2.4 \text{ in.} \\ \\ \dfrac{s}{2} = \dfrac{24.0 - (2 \times 1.5) - (2 \times 0.375) - 1.0}{2} = 9.6 \text{ in.} \end{cases}$$

$$K_{tr} = \frac{40 A_{tr}}{sn} = \frac{40 \times (3 \times 0.11)}{8.0 \times 3} = 0.6$$ ACI Eq. (25.4.2.4b)

$$(c_b + K_{tr}) / d_b = (2.4 + 0.6) / 1.0 = 3.0 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0 \times \sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} \right) \times 1.0 = 28.5 \text{ in.} > 12.0 \text{ in.}$$

Step 2 – Determine the required lap splice length

It is evident from Figure 13.23 that some of the factored load combinations fall within the region of the interaction diagram where $f_s > 0.5 f_y$. Therefore, a Class B tension lap splice must be provided in accordance with ACI Table 10.7.5.2.2. The Class B tension lap splice length, ℓ_{st}, is determined from ACI Table 25.5.2.1:

$$\ell_{st} = 1.3 \ell_d = 1.3 \times 28.5 = 37.1 \text{ in.}$$

Provide a 38.0-in. lap splice length.

Reinforcement details for column D1 are given in Figure 13.25. The lap splice is located near the mid-height of the column, although locating it immediately above the floor slab is permitted.

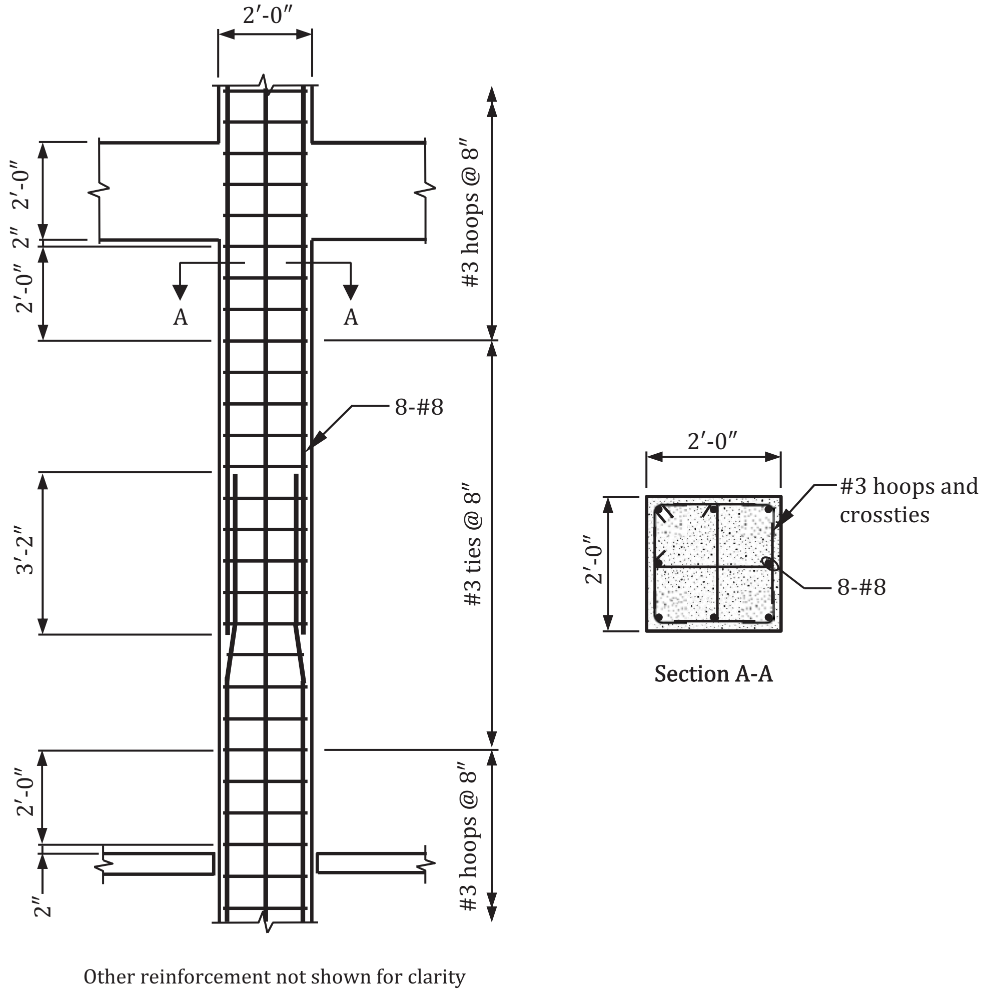


Figure 13.25 Reinforcement details for column D1 in Examples 13.5 through 13.7.

13.5.8 Example 13.8 – Check of Joint Shear Strength: Column in Building #1 (Framing Option B), Column is Part of the SFRS (Intermediate Moment Frame), SDC C

Check the joint shear strength in the north-south direction for column D1 in the first story of Building #1, Framing Option B assuming the column is part of the SFRS and the dimensions of the column are 24 in. by 24 in. (see Figure 1.1). The slab is 8.5 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. It is determined in Example 3.5, Part (c) that the building is assigned to SDC C where $S_{DS} = 0.192$.

Step 1 – Determine the factored horizontal joint shear force ACI 15.4.1.1

A free-body diagram of the joint is given in Figure 13.26. The negative and positive flexural reinforcement in the beams on both sides of the joint are the same, that is, 10-#6 top bars and 6-#6 bottom bars are used in both beams (see Example 13.4).

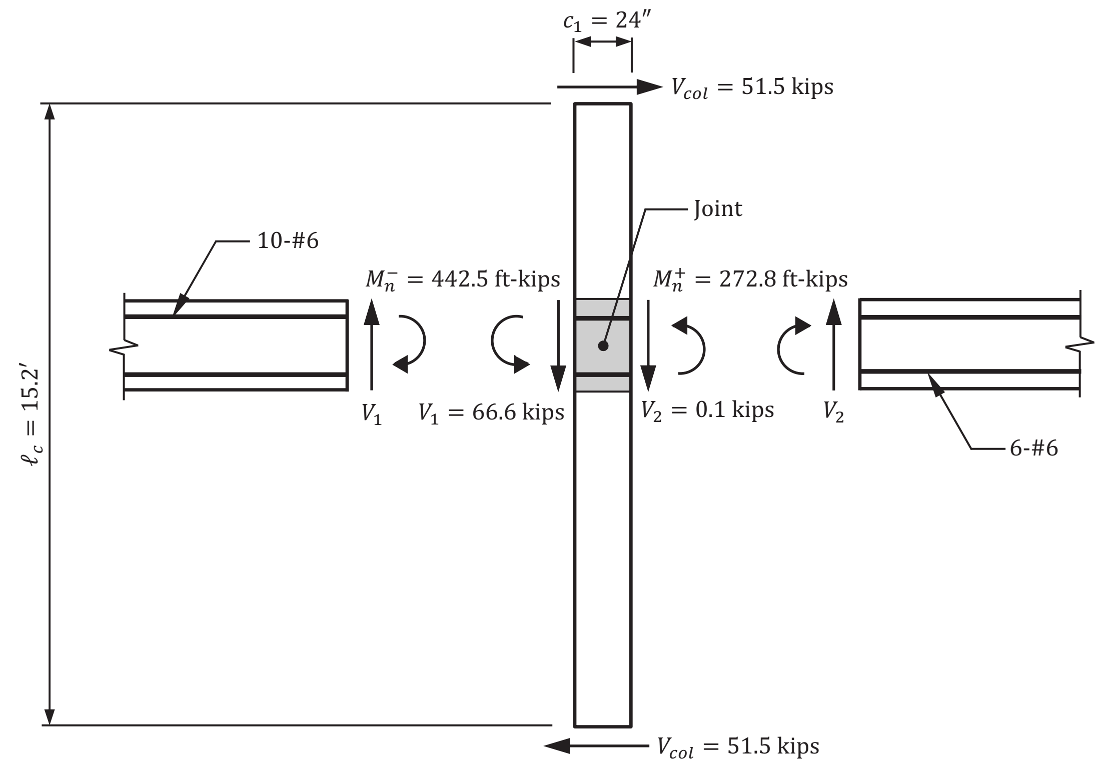


Figure 13.26 Free-body diagram of the joint at column D1.

Determine the shear force in the column, V_{col}, for an edge column with the direction of analysis parallel to the edge:

$$V_{col} = \frac{M_n^- + M_n^+}{\ell_c} + \frac{(V_1 + V_2) \times (c_1 / 2)}{\ell_c}$$ Table 13.5

$$M_n^- = A_s^- f_y \left(d - \frac{A_s^- f_y}{1.7 f_c' b} \right) = (10 \times 0.44) \times 60 \times \left[21.5 - \frac{(10 \times 0.44) \times 60}{1.7 \times 4.0 \times 28.0} \right] / \, 12 = 442.5 \text{ ft-kips}$$

$$M_n^+ = A_s^+ f_y \left(d - \frac{A_s^+ f_y}{1.7 f_c' b} \right) = (6 \times 0.44) \times 60 \times \left[21.5 - \frac{(6 \times 0.44) \times 60}{1.7 \times 4.0 \times 28.0} \right] / \, 12 = 272.8 \text{ ft-kips}$$

From analysis of the structure, the point of inflection in the column above the joint occurs approximately 5.9 ft from the center of the joint and the point of inflection in the column below the joint occurs approximately 9.3 ft from the center of the joint. Therefore, $\ell_c = 5.9 + 9.3 = 15.2$ ft.

$$V_{col} = \frac{M_n^- + M_n^+}{\ell_c} + \frac{(V_1 + V_2)(c_1 / 2)}{\ell_c} = \frac{442.5 + 272.8}{15.2} + \frac{(66.6 - 0.1) \times (2.0 / 2)}{15.2} = 47.1 + 4.4 = 51.5 \text{ kips}$$ Table 13.5

The factored shear force in the joint, V_u, is equal to the following:

$$V_u = (A_s^- + A_s^+) f_y - V_{col} = \{[(10 \times 0.44) + (6 \times 0.44)] \times 60.0\} - 51.5 = 422.4 - 51.5 = 370.9 \text{ kips}$$

Step 2 – Determine the design shear strength of the joint

Columns frame into the bottom and top of the joint (that is, the column is continuous). The beam in the direction of analysis is continuous. There are no beams in the transverse direction. Therefore, the joint type is I-B (see Figure 13.15).

Because the beam in the direction of analysis is slightly larger than the width of the column, the effective cross-sectional area within the joint is equal to the area of the column:

$A_j = c_1 \times c_2 = 24.0 \times 24.0 = 576.0 \text{ in.}^2$ Figure 11.13

The design shear strength of the joint is equal to the following for a Type I-B joint:

$\phi V_n = \phi 15\lambda\sqrt{f_c'}A_j = 0.75 \times 15 \times 1.0 \times \sqrt{4{,}000} \times 576.0 / 1{,}000 = 409.8 \text{ kips} > V_u = 370.9 \text{ kips}$ Figure 13.15

Step 3 – Determine the detailing for the joint

Because the joint is part of the SFRS, the detailing requirements in ACI 15.3.1.2, 15.3.1.3, and 18.4.4.2 through 18.4.4.5 must be satisfied (ACI 18.4.4.1).

Providing #3 ties and crossties at a spacing of 8.0 in. in the joint, which matches the spacing of the hoops in the plastic hinge region, satisfies ACI 18.4.4.4 (see Figure 13.25).

13.5.9 Example 13.9 – Determination of Flexural Reinforcement: Two-way Slab in Building #1 (Framing Option A), Two-way Slab is Part of the SFRS (Intermediate Moment Frame), SDC C

Determine the required flexural reinforcement in an end span of an interior design strip along column line 3 in Building #1, Framing Option A, at the second-floor level assuming the two-way slab is part of the SFRS (see Figure 1.1). The slab is 9.5 in. thick and the columns are 24.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000 \text{ psi}$ and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. The building is assigned to SDC C where $S_{DS} = 0.192$.

Step 1 – Determine the factored bending moments along the span

A summary of the service bending moments due to the dead and live loads at the critical sections in the column strip and middle strip in an end span is given in Table 13.13 (see Table 5.25 in Example 5.7).

Table 13.13 Bending Moments (ft-kips) due to Service Dead and Live Loads at the Second-Floor Level for the Flat Plate System in Example 13.9

Design Strip	Exterior Negative	Positive	First Interior Negative
Column strip	$M_D^- = -48.4$ $M_L^- = -24.4$	$M_D^+ = 57.7$ $M_L^+ = 29.1$	$M_D^- = -98.6$ $M_L^- = -49.8$
Middle strip	0	$M_D^+ = 39.1$ $M_L^+ = 19.7$	$M_D^- = -31.6$ $M_L^- = -16.0$

The bending moments due to earthquake loads are assigned to the column strip and are given in Table 13.14 assuming all frames in the direction of analysis are part of the SFRS. The “plus-minus” sign preceding the tabulated values signifies the earthquake loads can act in both the north direction and the south direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the east elevation of the building). The bending moments due to wind loads are smaller than those due to the earthquake loads and are not considered in this example.

Table 13.14 Bending Moments (ft-kips) due to Earthquake Loads at the Second-Floor Level for the Flat Plate System in Example 13.9

Exterior Negative	Positive	First Interior Negative
± 38.0	—	± 31.4

The design bending moments from the governing load combinations are given in Table 13.15 (see Table 13.2).

Table 13.15 Design Bending Moments (ft-kips) at the Second-Floor Level for the Flat Plate System in Example 13.9

Load Combination		Location		Exterior Negative	Positive	First Interior Negative
ACI Eq. (5.3.1a)	$1.4D$	Column strip		-67.8	80.8	-138.0
		Middle strip		0	54.7	-44.2
ACI Eq. (5.3.1b)	$1.2D + 1.6L$	Column strip		-97.1	115.8	-198.0
		Middle strip		0	78.4	-63.5
ACI Eq. (5.3.1e)	$1.24D + Q_E + 0.5L$	Column strip	SSR	-110.2	86.1	-115.8
			SSL	-34.2	86.1	-178.6
		Middle strip		0	58.3	-47.2
ACI Eq. (5.3.1g)	$0.86D + Q_E$	Column strip	SSR	-79.6	49.6	-53.4
			SSL	-3.6	49.6	-116.2
		Middle strip		0	33.6	-27.2

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Equation (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = Q_E + (0.2 \times 0.192)D = Q_E + 0.04D$ where, as noted in Section 3.5 of this publication, the redundancy factor, ρ, is permitted to be taken as 1.0 for buildings assigned to SDC C (ASCE/SEI 12.3.4.1). Therefore, this load combination becomes the following:

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + (Q_E + 0.04D) = 1.24D + Q_E + 0.5L$$

Similarly, in ACI Equation (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E + 0.2S_{DS}D = Q_E - (0.2 \times 0.192)D = Q_E - 0.04D$, and this load combination becomes the following:

$$0.9D + 1.0E = 0.9D + (Q_E - 0.04D) = 0.86D + Q_E$$

Step 2 – Determine the required flexural reinforcement at the critical sections

The required area of flexural reinforcement, A_s, at the critical sections is determined by the following equations, which are given in Section 6.5.1 of this publication for rectangular sections with a single layer of tension reinforcement where b is the width of the column strip or middle strip and $d = 9.5 - 1.25 = 8.25$ in.:

$$R_n = \frac{M_u}{\phi b d^2}$$

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right]$$

A summary of the required reinforcement is given in Table 13.16. It is evident that all sections are tension-controlled because $A_s < A_{s,t}$.

Table 13.16 Required Flexural Reinforcement at the Second-Floor Level for the Flat Plate System in Example 13.9

Location		M_u (ft-kips)	b (in.)	R_n (psi)	A_s (in.^{2})*	Reinforcement*
Column strip	Exterior Negative	−110.2	141.0	153	3.04	10-#5
	Positive	115.8	141.0	161	3.20	11-#5
	First Interior Negative	−198.0	141.0	275	5.57	18-#5
Middle strip	Exterior Negative	0	159.0	0	2.72	9-#5
	Positive	78.4	159.0	97	2.72	9-#5
	First Interior Negative	−63.5	159.0	78	2.72	9-#5

* *Min.* A_s *for the column strip* $= 0.0018 \times 141.0 \times 9.5 = 2.41$ in.^{2}
Min. A_s *for the middle strip* $= 0.0018 \times 159.0 \times 9.5 = 2.72$ in.^{2}
Max. spacing = lesser of $(2h,\ 18.0$ in.$) = 18.0$ in.
For $b = 141.0$ in., $141.0 / 18.0 = 7.8$ spaces, say minimum of 8 bars
For $b = 159.0$ in., $159.0 / 18.0 = 8.8$ spaces, say minimum of 9 bars
For the column strip: $A_{s,t} = 0.316\beta_1 f_c' bd / f_y = 20.8$ in.^{2}
For the middle strip: $A_{s,t} = 0.316\beta_1 f_c' bd / f_y = 23.5$ in.^{2}

Step 3 – Check that the flexural reinforcement is adequate for moment transfer requirements — ACI 8.4.2.2

- Edge columns

  Determine the required reinforcement based on the largest transfer moment at the edge columns.

  $M_{sc} = 110.2$ ft-kips — Table 13.16

  $$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.62$$ ACI Eq. (8.4.2.2.2)

  where

  $b_1 = c_1 + (d/2) = 24.0 + (8.25/2) = 28.13$ in. — Table 5.11, Case 3

  $b_2 = c_2 + d = 24.0 + 8.25 = 32.25$ in.

  Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

The moment $\gamma_f M_{sc} = 0.62 \times 110.2 = 68.3$ ft-kips must be transferred over the effective slab width b_{slab}:

$b_{slab} = c_2 + 3h = 24.0 + (3 \times 9.5) = 52.5$ in., say 52.0 in. ACI 18.4.5.2

Determine the required A_s:

$$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{68.3 \times 12}{0.9 \times 52.0 \times 8.25^2} = 0.257 \text{ ksi}$$

$$A_s = \frac{0.85 f'_c b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right] = \frac{0.85 \times 4 \times 52.0 \times 8.25}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.257}{0.85 \times 4}}\right] = 1.91 \text{ in.}^2$$

This required area of steel is equivalent to 7-#5 bars. Provide the 7-#5 bars by concentrating 7 of the 10-#5 column strip bars within the 52.0-in. width over the column (see Table 13.16). For symmetry, add 1-#5 bar and check the bar spacing:

For 7-#5 within the 52.0-in. width, $\text{bar spacing} = 52.0 / 7 = 7.4 \text{ in.} < 18.0 \text{ in.}$

For 4-#5 within the $141.0 - 52.0 = 89.0\text{-in. width, bar spacing} = 89.0 / 4 = 22.3 \text{ in.} > 18.0 \text{ in.}$

Therefore, provide 2 additional bars within the 89.0-in. width to satisfy maximum spacing requirements. A total of 13-#5 bars is required at the edge columns within the column strip, with 7 of the 13-#5 bars concentrated within a width of 52.0 in.

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2 q_D + 1.6 q_L = \left[1.2 \times (118.8 + 10.0)\right] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$$

$$d = 9.5 - 1.25 = 8.25 \text{ in.}$$

$$A_c = (2b_1 + b_2)d = [(2 \times 28.13) + 32.25)] \times 8.25 = 730.2 \text{ in.}^2$$ Table 5.11, Case 3

Factored shear force at the critical section:

$$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) = 258.6 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 32.25}{144}\right)\right\} / 1{,}000 = 80.8 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_u}{A_c} = \frac{80{,}800}{730.2} = 110.7 \text{ psi} > \phi 2 \lambda_s \lambda \sqrt{f'_c} = 0.75 \times 2 \times 1.0 \times 1.0 \sqrt{4{,}000} = 94.9 \text{ psi}$$ ACI 8.6.1.2

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (8.25 / 10)}} = 1.1 > 1.0, \text{ use } 1.0$$ ACI Eq. (22.5.5.1.3)

and

$\lambda = 1.0$ for normalweight concrete

Thus,

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2 \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi\alpha_s f_y} = \dfrac{5 \times 110.7 \times 52.0 \times [(2 \times 28.13) + 32.25)]}{0.75 \times 30 \times 60{,}000} = 1.89 \text{ in.}^2 \end{cases}$$ ACI 8.6.1.2

Provided A_s within $b_{slab} = 7 \times 0.31 = 2.17 \text{ in.}^2 > A_{s,min} = 1.89 \text{ in.}^2$

(2) Load combination: $1.24D + Q_E + 0.5L$

$$q_u = 1.24q_D + 0.5q_L = [1.24 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 192.2 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 192.2 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 32.25}{144}\right)\right\} / 1{,}000 = 60.1 \text{ kips}$$

$$V_{u(Q_E)} = (38.0 + 31.4) / 21.5 = 3.2 \text{ kips}$$ Table 13.14

Therefore,

$$v_{uv} = \frac{V_{u(D+L)} + V_{u(Q_E)}}{A_c} = \frac{60{,}100 + 3{,}200}{730.2} = 86.7 \text{ psi}$$

ACI 8.6.1.2

$$< \phi 2\lambda_s\lambda\sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0\sqrt{4{,}000} = 94.9 \text{ psi}$$

Provided A_s within $b_{slab} = 7 \times 0.31 = 2.17 \text{ in.}^2 > A_{s,min} = 0.89 \text{ in.}^2$

According to ACI 18.4.5.3, at least one-half of the reinforcement in the column strip at the support must be placed within the 52.0-in. effective slab width. Providing 7 of the 13-#5 bars within the effective slab width satisfies this requirement.

Reinforcement details for the top bars at the edge columns are given in Figure 13.27.

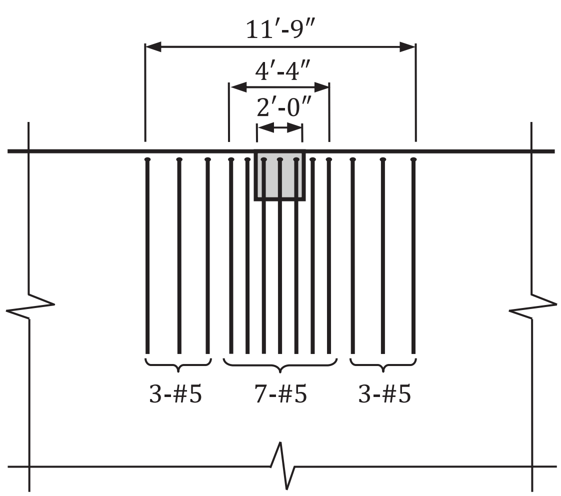


Figure 13.27 Reinforcement details for the top reinforcing bars at the edge column in the flat plate system in Example 13.9.

- Interior columns

Determine reinforcement based on the largest transfer moment at the interior columns, which occurs for the load combination $1.24D + Q_E + 0.5L$.

In lieu of a more exact analysis, the factored slab moment, M_{sc}, transferred to an interior column due to the gravity load effects can be determined from the following equation where the spans in the direction of analysis and perpendicular to the direction of analysis are equal:

$$M_{sc} = 0.035 q_{Lu} \ell_2 \ell_n^2 = 0.035 \times (0.5 \times 65.0) \times 25.0 \times 21.5^2 / 1{,}000 = 13.2 \text{ ft-kips}$$ Eq. (5.20)

At the first interior column, $M_{sc} = 31.4 + 28.1 = 59.5$ ft-kips due to seismic load effects.

Therefore, total $M_{sc} = 13.2 + 59.5 = 72.7$ ft-kips

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}} = 0.60$$ ACI Eq. (8.4.2.2.2)

where

$$b_1 = c_1 + d = 24.0 + 8.25 = 32.25 \text{ in.}$$ Table 5.11, Case 1

$$b_2 = c_2 + d = 24.0 + 8.25 = 32.25 \text{ in.}$$

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

The moment $\gamma_f M_{sc} = 0.60 \times 72.7 = 43.6$ ft-kips must be transferred over the effective slab width b_{slab}:

$$b_{slab} = c_2 + 3h = 24.0 + (3 \times 9.5) = 52.5 \text{ in., say } 52.0 \text{ in.}$$ ACI Table 8.4.2.2.3

Determine the required A_s:

$$R_n = \frac{\gamma_f M_{sc}}{\phi b d^2} = \frac{43.6 \times 12}{0.9 \times 52.0 \times 8.25^2} = 0.164 \text{ ksi}$$

$$A_s = \frac{0.85 f_c' b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right] = \frac{0.85 \times 4 \times 52.0 \times 8.25}{60} \times \left[1 - \sqrt{1 - \frac{2 \times 0.164}{0.85 \times 4}}\right] = 1.20 \text{ in.}^2$$

This required area of steel is equivalent to 4-#5 bars. With a uniform bar spacing in the column strip, 6 of the 18-#5 bars are within the 52.0-in. effective slab width at the first interior columns, so no additional reinforcement is required to satisfy moment transfer requirements at these locations.

Check $A_{s,min}$ within b_{slab}: ACI 8.6.1.2

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2 q_D + 1.6 q_L = \left[1.2 \times (118.8 + 10.0)\right] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$$

$$A_c = 2(b_1 + b_2)d = 2 \times (2 \times 32.25) \times 8.25 = 1{,}064.3 \text{ in.}^2$$ Table 5.11, Case 1

Factored shear force at the critical section:

$$V_u = q_u(\ell_1 \ell_2 - b_1 b_2) = 258.6 \times \left[(23.5 \times 25.0) - \left(\frac{32.25}{12}\right)^2\right] / 1{,}000 = 150.1 \text{ kips}$$

Therefore,

$$v_{uv} = \frac{V_u}{A_c} = \frac{150{,}100}{1{,}064.3} = 141.0 \text{ psi} > \phi 2\lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0\sqrt{4{,}000} = 94.9 \text{ psi}$$

where

$$\lambda_s = \sqrt{\frac{2}{1 + (d / 10)}} = \sqrt{\frac{2}{1 + (8.25 / 10)}} = 1.1 > 1.0, \text{ use } 1.0$$ ACI Eq. (22.5.5.1.3)

and

$\lambda = 1.0$ for normalweight concrete

Thus,

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2 \\ \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi \alpha_s f_y} = \dfrac{5 \times 141.0 \times 52.0 \times [2 \times (2 \times 32.25)]}{0.75 \times 40 \times 60{,}000} = 2.63 \text{ in.}^2 \end{cases}$$ ACI 8.6.1.2

At the first interior columns: provided A_s within $b_{slab} = 6 \times 0.31 = 1.86 \text{ in.}^2 < A_{s,min} = 2.63 \text{ in.}^2$

(2) Load combination: $1.24D + Q_E + 0.5L$

$$q_u = 1.24q_D + 0.5q_L = \left[1.24 \times (118.8 + 10.0)\right] + (0.5 \times 65.0) = 192.2 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1 \ell_2 - b_1 b_2) = 192.2 \times \left[(23.5 \times 25.0) - \left(\frac{32.25}{12}\right)^2\right] / 1{,}000 = 111.5 \text{ kips}$$

At the first interior columns: $V_{u(Q_E)} = (38.0 + 31.4) / 21.5 = 3.2$ kips

Therefore, maximum shear stress is equal to the following:

$$v_{uv} = \frac{V_{u(D+L)} + V_{u(Q_E)}}{A_c} = \frac{111{,}500 + 3{,}200}{1{,}064.3}$$

$$= 107.8 \text{ psi} > \phi 2\lambda_s \lambda \sqrt{f_c'} = 0.75 \times 2 \times 1.0 \times 1.0\sqrt{4{,}000} = 94.9 \text{ psi}$$ ACI 8.6.1.2

Thus,

$$A_{s,min} = \text{greater of} \begin{cases} 0.0018hb_{slab} = 0.0018 \times 9.5 \times 52.0 = 0.89 \text{ in.}^2 \\ \\ \dfrac{5v_{uv}b_{slab}b_o}{\phi \alpha_s f_y} = \dfrac{5 \times 107.8 \times 52.0 \times [2 \times (2 \times 32.25)]}{0.75 \times 40 \times 60{,}000} = 2.01 \text{ in.}^2 \end{cases}$$

It is evident that the $1.2D + 1.6L$ load combination governs for the determination of minimum reinforcement.

To satisfy minimum reinforcement requirements within b_{slab}, 9-#5 bars must be placed within the 52.0-in. effective slab width $(A_{s,provided} = 9 \times 0.31 = 2.79 \text{ in.}^2 > A_{s,min} = 2.63 \text{ in.}^2)$ at the first interior columns, with the remaining 9-#5 bars to be placed within the $141.0 - 52.0 = 89.0\text{-in.}$ width of the column strip. For symmetry, add 1-#5 bar to the 89.0-in. width. Thus, at least 19-#5 bars must be provided within the column strip at the first interior columns with 9-#5 bars concentrated within the 52.0-in. effective slab width. According to ACI 18.4.5.3, at least one-half of the reinforcement in the column strip must be placed within the 52.0-in. effective slab width. Providing 9 of the 19-#5 bars within the effective slab width does not satisfy this requirement. Therefore, provide 10-#5 within the 52.0-in. effective width and 10-#5 bars within the 89.0-in. width of the column strip. The reinforcement detail for this case is shown in Figure 13.28.

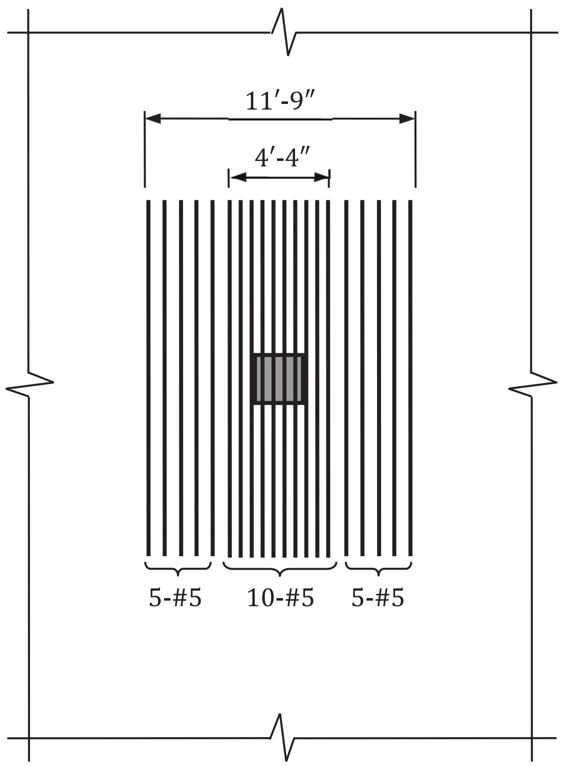


Figure 13.28 Reinforcement details for the top reinforcing bars at the first interior columns in the flat plate system in Example 13.9.

Step 4 – Determine the reinforcement details

The lengths of the top reinforcing bars in ACI Figure 8.7.4.1.3 cannot be used in the column strip because of the effects due to earthquake loads. According to ACI 18.4.5.4, at least 25 percent of the top bars in the column strips must be made continuous over the spans. In this case, the maximum amount of top reinforcement occurs at the first interior supports (20-#5 bars), so 5-#5 bars are made continuous.

The remaining bars in the column strip and the bars in the middle strip can be terminated at the locations identified in ACI Figure 8.7.4.1.3; this figure also includes the structural integrity requirements of ACI 8.7.4.2.

A Class B tension lap splice is provided over the supports for the bottom bars in the column strip. The required lap splice length is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.} \qquad \text{ACI Eq. (25.4.2.4a)}$$

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #5 reinforcing bars, $\psi_s = 0.8$

For less than 12 in. of fresh concrete cast below the positive reinforcement, $\psi_t = 1.0$

$$c_b = \text{lesser of } \begin{cases} \text{cover} + (d_b / 2) = 0.75 + (0.625 / 2) = 1.1 \text{ in.} \\ \\ s / 2 = 12.8 / 2 = 6.4 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (1.1 + 0) / 0.625 = 1.8 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 0.8 \times 1.0}{1.8} \right) \times 0.625 = 19.8 \text{ in.} > 12.0 \text{ in.}$$

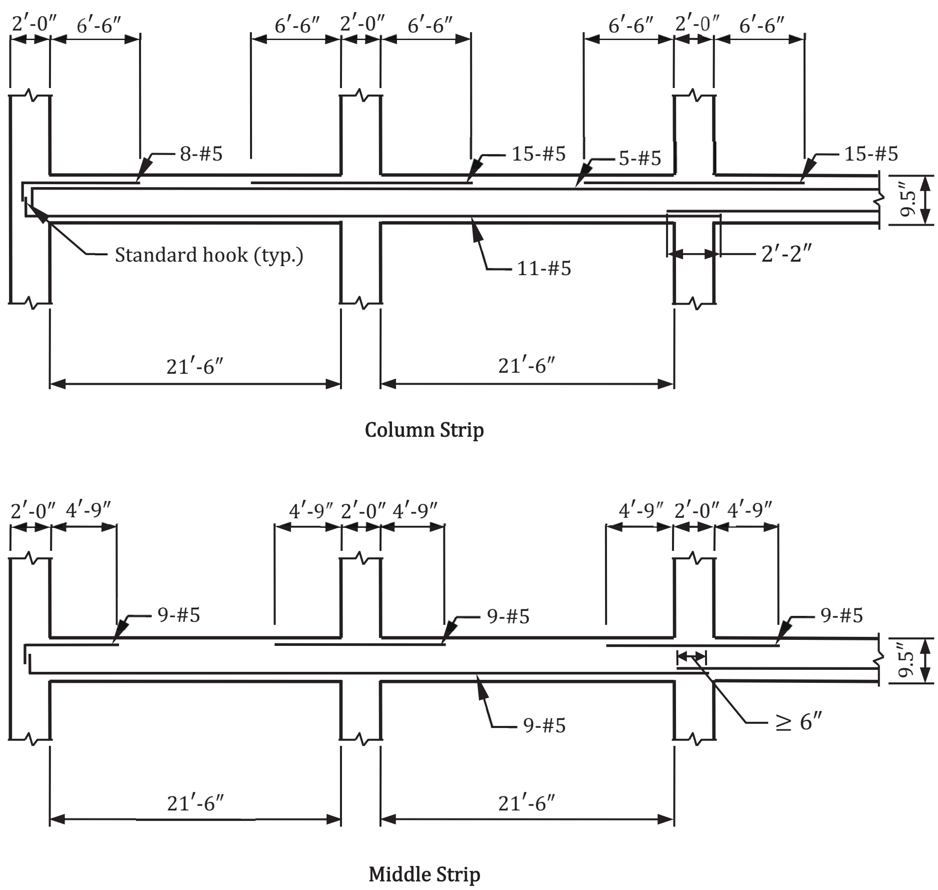


Figure 13.29 Reinforcement details for the flat plate system in Example 13.9.

$\text{Class B lap splice length} = 1.3\ell_d = 1.3 \times 19.8 = 25.7 \text{ in.}$ ACI Table 25.5.2.1

Provide a 2 ft-2 in. lap splice length.

Reinforcement details for the columns strip and middle strip in accordance with the provisions of ACI 18.4.5.4 through 18.4.5.7 are given in Figure 13.29 (see Figure 13.18).

13.5.10 Example 13.10 – Check of Two-way Shear Strength Requirements: Two-way Slab in Building #1 (Framing Option A), Two-way Slab is Part of the SFRS (Intermediate Moment Frame), SDC C

Check the two-way shear strength requirements at the edge columns in the interior design strip along column line 3 in Building #1, Framing Option A, at the second-floor level assuming the two-way slab is part of the SFRS (see Figure 1.1). The slab is 9.5 in. thick and the columns are 24.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000 \text{ psi}$ and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. The building is assigned to SDC C where $S_{DS} = 0.192$.

Step 1 – Check two-way shear strength requirements ACI 8.4.4

$\text{Dead load of slab} = (9.5 / 12) \times 150.0 = 118.8 \text{ lb/ft}^2$

$\text{Superimposed dead load} = 10.0 \text{ lb/ft}^2$

$\text{Live load} = 65.0 \text{ lb/ft}^2$

The critical section is located a distance $d / 2$ from the face of the column.

(1) Load combination: $1.2D + 1.6L$

$$q_u = 1.2q_D + 1.6q_L = [1.2 \times (118.8 + 10.0)] + (1.6 \times 65.0) = 258.6 \text{ lb/ft}^2$$

$$d = 9.5 - 1.25 = 8.25 \text{ in.}$$

$$b_1 = c_1 + (d / 2) = 24.0 + (8.25 / 2) = 28.13 \text{ in.}$$

$$b_2 = c_2 + d = 24.0 + 8.25 = 32.25 \text{ in.}$$

Factored shear force at the critical section:

$$V_u = q_u(\ell_1\ell_2 - b_1b_2) = 258.6 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 32.25}{144}\right)\right\} / 1{,}000 = 80.8 \text{ kips}$$

For the gravity load combination where the Direct Design Method (DDM) is used to determine design bending moments, $M_{sc} = 0.30M_o$ (see Sect. 5.3.4 of this publication).

$$\text{Dead load: } M_{oD} = \frac{q_D\ell_2\ell_n^2}{8} = \frac{(118.8 + 10.0) \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 186.1 \text{ ft-kips}$$

$$\text{Live load: } M_{oL} = \frac{q_L\ell_2\ell_n^2}{8} = \frac{65.0 \times 25.0 \times 21.5^2}{8 \times 1{,}000} = 93.9 \text{ ft-kips}$$

$$M_o = 1.2M_{oD} + 1.6M_{oL} = (1.2 \times 186.1) + (1.6 \times 93.9) = 373.6 \text{ ft-kips}$$

$$M_{sc} = 0.30 \times 373.6 = 112.1 \text{ ft-kips}$$

$$A_c = (2b_1 + b_2)d = [(2 \times 28.13) + 32.25)] \times 8.25 = 730.2 \text{ in.}^2$$ Table 5.11, Case 3

Determining whether γ_f can be increased to the values in ACI Table 8.4.2.2.4 is not considered in this example.

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} = 1 - 0.62 = 0.38$$ ACI Eq. (8.4.4.2.2)

$$\frac{J_c}{c_{AB}} = \frac{2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)}{6b_1} = 7,460 \text{ in.}^3$$ Table 5.11, Case 3

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{80,800}{730.2} + \frac{0.38 \times 112.1 \times 12,000}{7,460} = 110.7 + 68.5 = 179.2 \text{ psi}$$

$$\phi v_c = \text{ least of } \begin{cases} \phi 4\lambda_s \lambda\sqrt{f'_c} = 189.7 \text{ psi} > v_{u|AB} = 179.2 \text{ psi} \\ \phi\left(2 + \dfrac{4}{\beta}\right)\lambda_s\lambda\sqrt{f'_c} = 284.6 \text{ psi} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c} = 227.5 \text{ psi} \end{cases}$$ ACI Table 22.6.5.2

where $\beta = 24.0 / 24.0 = 1.0,\ \alpha_s = 30,$ and $b_o = (2 \times 28.13) + 32.25 = 88.5$ in.

The factored two-way shear stress caused by factored gravity loads without moment transfer must be less than or equal to $0.4\phi v_c$ (ACI 18.4.5.8):

$$v_{uv} = \frac{V_u}{A_c} = \frac{80,800}{730.2} = 110.7 \text{ psi} > 0.4\phi v_c = 0.4 \times 189.7 = 75.9 \text{ psi}$$

In lieu of increasing the column size at this point, determine if the requirements of ACI 18.14.5 are satisfied. This check is given in Step 2 below.

(2) Load combination: $1.24D + Q_E + 0.5L$

$$q_u = 1.24q_D + 0.5q_L = [1.24 \times (118.8 + 10.0)] + (0.5 \times 65.0) = 192.2 \text{ lb/ft}^2$$

$$V_{u(D+L)} = q_u(\ell_1\ell_2 - b_1b_2) = 192.2 \times \left\{\left[25.0 \times \left(\frac{23.5}{2} + \frac{24.0}{2 \times 12}\right)\right] - \left(\frac{28.13 \times 32.25}{144}\right)\right\} / 1,000 = 60.1 \text{ kips}$$

$$V_{u(Q_E)} = (38.0 + 31.4) / 21.5 = 3.2 \text{ kips}$$ Table 13.14

Total factored shear force:

$$V_u = V_{u(D+L)} + V_{u(Q_E)} = 60.1 + 3.2 = 63.3 \text{ kips}$$

For other than gravity load combinations, M_{sc} is equal to that from the actual load combination:

$M_{sc} = 110.2$ ft-kips — Table 13.15

Total factored shear stress:

$$v_{u|AB} = \frac{V_u}{A_c} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = \frac{63{,}300}{730.2} + \frac{0.38 \times 110.2 \times 12{,}000}{7{,}460} = 86.7 + 67.4 = 154.1 \text{ psi}$$

$$\phi v_c = \phi 4 \lambda_s \lambda \sqrt{f'_c} = 189.7 \text{ psi} > v_{u|AB} = 154.1 \text{ psi}$$

The factored two-way shear stress caused by factored gravity loads without moment transfer must be less than or equal to $0.4\phi v_c$ (ACI 18.4.5.8):

$$v_{uv} = \frac{V_u}{A_c} = \frac{63{,}300}{730.2} = 86.7 \text{ psi} > 0.4\phi v_c = 0.4 \times 189.7 = 75.9 \text{ psi}$$

Step 2 – Check two-way shear strength requirements of ACI 18.14.5

Because $v_{uv} > 0.4\phi v_c$, determine if the requirements of ACI 18.14.5 are satisfied. — ACI 18.4.5.8

From analysis, lateral displacement at the second-floor level due to the code-prescribed seismic forces is $\delta_{xe} = 0.46$ in.

Design displacement $\Delta_x = C_d \delta_{xe} / I_e = (4.5 \times 0.46)/1.0 = 2.1$ in. — ASCE/SEI Eq. (12.8-15)

where C_d is the deflection amplification factor, which is equal to 4.5 for intermediate moment frames (see Table 12.3 of this publication).

$$\frac{\Delta_x}{h_{sx}} = \frac{2.1}{14.0 \times 12} = 0.0125 > 0.035 - \frac{v_{uv} / \phi v_c}{20} = 0.035 - \frac{86.7 / 189.7}{20} = 0.0121$$

Therefore, provide shear reinforcement in accordance with ACI 18.14.5.3.

Required shear reinforcement must provide $v_s \geq 3.5\sqrt{f'_c} = 221.4$ psi.

Try ¾-in. diameter headed shear studs ($A_b = 0.442$ in.^{2}) with $f_{yt} = 51{,}000$ psi.

In the direction parallel to the column face, the maximum spacing between adjacent lines of headed shear studs $= 2d = 16.5$ in. (ACI Table 8.7.7.1.2).

For a 24-in.-wide column, use 3 lines of headed shear studs on each face of the column:

Maximum spacing $\cong 24.0 / 2 = 12.0$ in. < 16.5 in.

$b_o = 2b_1 + b_2 = (2 \times 28.13) + 32.25 = 88.5$ in.

In the direction perpendicular to the column face, maximum spacing $= 0.75d = 6.2$ in. (ACI Table 8.7.7.1.2).

Therefore,

$$v_s = \frac{A_b f_{yt}}{b_o s} = \frac{(9 \times 0.442) \times 51{,}000}{88.5 \times 6.0} = 382.1 \text{ psi} > 3.5\sqrt{f'_c} = 221.4 \text{ psi}$$

In accordance with ACI 18.14.5.3, the headed shear studs must extend at least $4h = 38.0$ in. from the face of the column. Provide 7 headed shear studs in each line on each face of the column.

Headed shear stud reinforcement details for the edge columns are given in Figure 13.30.

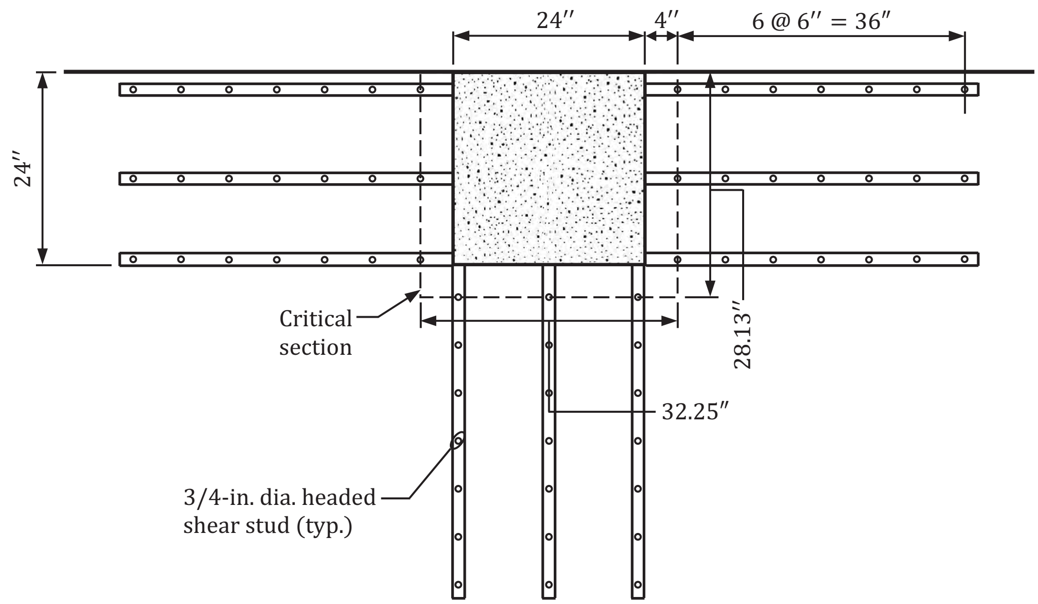


Figure 13.30 Headed shear stud reinforcement at the edge columns in Example 13.10.

13.5.11 Example 13.11 – Design of Foundation Seismic Tie: Column in Building #1 (Framing Option B), Column is Part of the SFRS (Intermediate Moment Frame), SDC C

Design a foundation seismic tie for the caisson supporting column D1 in Building #1, Framing Option B (see Figure 1.1). Assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. The building is assigned to SDC C where $S_{DS} = 0.192$.

Step 1 – Determine the seismic tie force ACI 18.13.4.3

The seismic tie must have a design strength in tension and compression of at least $0.1S_{DS}$ times the factored dead load plus factored live load in accordance with the load combination on column D1 that includes earthquake effects (assuming the axial forces on the adjacent columns are less than or equal to that on column D1).

From Table 13.11 in Example 13.5, the largest axial force corresponds to the load combination in ACI Eq. (5.3.1e) for sidesway to the right:

$$P_u = 1.24D + Q_E + 0.5L = 429.3 \text{ kips}$$

Therefore, the required axial tension and compression force in the seismic tie $= 0.1 \times 0.192 \times 429.3 = 8.2$ kips

Step 2 – Determine the required reinforcement in the seismic tie ACI 18.13.4.3

Assume the seismic tie is a 12 in. by 24 in. reinforced concrete beam and the tie beam is not subjected to any gravity or seismic load effects transmitted by the column or any other structural members.

Also assume the tie beam has a longitudinal reinforcement ratio $= 0.005$.

Thus, $A_{st} = 0.005 \times 12.0 \times 24.0 = 1.44 \text{ in.}^2$

Try 4-#6 longitudinal bars $(A_{st,provided} = 1.76 \text{ in.}^2)$.

Required longitudinal reinforcement for the axial tension force in the seismic tie $= \dfrac{8.2}{0.9 \times 60} = 0.15 \text{ in.}^2 < 1.76 \text{ in.}^2$

Assuming the tie beam has transverse reinforcement consisting of ties conforming to ACI 22.4.2.4, the design axial strength, $\phi P_{n,max}$, of the section is determined using ACI Table 22.4.2.1 and ACI Eq. (22.4.2.2):

$$\phi P_{n,max} = \phi 0.80[0.85 f_c'(A_g - A_{st}) + f_y A_{st}] = 561.0 \text{ kips} > 8.2 \text{ kips}$$

Use 4-#6 longitudinal bars with #3 ties spaced 12 in. on center over the length of the seismic tie beam.

Reinforcement details for the seismic tie beam in the east-west direction are similar to those in Figure 13.19. The #6 longitudinal bars are developed for tension and compression in the caisson cap.

Chapter 14
EARTHQUAKE-RESISTANT STRUCTURES — SDC D, E and F

14.1 Overview

This chapter covers the design and detailing requirements in ACI Chapter 18 for the following:

- Frame members (ACI 18.6 through 18.9)
- Structural walls and coupling beams (ACI 18.10)
- Diaphragms and trusses (ACI 18.12)
- Foundations (ACI 18.13)
- Frame members not designated as part of the seismic-force-resisting system (SFRS) [ACI 18.14]

Gravity and lateral load effects on structural members are obtained from an analysis method in ACI 6.2 considering the analysis and proportioning requirements in ACI 18.2.2. Design strength load combinations for structures assigned to SDC D, E, or F are given in Table 14.1 for gravity, wind, and seismic load effects (see Section 3.5 of this publication).

Table 14.1 Strength Design Load Combinations for Structures Assigned to SDC D, E, or F

ACI Equation Number	ASCE/SEI 7 Load Combination	Load Combination
5.3.1a	1	$U = 1.4D$
5.3.1b	2	$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
5.3.1c	3	$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$
5.3.1d	4	$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
5.3.1e	6	$U = (1.2 + 0.2S_{DS})D + \rho Q_E + 1.0L + 0.2S$
5.3.1f	5	$U = 0.9D + 1.0W$
5.3.1g	7	$U = (0.9 - 0.2S_{DS})D + \rho Q_E$

The seismic load combinations for the design of structural members where seismic load effects including overstrength are required are given in Table 14.2.

Table 14.2 Seismic Load Combinations Where Seismic Loads Including Overstrength are Required

ACI Equation Number	ASCE/SEI 7 Load Combination	Load Combination
5.3.1e	6	$U = (1.2 + 0.2S_{DS})D + \Omega_o Q_E + 1.0L + 0.2S$
5.3.1g	7	$U = (0.9 - 0.2S_{DS})D + \Omega_o Q_E$

Strength reduction factors (ACI 18.2.4) and requirements for concrete (ACI 18.2.5) and reinforcement (ACI 18.2.6) in special moment frames and special structural walls are given in Chapter 2 of this publication. Requirements for mechanical and welded splices in special moment frames and special structural walls are given in ACI 18.2.7 and 18.2.8, respectively.

14.2 Beams of Special Moment Frames

14.2.1 Overview

Design and detailing requirements for beams designated part of the SFRS in special moment frames are given in ACI 18.6. The provisions in this section have been developed based on the assumption that beams in a special moment frame are designed primarily for flexure and shear (ACI 18.6.1.1).

It is also assumed that a special moment frame consists of horizontal beams framing into vertical columns where the columns satisfy the requirements of ACI 18.7 (ACI 18.6.1.2). It is acceptable for the beams and columns to be inclined from the horizontal and vertical, respectively, provided the system behaves as a frame (that is, lateral resistance is provided primarily by moment transfer between beams and columns rather than by other means, such as strut or brace action). It is also acceptable for beams of special moment frames to (1) be designed as chords or collectors of a diaphragm (that is, be designed to resist combined moment and axial forces) and (2) frame into a boundary element of a wall provided the boundary is reinforced as a column in a special moment frame in accordance with ACI 18.7. Beams that cantilever beyond columns of special moment frames are not considered to be part of the SFRS.

14.2.2 Dimensional Limits

The required size of a beam in a special moment frame for strength and serviceability are determined using the provisions in ACI Chapter 9 for the combined effects due to gravity and lateral forces (see Chapter 6 of this publication) and the dimensional limits in ACI 18.6.2.1. The requirements in ACI 18.6.2.1 are given in Figure 14.1.

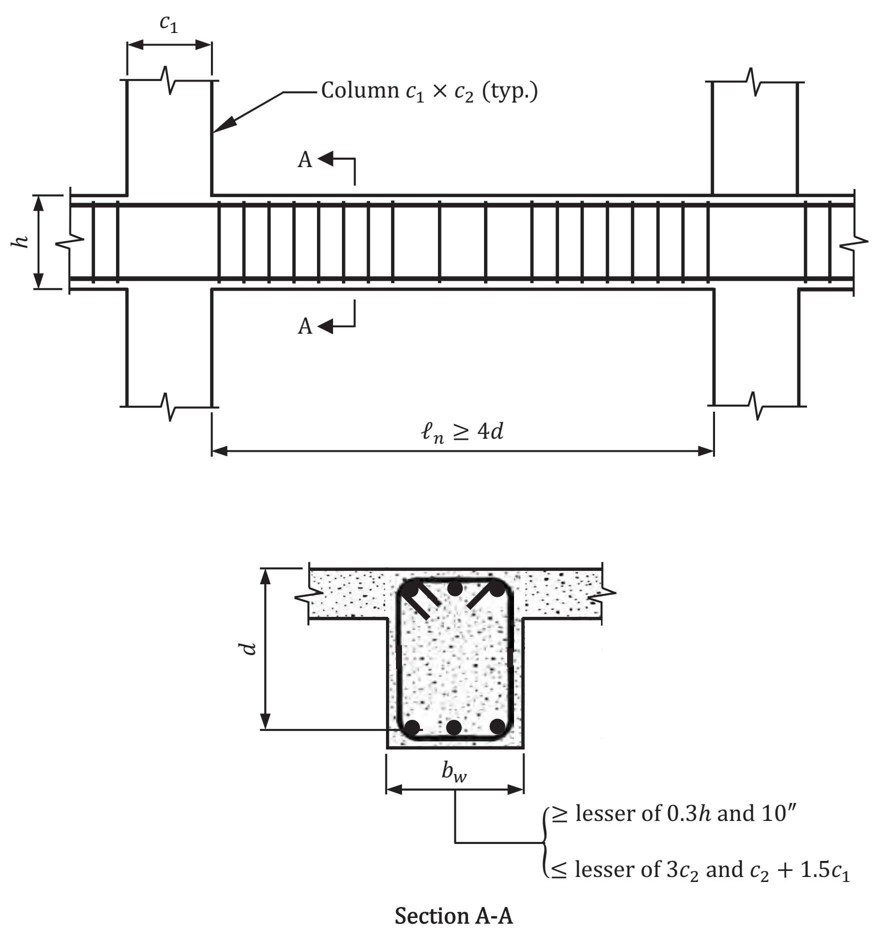


Figure 14.1 Dimensional limits for beams in special moment frames.

For beams wider than the columns that they frame into, the maximum beam width that can effectively transfer forces into a beam-column joint is the lesser of $3c_2$ and $c_2 + 1.5c_1$ (see ACI 18.6.2.1(c) and ACI Figure R18.6.2). Transverse reinforcement is required in the portions of the beam on either side of the column and through the column to confine the longitudinal reinforcement in the beam passing outside of the column core.

14.2.3 Longitudinal Reinforcement

Determining the Required Flexural Reinforcement

Once the cross-sectional dimensions of the beam have been determined, the required area of flexural reinforcement can be calculated at the critical sections for the combined effects due to gravity and lateral forces using the provisions in ACI Chapter 9 (see Chapter 6 of this publication). In addition to the flexural requirements in ACI Chapter 9, the requirements in ACI 18.6.3.1 and 18.6.3.2 must also be satisfied (see Figure 14.2).

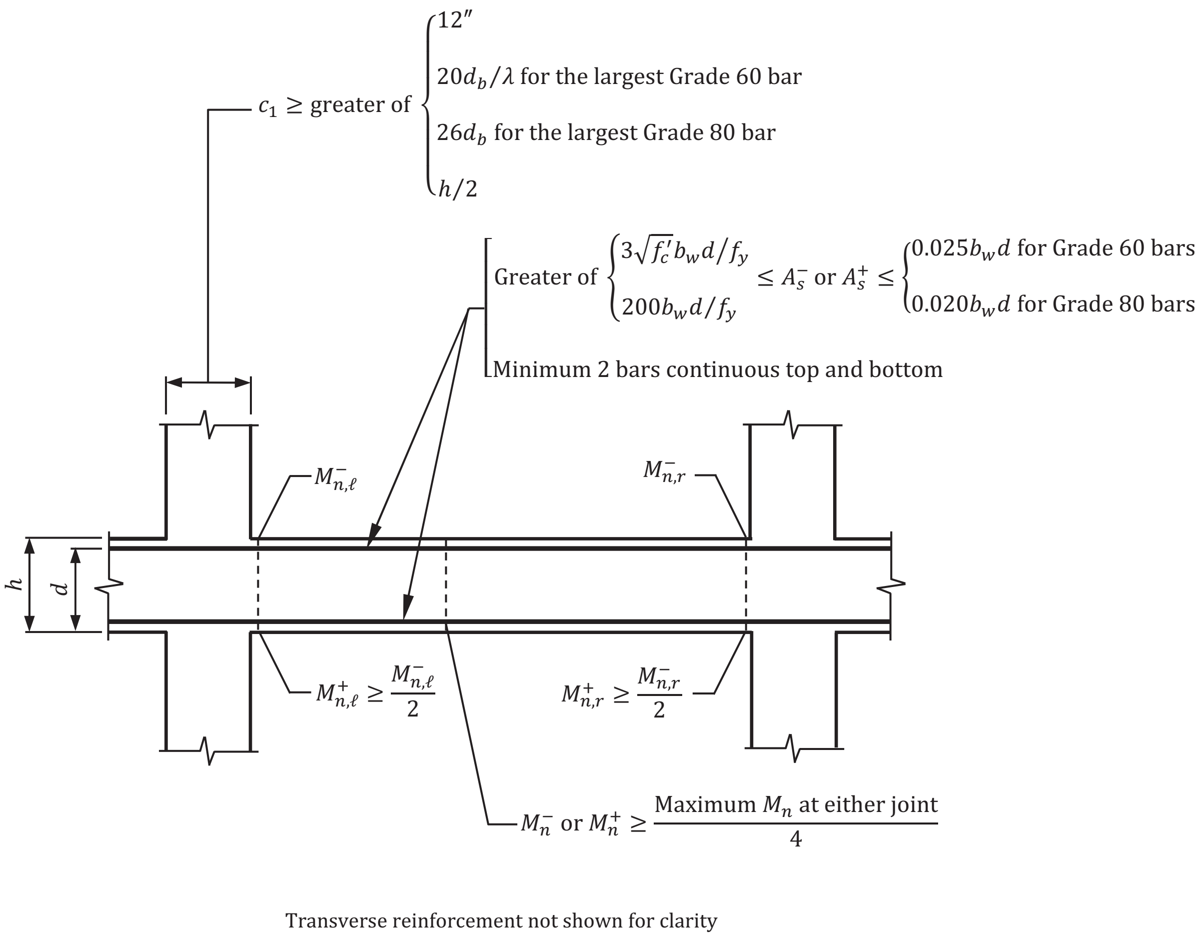


Figure 14.2 Flexural requirements for beams in special moment frames.

The minimum reinforcement must be in accordance with ACI 9.6.1.2, and the upper limit for the amount of flexural reinforcement is $\rho = 0.025$ for Grade 60 reinforcement and $\rho = 0.020$ for Grade 80 reinforcement (ACI 18.6.3.1). The purpose of the maximum reinforcement ratio is to help reduce congestion and to limit shear stresses. A practical reinforcement ratio for beams in special moment frames is 0.010.

Requirements having a possible impact on the amount of positive flexural reinforcement in beams that are part of a special moment frame are given in ACI 18.6.3.2. At the faces of the joints, the positive moment strength, M_n^+, must be at least equal to 50 percent of the corresponding negative moment strength, M_n^-, at that joint. The area of positive flexural reinforcement may need to be increased to satisfy this requirement. Also, at any section along the length of a beam, the negative and positive moment strength must be equal to at least 25 percent of the maximum moment strength provided at the face of either joint. This requirement ensures strength and ductility under large lateral displacements.

Once the required area of flexural reinforcement has been determined, the size and number of reinforcing bars can be selected using Tables 6.8 and 6.9 of this publication. At least two continuous bars must be provided at both the top and bottom faces of a beam (ACI 18.6.3.1).

The requirements of ACI 18.8.2.3 must also be satisfied when selecting the size of the longitudinal bars in a beam extending through an interior joint in a special moment frame (see Figure 14.2): The dimension of the joint (column) parallel to the longitudinal reinforcement in a beam must be at least $(20 / \lambda)$ times the diameter of the largest longitudinal beam bar for Grade 60 reinforcement where λ is equal to 0.75 for lightweight concrete and 1.0 for all other cases. For Grade 80 reinforcement, the minimum joint dimension is 26 times the diameter of the largest longitudinal bar (normalweight concrete must be used in joints with Grade 80 reinforcement; see ACI 18.8.2.3.1). This means the size of the longitudinal bars in a beam must be selected such that $d_b \leq \lambda c_1 / 20$ for Grade 60 reinforcement and $d_b \leq c_1 / 26$ for Grade 80 reinforcement. The purpose of these requirements is to limit the amount of potential slip of the longitudinal bars in a beam-column joint during a design-level earthquake event, thereby ensuring adequate development of the bars. The minimum dimensions of $h / 2$ prescribed in ACI 18.8.2.3 and 12 in. prescribed in ACI 18.7.2.1(a) for columns of special moment frames are also given in Figure 14.2.

Detailing the Flexural Reinforcement

Development of Flexural Reinforcement

Beam longitudinal reinforcement terminated in a joint must extend to the far face of the joint core and must be developed in tension in accordance with the tension development length requirements of ACI 18.8.5 and in compression in accordance with ACI 25.4.9 (ACI 18.8.2.2).

Hooked Bars in Tension. An example of an edge or corner column where the longitudinal reinforcement from the beam is terminated in a standard 90-degree hook is illustrated in Figure 14.3. In accordance with ACI 18.8.5.1, the hook must be located within the confined core of the column and it must be bent into the joint. The critical section for development of the hooked bars is taken at the outside edge of the joint transverse reinforcement and not at the face of the column because the concrete outside of the confined core may spall during a design-level earthquake event.

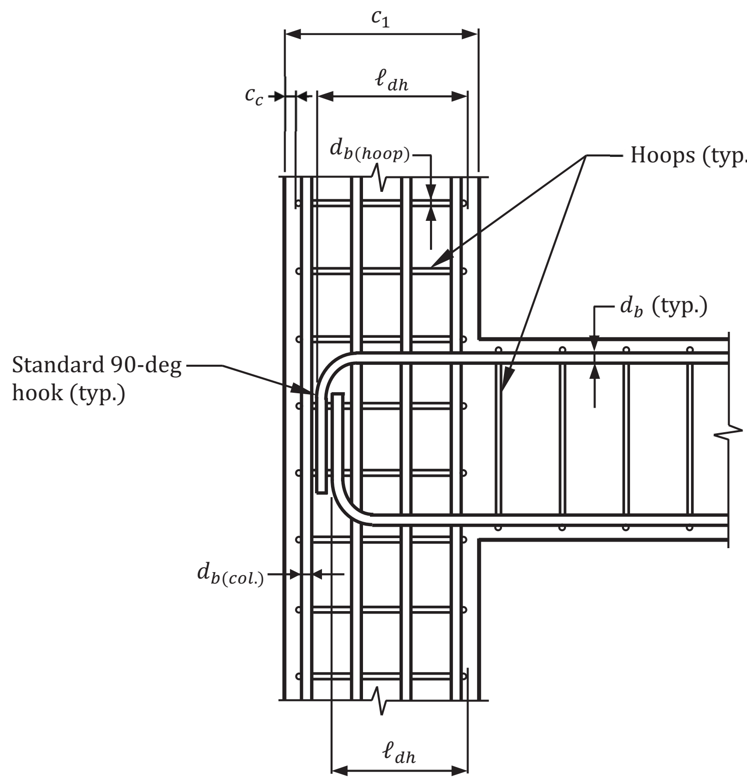
The development length of a deformed #3 through #11 reinforcing bar with a standard hook in tension, ℓ_{dh}, is determined by ACI Equation (18.8.5.1). The modification factor, λ, is equal to 0.75 for lightweight concrete and 1.0 otherwise. The following equations can be used to determine ℓ_{dh} based on the requirements in ACI 18.8.5.1:

For normalweight concrete:

$$\ell_{dh} = \text{greater of} \begin{cases} f_y d_b / 65\sqrt{f_c'} \\ 8d_b \\ 6 \text{ in.} \end{cases} \tag{14.1}$$

For lightweight concrete (Grade 60 reinforcement only):

$$\ell_{dh} = \text{greater of} \begin{cases} f_y d_b / (65 \times 0.75)\sqrt{f_c'} \\ 10d_b \\ 7.5 \text{ in.} \end{cases} \tag{14.2}$$



Normalweight concrete	ℓ_{dh} = greater of $\begin{cases} f_y d_b / 65\sqrt{f'_c} \\ 8d_b \\ 6'' \end{cases}$
Lightweight concrete*	ℓ_{dh} = greater of $\begin{cases} f_y d_b / (65 \times 0.75)\sqrt{f'_c} \\ 10d_b \\ 7.5'' \end{cases}$
For epoxy-coated or zinc and epoxy dual-coated bars, multiply ℓ_{dh} by 1.2.	

* Applicable to Grade 60 reinforcement only

Figure 14.3 Development of flexural reinforcement with standard hooks in beams of special moment frames.

These development lengths are measured from the outside edge of the joint transverse reinforcement to the outside face of the hook (see Figure 14.3). In order for the hooked longitudinal bars to be fully developed in the joint for tension, the available development length must be greater than or equal to ℓ_{dh}:

$$\text{Available development length} = c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} \geq \ell_{dh} \tag{14.3}$$

In this equation, c_c is the clear cover to the hoop reinforcement in the joint, $d_{b(hoop)}$ is the diameter of the hoop reinforcement in the joint, and $d_{b(col.)}$ is the diameter of the longitudinal reinforcement in the column.

Straight Bars in Tension. In lieu of hooked bars, #3 through #11 straight bars may be used provided the bars are properly developed in accordance with ACI 18.8.5.3 and 18.8.5.4.

The required development lengths of #3 through #11 straight bars are multiples of the hooked bar development lengths in ACI 18.8.5.1:

- Minimum development length $= 2.5\ell_{dh}$ where the depth of the concrete cast in one lift beneath the bar is less than or equal to 12 in. (typically bottom bars).
- Minimum development length $= 3.25\ell_{dh}$ where the depth of the concrete cast in one lift beneath the bar is greater than 12 in. (typically top bars).

Note that #14 and #18 bars are not included because of the lack of information pertaining to anchorage of these bar sizes when subjected to load reversals.

Where the required straight development length of the longitudinal beam bars extends beyond the confined core of the column, the required development length must be increased because the limiting bond stress outside the confined region is less than that inside the confined region. The length of the bars outside of the confined core must be increased by 1.6 (ACI 18.8.5.4). The equations in ACI R18.8.5.4 can be used to determine the required development length in such cases.

Headed Bars in Tension. An example of an edge or corner column with headed deformed bars is illustrated in Figure 14.4. Such bars are permitted to be used when the conditions of ACI 25.4.4.1 are satisfied:

(a) Bars must conform to ACI 20.2.1.6
(b) Bar size must be #11 or smaller
(c) Net bearing area of the head, A_{brg}, must be at least $4A_b$ where A_b is the area of the bar
(d) Concrete must be normalweight
(e) Clear cover to the bar must be greater than or equal to $2d_b$ where d_b is the nominal diameter of the bar
(f) Center-to-center spacing of the bars must be greater than or equal to $3d_b$

According to ACI 18.8.5.2, the development length of a headed deformed reinforcing bar in tension, ℓ_{dt}, is to be calculated in accordance with ACI 25.4.4 where $1.25f_y$ is substituted for f_y :

$$\ell_{dt} = \text{greater of} \begin{cases} \left(\dfrac{1.25f_y\psi_e\psi_p\psi_o\psi_c}{75\sqrt{f_c'}}\right)d_b^{1.5} \\ 8d_b \\ 6\text{ in.} \end{cases} \tag{14.4}$$

[Note: The development lengths of headed bars determined in accordance with ACI 18.8.5.2 are greater than the development lengths of hooked bars determined in accordance with ACI 18.5.5.1, which is not correct. ACI Committee 318 is aware of this and is working to modify the provisions in ACI 18.8.5.2 accordingly.]

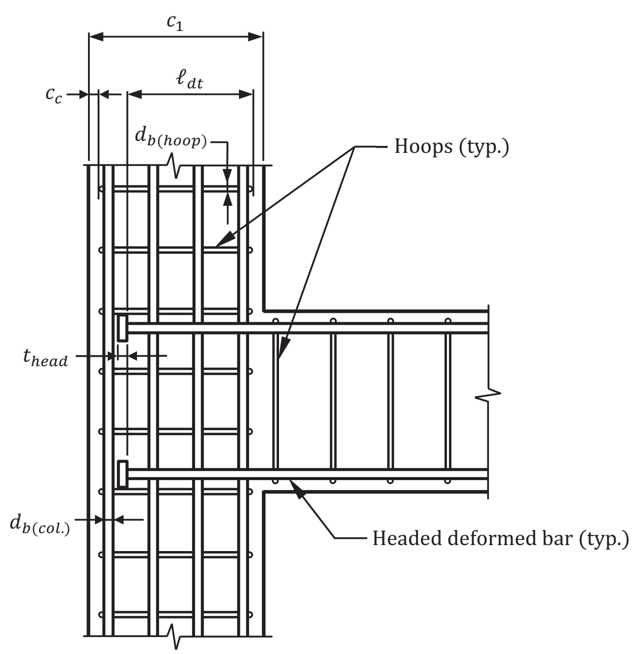
The modification factors in Equation (14.4) are given in Table 14.3 (see ACI Table 25.4.4.3).

The term ψ_p accounts for confining effects provided by transverse reinforcement oriented parallel to the development length of headed bars. At beam-column joints, the total cross-sectional area of transverse reinforcement, A_{tt}, must be located within $8d_b$ of the centerline of the headed bar toward the middle of the joint where d_b is the nominal diameter of the headed bar (see ACI Figure R25.4.4.4). Where A_{tt} is greater than or equal to $0.3A_{hs}$ or where the center-to center spacing of the headed bars is greater than or equal to $6d_b$, $\psi_p = 1.0$. The term A_{hs} is the total cross-sectional area of the headed bars being developed.

Table 14.3 Modification Factors for Development of Headed Bars in Tension

Modification Factor	Condition	Value of Factor
Epoxy, ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Parallel tie reinforcement, ψ_p	For #11 and smaller bars with $A_{tt} \geq 0.3A_{hs}$ or $s \geq 6d_b$	1.0
	Other	1.6
Location, ψ_o	For headed bars (1) terminating inside a column core with side cover to bar ≥ 2.5 in. or (2) with side cover to bar $\geq 6d_b$	1.0
	Other	1.25
Concrete strength, ψ_c	$f'_c < 6{,}000$ psi	$(f'_c / 15{,}000) + 0.6$
	$f'_c \geq 6{,}000$ psi	1.0

Headed bars used as negative reinforcement in beams terminated in a joint where there is no column above (such as, in the top story of a building) require confinement along the top face of the joint. Two confinement options are provide in ACI 18.4.4.5: (a) the column below the joint must be extended above the top of the joint a distance equal to at least the depth of the joint in the direction of analysis [see Figure 13.13(a) of this publication] or (b) vertical joint reinforcement must be provided that hooks around the headed bars and extends downward into the joint in addition to the column longitudinal reinforcement [see Figure 13.13(b) of this publication]. Design recommendations for the vertical joint reinforcement are given in Reference 26 [see also ACI Figure R25.4.4.4(b)].



$$\ell_{dt} = \text{greater of} \begin{cases} \left(\dfrac{1.25 f_y \psi_e \psi_p \psi_o \psi_c}{75\sqrt{f'_c}}\right) d_b^{1.5} \\ 8d_b \\ 6'' \end{cases}$$

Figure 14.4 Development of flexural reinforcement with headed bars in beams of special moment frames.

In order for the headed longitudinal bars to be fully developed in the joint for tension, the available development length must be greater than or equal to ℓ_{dt} :

$$\text{Available development length} = c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} - t_{head} \geq \ell_{dt} \tag{14.5}$$

where t_{head} is the thickness of the head.

Deformed Bars in Compression. The development length, ℓ_{dc}, for deformed bars in compression is determined in accordance with ACI 25.4.9:

$$\ell_{dc} = \text{ greater of } \begin{cases} \left(\dfrac{f_y \psi_r}{50\lambda\sqrt{f'_c}}\right) d_b \\ \\ 0.0003 f_y \psi_r d_b \\ \\ 8 \text{ in.} \end{cases} \tag{14.6}$$

The modification factors in Equation (14.6) are given in Table 14.4 (see ACI Table 25.4.9.3).

Table 14.4 Modification Factors for Development of Deformed Bars in Compression

Modification Factor	Condition	Value of Factor
Lightweight, λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Confining reinforcement, ψ_r	Reinforcement enclosed within the following: • A spiral • A circular continuously wound tie with $d_b \geq 1/4$ in. and pitch of 4 in. • #4 bar ties in accordance with ACI 25.7.2 spaced ≤ 4 in. on center • Hoops in accordance with ACI 25.7.4 spaced ≤ 4 in. on center	0.75
	Other	1.0

This development length corresponds to the straight portion of a hooked or headed bar measured from the critical section (in this case, the outside edge of the joint transverse reinforcement) to the onset of the bend for hooked bars and from the critical section to the head for headed bars.

For hooked longitudinal bars to be fully developed in the joint for compression, the available development length must be greater than or equal to ℓ_{dc} :

$$\text{Available development length} = c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} - d_b - r \geq \ell_{dc} \tag{14.7}$$

In this equation, c_c is the clear cover to the hoop reinforcement in the joint, $d_{b(hoop)}$ is the diameter of the hoop reinforcement in the joint, $d_{b(col.)}$ is the diameter of the longitudinal reinforcement in the column, d_b is the diameter of the longitudinal reinforcement in the beam, and r is the bend radius of the beam longitudinal bar determined in accordance with ACI Table 25.3.1 for standard 90-degree hooks.

For headed bars to be fully developed in the joint for compression, the following equation must be satisfied:

$$\text{Available development length} = c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} - t_{head} \geq \ell_{dc} \tag{14.8}$$

Cutoff Points of Flexural Reinforcement. Flexural bars may be terminated along the span in accordance with the requirements in ACI 9.7.3. The factored load used to determine the cutoff points is $w_u = (0.9 - 0.2S_{DS})w_D$ in combination with the negative and positive probable flexural strengths M_{pr}^- and M_{pr}^+ at the ends of the beam because this combination produces the longest bar lengths (see Figure 14.5).

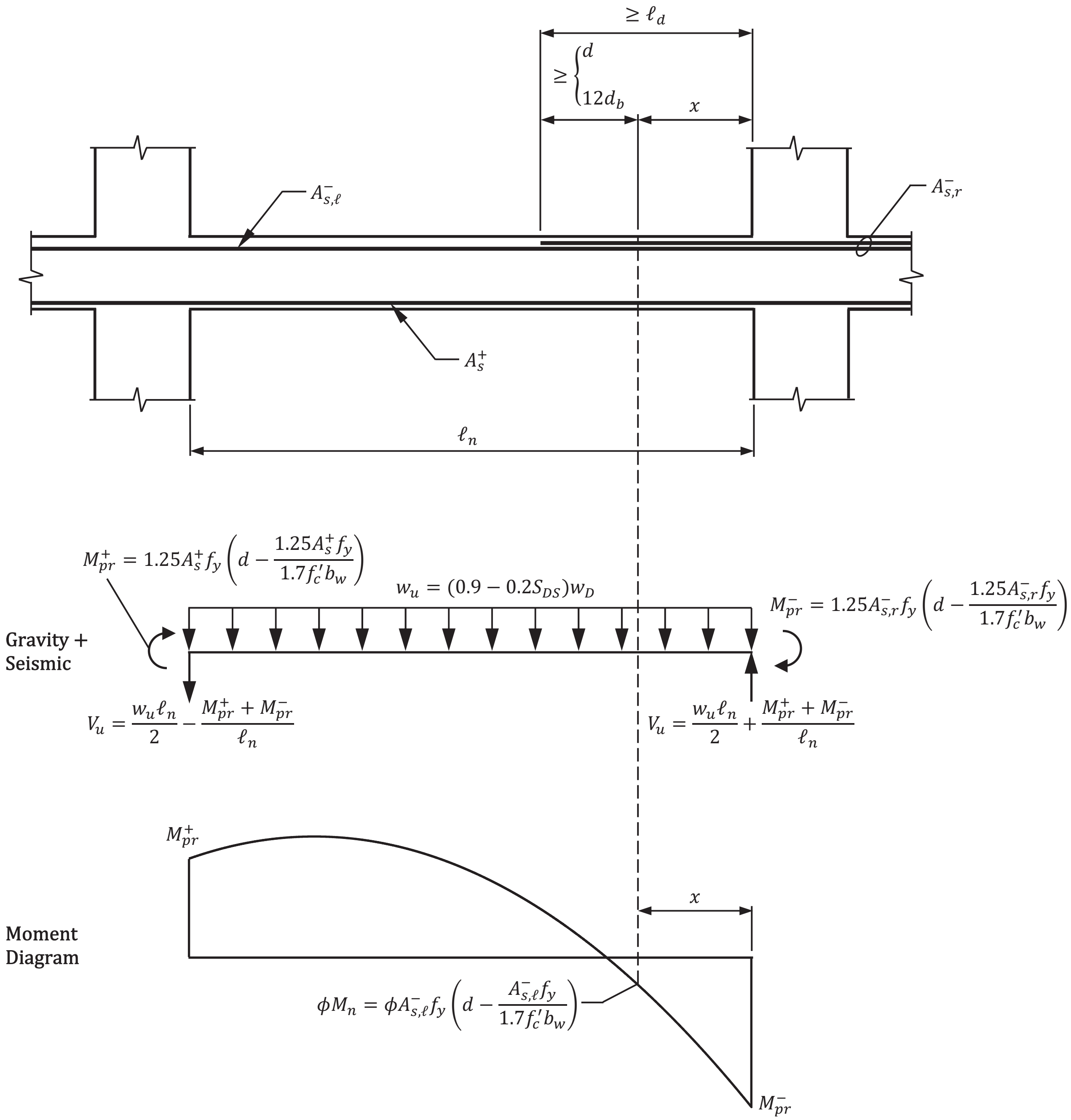


Figure 14.5 Cutoff point of negative flexural reinforcement for beams in special moment frames.

The probable flexural strength, M_{pr}, which is associated with plastic hinging in a beam, is defined as the strength of a flexural member based on the properties of the member at the joint faces assuming the tensile stress in the longitudinal reinforcement is equal to $1.25f_y$ and the strength reduction factor, ϕ, is equal to 1.0 (ACI 2.2):

$$M_{pr} = A_s(1.25f_y)\left(d - \frac{a}{2}\right) \tag{14.9}$$

where $a = A_s(1.25f_y) / 0.85f'_c b_w$.

The reasons for using $1.25f_y$ are the following: (1) the actual yield strength of the reinforcement may exceed the specified yield strength and (2) strain hardening of the longitudinal reinforcement is likely to occur at a joint undergoing large rotations, which would be expected during a design-level earthquake event.

The following equation can be used to calculate the theoretical cutoff point x from the face of the support for the negative reinforcing bars (see Figure 14.5):

$$\frac{w_u x^2}{2} - \left(\frac{w_u \ell_n}{2} + \frac{M_{pr}^+ + M_{pr}^-}{\ell_n}\right)x + M_{pr}^- = \phi M_n \tag{14.10}$$

where ϕM_n is the design flexural strength of the beam based on the area of negative reinforcement, $A_{s,\ell}^-$, at the left support. The distance x is the location where the total area of negative reinforcement $A_{s,r}^-$ at the right support is no longer needed, that is, it is the point where $A_{s,\ell}^-$ alone is sufficient to resist the required bending moment.

Once a theoretical cutoff point is determined, the bars must extend a distance equal to the larger of d or $12d_b$ beyond that point (ACI 9.7.3.3; see Figure 14.5). Additionally, the total length of the bars from the face of the support must be at least equal to the development length, ℓ_d, determined by ACI 25.4.2.

Flexural reinforcement is not permitted to be terminated in a tension zone unless one or more of the conditions of ACI 9.7.3.5 is satisfied.

Lap Splices

Requirements for lap splices in beams of special moment frames are given in ACI 18.6.3.3:

1. Hoop or spiral reinforcement spaced on center no more than the lesser of $d / 4$ or 4 in. must be provided over the lap splice length

Lap splices must not be located within the following regions (see Figure 14.6):

(a) Within the joints
(b) Within a distance of $2h$ from the face of the joint where h is the overall depth of the beam
(c) Within a distance of $2h$ from critical sections where flexural yielding is likely to occur as a result of lateral displacements beyond the elastic range of behavior

Like intermediate moment frames, the plastic hinge regions in beams of special moment frames are expected to occur over a distance of $2h$ at both ends of a beam. Yielding will typically take place at these locations for relatively short beams where bending moments due to gravity loads are low compared to those from the earthquake load effects.

Where gravity load bending moments are large compared to the bending moments from earthquake load effects, it is possible for a plastic hinge to form away from the faces of the columns. For a beam subjected to a factored uniformly distributed gravity load w_u that has a constant flexural strength along the entire span length, plastic hinges may form away from the ends of beam at locations where $M_{pr}^- + M_{pr}^+ < w_u \ell_n^2 / 2$.

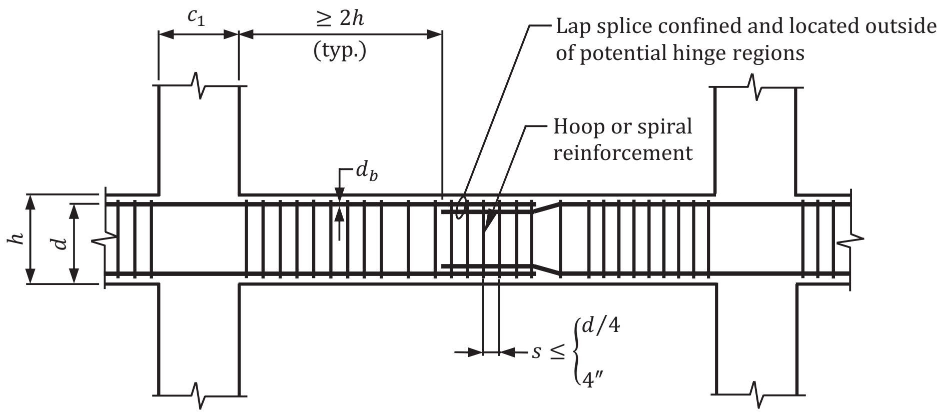


Figure 14.6 Lap splice requirements for beams in special moment frames.

Mechanical and Welded Splices

Mechanical and welded splices of the longitudinal reinforcement in beams of special moment frames must conform to the requirements in ACI 18.2.7 and 18.2.8, respectively.

Type 1 mechanical splices must conform to ACI 25.5.7 and must be able to develop in tension or compression at least $1.25f_y$ of the bar. The specified tensile strength of the spliced bars must be developed by Type 2 mechanical splices. Except for Type 2 mechanical splices on Grade 60 reinforcement, mechanical splices are not permitted to be located within $2h$ from the face of the column or from critical sections where yielding of the reinforcement is likely to occur as a result of lateral displacements beyond the linear range of behavior. A Type 2 mechanical splice on Grade 60 reinforcement is permitted at any location in a cast-in-place reinforced concrete beam.

Welded splices must conform to ACI 25.5.7 and must be able to develop in tension or compression at least $1.25f_y$ of the bar. Such splices are not permitted to be located within $2h$ from the face of the column or from critical sections where yielding of the reinforcement is likely to occur as a result of lateral displacements beyond the linear range of behavior.

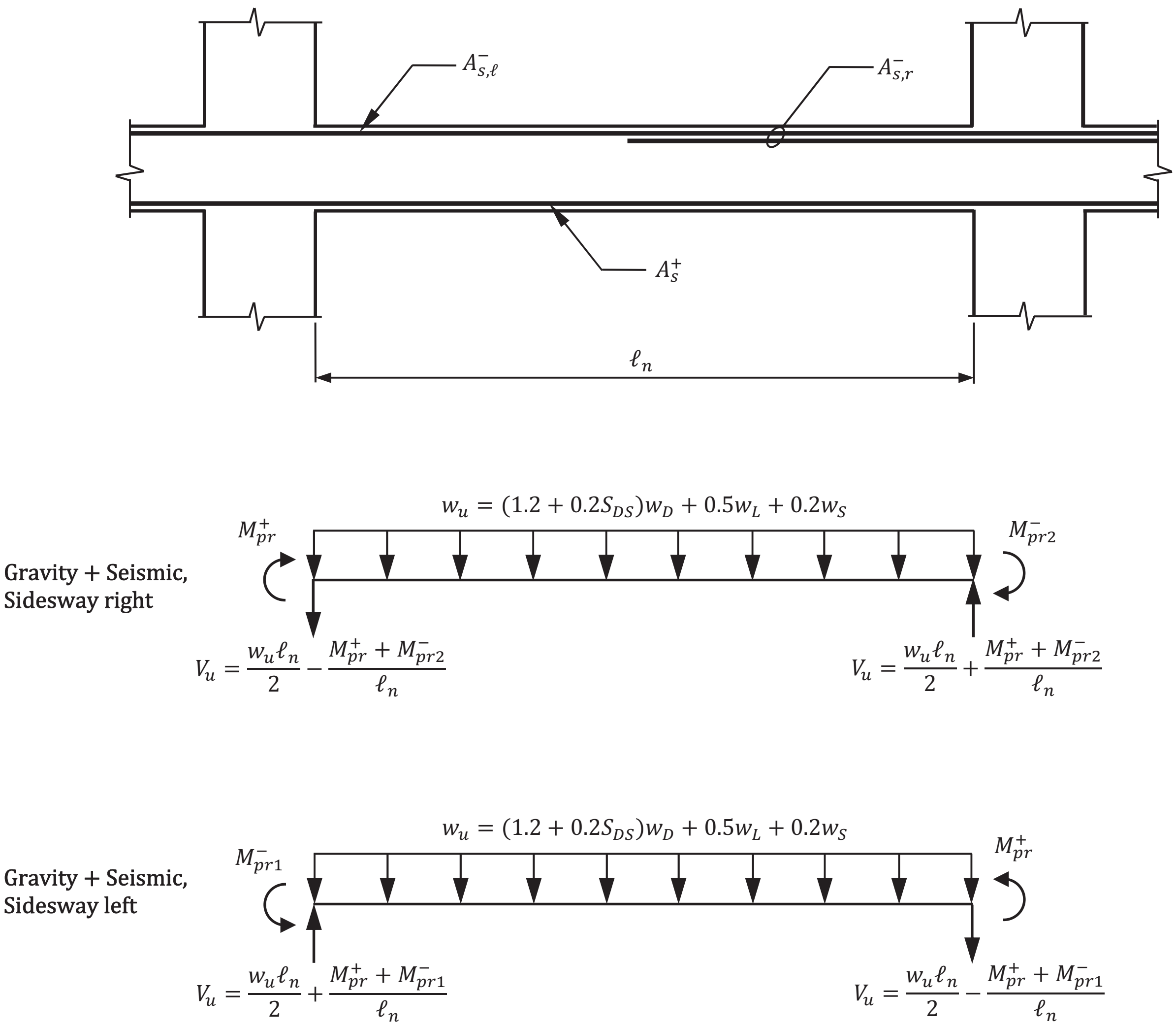
According to ACI 18.2.8.2, it is not permitted to weld stirrups, ties, inserts, or other similar elements to the longitudinal reinforcement in a beam that is part of a special moment frame. Such welding can lead to local embrittlement of the reinforcing steel.

14.2.4 Transverse Reinforcement

Determining the Required Transverse Reinforcement

In order to assure that a beam in a special moment frame yields in flexure before it fails in shear, the beam is subjected to a required shear force based on the maximum shear that can develop in the beam. Instead of designing the required shear reinforcement based solely on the results from a structural analysis using factored load combinations, the design shear forces must be determined using the factored gravity and vertical earthquake loads plus the shear forces associated with the development of the negative and positive probable flexural strengths, M_{pr}^- and M_{pr}^+, at the ends of the beam, which are determined by Equation (14.9) [ACI 18.6.5.1]. The determination of the factored shear forces, V_u, based on this provision is illustrated in Figure 14.7 for a beam subjected to a factored uniform load, w_u. ACI Equation (5.3.1e) is used to determine w_u : $0.2S_{DS}D$ is combined with $1.2D$, $0.5L$, and $0.2S$ where it is assumed the load factor on L can be taken as 0.5 in accordance with ACI 5.3.3.

Because earthquake effects can act in either direction, both sidesway to the right and sidesway to the left must be considered. The total factored shear forces, V_u, due to gravity and earthquake load effects are determined from statics. In cases where the top reinforcement is the same at both ends of a beam (that is, where $A_{s,\ell}^- = A_{s,r}^-$) and the bottom reinforcement is continuous along the entire span, maximum V_u is the same for both sidesway to the right and to the left.



$$M_{pr}^+ = 1.25A_s^+ f_y \left(d - \frac{1.25A_s^+ f_y}{1.7f_c' b_w}\right)$$

$$M_{pr1}^- = 1.25A_{s,\ell}^- f_y \left(d - \frac{1.25A_{s,\ell}^- f_y}{1.7f_c' b_w}\right) \qquad M_{pr2}^- = 1.25A_{s,r}^- f_y \left(d - \frac{1.25A_{s,r}^- f_y}{1.7f_c' b_w}\right)$$

Figure 14.7 Design shear forces for beams in special moment frames.

Hoops are required in regions where flexural yielding is expected (ACI 18.6.4.1):

- Over a length equal to $2h$ measured from the face of the supporting column toward midspan, at both ends of the beam.
- Over lengths equal to $2h$ on both sides of a section where flexure yielding is likely to occur as a result of lateral displacements beyond the elastic range of behavior.

Hoops are defined in ACI 2.3 as closed ties or continuously wound ties, made up of one or several reinforcement elements each having seismic hooks at each end conforming to ACI 25.3.4 (ACI 25.7.4; see Figure 14.8). The ends of the reinforcement elements must engage a longitudinal bar in the beam and the extensions must project into the interior of the hoop. Hoops formed by stirrups with seismic hooks and crossties (Details B and C in Figure 14.8) are preferred over those formed by closed stirrups with seismic hooks (Detail A) because the longitudinal bars in the beam can be placed more easily and efficiently. Crossties must conform to ACI 25.3.5. Note that hoops made up of interlocking headed deformed bars are not permitted (ACI 25.7.4.2).

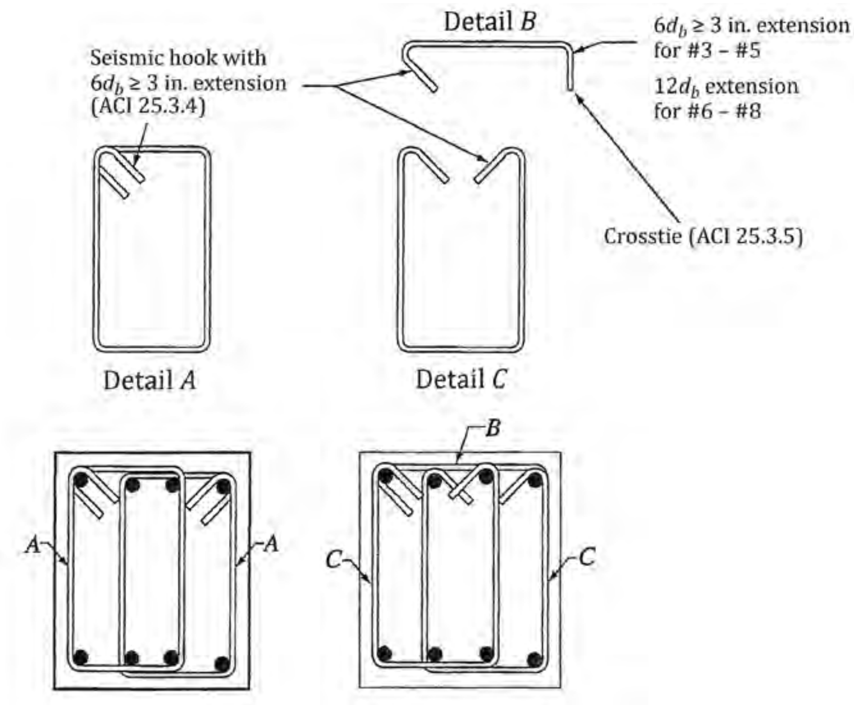


Figure 14.8 Examples of hoops and overlapping hoops.

In general, shear strength in the regions of flexural yielding is provided by both concrete (V_c) and transverse reinforcement (V_s) in the form of hoops. The size and spacing of the hoops can be determined by the following equation (ACI 22.5.8.5):

$$\frac{A_v}{s} \geq \frac{(V_u - \phi V_c)}{\phi f_{yt} d} \tag{14.11}$$

where $\phi = 0.75$ for shear (see ACI Table 21.2.1 and ACI 21.2.4). The nominal shear strength of the concrete, V_c, is determined in accordance with ACI 22.5.5. Assuming the provided transverse reinforcement is greater than or equal to the minimum required transverse reinforcement, $A_{v,min}$, given in ACI Table 9.6.3.4 and the beam is not subjected to an axial force, V_c is determined as follows (see ACI Table 22.5.5.1):

$$V_c = \text{either of} \begin{cases} 2\lambda\sqrt{f'_c} b_w d \\ \\ 8\lambda(\rho_w)^{1/3}\sqrt{f'_c} b_w d \end{cases} \leq 5\lambda\sqrt{f'_c} b_w d \tag{14.12}$$

where $\rho_w = A_s / b_w d$ and A_s is the area of longitudinal reinforcement in the beam located more than $2h/3$ away from the extreme compression fiber.

According to ACI 18.6.5.2, V_c must be taken equal to zero in Equation (14.11) over the lengths identified in ACI 18.6.4.1 when both of the following two conditions occur:

1. The earthquake-induced shear force $(M^+_{pr} + M^-_{pr}) / \ell_n$ is greater than or equal to one-half of the maximum required shear force, V_u.
2. The factored axial compressive force, P_u, on the beam, which includes earthquake effects, is less than $A_g f'_c / 20$.

These conditions must be checked over the entire lengths identified in ACI 18.6.4.1. Because the shear force due to earthquake effects is constant along the span, it is possible that at the faces of the supports where shear forces due to gravity loads are relatively large, $(M_{pr}^{+} + M_{pr}^{-}) / \ell_n < V_u / 2$, which means V_c is not equal to zero. However, at sections within $2h$ and away from the faces of the supports where shear forces due to gravity loads are smaller, it is possible that $(M_{pr}^{+} + M_{pr}^{-}) / \ell_n \geq V_u / 2$, which means V_c is equal to zero. In such cases, V_c must be set equal to zero over the entire lengths identified in ACI 18.6.4.1. Note that V_s is limited to $8\sqrt{f_c'}b_w d$ regardless if V_c is included or not (ACI 22.5.1.2).

Detailing the Transverse Reinforcement

Once the size and spacing of the hoops are determined based on the governing V_u, the required spacing must be checked against the maximum spacing requirements in ACI 18.6.4.4. The calculated hoop spacing, s, within $2h$ must be less than or equal to the smallest of the following (see Figure 14.9):

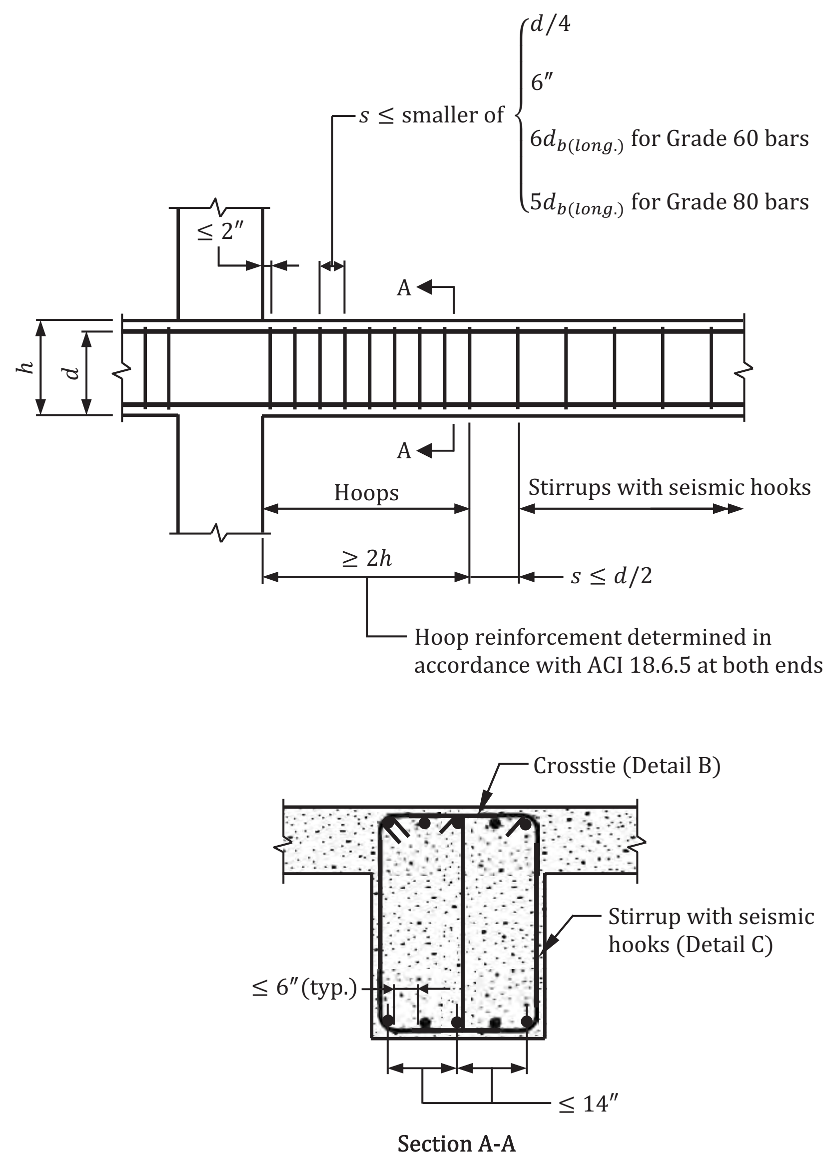


Figure 14.9 Transverse reinforcement requirements for beams in special moment frames.

$$s \leq \text{smallest of} \begin{cases} d/4 \\ 6 \text{ in.} \\ 6 \times \text{diameter of the smallest longitudinal bar enclosed for Grade 60 bars} \\ 5 \times \text{diameter of the smallest longitudinal bar enclosed for Grade 80 bars} \end{cases} \tag{14.13}$$

Longitudinal skin reinforcement required by ACI 9.7.2.3 is not included when calculating s based on the smallest longitudinal bar in the section.

In the regions where hoops are required, the primary longitudinal reinforcing bars in the beam closest to the tension and compression faces of the member must have the same lateral support required in ACI 25.7.2.3 and 25.7.2.4 for the longitudinal bars in columns with rectilinear ties or circular ties, respectively (ACI 18.6.4.2; see Figure 14.9 for a beam with rectilinear hoops). Additionally, the center-to-center spacing of the transversely supported longitudinal bars in the beam must be less than or equal to 14 in.; additional vertical crossties are required where this requirement is not satisfied. Note that skin reinforcement required by ACI 9.7.2.3 need not be laterally supported.

Where hoops are not required, it is permitted to use stirrups with seismic hooks at both ends spaced no greater than $d/2$ on center (ACI 18.6.4.6). The factored shear force V_u is calculated at the location where the hoops are terminated and the size and spacing of the stirrups are determined using Equation (14.11). There are no restrictions on the nominal shear strength of the concrete outside of the plastic hinge regions, so the contribution of V_c is included in the nominal shear strength and is determined by Equation (14.12).

For beams subjected to a factored axial compressive force greater than $A_g f_c' / 10$, the transverse reinforcement requirements in ACI 18.7.5.2 through 18.7.5.4 for columns in special moment frames must be satisfied along the lengths of the anticipated plastic hinge regions given in ACI 18.6.4.1 (ACI 18.6.4.7). Outside of these regions, hoops conforming to ACI 18.7.5.2 must be provided at a spacing not to exceed the least of 6 in., $6d_b$ of the smallest Grade 60 enclosed longitudinal beam bar, and $5d_b$ of the smallest Grade 80 enclosed longitudinal beam bar. In beams where the concrete cover over the transverse reinforcement exceeds 4 in., additional transverse reinforcement having a cover of 4 in. or less and a spacing of 12 in. or less must be provided. These additional requirements are intended to provide lateral support for the longitudinal reinforcement subjected to factored axial compressive forces greater than the prescribed limit.

14.3 Columns of Special Moment Frames

14.3.1 Overview

Design and detailing requirements for columns designated part of a SFRS in special moment frames are given in ACI 18.7. These requirements are applicable to any column that is part of a special moment frame regardless of the magnitude of the axial force on the column.

Additional analysis requirements must be satisfied for columns that are part of special moment frames in buildings assigned to SDC D through F. Any column (1) part of two or more intersecting special moment frames and (2) subjected to an axial force due to seismic forces acting along either principal plan axis greater than or equal to 20 percent of the axial design strength of the column must be designed for the most critical load effects due to application of the seismic forces in any direction (ASCE/SEI 12.5.4). Either of the procedures in ASCE/SEI 12.5.3 are permitted to be used to satisfy this requirement. In the first procedure, the columns (and their foundations) are designed for 100 percent of the seismic forces in one direction plus 30 percent of the seismic forces in the perpendicular direction using any of the analysis procedures in ASCE/SEI 12.5.3.1a. In the second procedure, the structure is analyzed using the analysis methods in ASCE/SEI 12.5.3.1b where orthogonal pairs of ground motion acceleration histories are applied simultaneously to the structure.

The required column size and the required longitudinal reinforcement for columns in special moment frames are determined using the provisions in ACI Chapter 10 for the combined effects due to gravity and lateral forces (see Chapter 7 of this publication). The following requirements must also be satisfied:

1. Dimensional limits (ACI 18.7.2)
2. Minimum flexural strength of columns (ACI 18.7.3)
3. Detailing of longitudinal reinforcement (ACI 18.7.4)
4. Detailing of transverse reinforcement (ACI 18.7.5)
5. Shear strength (ACI 18.7.6)

14.3.2 Dimensional Limits

The dimensional limits in ACI 18.7.2 must be satisfied for columns in special moment frames (see Figure 14.10). The 0.4 limit on the cross-sectional dimensions of a column results in a more compact section as opposed to a long, rectangular section, like a wall. The prescribed 12-in. minimum column dimension is not practical in most special moment frames; a larger column size is usually required based on strength requirements for the column and/or strength requirements for the joint. The dimensional limits for beam-column joints where longitudinal beam reinforcement extends through the joint are also given in Figure 14.10 (see ACI 18.8.2.3).

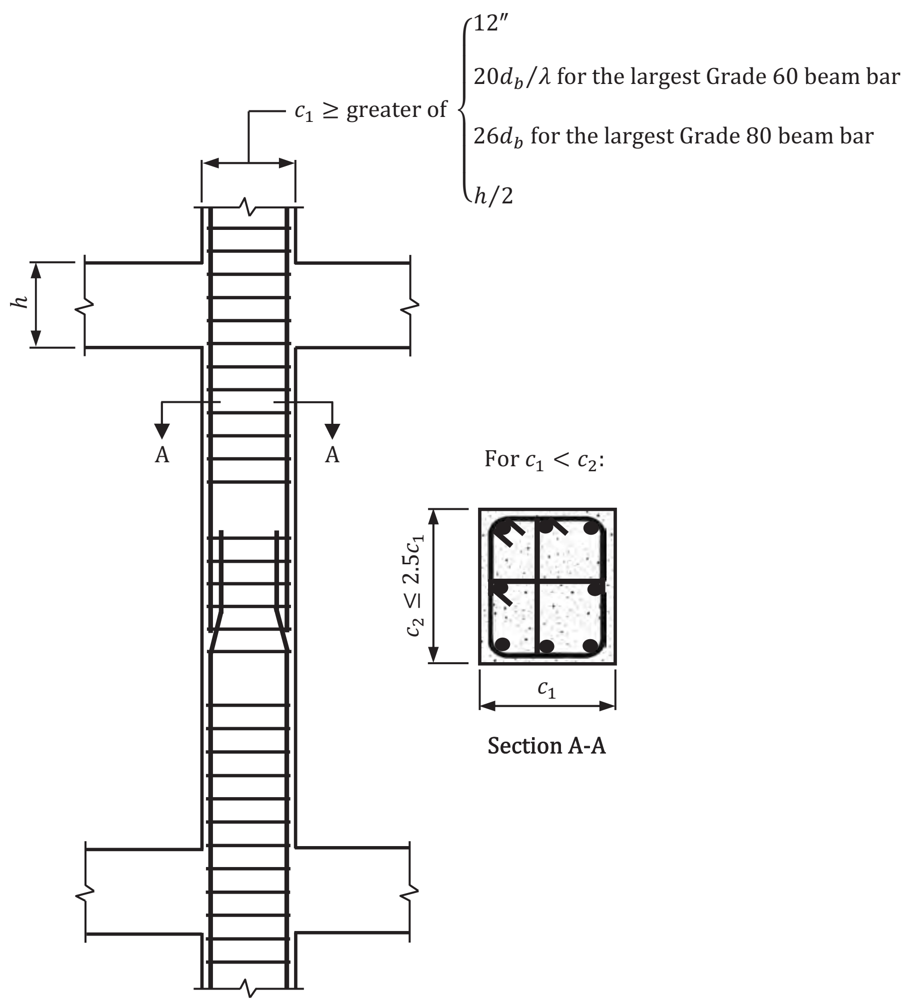


Figure 14.10 Dimensional limits of columns in special moment frames.

In addition to ACI 18.7.2, the minimum flexural strength provisions for columns in special moment frames (ACI 18.7.3) and the shear strength requirements for joints in special moment frames (ACI 18.8.4) may have an impact on column size. These requirements are covered below.

The general guidelines to achieve overall economy given in Chapter 7 of this publication are also valid for columns in special moment frames. Using a larger column size with a longitudinal reinforcement ratio between 1 and 2 percent helps in alleviating congestion problems and in satisfying shear strength requirements at beam-column joints. Specifying larger longitudinal bar sizes and/or Grade 80 longitudinal bars can also help with congestion and reduces the number of pieces that must be handled on site.

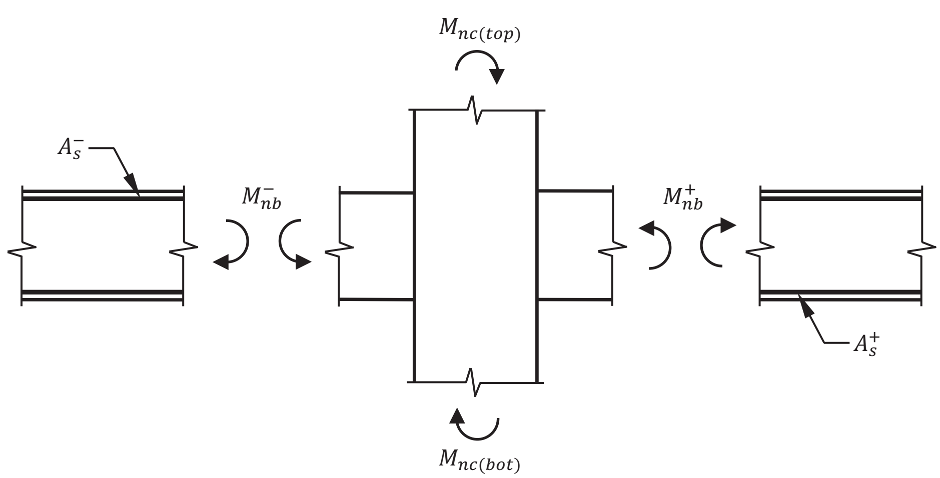
14.3.3 Minimum Flexural Strength of Columns

The requirements in ACI 18.7.3 must be satisfied at joints in special moment frames except at connections where (1) the column is discontinuous above the connection (like in the top story of a building) and (2) the column factored axial compressive force, P_u, under load combinations including earthquake effect, E, is less than $A_g f_c' / 10$.

In lieu of satisfying ACI 18.7.3.3, the flexural strength of the columns at a joint in a special moment frame must satisfy ACI Equation (18.7.3.2):

$$\sum M_{nc} \geq (6/5)\sum M_{nb} \tag{14.14}$$

In this equation, ΣM_{nc} is the sum of the nominal flexural strengths of the columns framing into the joint and ΣM_{nb} is the sum of the nominal flexural strengths of the beams framing into the joint. Both the column and beam nominal flexural strengths are evaluated at the faces of the joint. This requirement is illustrated in Figure 14.11 for sidesway to the right (sidesway to the left must also be checked in this direction, and sidesway to the right and left must be checked independently in the direction perpendicular to the one shown in Figure 14.11).



$$M_{nc(top)} + M_{nc(bot)} \geq (6/5)(M_{nb}^- + M_{nb}^+)$$

Figure 14.11 Minimum flexural strength requirements for columns in special moment frames.

The main intent of this requirement is to reduce the possibility of yielding in the columns during a design earthquake event; that is, yielding is more likely to occur at the ends of the beams. This is a more desirable situation than the one where yielding would occur in the columns: If yielding takes place at both ends of all the columns in a story of a building, a story mechanism can occur, which could lead to collapse.

The nominal flexural strength of a column, M_{nc}, is dependent on the magnitude of the axial force in the column. According to ACI 18.7.3.2, M_{nc} is to be calculated for the factored axial force consistent with the direction of analysis resulting in the lowest flexural strength. The factored axial forces from ACI Equations (5.3.1e) and (5.3.1g), which include earthquake effects, E, and the corresponding nominal flexural strengths are indicated on the nominal strength interaction diagram in Figure 14.12. In this case, the lowest nominal flexural strength is equal to $M_{nc|5.3.1g}$, which corresponds to the axial force associated with ACI Equation (5.3.1g); this nominal flexural strength is to be used in Equation (14.14).

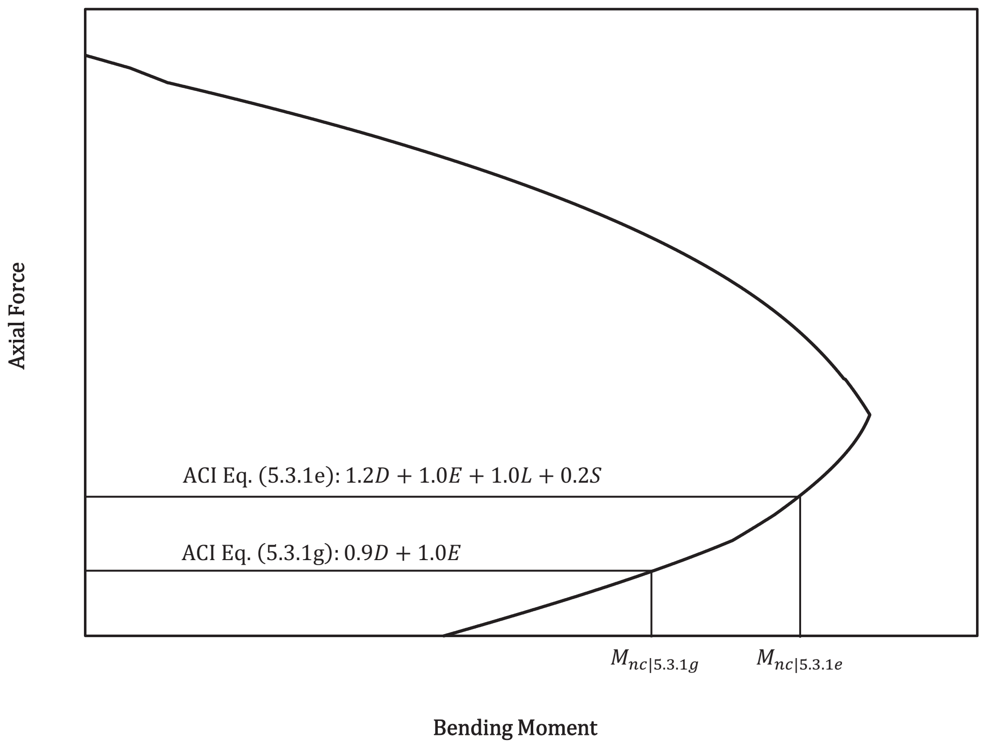


Figure 14.12 Determination of nominal flexural strength for a column in a special moment frame.

The slab reinforcement within the effective width defined in ACI 6.3.2 must be included when calculating the nominal flexural strengths of the beams, M_{nb} (ACI 18.7.3.2; see Figure 14.13 for a beam with a slab on each side of the web and a beam with a slab on one side of the web). The contribution of this reinforcement to M_{nb}, even where the minimum required amount is present, can be considerable and must be included in all cases.

If Equation (14.14) is not satisfied at a joint, either (1) or (2) must be satisfied:

(1) The nominal flexural strength of the column must be increased by increasing the (a) size of the column, (b) amount of flexural reinforcement, and/or (c) concrete compressive strength.

(2) Ignore any contribution the column has to the lateral strength and stiffness of the structure, and design and detail the column in accordance with the requirements of ACI 18.14 for members that are not part of the SFRS (ACI 18.7.3.3).

The columns under item 2 above must not be ignored when calculating the seismic base shear, and the overall torsional effects must not be reduced because the stiffness of these columns have been disregarded in the overall model of the structure.

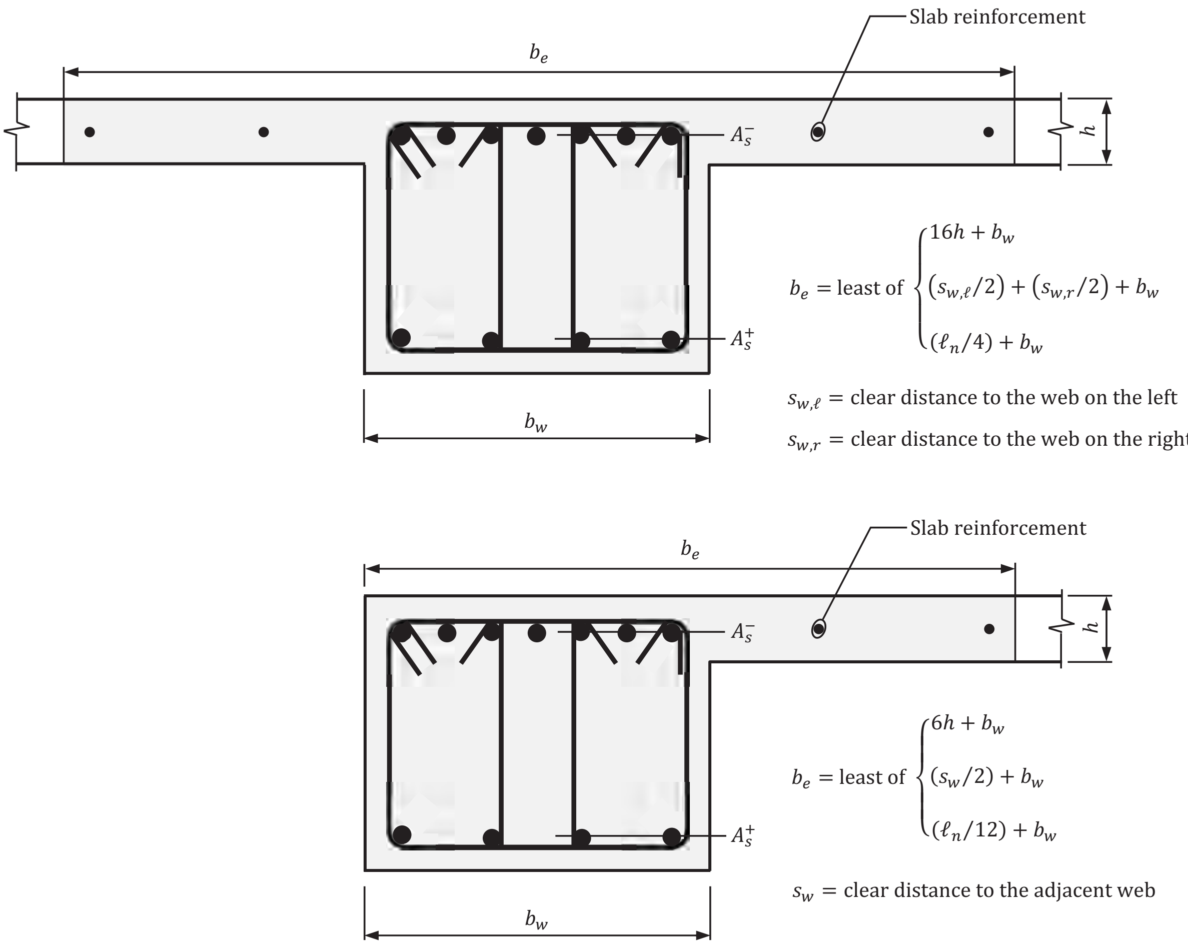


Figure 14.13 Determination of nominal flexural strength for a beam in a special moment frame.

14.3.4 Longitudinal Reinforcement

Determining the Required Longitudinal Reinforcement

The methods given in Chapter 7 of this publication can be used in determining the required area of longitudinal reinforcement for reinforced concrete columns in special moment frames subjected to combined flexure and axial forces. The area of longitudinal reinforcement is limited to $0.06A_g$ instead of $0.08A_g$ (ACI 18.7.4.1); the purpose of this is to help control congestion and the development of high shear stresses. As noted above, it is good practice to limit the area of longitudinal reinforcement to $0.01A_g$ to $0.02A_g$. Providing larger areas of reinforcement is usually not practical or economical, and can result in very congested joints and splice locations.

The minimum flexural strength provisions in ACI 18.7.3 may have an impact on the area of longitudinal reinforcement required in a column that is part of a special moment frame.

Reference 16 contains 900 design strength interaction diagrams for the following:

- Tied, rectangular columns ranging in size from 12 to 48 in., inclusive;
- Tied and spiral circular columns ranging in diameter from 12 to 48 in., inclusive;
- Grade 60 and Grade 80 longitudinal reinforcement;
- Concrete compressive strengths from 4,000 psi to 14,000 psi, inclusive; and,
- Longitudinal reinforcement ratios from 1 percent to less than 2.5 percent

These interaction diagrams can be used to facilitate selection of the column size and area of longitudinal reinforcement.

Detailing the Longitudinal Reinforcement

The longitudinal reinforcement requirements of ACI 18.7.4 are given in Figure 14.14. Lap splices are required to be located within the center half of the column length, away from the ends of the column where spalling of the concrete shell surrounding the transverse reinforcement is likely to occur due to high bending moments (ACI 18.7.4.4). These splices must be designed as tension lap splices in accordance with ACI 25.5.2. Transverse reinforcement conforming to ACI 18.7.5.2 and 18.7.5.3, which is applicable at the ends of the column, must be provided over the lap splice length mainly to ensure that the splice performs as intended when subjected to stress reversals.

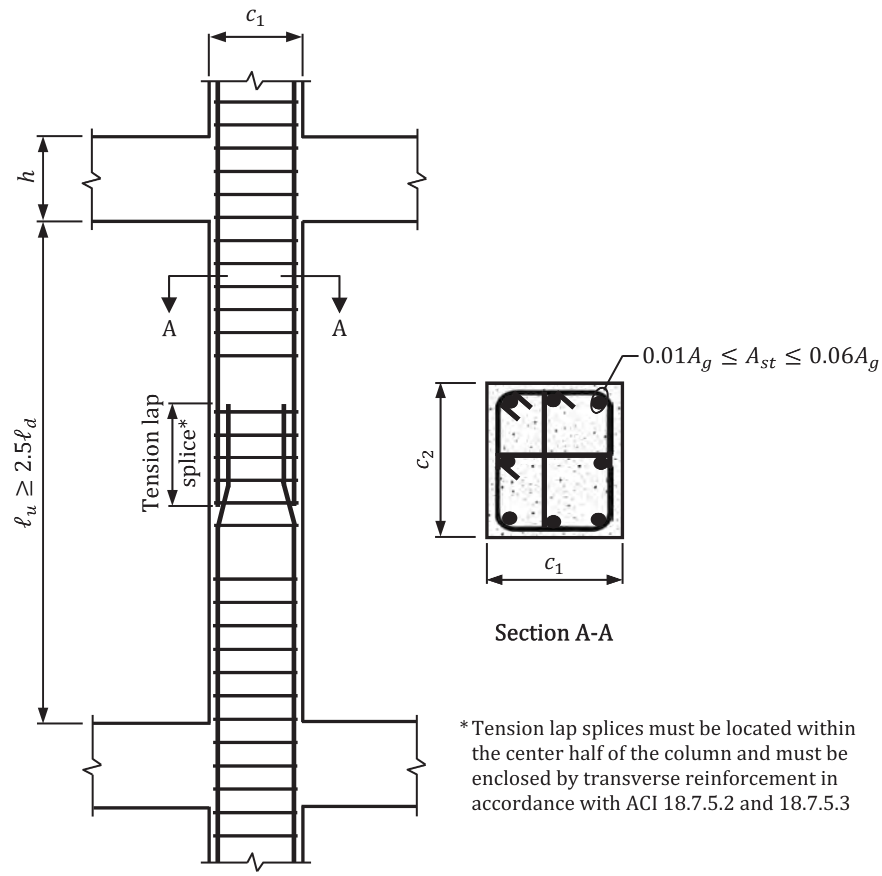


Figure 14.14 Longitudinal reinforcement requirements for columns in special moment frames.

The longitudinal reinforcement must be selected such that the clear height of the column, ℓ_u, is greater than or equal to 2.5 times the tension development length of the bars, ℓ_d, determined in accordance with ACI 25.4.2 (ACI 18.7.4.3). The purpose of this requirement is to reduce the likelihood of bond splitting failure along the longitudinal bars within the clear column height. Where this requirement is not satisfied, ℓ_d can be reduced by using smaller longitudinal bars, increasing the amount of transverse reinforcement, increasing the compressive strength of the concrete, or a combination of one or more of the above.

Mechanical and welded splices conforming to ACI 18.2.7 and 18.2.8, respectively, may be used instead of lap splices (ACI 18.7.4.4).

At least six longitudinal bars are required in columns of special moment frames utilizing circular hoops as the transverse reinforcement (ACI 18.7.4.2).

14.3.5 Transverse Reinforcement

Determining the Required Transverse Reinforcement

Plastic hinges can form at the ends of a column in a special moment frame during a design-level earthquake event and the concrete must be properly confined within these regions. According to ACI 18.7.5.1, transverse reinforcement in the form of spirals, circular hoops, or rectilinear hoops with or without crossties satisfying the requirements of ACI 18.7.5.2 through 18.7.5.4 must be provided over a length ℓ_o from each joint face and on both sides of any section where flexural yielding is likely to occur where ℓ_o is the assumed length of the anticipated plastic hinge region:

$$\ell_o \geq \text{greatest of} \begin{cases} \text{Maximum cross-sectional dimension of the column} \\ \text{Clear span of the column}/6 \\ 18 \text{ in.} \end{cases} \tag{14.15}$$

Similar to beams, columns in special moment frames must be designed for the shear forces generated by the probable flexural strengths, M_{pr}, at the ends of the column, which are associated with the range of factored axial forces, P_u, that include earthquake effects, E. The determination of the factored shear force, V_u, based on this requirement is given in Figure 14.15 for a column subjected to a factored axial force P_u. It is evident from the figure that V_u is constant over the height of the column. In order to obtain the maximum shear force, sidesway to the right and sidesway to left must both be considered.

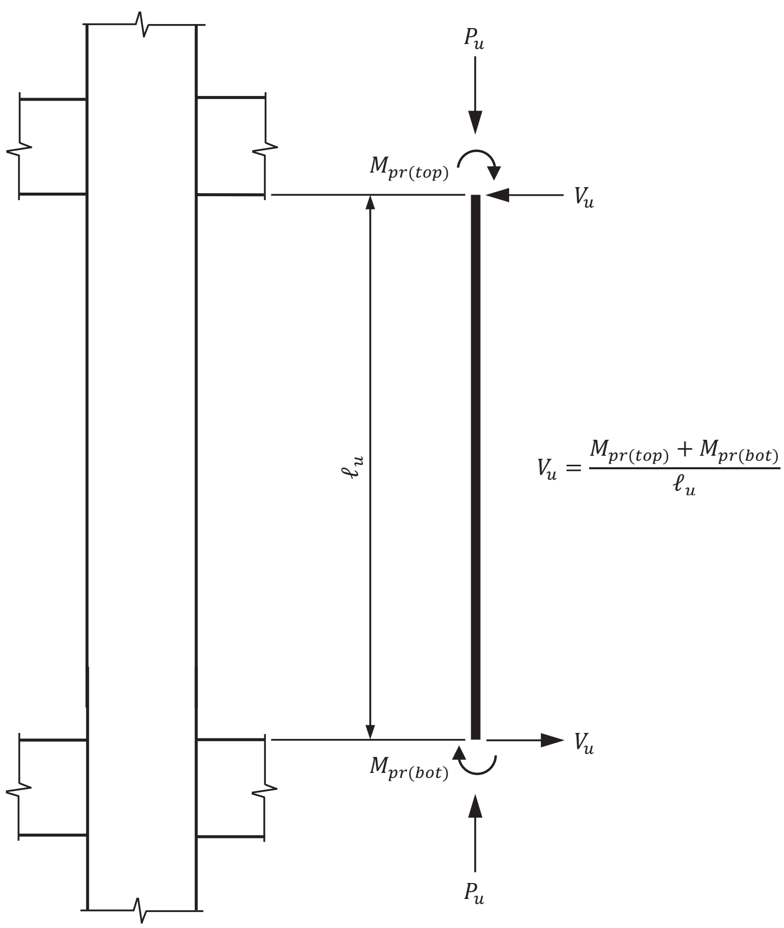


Figure 14.15 Design shear force for columns in special moment frames.

Interaction diagrams for a column where the tensile stress in the longitudinal reinforcement is equal to $1.25f_y$ and the strength reduction factor ϕ is equal to 1.0 are given in Figure 14.16; these diagrams are a representation of the probable flexural strengths, M_{pr}, of a column as a function of factored axial forces. For the range of axial forces shown in Figure 14.16(a), the largest M_{pr} occurs at the balanced point even though the factored axial force corresponding to this probable flexural strength is not obtained from either of the two applicable load combinations that contain E [that is, ACI Equations (5.3.1e) and (5.3.1g)]. In this case, M_{pr} at the balanced point must be used in the calculation of V_u. For the range of axial forces in Figure 14.16(b), the largest M_{pr} is equal to the probable flexural strength associated with the factored axial force from ACI Equation (5.3.1e) because that probable flexural strength is greater than the probable flexural strengths associated with any of the other factored axial forces in that range.

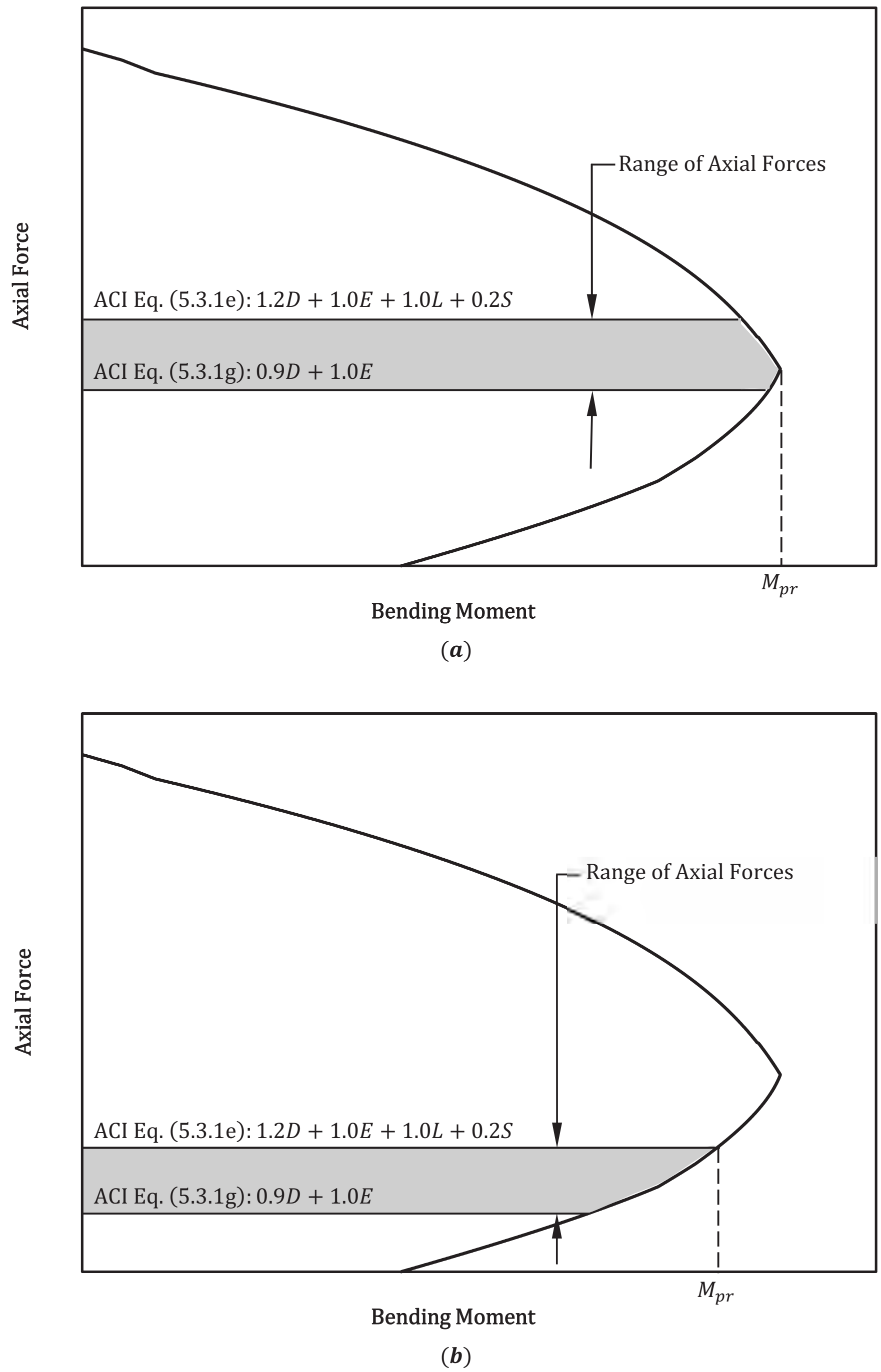


Figure 14.16 Nominal strength interaction diagrams for columns with $f_s = 1.25f_y$ and $\phi = 1.0$.

It is conservative to assume that M_{pr} develops at both ends simultaneously in a column in a story above the first story, especially in cases where Equation (14.14) is satisfied. It is for this reason that the earthquake-induced shear force is permitted to be taken as the shear force calculated from joint shear strengths based on the probable flexural strengths of the beams framing into the joint (ACI 18.7.6.1.1). The determination of the shear force based on M_{pr} of the beams is indeterminate and an analysis must be performed to obtain it (see, for example, the analysis method in Reference 27). For columns in the first story, it is possible to develop M_{pr} of the column at its base and the earthquake-induced shear force must be determined based on that possibility. In any case, the design shear force must not be taken less than that determined from analysis of the building subjected to the code-prescribed seismic forces.

In general, shear strength in the regions of flexural yielding is provided by both concrete (V_c) and transverse reinforcement (V_s). The size and spacing of the transverse reinforcement can be determined by the following equation (ACI 22.5.8.5.1):

$$\frac{A_v}{s} \geq \frac{(V_u - \phi V_c)}{\phi f_{yt} d} \tag{14.16}$$

where $\phi = 0.75$ for shear (see ACI Table 21.2.1 and ACI 21.2.4). The nominal shear strength of the concrete, V_c, is determined in accordance with ACI 22.5.5. Assuming the provided transverse reinforcement is greater than or equal to the minimum required transverse reinforcement, $A_{v,min}$, given in ACI 10.6.2, V_c is determined as follows (see ACI Table 22.5.5.1):

$$V_c = \text{either of} \begin{cases} \left(2\lambda\sqrt{f_c'} + \dfrac{N_u}{6A_g}\right)b_w d \\ \\ \left(8\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \dfrac{N_u}{6A_g}\right)b_w d \end{cases} \leq 5\lambda\sqrt{f_c'}b_w d \tag{14.17}$$

where $\rho_w = A_s / b_w d$, A_s is the area of longitudinal reinforcement in the column located more than $2h / 3$ away from the extreme compression fiber, and N_u is the factored axial force on the column corresponding to the load combination used to determine V_u. Note that N_u is positive for compression and negative for tension, and $N_u / 6A_g$ in Equation (14.17) must not be taken greater than $0.05f_c'$ (ACI 22.5.5.1.2).

For rectilinear hoops, A_v is equal to the area of all hoop legs or all hoop legs plus crosstie legs in the direction of analysis within the longitudinal center-to-center spacing, s. For circular ties, A_v is equal to two times the area of the tie bar within s (ACI 22.5.8.5.6). Similarly, for spirals, A_v is equal to two times the area of the spiral bar within the spiral pitch, s.

Like in the case of beams in special moment frames, V_c must be taken equal to zero in Equation (14.16) over the lengths ℓ_o when both of the following two conditions occur (ACI 18.7.6.2.1):

1. The earthquake-induced shear force $[M_{pr(top)} + M_{pr(bot)}] / \ell_u$ is greater than or equal to one-half of the maximum required shear force, V_u.
2. The factored axial compressive force, P_u, on the column, which includes earthquake effects, is less than $A_g f_c' / 20$.

It is typical for the earthquake-induced shear force to be greater than the shear forces from gravity loads (especially at interior columns), so the first condition is usually met. The minimum factored axial force obtained from ACI Equations (5.3.1e) and (5.3.1g) is used to determine if the second condition is met or not.

If the shear strength requirements are satisfied based on the probable flexural strengths at the ends of a column, a more detailed analysis to determine the factored shear forces based on the probable flexural strengths that can develop at the ends of the beams need not be performed.

Beyond the length ℓ_o, the nominal shear strength of the concrete, V_c, is permitted to be used and can be determined by Equation (14.17).

The usual procedure for determining column transverse reinforcement is to calculate the area and spacing based on the transverse reinforcement requirements of ACI 18.7.5 (see below) and then check that the shear strength requirements of ACI 18.7.6 outlined above are satisfied using that area and spacing.

Detailing the Transverse Reinforcement

Hoop Reinforcement

Transverse reinforcement in the form of rectilinear hoops (with or without crossties) or circular hoops in accordance with ACI 18.7.5 must be provided along the entire length of columns in special moment frames. Within a distance ℓ_o from each end of a column, the hoops must satisfy the requirements of ACI 18.7.5.2 through 18.7.5.4 (ACI 18.7.5.1). Beyond the length ℓ_o, hoops satisfying ACI 25.7.2 and 25.7.4 are permitted at a spacing less than or equal to that in ACI 18.7.5.5. A summary of these requirements is given in Table 14.5. Transverse reinforcement requirements for columns with rectilinear hoops where $P_u \leq 0.3A_g f_c'$ and $f_c' \leq 10{,}000$ psi are given in Figure 14.17. Requirements where $P_u > 0.3A_g f_c'$ and/or $f_c' > 10{,}000$ psi are given in Figure 14.18.

Table 14.5 Design and Detailing Requirements for Hoop Reinforcement in Columns of Special Moment Frames

	Requirement	ACI Section No.
Within ℓ_o	Bends of rectilinear hoops and crossties must engage peripheral longitudinal reinforcing bars.	18.7.5.2(b)
	Crossties of the same or smaller bar size as the hoops are permitted, subject to the limitation of ACI 25.7.2.2. Consecutive crossties must be alternated end for end along the longitudinal reinforcement and around the perimeter of the cross-section.	18.7.5.2(c)
	Rectilinear hoops or crossties must provide lateral support to longitudinal reinforcement in accordance with ACI 25.7.2.2 and 25.7.2.3.	18.7.5.2(d)
	Center-to-center spacing, h_x, of longitudinal bars laterally supported by the corner of a crosstie or hoop leg must be less than or equal to 14 in. around the perimeter of the column.	18.7.5.2(e)
	In columns with rectilinear hoops where the factored axial compressive force $P_u > 0.3A_g f_c'$ or where $f_c' > 10{,}000$ psi, every longitudinal bar or bundle of bars around the perimeter of the column core must have lateral support provided by the corner of a hoop or by a seismic hook. The value of h_x in such cases must be less than or equal to 8 in. The factored axial force P_u is the largest value obtained from the load combinations that include the earthquake effect, E.	18.7.5.2(f)
	The center-to-center spacing of the hoops must not exceed the least of the following: (a) Minimum column dimension/4 (b) For Grade 60 bars, $6d_b$ of the smallest longitudinal bar (c) For Grade 80 bars, $5d_b$ of the smallest longitudinal bar (d) s_o where $4 \text{ in.} \leq s_o = 4 + [(14 - h_x)/3] \leq 6 \text{ in.}$	18.7.5.3

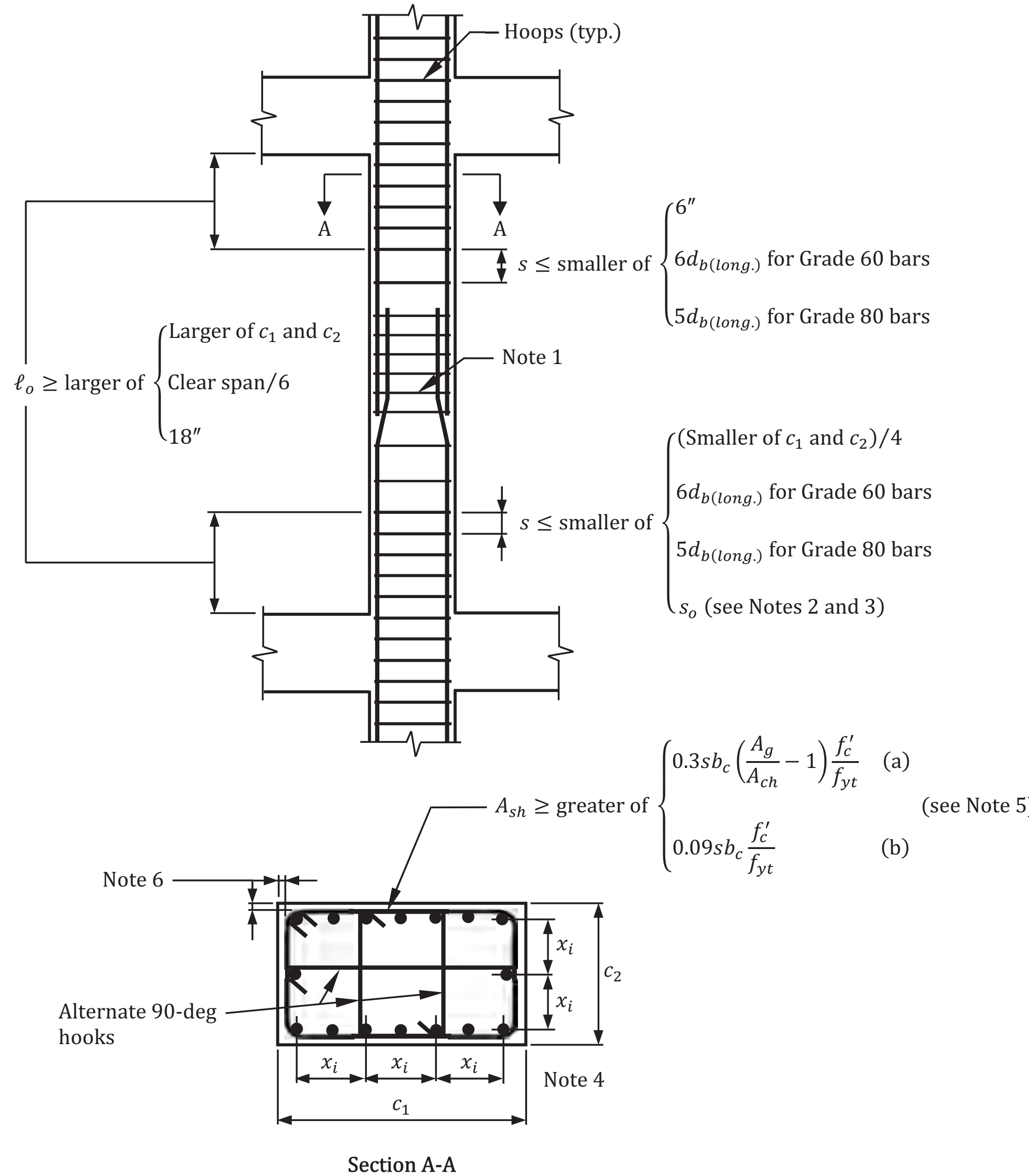
(table continued on next page)

Table 14.5 Design and Detailing Requirements for Hoop Reinforcement in Columns of Special Moment Frames (cont.)

	Requirement			ACI Section No.
Within ℓ_o	Minimum transverse reinforcement is determined in accordance with ACI Table 18.7.5.4			18.7.5.4
	Hoop Type	$P_u \le 0.3A_g f'_c$ and $f'_c \le 10{,}000$ psi	$P_u > 0.3A_g f'_c$ and/or $f'_c > 10{,}000$ psi	
	Rectilinear (A_{sh})	$A_{sh} \ge$ greater of $\left\{\begin{array}{l} 0.3sb_c\left(\dfrac{A_g}{A_{ch}}-1\right)\dfrac{f'_c}{f_{yt}} \\ 0.09sb_c\dfrac{f'_c}{f_{yt}} \end{array}\right.$	$A_{sh} \ge$ greater of $\left\{\begin{array}{l} 0.3sb_c\left(\dfrac{A_g}{A_{ch}}-1\right)\dfrac{f'_c}{f_{yt}} \\ 0.09sb_c\dfrac{f'_c}{f_{yt}} \\ 0.2sb_c k_f k_n \dfrac{P_u}{f_{yt}A_{ch}} \end{array}\right.$	
	Circular (ρ_s)	$\rho_s \ge$ greater of $\left\{\begin{array}{l} 0.45\left(\dfrac{A_g}{A_{ch}}-1\right)\dfrac{f'_c}{f_{yt}} \\ 0.12\dfrac{f'_c}{f_{yt}} \end{array}\right.$	$\rho_s \ge$ greater of $\left\{\begin{array}{l} 0.45\left(\dfrac{A_g}{A_{ch}}-1\right)\dfrac{f'_c}{f_{yt}} \\ 0.12\dfrac{f'_c}{f_{yt}} \\ 0.35k_f \dfrac{P_u}{f_{yt}A_{ch}} \end{array}\right.$	
Outside ℓ_o	Hoop reinforcement satisfying the provisions of ACI 25.7.2 and 25.7.4 must be provided.			18.7.5.5
	The spacing of the transverse reinforcement, s, must not exceed the lesser of the following: (a) 6 in. (b) For Grade 60 bars, $6d_b$ of the smallest longitudinal bar (c) For Grade 80 bars, $5d_b$ of the smallest longitudinal bar			
	A larger amount of transverse reinforcement may be required based on the provisions of ACI 18.7.4.4 or 18.7.6.			

The minimum area of transverse reinforcement, A_{sh}, that must be provided within the length ℓ_o is equal to the greater of that determined by Equations (a) and (b) in Figure 14.17 or by Equations (a), (b), and (c) in Figure 14.18. The term A_{ch} in these equations is equal to the area of the section measured to the outside of the transverse reinforcement, which is the area of the confined core of the column. The term b_c is the dimension of the confined core perpendicular to the legs of the transverse reinforcement that make up A_{sh}. Identified in Figure 14.19 are the appropriate b_c to determine A_{sh} for the column depicted in the figure.

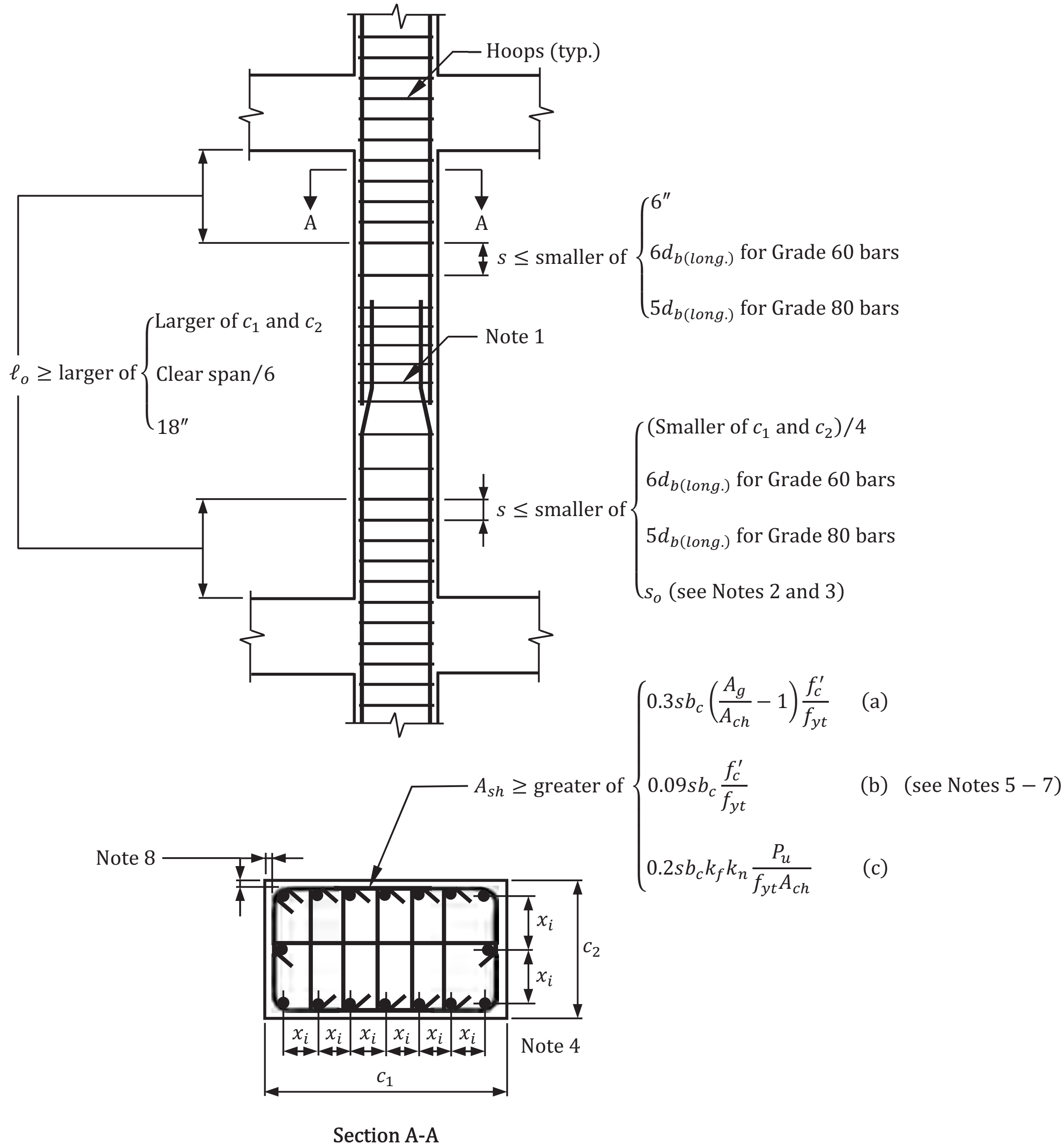
The term k_f in Equation (c) is equal to $(f'_c / 25{,}000) + 0.6 \ge 1.0$. This term increases the minimum required area of transverse reinforcement where f'_c exceeds 10,000 psi and is meant to help in preventing brittle failure of the column. ACI R18.7.5.4 recommends concrete strengths greater than 15,000 psi should be used with caution in columns of special moment frames because of the limited test data available on their overall performance. The term k_n in Equation (c) is equal to $n_l / (n_l - 2)$ where n_l is the number of longitudinal bars around the perimeter of the column core laterally supported by the corner of hoops or by seismic hooks.



Notes

1. Transverse reinforcement in accordance with ACI 18.7.5.2 and 18.7.5.3 over the lap splice length.
2. $4'' \leq s_o = 4 + [(14 - h_x)/3] \leq 6''$
3. h_x = largest value of x_i
4. $x_i \leq 14''$
5. In determining A_{sh}, provisions of ACI 18.7.6 must also be satisfied.
6. Where the concrete cover > 4″, provide additional transverse reinforcement having cover ≤ 4 in. and spacing no greater than 12″.

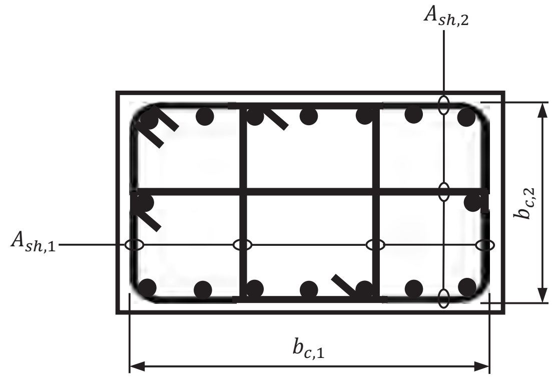
Figure 14.17 Requirement for rectilinear hoops in columns of special moment frames where $P_u \leq 0.3A_g f_c'$ and $f_c' \leq 10{,}000$ psi.



<u>Notes</u>

1. Transverse reinforcement in accordance with ACI 18.7.5.2 and 18.7.5.3 over the lap splice length.
2. 4 in. $\leq s_o = 4 + [(14 - h_x)/3] \leq 6$ in.
3. h_x = largest value of x_i
4. $x_i \leq 8$ in.
5. In determining A_{sh}, provisions of ACI 18.7.6 must also be satisfied.
6. $k_f = (f_c'/25{,}000) + 0.6 \geq 1.0$
7. $k_n = n_l/(n_l - 2)$
8. Where the concrete cover > 4 in., provide additional transverse reinforcement having cover ≤ 4 in. and spacing no greater than 12 in.

Figure 14.18 Requirement for rectilinear hoops in columns of special moment frames where $P_u > 0.3A_gf_c'$ and/or $f_c' > 10{,}000$ psi.

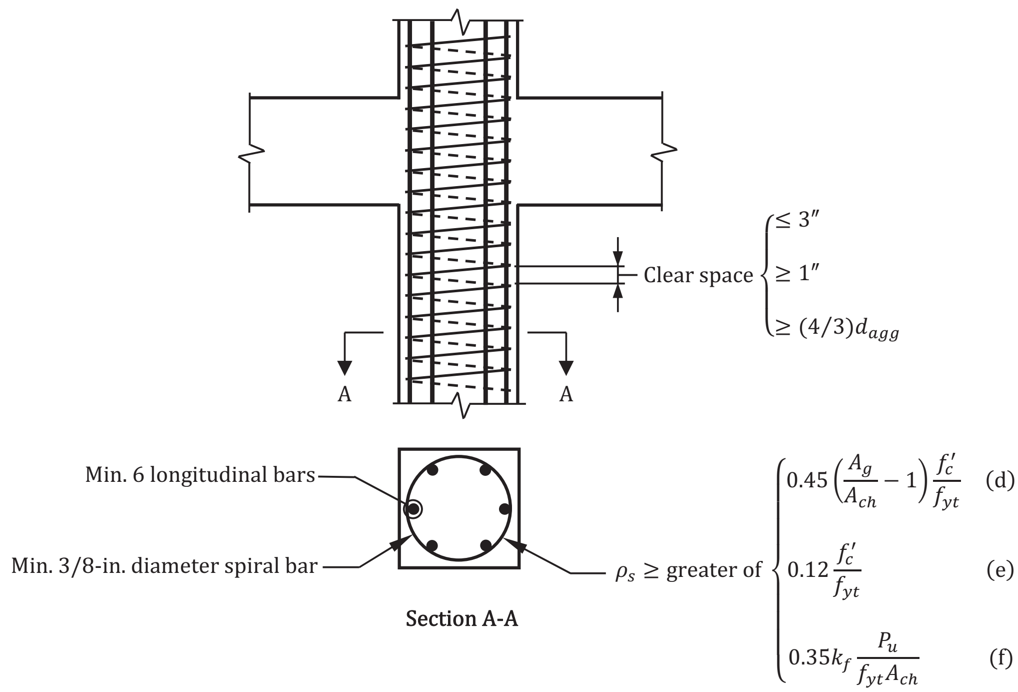


$A_{ch} = b_{c,1} \times b_{c,2}$

Figure 14.19 Confined core dimensions for columns with rectilinear hoop reinforcement in special moment frames.

Spiral Reinforcement

Transverse reinforcement requirements for columns with spiral reinforcement are given in ACI 18.7.5.4 (see Figure 14.20). Spiral reinforcement is generally the most efficient type of confinement reinforcement; however, the spirals within the beam-column joints may cause constructability issues, especially when placing longitudinal bars from the beams through the joint.



<u>Notes</u>

1. Equation (f) for ρ_s is applicable only where $P_u > 0.3A_g f_c'$ and/or $f_c' > 10{,}000$ psi.
2. In determining A_{sh}, provisions of ACI 18.7.6 must also be satisfied.
3. $k_f = (f_c'/25{,}000) + 0.6 \geq 1.0$

Figure 14.20 Requirements for spiral reinforcement in columns of special moment frames.

Columns Supporting Reactions from Discontinuous Stiff Members

Where a stiff member, such as a wall, is discontinued and supported on columns instead of on a foundation, the columns must contain transverse reinforcement conforming to the requirements in ACI 18.7.5.2 through 18.7.5.4 for columns in special moment frames over the full height at all levels beneath the discontinuity where the factored axial compressive force related to earthquake effects in the columns exceeds $A_g f_c' / 4$ (ACI 18.7.5.6; see Figure 14.21). The limit of $A_g f_c' / 4$ pertains to columns designed using load combinations that include the overstrength factor Ω_o, which are given in ASCE/SEI 2.3.6 (see Table 14.2 of this publication). Otherwise, the limit is $A_g f_c' / 10$. The transverse reinforcement must extend above and below the column in accordance with ACI 18.7.5.6(b).

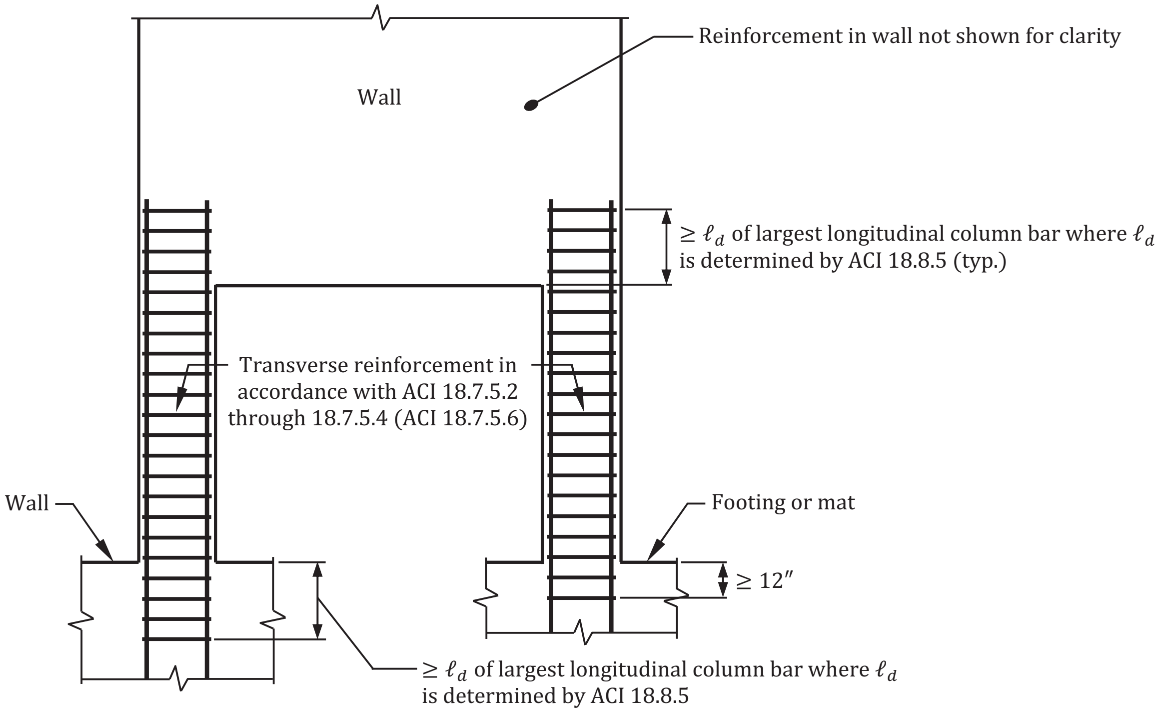


Figure 14.21 Transverse reinforcement requirements for columns supporting discontinuous stiff members in special moment frames.

As noted previously, the required reinforcement details presented above must be provided even where the effects from wind loads govern the design of a column.

14.4 Joints of Special Moment Frames

14.4.1 Overview

Requirements for beam-column joints in special moment frames are given in ACI 18.8.

The overall integrity and performance of a special moment frame is dependent on the behavior of the beam-column joints in the frames. Inelastic rotations at the faces of the joints produce strains in the beam longitudinal reinforcement well in excess of the strain corresponding to the yield strength of the reinforcement. As such, joint shear forces must be calculated using a stress equal to $1.25 f_y$ in the beam longitudinal reinforcement passing through or anchored in the joint (ACI 18.8.2.1).

Slippage of the beam longitudinal reinforcement through a joint can lead to an increase in joint rotation. At edge and corner columns where the reinforcement does not continue through the joint, the terminated bars must extend to the far face of the confined column core and must be developed in tension in accordance with ACI 18.8.5 and in compression in accordance with ACI 25.4.9 (ACI 18.8.2.2). At interior joints, the minimum joint (column) dimension requirements of ACI 18.8.2.3 must be satisfied (see Figure 14.2).

Because test data for the combination of lightweight concrete and Grade 80 longitudinal reinforcement in joints are not available, normalweight concrete must be used in joints where Grade 80 longitudinal reinforcement is specified (ACI 18.8.2.3.1).

14.4.2 Transverse Reinforcement

Transverse reinforcement in a beam-column joint is required to confine the concrete so that it behaves in a ductile manner and maintains it ability to carry vertical loads even after the outer concrete shell were to spall off. The transverse reinforcement specified in ACI 18.7.5.2, 18.7.5.3, 18.7.5.4, and 18.7.5.7 is required as a minimum in a joint regardless of the magnitude of the shear force (ACI 18.8.3.1); this is the same amount of transverse reinforcement required over the length ℓ_o at the ends of columns in special moment frames. An exception to this requirement occurs where a joint is confined by beams on all four sides of the joint (beams are defined as providing confinement to a joint where the width of the beam is at least three-fourths the width of the column): the amount of transverse reinforcement required by ACI 18.7.5.4 is permitted to be reduced by 50 percent and the spacing required by ACI 18.7.5.3 is permitted to be increased to 6 in. within the overall depth of the shallowest beam framing into the joint (ACI 18.8.3.2). For simpler detailing, it is common for the transverse reinforcement at the ends of the column to continue through the joint regardless of any confinement provided by beams.

Where beams are wider than the joint that they frame into, the beam longitudinal reinforcement located outside of the confined core of the column must be confined by transverse reinforcement passing through the column satisfying the spacing requirements of ACI 18.6.4.2, 18.6.4.3, and 18.6.4.4 where transverse beams do not provide confinement in the joint (ACI 18.8.3.3). Where transverse beams with widths at least three-fourths the column width frame into a joint, only the spacing requirements of ACI 18.6.4.4 must be satisfied. An example of such transverse reinforcement is shown in ACI Figure R18.6.2.

14.4.3 Shear Strength

The design shear strength of cast-in-place beam-column joints in special moment frames must satisfy the following equation (ACI 18.8.4):

$$\phi V_n \geq V_u \tag{14.18}$$

The horizontal joint shear force, V_u, is determined at the mid-height of the joint in the direction of analysis from statics using tensile and compressive beam forces determined in accordance with ACI 18.8.2.1 and column shear consistent with the negative and positive probable flexural strengths of the beam at the face of the joint, M_{pr}^- and M_{pr}^+, where M_{pr} is determined by Equation (14.9).

Free-body diagrams of (1) an interior column or an edge column with the direction of analysis parallel to the edge and (2) an edge column with the direction of analysis perpendicular to the edge or a corner column in a moment frame subjected to gravity and earthquake loads are given in Figure 14.22 for sidesway to the left where probable flexural strengths of the beams are used to determine V_u.

For an interior column or an edge column with the direction of analysis parallel to the edge, the shear force in the column, V_{col}, can be obtained from equilibrium by summing moments about the center of the joint assuming the point of inflection is at the mid-height of the column, which is a reasonable assumption for columns above the first story and below the top story (see Figure 14.22):

$$V_{col} = \frac{M_{pr}^- + M_{pr}^+}{\ell_c} + \frac{(V_1 + V_2) \times (c_1 / 2)}{\ell_c} \tag{14.19}$$

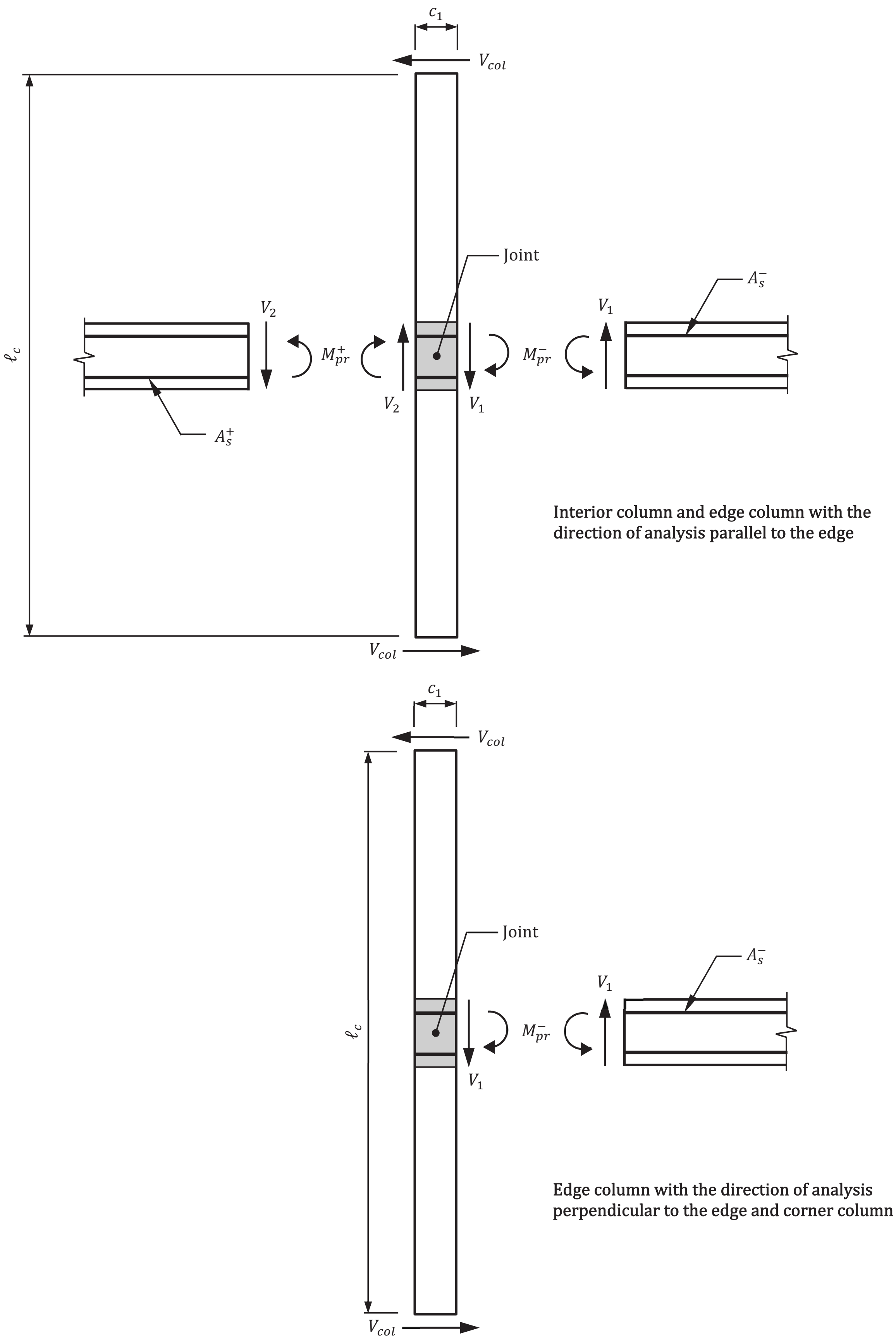


Figure 14.22 Free-body diagrams of columns in a special moment frame subjected to gravity and earthquake loads.

where M_{pr}^{-} and M_{pr}^{+} are determined by the following equations:

$$M_{pr}^{-} = A_s^{-}(1.25f_y)\left(d - \frac{a}{2}\right) = 1.25A_s^{-}f_y\left(d - \frac{1.25A_s^{-}f_y}{1.7f_c'b_w}\right) \quad (14.20)$$

$$M_{pr}^{+} = A_s^{+}(1.25f_y)\left(d - \frac{a}{2}\right) = 1.25A_s^{+}f_y\left(d - \frac{1.25A_s^{+}f_y}{1.7f_c'b_w}\right) \quad (14.21)$$

Similarly, for an edge column with the direction of analysis perpendicular to the edge or a corner column, the shear force in the column, V_{col}, can be obtained from the following equation (see Figure 14.22):

$$V_{col} = \frac{M_{pr}^{-}}{\ell_c} + \frac{V_1 \times (c_1 / 2)}{\ell_c} \quad (14.22)$$

Free-body diagrams of the joints for these two cases with only horizontal forces are given in Figure 14.23 where it is assumed the axial forces on the beams are negligible.

For an interior column or an edge column with the direction of analysis parallel to the edge, the horizontal shear force in the joint, V_u, is obtained from equilibrium by summing forces in the horizontal direction:

$$V_u = T_1 + T_2 - V_{col} = 1.25(A_s^{-} + A_s^{+})f_y - V_{col} \quad (14.23)$$

where the force in the reinforcement is increased by 1.25 in accordance with ACI 18.8.2.1.

Similarly, for an edge column with the direction of analysis perpendicular to the edge or a corner column, the horizontal shear force in the joint, V_u, is equal to the following:

$$V_u = T_1 - V_{col} = 1.25A_s^{-}f_y - V_{col} \quad (14.24)$$

When calculating V_{col}, it is conservative to disregard the term(s) associated with the beam shear force(s). Similarly, it is conservative to disregard V_{col} when calculating V_u.

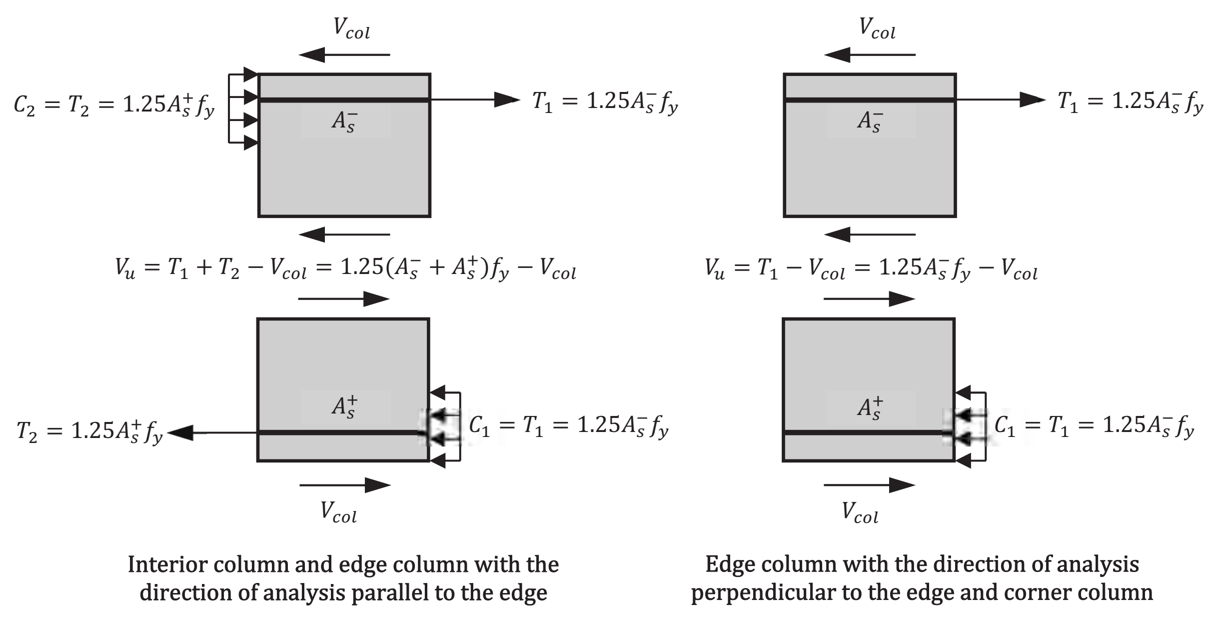


Figure 14.23 Free-body diagrams of the joints in special moment frames.

The strength reduction factor, ϕ, is equal to 0.85 for shear in beam-column joints of special moment frames (see ACI 18.8.4.2 and 21.2.4.4).

The nominal joint shear strength, V_n, in a special moment frame is determined using the requirements of ACI 18.8.4.3. The continuity of the columns and beams framing into the joint and whether the joint is confined or not by transverse beams have an influence on V_n. Continuous columns and beams refer to members present on both sides of a joint. Alternatively, column and beam extensions are assumed to provide continuity through a beam-column joint provided the requirements in ACI 15.2.6 are 15.2.7 are satisfied, respectively (see Figures 11.1 and 11.2 of this publication). A joint is considered to be laterally confined where two transverse beams (that is, two beams framing into the column in the direction perpendicular to the direction of analysis) satisfy the three requirements in ACI 15.2.8 (see Figure 11.3 of this publication). The nominal joint shear strengths in ACI Table 18.8.4.3 are given in Table 14.6.

Table 14.6 Nominal Joint Shear Strength, V_n

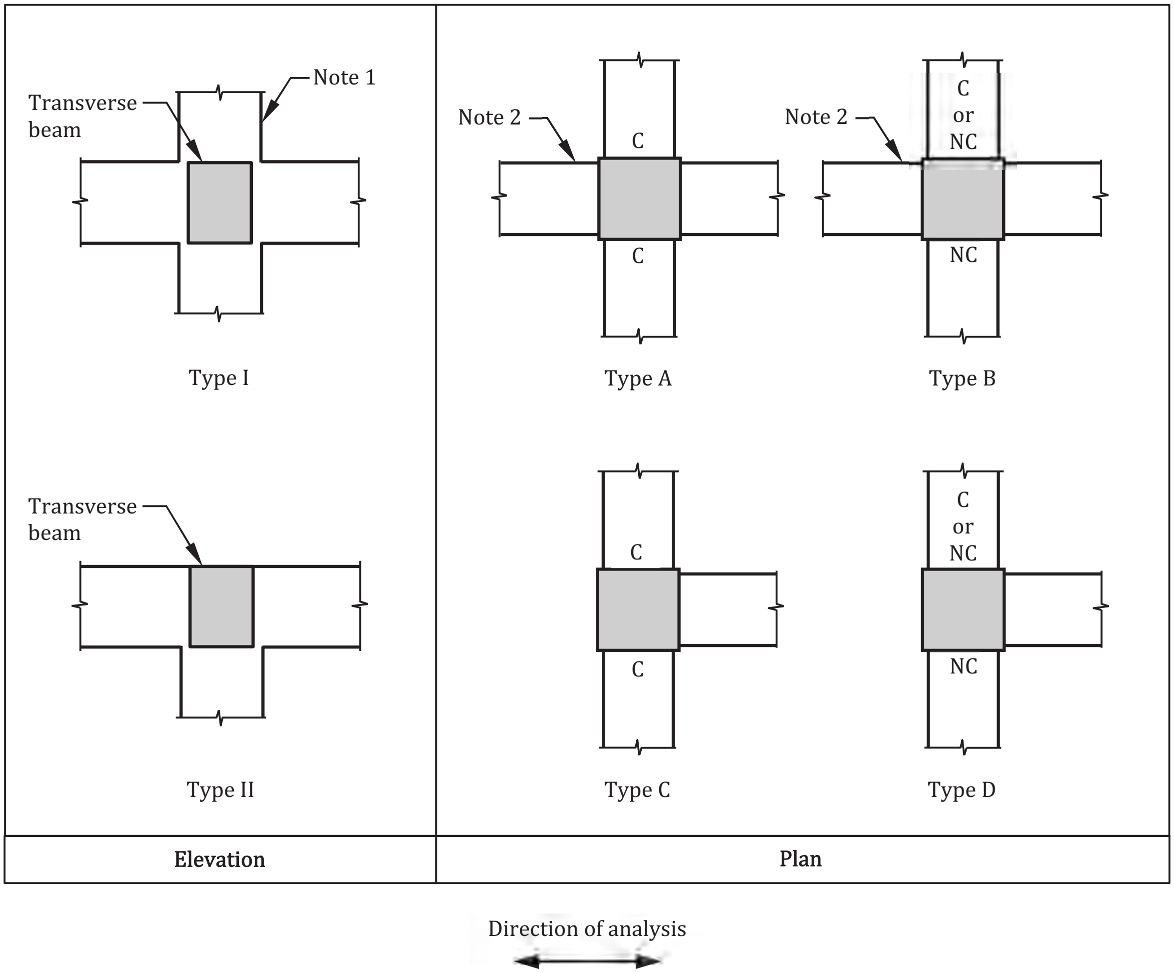
Column	Beam in Direction of Analysis	Confinement by Transverse Beams*	V_n (lbs)**
Continuous or a column extension is provided that satisfies ACI 15.2.6	Continuous or a beam extension is provided that satisfies ACI 15.2.7	Confined	$20\lambda\sqrt{f'_c}A_j$
		Not confined	$15\lambda\sqrt{f'_c}A_j$
	Other	Confined	$15\lambda\sqrt{f'_c}A_j$
		Not confined	$12\lambda\sqrt{f'_c}A_j$
Other	Continuous or a beam extension is provided that satisfies ACI 15.2.7	Confined	$15\lambda\sqrt{f'_c}A_j$
		Not confined	$12\lambda\sqrt{f'_c}A_j$
	Other	Confined	$12\lambda\sqrt{f'_c}A_j$
		Not confined	$8\lambda\sqrt{f'_c}A_j$

** Transverse beams that satisfy the requirements of ACI 15.2.8 are considered to provide confinement (see Figure 11.3 of this publication). Examples of the various joint types in this table are given in Figure 14.24.*

*** The modification factor, λ, that reflects the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength is equal to 0.75 for lightweight concrete and 1.0 for normalweight concrete.*

Examples of the various joint types in Table 14.6 and the corresponding V_n are given in Figure 14.24. Joints with continuous columns (that is, columns frame into the top and bottom of the joint) or column extensions satisfying the requirements of ACI 15.2.6 are designated Type I. Joints with noncontinuous columns (that is, joints where columns frame into the bottom of the joints only, such as those in the top story of a building), are designated Type II. Type A and Type B designations in Figure 14.24 correspond to joints with continuous beams in the direction of analysis or beam extensions satisfying the requirements in ACI 15.2.7. Type C and Type D designations correspond to joints with noncontinuous beams in the direction of analysis. A designation of “C” at the face of a joint means a transverse beam provides confinement at that face in accordance with ACI 15.2.8. An “NC” designation means no transverse beam is present at that face or a transverse beam is present but does not satisfy the confinement requirements in ACI 15.2.8.

The nominal shear strengths, V_n, can be obtained from Figure 14.24 for the various combinations of joint types. For example, a Type I-A joint consists of a continuous column and continuous beams in the direction of analysis with confinement provided by transverse beams on both faces; in this case, $V_n = 20\lambda\sqrt{f'_c}A_j$. This joint type corresponds to an interior column located in the structure other than the top story. Similarly, a corner column in the top story of a structure corresponds to a Type II-D joint, and $V_n = 8\lambda\sqrt{f'_c}A_j$.



Joint Type	V_n
I-A	$20\lambda\sqrt{f_c'}A_j$
I-B	$15\lambda\sqrt{f_c'}A_j$
I-C	$15\lambda\sqrt{f_c'}A_j$
I-D	$12\lambda\sqrt{f_c'}A_j$
II-A	$15\lambda\sqrt{f_c'}A_j$
II-B	$12\lambda\sqrt{f_c'}A_j$
II-C	$12\lambda\sqrt{f_c'}A_j$
II-D	$8\lambda\sqrt{f_c'}A_j$

Notes

1. Column is continuous or a column extension satisfying ACI 15.2.6 is provided (see Figure 11.1 of this publication).
2. Beam in direction of analysis is continuous or a beam extension satisfying ACI 15.2.7 is provided (see Figure 11.2 of this publication).
3. C – Transverse beam provides confinement in accordance with ACI 15.2.8 (see Figure 11.3 of this publication).
4. NC – Transverse beam is not provided or transverse beam does not provide confinement in accordance with ACI 15.2.8 (see Figure 11.3 of this publication).

Figure 14.24 Examples of joint types and the corresponding nominal joint shear strength, V_n.

The term A_j is the effective cross-sectional area within a joint, which is given in ACI 15.4.2.4. By definition, A_j is equal to the product of the joint depth and the effective joint width. The depth of the joint is always equal to the depth of the column parallel to the direction of analysis, c_1 (see Figure 14.25). The effective joint width, w, is equal to the width of the column perpendicular to the direction of analysis where the beams in the direction of analysis are as wide as or wider than the column. In such cases, $A_j = c_1 \times c_2$. For beams not as wide as the column, w is equal to the lesser of $b + c_1$ and $b + 2x$ where b is the beam width and x is the perpendicular distance from the edge of the beam to the nearest side face of the column.

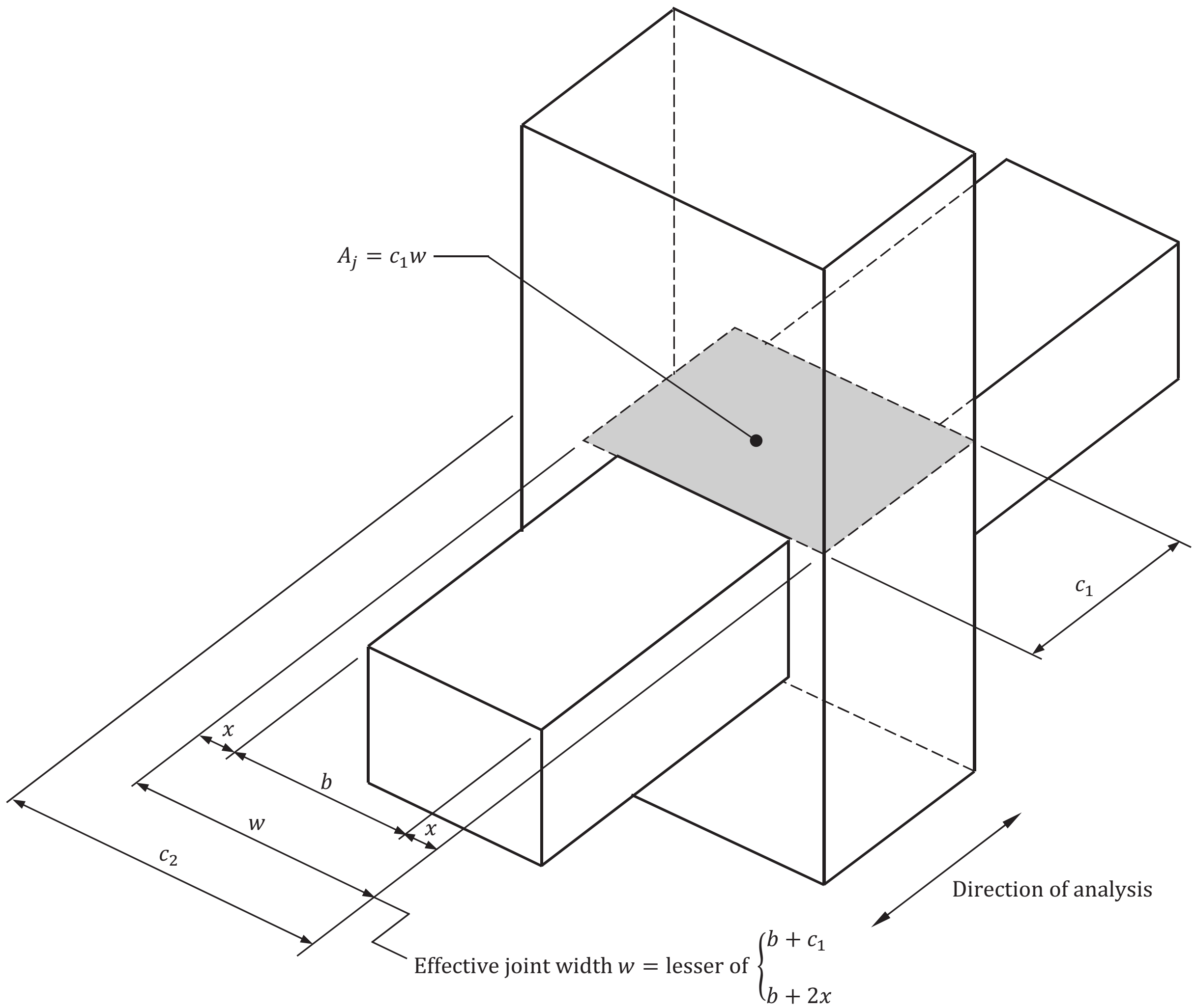


Figure 14.25 Effective cross-sectional area within a joint, A_j.

14.4.4 Development Length of Bars in Tension

Requirements for the development length of hooked bars and headed bars terminating in a joint and the development length of straight bars terminated at a joint are given in ACI 18.8.5.

Detailed information on these requirements is given in Section 14.2.3 of this publication.

14.5 Special Structural Walls

14.5.1 Overview

Design and detailing requirements for special structural walls, including ductile coupled walls and all components of special structural walls (including coupling beams and wall piers that are part of the SFRS) are given in ACI 18.10. According to ACI 2.3, a wall is a vertical structural element designed to resist axial load, lateral load, or both, with an overall horizontal length-to-thickness ratio greater than 3.

Structural walls can be configurated in a number of ways, including the following (see Figure 14.26): (a) rectangular walls, (b) rectangular walls with columns incorporated at the ends (sometimes referred to as a "barbell" wall configuration), (c) intersecting rectangular walls, and (d) core walls with coupling beams.

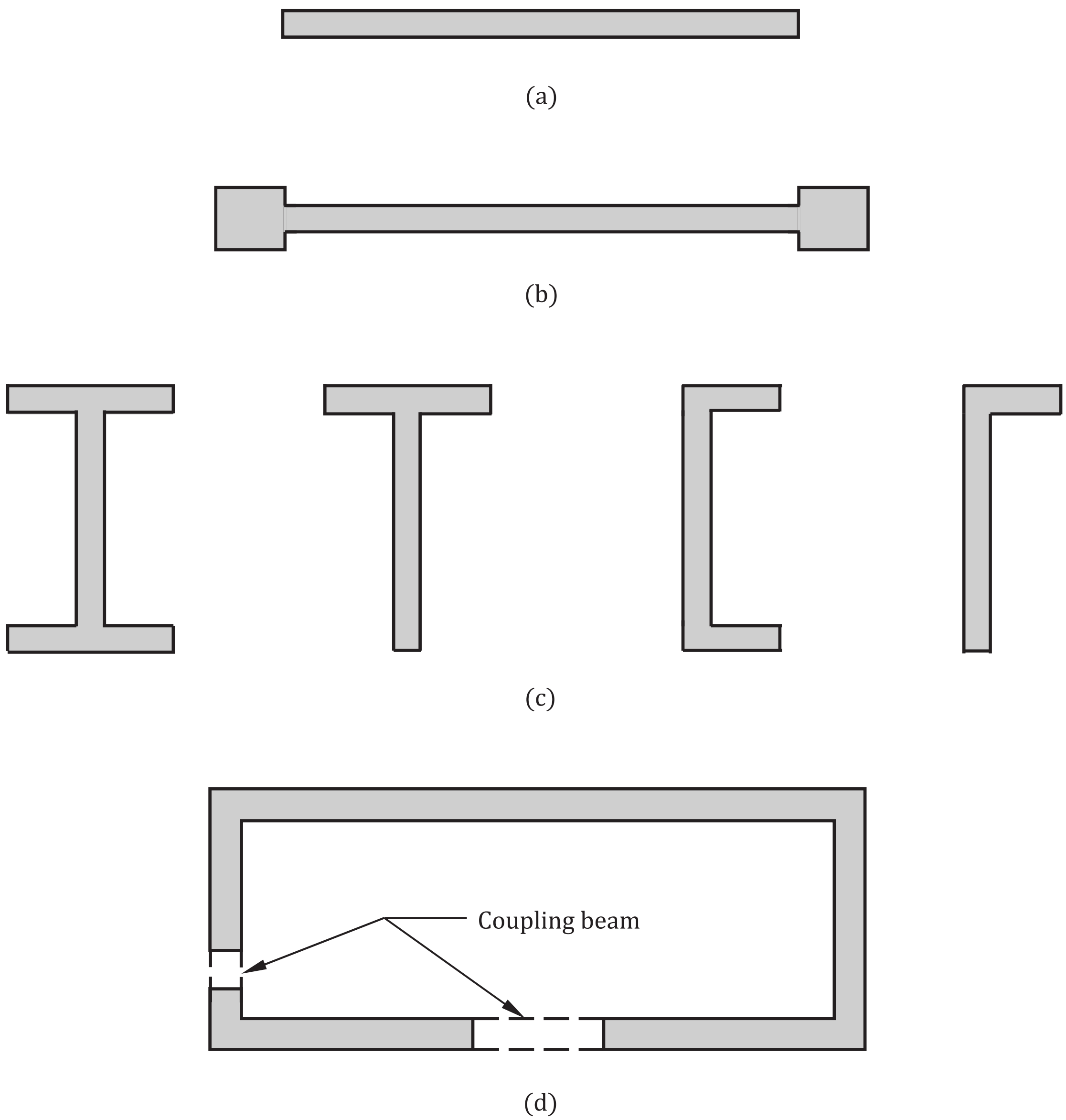
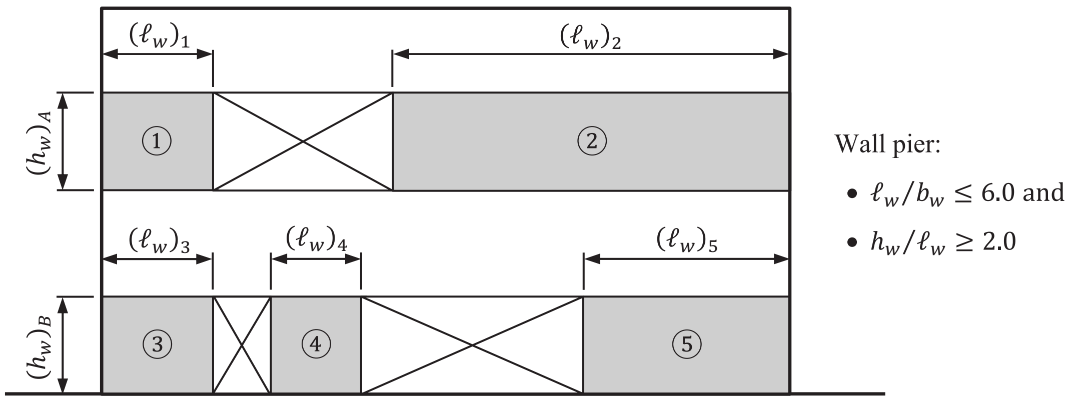


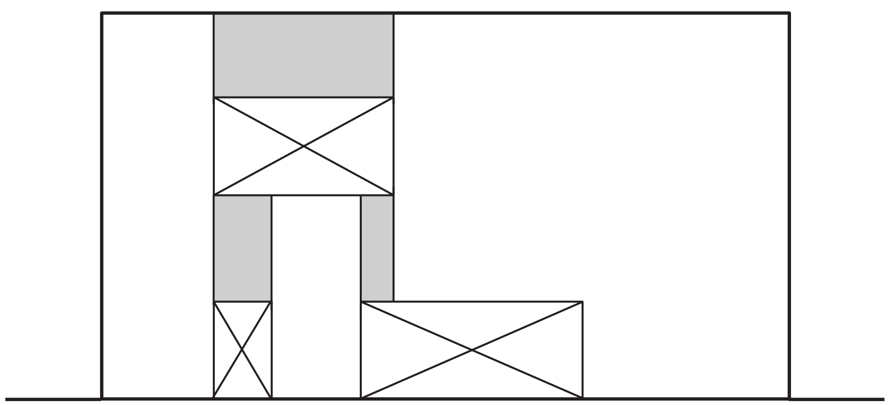
Figure 14.26 Structural wall configurations. (a) Rectangular wall. (b) Rectangular wall with columns. (c) Intersecting rectangular walls. (d) Core walls.

Walls with openings consist of vertical and horizontal wall segments. A vertical wall segment is a segment of a structural wall bounded horizontally by two openings or by an opening and an edge (see Figure 14.27). Wall piers are defined in ACI 2.3 as vertical wall segments satisfying the following dimensional ratios: (1) horizontal length to wall thickness $\ell_w / b_w \le 6.0$ and (2) clear height to horizontal length $h_w / \ell_w \ge 2.0$.

A horizontal wall segment is a segment of a structural wall bounded vertically by two openings or by an opening and an edge (see Figure 14.27).



Vertical wall segments (shaded)



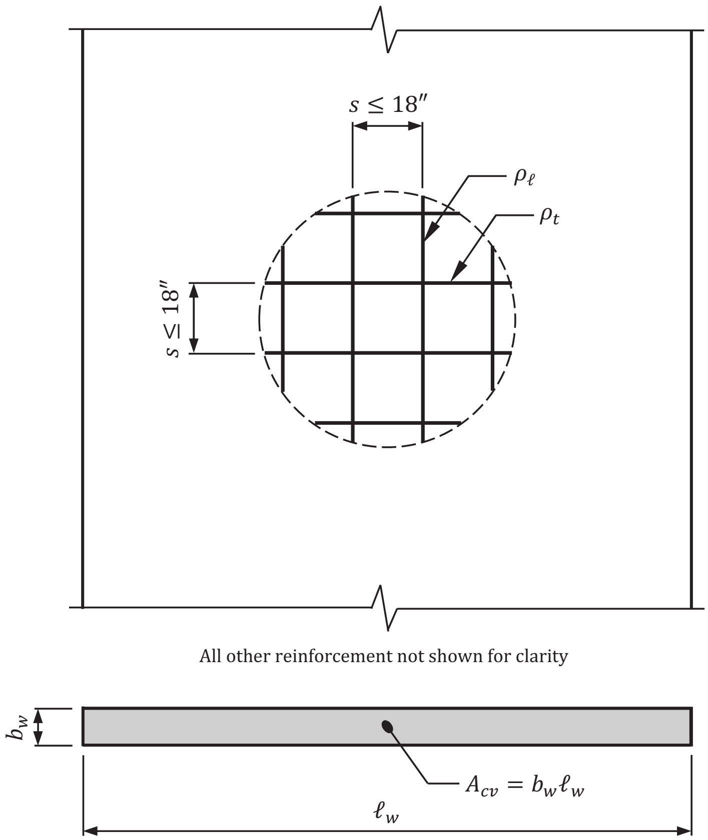
Horizontal wall segments (shaded)

Figure 14.27 Vertical and horizontal wall segments in a structural wall.

14.5.2 Reinforcement

Minimum Reinforcement Requirements

Special structural walls must have reinforcement in two orthogonal directions in the plane of the wall. The minimum reinforcement requirements of ACI 18.10.2.1 are summarized in Figure 14.28 for reinforcement with $f_y \ge 60{,}000$ psi where ρ_ℓ is the ratio of the area of the distributed longitudinal reinforcement in the wall to the gross concrete area perpendicular to that reinforcement and ρ_t is the ratio of the area of the distributed transverse reinforcement in the wall to the gross concrete area perpendicular to that reinforcement.



V_u*	ρ_t		ρ_ℓ
	#5 and smaller	#6 and larger	
$\le \lambda\sqrt{f'_c}A_{cv}$	≥ 0.0020	≥ 0.0025	≥ 0.0025
$> \lambda\sqrt{f'_c}A_{cv}$	≥ 0.0025		

* For $V_u > 2\lambda\sqrt{f'_c}A_{cv}$ or $h_w/\ell_w \ge 2.0$, provide two curtains of reinforcement where h_w = height of entire wall.

Figure 14.28 Web reinforcement requirements for special structural walls.

Tension Development and Splice Requirements

Longitudinal Bar Termination

The tension development and splice requirements in ACI 18.10.2.3 are applicable to any special structural wall. Longitudinal bar cutoff points are given in ACI 18.10.2.3(a). Where longitudinal reinforcement is no longer required to resist the combined effects of flexure and axial forces, it must extend at least 12 ft above the theoretical cutoff point but need not extend more than the tension development length, ℓ_d, determined in accordance with ACI 25.4.2 above the next floor level. This requirement is illustrated in Figure 14.29 where it has been determined by analysis that the theoretical cutoff point for the bars labelled "A" in the figure is located h_{cutoff} from the base of the wall. In order to satisfy the requirements in ACI 18.10.2.3(a), the length of the "A" bars must be equal to the following:

$$\text{Length of "A" bars} = h_{cutoff} + 12\text{ ft} \le h_x + \ell_d \tag{14.25}$$

where h_x is the distance from the base of the wall to the top of the floor level above the theoretical cutoff point.

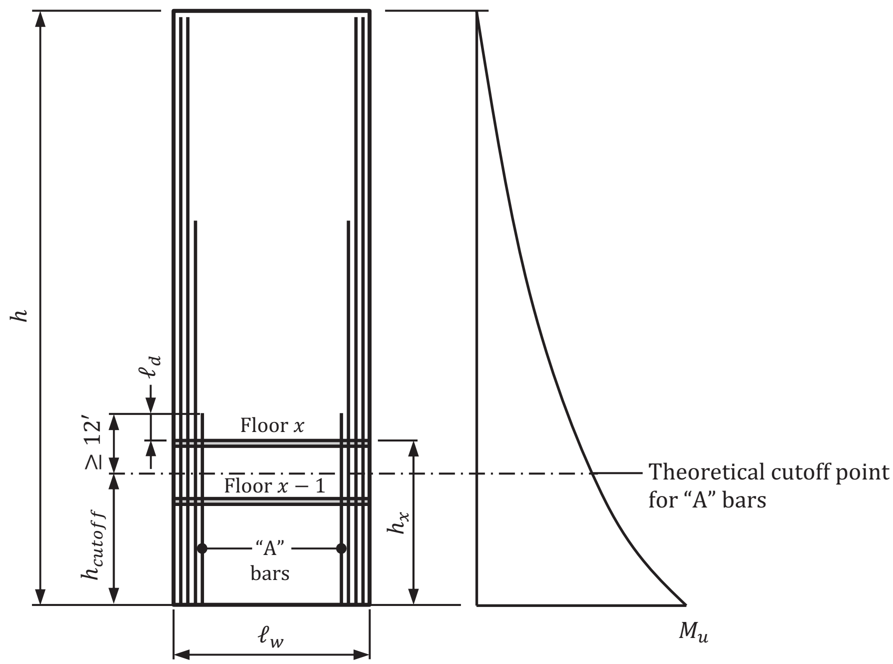


Figure 14.29 Longitudinal bar cutoff points for special structural walls.

The longitudinal reinforcement at the top of a wall need not satisfy ACI 18.10.2.3(a). In general, longitudinal reinforcement should be terminated gradually over the height of a wall and should not occur near critical sections where yielding of the longitudinal reinforcement is expected (such as at the base of a wall where plastic hinges are likely to form).

When determining tension development lengths at locations where yielding of the longitudinal reinforcement is likely to occur, ℓ_d must be multiplied by 1.25 [ACI 18.10.2.3(b)]. This multiplier accounts for (1) the likelihood that the actual yield strength exceeds the specified yield strength of the bars, (2) strain hardening, and (3) cyclic load reversals.

Lap Splices

Requirements where lap splices can occur within boundary regions of special structural walls are given in ACI 18.10.2.3(c). In general, boundary regions are those regions at the ends of a wall and at the edges adjacent to openings subjected to large compressive forces when the wall undergoes cyclic deformations caused by a design-level earthquake event. Such regions typically require special transverse reinforcement to confine the concrete and to restrain the longitudinal reinforcement in the wall so buckling of the bars does not occur. The horizontal length of a boundary element, ℓ_{be}, is defined in ACI 18.10.6.4(a) [see below].

Lap splices of longitudinal wall reinforcement within boundary regions are not permitted over the following regions: (1) a story height h_{sx} or 20 ft, whichever is less, above a critical section where yielding is likely to occur as a result of lateral displacements and (2) a tension development length, ℓ_d, below a critical section where yielding is likely to occur as a result of lateral displacements.

A critical section located at the top of a floor slab is depicted in Figure 14.30. The horizontal length of a boundary region, ℓ_{be}, at the ends of walls and around openings is equal to the greater of $c - 0.1\ell_w$ and $c / 2$, which is measured from the extreme compression fiber or from the edge of an opening. The term c is the largest neutral axis depth calculated from a strain compatibility analysis for the factored axial force and nominal flexural strength consistent with the design displacement, δ_u, at the top of the wall. At wall intersections, boundary regions occur within a length equal to the wall thickness measured beyond the interesting region(s) of the connected walls (see Figure 14.30).

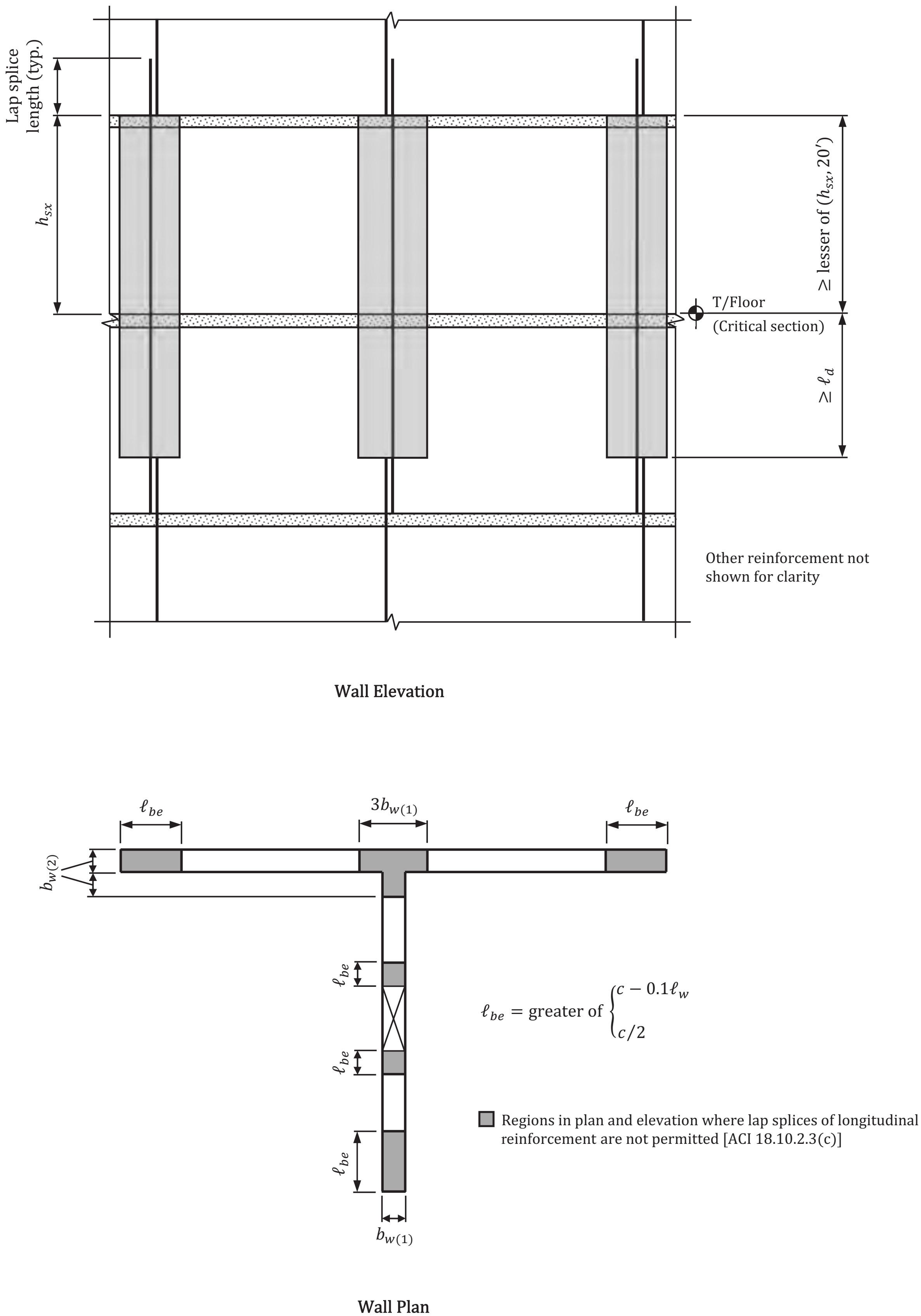


Figure 14.30 Regions where lap splices of longitudinal reinforcement are not permitted in special structural walls.

Mechanical and Welded Splices

Mechanical and welded splices of the reinforcement in special structural walls must conform to the requirements in ACI 18.2.7 and 18.2.8, respectively [ACI 18.10.2.3(d)].

Type 1 mechanical splices must be able to develop in tension or compression at least $1.25f_y$ of the bar. The specified tensile strength of the spliced bars must be developed by Type 2 mechanical splices. Except for Type 2 mechanical splices on Grade 60 reinforcement, mechanical splices are not permitted to be located in regions where yielding of the reinforcement is likely to occur as a result of lateral displacements beyond the linear range of behavior. A Type 2 mechanical splice on Grade 60 reinforcement is permitted at any location in a cast-in-place special structural wall.

Welded splices must be able to develop in tension or compression at least $1.25f_y$ of the bar; these types of splices are not permitted to be located within regions where yielding of the reinforcement is likely to occur as a result of lateral displacements beyond the linear range of behavior.

According to ACI 18.2.8.2, it is not permitted to weld stirrups, ties, inserts, or other similar elements to the longitudinal reinforcement in a special structural wall. Such welding can lead to local embrittlement of the reinforcing steel.

Minimum End Reinforcement for Walls or Piers with a Single Critical Section for Flexure and Axial Loads

A minimum amount of longitudinal reinforcement is required at the ends of walls or walls piers that meet the following conditions:

- $h_w / \ell_w \geq 2.0$
- The wall or pier is essentially continuous from the base of the structure to the top of the wall or pier
- The wall or pier is designed to have a single critical section for flexure and axial loads (such as a cantilever wall)

The following requirements in ACI 18.10.2.4 must be satisfied in such cases:

1. The longitudinal reinforcement ratio within $0.15\ell_w$ from the end of a vertical wall segment and over a width equal to the wall thickness, b_w, must be greater than or equal to $6\sqrt{f'_c} / f_y$.
2. The minimum longitudinal reinforcement prescribed in item 1 above must extend vertically above and below the critical section a length equal to at least the greater of ℓ_w and $M_u / 3V_u$ (which is the length over which yielding is expected).
3. No more than 50 percent of the minimum longitudinal reinforcement prescribed in item 1 above is permitted to be terminated at any one section.

The requirements of ACI 18.10.2.4 are illustrated in Figure 14.31 for a rectangular, cantilever wall that satisfies the conditions noted above. The minimum area of longitudinal reinforcement, $A_{\ell(end)}$, within $0.15\ell_w$ from the ends of the wall is equal to the following based on the requirement in ACI 18.10.2.4(a):

$$A_{\ell(end)} = (6\sqrt{f'_c} / f_y) \times 0.15\ell_w \times b_w = 0.9b_w\ell_w\sqrt{f'_c} / f_y \tag{14.26}$$

Locations where minimum longitudinal reinforcement is required by ACI 18.10.2.4(a) for different wall configurations are given in ACI Figure R18.10.2.4.

Development of Reinforcement in Coupling Beams

In addition to the development requirements in ACI 25.4 and 25.5, the reinforcement in coupling beams must be developed in tension in accordance with the following requirements in ACI 18.10.2.5:

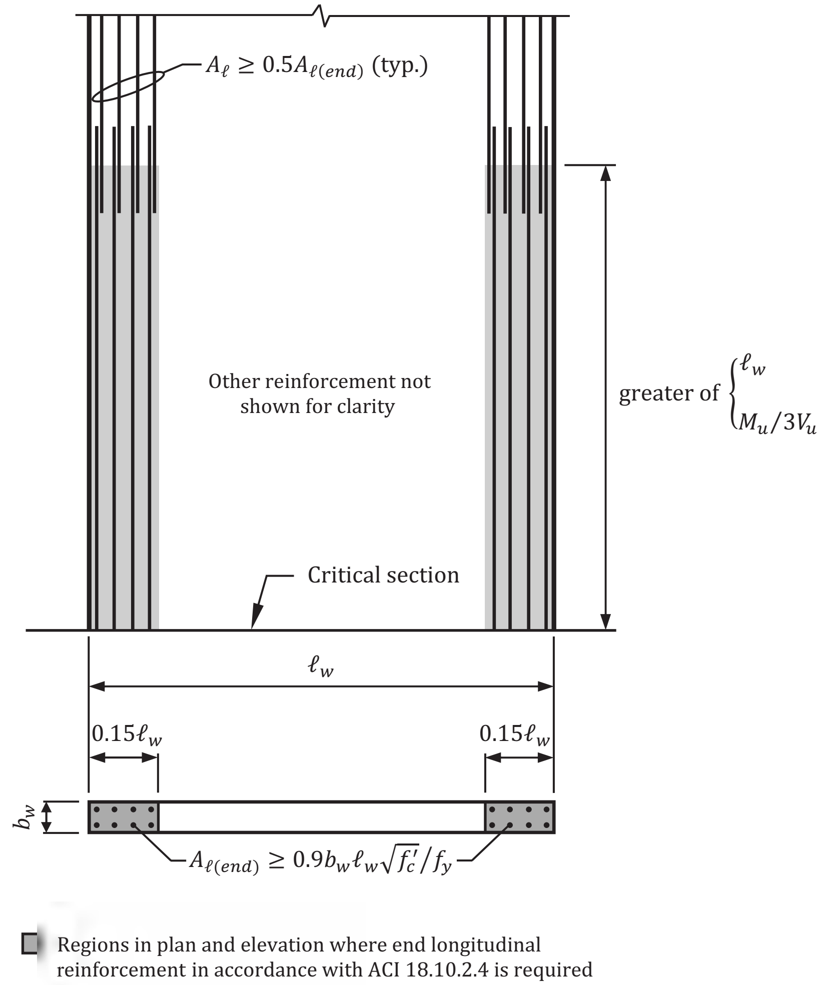


Figure 14.31 End longitudinal reinforcement requirements for walls and piers in accordance with ACI 18.10.2.4.

1. For coupling beams reinforced with longitudinal reinforcement in accordance with the requirements in ACI 18.6.3.1 for beams in special moment frames, the tension development length of the longitudinal reinforcement must be determined using $1.25f_y$.
2. For coupling beams reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan of the beam in accordance with ACI 18.10.7.4, the tension development length of the diagonal reinforcement must be determined using $1.25f_y$.

14.5.3 Design Shear Force

For relatively slender walls (that is, walls with an overall height to length ratio greater than 2.0) without significant openings, the behavior is much like a flexural cantilever where the critical section for flexure and axial forces is at the base of the wall where flexural yielding can occur. Due to material overstrength and strain hardening of the reinforcement, a wall yielding in flexure will likely develop a moment at the critical section equal to the probable flexural strength, M_{pr}. Thus, the shear capacity of the wall must be sufficient so that M_{pr} can develop at the critical section prior to shear failure. For nonslender walls, flexural strength is relatively high and inelastic response occurs in shear rather than flexural yielding.

Consider the wall in Figure 14.32 where it is assumed that the height of the entire wall above the critical section, h_{wcs}, is at least 2.0 times the length of the wall, ℓ_w. For purposes of discussion, assume the wall is subjected to a triangular lateral load distribution, which approximates the distribution of code-prescribed, static earthquake forces over the height of the wall (the following discussion is equally valid where a parabolic distribution of earthquake forces is required). The factored shear and moment diagrams are also shown in the figure and are obtained from the appropriate load combinations. Because multiple load combinations must be considered, flexural overstrength occurs at the critical section, that is, M_{pr} is greater than M_u at the base of the wall for one or both of the load combinations that include earthquake effects, E. Therefore, in order for the wall to yield in flexure at the critical section, the lateral forces applied to the wall must be greater than the code-prescribed forces (see Figure 14.32). This means the shear forces in the wall must be greater as well.

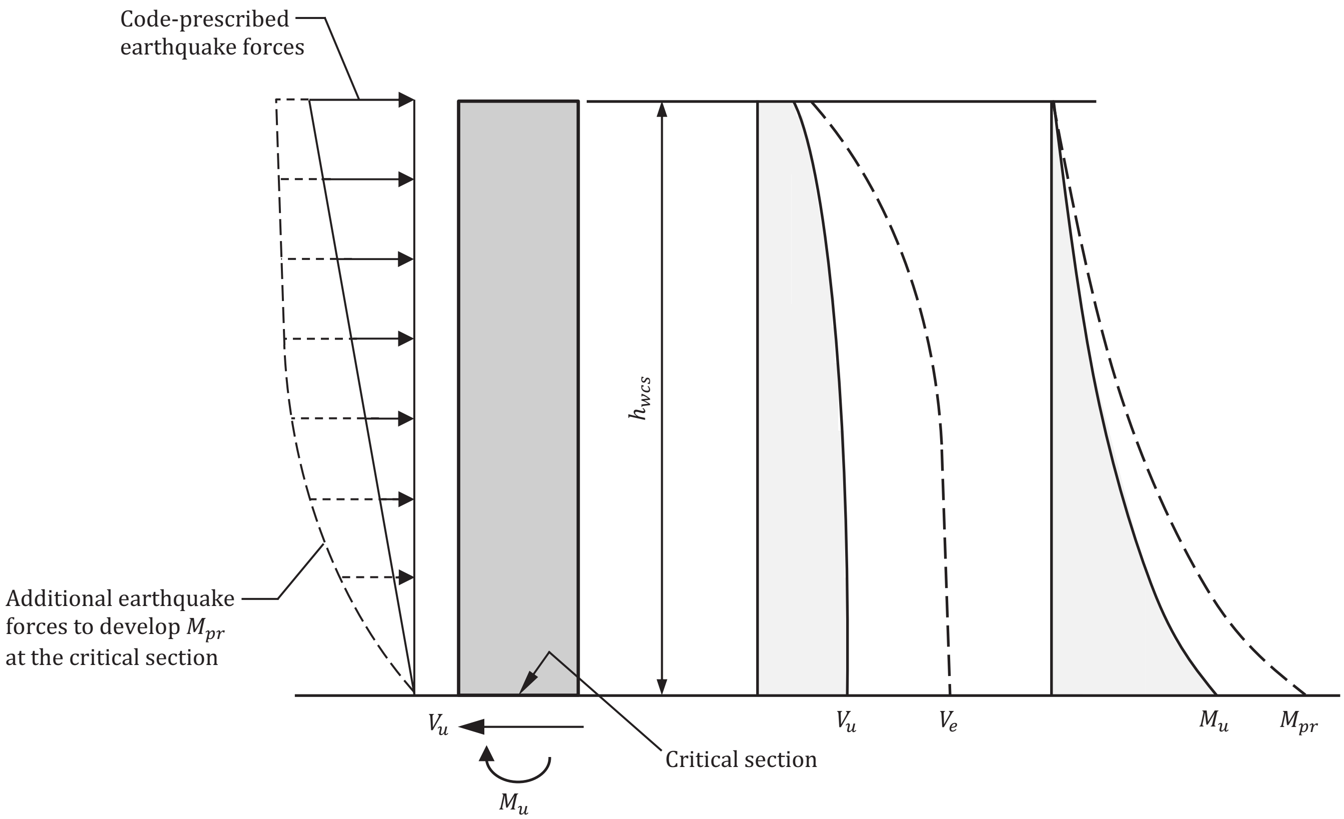


Figure 14.32 Determination of shear demand in a slender wall in accordance with ACI 18.10.3.

The ratio M_{pr} / M_u is a measure of flexural overstrength built into the wall. In general, the shear force corresponding to the development of M_{pr} at the critical section is equal to the design shear force, V_u, due to the code-prescribed forces multiplied by M_{pr} / M_u and a factor to account for the dynamic response of a building.

According to ACI 18.10.3.1, special structural walls must be designed for shear using the design shear force, V_e, determined by ACI Equation (18.10.3.1):

$$V_e = \Omega_v \omega_v V_u \leq 3V_u \tag{14.27}$$

The overstrength factor, Ω_v, is obtained from ACI Table 18.10.3.1.2 and is equal to 1.0 where $h_{wcs} / \ell_w \leq 1.5$ (that is, for nonslender walls). In cases where $h_{wcs} / \ell_w > 1.5$, Ω_v is equal to the following:

$$\Omega_v = \text{greater of} \begin{cases} M_{pr} / M_u \\ \\ 1.5 \end{cases} \tag{14.28}$$

Because M_{pr} depends on the magnitude of the axial force at the critical section, which is different for different load combinations, the ratio M_{pr} / M_u resulting in the largest Ω_v must be used.

The dynamic response of a multistory building produces changing patterns of lateral inertial forces. One or more patterns can shift the centroid of the lateral forces downward, thereby increasing the shear force at the critical section. The term ω_v in Equation (14.27) accounts for this dynamic shear amplification. For walls where $h_{wcs} / \ell_w < 2.0$, $\omega_v = 1.0$ (ACI 18.10.3.1.3); otherwise, ω_v is determined by ACI Equation (18.10.3.1.3):

$$\omega_v = \begin{cases} 0.9 + (n_s / 10) \text{ for } n_s \leq 6 \\ \\ [1.3 + (n_s / 30)] \leq 1.8 \text{ for } n_s > 6 \end{cases} \tag{14.29}$$

In this equation, n_s is the number of stories above the critical section, which must be taken greater than or equal to $0.007h_{wcs}$. This limit is imposed on n_s to account for buildings with relatively large story heights.

It is permitted to use $\phi = 0.75$ when determining the design shear strength of a slender special structural wall where V_e is determined by Equation (14.27) [see ACI 21.2.4]. For short, squat walls where it is impractical to design for the shear force corresponding to the development of M_{pr}, $\phi = 0.60$.

14.5.4 Shear Strength

According to ACI 18.10.4.1, the nominal shear strength of special structural walls, V_n, is determined by ACI Equation (18.10.4.1):

$$V_n = (\alpha_c \lambda \sqrt{f_c'} + \rho_t f_{yt}) A_{cv} \tag{14.30}$$

where α_c is equal to the following:

$$\alpha_c = \begin{cases} 3 \text{ for } h_w / \ell_w \leq 1.5 \\ \\ 6 - (2h_w / \ell_w) \text{ for } 1.5 < h_w / \ell_w < 2.0 \\ \\ 2 \text{ for } h_w / \ell_w \geq 2.0 \end{cases} \tag{14.31}$$

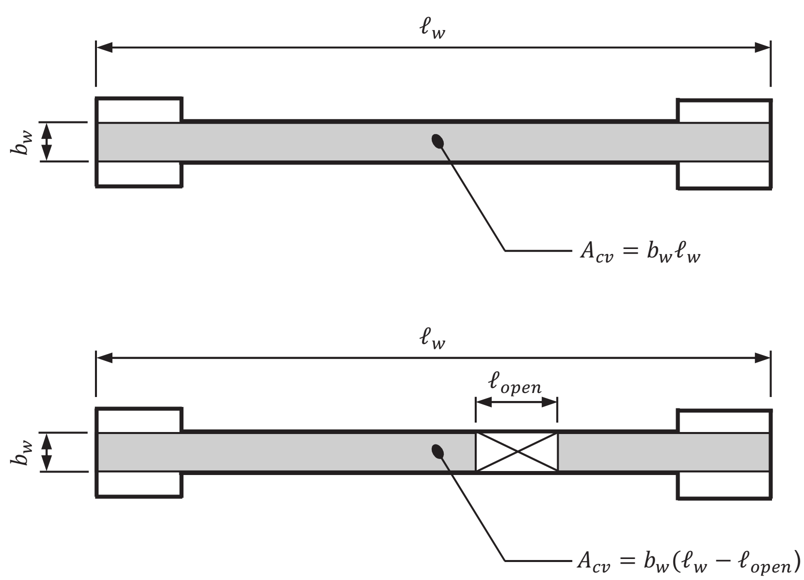


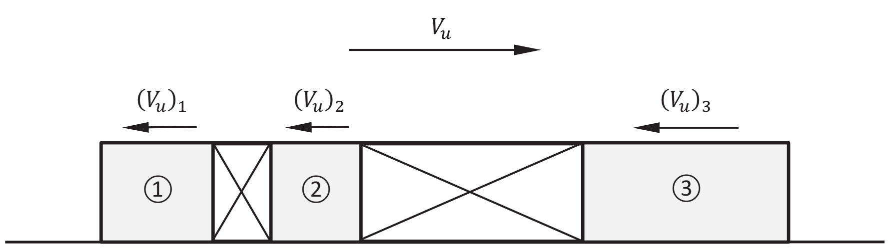
Figure 14.33 Area of wall resisting shear force.

The term A_{cv} is the gross area of the cross-section of the wall resisting the shear force. For a rectangular wall with no openings, A_{cv} is equal to the product of the thickness of the web and the length of the web in the direction of analysis. If a wall section has openings, the area of the openings must not be included in A_{cv}. The areas A_{cv} to be used in Equation (14.30) where the ends of a wall are enlarged are given in Figure 14.33.

Transverse and longitudinal shear reinforcement must be appropriately distributed along the height and length of a wall to effectively restrain inclined cracks. Wherever practical, shear reinforcement should be uniformly distributed and at a relatively small spacing. The longitudinal reinforcement ratio, ρ_ℓ, must be greater than or equal to the transverse reinforcement ratio, ρ_t, where the overall height-to-length ratio $h_w / \ell_w \leq 2.0$ (ACI 18.10.4.3). Any concentrated reinforcement near the ends of the wall provided primarily for resisting bending moments is not to be included when determining ρ_ℓ and ρ_t.

Nominal shear strength requirements for walls with openings are given in ACI 18.10.4.4 and 18.10.4.5. Where several vertical wall segments all resist a common factored shear force, V_u, the nominal shear strength, V_n, is limited to $8\sqrt{f_c'}A_{cv}$ (ACI 18.10.4.4).

When designing an individual vertical wall segment, V_n is calculated by Equation (14.30) where ρ_t is determined using the transverse reinforcement in that segment and α_c is obtained using the larger of the following two ratios: (1) h_w / ℓ_w based on the dimensions of the entire wall and (2) h_w / ℓ_w based on the dimensions of the wall segment. The intent of this requirement is to ensure that a vertical wall segment will not have a unit shear strength greater than that of the entire wall; however, the vertical segment may have a lower unit shear strength if h_w / ℓ_w of the vertical segment is greater than that of the entire wall. The nominal shear strength, V_n, is limited to $10\sqrt{f_c'}A_{cw}$ for any one of the vertical wall segments in the group where A_{cw} is the area of the concrete section of the individual vertical wall segment. The portion of the total shear force V_u resisted by an individual vertical wall segment is based on the flexural and shear rigidities of that segment, and that shear force must be less than or equal to the design shear strength, ϕV_n, for that segment (see Figure 14.34).



$V_u = (V_u)_1 + (V_u)_2 + (V_u)_3$

$(V_u)_1 \leq \phi\left[(\alpha_c)_1\lambda\sqrt{f_c'} + (\rho_t)_1 f_{yt}\right](A_{cv})_1 \leq \phi 10\sqrt{f_c'}(A_{cw})_1$

$(V_u)_2 \leq \phi\left[(\alpha_c)_2\lambda\sqrt{f_c'} + (\rho_t)_2 f_{yt}\right](A_{cv})_2 \leq \phi 10\sqrt{f_c'}(A_{cw})_2$

$(V_u)_3 \leq \phi\left[(\alpha_c)_3\lambda\sqrt{f_c'} + (\rho_t)_3 f_{yt}\right](A_{cv})_3 \leq \phi 10\sqrt{f_c'}(A_{cw})_3$

For combined vertical wall segments: $\phi V_n \leq \phi 8\sqrt{f_c'}A_{cv}$
where $A_{cv} = (A_{cw})_1 + (A_{cw})_2 + (A_{cw})_3$

Figure 14.34 Shear strength requirements for vertical wall segments.

In the case of horizontal wall segments and coupling beams, V_n is limited to $10\sqrt{f_c'}A_{cw}$ where A_{cw} is the area of the concrete section of the horizontal wall segment or the area of the coupling beam (ACI 18.10.4.5).

The requirements in ACI 21.2.4.1 where $\phi = 0.60$ for shear are not applicable to walls or wall piers designed in accordance with ACI 18.10.6.2 because these members are controlled by flexural yielding and the code-prescribed shear forces have been amplified (ACI 18.10.4.6). As noted previously, $\phi = 0.75$ in such cases.

14.5.5 Design for Flexure and Axial Force

Structural walls are designed for combined flexure and axial forces in accordance with the provisions in ACI 22.4 (ACI 18.10.5.1). An interaction diagram for a wall can be constructed using a strain compatibility analysis based on the longitudinal reinforcement in the section. Just like in the design of reinforced concrete columns, all the combined factored axial force and bending moment points obtained from the applicable load combinations in ACI Chapter 5 must fall within or on the design strength interaction diagram in order for strength requirements to be satisfied.

Openings in walls must be considered when determining the strength requirements of a special structural wall. Relatively large openings can dramatically change the overall behavior and it is important to understand how a wall will perform when one or more openings are introduced. Additional reinforcement around the openings is typically provided to counteract large tensile forces that may develop, especially at the corners.

For walls with flanges like those depicted in Figure 14.26, a portion of the flange is considered to be effective in resisting the effects from combined axial force and bending moment. In general, the effective width of a flange depends on the magnitude of the axial force on the wall, the amount of lateral drift (effective flange width increases with increasing drift), and whether the flange is in tension or compression. In lieu of a rational analysis that takes these variables into account, ACI 18.10.5.2 contains a simple method to determine the effective flange width, which is illustrated in Figure 14.35 where $h_{w,above}$ is the total wall height above the section under consideration. Because the effective compression flange width has little effect on the strength and deformation capacity of a wall, the provisions in ACI 18.10.5.2 are based on the effective tension flange width only. It is assumed the effective flange and the reinforcement in it fully contribute to the nominal strength of the wall section.

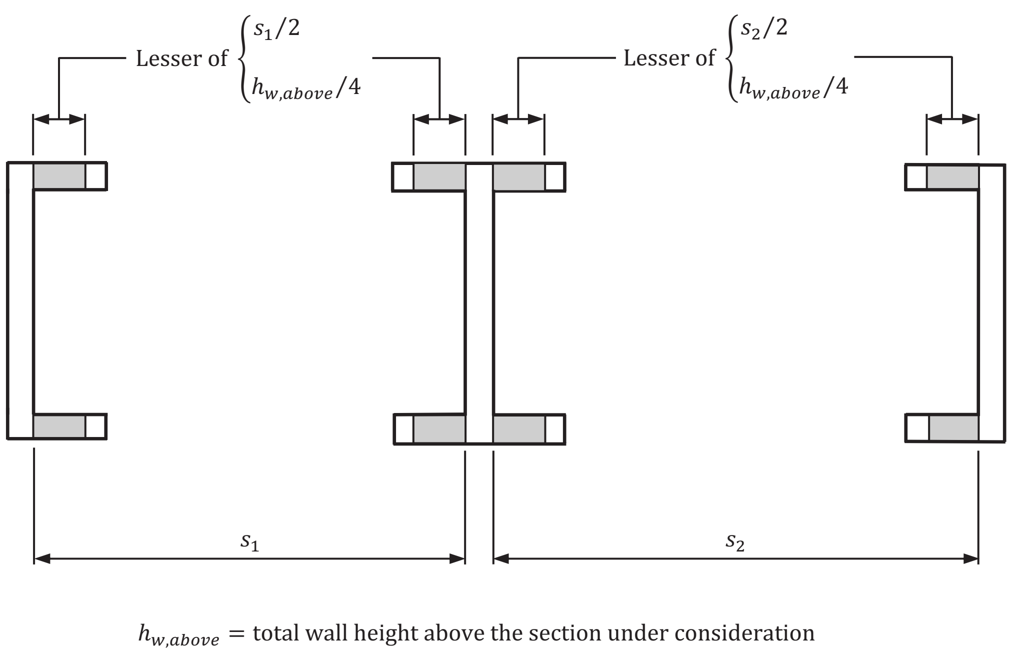


Figure 14.35 Effective flange width for special structural walls.

14.5.6 Boundary Elements of Special Structural Walls

Overview

The ends of a structural wall and the edges adjacent to openings in structural walls can be subjected to large compressive forces when the wall undergoes cyclic deformations caused by a design-level earthquake event. Special transverse reinforcement may be required at these locations to confine the concrete and to restrain the longitudinal reinforcement in the wall so buckling of the bars does not occur.

Two design approaches for evaluating the need of special elements are given in ACI 18.10.6: (1) displacement-based approach and (2) compressive stress approach. The first approach is applicable to slender walls or wall piers $(h_{wcs} / \ell_w \geq 2.0)$ effectively continuous from the base of the structure to the top of the wall and are designed to have one critical section for flexure and axial forces (which typically occurs at the base of the wall). The compressive stress approach can be used to assess the need for special boundary elements for any structural wall.

Displacement-Based Approach (ACI 18.10.6.2)

In the displacement-based approach, special transverse reinforcement must be provided in compression zones of special structural walls where the strain at the extreme compression fiber exceeds a critical value when the wall is subjected to 1.5 times the design displacement, δ_u, which occurs at the top of the wall. According to ASCE/SEI 12.8.6, $\delta_u = \delta_x = C_d \delta_{xe} / I_e$ where C_d is the deflection amplification factor given in ASCE/SEI Table 12.2-1, δ_{xe} is the deflection at the top of the wall determined from analysis where the code-prescribed lateral earthquake forces are applied over the height of the structure, and I_e is the earthquake importance factor determined in accordance with ASCE/SEI 11.5.1.

Compression zones must be reinforced with special boundary elements where ACI Equation (18.10.6.2a) is satisfied:

$$\frac{1.5\delta_u}{h_{wcs}} \geq \frac{\ell_w}{600c} \tag{14.32}$$

where δ_u / h_{wcs} must be taken greater than or equal to 0.005. The lower limit of 0.005 on the ratio δ_u / h_{wcs} is specified to require moderate wall deformation capacity for stiff buildings.

The neutral axis depth, c, in Equation (14.32) corresponds to the largest neutral axis depth calculated for the factored axial force and nominal moment strength of the wall when it is displaced in the same direction as δ_u. In general, c can be obtained from a strain compatibility analysis for each load combination that includes earthquake effects, considering sidesway to the left and to the right. The wall dimensions, the material properties, and the amount and distribution of the longitudinal reinforcement in the wall are required to perform such an analysis.

Equation (14.32) is derived from the relationships shown in Figure 14.36. It is assumed the displacement $1.5\delta_u$ at the top of the wall is due entirely to the curvature, ϕ_u, which is centered on the critical section of the wall at its base. It is also assumed the length of the plastic hinge that forms at the base is equal to one-half the length of the wall, ℓ_w.Assuming small curvatures, $\phi_u = \varepsilon_{cu} / c = 0.003 / c$. Therefore, $1.5\delta_u = 0.003 h_{wcs} \ell_w / 2c$. Rearranging terms and rounding down the numerical constant in the denominator results in Equation (14.32).

Where special boundary elements are required by Equation (14.32), the requirements in (1) and either (2) or (3) must be satisfied:

1. Special boundary transverse reinforcement must extend vertically above and below the critical section at least the greater of ℓ_w and $M_u / 4V_u$ (that is, the anticipated plastic hinge length), except as permitted by ACI 18.10.6.4(j).
2. Width of the flexural compression zone, $b \geq \sqrt{0.025 c \ell_w}$.
3. $\dfrac{\delta_c}{h_{wcs}} \geq \dfrac{1.5\delta_u}{h_{wcs}}$ where $\dfrac{\delta_c}{h_{wcs}} = \dfrac{1}{100}\left[4 - \dfrac{1}{50}\left(\dfrac{\ell_w}{b}\right)\left(\dfrac{c}{b}\right) - \dfrac{V_e}{8\sqrt{f'_c} A_{cv}}\right] \geq 0.015$

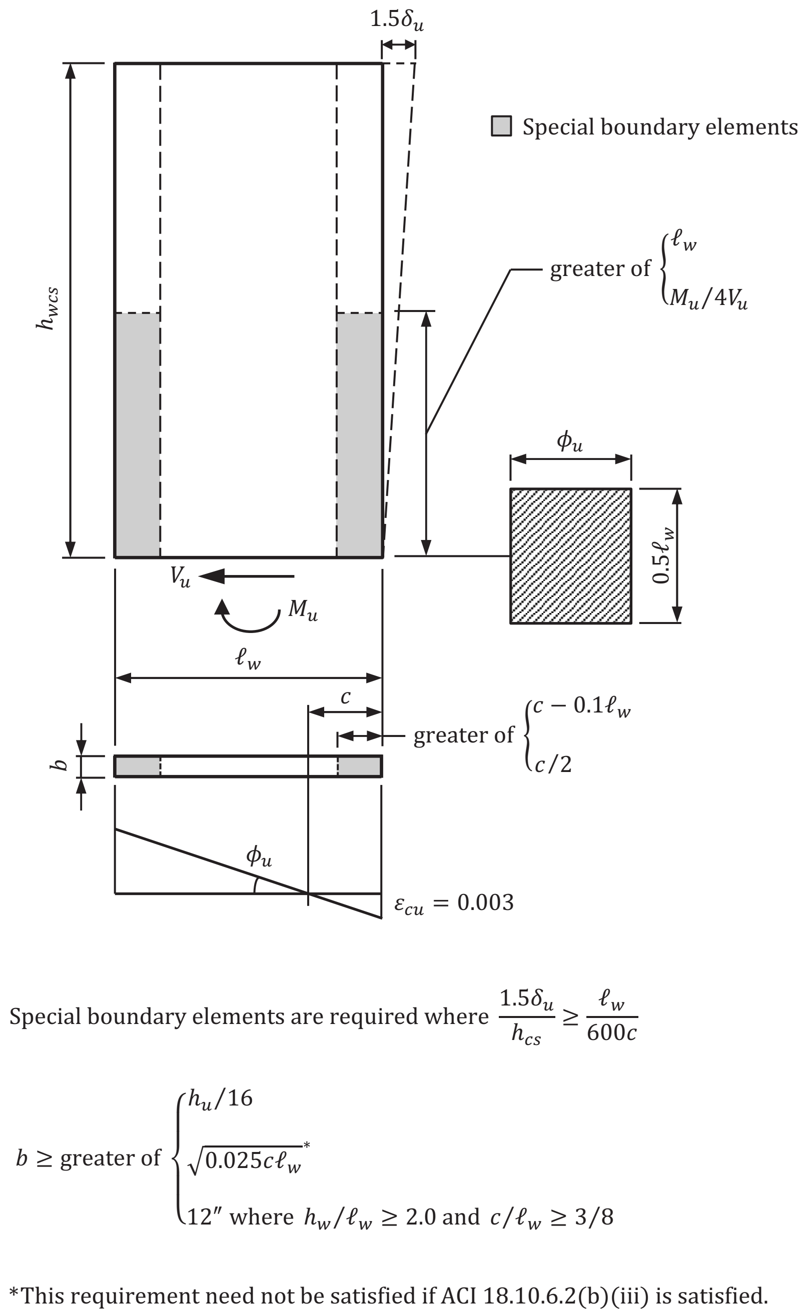


Figure 14.36 Special boundary element requirements in accordance with ACI 18.10.6.2.

The term δ_c in (3) is the wall displacement capacity at the top of the wall at a 20 percent loss of lateral strength. Thus, (3) requires the drift capacity of a wall to exceed 1.5 times the drift demand. The expression for b in (2) is derived from the equation in (3) assuming $\delta_u / h_{wcs} \cong 0.015$ and $(V_e / 8\sqrt{f_c'}A_{cv}) \cong 1.0$.

Compressive Stress Approach (ACI 18.10.6.3)

The compressive stress approach in ACI 18.10.6.3 can be used to assess the need for special boundary elements for any structural wall. In this method, the wall is subjected to gravity loads and the maximum bending moments and shear forces due to earthquake effects in a given direction (see Figure 14.37). The combined compressive stress due to gravity loads and bending moments is calculated assuming a linearly elastic model and gross section properties

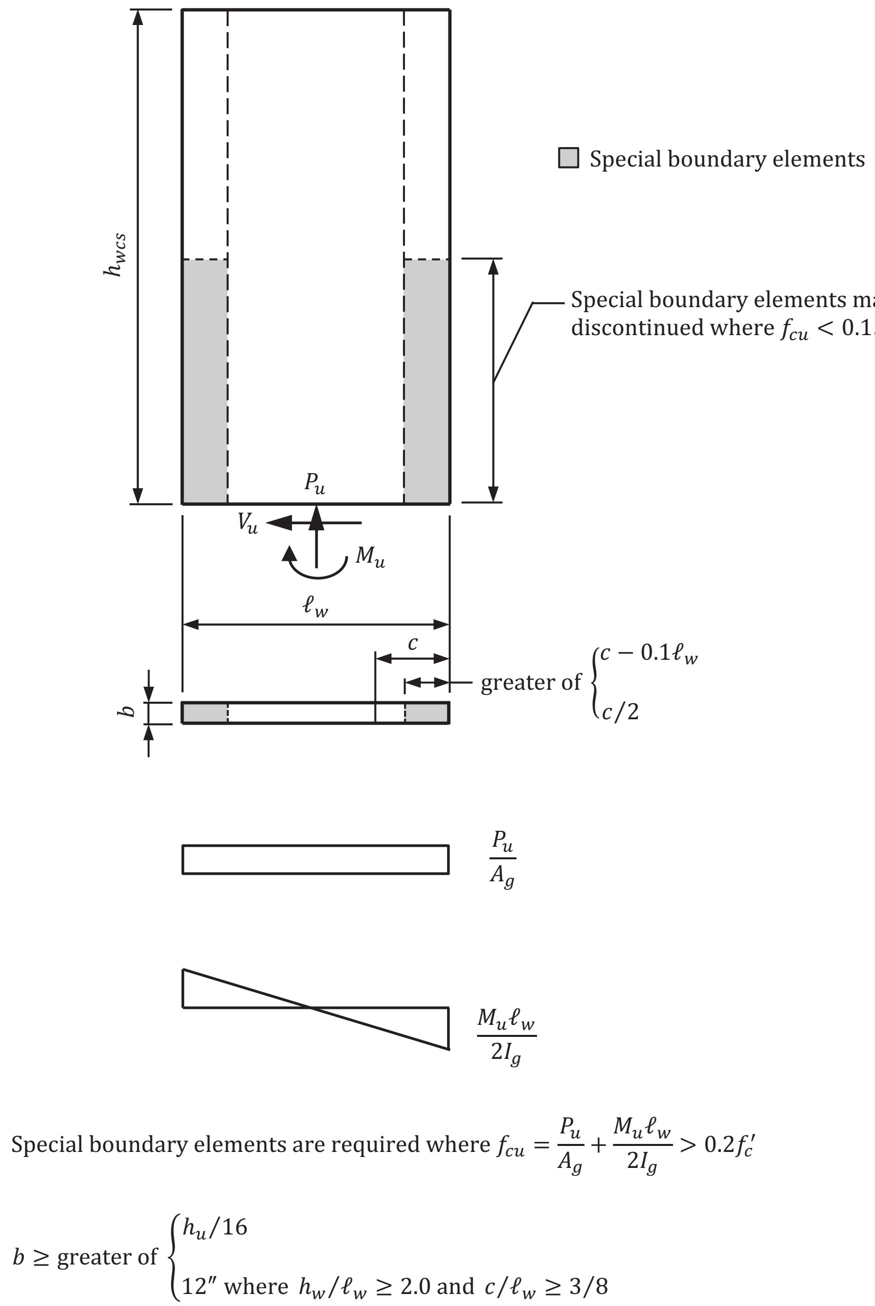


Figure 14.37 Special boundary element requirements in accordance with ACI 18.10.6.3.

of the wall considering the effective flange width requirements in ACI 18.10.5.2 for walls with flanges (see Figure 14.35). Special boundary elements are required at the ends of the wall or at edges around openings where the maximum compressive stress, f_{cu}, exceeds $0.2f'_c$:

$$f_{cu} = \frac{P_u}{A_g} + \frac{M_u \ell_w}{2I_g} > 0.2f'_c \tag{14.33}$$

The special boundary elements may be discontinued where the combined stress is less than $0.15f'_c$.

Design and Detailing Requirements for Special Boundary Elements

The design and detailing requirements of ACI 18.10.6.4 must be satisfied where special boundary elements are required by ACI 18.10.6.2 or 18.10.6.3. A summary of these requirements is given in Table 14.7 and in Figure 14.38 for the case of a rectangular wall with (1) a perimeter rectilinear hoop and crossties where the web reinforcement is developed by standard hooks at the ends and (2) overlapping hoops and crossties where the web reinforcement is developed by heads. In the figure, ℓ_1 and ℓ_2 are the lengths of the hoop legs in the direction of analysis. It is assumed the wall sections in Figure 14.38 are within the plastic hinge region defined in ACI 18.10.6.2(b) so that the web vertical reinforcement outside of the special boundary element must have lateral support in accordance with ACI 18.10.6.4(i).

Table 14.7 Design and Detailing Requirements for Special Boundary Elements

Requirement	ACI Section Number	Figure Number(s)
Horizontal length of a special boundary element, ℓ_{be}, at each end of a wall or from the edge of an opening must be greater than or equal to the greater of $c - 0.1\ell_w$ and $c / 2$.	18.10.6.4(a)	14.36, 14.37
Width of the flexural compression zone, b, over ℓ_{be}, including a flange if present, must be greater than or equal to $h_u / 16$ where h_u is the laterally unsupported height of a wall or pier at the extreme compression fiber.	18.10.6.4(b)	14.36, 14.37
Width of the flexural compression zone, b, over ℓ_{be} must be greater than or equal to 12 in. where all the following are satisfied: • the wall or wall pier is effectively continuous from the base of the structure to the top of the wall • the wall or wall pier is designed to have a single critical section for flexure and axial loads • $h_w / \ell_w \geq 2.0$ • $c / \ell_w \geq 3 / 8$	18.10.6.4(c)	14.36, 14.37
For walls with flanges, the boundary element must include the effective flange width defined in ACI 18.10.5.2 and must extend into the web at least 12 in.	18.10.6.4(d)	14.35
Transverse reinforcement in the boundary element must satisfy the detailing requirements of ACI 18.7.5.2(a) through (d) and ACI 18.7.5.3, which are applicable to columns in special moment frames, except the vertical spacing limit in ACI 18.7.5.3(a) must be one-third of the least dimension of the boundary element. The maximum vertical spacing requirements in ACI Table 18.10.6.5(b) for transverse reinforcement where special boundary elements are not required must also be satisfied.	18.10.6.4(e)	14.38
The maximum center-to-center spacing, h_x, of the laterally supported longitudinal bars around the perimeter of the boundary element must be less than or equal to the lesser of 14 in. and two-thirds the thickness of the boundary element. Lateral support must be provided by a seismic hook of a crosstie or a corner of a hoop. The length of the hoop leg must be less than or equal to 2 times the thickness of the confined core. Adjacent hoops must overlap at least the lesser of 6 in. and two-thirds of the boundary element thickness.	18.10.6.4(f)	14.38

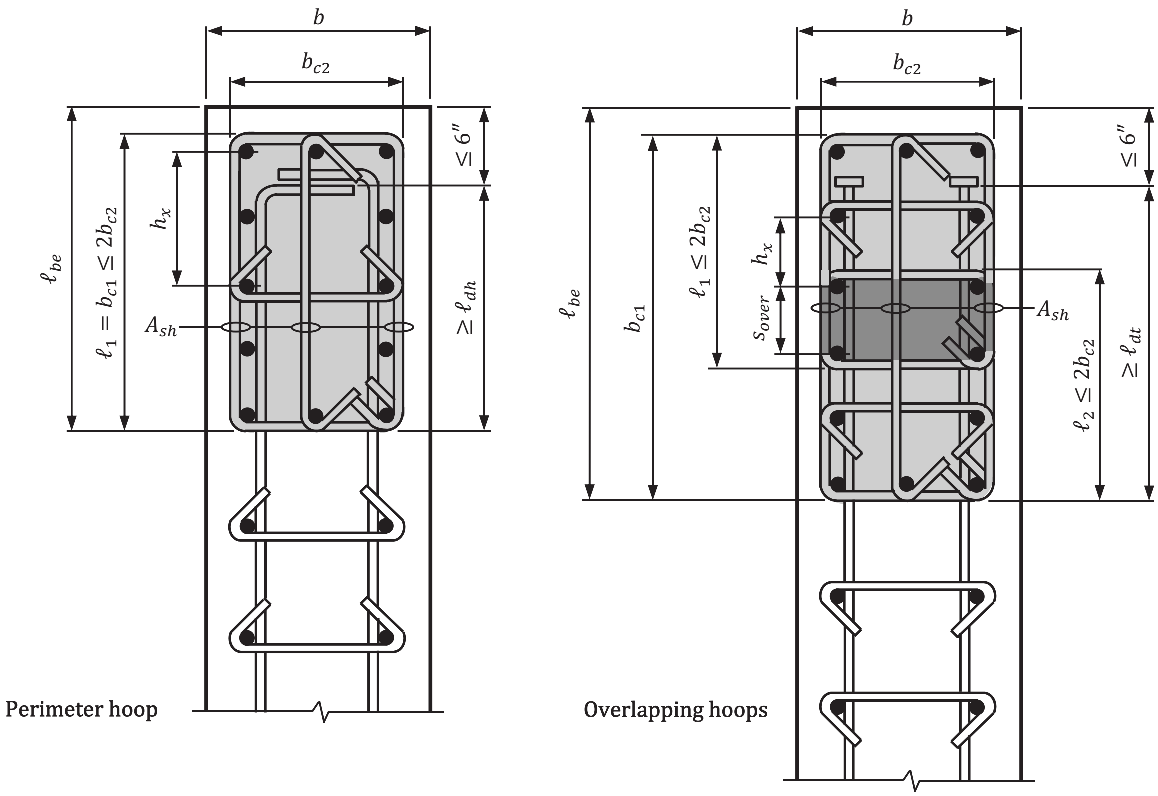
(table continued on next page)

Table 14.7 Design and Detailing Requirements for Special Boundary Elements (cont.)

Requirement	ACI Section Number	Figure Number(s)
Minimum transverse reinforcement must be in accordance with ACI Table 18.10.6.4(g).	18.10.6.4(g)	14.38
The specified compressive strength of the concrete within the thickness of the floor system must be at least 70 percent of f_c' of the wall.	18.10.6.4(h)	—
Web vertical reinforcement must have lateral support provided by the corner of hoop or by a crosstie with seismic hooks at each end for a distance above and below the critical section specified in ACI 18.10.6.2(b). The hoops or crossties must have a diameter of at least that specified in ACI 25.7.2.2 for ties and the vertical spacing must be less than or equal to 12 in.	18.10.6.4(i)	14.38
Where the critical section occurs at the base of a wall, special boundary element transverse reinforcement must extend into the support at least ℓ_d of the largest longitudinal reinforcement in the special boundary element where ℓ_d is determined in accordance with ACI 18.10.2.3. Special boundary element transverse reinforcement must extend at least 12 in. into footings, mats, or piles caps, unless a greater extension is required by ACI 18.13.2.4.	18.10.6.4(j)	14.40, 14.41
Horizontal reinforcement in a wall web must extend to within 6 in. of the end of the wall and it must be anchored to develop f_y within the confined core of the boundary element using standard hooks or heads. It is permitted to terminate the horizontal web reinforcement without a standard hook or head where (1) the confined boundary element has sufficient length to develop the horizontal web reinforcement and (2) $A_s f_y / s$ of the horizontal web reinforcement is less than or equal to $A_s f_{yt} / s$ of the boundary element transverse reinforcement parallel to the horizontal web reinforcement.	18.10.6.4(k)	14.38

Design and Detailing Requirements Where Special Boundary Elements Are Not Required

Even though special boundary elements may not be required by ACI 18.10.6.2 or 18.10.6.3, transverse reinforcement in accordance with ACI 18.10.6.5 may need to be provided at the ends of a wall. In particular, transverse reinforcement applicable to columns in special moment frames [ACI 18.7.5.2(a) through (e)] must be provided over the distance calculated in accordance with ACI 18.10.6.4(a) at any location over the height of the wall where the longitudinal reinforcement ratio at the wall boundary, ρ, is greater than $400 / f_y$. Methods to determine ρ for two cases are given in Figure 14.39: (1) longitudinal bars at the ends of the wall are larger than the uniformly distributed web longitudinal bars (left part of the figure) and (2) uniformly distributed web longitudinal bars of the same size and spacing are provided throughout the length of the wall (right part of the figure). It has been shown that walls with relatively large amounts of longitudinal reinforcement at their ends can attract relatively large compressive forces; thus, the purpose of these provisions is to help prevent these bars from buckling.

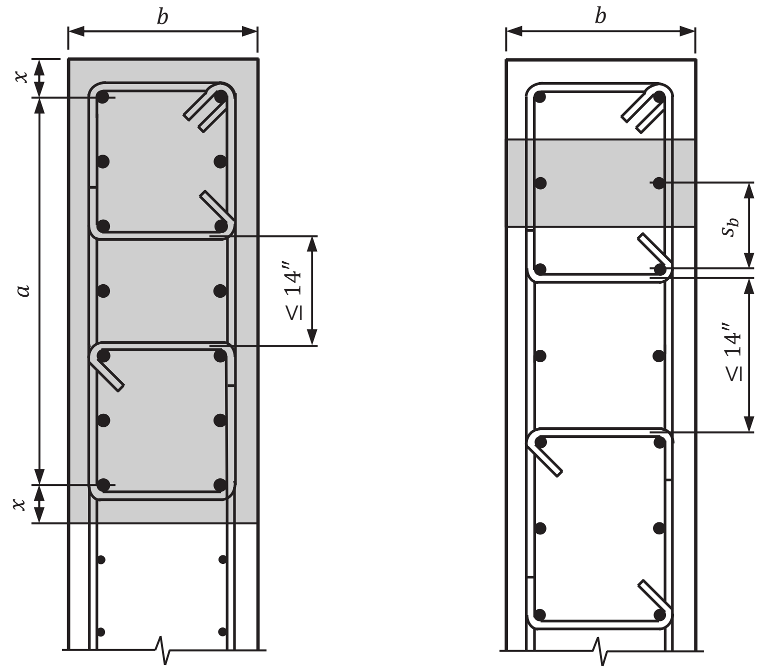


Length of boundary element, ℓ_{be}		$\ell_{be} \geq$ greater of $\begin{cases} c - 0.1\ell_w \\ c/2 \end{cases}$
Width of flexural compression zone, b		$b \geq$ greater of $\begin{cases} h_u/16 \\ \sqrt{0.025c\ell_w} \text{ where ACI 18.10.6.2 is used} \\ 12'' \text{ where } h_w/\ell_w \geq 2.0 \text{ and } c/\ell_w \geq 3/8 \end{cases}$
Horizontal spacing, h_x		$h_x \leq$ lesser of $\begin{cases} 14'' \\ 2b/3 \end{cases}$
Transverse reinforcement	Vertical spacing, s	$s \leq$ lesser of $\begin{cases} \text{lesser of } b/3 \text{ and } \ell_{be}/3 \\ \text{lesser of } 6d_b \text{ of the smallest long. bar and } 6'' \text{ for Grade 60 bars} \\ \text{lesser of } 5d_b \text{ of the smallest long. bar and } 6'' \text{ for Grade 80 bars} \\ \text{lesser of } 4d_b \text{ of the smallest long. bar and } 6'' \text{ for Grade 100 bars} \\ 4'' \leq s_o = 4 + \left(\frac{14 - h_x}{3}\right) \leq 6'' \end{cases}$
	Area, A_{sh}	$A_{sh} =$ greater of $\begin{cases} 0.3sb_{c2}\left(\frac{b\ell_{be}}{b_{c1}b_{c2}} - 1\right)\frac{f'_c}{f_{yt}} \\ 0.09sb_{c2}\frac{f'_c}{f_{yt}} \end{cases}$
Hoop overlap, s_{over}		$s_{over} \geq$ lesser of $\begin{cases} 6'' \\ 2b/3 \end{cases}$

Figure 14.38 Detailing requirements for special boundary elements with rectilinear hoops and crossties.

A_{sb} = area of longitudinal reinforcement in A_{cb}

$$\rho = \frac{A_{sb}}{A_{cb}} > \frac{400}{f_y}$$



$A_{cb} = b(2x + a)$ $\qquad$ $A_{cb} = bs_b$

Notes:

1. Where $V_u \geq \lambda\sqrt{f_c'}A_{cv}$, provide either of the following:
   a. Standard hooks at the ends of the horizontal reinforcement engaging the end edge reinforcement.
   b. U-stirrups spliced to the horizontal reinforcement with the same size and spacing as the horizontal reinforcement.
2. Size of transverse reinforcement must satisfy the requirements of ACI 25.7.2.2: (a) #3 bars for #10 and smaller longitudinal bars and (b) #4 bars for #11 and larger longitudinal bars.
3. Vertical spacing, s, of transverse reinforcement at the wall boundary must be in accordance with ACI Table 18.10.6.5(b):

Location	Maximum Vertical Spacing of Transverse Reinforcement, s		
	Grade of Primary Flexural Reinforcing Bar		
	60	80	100
Within the greater of ℓ_w and $M_u/4V_u$ above and below critical sections	Lesser of $\begin{cases} 6d_b \\ 6'' \end{cases}$	Lesser of $\begin{cases} 5d_b \\ 6'' \end{cases}$	Lesser of $\begin{cases} 4d_b \\ 6'' \end{cases}$
Other locations	Lesser of $\begin{cases} 8d_b \\ 8'' \end{cases}$	Lesser of $\begin{cases} 6d_b \\ 6'' \end{cases}$	Lesser of $\begin{cases} 6d_b \\ 6'' \end{cases}$

d_b = diameter of the smallest primary flexural reinforcing bar

Figure 14.39 Transverse reinforcement requirements where special boundary elements are not required.

Transverse reinforcement in this case must extend over the same distance prescribed in ACI 18.10.6.4(a) for walls where special boundary elements are required. The maximum vertical spacing for the transverse reinforcement at the wall boundary are given in ACI Table 18.10.6.5(b) (see Figure 14.39).

Where in-plane shear forces are relatively large (that is, where $V_u > \lambda\sqrt{f_c'}A_{cv}$), standard hooks must be provided at the ends of the horizontal wall reinforcement so that they can be effective in resisting the required shear force. In lieu of providing hooks on these bars, U-stirrups can be provided, which enclose the edge reinforcement. The U-stirrups must have the same size and spacing as the horizontal reinforcement in the wall, and must be spliced to the horizontal wall bars.

Summary of Boundary Element Requirements for Special Structural Walls

A summary of the requirements for special structural walls with $h_{wcs} / \ell_w \geq 2.0$ that are effectively continuous from the base of the structure to the top of the wall and are designed to have a single critical section for flexure and axial loads is given in Figure 14.40 (ACI 18.10.6.2, 18.10.6.4, and 18.10.6.5). Included are the transverse reinforcement requirements that must satisfied at foundations in accordance with ACI 18.10.6.4(j) and 18.13.2.4.

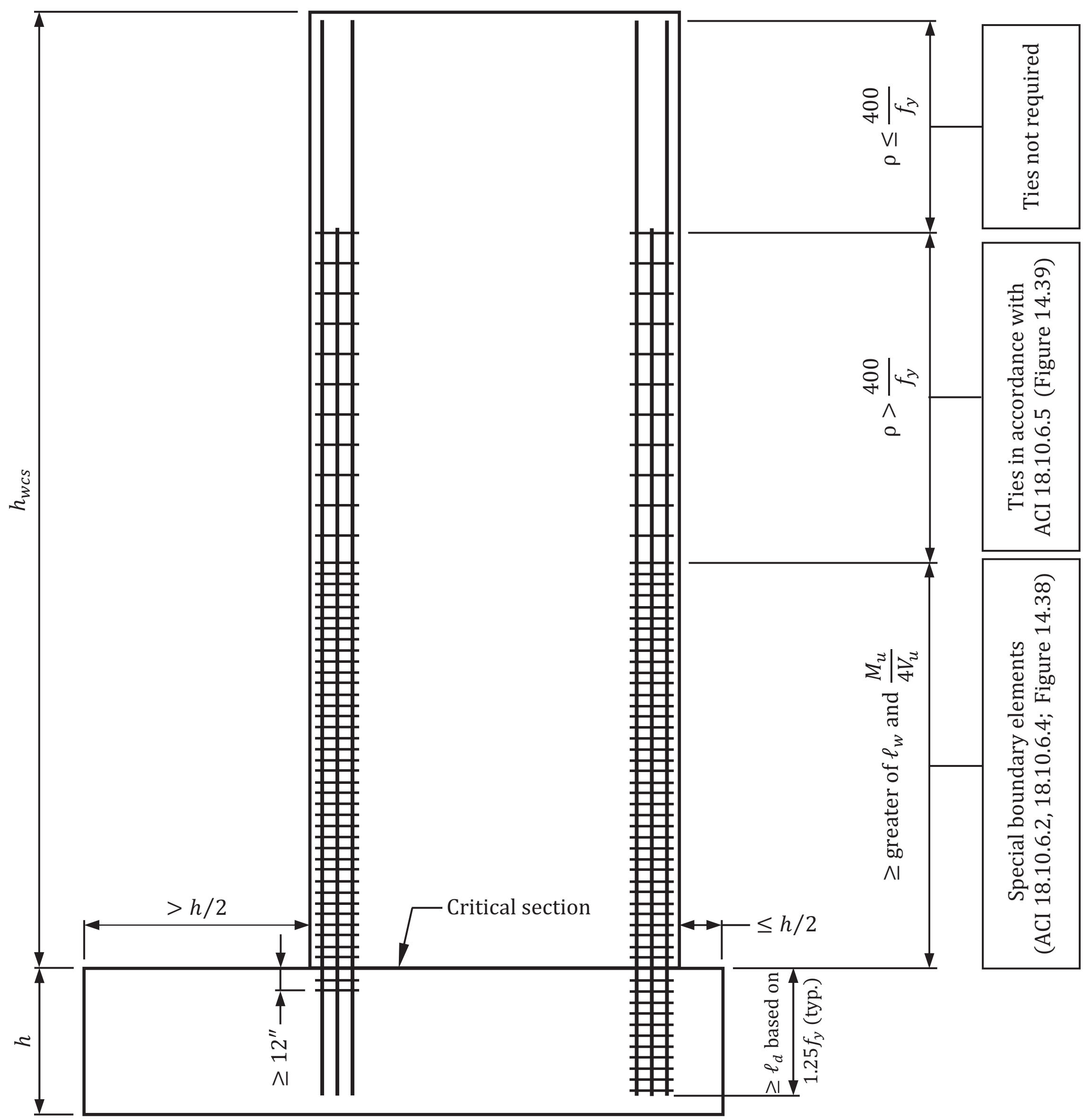
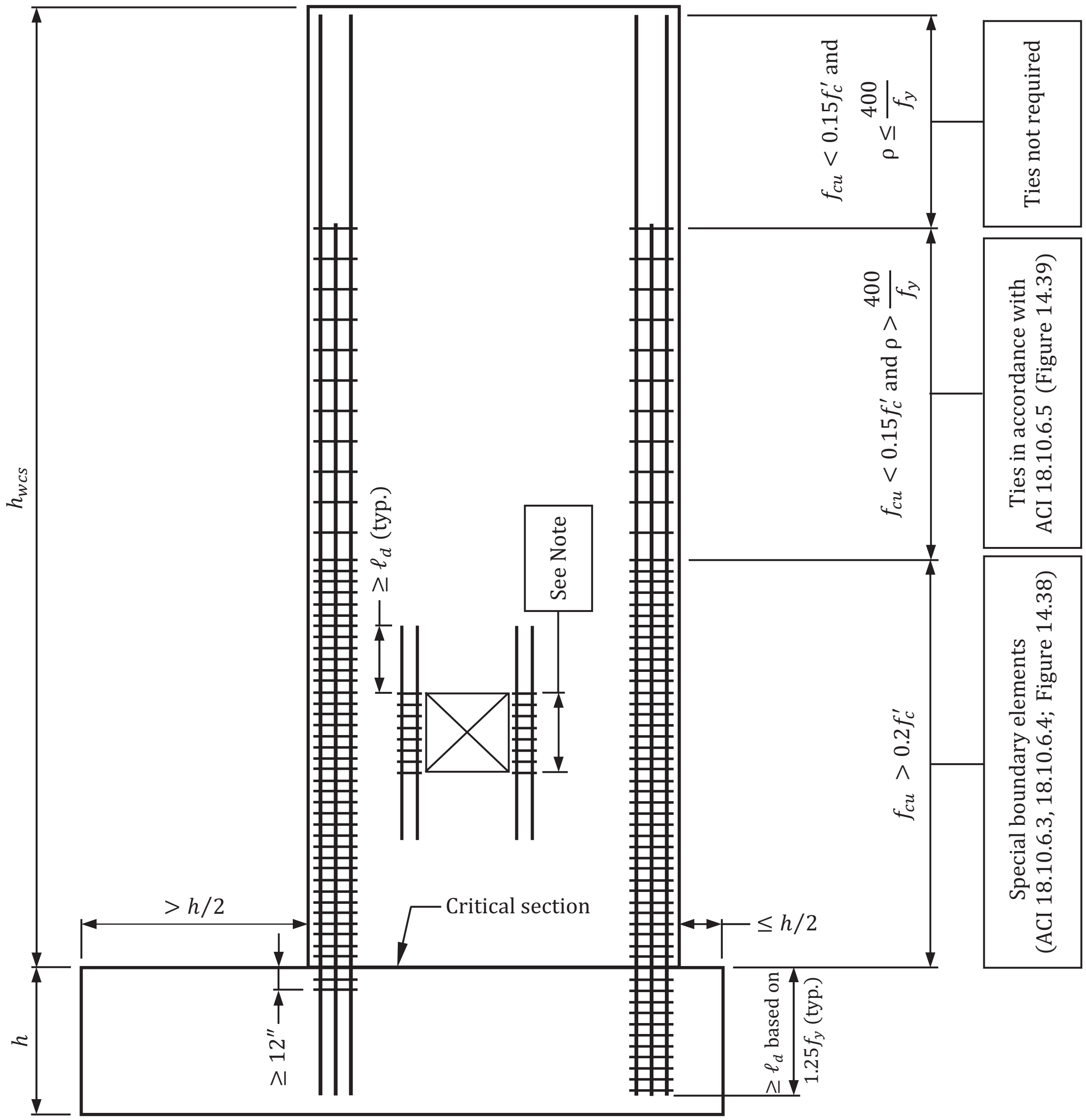


Figure 14.40 Summary of requirements for special structural walls designed in accordance with ACI 18.10.6.2.



Note:

For $f_{cu} > 0.2f'_c$: provide special boundary elements

For $f_{cu} < 0.15f'_c$ and $\rho > 400/f_y$: provide ties in accordance with ACI 18.10.6.5

Figure 14.41 Summary requirements for special structural walls designed in accordance with ACI 18.10.6.3.

Similarly, a summary of the requirements for special structural walls designed in accordance with ACI 18.10.6.3 is given in Figure 14.41.

14.5.7 Coupling Beams

Overview

Coupling beams are beams connecting two structural walls together and are typically aligned vertically above openings in the wall over its height (see Figure 14.42). When properly designed and detailed, such beams can provide an efficient means of energy dissipation when the structure is subjected to earthquake ground motion.

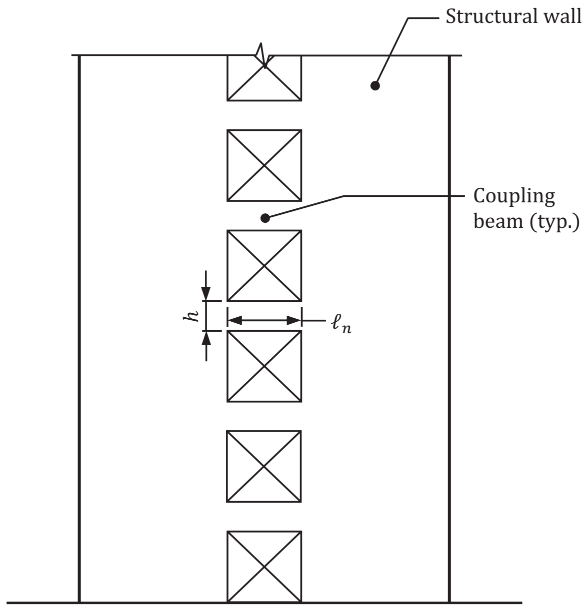


Figure 14.42 Structural wall with coupling beams.

Because of geometric constraints, coupling beams usually have relatively large depth to span ratios. As such, the ends of the beams are typically subjected to large inelastic rotations and large shear forces. Sufficient shear reinforcement and proper detailing and confinement of all the reinforcement in the beam are needed to prevent shear failure and to ensure ductility and energy dissipation. Tests have shown that adequate resistance can be achieved by using confined diagonal reinforcement in deep coupling beams.

Design and Detailing Requirements

The design and detailing requirements in ACI 18.10.7 depend on the clear span length to depth ratio, ℓ_n / h, of the beam and the shear demand. A summary of the requirements is given in Table 14.8.

Table 14.8 Design and Detailing Requirements for Coupling Beams

Case	Requirements
$\ell_n / h \geq 4$ (ACI 18.10.7.1)	• Coupling beams are designed and detailed for flexure and shear in accordance with the requirements in ACI 18.6 for beams in special moment frames. • Wall boundaries take the place of columns. • The dimensional limits in ACI 18.6.2.1(b) and (c) need not be satisfied where analysis shows that beams have adequate lateral stability. • The use of diagonal reinforcement in this case is not required because the beams are considered to be too shallow for the diagonal bars to be efficient. • The longitudinal reinforcement must be embedded into the wall boundaries in accordance with the provisions in ACI 18.8.5 for development of straight bars.
$\ell_n / h < 2$ and $V_u \geq 4\lambda\sqrt{f_c'}A_{cw}$ (ACI 18.10.7.2)	• Reinforcement must consist of two intersecting groups of diagonally placed bars symmetrical about the midspan of the beam unless it can be demonstrated that safety and stability are not compromised. • The diagonally placed bars are intended to provide both the shear and flexural strength of the beam. • Nominal shear strength and reinforcement details are given in ACI 18.10.7.4.

(table continued on next page)

Table 14.8 Design and Detailing Requirements for Coupling Beams (cont.)

Case	Requirements
All other cases (ACI 18.10.7.3)	Coupling beams that are not in the other cases may be reinforced with either two intersecting groups of diagonally placed bars symmetrical about the midspan of the beam or in accordance with the longitudinal and transverse reinforcement requirements of ACI 18.6.3 through 18.6.5 for beams in special moment frames with the wall boundaries acting as the columns. The choice between these reinforcement options leading to the most efficient design typically depends on geometry and the magnitude of the factored shear force.

Coupling beams reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan of the beam must satisfy the requirements in ACI 18.10.7.4(a) and (b) and either ACI 18.10.7.4(c) or (d). The requirements in ACI 9.9 for deep beams need not be satisfied.

The nominal shear strength of a coupling beam reinforced with two intersecting groups of diagonally placed bars is determined by ACI Equation (18.10.7.4):

$$V_n = 2A_{vd}f_y \sin\alpha \leq 10\sqrt{f'_c}A_{cw} \tag{14.34}$$

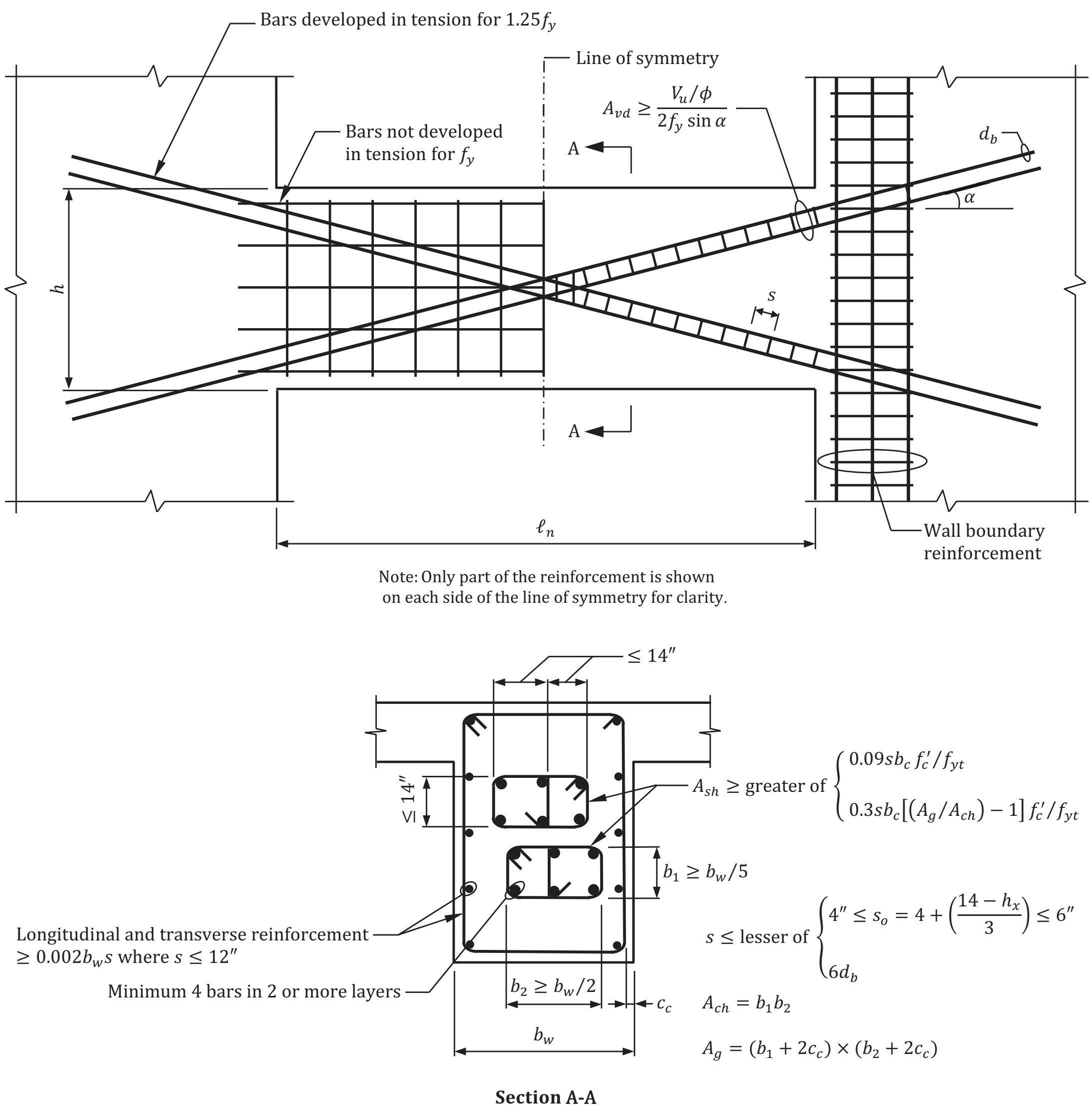


Figure 14.43 Detailing requirements for coupling beams with confinement of individual diagonals.

where A_{vd} is the total area of reinforcement in each group of diagonals bars, α is the angle between the diagonal bars and the longitudinal axis of the coupling beam, and A_{cw} is the gross area of the coupling beam. Each group of diagonal bars must have a minimum of four bars provided in two or more layers and the diagonal bars must be developed in tension in the wall boundary for $1.25f_y$. Headed bars can be used to shorten development lengths and to help alleviate congestion, if required.

The required shear strength, V_u, must be less than or equal to the design shear strength, ϕV_n, where $\phi = 0.85$ for diagonally reinforced coupling beams (ACI 21.2.4.4). Moment resistance is automatically provided by the diagonal bars as well.

Two options are given regarding confinement of the diagonal bars. In the first option, each group of diagonal bars is confined by rectilinear transverse reinforcement satisfying the requirements of ACI 18.10.7.4(c). These requirements are illustrated in Figure 14.43. The transverse reinforcement around the diagonal bars must have out-to-out dimensions of at least $b_w / 2$ in the direction parallel to b_w and $b_w / 5$ along the other sides where b_w is the web width of the coupling beam. The required area of transverse reinforcement, A_{sh}, must be determined in each direction where b_c is the out-to-out cross-sectional dimension of the confined core perpendicular to A_{sh}. Equations to determine A_{ch} and A_g are given in Figure 14.43 based on the requirements in ACI 18.10.7.4(c). A minimum amount of transverse and longitudinal reinforcement must be provided around the beam section. The longitudinal reinforcement should not be fully developed into the wall so that it does not develop significant tensile stresses.

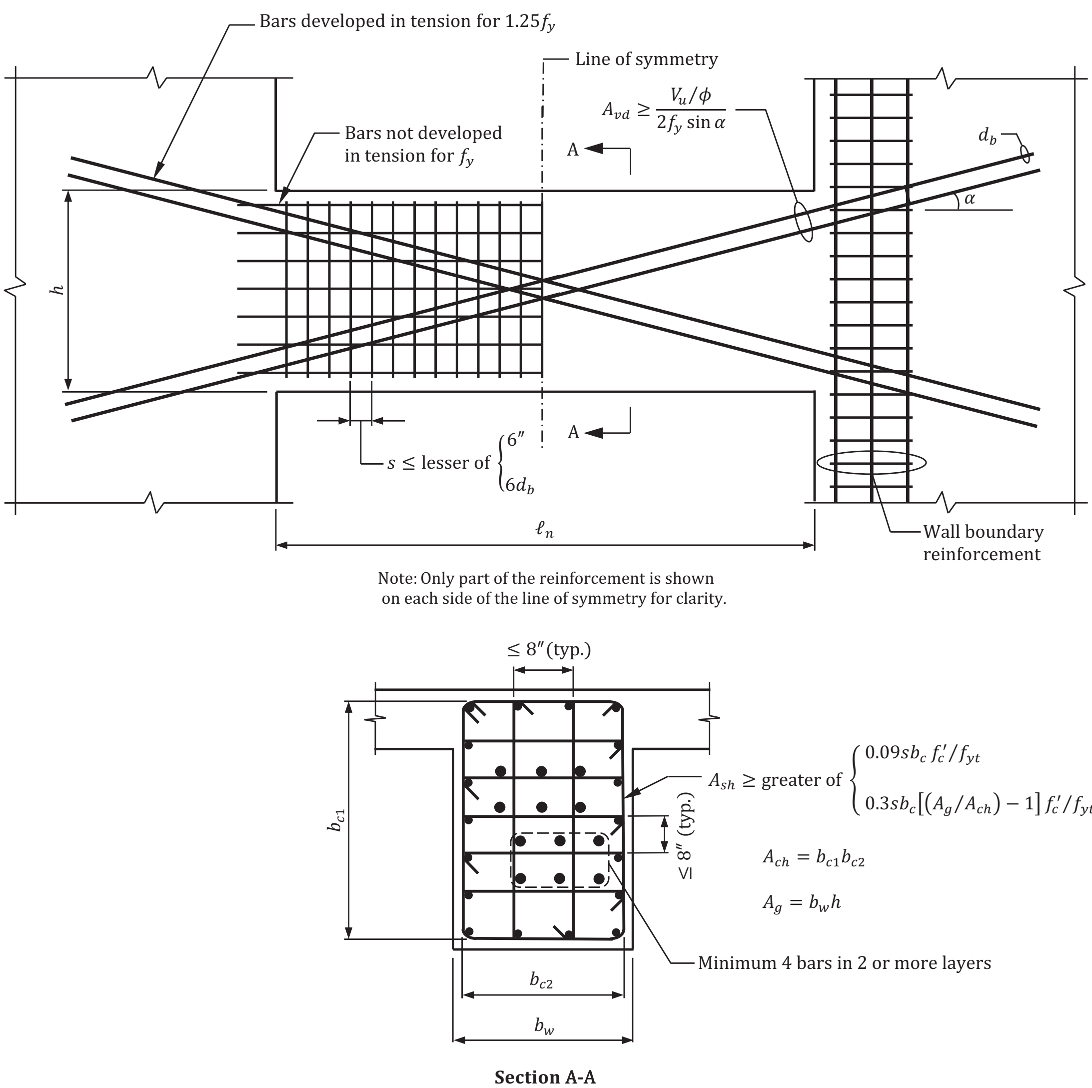


Figure 14.44 Detailing requirements for coupling beams with confinement of the entire cross-section.

In the second option, transverse reinforcement in accordance with ACI 18.10.7.4(d) is provided for the entire cross-section of the coupling beam (see Figure 14.44). This option helps facilitate placement of the reinforcing bars in the field, especially at the location where the diagonal bars intersect each other. Like in the first option, A_{sh} must be determined in each direction using the out-to-out dimensions b_{c1} and b_{c2} of the confined core.

Regardless of the type of coupling beam utilized as part of the SFRS, it is important to ensure that all the required reinforcement can be accommodated within the section and the wall boundary elements. In the case of diagonally reinforced coupling beams, a minimum width of 14 in. is suggested.

14.5.8 Wall Piers

As noted previously, a wall pier is a vertical wall segment within a structural wall bounded horizontally by two openings or by an opening and an edge and satisfying the following: (1) horizontal length to wall thickness $\ell_w / b_w \leq 6.0$ and (2) clear height to horizontal length $h_w / \ell_w \geq 2.0$ (see Figure 14.27). These members are essentially columns; however, the dimensions do not satisfy the requirements of columns in special moment frames.

Wall piers are designed using the requirements for vertical wall segments in Section 14.5.4 of this publication and the following additional design and detailing requirements applicable for columns in special moment frames: (1) ACI 18.7.4 (longitudinal reinforcement), (2) ACI 18.7.5 (transverse reinforcement) and (3) ACI 18.7.6 (shear strength). The joint faces are taken at the top and bottom of the clear height of the wall pier, h_w.

Alternatively, wall piers with $\ell_w / b_w > 2.5$ are permitted to be designed and detailed according to the provisions of ACI 18.10.8.1(a) through (f):

- The factored shear force, V_u, is equal to the lesser of the following: (1) the shear force corresponding to the development of the probable flexural strength, M_{pr}, at both ends of the pier and (2) the shear force determined from analysis using code-prescribed earthquake forces multiplied by the overstrength factor, Ω_o.
- The design shear strength, ϕV_n, is determined in accordance with ACI 18.10.4 for structural walls and must be greater than or equal to V_u.
- Transverse reinforcement is required to be hoops. However, where one curtain of shear reinforcement is provided, which, according to ACI 18.10.2.2, is permitted only if $V_u \leq 2\lambda\sqrt{f_c'}A_{cv}$, single-leg horizontal shear reinforcement parallel to ℓ_w with 180-degree bends at each end engaging the wall pier boundary longitudinal reinforcement may be used.
- The vertical spacing of the transverse reinforcement is limited to 6 in.
- The transverse reinforcement must extend at least 12 in. above and below the clear height of the wall pier.
- Special boundary elements at the ends of the pier must be provided if required by ACI 18.10.6.3.

For wall piers located at the edge of wall, as illustrated in ACI Figure R18.10.8, horizontal reinforcement is required in the adjacent wall segments above and below the wall pier to transfer the design shear force from the wall pier into the adjacent wall segments. The required length of the reinforcement into the adjacent wall is equal to the greater of the development length of the bars in tension and the shear strength of the wall segment. In the latter case, the total embedment length is equal to the factored shear force V_u in the wall pier divided by the design unit shear strength ϕv_n in the adjacent wall.

14.5.9 Ductile Coupled Structural Walls

Ductile coupled structural walls is a type of SFRS where limits are given for the aspect ratios of the structural walls and coupling beams and for the development lengths so as to induce an energy dissipation mechanism associated with inelastic deformation reversal of the coupling beams. To achieve this intended behavior, the following requirements must be satisfied (ACI 18.10.9):

- The height-to-length aspect ratio of an individual wall, h_{wcs} / ℓ_w, must be greater than or equal to 2.
- An individual wall must satisfy the applicable requirements of ACI 18.10 for special structural walls.

- At all levels of the building, the length-to-height aspect ratios of the coupling beams, ℓ_n / h, must be greater than or equal to 2.
- All coupling beams at a floor level must have $\ell_n / h \leq 5$ in at least 90 percent of the levels of the building.
- The development length requirements in ACI 18.10.2.5 must be satisfied at both ends of all coupling beams.

14.5.10 Construction Joints

According to ACI 18.10.10.1, construction joints in special structural walls are to be specified in accordance with ACI 26.5.6. The hardened concrete at the joint must be clean, free of laitance, and intentionally roughened to a full amplitude of ¼ in. [ACI Table 22.9.4.2(b)].

14.5.11 Discontinuous Walls

For special structural walls supported on columns instead of a foundation, the columns must be designed and detailed in accordance with ACI 18.7.5.6 (ACI 18.10.11.1). Requirements for these columns are given in Section 14.3.5 of this publication.

14.6 Diaphragms

14.6.1 Overview

Diaphragms and collectors that are part of the SFRS in buildings assigned to SDC D through F must be designed and detailed in accordance with the provisions in ACI 18.12. Diaphragm design forces are prescribed in ASCE/SEI 12.10.1.1, and collectors must be designed to resist the maximum of the three forces given in ASCE/SEI 12.10.2.1 (see Reference 21). The diaphragm modeling and analysis methods presented in Section 9.3.3 of this publication can be used to determine the in-plane design bending moments, shear forces, and axial forces in a diaphragm.

Information on how to determine the required chord, shear, and shear transfer reinforcement in a diaphragm is given in Section 9.6 of this publication and in Reference 21.

14.6.2 Minimum Thickness

Diaphragms must have sufficient thickness so that all applicable strength and serviceability requirements are satisfied. Methods to determine minimum slab thickness based on serviceability requirements, out-of-plane strength requirements, and in-plane strength requirements are given in Section 9.2 of this publication.

A 2-in. minimum thickness is prescribed in ACI 18.12.6 for concrete slabs serving as diaphragms. It is important to note that fire resistance requirements must also be considered when selecting a minimum slab thickness. Typically, the slab thickness required to satisfy strength and serviceability requirements is adequate for fire resistance.

14.6.3 Reinforcement

Minimum Reinforcement

The minimum reinforcement that must be provided in diaphragms corresponds to the minimum shrinkage and temperature reinforcement in ACI 24.4.3.2, which is equal to 0.0018 times the gross concrete area (ACI 18.12.7.1). The maximum spacing of the reinforcement is 18 in., which is intended to control the width of inclined cracks that may form in the diaphragm.

Where reinforcement is required for shear strength, the reinforcement must be continuous and must be distributed uniformly across the shear plane.

Development and Splices

Tension development lengths and lap splice lengths in diaphragms and collectors are to be determined in accordance with ACI Chapter 25. Reduction in development or lap splice lengths based on excess reinforcement in accordance with ACI 25.4.10.1 is not permitted for members of the SFRS in buildings assigned to SDC C, D, E, or F [ACI 25.4.10.2(e)].

Type 2 splices in accordance with ACI 18.2.7.1 are required where mechanical splices on Grade 60 reinforcement are used to transfer forces between the diaphragm and the vertical elements of the SFRS (ACI 18.12.7.4). It is not permitted to mechanically splice Grade 80 and Grade 100 reinforcement for this application.

Collectors

Collectors must be designed to resist the combined effects of flexure, shear, and axial compression and tension forces caused by the combination of gravity and earthquake load effects. Tension and compression requirements are given in ACI 18.12.7.5 and 18.12.7.6, respectively. To help control cracking over the length of a collector, the longitudinal reinforcement must be proportioned such that the average tensile stress over the following lengths in (a) or (b) does not exceed ϕf_y where f_y is limited to 60,000 psi: (a) the length between the end of a collector and the location where transfer of load to the vertical element of the SFRS begins or (b) the length between two vertical elements of the SFRS (ACI 18.12.7.5). Note that the calculation of the average tensile stress along the length is not necessary if the collector is designed using 60,000 psi for f_y even if Grade 80 reinforcement is specified [which is permitted in accordance with ACI Table 20.2.2.4(a)].

At any section in a collector where the combined compressive stress exceeds $0.2f'_c$, transverse reinforcement conforming to ACI 18.7.5.2(a) through (e) and ACI 18.7.5.3 for columns of special moment frames must be provided, with the exception that the spacing limit of ACI 18.7.5.3(a) must be one-third the least dimension of the collector (ACI 18.12.7.6). The combined compressive stress is calculated using the load combinations in ACI Chapter 5 (see Section 14.1 of this publication) and a linearly elastic model based on gross section properties. The required amount of transverse reinforcement is given in ACI Table 18.12.7.6; it is permitted to discontinue this reinforcement where the combined compressive stress is less than $0.15f'_c$. In cases where the forces have been amplified by the overstrength factor, Ω_o, the limits of $0.2f'_c$ and $0.15f'_c$ are to be increased to $0.5f'_c$ and $0.4f'_c$, respectively. To avoid triggering the requirements for transverse reinforcement, the collector can be sized so that the compressive stress is less than $0.15f'_c$, that is, $A_g \geq P_u / 0.15f'_c$ where A_g is the gross area of the collector and P_u is the maximum factored axial compressive force in the collector. Where Ω_o has been used to determine the forces, $A_g \geq P_u / 0.4f'_c$.

Detailing requirements at splice and anchorage zone locations for the longitudinal reinforcement in collectors are given in ACI 18.12.7.7. The requirements in either ACI 18.12.7.7(a) or (b) need to be satisfied, not both. The purpose of these requirements is to reduce the possibility of longitudinal bar buckling and to provide adequate bar development in these regions. The minimum area of transverse reinforcement in ACI 18.12.7.7(b) corresponds to that for beams in ACI Table 9.6.3.4 at sections where the provisions of ACI 18.12.7.6 do not govern.

Detailing requirements for diaphragms and collectors with rectilinear transverse reinforcement are given in Figure 14.45.

Elements of a structural diaphragm system subjected primarily to axial forces and used to transfer diaphragm shear or flexural forces around openings or other discontinuities must satisfy the requirements in ACI 18.12.7.6 and 18.12.7.7 for collectors (ACI 18.12.3.2). An example of an element adjacent to an opening where these requirements must be satisfied is given in ACI Figure R18.12.3.2.

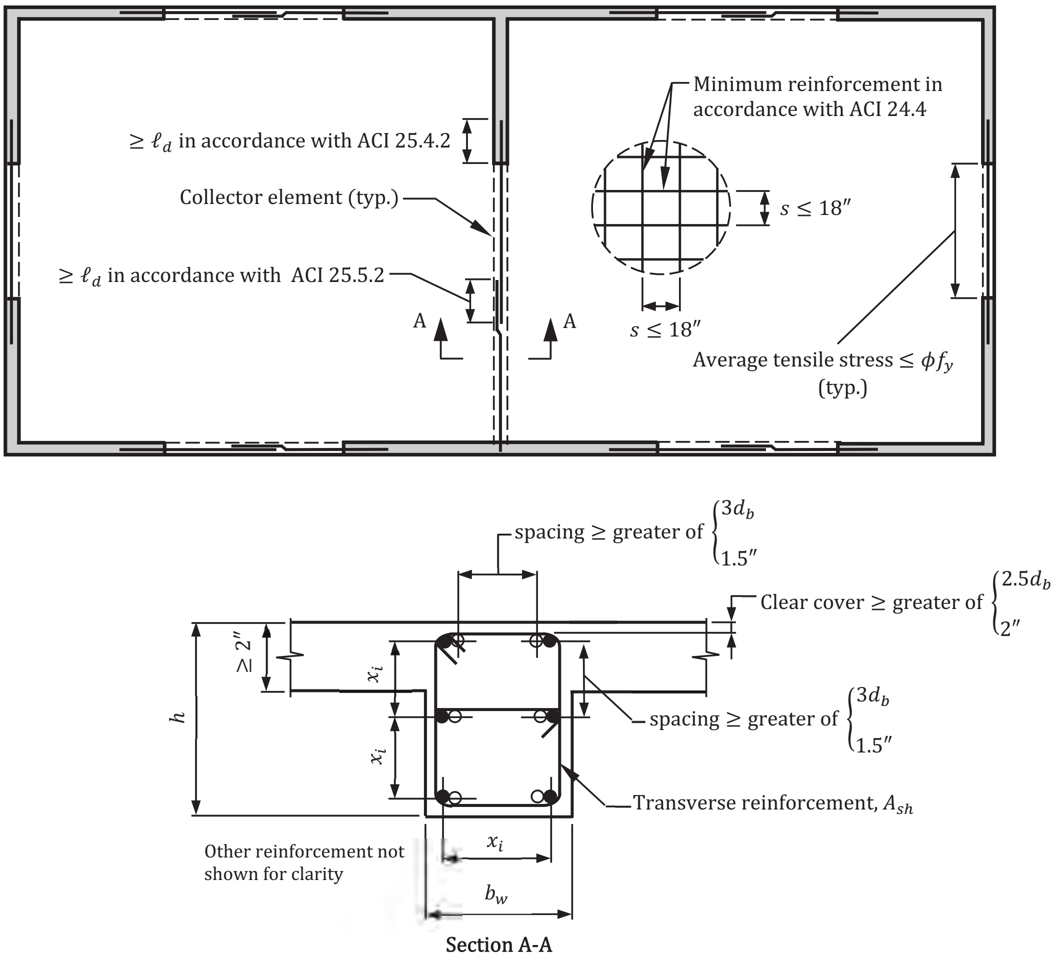
14.6.4 Flexural Strength

Diaphragms are designed for flexure using the same assumptions used in the design of beams, columns, and walls, including the assumption that strains vary linearly along the depth of the diaphragm (ACI 18.12.8.1). In particular, the flexural strength is calculated using the provisions in ACI Chapter 12, and the required strength is determined using the factored load combinations in ACI Chapter 5. Additional information can be found in Chapter 9 of this publication.

14.6.5 Shear Strength

The nominal shear strength, V_n, of a diaphragm is determined by ACI Equation (18.12.9.1), which is the same as ACI Equation (12.5.3.3):

$$V_n = A_{cv}(2\lambda\sqrt{f'_c} + \rho_t f_y) \leq 8\sqrt{f'_c}A_{cv} \tag{14.35}$$



Compressive stress*	Transverse reinforcement, A_{sh}	Spacing, s
$> 0.2f_c'$	$A_{sh} \geq 0.09 s b_c f_c'/f_{yt}$	$s \leq$ lesser of $\begin{cases} (\text{lesser of } h \text{ and } b_w)/3 \\ 6d_b \text{ for Grade 60 reinforcement} \\ 5d_b \text{ for Grade 80 reinforcement} \\ 4'' \leq s_o = 4 + [(14 - h_x)/3] \leq 6'' \end{cases}$ $h_x =$ maximum of $x_i \leq 14''$
$< 0.15f_c'$	$A_{sh} \geq$ greater of $\begin{cases} 0.75\sqrt{f_c'}(b_w s/f_{yt}) \\ 50\, b_w s/f_{yt} \end{cases}$	s determined in accordance with ACI 22.5 $\leq$ maximum spacing in ACI Table 9.7.6.2.2

* Where design forces have been amplified by Ω_o, limits of $0.2f_c'$ and $0.15f_c'$ are increased to $0.5f_c'$ and $0.4f_c'$, respectively.

Figure 14.45 Detailing requirements for diaphragms and collectors.

In this equation, A_{cv} is the gross area of the diaphragm and is limited to the thickness of the diaphragm times the width of the diaphragm in the direction of analysis; this corresponds to the gross area of the deep beam that forms the diaphragm. The reinforcement ratio, ρ_t, that contributes to V_n is the reinforcement parallel to the in-plane shear. If required for shear strength, shear reinforcement for diaphragms must be in addition to reinforcement in the slab required to resist other load effects (ACI 12.6.3). Reinforcement designed to resist shrinkage and temperature load effects is permitted to also resist in-plane shear forces in the diaphragm.

According to ACI 21.2.4.2, the strength reduction factor, ϕ, for shear in diaphragms must not exceed the minimum ϕ used in the shear design of the vertical elements of the SFRS connected to the diaphragm. For example, in a reinforced concrete building with nonslender walls where the shear strength of the walls has been determined using $\phi = 0.60$, the shear strength of the diaphragm must be determined using $\phi = 0.60$.

14.6.6 Construction Joints

According to ACI 18.12.10.1, construction joints in diaphragms are to be specified in accordance with ACI 26.5.6. The hardened concrete at the joint must be clean, free of laitance, and intentionally roughened to a full amplitude of ¼ in. [ACI Table 22.9.4.2(b)].

14.7 Foundations

14.7.1 Overview

Requirements for foundations supporting buildings assigned to SDC D, E, or F are given in ACI 18.13. Provisions are provided for footings, foundation mats, pile caps, grade beams, slabs-on-ground, foundation seismic ties, piles, piers, and caissons. It is desirable for foundations not to undergo any significant inelastic response when subjected to strong ground motion because repair of foundations is either difficult or impossible. ACI R18.13.1 discusses methods that can be implemented to achieve this goal.

According to the second exception in ASCE/SEI 12.4.2.2, the vertical seismic load effect $E_v = 0.2S_{DS}D$ is permitted to be taken as zero in ASCE/SEI Equation (12.4-2) when determining demands on the soil-structure interface of foundations. This equation is applicable when gravity and earthquake effects counteract, and is used when checking overturning resistance of a footing.

ASCE/SEI 12.7.1 permits a structure to be fixed at its base for purposes of determining earthquake forces. This assumption is generally conservative because considering soil-structure interaction may reduce the effects of the ground motion, increase the period of the building, or increase damping, any of which can reduce the earthquake effects on the structure except, possibly, for the lateral displacements.

A geotechnical report provides important information on allowable soil bearing capacities under gravity (static) and gravity plus lateral (static plus transient) loading; coefficient of friction for sliding resistance; passive resistance pressure; vertical and horizontal modulus of subgrade reaction; anticipated differential and total settlements; and any potential geological and seismic hazards (such as soil liquefaction), to name a few. A clear understanding of the dynamic properties of the soil is essential in the design of any foundation system for buildings assigned to SDC D through F.

14.7.2 Footings, Foundation Mats, and Pile Caps

Longitudinal reinforcement of columns and structural walls that are part of the SFRS must be fully developed for tension into the footings, mats, and pile caps supporting them (ACI 18.13.2.2). Where supported members (columns and walls) are assumed to be fixed at the top of the foundation, tests have demonstrated that longitudinal reinforcement developed into the foundation from the supported member must have 90-degree hooks turned inwards towards the axis of the member in order for the joint to be able to resist flexure at this location (ACI 18.13.2.3).

Requirements for columns or boundary elements of special structural walls located near the edge of the foundation are given in ACI 18.13.2.4. These requirements are illustrated in Figures 14.40 and 14.41 for a foundation supporting a structural wall and in Figure 14.46.

Flexural reinforcement must be provided in the top of the foundation where uplift forces are generated in columns and boundary elements of special structural walls (ACI 18.13.2.5). The reinforcement must be designed for the applicable load combinations in ACI Chapter 5 and must be greater than or equal to the applicable minimum reinforcement for one-way slabs in ACI 7.6.1 or for beams in ACI 9.6.1.

A summary of the provisions in this section is illustrated in Figure 14.46.

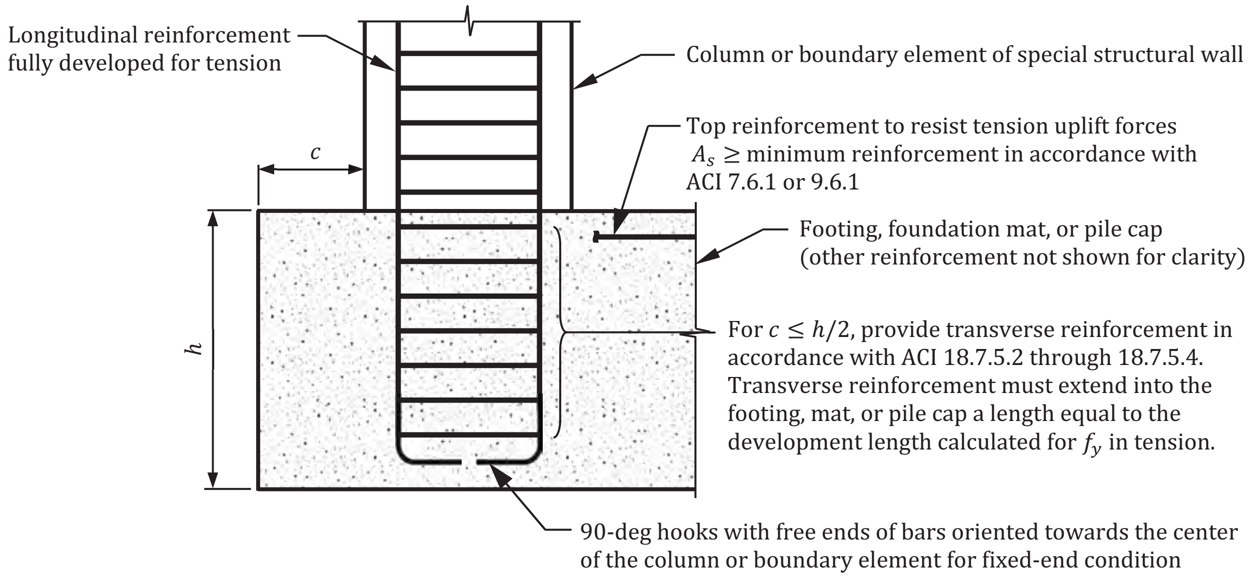


Figure 14.46 Requirements for footings, foundation mats, and pile caps

14.7.3 Grade Beams and Slabs-on-ground

Grade beams and beams that are part of a mat foundation supporting columns that are part of the SFRS must be designed and detailed in accordance with ACI 18.6 for beams in special moment frames (ACI 18.13.3.1; see Section 14.2 of this publication).

A slab-on-ground not subjected to earthquake effects is generally considered nonstructural, and ACI 318 does not govern its design and construction unless the slab transmits vertical load or lateral forces from other portions of the structure to the soil (ACI 1.4.8). However, for structures assigned to SDC D, E, or F, a slab-on-ground is often part of the SFRS, acting as a diaphragm that holds the structure together at the ground level thereby minimizing the effects of out-of-phase ground motion that may occur over the footprint of the structure. In such cases, it must be designed and detailed in accordance with the provisions of ACI 18.12 for diaphragms (ACI 18.13.3.2; see Section 14.6 of this publication). Construction documents must clearly indicate the slabs-on-ground that are part of the SFRS so as to prohibit saw cutting of the slab.

14.7.4 Foundation Seismic Ties

Overview

Individual pile caps, piers, or caissons must be interconnected by foundation seismic ties in orthogonal directions where equivalent restraint is not provided by other means (ACI 18.13.4.1). The purpose of this requirement is to minimize the movement of the supported member relative to the movement of the foundation.

Individual spread footings founded on Site Class E or F soil (see ASCE/SEI Chapter 20) must be interconnected by foundation seismic ties (ACI 18.13.4.2). The ties interconnect the footings so that they can act as a unit, thereby minimizing the movement of the individually supported members relative to the movement of the footings.

Design and Detailing Requirements

Design and detailing requirements for foundation seismic ties are given in ACI 18.13.4.3 and 18.13.4.4. Ties are required to have a design strength in compression or tension greater than or equal to $0.1S_{DS}$ times the greater of the pile cap or column factored dead load plus factored live load occurring on either end of the tie. This requirement need not be satisfied if it can be demonstrated that equivalent restraint is provided by at least one of the ways in ACI 18.13.4.3(a) through (d).

Dimensional and minimum closed tie requirements for grade beams designed as horizontal foundation seismic ties between pile caps or footings are given in ACI 18.13.4.4 (see Figure 14.47 for the case of columns supported on footings with a grade beam at the same top elevation as the footings). Longitudinal reinforcement in the grade beam must be continuous, and at discontinuous locations (for example, at perimeter pile caps or footings), the longitudinal reinforcement must be either developed within or beyond the supported column or boundary element or anchored within the pile cap or footing (straight or hooked bar development).

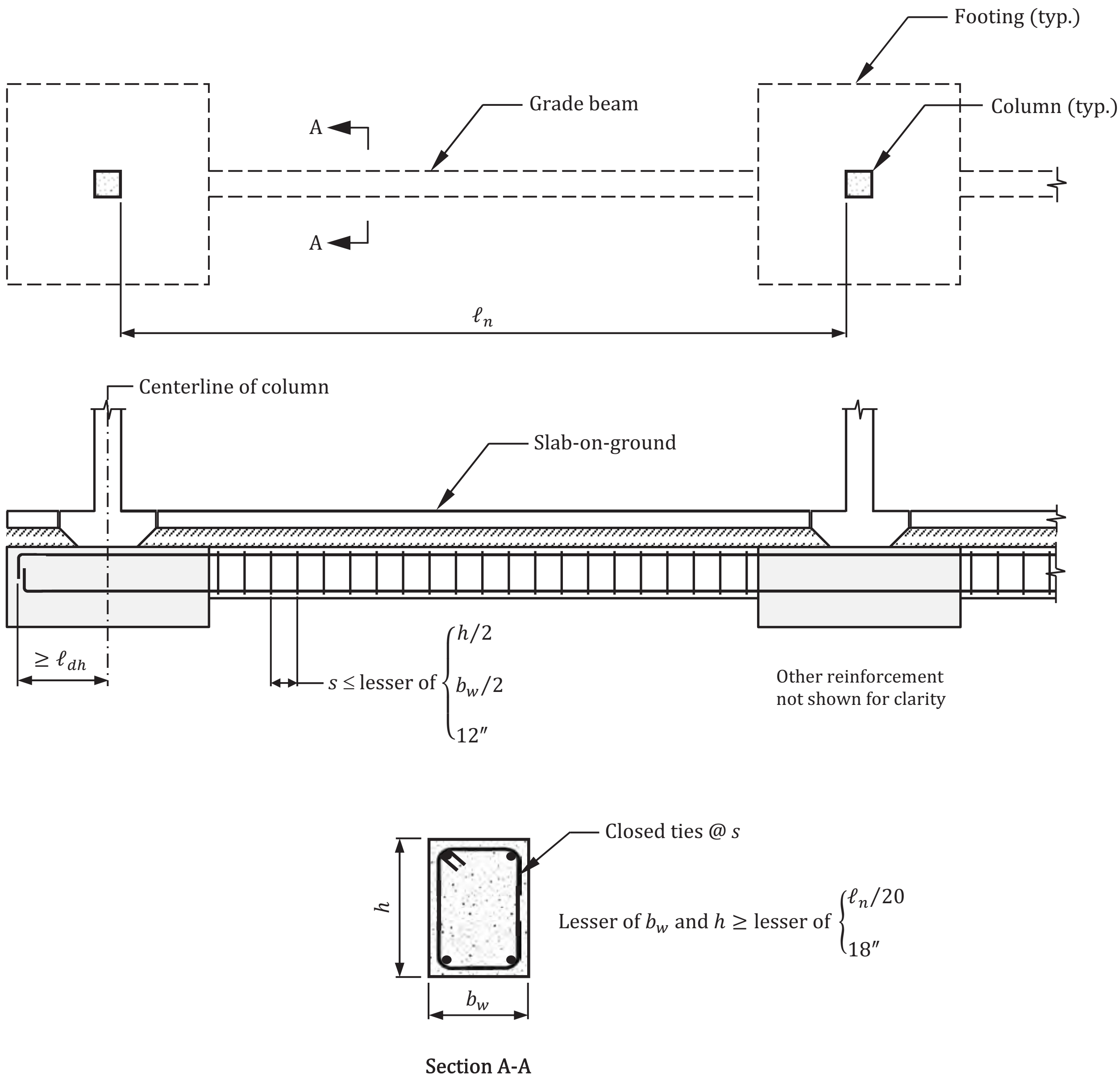


Figure 14.47 Dimensional and detailing requirements for grade beams designed as foundation seismic ties.

14.7.5 Deep Foundations

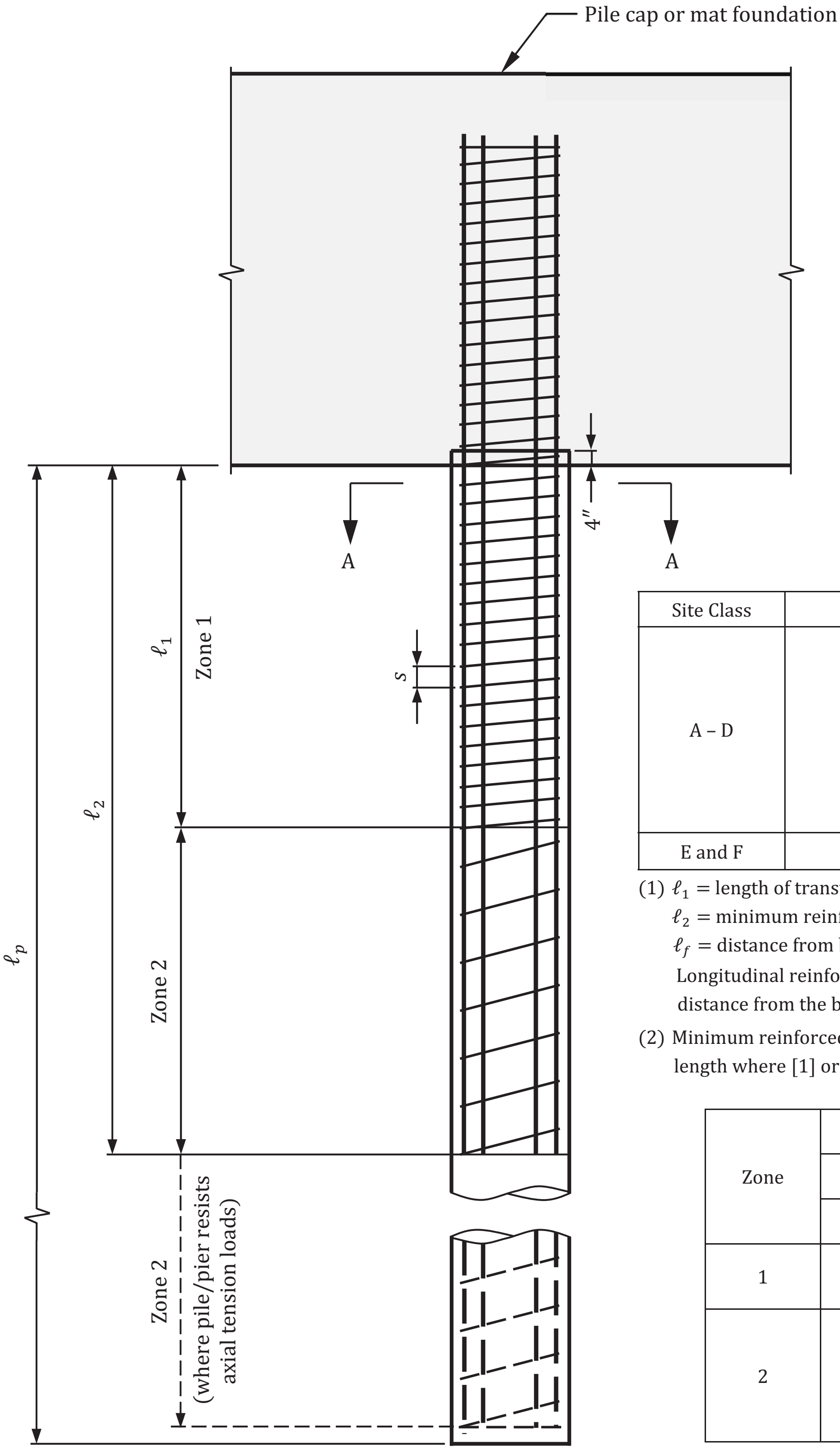
Requirements for deep foundations supporting structures assigned to SDC D, E, or F are given in ACI 18.13.5 and are applicable to uncased cast-in-place concrete drilled piles (caissons), uncased augered piles, metal cased concrete piles, concrete filled pipe piles, and precast concrete piles. A summary of the general requirements is given in Table 14.9, which also includes the specific requirements for uncased drilled piers (caissons), piles, and augered piles in ACI 18.13.5.7.

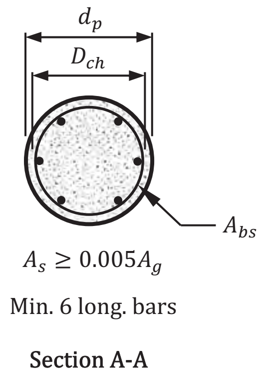
Table 14.9 Requirements for Deep Foundations Supporting Structures Assigned to SDC D, E, or F

Requirement	ACI Section No.
Piles, piers, or caissons resisting design tension forces must have continuous longitudinal reinforcement over their length.	18.13.5.2
Minimum longitudinal and transverse reinforcement required in ACI 18.13.5.7 through 18.13.5.10 must be extended over the entire unsupported length for the portion of the deep foundation member in air or water, or in soil not capable of providing adequate lateral restraint to prevent buckling throughout this length.	18.13.5.3
Hoops, spirals, and ties in deep foundation members must be terminated with seismic hooks.	18.13.5.4
Deep foundation members supporting members structures assigned to SDC D, E, or F or located in Site Class E or F must have transverse reinforcement in accordance with ACI 18.7.5.2, 18.7.5.3, and ACI Table 18.7.5.4 Item (e) within seven member diameters above and below the interfaces between strata that are hard or stiff and strata that are liquefiable or soft.	18.13.5.5
Longitudinal and transverse reinforcement in uncased cast-in-place caissons or augered concrete piles or piers must be provided in accordance with ACI Table 18.13.5.7.1.	18.13.5.7.1
Minimum longitudinal and transverse reinforcement must be provided along minimum reinforced lengths measured from the top of the pile (caisson) in accordance with ACI Table 18.13.5.7.1.	18.13.5.7.2
Longitudinal reinforcement must extend at least the tension development length, ℓ_d, beyond the flexural length of the pile, which is defined in ACI Table 18.13.5.7.1 as the distance from the bottom of the pile cap to where 40 percent of the cracking moment, M_{cr}, is greater than the factored moment, M_u.	18.13.5.7.3
Longitudinal reinforcement in piles, piers, or caissons resisting tension forces must be detailed to transfer the tension forces within the pile cap to the supported structural members.	18.13.6.1
Concrete piles and concrete filled pipe piles must be connected to the pile cap by (1) embedding the pile reinforcement in pile cap a distance equal to the tension or compression development length for piles subjected to uplift and compression, respectively or (2) the use of field-placed dowels anchored in the concrete pile.	18.13.6.2
Where tension forces induced by earthquake effects are transferred between a pile cap or mat foundation and a precast pile by reinforcing bars grouted or post-installed in the top of the pile, the grouting system must develop at least $1.25f_y$ of the bar, which must be demonstrated by testing.	18.13.6.3

Requirements for uncased cast-in-place or augered concrete piles or piers are illustrated in Figure 14.48 for structures located in Site Classes A through D and in Site Class E and F for a circular pile or pier with spiral reinforcement where it is assumed a hard or stiff layer of soil is present over the entire length of the pile or pier. Similar details are given in Figure 14.49 where a liquefiable or soft layer of soil and a hard or stiff layer of soil are present.

In the figures, ℓ_1 is the length of the required transverse confinement reinforcement zone and ℓ_2 is the minimum length where reinforcement is required. These lengths are defined in ACI Table 18.13.5.7.1 based on Site Class. Zone 1 in the figures corresponds to the segment where minimum transverse confinement reinforcement is required over the length ℓ_1. In Figure 14.48, Zone 2 corresponds to the segment where minimum transverse reinforcement is required in the remainder of the required reinforcement length (that is, in the region outside of Zone 1 where the length is equal to $\ell_2 - \ell_1$). The dashed portion of Zone 2 corresponds to the case for Site Classes E and F where reinforcement is required over the full length of the member.





Site Class	ℓ_1	ℓ_2
A – D	$\geq 3d_p$	$\geq$ longest of $\begin{cases} \ell_p/2 \\ 10' \\ 3d_p \\ \ell_f \end{cases}$
E and F	$\geq 7d_p$	Full length of pile(2)

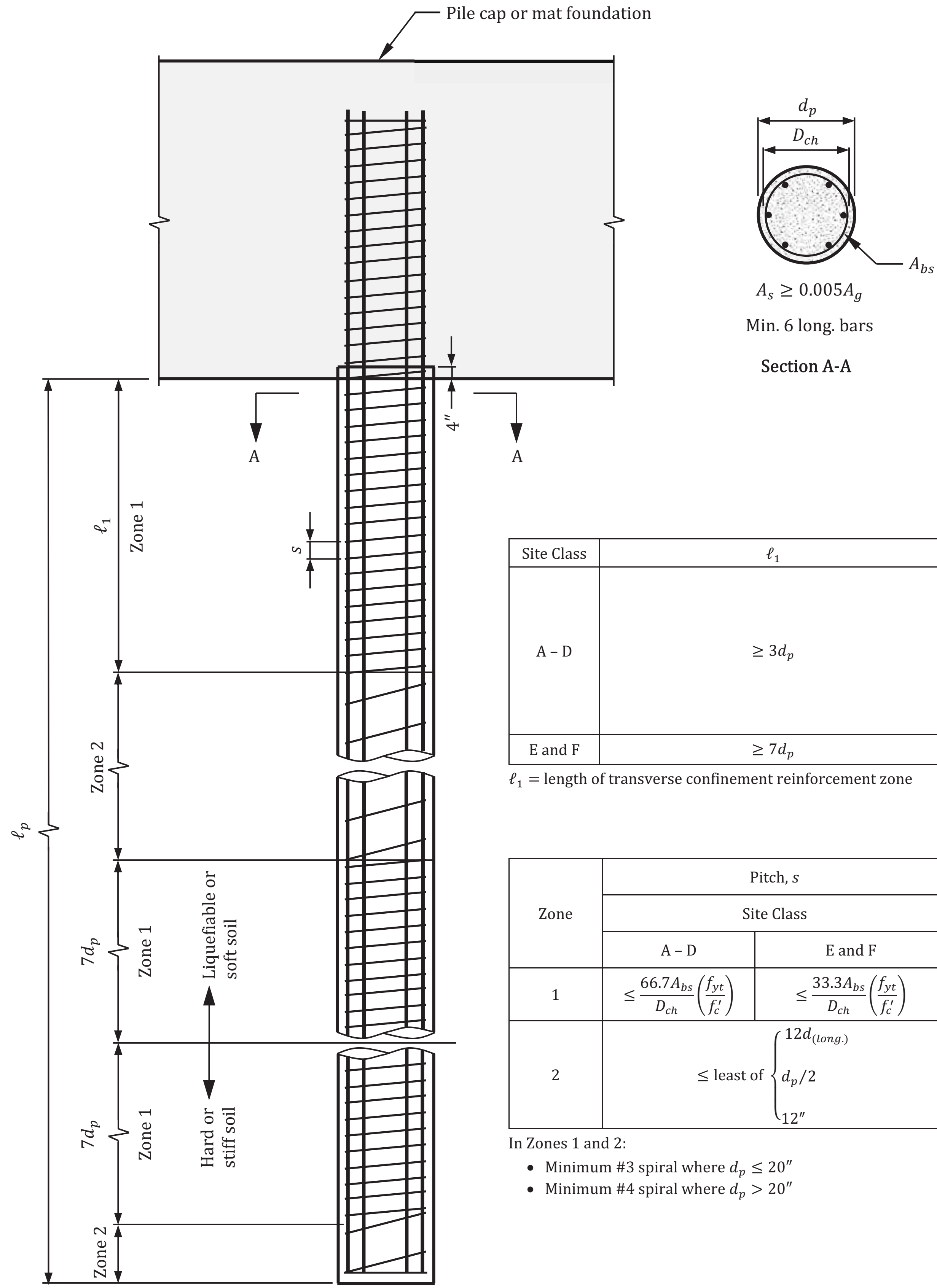
(1) ℓ_1 = length of transverse confinement reinforcement zone
ℓ_2 = minimum reinforced pier/pile length
ℓ_f = distance from bottom of pile cap to where $0.4M_{cr} > M_u$
Longitudinal reinforcement must extend at least ℓ_d beyond the distance from the bottom of the pile cap to where $0.4M_{cr} > M_u$.

(2) Minimum reinforced pier/pile length need not extend the full length where [1] or [2] in ACI Table 18.13.5.7.1 are satisfied.

Zone	Pitch, s	
	Site Class	
	A – D	E and F
1	$\leq \dfrac{66.7A_{bs}}{D_{ch}}\left(\dfrac{f_{yt}}{f'_c}\right)$	$\leq \dfrac{33.3A_{bs}}{D_{ch}}\left(\dfrac{f_{yt}}{f'_c}\right)$
2	$\leq$ least of $\begin{cases} 12d_{b(long.)} \\ d_p/2 \\ 12'' \end{cases}$	

In Zones 1 and 2:
- Minimum #3 spiral where $d_p \leq 20''$
- Minimum #4 spiral where $d_p > 20''$

Figure 14.48 Requirements for uncased cast-in-place or augered concrete piles or piers supporting structures assigned to SDC D, E, or F – Hard or stiff soil over entire length.



Site Class	ℓ_1
A – D	$\geq 3d_p$
E and F	$\geq 7d_p$

ℓ_1 = length of transverse confinement reinforcement zone

Zone	Pitch, s	
	Site Class	
	A – D	E and F
1	$\leq \dfrac{66.7A_{bs}}{D_{ch}}\left(\dfrac{f_{yt}}{f_c'}\right)$	$\leq \dfrac{33.3A_{bs}}{D_{ch}}\left(\dfrac{f_{yt}}{f_c'}\right)$
2	$\leq$ least of $\begin{cases} 12d_{(long.)} \\ d_p/2 \\ 12'' \end{cases}$	

In Zones 1 and 2:

- Minimum #3 spiral where $d_p \leq 20''$
- Minimum #4 spiral where $d_p > 20''$

Figure 14.49 Requirements for uncased cast-in-place or augered concrete piles or piers supporting structures assigned to SDC D, E, or F – Liquefiable or soft soil layer adjacent to hard or stiff soil layer.

The maximum spiral pitch, s, within the transverse confinement reinforcement zone (Zone 1) is based on one-half the requirement of ACI Table 18.7.5.4(e) for Site Class A through D and is determined by equating one-half of ρ_s from the equation in that table with the general expression for the volume of spiral reinforcement within s :

$$\rho_s = 0.5 \times 0.12 \frac{f_c'}{f_{yt}} = \frac{4A_{bs}}{D_{ch}s} \tag{14.36}$$

In this equation, A_{bs} is the area of the spiral bar and D_{ch} is the diameter of the pile/pier core measured to the outside of the spiral reinforcement.

Thus, the maximum pitch must be less than or equal to the following for Site Classes A through D in Zone 1:

$$s \leq \frac{66.7A_{bs}}{D_{ch}}\left(\frac{f_{yt}}{f_c'}\right) \tag{14.37}$$

Similarly, for Site Classes E and F where s is based on the requirement of ACI Table 18.7.5.4(e), the maximum pitch must be less than or equal to the following in Zone 1:

$$s \leq \frac{33.3A_{bs}}{D_{ch}}\left(\frac{f_{yt}}{f_c'}\right) \tag{14.38}$$

For piers or piles with rectilinear transverse reinforcement, the transverse reinforcement requirements of ACI 18.7.5.3 must also be satisfied in Zone 1.

14.8 Members Not Designated as Part of the SFRS

14.8.1 Overview

Not all the structural members in a building are usually assigned to the SFRS. In a building frame system, it is assumed the effects due to the applied earthquake forces are resisted solely by special structural walls. In structures utilizing special moment frames, it is typical that only some of the frames in the building are designated to be part of the SFRS in both orthogonal directions.

Regardless of the type of the SFRS utilized and the members selected to be part of it, all members in a structure are subjected to the effects of an earthquake because all are tied together by the diaphragm at each level. During an earthquake event, the entire structure undergoes displacements and all members are subjected to the effects of these displacements, including those that have not been designated to be part of the SFRS.

Members in a structure that are not part of SFRS must satisfy the deformation compatibility requirements in ASCE/SEI 12.12.5. The purpose of these requirements is to ensure that these members can support their respective gravity loads when subjected to the design displacements, δ_u, caused by the design-level earthquake, which are calculated in accordance with ASCE/SEI 12.8.6:

$$\delta_u = \frac{C_d \delta_{xe}}{I_e} \tag{14.39}$$

In this equation, C_d is the deflection amplification factor given in ASCE/SEI Table 12.2-1, δ_{xe} are the deflections at the floor levels where the code-prescribed lateral earthquake forces are applied over the height of the structure, and I_e is the earthquake importance factor determined in accordance with ASCE/SEI 11.5.1. In ASCE/SEI 12.8.6, $\delta_x = \delta_u$.

The provisions of ACI 18.14 must be satisfied for reinforced concrete members that have not been designated as part of the SFRS. Requirements are provided for beams, columns, joints, slab-column connections, and wall piers. The purpose of these of these requirements is to enable ductile flexural yielding of these members (by providing sufficient confinement and shear strength) when subjected to the effects caused by the design displacements.

A structural analysis is performed to determine axial forces, bending moments, and shear forces in the members that are not part of the SFRS due to the design displacements δ_u, which are applied at each floor level. Only the members that are not part of the SFRS are included in the structural model in this analysis. According to ACI 18.14.2.1, the effects caused by δ_u and the vertical ground motion are to be combined with factored gravity load effects determined by the load combinations in ACI 5.3. Therefore, each member that is not part of the SFRS must be designed for the following load combinations.

$$(1.2 + 0.2S_{DS})D + Q_E + 1.0L + 0.2S \tag{14.40}$$

$$(0.9 - 0.2S_{DS})D + Q_E \tag{14.41}$$

In these equations, Q_E are the effects in the members due to δ_u.

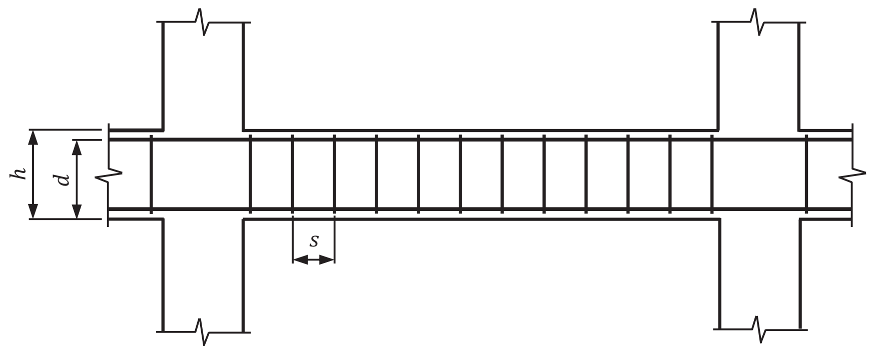
The design and detailing requirements for beams and columns depend on the magnitude of the bending moments and shear forces induced in those members when they are subjected to δ_u. One set of requirements is applicable where the induced bending moments and shear forces do not exceed the corresponding design moment and shear strengths, and another set must be satisfied where the design strengths are exceeded. In cases where it has been decided that the induced moments and shears caused by δ_u are not going to be calculated, the set of requirements where design strengths are exceeded must be satisfied. It is assumed that beams and columns yield in cases where the combined effects of factored gravity loads and design displacements exceed the design strengths, or where the effects of design displacements are not calculated. As expected, the requirements where the members yield are more stringent than those where yielding is assumed not to occur (that is, where the design strengths are satisfied).

14.8.2 Beams

A summary of the requirements for beams that are not part of the SFRS is given in Table 14.10. Where the induced moments and shears do not exceed the design moment and shear strengths, the requirements of ACI 18.14.3.2(a) must be satisfied, which are illustrated in Figure 14.50.

Table 14.10 Detailing Requirements for Beams That are Not Part of the SFRS

	$M_u \le \phi M_n$ and $V_u \le \phi V_n$		$M_u > \phi M_n$ or $V_u > \phi V_n$ or M_u and V_u not calculated	
Longitudinal Reinforcement	Requirements of ACI 18.6.3.1 must be satisfied		Requirements of ACI 18.6.3.1 must be satisfied	
	No restrictions on mechanical and welded splices		Mechanical and welded splices must satisfy the requirements of ACI 18.2.5 through 18.2.8	
	No restrictions on location of lap splices		No restrictions on location of lap splices	
Transverse Reinforcement	$P_u \le A_g f'_c / 10$	$P_u > A_g f'_c / 10$	$P_u \le A_g f'_c / 10$	$P_u > A_g f'_c / 10$
	Size and spacing of stirrups determined in accordance with ACI 22.5.8	Size and spacing of hoops and cross-ties satisfying ACI 18.7.5.2 determined in accordance with ACI 22.5.8	Size and spacing of stirrups with seismic hooks determined in accordance with ACI 18.6.5	Size and spacing of hoops and cross-ties satisfying ACI 18.7.5.2 determined in accordance with ACI 18.6.5
	Spacing $\le d / 2$ throughout the length of the beam	Spacing $\le$ lesser of $6d_{b(long.)}$ and 6 in. throughout the length of the beam	Spacing $\le d / 2$ throughout the length of the beam	Spacing $\le$ lesser of $6d_{b(long.)}$ and 6 in. throughout the length of the beam



Longitudinal reinforcement

- Greater of $\begin{cases} 3\sqrt{f_c'}\,b_w d/f_y \\ 200 b_w d/f_y \end{cases} \le A_s^- \text{ or } A_s^+ \le \begin{cases} 0.025 b_w d \text{ for Grade 60 bars} \\ 0.020 b_w d \text{ for Grade 80 bars} \end{cases}$
- Minimum 2 bars continuous top and bottom
- No restrictions on lap splice locations
- No restrictions on mechanical and welded splices

Transverse reinforcement

- For $P_u \le A_g f_c'/10$: Stirrups determined by ACI 22.5.8 with $s \le d/2$
- For $P_u > A_g f_c'/10$: Hoops satisfying ACI 18.7.5.2 determined by ACI 22.5.8 with $s \le$ lesser of $\begin{cases} 6d_{b(long.)} \\ 6'' \end{cases}$

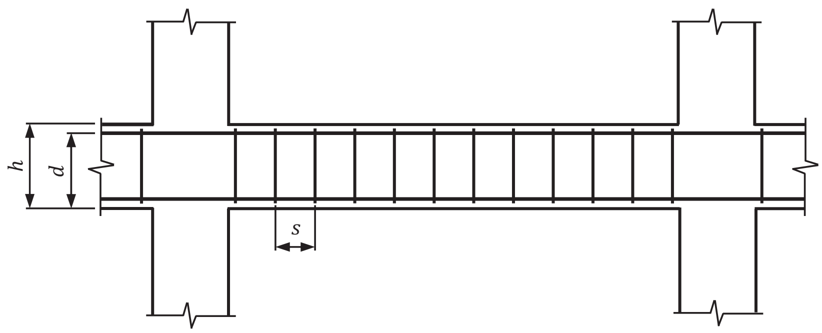
$$\boxed{\boldsymbol{M_u \le \phi M_n} \textbf{ and } \boldsymbol{V_u \le \phi V_n}}$$

Figure 14.50 Requirements for beams that are not part of the SFRS in accordance with ACI 18.14.3.2(a).

Where the induced moments or shears exceed the respective design strengths, or where the induced moments and shear due to δ_u are not calculated, the requirements of ACI 18.14.3.3(a) and (b) must be satisfied (see Figure 14.51).

14.8.3 Columns

A summary of the requirements for columns that are not part of the SFRS is given in Table 14.11. Where the induced moments and shears do not exceed the design moment and shear strengths, the requirements of ACI 18.14.3.2(b) and (c) must be satisfied. The requirements for transverse reinforcement consisting of rectilinear hoops are illustrated in Figure 14.52.



Longitudinal reinforcement

- Greater of $\begin{cases} 3\sqrt{f_c'}b_w d/f_y \\ 200 b_w d/f_y \end{cases} \leq A_s^- \text{ or } A_s^+ \leq \begin{cases} 0.025 b_w d \text{ for Grade 60 bars} \\ 0.020 b_w d \text{ for Grade 80 bars} \end{cases}$
- Minimum 2 bars continuous top and bottom
- No restrictions on lap splice locations
- Type 1 mechanical splices must be located outside of $2h$ from column face
- Type 2 mechanical splices on Grade 60 bars are permitted at any location
- Welded splices must be located outside of $2h$ from column face

Transverse reinforcement

- For $P_u \leq A_g f_c'/10$: Stirrups with seismic hooks determined by ACI 18.6.5 with $s \leq d/2$
- For $P_u > A_g f_c'/10$: Hoops satisfying ACI 18.7.5.2 determined by ACI 18.6.5 with $s \leq$ lesser of $\begin{cases} 6d_{b(long.)} \\ 6'' \end{cases}$

$M_u > \phi M_n$ or $V_u > \phi V_n$ or

Induced moments and shears not calculated

Figure 14.51 Requirements for beams that are not part of the SFRS in accordance with ACI 18.14.3.3(a) and (b).

Table 14.11 Detailing Requirements for Columns That are Not Part of the SFRS

	$M_u \leq \phi M_n$ and $V_u \leq \phi V_n$	$M_u > \phi M_n$ or $V_u > \phi V_n$ or M_u and V_u not calculated
Longitudinal Reinforcement	Requirements of ACI 18.7.4.1 must be satisfied	Requirements of ACI 18.7.4 for columns in special moment frames must be satisfied
	No restrictions on mechanical and welded splices	Mechanical and welded splices must satisfy the requirements of ACI 18.7.4.4
	No restrictions on location of tension lap splices	Tension lap splices are permitted only within the center half of the member length

(table continued on next page)

Table 14.11 Detailing Requirements for Columns That are Not Part of the SFRS (cont.)

<table>
<tr><th></th><th>$M_u \leq \phi M_n$ and $V_u \leq \phi V_n$</th><th>$M_u > \phi M_n$ or $V_u > \phi V_n$ or
M_u and V_u not calculated</th></tr>
<tr><td rowspan="5">Transverse Reinforcement</td><td>Size and spacing of transverse reinforcement determined in accordance with ACI 18.7.6</td><td rowspan="5">Size and spacing of transverse reinforcement must be determined in accordance with the requirements of ACI 18.7.5 and 18.7.6 for columns in special moment frames</td></tr>
<tr><td>Spiral reinforcement satisfying ACI 25.7.3 or hoop reinforcement satisfying ACI 25.7.4 must be provided over the full length of the column</td></tr>
<tr><td>Spacing $\leq$ lesser of $6d_{b(long.)}$ and 6 in.</td></tr>
<tr><td>Transverse reinforcement satisfying ACI 18.7.5.2(a) through (e) must be provided over the length ℓ_o defined in ACI 18.7.5.1 from each joint face</td></tr>
<tr><td>In addition, where $P_u > 0.35P_o$:
• Transverse reinforcement requirements of ACI 18.7.5.7 must be satisfied
• For columns with rectilinear hoops, minimum transverse reinforcement must be at least one-half the greater of (a) and (b) in ACI Table 18.7.5.4 over the length ℓ_o defined in ACI 18.7.5.1 from each joint face
• For columns with spirals or circular hoops, minimum transverse reinforcement must be at least one-half the greater of (d) and (e) in ACI Table 18.7.5.4 over the length ℓ_o defined in ACI 18.7.5.1 from each joint face</td></tr>
</table>

Where the induced moments or shears exceed the respective design strengths, or where the induced moments and shear due to δ_u are not calculated, the requirements of ACI 18.14.3.3(c) must be satisfied, which are the same as those for columns in special moment frames. These requirements are illustrated in Figures 14.14, 14.17, and 14.18 for the case of rectilinear hoops and crossties and in Figure 14.20 for spirals.

14.8.4 Joints

For joints that are not part of the SFRS where the induced moments and shears do not exceed the design moment and shear strengths, the requirements in ACI Chapter 15 must be satisfied [ACI 18.14.3.2(d)]. These requirements are covered in Chapter 11 of this publication.

Where the induced moments or shears exceed the respective design strengths, or where the induced moments and shear due to δ_u are not calculated, the requirements of ACI 18.4.4.1 must be satisfied [ACI 18.14.3.3(d)]. These requirements are covered in Section 13.3.4 of this publication.

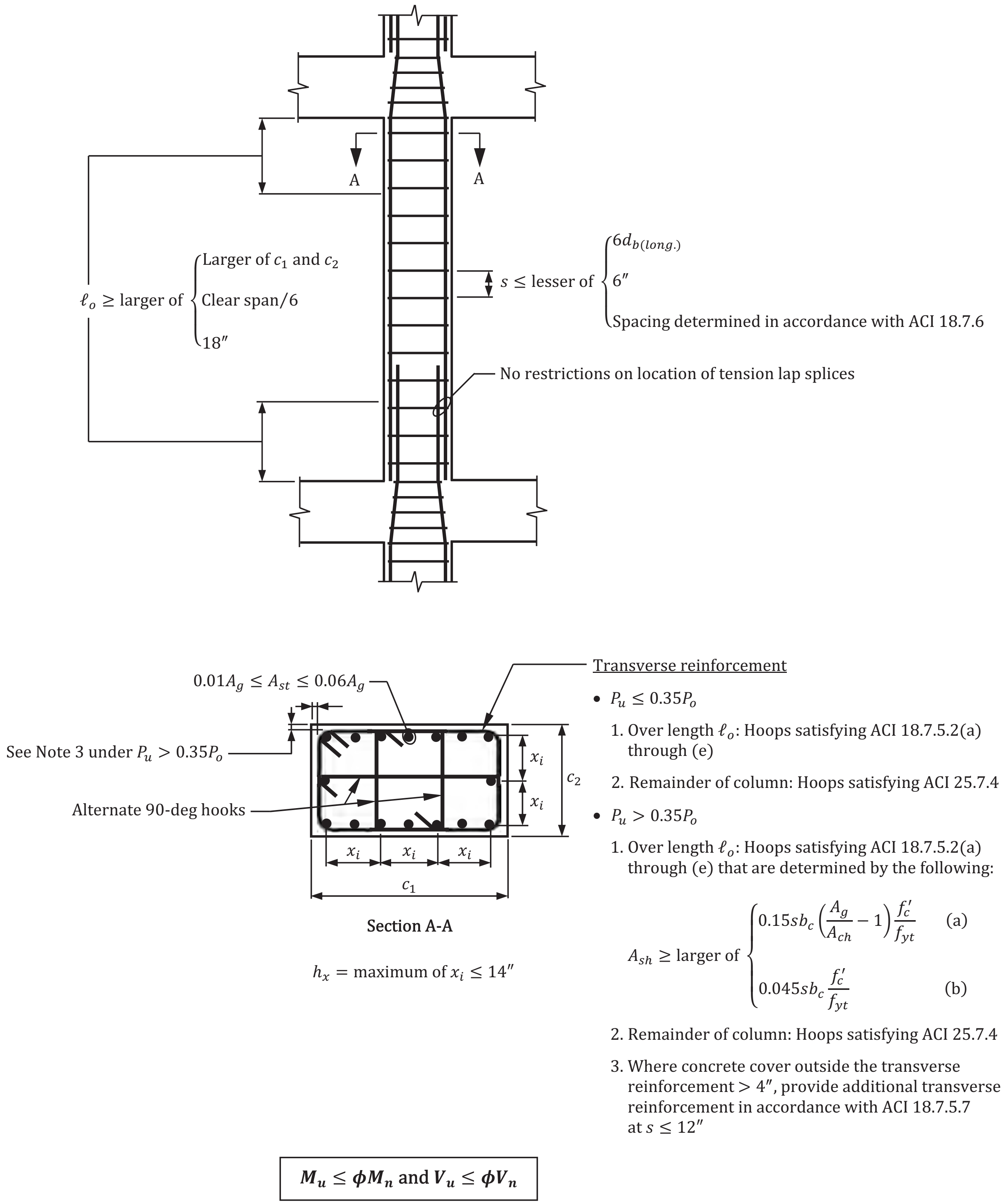


Figure 14.52 Requirements for columns that are not part of the SFRS in accordance with ACI 18.14.3.2(b) and (c).

14.8.5 Slab-Column Connections

Provisions for slab-column connections in two-way slab systems without beams are given in ACI 18.14.5. The purpose of these requirements is to reduce the likelihood of two-way shear failure when the structure is subjected to the design displacements δ_u caused by the design-level earthquake.

Slab shear reinforcement must be provided at all slab-column connections of nonprestressed two-way slabs without beams where the following equation is satisfied:

$$\frac{\Delta_x}{h_{sx}} \geq 0.035 - \frac{1}{20}\left(\frac{v_{uv}}{\phi v_c}\right) \tag{14.42}$$

where $\Delta_x =$ relative lateral deflection between the top and bottom of a story (story drift)

$h_{sx} =$ story height under consideration

$v_{uv} =$ factored shear stress at the slab critical section due to gravity loads without moment transfer where the factored gravity shear force is determined using the load combinations that include E

$v_c =$ nominal two-way shear strength provided by the concrete calculated in accordance with ACI 22.6.5

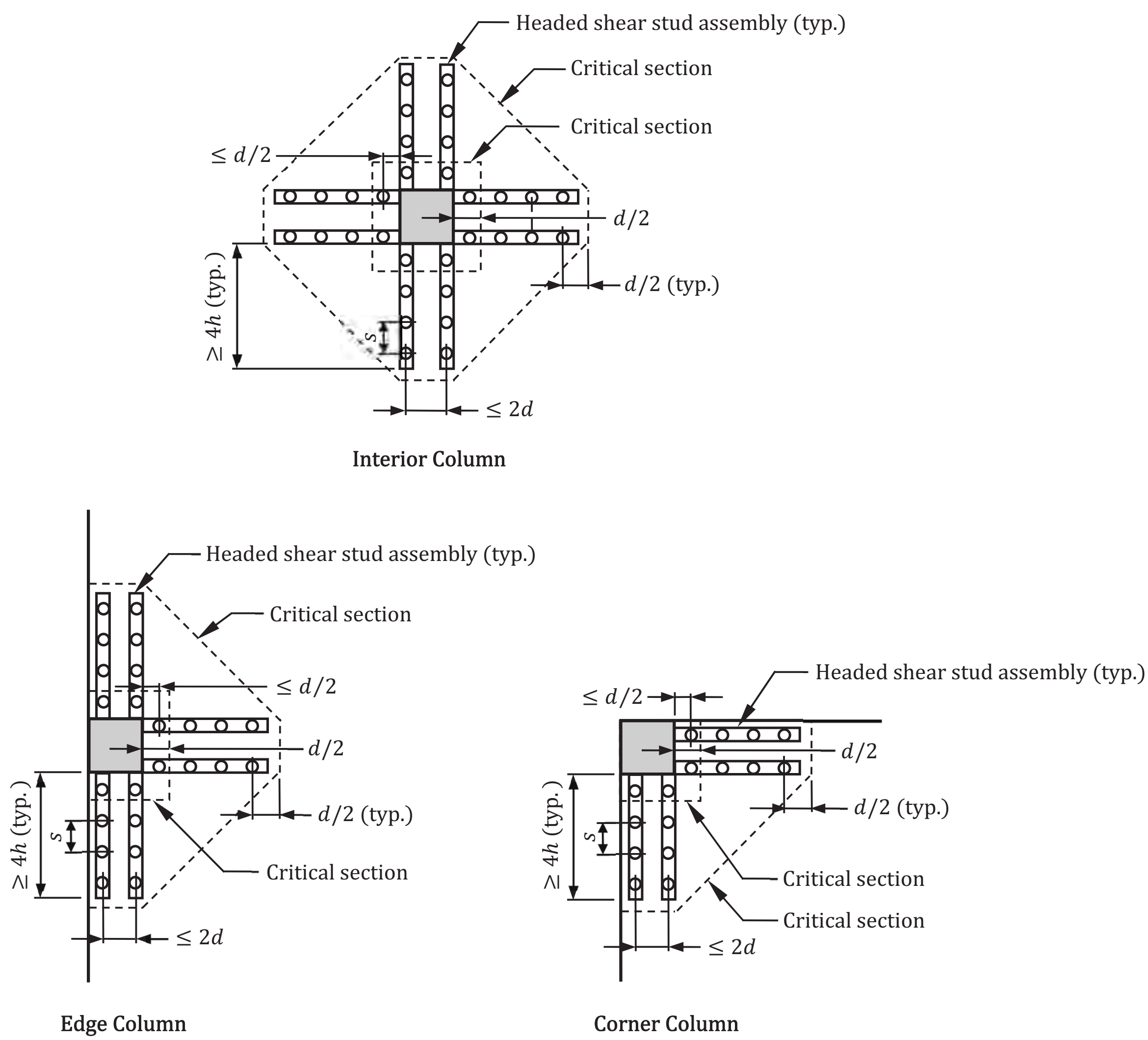


Figure 14.53 Headed shear stud reinforcement at slab-column connections that are not part of the SFRS.

The value of Δ_x / h_{sx} to use in Equation (14.42) is the greater of the values of the adjacent stories above and below the slab-column connection under consideration. Note that the shear reinforcement requirements of this section are not applicable where $\Delta_x / h_{sx} \le 0.005$ [ACI 18.14.5.2(a)]. The conditions where shear reinforcement is required and not required are given in ACI Figure R18.14.5.1.

Where Equation (14.42) is satisfied, stirrups conforming to ACI 8.7.6 or headed shear studs conforming to ACI 8.7.7 providing a nominal shear strength, v_s, greater than or equal to $3.5\sqrt{f_c'}$ must be utilized at the slab critical section (ACI 18.14.5.1 and 18.14.5.3). This reinforcement must extend a minimum of four times the slab thickness from the face of the column in both principal directions. Typical details for headed shear stud reinforcement at interior, edge, and corner columns; the details for stirrups are similar are given in Figure 14.53.

14.8.6 Wall Piers

Wall piers not designated part of the SFRS must satisfy the requirements in ACI 18.10.8 (ACI 18.14.6). The design shear force must be determined using the probable flexural strength, M_{pr}, with the joint faces taken at the top and bottom of the clear height of the wall pier (ACI 18.7.6.1).

In lieu of determining the design shear force in accordance with ACI 18.14.6.1, it is permitted to calculate V_u as the product of the shear induced in the pier due to the design displacement δ_u and the overstrength factor, Ω_o.

14.9 Examples

14.9.1 Example 14.1 – Determination of Flexural Reinforcement: Beam in Building #1 (Framing Option C), Beam is Part of the SFRS (Special Moment Frame), SDC D

Determine the required flexural reinforcement for the beam along column line C between column lines 1 and 2 in Building #1, Framing Option C, at the second-floor level assuming the beam is part of the SFRS and the dimensions of the beam are 28 in. by 24 in. (see Figure 1.1). The slab is 7.0 in. thick and the columns are 28.0 in. by 28.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. The building is assigned to SDC D based on the following:

- Site Class D (determined)
- $S_S = 1.386,\ S_1 = 0.483$
- $S_{DS} = 0.924,\ S_{D1} = 0.586$

Step 1 – Check the dimensional limits — ACI 18.6.2

Prior to determining the design bending moments, check to ensure the dimensional limits of ACI 18.6.2 are satisfied where $d = 24.0 - 2.5 = 21.5$ in. and $\ell_n = 25.0 - (28.0/12) = 22.7$ ft:

- $\ell_n = 22.7 \text{ ft} > 4d = 4 \times (21.5/12) = 7.2 \text{ ft}$ — Figure 14.1
- $b_w = 28.0 \text{ in.} > \text{lesser of} \begin{cases} 0.3h = 0.3 \times 24.0 = 7.2 \text{ in.} \\ 10 \text{ in.} \end{cases}$
- $b_w = 28.0 \text{ in.} < \text{lesser of} \begin{cases} 3c_2 = 3 \times 28.0 = 84.0 \text{ in.} \\ c_2 + 1.5c_1 = 28.0 + (1.5 \times 28.0) = 70.0 \text{ in.} \end{cases}$

Therefore, the dimensions of the beam satisfy the requirements of ACI 18.6.2.

Step 2 – Determine the factored bending moments along the span

A summary of the service bending moments due to the dead and live loads at the critical sections of the beam is given in Table 14.12.

Table 14.12 Bending Moments (ft-kips) due to Service Dead and Live Loads at the Second-Floor Level for the Beam in Example 14.1

Negative – Line 1	Positive	Negative – Line 2
$M_D^- = -88.9$	$M_D^+ = 77.8$	$M_D^- = -109.4$
$M_L^- = -38.7$	$M_L^+ = 36.5$	$M_L^- = -49.8$

A three-dimensional analysis was performed for seismic forces in the east-west direction where it is assumed the frames along column lines A, C, and E are part of the SFRS. In the north-south direction, the frames along column lines 1, 3, 5, and 7 are part of the SFRS. Rigid diaphragms are assigned at each level and to account for cracking, the following reduced moments of inertia are used for the beams and columns [see ACI Table 6.6.3.1.1(a)]:

- Beams: $I = 0.35I_g$
- Columns: $I = 0.70I_g$

The bending moments in the beam due to earthquake loads are given in Table 14.13. The "plus-minus" sign preceding the tabulated values signifies the earthquake loads can act in both the east direction and the west direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the south elevation of the building). The bending moments due to wind loads are smaller than those due to the earthquake loads and are not considered in this example.

Table 14.13 Bending Moments (ft-kips) due to Earthquake Loads at the Second-Floor Level for the Beam in Example 14.1

Negative – Line 1	Positive	Negative – Line 2
± 182.9	—	± 175.9

The design bending moments from the governing load combinations are given in Table 14.14 (see Table 14.1). It can be determined that the redundancy factor, ρ, is equal to 1.0 in accordance with ASCE/SEI 12.3.4.2.

Table 14.14 Design Bending Moments (ft-kips) at the Second-Floor Level for the Beam in Example 14.1

Load Combination			Negative – Line 1	Positive	Negative – Line 2
ACI Eq. (5.3.1a)	$1.4D$		−124.5	108.9	−153.2
ACI Eq. (5.3.1b)	$1.2D + 1.6L$		−168.6	151.8	−211.0
ACI Eq. (5.3.1e)	$1.39D + Q_E + 0.5L$	SSR	40.0	126.4	−352.9
		SSL	−325.8	126.4	−1.1
ACI Eq. (5.3.1g)	$0.71D + Q_E$	SSR	119.8	55.2	−253.6
		SSL	−246.0	55.2	98.2

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Equation (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = Q_E + (0.2 \times 0.924)D = Q_E + 0.19D$. Therefore, this load combination becomes the following:

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + (Q_E + 0.19D) = 1.39D + Q_E + 0.5L$$

Similarly, in ACI Equation (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E + 0.2S_{DS}D = Q_E - (0.2 \times 0.924)D = Q_E - 0.19D$, and this load combination becomes the following:

$$0.9D + 1.0E = 0.9D + (Q_E - 0.19D) = 0.71D + Q_E$$

Step 3 – Determine the required flexural reinforcement at the critical sections

The required area of flexural reinforcement, A_s, at the critical sections is determined by the following equations, which are given in Section 6.5.1 of this publication for rectangular sections with a single layer of tension reinforcement (an effective slab width at positive moment critical sections is not considered in this example):

$$R_n = \frac{M_u}{\phi b d^2}$$

$$A_s = \frac{0.85 f'_c b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right]$$

A summary of the required flexural reinforcement is given in Table 14.15. It is evident that all sections are tension-controlled because $A_s < A_{s,t}$. Also, the required reinforcement is less than the maximum amount prescribed in ACI 18.6.3.1 for Grade 60 reinforcement, which is based on $\rho = 0.025$.

Table 14.15 Required Flexural Reinforcement for the Beam in Example 14.1

Location		M_u (ft-kips)	R_n (psi)	A_s (in.^{2})*
Negative – Line 1	SSR	119.8	123	2.01
	SSL	−325.8	336	3.55
Positive		151.8	156	2.01
Negative – Line 2	SSR	−352.9	364	3.87
	SSL	98.2	101	2.01

* *Min.* $A_s = 200 b_w d / f_y = 200 \times 28.0 \times 21.5 / 60{,}000 = 2.01$ in.^{2}
Max. $A_s = 0.025 b_w d = 0.025 \times 28.0 \times 21.5 = 15.05$ in.^{2} for Grade 60 reinforcement
$A_{s,t} = 0.018 b_w d = 0.018 \times 28.0 \times 21.5 = 10.84$ in.^{2}

Step 4 – Select the flexural reinforcement

Select the size and number of reinforcing bars based on the maximum and minimum spacing requirements in ACI 24.3 and 25.2, respectively.

Negative reinforcement – Line 1:

Use 6-#7 bars ($A_{s,provided} = 3.60$ in.$^2 > 3.55$ in.^{2}; maximum and minimum number of longitudinal bars in a single layer for a 28.0-in.-wide beam are equal to 11 and 4 from Tables 6.8 and 6.9 of this publication, respectively).

Positive reinforcement:

Use 4-#7 bars ($A_{s,provided} = 2.40$ in.$^2 > 2.01$ in.^{2}; maximum and minimum number of longitudinal bars in a single layer for a 28.0-in.-wide beam are equal to 11 and 4 from Tables 6.8 and 6.9 of this publication, respectively).

The 4-#7 bars ($\phi M_n = 224.0$ ft-kips) are also adequate for the 119.8 ft-kip and 98.2 ft-kip positive moments at the faces of the supports at column lines 1 and 2, respectively (see Table 14.15).

Negative reinforcement – Line 2:

Use 7-#7 bars ($A_{s,provided} = 4.20$ in.$^2 > 3.87$ in.^{2}; maximum and minimum number of longitudinal bars in a single layer for a 28.0-in.-wide beam are equal to 11 and 4 from Tables 6.8 and 6.9 of this publication, respectively).

Step 5 – Check the dimensional requirements of ACI 18.8.2.3

Check if the #7 bars satisfy the dimensional requirements in ACI 18.8.2.3:

$$c_1 = 28.0 \text{ in.} > \text{greater of} \begin{cases} 20d_b / \lambda = (20 \times 0.875) / 1.0 = 17.5 \text{ in. for Grade 60 bars} \\ h / 2 = 24.0 / 2 = 12.0 \text{ in.} \end{cases}$$ Figure 14.2

Step 6 – Check the nominal strength requirements of ACI 18.6.3.2

The positive moment strength at the face of a joint be greater than or equal to one-half the negative moment strength provided at that location (ACI 18.6.3.2).

At the interior joint:

$$M_n^+ \text{ (4-\#7)} = A_s^+ f_y \left(d - \frac{A_s^+ f_y}{1.7 f_c' b_w} \right) = (2.40 \times 60) \times \left(21.5 - \frac{2.40 \times 60}{1.7 \times 4.0 \times 28.0} \right) / 12 = 248.9 \text{ ft-kips}$$

$$> \frac{M_n^- \text{ (7-\#7)}}{2} = \frac{A_s^- f_y}{2} \left(d - \frac{A_s^- f_y}{1.7 f_c' b_w} \right) = \frac{(4.20 \times 60)}{2} \times \left(21.5 - \frac{4.20 \times 60}{1.7 \times 4.0 \times 28.0} \right) / 12 = \frac{423.7}{2} = 211.9 \text{ ft-kips}$$

This requirement is also satisfied at the exterior joint where the negative reinforcement is 6-#7 bars.

Also, the negative or positive moment strength at any section of the beam must be greater than or equal to one-fourth the maximum moment strength provided at the face of either joint, which in this example is equal to $423.7 / 4 = 105.9$ ft-kips. Providing 4-#7 continuous top and bottom bars ($M_n = 248.9$ ft-kips) satisfies this requirement.

Step 7 – Check if the flexural reinforcement can be fully developed in the column at column line 1 ACI 18.8.2.2

The #7 bars must be developed in tension in accordance with ACI 18.8.5 and in compression in accordance with ACI 25.4.9.

The required development length of the hooked bars in tension is equal to the following for normalweight concrete:

$$\ell_{dh} = \text{greater of} \begin{cases} f_y d_b / 65\sqrt{f_c'} = (60{,}000 \times 0.875) / (65 \times \sqrt{4{,}000}) = 12.8 \text{ in.} \\ 8d_b = 8 \times 0.875 = 7.0 \text{ in.} \\ 6 \text{ in.} \end{cases}$$ Eq. (14.1)

The available development length is equal to the following assuming #5 hoops and #10 longitudinal bars in the column:

$$c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} = 28.0 - (2 \times 1.5) - 0.625 - 1.27 = 23.1 \text{ in.} > \ell_{dh} = 12.8 \text{ in.} \qquad \text{Eq. (14.3)}$$

The required development length of the #7 bars in compression is equal to the following:

$$\ell_{dc} = \text{ greater of } \begin{cases} \left(\dfrac{f_y \psi_r}{50\lambda\sqrt{f_c'}}\right) d_b = \left(\dfrac{60{,}000 \times 1.0}{50 \times 1.0 \times \sqrt{4{,}000}}\right) \times 0.875 = 16.6 \text{ in.} \\ 0.0003 f_y \psi_r d_b = 0.0003 \times 60{,}000 \times 1.0 \times 0.875 = 15.8 \text{ in.} \\ 8 \text{ in.} \end{cases} \qquad \text{Eq. (14.6)}$$

The available development length is equal to the following assuming #5 hoops and #10 longitudinal bars in the column:

$$\begin{aligned} c_1 - 2c_c - d_{b(hoop)} - d_{b(col.)} - d_b - r &= 28.0 - (2 \times 1.5) - 0.625 - 1.27 - 0.875 - (3 \times 0.875) \\ &= 19.6 \text{ in.} > \ell_{dc} = 16.6 \text{ in.} \end{aligned} \qquad \text{Eq. (14.7)}$$

Therefore, the #7 bars can be adequately developed in the column at column line 1.

14.9.2 Example 14.2 – Determination of Shear Reinforcement: Beam in Building #1 (Framing Option C), Beam is Part of the SFRS (Special Moment Frame), SDC D

Determine the required shear reinforcement for the beam along column line C between column lines 1 and 2 in Building #1, Framing Option C, at the second-floor level assuming the beam is part of the SFRS and the dimensions of the beam are 28 in. by 24 in. (see Figure 1.1). The slab is 7.0 in. thick and the columns are 28.0 in. by 28.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 14.1.

Step 1 – Determine the factored shear forces ACI 18.6.5

Design shear forces are determined from statics assuming the beam is loaded with the factored gravity and vertical earthquake loads along the span and moments of opposite sign corresponding to the probable flexural strength, M_{pr}, acting at the joint faces.

The largest shear force associated with earthquake effects is obtained from ACI Eq. (5.3.1e). The distributed load along the length of the beam is trapezoidal (see Figure 5.26 of this publication), and the maximum factored distributed load in the center 1.5-ft segment of the beam is determined as follows:

$$\text{Slab dead load} = (7.0 / 12) \times 150.0 \times 23.5 = 2{,}056 \text{ lb/ft}$$

$$\text{Superimposed dead load} = 10.0 \times 23.5 = 235 \text{ lb/ft}$$

$$\text{Weight of beam stem} = \frac{28.0 \times (24.0 - 7.0)}{144} \times 150.0 = 496 \text{ lb/ft}$$

$$\text{Live load} = 65.0 \times 23.5 = 1{,}528 \text{ lb/ft}$$

Therefore,

$$w_u = 1.39w_D + 0.5w_L = \frac{[1.39\times(2{,}056+235+496)]+(0.5\times1{,}528)}{1{,}000} = 4.64 \text{ kips/ft}$$

Shear forces due to the combination of w_u and M_{pr} for sidesway to the right and to the left are given in Figure 14.54 where the positive and negative probable flexural strengths are determined by Eq. (14.9):

$$M_{pr}^{-}(6\text{-}\#7) = 1.25\times3.60\times60\times\left(21.5-\frac{1.25\times3.60\times60}{1.7\times4.0\times28.0}\right)/12 = 451.8 \text{ ft-kips}$$

$$M_{pr}^{+}(4\text{-}\#7) = 1.25\times2.40\times60\times\left(21.5-\frac{1.25\times2.40\times60}{1.7\times4.0\times28.0}\right)/12 = 308.3 \text{ ft-kips}$$

$$M_{pr}^{-}(7\text{-}\#7) = 1.25\times4.20\times60\times\left(21.5-\frac{1.25\times4.20\times60}{1.7\times4.0\times28.0}\right)/12 = 521.0 \text{ ft-kips}$$

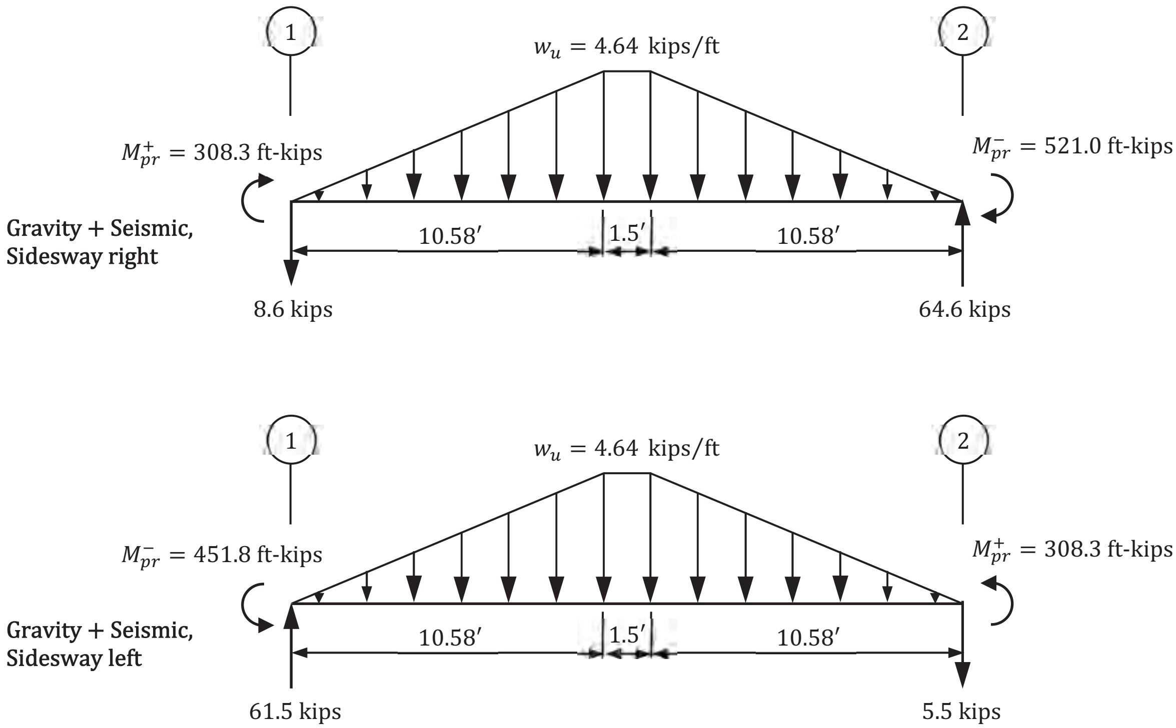


Figure 14.54 Factored shear forces on the beam in Example 14.2.

The maximum V_u is equal to 64.6 kips, which is greater than the maximum V_u from analysis.

Step 2 – Determine the required transverse reinforcement

According to ACI 18.6.4.1, hoops are required over a length equal to $2h = 48.0$ in. from the face of the columns at both ends of the beam.

The size and spacing of the hoops can be determined by the following equation:

$$\frac{A_v}{s} \geq \frac{(V_u - \phi V_c)}{\phi f_{yt} d} \qquad \text{Eq. (14.11)}$$

Check if V_c must be taken equal to zero: ACI 18.6.5.2

At the faces of the supports, the maximum earthquake-induced shear force $= (308.3 + 521.0) / 22.7 = 36.5$ kips

One-half of the maximum shear force $= 64.6 / 2 = 32.3 \text{ kips} < 36.5 \text{ kips}$

Factored axial compressive force on the beam including earthquake effects $< A_g f_c' / 20$

Therefore, V_c must be taken equal to zero.

Assuming #3 hoops with 2 crossties, the required hoop spacing is equal to the following (2 crossties must be used because the clear spacing of the bottom bars is greater than 6 in.; see ACI 18.6.4.2 and 25.7.2.3):

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (4 \times 0.11) \times 60 \times 21.5}{64.6 - 0} = 6.6 \text{ in.}$$

Check maximum hoop spacing along the length: ACI Table 9.7.6.2.2

$$V_s = (V_u / \phi) - V_c = (64.6 / 0.75) - 0 = 86.1 \text{ kips} < 4\sqrt{f_c'} b_w d = 152.3 \text{ kips}$$

Therefore, maximum $s = d / 2 = 10.8$ in.

Also,

$$s \leq \text{smallest of} \begin{cases} d / 4 = 21.5 / 4 = 5.4 \text{ in.} \\ 6 \text{ in.} \\ \text{For Grade 60 bars, } 6d_b = 6 \times 0.875 = 5.3 \text{ in.} \end{cases} \qquad \text{ACI 18.6.4.4, Figure 14.9}$$

Use 11-#3 hoops with 2 crossties at each end of the beam spaced 5.0 in. on center with the first set located 2.0 in. from the face of the support (ACI 18.6.4.4).

Where hoops are no longer required, stirrups with seismic hooks at both ends may be used instead of hoops (ACI 18.6.4.6). At a distance of 52.0 in. from the face of the support at column line 2 for sidesway to the right, $V_u = 60.5$ kips. The required spacing of #3 stirrups is equal to the following where it is permitted to use V_c in this region of the beam (the maximum spacing of stirrup legs across the width of the beam $= d = 21.5$ in., so 3 stirrup legs are required; see ACI Table 9.7.6.2.2):

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (3 \times 0.11) \times 60 \times 21.5}{60.5 - (0.75 \times 2 \times \sqrt{4{,}000} \times 28.0 \times 21.5 / 1{,}000)} = 94.2 \text{ in.}$$

$$\leq \frac{A_v f_{yt}}{0.75\sqrt{f_c'} b_w} = \frac{(3 \times 0.11) \times 60{,}000}{0.75 \times \sqrt{4{,}000} \times 28.0} = 14.9 \text{ in.} \qquad \text{ACI Table 9.6.3.4}$$

$$\leq \frac{A_v f_{yt}}{50 b_w} = \frac{(3 \times 0.11) \times 60{,}000}{50 \times 28.0} = 14.1 \text{ in.}$$

Also,

$s = d / 2 = 21.5 / 2 = 10.8$ in. ACI 18.6.4.6

Use #3 stirrups with one crosstie spaced at 10.0 in. on center in the region of the beam outside of the anticipated plastic hinge regions.

14.9.3 Example 14.3 – Determination of Cutoff Points of Flexural Reinforcement: Beam in Building #1 (Framing Option C), Beam is Part of the SFRS (Special Moment Frame), SDC D

Determine the location where 3 of the 7-#7 top bars at the interior support can be terminated for the beam along column line C between column lines 1 and 2 in Building #1, Framing Option C, at the second-floor level assuming the beam is part of the SFRS and the dimensions of the beam are 28 in. by 24 in. (see Figure 1.1). The slab is 7.0 in. thick and the columns are 28.0 in. by 28.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 14.1 and 14.2.

Step 1 – Determine the load combination to determine the cutoff point

The load combination used to determine the cutoff point is $(0.9 - 0.2S_{DS})w_D = 0.71w_D = 1.98$ kip/ft in combination with the probable flexural strengths at the ends of the beam for sidesway to the right because this combination produces the longest bar lengths.

Step 2 – Determine the theoretical cutoff point

Three of the 7-#7 top bars can be theoretically cutoff at the location where the factored bending moment, M_u, is equal to the design moment strength of the section, ϕM_n, with 4-#7 bars, which is equal to 224.0 ft-kips.

The distance x from the face of the support at column line 2 to the location where the moment is equal to 224.0 ft-kips can be determined by summing moments about section $a - a$ in Figure 14.55:

$$\left(\frac{1.98x}{10.58}\right)\left(\frac{x}{2}\right)\left(\frac{x}{3}\right) - 48.5x + 521.0 - 224.0 = 0$$

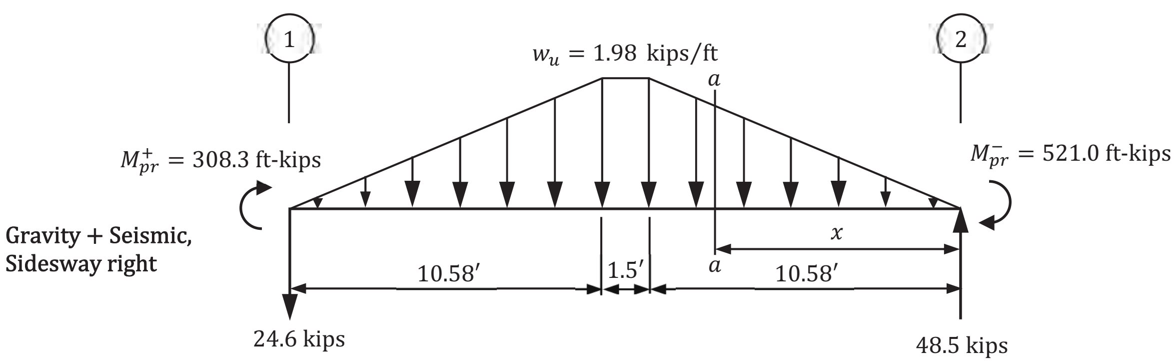


Figure 14.55 Cutoff location for the top bars at the interior support for the beam in Example 14.3.

Solving for x gives a distance of 6.3 ft from the face of the support.

The 3-#7 bars must extend a distance equal to the greater of the following beyond x: ACI 9.7.3.3

$d = 21.5$ in.

$12d_b = 10.5$ in.

Thus, the total bar length from the face of the support must be at least equal to $6.3 + (21.5 / 12) = 8.1$ ft.

Additionally, the bars must extend at least ℓ_d beyond the face of the support: ACI 9.7.3.4

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$

ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #7 reinforcing bars, $\psi_s = 1.0$

For more than 12 in. of fresh concrete cast below the negative reinforcement, $\psi_t = 1.3$

$$c_b = \text{ lesser of } \begin{cases} c_c + (d_b)_{hoop} + 0.5(d_b)_{long.} = 1.5 + 0.375 + (0.5 \times 0.875) = 2.3 \text{ in.} \\ \\ \dfrac{s}{2} = \dfrac{28.0 - [2 \times (1.5 + 0.375)] - 0.875}{2 \times 6} = 2.0 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.0 + 0) / 0.875 = 2.3 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.3 \times 1.0 \times 1.0 \times 1.0}{2.3} \right) \times 0.875 = 35.2 \text{ in.} = 2.9 \text{ ft} < 8.1 \text{ ft}$$

According to ACI 9.7.3.5, flexural reinforcement is not permitted to be terminated in a tension zone unless one or more of the conditions in that section are satisfied. In this example, the point of inflection in the bending moment diagram occurs approximately 11.8 ft from the face of the support, which is greater than 8.1 ft, which means the #7 bars cannot be terminated at that location because they are in a tension zone.

Check if ACI 9.7.3.5(a) is satisfied at the inflection point, which would permit the #7 bars to be terminated at this location:

At 8.1 ft from the face of the support, $V_u = 48.5 - (0.5 \times 1.98 \times 8.1^2 / 10.58) = 42.4$ kips

$V_c = 2 \times \sqrt{4{,}000} \times 28.0 \times 21.5 / 1{,}000 = 76.2$ kips

Within this region of the beam, #3 stirrups with one crosstie spaced at 10.0 in. on center are provided; thus,

$$V_s = \frac{A_v f_{yt} d}{s} = \frac{(3 \times 0.11) \times 60 \times 21.5}{10.0} = 42.6 \text{ kips}$$

$V_u = 42.4 \text{ kips} < 2\phi V_n / 3 = 2 \times 0.75 \times (76.2 + 42.6) / 3 = 59.4 \text{ kips}$

Therefore, the #7 bars are permitted to be terminated 8.1 ft from the face of the support.

Similar calculations can be performed for the cutoff point for two of the 6-#7 top bars at the exterior support at column line 1. Two of the #7 bars can be terminated 6.9 ft from the face of that support.

Reinforcement details for this beam are given in Figure 14.56.

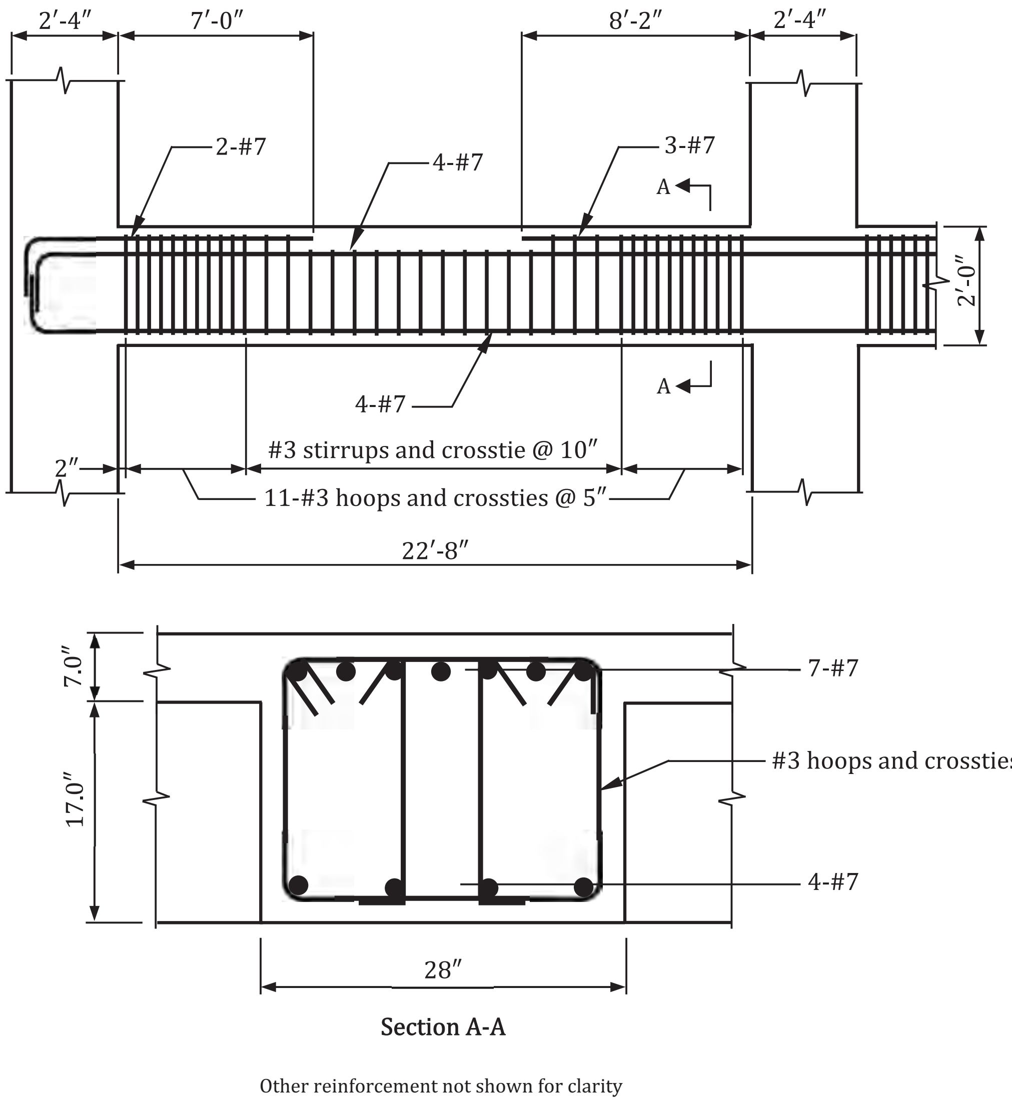


Figure 14.56 Reinforcement details for the beam in Examples 14.1, 14.2, and 14.3.

14.9.4 Example 14.4 – Determination of Longitudinal Reinforcement: Interior Column in Building #1 (Framing Option C), Column is Part of the SFRS (Special Moment Frame), SDC D

Determine the required longitudinal reinforcement for column C2 in Building #1, Framing Option C, in the first story assuming the column is part of the SFRS and the dimensions of the column are 28 in. by 28 in. (see Figure 1.1). The slab is 7.0 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000 \text{ psi}$ and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. The building is assigned to SDC D based on the following:

- Site Class D (determined)
- $S_S = 1.386,\ S_1 = 0.483$
- $S_{DS} = 0.924,\ S_{D1} = 0.586$

Step 1 – Check the dimensional limits ACI 18.7.2

Shortest cross-sectional dimension $28.0 \text{ in.} > 12.0 \text{ in.}$ Figure 14.10

Ratio of shortest cross-sectional dimension to the perpendicular dimension $= 28.0 / 28.0 = 1.0 > 0.4$

Therefore, the provided cross-sectional dimensions are adequate.

Step 2 – Determine the factored load combinations

A three-dimensional analysis was performed for gravity and earthquake effects in the east-west direction where it is assumed the frames along column lines A, C, and E are part of the SFRS. In the north-south direction, the frames along column lines 1, 3, 5, and 7 are part of the SFRS. Rigid diaphragms are assigned at each level and to account for cracking, the following reduced moments of inertia are used for the beams and columns [see ACI Table 6.6.3.1.1(a)]:

- Beams: $I = 0.35I_g$
- Columns: $I = 0.70I_g$

This column is part of the SFRS in the east-west direction only, which means it need not be designed for orthogonal load effects in accordance with ASCE/SEI 12.5.4. Also, P-delta effects were automatically accounted for in the analysis and were found to be negligible.

The gravity and earthquake axial forces and bending moments at the top and bottom of column A2 are given in Table 14.16. Bending moments due to gravity load effects are negligible compared to those from earthquake effects and are taken equal to zero. The "plus-minus" sign preceding the tabulated seismic axial force, bending moments, and shear force signifies the earthquake loads can act in both the east direction and the west direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the south elevation of the building). The effects due to wind loads are smaller than those due to the earthquake loads and are not considered in this example.

Table 14.16 Design Axial Forces, Bending Moments, and Shear Forces for Column C2 in Example 14.4

Load Case		Axial Force (kips)	Bending Moment (ft-kips)		Shear Force (kips)
			Bottom	Top	
Dead (D)		457.2	0	0	0
Live (L)		152.8	0	0	0
Roof live load (L_r)		11.8			
Seismic (Q_E)		± 3.6	± 526.3	± 72.6	± 49.9
Load Combination					
ACI Eq. (5.3.1a)	$1.4D$	640.1	0	0	0

(table continued on next page)

Table 14.16 Design Axial Forces, Bending Moments, and Shear Forces for Column C2 in Example 14.4 (cont.)

Load Case			Axial Force (kips)	Bending Moment (ft-kips)		Shear Force (kips)
				Bottom	Top	
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		799.0	0	0	0
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		643.9	0	0	0
ACI Eq. (5.3.1e)	$1.39D + 0.5L + Q_E$	SSR	708.3	−526.3	72.6	−49.9
		SSL	715.5	526.3	−72.6	49.9
ACI Eq. (5.3.1g)	$0.71D + Q_E$	SSR	321.0	−526.3	72.6	−49.9
		SSL	328.2	526.3	−72.6	49.9

SSR = sidesway to the right, SSL = sidesway to the left

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Equation (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = Q_E + (0.2 \times 0.924)D = Q_E + 0.19D$. Therefore, this load combination becomes the following:

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + (Q_E + 0.19D) = 1.39D + Q_E + 0.5L$$

Similarly, in ACI Equation (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E + 0.2S_{DS}D = Q_E - (0.2 \times 0.924)D = Q_E - 0.19D$, and this load combination becomes the following:

$$0.9D + 1.0E = 0.9D + (Q_E - 0.19D) = 0.71D + Q_E$$

Step 3 – Determine the required longitudinal reinforcement

The design strength interaction diagram for this column reinforced with 12-#10 bars $(0.010 < \rho_{st} = 0.019 < 0.060)$ is given in Figure 14.57. All load combinations fall within the boundary of the design strength interaction diagram for this column.

Try 12-#10 bars.

Step 4 – Check the minimum flexural strength requirements — ACI 18.7.3

The minimum flexural strength requirements of ACI 18.7.3 are checked for this column in the east-west direction. The nominal flexural strengths of the columns and beams are determined in accordance with ACI 18.7.3.2. Only load combinations that include earthquake effects need to be considered when checking ACI Eq. (18.7.3.2).

The flexural reinforcement in the beams framing into the joint is determined in Example 14.1 (see Figure 14.56). The negative flexural strength of the beam must include the contribution of the reinforcement in the slab within the effective slab width defined in ACI 6.3.2 (the slab is in tension). It can be determined that 11-#4 bars are required for both the positive and negative reinforcement within the 11.75-ft column strip. The effective slab width in accordance with ACI 6.3.2 is equal to the following:

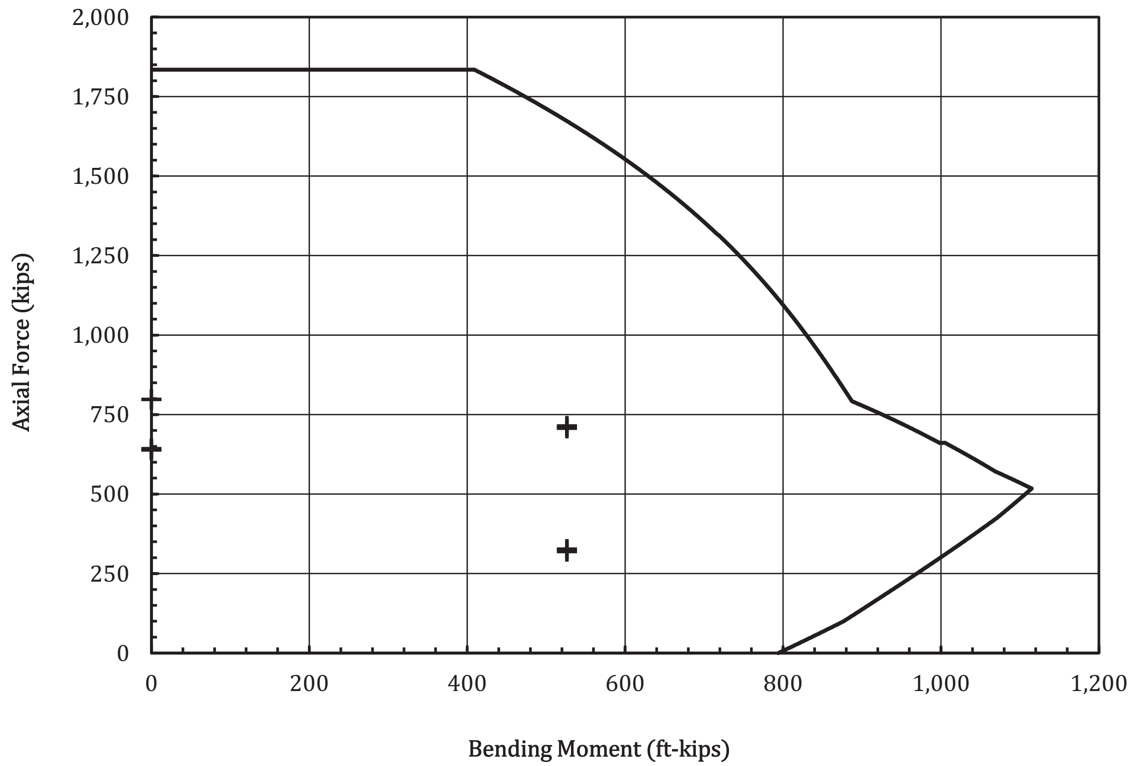


Figure 14.57 Design strength interaction diagram for column C2 in Example 14.4.

$$b_e = \text{least of} \begin{cases} 16h + b_w = (16 \times 7.0) + 28.0 = 140.0 \text{ in.} \\ \text{Center-to-center beam spacing} = 23.5 \text{ ft} = 282.0 \text{ in.} \\ (\ell_n / 4) + b_w = (22.67 \times 12 / 4) + 28.0 = 96.0 \text{ in.} \end{cases}$$ Figure 14.13

The beam used in determining the negative flexural strength is given in Figure 14.58.

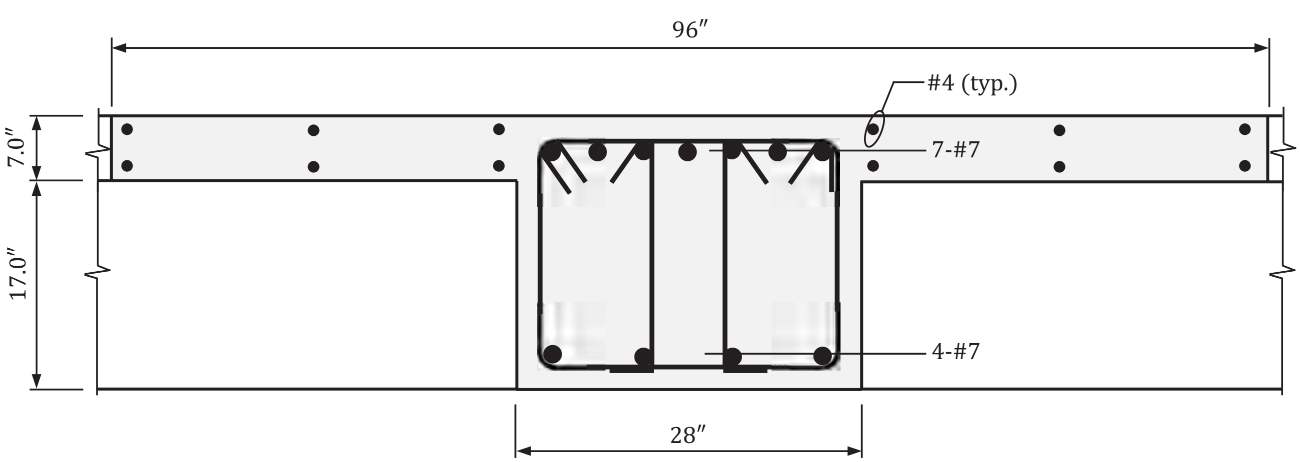


Figure 14.58 Beam framing into the interior joint at column C2 in Example 14.4.

A strain compatibility analysis of the section with the 96.0-in. effective width, the 7-#7 top bars in the beam, and the #4 bars in the slab yields $M_{nb}^{-} = 632.9$ ft-kips. Also, $M_{nb}^{+} = 283.6$ ft-kips.

The nominal flexural strength of the column is determined for the factored axial compressive force resulting in the lowest flexural strength, consistent with the direction of analysis. For the column below the joint, the axial forces in Table 14.16 are applicable. These forces are indicated in Figure 14.59, which contains the nominal strength interaction diagram for this column.

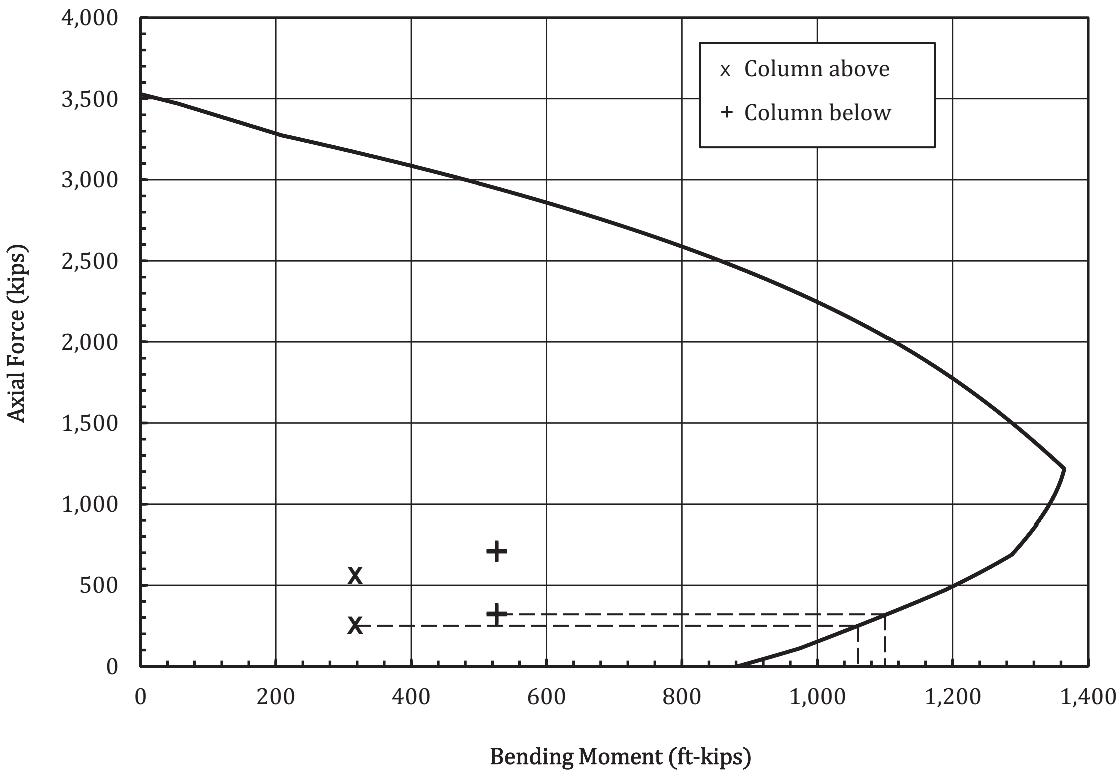


Figure 14.59 Nominal strength interaction diagram for column C2 in Example 14.4.

The factored axial compressive forces and corresponding bending moments for the load combinations with earthquake effects are also indicated in Figure 14.59 for the column above the joint where it is assumed the column above the joint has the same cross-section, concrete compressive strength, and longitudinal reinforcement as the column below the joint.

From Figure 14.59, the minimum flexural strengths for the columns above and below the joint are equal to the following:

$$M_{nc(top)} = 1{,}063.1 \text{ ft-kips}$$

$$M_{nc(bot)} = 1{,}102.8 \text{ ft-kips}$$

Check ACI Eq. (18.7.2.3): Figure 14.11

$$\Sigma M_{nc} = 1{,}063.1 + 1{,}102.8 = 2{,}165.9 \text{ ft-kips} > \frac{6}{5}\Sigma M_{nb} = \frac{6}{5} \times (632.9 + 283.6) = 1{,}099.8 \text{ ft-kips}$$

Therefore, the minimum flexural strength requirements of ACI 18.7.3 are satisfied.

Step 5 – Check the development length requirements of ACI 18.7.4.3

Unsupported length of column: $\ell_u = 14.0 - (24.0 / 24) = 13.0$ ft

Determine the tension development length, ℓ_d, of the #10 longitudinal bars:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #10 reinforcing bars, $\psi_s = 1.0$

$\psi_t = 1.0$

Assuming #4 hoops:

$$c_b = \text{ lesser of } \begin{cases} c_c + (d_b)_{hoop} + 0.5(d_b)_{long.} = 1.5 + 0.5 + (0.5 \times 1.27) = 2.6 \text{ in.} \\ \\ \dfrac{s}{2} = \dfrac{28.0 - [2 \times (1.5 + 0.5)] - 1.27}{2 \times 3} = 3.8 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.6 + 0) / 1.27 = 2.1 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.1} \right) \times 1.27 = 43.0 \text{ in.} = 3.6 \text{ ft}$$

$\ell_u = 13.0 \text{ ft} > 2.5\ell_d = 2.5 \times 3.6 = 9.0$ ft Figure 14.14

Therefore, the requirements of ACI 18.7.4.3 are satisfied.

Use 12-#10 bars.

14.9.5 Example 14.5 – Determination of Transverse Reinforcement: Interior Column in Building #1 (Framing Option C), Column is Part of the SFRS (Special Moment Frame), SDC D

Determine the required transverse reinforcement for column C2 in Building #1, Framing Option C, in the first story assuming the column is part of the SFRS and the dimensions of the column are 28 in. by 28 in. (see Figure 1.1). The slab is 7.0 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 14.4.

Step 1 – Determine the length over which flexural yielding is likely to occur ACI 18.7.5.1

$$\ell_o \geq \text{greatest of} \begin{cases} \text{Maximum cross-sectional dimension of the column} = 28.0 \text{ in.} \\ \text{Clear span of the column}/6 = [(14.0 \times 12) - 12.0] / 6 = 26.0 \text{ in.} \\ 18 \text{ in.} \end{cases}$$ Eq. (14.015)

Step 2 – Check if the requirements of ACI 18.7.5.2(f) must be satisfied

The largest factored axial force including earthquake effects is equal to 715.5 kips. Table 14.16

$0.3f_c'A_g = 0.3 \times 4 \times 28^2 = 940.8 \text{ kips} > 715.5 \text{ kips}$

Also, $f_c' = 4{,}000 \text{ psi} < 10{,}000 \text{ psi}$.

Therefore, the requirements of ACI 18.7.5.2(f) need not be satisfied.

Step 3 – Determine the maximum hoop spacing based on the requirements of ACI 18.7.5.3

The maximum hoop spacing within ℓ_o from each end of the column is equal to the following:

$$s = \text{smaller of} \begin{cases} \text{Minimum column dimension} / 4 = 28.0 / 4 = 7.0 \text{ in.} \\ \text{For Grade 60 longitudinal bars, } 6d_b = 6 \times 1.27 = 7.6 \text{ in.} \\ s_o = 4 + [(14 - h_x) / 3] = 4 + [(14 - 7.6) / 3] = 6.1 \text{ in.} > 6.0 \text{ in., use } 6.0 \text{ in.} \end{cases}$$ Figure 14.17

In the equation for s_o, the maximum center-to-center spacing of the laterally supported longitudinal bars, h_x, is determined as follows assuming #4 hoops with crossties around all the longitudinal bars:

$$h_x = \frac{28.0 - [2 \times (1.5 + 0.5)] - 1.27}{3} = 7.6 \text{ in.}$$

Step 4 – Determine the minimum area of hoop reinforcement based on the requirements of ACI 18.7.5.4

In both directions, $b_c = 28.0 - (2 \times 1.5) = 25.0$ in.

$A_{ch} = 25.0 \times 25.0 = 625.0 \text{ in.}^2$

$$A_{sh} = \text{larger of} \begin{cases} 0.3sb_c\left(\dfrac{A_g}{A_{ch}} - 1\right)\dfrac{f_c'}{f_{yt}} = 0.3 \times 6.0 \times 25.0 \times \left(\dfrac{28.0^2}{625.0} - 1\right) \times \dfrac{4}{60} = 0.76 \text{ in.}^2 \\ 0.09sb_c\dfrac{f_c'}{f_{yt}} = 0.09 \times 6.0 \times 25.0 \times \dfrac{4}{60} = 0.90 \text{ in.}^2 \end{cases}$$ Figure 14.17

#4 hoops with two #4 crossties spaced at 6.0 in. provides $A_{sh} = 4 \times 0.20 = 0.80 \text{ in.}^2 < 0.90 \text{ in.}^2$

#5 hoops with two #5 crossties spaced at 6.0 in. provides $A_{sh} = 4 \times 0.31 = 1.24 \text{ in.}^2 > 0.90 \text{ in.}^2$

Try #5 hoops and crossties spaced at 6.0 in. on center within ℓ_o (note: recalculating s_o with #5 hoops and crossties instead of #4 hoops and crossties results in the same value for s_o).

Step 5 – Check if the size and spacing of the hoop reinforcement determined in Step 4 are adequate for shear

Design shear force, V_u, is determined using the probable flexural strengths, M_{pr}, at the ends of the column associated with the range of factored axial forces acting on the column (ACI 18.7.6.1). The design strength interaction diagram for this column with $f_y = 1.25 \times 60 = 75$ ksi and $\phi = 1.0$ is given in Figure 14.60. It is evident from the figure that the largest M_{pr} is equal to 1,402.8 ft-kips, which corresponds to an axial force of 715.5 kips for sidesway to the left.

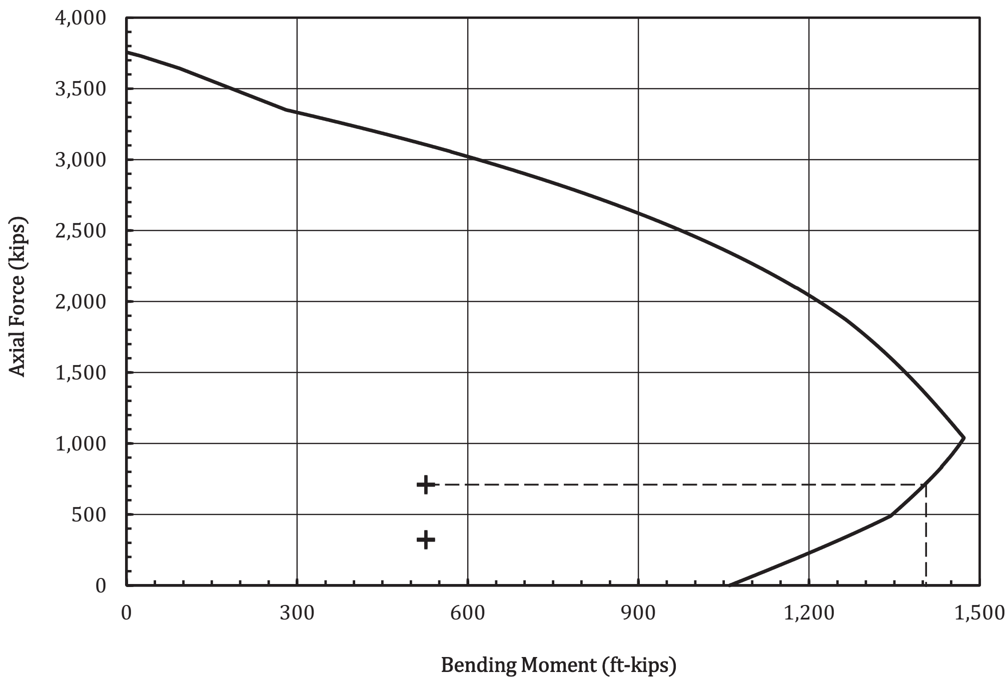


Figure 14.60 Nominal strength interaction diagram for column C2 with $f_y = 75$ **ksi** and $\phi = 1.0$.

Therefore,

$$V_u = \frac{2M_{pr}}{\ell_u} = \frac{2 \times 1,402.8}{14.0 - 1.0} = 215.8 \text{ kips}$$ Figure 14.15

This shear force is greater than the maximum shear force of 49.9 kips determined from analysis (see Table 14.16).

The earthquake-induced shear force is the total shear force on the column. Also, the minimum factored axial force including earthquake effects is equal to 321.0 kips, which is greater than $A_g f_c' / 20 = 156.8$ kips. Therefore, V_c can be included when determining the required shear reinforcement (ACI 18.7.6.2.1):

$$\frac{N_u}{6A_g} = \frac{715.5}{6 \times 28.0^2} = 0.15 \text{ ksi} < 0.05 f_c' = 0.20 \text{ ksi}$$

$$V_c = \left(2\lambda\sqrt{f_c'} + \frac{N_u}{6A_g}\right) b_w d = \left(\frac{2 \times 1.0 \times \sqrt{4,000}}{1,000} + 0.15\right) \times 28.0 \times 22.8 = 176.5 \text{ kips}$$

Eq. (14.17)

$$< 5\lambda\sqrt{f_c'} b_w d = 201.9 \text{ kips}$$

where d is determined from a strain compatibility analysis of the section for the axial compressive force $N_u = 715.5$ kips, which corresponds to M_{pr} used to determine V_u.

The required spacing of the #5 hoops and crossties in the direction of analysis is equal to the following:

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (4 \times 0.31) \times 60 \times 22.8}{215.8 - (0.75 \times 176.5)} = 15.3 \text{ in.} > 6.0 \text{ in.}$$

Thus, the size and spacing of the hoops determined in Step 4 are adequate for shear.

Use #5 hoops with two crossties spaced at 6.0 in. on center over a distance of at least $\ell_o = 28.0$ in. from each end of the column.

Step 6 – Determine the size and spacing of the hoop reinforcement in the remainder of the column

Outside of ℓ_o, the spacing of the hoops is equal to the following:

$$s = \text{smaller of} \begin{cases} 6 \text{ in.} \\ \text{For Grade 60 longitudinal bars, } 6d_b = 6 \times 1.27 = 7.6 \text{ in.} \end{cases}$$ Figure 14.17

The same transverse reinforcement required within ℓ_o is used over the entire length of the column, including the segment over the lap splice length.

A Class B lap splice must be used. The length of the lap splice $= 1.3\ell_d = 1.3 \times 3.6 = 4.7$ ft (see Example 14.4 for the calculation of ℓ_d).

Use a 4 ft-9 in. lap splice length, which must be located near the mid-height of the column.

Reinforcement details for this column are given in Figure 14.61.

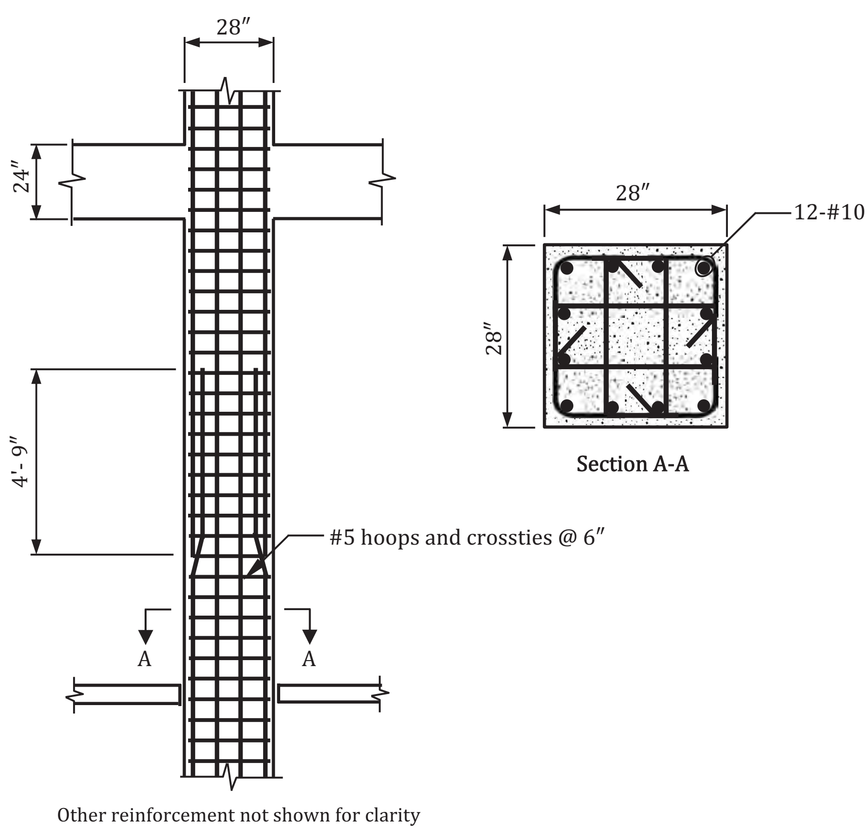


Figure 14.61 Reinforcement details for column A2 in Examples 14.4 and 14.5.

14.9.6 Example 14.6 – Check of Joint Shear Strength: Interior Column in Building #1 (Framing Option C), Column is Part of the SFRS (Special Moment Frame), SDC D

Check the joint shear strength in the east-west direction for column C2 in Building #1, Framing Option C, at the second-floor level assuming the column is part of the SFRS and the dimensions of the column are 28 in. by 28 in. (see Figure 1.1). The slab is 7.0 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 14.1 through 14.5.

Step 1 – Determine the factored horizontal joint shear force ACI 18.8.4.1

Free-body diagrams of the joint are given in Figure 14.62 for sidesway to the right and sidesway to the left. The negative and positive flexural reinforcement in the beams on both sides of the joint are the same, that is, 7-#7 top bars and 4-#7 bottom bars are used in both beams (see Example 14.1).

It is determined in Example 14.2 that $M_{pr}^{+}(4\text{-}\#7) = 308.3$ ft-kips and $M_{pr}^{-}(7\text{-}\#7) = 521.0$ ft-kips. The shear forces at the ends of the beams are given in Figure 14.54 for sidesway to the right and to the left.

From analysis of the structure, the point of inflection in the column above the joint occurs approximately 6.0 ft from the center of the joint and the point of inflection in the column below the joint occurs approximately 3.4 ft from the center of the joint. Therefore, $\ell_c = 6.0 + 3.4 = 9.4$ ft.

The shear force in the column is obtained by summing moments about the center of the joint for sidesway to the right and to the left.

Sidesway to the right:

$$V_{col} = \frac{M_{pr}^{-} + M_{pr}^{+}}{\ell_c} + \frac{(V_1 + V_2)\times(c_1 / 2)}{\ell_c} \quad \text{Eq. (14.19)}$$

$$= \frac{521.0 + 308.3}{9.4} + \frac{(8.6 + 64.6)\times(28.0 / 24)}{9.4} = 88.2 + 9.1 = 97.3 \text{ kips}$$

Sidesway to the left:

$$V_{col} = \frac{M_{pr}^{-} + M_{pr}^{+}}{\ell_c} + \frac{(V_1 + V_2)\times(c_1 / 2)}{\ell_c}$$

$$= \frac{521.0 + 308.3}{9.4} + \frac{(64.6 + 5.5)\times(28.0 / 24)}{9.4} = 88.2 + 8.7 = 96.9 \text{ kips}$$

The shear force in the column for sidesway to the left will be used in determining the joint shear force because this results in a larger joint shear force, V_u :

$$V_u = 1.25(A_s^{-} + A_s^{+})f_y - V_{col} = 1.25\times[(4.20 + 2.40)\times 60] - 96.9 = 495.0 - 96.9 = 398.1 \text{ kips} \quad \text{Eq. (14.23)}$$

Step 2 – Determine the design shear strength of the joint

Columns frame into the bottom and top of the joint (that is, the column is continuous). The beam in the direction of analysis is continuous. There are beams in the transverse direction that have the same width as the joint, which provide confinement. Therefore, the joint type is I-A (see Figure 14.24).

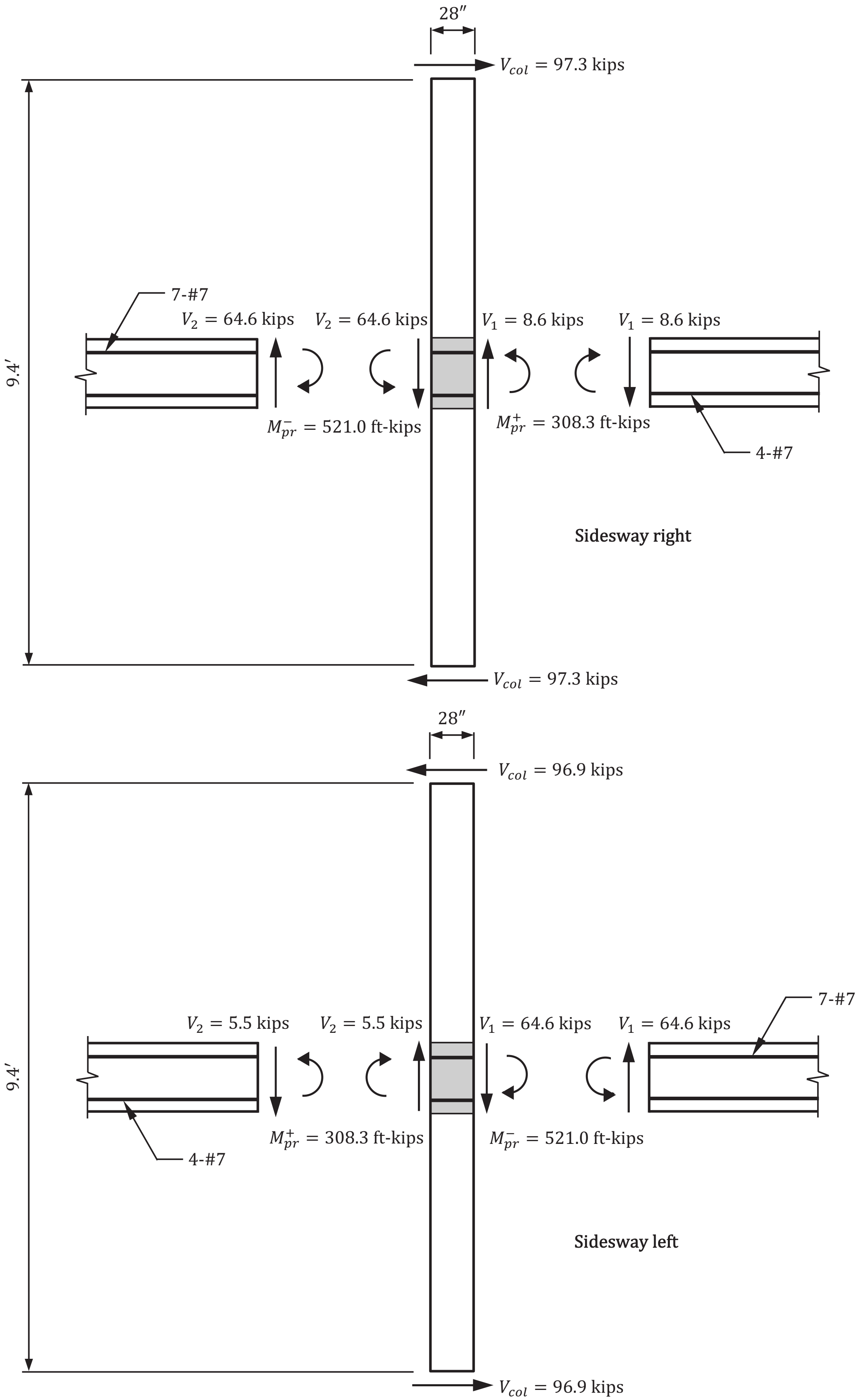


Figure 14.62 Free-body diagram of the joint at column C2.

Because the beam in the direction of analysis is the same the width as the column, the effective cross-sectional area within the joint is equal to the area of the column:

$A_j = c_1 \times c_2 = 28.0 \times 28.0 = 784.0 \text{ in.}^2$ Figure 14.25

The design shear strength of the joint is equal to the following for a Type I-A joint:

$\phi V_n = \phi 20\lambda\sqrt{f_c'}A_j = 0.85 \times 20 \times 1.0 \times \sqrt{4{,}000} \times 784.0 / 1{,}000 = 842.9 \text{ kips} > V_u = 398.1 \text{ kips}$ Figure 14.24

Step 3 – Determine the detailing for the joint

Because beams that have the same width as the column frame into all four sides of the joint, one-half the amount of transverse reinforcement required by ACI 18.7.5.4 is permitted in the joint (ACI 18.8.3.2). For simpler detailing, the same transverse reinforcement provided at the ends of the column is provided in the joint (see Figure 14.61).

14.9.7 Example 14.7 – Determination of Longitudinal Reinforcement: Corner Column in Building #1 (Framing Option C), Column is Part of the SFRS (Special Moment Frame), SDC D

Determine the required longitudinal reinforcement for column E1 in Building #1, Framing Option C, in the first story assuming the column is part of the SFRS and the dimensions of the column are 28 in. by 28 in. (see Figure 1.1). The slab is 7.0 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1.

Step 1 – Check the dimensional limits ACI 18.7.2

Shortest cross-sectional dimension $28.0 \text{ in.} > 12.0 \text{ in.}$ Figure 14.10

Ratio of shortest cross-sectional dimension to the perpendicular dimension $= 28.0 / 28.0 = 1.0 > 0.4$

Therefore, the provided cross-sectional dimensions are adequate.

Step 2 – Determine the factored load combinations

A three-dimensional analysis was performed for gravity and earthquake effects in the north-south and east-west directions where it is assumed the frames along column lines 1, 3, 5, and 7 in the north-south direction and the frames along column lines A, C, and E in the east-west direction are part of the SFRS. Rigid diaphragms are assigned at each level and to account for cracking, the following reduced moments of inertia are used for the beams and columns [see ACI Table 6.6.3.1.1(a)]:

- Beams: $I = 0.35I_g$
- Columns: $I = 0.70I_g$

This column is part of the SFRS in both the north-south and east-west directions, which means it may need to be designed for orthogonal load effects in accordance with ASCE/SEI 12.5.4. The maximum axial force due to earthquake effects is equal to 72.7 kips based on earthquake forces in the north-south direction. At this stage in the design, the required area of longitudinal reinforcement is not known. Assume a 1 percent longitudinal reinforcement ratio and determine the design axial strength of the column:

$$\phi P_{n(max)} = \phi 0.80 P_o = \phi 0.80[0.85 f_c'(A_g - 0.01A_g) + f_y(0.01A_g)]$$

ACI 22.4.2

$$= 0.65 \times 0.80 \times \{[0.85 \times 4 \times (784.0 - 7.84)] + (60 \times 7.84)\} = 1{,}616.9 \text{ kips}$$

$$0.2\phi P_{n(max)} = 0.2 \times 1{,}616.9 = 323.4 \text{ kips} > 72.7 \text{ kips}$$

Therefore, in accordance with ASCE/SEI 12.5.4, orthogonal load effects due to earthquake forces need not be considered. This requirement will be checked again once the required area of longitudinal reinforcement is determined.

Also, P-delta effects were automatically accounted for in the analysis and were found to be negligible.

The gravity and earthquake axial forces and bending moments at the top and bottom of column E1 are given in Table 14.17 for earthquake forces in the east-west direction. The "plus-minus" sign preceding the tabulated seismic axial force, bending moments, and shear force signifies the earthquake loads can act in both the east direction and the west direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the south elevation of the building). The effects due to wind loads are smaller than those due to the earthquake loads and are not considered in this example.

Table 14.17 Design Axial Forces, Bending Moments, and Shear Forces for Column E1 in Example 14.7

Load Case			Axial Force (kips)	Bending Moment (ft-kips)		Shear Force (kips)
				Bottom	Top	
Dead (D)			195.8	31.0	−51.0	6.8
Live (L)			46.0	14.2	−23.4	3.1
Roof live load (L_r)			3.5			
Seismic (Q_E)			±72.4	±480.4	±3.0	±39.8
Load Combination						
ACI Eq. (5.3.1a)	$1.4D$		274.1	43.4	−71.4	9.5
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		310.3	59.9	−98.6	13.1
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		263.6	44.3	−72.9	9.7
ACI Eq. (5.3.1e)	$1.39D + 0.5L + Q_E$	SSR	222.8	−430.29	−85.6	−28.8
		SSL	367.6	530.6	−79.6	50.8
ACI Eq. (5.3.1g)	$0.71D + Q_E$	SSR	66.6	−458.4	−39.2	−35.0
		SSL	211.4	502.4	−33.2	44.6

SSR = sidesway to the right, SSL = sidesway to the left

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Equation (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = Q_E + (0.2 \times 0.924)D = Q_E + 0.19D$. Therefore, this load combination becomes the following:

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + (Q_E + 0.19D) = 1.39D + Q_E + 0.5L$$

Similarly, in ACI Equation (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E + 0.2S_{DS}D = Q_E - (0.2 \times 0.924)D = Q_E - 0.19D$, and this load combination becomes the following:

$$0.9D + 1.0E = 0.9D + (Q_E - 0.19D) = 0.71D + Q_E$$

Step 3 – Determine the required longitudinal reinforcement

The design strength interaction diagram for this column reinforced with 12-#8 bars $(0.010 < \rho_{st} = 0.012 < 0.060)$ is given in Figure 14.63. All load combinations fall within the boundary of the design strength interaction diagram for this column.

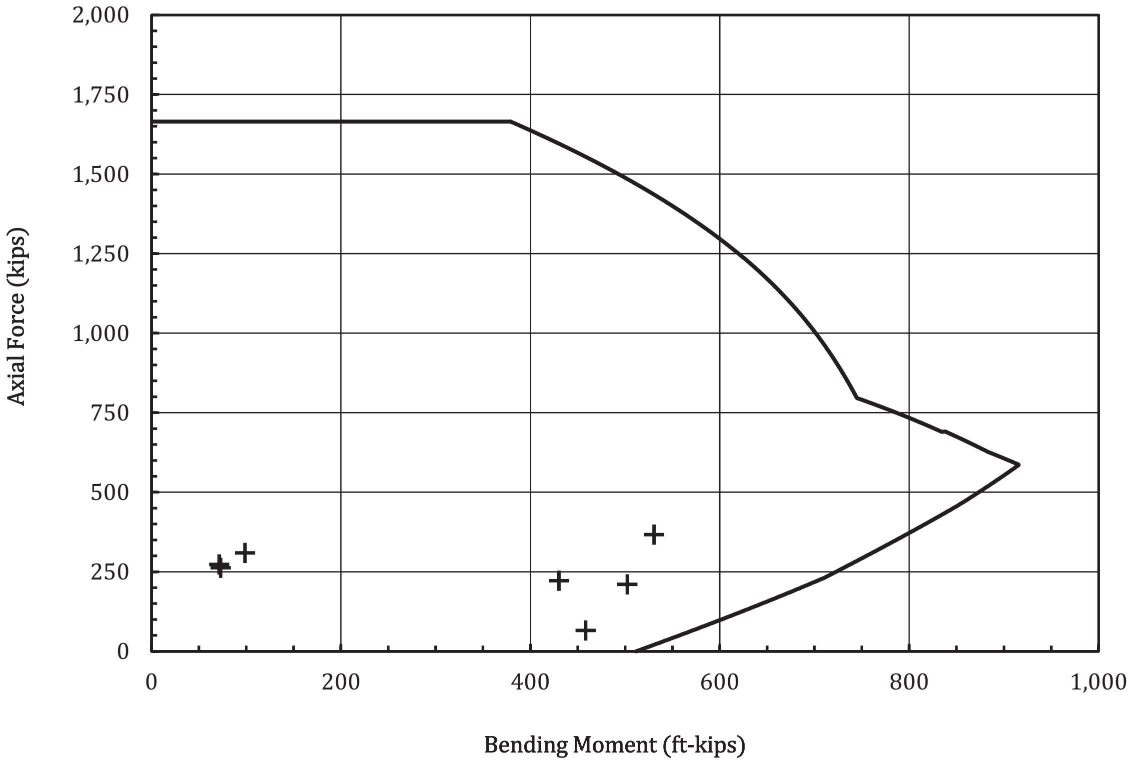


Figure 14.63 Design strength interaction diagram for column E1 in Example 14.7.

Try 12-#8 bars.

Note that orthogonal load effects in accordance with ASCE/SEI 12.5.4 need not be considered based on the 12-#8 longitudinal bars in the column.

Step 4 – Check the minimum flexural strength requirements ACI 18.7.3

The minimum flexural strength requirements of ACI 18.7.3 are checked for this column in the east-west direction. The nominal flexural strengths of the columns and beam are determined in accordance with ACI 18.7.3.2. Only load combinations that include earthquake effects need to be considered when checking ACI Eq. (18.7.3.2).

The flexural reinforcement in the beam framing into the joint in the direction of analysis is the following:

At column line 1: 5-#7 bars

At midspan: 4-#7 bars

At column line 2: 6-#7 bars

The negative flexural strength of the beam must include the contribution of the reinforcement in the slab within the effective slab width defined in ACI 6.3.2 (the slab is in tension). The effective slab width in accordance with ACI 6.3.2 is equal to the following:

$$b_e = \text{least of} \begin{cases} 6h + b_w = (6 \times 7.0) + 28.0 = 70.0 \text{ in.} \\ (s_w / 2) + b_w = (21.17 \times 12 / 2) + 28.0 = 155.0 \text{ in.} \\ (\ell_n / 12) + b_w = (22.67 \times 12 / 12) + 28.0 = 50.7 \text{ in.} \end{cases}$$ Figure 14.13

The beam used in determining the negative flexural strength is given in Figure 14.64.

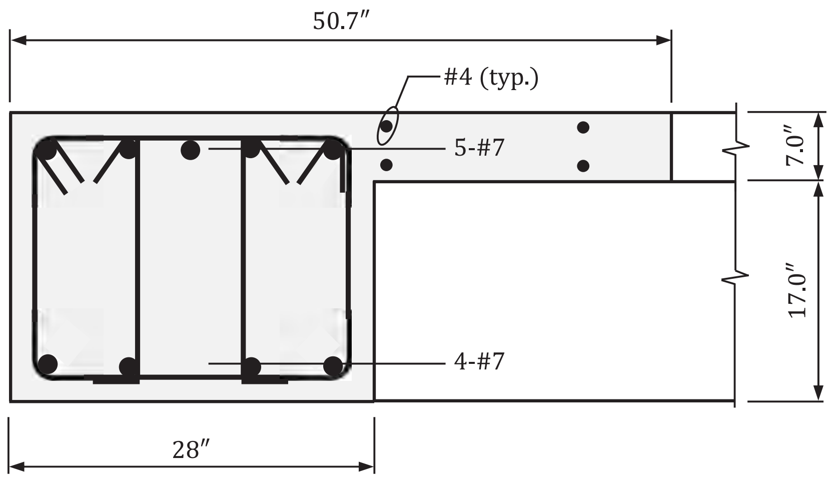


Figure 14.64 Beam framing into the corner joint at column E1 in Example 14.7.

A strain compatibility analysis of the section with the 50.7-in. effective width, the 5-#7 top bars in the beam, and the #4 bars in the slab yields $M_{nb}^{-} = 370.0$ ft-kips.

The nominal flexural strength of the column is determined for the factored axial compressive force resulting in the lowest flexural strength, consistent with the direction of analysis. For the column below the joint, the axial forces in Table 14.17 are applicable. These forces are indicated in Figure 14.65, which contains the nominal strength interaction diagram for this column.

The factored axial compressive forces and corresponding bending moments for the load combinations with earthquake effects are also indicated in Figure 14.65 for the column above the joint where it is assumed the column above the joint has the same cross-section, concrete compressive strength, and longitudinal reinforcement as the column below the joint.

From Figure 14.65, the minimum flexural strengths for the columns above and below the joint are equal to the following:

$$M_{nc(top)} = 619.5 \text{ ft-kips}$$

$$M_{nc(bot)} = 628.8 \text{ ft-kips}$$

Check ACI Eq. (18.7.2.3): Figure 14.11

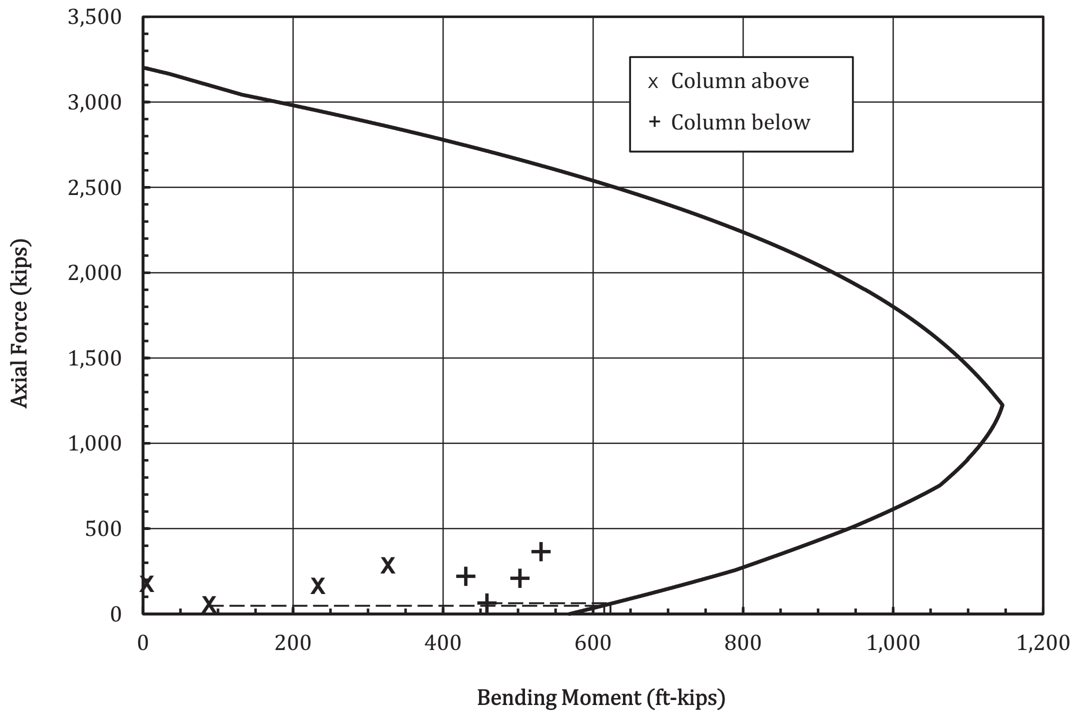


Figure 14.65 Nominal strength interaction diagram for column E1 in Example 14.7.

$$\Sigma M_{nc} = 619.5 + 628.8 = 1,248.3 \text{ ft-kips} > \frac{6}{5}\Sigma M_{nb} = \frac{6}{5} \times 370.0 = 444.0 \text{ ft-kips}$$

Therefore, the minimum flexural strength requirements of ACI 18.7.3 are satisfied.

Step 5 – Check the development length requirements of ACI 18.7.4.3

Unsupported length of column: $\ell_u = 14.0 - (24.0 / 24) = 13.0$ ft

Determine the tension development length, ℓ_d, of the #8 longitudinal bars:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #8 reinforcing bars, $\psi_s = 1.0$

$\psi_t = 1.0$

Assuming #4 hoops:

$$c_b = \text{ lesser of } \begin{cases} c_c + (d_b)_{hoop} + 0.5(d_b)_{long.} = 1.5 + 0.5 + (0.5 \times 1.0) = 2.5 \text{ in.} \\ \dfrac{s}{2} = \dfrac{28.0 - [2 \times (1.5 + 0.5)] - 1.0}{2 \times 3} = 3.8 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.5 + 0) / 1.0 = 2.5$

Therefore,a

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{4{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} \right) \times 1.0 = 28.5 \text{ in.} = 2.4 \text{ ft}$$

$\ell_u = 13.0 \text{ ft} > 2.5\ell_d = 2.5 \times 2.4 = 6.0 \text{ ft}$ Figure 14.14

Therefore, the requirements of ACI 18.7.4.3 are satisfied.

Use 12-#8 bars.

Comments. The 12-#8 longitudinal bars are also adequate for biaxial bending of the column due to wind forces.

14.9.8 Example 14.8 – Determination of Transverse Reinforcement: Corner Column in Building #1 (Framing Option C), Column is Part of the SFRS (Special Moment Frame), SDC D

Determine the required transverse reinforcement for column E1 in Building #1, Framing Option C, in the first story assuming the column is part of the SFRS and the dimensions of the column are 28 in. by 28 in. (see Figure 1.1). The slab is 7.0 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f'_c = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Example 14.7.

Step 1 – Determine the length over which flexural yielding is likely to occur ACI 18.7.5.1

$$\ell_o \geq \text{greatest of} \begin{cases} \text{Maximum cross-sectional dimension of the column} = 28.0 \text{ in.} \\ \text{Clear span of the column}/6 = [(14.0 \times 12) - 12.0] / 6 = 26.0 \text{ in.} \\ 18 \text{ in.} \end{cases}$$ Eq. (14.15)

Step 2 – Check if the requirements of ACI 18.7.5.2(f) must be satisfied

The largest factored axial force including earthquake effects is equal to 367.6 kips. Table 14.17

$0.3 f'_c A_g = 0.3 \times 4 \times 28^2 = 940.8 \text{ kips} > 367.6 \text{ kips}$

Also, $f'_c = 4{,}000 \text{ psi} < 10{,}000 \text{ psi}$.

Therefore, the requirements of ACI 18.7.5.2(f) need not be satisfied.

Step 3 – Determine the maximum hoop spacing based on the requirements of ACI 18.7.5.3

The maximum hoop spacing within ℓ_o from each end of the column is equal to the following:

$$s = \text{smaller of} \begin{cases} \text{Minimum column dimension} / 4 = 28.0 / 4 = 7.0 \text{ in.} \\ \text{For Grade 60 longitudinal bars, } 6d_b = 6 \times 1.0 = 6.0 \text{ in.} \\ s_o = 4 + [(14 - h_x) / 3] = 4 + [(14 - 7.7) / 3] = 6.1 \text{ in.} > 6.0 \text{ in., use } 6.0 \text{ in.} \end{cases}$$ Figure 14.17

In the equation for s_o, the maximum center-to-center spacing of the laterally supported longitudinal bars, h_x, is determined as follows assuming #4 hoops with #4 crossties around all the longitudinal bars:

$$h_x = \frac{28.0 - [2 \times (1.5 + 0.5)] - 1.0}{3} = 7.7 \text{ in.}$$

Step 4 – Determine the minimum area of hoop reinforcement based on the requirements of ACI 18.7.5.4

In both directions, $b_c = 28.0 - (2 \times 1.5) = 25.0$ in.

$$A_{ch} = 25.0 \times 25.0 = 625.0 \text{ in.}^2$$

$$A_{sh} = \text{larger of} \begin{cases} 0.3sb_c\left(\dfrac{A_g}{A_{ch}} - 1\right)\dfrac{f_c'}{f_{yt}} = 0.3 \times 6.0 \times 25.0 \times \left(\dfrac{28.0^2}{625.0} - 1\right) \times \dfrac{4}{60} = 0.76 \text{ in.}^2 \\ 0.09sb_c\dfrac{f_c'}{f_{yt}} = 0.09 \times 6.0 \times 25.0 \times \dfrac{4}{60} = 0.90 \text{ in.}^2 \end{cases}$$ Figure 14.17

#4 hoops with two #4 crossties spaced at 6.0 in. provides $A_{sh} = 4 \times 0.20 = 0.80 \text{ in.}^2 < 0.90 \text{ in.}^2$

#5 hoops with two #5 crossties spaced at 6.0 in. provides $A_{sh} = 4 \times 0.31 = 1.24 \text{ in.}^2 > 0.90 \text{ in.}^2$

Try #5 hoops and crossties spaced at 6.0 in. on center within ℓ_o (note: recalculating s_o with #5 hoops and crossties instead of #4 hoops and crossties results in the same value for s_o).

Step 5 – Check if the size and spacing of the hoop reinforcement determined in Step 4 is adequate for shear

Design shear force, V_u, is determined using the probable flexural strengths, M_{pr}, at the ends of the column associated with the range of factored axial forces acting on the column (ACI 18.7.6.1). The design strength interaction diagram for this column with $f_y = 1.25 \times 60 = 75$ ksi and $\phi = 1.0$ is given in Figure 14.66. It is evident from the figure that the largest M_{pr} is equal to 957.1 ft-kips, which corresponds to an axial force of 367.6 kips for sidesway to the left.

Therefore,

$$V_u = \frac{2M_{pr}}{\ell_u} = \frac{2 \times 957.1}{14.0 - 1.0} = 147.3 \text{ kips}$$ Figure 14.15

This shear force is greater than the maximum shear force of 50.8 kips determined from analysis (see Table 14.17).

The earthquake-induced shear force is the total shear force on the column. Also, the minimum factored axial force including earthquake effects is equal to 66.6 kips, which is less than $A_g f_c' / 20 = 156.8$ kips. Therefore, V_c must be taken equal to zero (ACI 18.7.6.2.1).

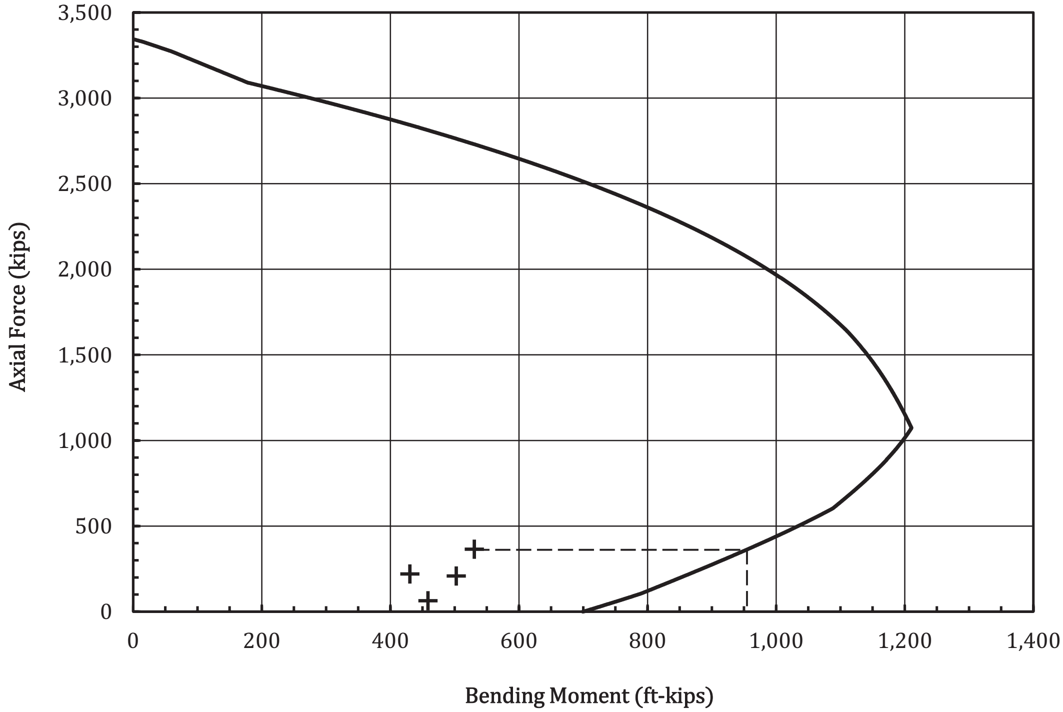


Figure 14.66 Nominal strength interaction diagram for column E1 with $f_y = 75$ ksi and $\phi = 1.0$.

The required spacing of the #5 hoops and crossties in the direction of analysis is the following:

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (4 \times 0.31) \times 60 \times 19.7}{147.3 - 0} = 7.5 \text{ in.} > 6.0 \text{ in.}$$

where $d = 19.7$ in. is determined from a strain compatibility analysis using the axial load equal to 367.6 kips, which is the axial load used in determining the shear force on the column.

Thus, the size and spacing of the hoops determined in Step 4 are adequate for shear.

Use #5 hoops with two crossties spaced at 6.0 in. on center over a distance of at least $\ell_o = 28.0$ in. from each end of the column.

Step 6 – Determine the size and spacing of the hoop reinforcement in the remainder of the column

Outside of ℓ_o, the spacing of the hoops is equal to the following:

$$s = \text{smaller of} \begin{cases} 6 \text{ in.} \\ \text{For Grade 60 longitudinal bars, } 6d_b = 6 \times 1.0 = 6.0 \text{ in.} \end{cases}$$ Figure 14.17

The same transverse reinforcement required within ℓ_o is used over the entire length of the column, including the segment over the lap splice length.

A Class B lap splice must be used. The length of the lap splice $= 1.3\ell_d = 1.3 \times 2.4 = 3.1$ ft (see Example 14.7 for the calculation of ℓ_d).

Use a 3 ft-2 in. lap splice length, which must be located near the mid-height of the column.

Reinforcement details for this column are similar to those in Figure 14.61.

Comments. The transverse reinforcement determined above is also adequate for biaxial shear due to wind forces (see ACI 22.5.1.10).

14.9.9 Example 14.9 – Check of Joint Shear Strength: Corner Column in Building #1 (Framing Option C), Column is Part of the SFRS (Special Moment Frame), SDC D

Check the joint shear strength in the east-west direction for column E1 in Building #1, Framing Option C, at the second-floor level assuming the column is part of the SFRS and the dimensions of the column are 28 in. by 28 in. (see Figure 1.1). The slab is 7.0 in. thick and the beams are 28.0 in. by 24.0 in. Also assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.1. See Examples 14.7 and 14.8.

Step 1 – Determine the factored horizontal joint shear force — ACI 18.8.4.1

The joint shear force is determined based on 5-#7 negative reinforcing bars (see Example 14.7) and conservatively assuming $V_{col} = 0$:

$$V_u = 1.25A_s^- f_y - V_{col} = [1.25\times(5\times 0.6)\times 60] - 0 = 225.0 \text{ kips} \quad \text{Eq. (14.24)}$$

Step 2 – Determine the design shear strength of the joint

Columns frame into the bottom and top of the joint (that is, the column is continuous). The beam in the direction of analysis provides confinement on that face of the joint. There is a beam in the transverse direction that is the same width as the joint, which provides confinement on that face of the joint. Therefore, the joint type is I-D (see Figure 14.24).

Because the beam in the direction of analysis is the same the width as the column, the effective cross-sectional area within the joint is equal to the area of the column:

$$A_j = c_1 \times c_2 = 28.0\times 28.0 = 784.0 \text{ in.}^2 \quad \text{Figure 14.25}$$

The design shear strength of the joint is equal to the following for a Type I-D joint:

$$\phi V_n = \phi 12\lambda\sqrt{f_c'}A_j = 0.85\times 12\times 1.0\times\sqrt{4{,}000}\times 784.0/1{,}000 = 505.8 \text{ kips} > V_u = 225.0 \text{ kips} \quad \text{Figure 14.24}$$

Step 3 – Determine the detailing for the joint

Because the joint is confined on only two faces, the same transverse reinforcement provided at the ends of the column must be provided in the joint (ACI 18.8.3.1).

14.9.10 Example 14.10 – Design of Special Structural Wall: Building #3, Wall is Part of the SFRS (Building Frame System), SDC D, Displacement-Based Approach

Design the wall on column line 2 in Building #3 in the first story assuming the wall is part of the SFRS and the wall thickness is 24 in. with 36 in. by 36 in. enlarged segments at each end (see Figure 1.3). The slab is 8.0 in. thick. Also assume normalweight concrete with $f_c' = 7{,}500$ psi and ASTM A706 Grade 80 reinforcement.

Design data are given in Sect. 1.2.3. The building is assigned to SDC D based on the following:

- Site Class D (determined)
- $S_S = 1.386,\ S_1 = 0.483$
- $S_{DS} = 0.924,\ S_{D1} = 0.586$

Step 1 – Determine the factored load combinations

A three-dimensional analysis was performed for seismic forces in the north-south direction where the walls on column lines 2 and 4 are part of the SFRS. In the east-west direction, the walls on column lines A and D are part of the SFRS. Thus, in both directions the SFRS is a building frame system. Rigid diaphragms are assigned at each level and to account for cracking, 35 percent of the gross moment of inertia is used for the walls [see ACI Table 6.6.3.1.1(a)].

The gravity and earthquake axial forces, bending moments, and shear forces at the base of the wall are given in Table 14.18 (the critical section for this wall occurs at the base). The "plus-minus" sign preceding the tabulated seismic bending moment and shear force signifies the earthquake loads can act in both the north direction and the south direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the east elevation of the building). The effects due to wind loads are smaller than those due to the earthquake loads and are not considered in this example.

Table 14.18 Design Axial Forces, Bending Moments, and Shear Forces for the Wall in Example 14.10

Load Case			Axial Force (kips)	Bending Moment (ft-kips)	Shear Force (kips)
Dead (D)			2,584.1	0	0
Live (L)			208.5	0	0
Roof live load (L_r)			17.4		
Seismic (Q_E)			0	$\pm 75,571$	± 812.5
Load Combination					
ACI Eq. (5.3.1a)	$1.4D$		3,617.7	0	0
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		3,443.2	0	0
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		3,233.0	0	0
ACI Eq. (5.3.1e)	$1.39D + 0.5L + 1.3Q_E$	SSR	3,696.2	−98,242	−1,056.3
		SSL	3,696.2	98,242	1,056.3
ACI Eq. (5.3.1g)	$0.71D + 1.3Q_E$	SSR	1,834.7	$-98,242$	$-1,056.3$
		SSL	1,834.7	98,242	1,056.3

SSR = sidesway to the right, SSL = sidesway to the left

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Equation (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = 1.3Q_E + (0.2 \times 0.924)D = 1.3Q_E + 0.19D$ where $\rho = 1.3$ in accordance with ASCE/SEI 12.3.4.2. Therefore, this load combination becomes the following:

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + (1.3Q_E + 0.19D) = 1.39D + 0.5L + 1.3Q_E$$

Similarly, in ACI Equation (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E + 0.2S_{DS}D = 1.3Q_E - (0.2 \times 0.924)D = 1.3Q_E - 0.19D$, and this load combination becomes the following:

$$0.9D + 1.0E = 0.9D + (1.3Q_E - 0.19D) = 0.71D + 1.3Q_E$$

Step 2 – Determine the minimum reinforcement ratios for the web ACI 18.10.2.1

$$V_u = 1{,}056.3 \text{ kips} > \lambda\sqrt{f_c'}A_{cv} = 1.0 \times \sqrt{7{,}500} \times [24.0 \times (23.5 \times 12)] / 1{,}000 = 586.1 \text{ kips}$$

Therefore, minimum $\rho_\ell = \rho_t = 0.0025$.

Step 3 – Determine the number of curtains of reinforcement in the web ACI 18.10.2.2

$$h_w / \ell_w = 152.0 / 23.5 = 6.5 > 2.0$$

Therefore, 2 curtains of longitudinal and transverse reinforcement are required in the web.

Step 4 – Determine the required area of longitudinal reinforcement for flexure and axial forces ACI 18.10.5

The design strength interaction diagram for this wall is given in Figure 14.67. The longitudinal reinforcement in the web is 2-#7 bars spaced at 9.0 in. on center and the longitudinal reinforcement in each 36 in. by 36 in. segment is 28-#11 bars. All load combinations fall within the boundary of the design strength interaction diagram.

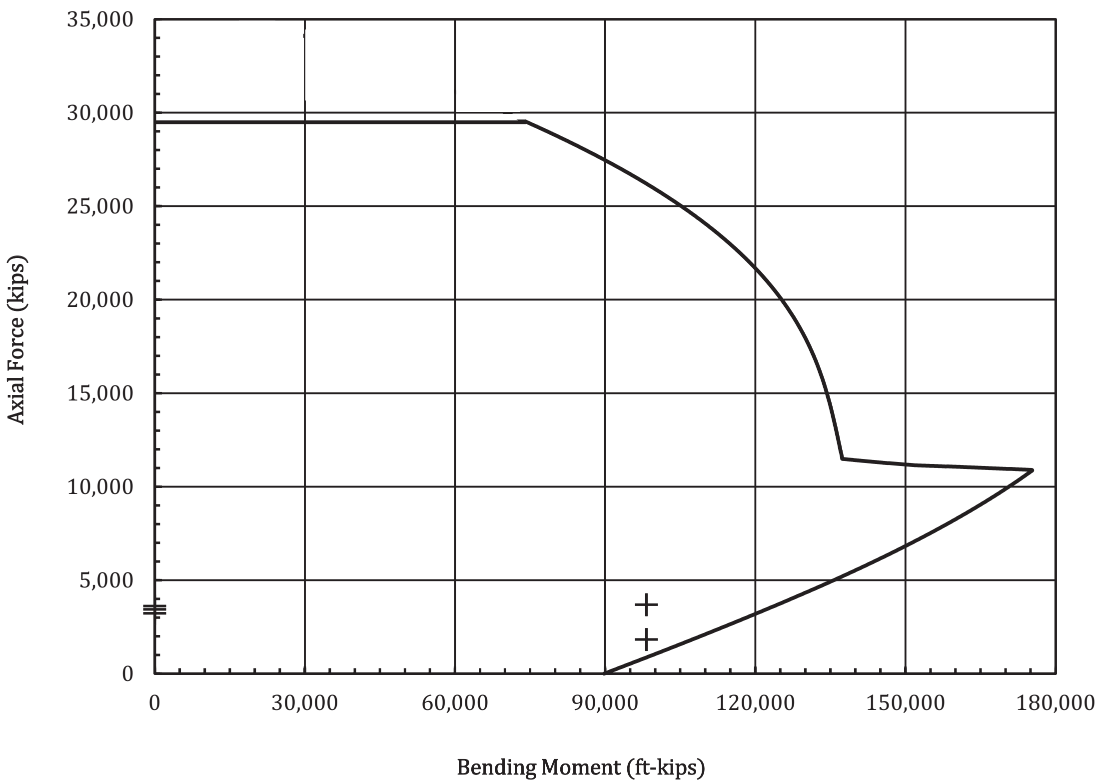


Figure 14.67 Design strength interaction diagram for the wall in Example 14.10.

Check the minimum longitudinal reinforcement requirements of ACI 18.10.2.4(a):

$0.15\ell_w = 0.15 \times 23.5 = 3.5 \text{ ft}$, which is equal to the depth of the 36 in. by 36 in. segments at the ends of the wall

Longitudinal reinforcement ratio within the 36 in. by 36 in. segment is equal to the following:

$$\frac{28 \times 1.56}{36.0 \times 36.0} = 0.0337 > \frac{6\sqrt{f_c'}}{f_y} = \frac{6 \times \sqrt{7{,}500}}{80{,}000} = 0.0065$$ ACI Fig. R18.10.2.4

Therefore, the requirements of ACI 18.10.2.4(a) are satisfied.

Step 5 – Determine the design shear force ACI 18.10.3

The design shear force, V_e, is determined in accordance with ACI 18.10.3.1:

$$V_e = \Omega_v \omega_v V_u \leq 3V_u \quad \text{Eq. (14.27)}$$

Because $h_{wcs} / \ell_w = 6.5 > 1.5$, Ω_v, is equal to the greater of M_{pr} / M_u and 1.5. ACI Table 18.10.3.1.2

The design strength interaction diagram for this wall with $f_y = 1.25 \times 80 = 100$ ksi and $\phi = 1.0$ is given in Figure 14.68. It is evident from the figure that the largest M_{pr} is equal to 143,338 ft-kips, which corresponds to an axial force of 3,696.2 kips (see Table 14.18).

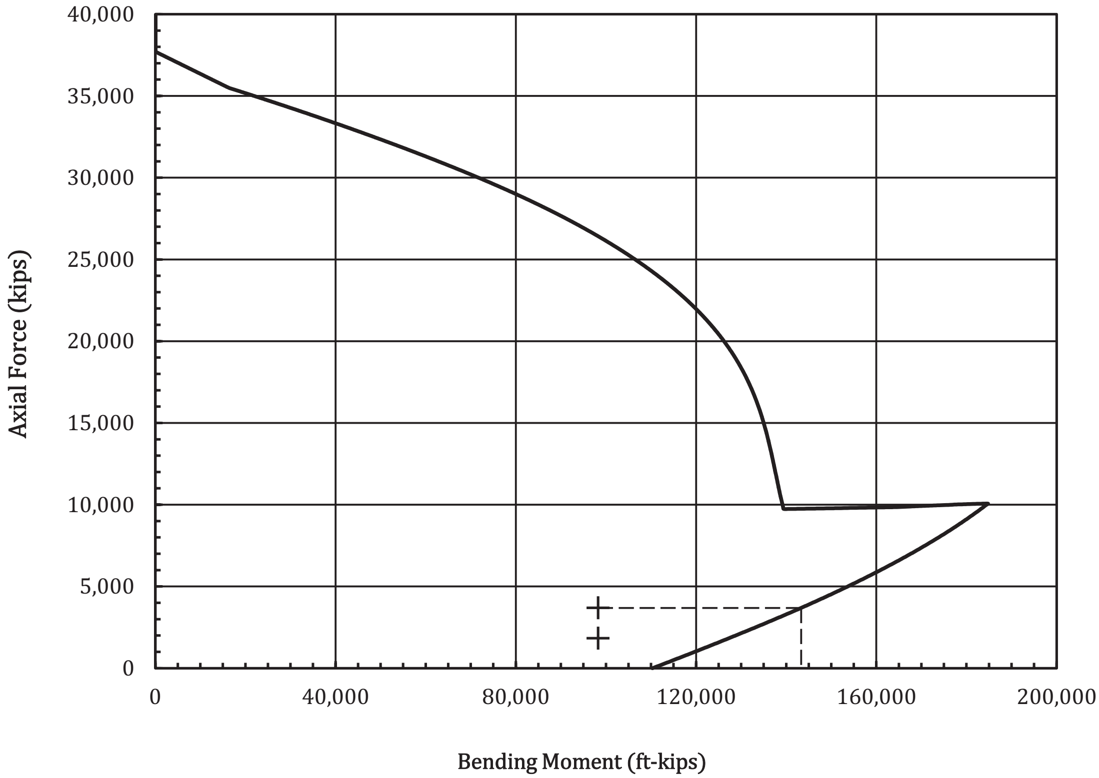


Figure 14.68 Nominal strength interaction diagram for the wall in Example 14.10 with f_y = **100 ksi** and ϕ = **1.0**.

$$M_{pr} / M_u = 143{,}338 / 98{,}242 = 1.46 < 1.5, \text{ use } 1.5$$

For this 16-story building, the dynamic shear amplification factor is equal to the following:

$$\omega_v = 1.3 + \frac{n_s}{30} = 1.3 + \frac{16}{30} = 1.83 > 1.8, \text{ use } 1.8 \quad \text{Eq. (14.29)}$$

Therefore,

$$V_e = 1.5 \times 1.8 \times 1{,}056.3 = 2{,}852.0 \text{ kips} < 3V_u = 3{,}168.9 \text{ kips}$$

Step 6 – Check shear strength requirements ACI 18.10.4

The design shear strength, ϕV_n, must be greater than or equal to V_e.

Determine the required transverse reinforcement ratio using Eqs. (14.30) and (14.31):

$$\phi V_n = \phi(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_{yt}) A_{cv} = 0.75 \times [(2 \times 1.0 \times \sqrt{7{,}500} \,/\, 1{,}000) + (\rho_t \times 80)] \times (24.0 \times 282.0) = V_e = 2{,}852.0 \text{ kips}$$

where $\alpha_c = 2$ for $h_w / \ell_w > 2.0$.

Therefore, required $\rho_t = 0.0049$.

Assuming two layers of #7 horizontal bars in the web, the required spacing is equal to the following:

$$s = \frac{2 \times 0.60}{24.0 \times 0.0049} = 10.2 \text{ in.}$$

Check the upper limit on shear strength based on 2-#7 bars with a center-to-center spacing of 10.0 in.:

$$\phi V_n = 0.75 \times [(2 \times 1.0 \times \sqrt{7{,}500} \,/\, 1{,}000) + (0.0050 \times 80)] \times (24.0 \times 282.0)$$

ACI 18.10.4.4

$$= 2{,}909.6 \text{ kips} < \phi 8 \sqrt{f'_c} A_{cv} = 3{,}516.8 \text{ kips}$$

Use 2-#7 horizontal bars spaced at 10.0 in. on center.

Step 7 – Determine if special boundary elements are required ACI 18.10.6.2

Special boundary elements are required where the following equation is satisfied:

$$\frac{1.5\delta_u}{h_{wcs}} \ge \frac{\ell_w}{600c} \qquad \text{Eq. (14.32)}$$

From the analysis of the structure with the code-prescribed seismic forces applied over the height of the building, the displacement at the top of the building, δ_{xe} is equal to 4.4 in. Therefore,

$$\delta_u = \frac{C_d \delta_{xe}}{I_e} = \frac{5 \times 4.4}{1.0} = 22.0 \text{ in.} \qquad \text{ASCE/SEI Eq. (12.8-15)}$$

where the deflection amplification factor $C_d = 5$ is obtained from ASCE/SEI Table 12.2-1 for a building frame system with special reinforced concrete shear walls.

$$\delta_u / h_{wcs} = (22.0 / 12) / 152.0 = 0.012 > 0.005 \qquad \text{ACI 18.10.6.2(a)}$$

From a strain compatibility analysis of the section, the largest neutral axis depth, c, is equal to 47.5 in., which corresponds to the axial force of 3,696.2 kips obtained from ACI Eq. (5.3.1e) [see Table 14.18].

Thus,

$$\frac{1.5\delta_u}{h_{wcs}} = \frac{1.5 \times (22.0 / 12)}{152.0} = 0.018 > \frac{\ell_w}{600c} = \frac{23.5}{600 \times (47.5 / 12)} = 0.010$$

Therefore, special boundary elements are required at the ends of the wall.

Step 8 – Determine the vertical extent of the special boundary element transverse reinforcement

Provide special boundary element transverse reinforcement vertically over at least the greater of the following lengths from the base of the wall:

$$\ell_w = 23.5 \text{ ft} \qquad \text{ACI 18.10.6.2(b)(i)}$$

$$M_u / 4V_u = 98{,}242 / (4 \times 1{,}056.3) = 23.3 \text{ ft}$$

Step 9 – Check if ACI 18.10.6.2(b)(ii) or (iii) is satisfied

Because b varies over c, an average or representative value of b can be used to determine if these requirements are satisfied or not. Alternatively, check these requirements using b equal to 24.0 in. and 36.0 in.

$$b = 24.0 \text{ in.} > \sqrt{0.025c\ell_w} = \sqrt{0.025 \times 47.5 \times 282.0} = 18.3 \text{ in.}$$ ACI 18.10.6.2(b)(ii)

$$\frac{\delta_c}{h_{wcs}} = \frac{1}{100}\left[4 - \frac{1}{50}\left(\frac{\ell_w}{b}\right)\left(\frac{c}{b}\right) - \frac{V_e}{8\sqrt{f'_c}A_{cv}}\right]$$

$$= \frac{1}{100}\left[4 - \frac{1}{50}\left(\frac{282.0}{24.0}\right)\left(\frac{47.5}{24.0}\right) - \frac{2,852.0}{8\sqrt{7,500} \times 6,768 / 1,000}\right] = 0.029 > 0.015$$ ACI 18.10.6.2(b)(iii)

$$> \frac{1.5\delta_u}{h_{wcs}} = 0.018$$

Therefore, both ACI 18.10.6.2(b)(ii) and (iii) are satisfied for $b = 24.0$ in. It can also be determined that these requirements are satisfied for $b = 36.0$ in., which means the requirements are satisfied for any average value of b.

Step 10 – Determine the horizontal length of the special boundary elements — ACI 18.10.6.4(a)

The special boundary elements must extend horizontally from the extreme compression fiber a distance equal to the following:

$$\ell_{be} = \text{greater of} \begin{cases} c - 0.1\ell_w = 47.5 - (0.1 \times 282.0) = 19.3 \text{ in.} \\ c / 2 = 47.5 / 2 = 23.8 \text{ in.} \end{cases}$$ Figure 14.38

For simpler detailing, provide special boundary element transverse reinforcement within the 36 in. by 36 in. segments at each end of the wall.

Step 11 – Check the width of the flexural compression zone — ACI 18.10.6.4(b) and (c)

$$b = 36.0 \text{ in.} > h_u / 16 = [(9.5 \times 12) - 4.0] / 16 = 6.9 \text{ in.}$$

$$c / \ell_w = 47.5 / 282.0 = 0.17 < 3 / 8$$

Therefore, the requirement of ACI 18.10.6.4(b) is satisfied and the requirement of ACI 18.10.6.4(c) is not applicable.

Step 12 – Determine the required special boundary element transverse reinforcement — ACI 18.10.6.4(e) and (f)

The maximum vertical spacing of the special boundary element transverse reinforcement is equal to the following:

$$s = \text{lesser of} \begin{cases} b / 3 = 36.0 / 3 = 12.0 \text{ in.} \\ \ell_{be} / 3 = 23.8 / 3 = 7.9 \text{ in.} \\ \text{For Grade 80 bars, lesser of } 5d_b = 5 \times 1.41 = 7.1 \text{ in. or } 6 \text{ in.} \\ s_o = 4 + \left(\dfrac{14 - 4.3}{3}\right) = 7.2 \text{ in.} > 6.0 \text{ in., use } 6.0 \text{ in.} \end{cases}$$ Figure 14.38

In the equation for s_o, the maximum center-to-center spacing of the laterally supported longitudinal bars, h_x, is determined as follows assuming #5 hoops with #5 crossties around all the longitudinal bars:

$$h_x = \frac{36.0 - [2 \times (1.5 + 0.625)] - 1.41}{7} = 4.3 \text{ in.}$$

In both directions, $b_c = 36.0 - (2 \times 1.5) = 33.0$ in.

$$A_{ch} = 33.0 \times 33.0 = 1{,}089.0 \text{ in.}^2$$

$$A_{sh} = \text{larger of} \begin{cases} 0.3sb_c\left(\dfrac{A_g}{A_{ch}} - 1\right)\dfrac{f_c'}{f_{yt}} = 0.3 \times 6.0 \times 33.0 \times \left(\dfrac{36.0^2}{1{,}089.0} - 1\right) \times \dfrac{7.5}{80} = 1.06 \text{ in.}^2 \\ \\ 0.09sb_c\dfrac{f_c'}{f_{yt}} = 0.09 \times 6.0 \times 33.0 \times \dfrac{7.5}{80} = 1.67 \text{ in.}^2 \end{cases}$$ ACI Table 18.10.6.4(g)

#5 hoops with six #5 crossties spaced at 6.0 in. provides $A_{sh} = 8 \times 0.31 = 2.48 \text{ in.}^2 > 1.67 \text{ in.}^2$

Use #5 hoops and #5 crossties around all the longitudinal bars spaced at 6.0 in. on center.

Step 13 – Determine if transverse reinforcement required by ACI 18.10.6.5 is needed at the ends of the wall

It can be determined that 28-#10 longitudinal bars in the 36.0 in. by 36.0 in. segments and 2-#7 longitudinal bars spaced at 9.0 in. in the web are adequate for flexure and axial forces at the section immediately above the top of the fourth story (which is above the required vertical extent of the special boundary elements; see Step 8). The reinforcement ratio in the 36.0 in. by 36.0 in. segments [that is, over the distance ℓ_{be} calculated in accordance with ACI 18.10.6.4(a)] is equal to the following:

$$\rho = \frac{A_{sb}}{A_{cb}} = \frac{28 \times 1.27}{36.0^2} = 0.027 > \frac{400}{f_y} = \frac{400}{80{,}000} = 0.005$$ Figure 14.39

Therefore, transverse reinforcement in accordance with ACI 18.10.6.5 is required at the ends of the wall. The size of the transverse reinforcement must satisfy the requirements of ACI 25.7.2.2; thus, use #3 hoops and #3 crossties around every other longitudinal bar in the 36.0 in. by 36.0 in. segments.

Because the section immediately above the top of the fourth story is not expected to yield (that is, this section is located above $\ell_w = 23.5$ ft; see Step 8), the maximum spacing of the transverse reinforcement is equal to the lesser of the following for Grade 80 reinforcement:

$$s = \text{lesser of} \begin{cases} 6d_b = 6 \times 1.27 = 7.6 \text{ in.} \\ \\ 6 \text{ in.} \end{cases}$$ Figure 14.39

At the section immediately above the top of the eighth story, it can be determined that 28-#8 longitudinal bars in the enlarged segments and 2-#5 longitudinal bars spaced at a 9.0 in. in the web are adequate for flexure and axial forces. Assuming the enlarged segments at the end of the wall are 30 in. by 30 in. in stories 9 through 16, the reinforcement ratio in the enlarged segments is equal to the following:

$$\rho = \frac{A_{sb}}{A_{cb}} = \frac{28 \times 0.79}{30.0^2} = 0.025 > \frac{400}{f_y} = \frac{400}{80{,}000} = 0.005$$

Therefore, use #3 hoops and #3 crossties around every other longitudinal bar at a 6.0-in. spacing in stories 9 through 16 [the number of provided #3 crossties satisfies the requirement in ACI 18.10.6.5(b) that the hoops conform to ACI 18.7.5.2(a) through (e) where $\rho > 400 / f_y$].

Step 14 – Detail the reinforcement

It can be determined that the 28-#11 longitudinal bars in the enlarged segments at the ends of the wall and the 2-#7 longitudinal bars in the web are no longer required for flexure and axial forces at a section slightly below the top of the third-floor level. According to ACI 18.10.2.3(a), longitudinal reinforcement must extend at least 12 ft above the point it is no longer required, which in this example, is approximately $19.0 + 12.0 = 31.0\ \text{ft}$ above the base of the wall, or equivalently, 2.5 ft above the top of the fourth-floor level. Also, the longitudinal bars need not extend more than ℓ_d above the next floor level, which is the fourth-floor level. Determine ℓ_d for the #11 longitudinal bars:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 80 reinforcement, $\psi_g = 1.15$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #11 reinforcing bars, $\psi_s = 1.0$

$\psi_t = 1.0$

$$c_b = \text{ lesser of } \begin{cases} c_c + (d_b)_{hoop} + 0.5(d_b)_{long.} = 1.5 + 0.625 + (0.5 \times 1.41) = 2.8 \text{ in.} \\ \\ \dfrac{s}{2} = \dfrac{36.0 - [2 \times (1.5 + 0.625)] - 1.41}{2 \times 7} = 2.2 \text{ in.} \end{cases}$$

Set $K_{tr} = 0.$ ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.2 + 0) / 1.41 = 1.6$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{80{,}000}{1.0\sqrt{7{,}500}} \frac{1.0 \times 1.0 \times 1.0 \times 1.15}{1.6} \right) \times 1.41 = 70.2 \text{ in.} = 5.9 \text{ ft}$$

Thus, the #11 longitudinal bars need not extend more than 2.5 ft above the top of the fourth-floor level.

For simpler detailing, extend the 28-#11 longitudinal bars to the top of the fifth-floor level where they can be spliced immediately above the fifth-floor slab with the 28-#10 longitudinal bars required in the enlarged segments above (see Step 13). The #7 bars in the web can be spliced in a similar fashion.

According to ACI 18.10.2.3(c), lap splices of longitudinal reinforcement within the special boundary regions are not permitted over a height equal to $h_{sx} = 9.5\ \text{ft} < 20\ \text{ft}$ above the critical sections and ℓ_d below the critical sections where yielding is likely to occur. In this example, the critical section occurs at the base of the wall, so the #11 longitudinal bars are permitted to be lap spliced to the #10 longitudinal bars immediately above the fifth-floor slab.

Where bars of different size are lap spliced in tension, the tension lap splice length, ℓ_{st}, is equal to the greater of ℓ_d of the larger bar and ℓ_{st} of the smaller bar (ACI 25.5.2.2). For the #11 bars, $\ell_d = 5.9\ \text{ft}$ (see above). Calculate ℓ_{st} for the #10 bars:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \psi_g}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 80 reinforcement, $\psi_g = 1.15$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #10 reinforcing bars, $\psi_s = 1.0$

$\psi_t = 1.0$

$$c_b = \text{ lesser of } \begin{cases} c_c + (d_b)_{hoop} + 0.5(d_b)_{long.} = 1.5 + 0.375 + (0.5 \times 1.27) = 2.5 \text{ in.} \\ \\ \dfrac{s}{2} = \dfrac{36.0 - [2 \times (1.5 + 0.375)] - 1.27}{2 \times 7} = 2.2 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (2.2 + 0) / 1.27 = 1.7$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{80,000}{1.0\sqrt{7,500}} \frac{1.0 \times 1.0 \times 1.0 \times 1.15}{1.7} \right) \times 1.27 = 59.5 \text{ in.} = 5.0 \text{ ft}$$

For a Class B tension lap splice:

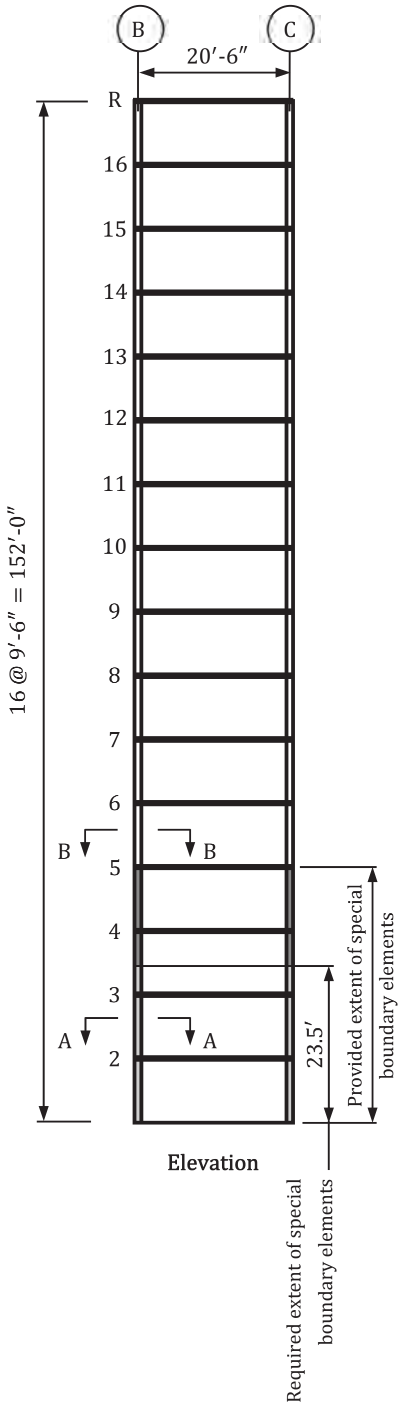
$\ell_{st} = 1.3 \times 5.0 = 6.5 \text{ ft} > 5.9 \text{ ft}$ ACI Table 25.5.2.1

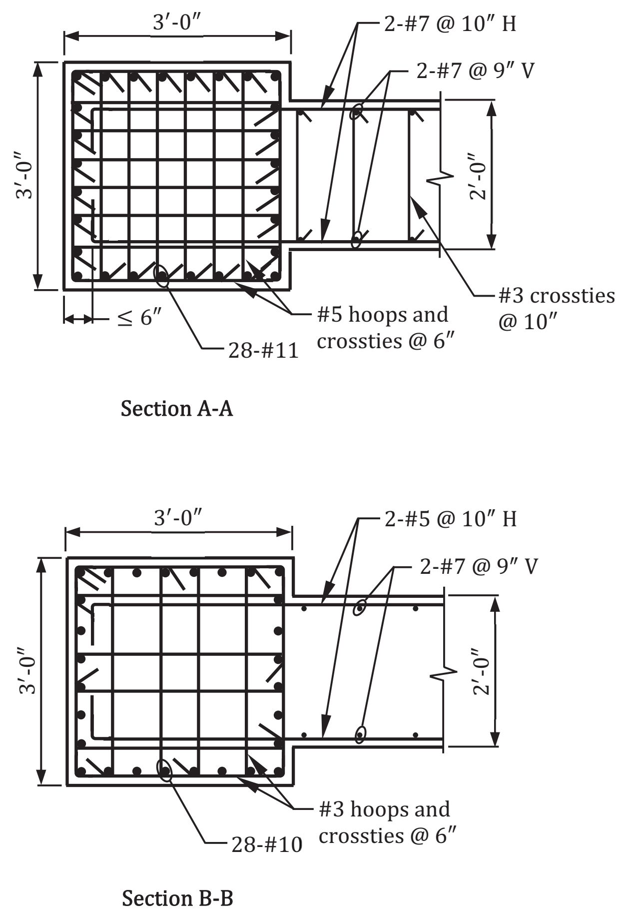
Therefore, provide a 6.5-ft long lap splice immediately above the fifth-floor slab.

The special boundary element longitudinal reinforcement must extend into the support a distance of at least ℓ_d [ACI 18.10.6.4(j)]. The extent of the special boundary element transverse reinforcement into the support depends on the type of support and the distance from the edge of the special boundary element to the edge of the support (see Figure 14.40).

Reinforcement details for this wall are given in Figure 14.69. In accordance with ACI 18.10.6.4(i), the web vertical reinforcement within the vertical extent of the special boundary elements must have lateral support provided by the corner of a hoop or by a crosstie with seismic hooks at each end. In this example, #3 crossties are provided at a 10 in. spacing, which matches the spacing of the horizontal web reinforcement. The #3 crossties satisfy the requirements of ACI 25.7.2.2 for the #7 vertical web bars and the provided spacing is less than the 12-in. maximum spacing specified in ACI 18.10.6.4(i).

Comments. For a construction sequence where (1) a wall is constructed to the underside of the slab, (2) the slab is constructed, and (3) the wall above the slab is constructed, the concrete within the thickness of the slab at the location of the special boundary elements must have a specified compressive strength of at least 0.7 times the specified compressive strength of the wall [ACI 18.10.6.4(h)]. In this example, the slab must have a minimum compressive strength of $0.7 \times 7,500 = 5,250$ psi. For a construction sequence where the wall is constructed ahead of the slab (for example, where slip forms are used to construct the wall), this requirement is not applicable.





Level	Height (ft)	Story	f'_c (psi)	Web	Enlarged Segments
R	152.0	16	5,000	18″ 2-#5 @ 9″ V 2-#5 @ 10″ H	30″ × 30″ 28-#8 #3 hoops and crossties @ 6″
16	142.5	15			
15	133.0	14			
14	123.5	13			
13	114.0	12			
12	104.5	11			
11	95.0	10			
10	85.5	9			
9	76.0	8	7,500	24″ 2-#7 @ 9″ V 2-#5 @ 10″ H	36″ × 36″ 28-#10 #3 hoops and crossties @ 6″
8	66.5	7			
7	57.0	6			
6	47.5	5			
5	38.0	4		24″ 2-#7 @ 9″ V 2-#7 @ 10″ H #3 crossties @ 10″	36″ × 36″ 28-#11 #5 hoops and crossties @ 6″
4	28.5	3			
3	19.0	2			
2	9.5	1			

Figure 14.69 Reinforcement details for the wall in Example 14.10.

14.9.11 Example 14.11 – Design of Special Structural Wall: Building #4, Wall is Part of the SFRS (Dual System), SDC D, Compressive Stress Approach

Design the wall on column line D in Building #4 in the first story assuming the wall is part of the SFRS (see Figure 1.4). The wall thickness is 24.0 in. in stories 1 through 10, 18 in. in stories 11 through 20, and 12 in. in stories 21 through 30. Assume normalweight concrete with $f_c' = 6{,}000\ \text{psi}$ in stories 1 through 10 and $f_c' = 5{,}000\ \text{psi}$ in stories 11 through 30. Also assume ASTM A706 Grade 80 reinforcement.

Design data are given in Sect. 1.2.4. The building is assigned to SDC D based on the following:

- Site Class D (default)
- $S_S = 0.583,\ S_1 = 0.192$
- $S_{DS} = 0.517,\ S_{D1} = 0.284$

Step 1 – Determine the factored load combinations

A three-dimensional analysis was performed for seismic forces in the east-west direction where the walls on column lines C, D, and E and the moment frames on column lines A and G are part of the SFRS. In the north-south direction, the walls on column lines 2 and 3 and the moment frames on column lines 1 and 4 are part of the SFRS. Thus, in both directions the SFRS is a dual system with special structural walls and special moment frames that are capable of independently resisting at least 25 percent of the prescribed seismic forces. Rigid diaphragms are assigned at each level and to account for cracking, 35 percent of the gross moment of inertia is used for the walls and beams and 70 percent for the columns [see ACI Table 6.6.3.1.1(a)].

Prior to determining the factored load combinations, calculate the effective width of the flanges at the ends of the wall in accordance with ACI 18.10.5.2; the effective flange width is used in determining the moment of inertia of the section permitted to resist the earthquake load effects (for determination of gravity load effects, the full wall area must be used). The effective flange width is equal to the lesser of the following:

$$\text{Effective flange width} = \text{lesser of} \begin{cases} s_1 / 2 = [25.0 - (24.0 / 12)] / 2 = 11.5 \text{ ft} \\ \\ h_{w,above} / 4 = 330.0 / 4 = 82.5 \text{ ft} \end{cases}$$ Figure 14.35

Total effective flange width $= 11.5 + (24.0 / 12) + 11.5 = 25.0 \text{ ft} > \text{Flange width} = 18.0 \text{ ft}$

Therefore, the total flange width is effective and may be used to determine the moment of inertia and to resist lateral earthquake effects.

The gravity and earthquake axial forces, bending moments, and shear forces at the base of the wall are given in Table 14.19 (the critical section for this wall occurs at the base). The "plus-minus" sign preceding the tabulated seismic bending moment and shear force signifies the earthquake loads can act in both the east direction and the west direction (that is, sidesway to the right (SSR) and sidesway to the left (SSL) when looking at the south elevation of the building). The effects due to wind loads are smaller than those due to the earthquake loads and are not considered in this example.

Table 14.19 Design Axial Forces, Bending Moments, and Shear Forces for the Wall in Example 14.11

Load Case			Axial Force (kips)	Bending Moment (ft-kips)	Shear Force (kips)
Dead (D)			15,766.1	0	0
Live (L)			4,594.4	0	0
Roof live load (L_r)			47.1		
Seismic (Q_E)			0	$\pm 163{,}801$	± 901.3
Load Combination					
ACI Eq. (5.3.1a)	$1.4D$		22,072.5	0	0
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		26,293.9	0	0
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		21,291.9	0	0
ACI Eq. (5.3.1e)	$1.3D + 0.5L + 1.3Q_E$	SSR	22,793.1	−212,941	−1,171.7
		SSL	22,793.1	212,941	1,171.7
ACI Eq. (5.3.1g)	$0.8D + 1.3Q_E$	SSR	12,612.9	−212,941	−1,171.7
		SSL	12,612.9	212,941	1,171.7

SSR = sidesway to the right, SSL = sidesway to the left

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Equation (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = 1.3Q_E + (0.2 \times 0.517)D = 1.3Q_E + 0.10D$ where $\rho = 1.3$ in accordance with ASCE/SEI 12.3.4.2. Therefore, this load combination becomes the following:

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + (1.3Q_E + 0.10D) = 1.3D + 0.5L + 1.3Q_E$$

Similarly, in ACI Equation (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E + 0.2S_{DS}D = 1.3Q_E - (0.2 \times 0.517)D = 1.3Q_E - 0.10D$, and this load combination becomes the following:

$$0.9D + 1.0E = 0.9D + (1.3Q_E - 0.10D) = 0.8D + 1.3Q_E$$

Step 2 – Determine the minimum reinforcement ratios for the web ACI 18.10.2.1

$$V_u = 1{,}171.7 \text{ kips} > \lambda\sqrt{f_c'}A_{cv} = 1.0 \times \sqrt{6{,}000} \times [24.0 \times (32.0 \times 12)] / 1{,}000 = 713.9 \text{ kips}$$

Therefore, minimum $\rho_\ell = \rho_t = 0.0025$.

Step 3 – Determine the number of curtains of reinforcement in the web ACI 18.10.2.2

$$h_w / \ell_w = 330.0 / 32.0 = 10.3 > 2.0$$

Therefore, 2 curtains of longitudinal and transverse reinforcement are required in the web.

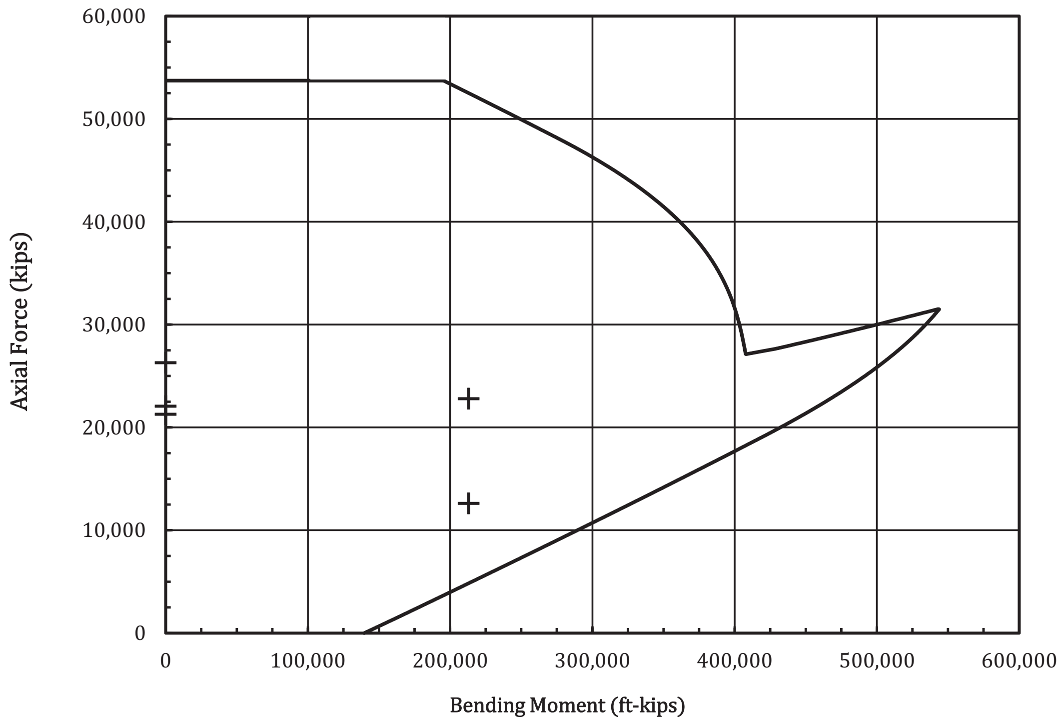


Figure 14.70 Design strength interaction diagram for the wall in Example 14.11.

Step 4 – Determine the required area of longitudinal reinforcement for flexure and axial force ACI 18.10.5

The design strength interaction diagram for this wall is given in Figure 14.70. The longitudinal reinforcement consists of 44-#8 bars in each flange and 2-#8 bars spaced at 10.0 in. on center in the web. Note that 4-#8 bars are required at the mid-depth of the flange (two bars each end) in order to satisfy the maximum spacing requirement in ACI 18.10.6.4(f) for bars around the perimeter of the special boundary element (the maximum spacing is the lesser of 14 in. and two-thirds the boundary element thickness, which is equal to 16.0 in.; see Steps 7 and 11 below). All load combinations fall within the boundary of the design strength interaction diagram.

Check the minimum longitudinal reinforcement requirements of ACI 18.10.2.4 (see Figure 14.71).

$$0.15\ell_w = 0.15 \times 18.0 = 2.7 \text{ ft}$$ ACI Figure R18.10.2.4

There are 7-#8 bars within the 2.7-ft width at each end of both flanges; therefore,

$$\frac{7 \times 0.79}{24.0 \times (2.7 \times 12)} = 0.0071 > \frac{6\sqrt{f_c'}}{f_y} = \frac{6 \times \sqrt{6,000}}{80,000} = 0.0058$$

There are 6-#8 bars within the 2.7-ft width in the web; therefore,

$$\frac{6 \times 0.79}{24.0 \times (2.7 \times 12)} = 0.0061 > \frac{6\sqrt{f_c'}}{f_y} = \frac{6 \times \sqrt{6,000}}{80,000} = 0.0058$$

Thus, the requirements of ACI 18.10.2.4 are satisfied for this wall.

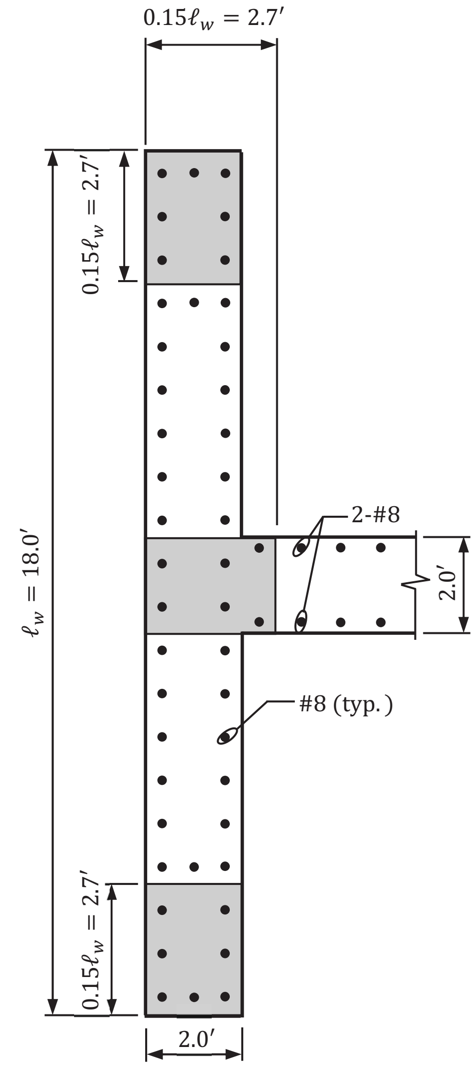


Figure 14.71 Minimum longitudinal reinforcement in accordance with ACI 18.10.2.4 (see Figure 14.71).

Step 5 – Determine the design shear force ACI 18.10.3

The design shear force, V_e, is determined in accordance with ACI 18.10.3.1:

$$V_e = \Omega_v \omega_v V_u \leq 3V_u \quad \text{Eq. (14.27)}$$

Because $h_{wcs} / \ell_w = 10.3 > 1.5$, Ω_v, is equal to the greater of M_{pr} / M_u and 1.5. ACI Table 18.10.3.1.2

The design strength interaction diagram for this wall with $f_y = 1.25 \times 80 = 100$ ksi and $\phi = 1.0$ is given in Figure 14.72. It is evident from the figure that the largest M_{pr} is equal to 520,152 ft-kips, which corresponds to an axial force of 22,793.1 kips (see Table 14.19).

$$M_{pr} / M_u = 520{,}152 / 212{,}941 = 2.4 > 1.5$$

For this 30-story building, the dynamic shear amplification factor is equal to the following:

$$\omega_v = 1.3 + \frac{n_s}{30} = 1.3 + \frac{30}{30} = 2.3 > 1.8, \text{ use } 1.8 \quad \text{Eq. (14.29)}$$

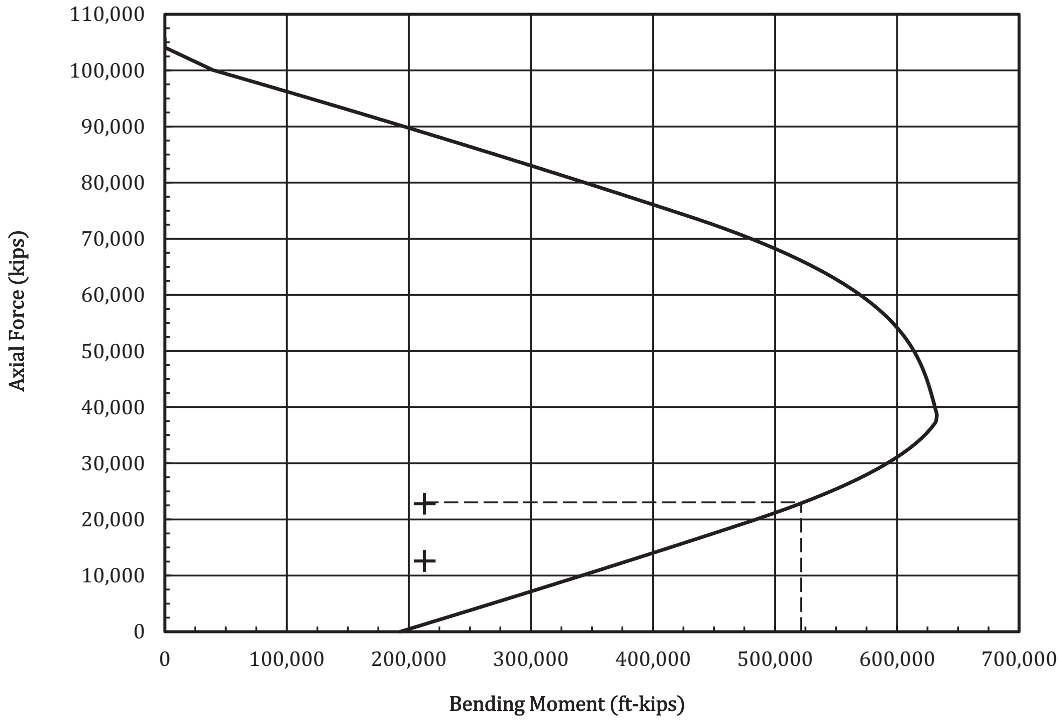


Figure 14.72 Nominal strength interaction diagram for the wall in Example 14.11 with $f_y = 100$ ksi and $\phi = 1.0$.

Therefore,

$V_e = 2.4 \times 1.8 \times 1{,}171.7 = 5{,}061.7 \text{ kips} > 3V_u = 3{,}515.1 \text{ kips, use } 3{,}515.1 \text{ kips}$

Step 6 – Check shear strength requirements ACI 18.10.4

The design shear strength, ϕV_n, must be greater than or equal to V_e.

Determine the required transverse reinforcement ratio using Eqs. (14.30) and (14.31):

$$\phi V_n = \phi(\alpha_c \lambda \sqrt{f_c'} + \rho_t f_{yt}) A_{cv} = 0.75 \times [(2 \times 1.0 \times \sqrt{6{,}000} / 1{,}000) + (\rho_t \times 80)] \times (24.0 \times 384.0) = V_e = 3{,}515.1 \text{ kips}$$

where $\alpha_c = 2$ for $h_w / \ell_w > 2.0$.

Therefore, required $\rho_t = 0.0044$.

Assuming two layers of #7 horizontal bars in the web, the required spacing is equal to the following:

$$s = \frac{2 \times 0.60}{24.0 \times 0.0044} = 11.4 \text{ in.}$$

Check the upper limit on shear strength based on 2-#7 bars with a center-to-center spacing of 11.0 in.:

$$\phi V_n = 0.75 \times [(2 \times 1.0 \times \sqrt{6{,}000} / 1{,}000) + (0.0045 \times 80)] \times (24.0 \times 384.0)$$

ACI 18.10.4.4

$$= 3{,}559.1 \text{ kips} < \phi 8 \sqrt{f_c'} A_{cv} = 4{,}283.2 \text{ kips}$$

Use 2-#7 horizontal bars spaced at 11.0 in. on center.

Step 7 – Determine if special boundary elements are required ACI 18.10.6.3

Special boundary elements are required where the following equation is satisfied:

$$f_{cu} = \frac{P_u}{A_g} + \frac{M_u \ell_w}{2I_g} > 0.2f_c' \quad \text{Eq. (14.33)}$$

From Table 14.19, the load combination that includes earthquake effects which results in the largest f_{cu} is ACI Eq. (5.3.1e):

$$f_{cu} = \frac{22,793.1 \times 1,000}{18,432} + \frac{212,941 \times 12,000 \times 384.0}{2 \times 412,286,976} = 1,236.6 + 1,190.0 = 2,426.6 \text{ psi} > 0.2f_c' = 1,200 \text{ psi}$$

Therefore, special boundary elements are required at the ends of the wall.

Step 8 – Determine the vertical extent of the special boundary element transverse reinforcement

Special boundary elements are permitted to be discontinued where the calculated $f_{cu} < 0.15f_c'$.

It can be determined that special boundary elements can be terminated above the twenty-third-floor level (the wall thickness is 12.0 in. and the compressive strength of the concrete is equal to 4,000 psi at this elevation, which makes the stress limit equal to $0.15f_c' = 600$ psi).

Step 9 – Determine the horizontal length of the special boundary elements ACI 18.10.6.4(a)

The special boundary elements must extend horizontally from the extreme compression fiber a distance equal to the following:

$$\ell_{be} = \text{greater of} \begin{cases} c - 0.1\ell_w = 61.8 - (0.1 \times 384.0) = 23.4 \text{ in.} \\ \\ c/2 = 61.8/2 = 30.9 \text{ in.} \end{cases} \quad \text{Figure 14.38}$$

The largest neutral axis depth, c, used in the determination of ℓ_{be} is calculated from a strain compatibility analysis using the axial force and nominal moment strength from ACI Eq. (5.3.1e) in Table 14.19.

It is evident that the calculated horizontal length of the special boundary element includes the 24.0-in.-thick flange and extends $30.9 - 24.0 = 6.9$ in. into the web. According to ACI 18.10.6.4(d), a special boundary element must extend at least 12 in. into the web for flanged sections. Based on the 10.0-in spacing of the #8 longitudinal bars in the web, use a 24-in. extension into the web.

Step 10 – Check the width of the flexural compression zone ACI 18.10.6.4(b) and (c)

$$b = \begin{cases} 216.0 \text{ in.} \\ \\ 24.0 \text{ in.} \end{cases} > h_u/16 = [(11.0 \times 12) - 28.5]/16 = 6.5 \text{ in.}$$

$$c/\ell_w = 61.8/384.0 = 0.16 < 3/8$$

Therefore, the requirement of ACI 18.10.6.4(b) is satisfied and the requirement of ACI 18.10.6.4(c) is not applicable.

Step 11 – Determine the required special boundary element transverse reinforcement ACI 18.10.6.4(e) and (f)

- Confinement of the flange

  The maximum vertical spacing of the special boundary element transverse reinforcement in the flange is equal to the following:

$$s = \text{ lesser of } \begin{cases} b\ /\ 3 = 24.0\ /\ 3 = 8.0 \text{ in.} \\ \\ \text{For Grade 80 bars, lesser of } 5d_b = 5 \times 1.0 = 5.0 \text{ in. or 6 in.} \\ \\ s_o = 4 + \left(\dfrac{14 - 11.3}{3} \right) = 4.9 \text{ in.} \end{cases}$$

Figure 14.38

In the equation for s_o, the maximum center-to-center spacing of the laterally supported longitudinal bars, h_x, is equal to approximately 11.3 in. assuming #5 transverse reinforcement confining all the longitudinal bars in the flange.

$b_{c,1} = 216.0 - (2 \times 0.75) = 214.5$ in.

$b_{c,2} = 24.0 - (2 \times 0.75) = 22.5$ in.

$A_{ch} = 214.5 \times 22.5 = 4{,}826.3$ in.^{2}

For confinement of the flange in the direction of analysis, $b_c = b_{c,1} = 214.5$ in.

$$A_{sh} = \text{larger of } \begin{cases} 0.3sb_c \left(\dfrac{A_g}{A_{ch}} - 1 \right) \dfrac{f'_c}{f_{yt}} = 0.3 \times 4.0 \times 214.5 \times \left(\dfrac{24.0 \times 216.0}{4{,}826.3} - 1 \right) \times \dfrac{6}{80} = 1.43 \text{ in.}^2 \\ \\ 0.09sb_c \dfrac{f'_c}{f_{yt}} = 0.09 \times 4.0 \times 214.5 \times \dfrac{6}{80} = 5.79 \text{ in.}^2 \end{cases}$$

ACI Table 18.10.6.4(g)

With #5 overlapping hoops and #5 crossties engaging all the longitudinal reinforcement in the flange, provided $A_{sh} = 20 \times 0.31 = 6.20 \text{ in.}^2 > 5.79 \text{ in.}^2$

Use #5 hoops and #5 crossties around all the longitudinal bars spaced at 4.0 in. on center in the flanges.

- Confinement of the web

  The maximum vertical spacing of the special boundary element transverse reinforcement in the web is equal to the following:

$$s = \text{ lesser of } \begin{cases} b\ /\ 3 = 24.0\ /\ 3 = 8.0 \text{ in.} \\ \\ \text{For Grade 80 bars, lesser of } 5d_b = 5 \times 1.0 = 5.0 \text{ in. or 6 in.} \\ \\ s_o = 4 + \left(\dfrac{14 - 10.0}{3} \right) = 5.3 \text{ in.} \end{cases}$$

Figure 14.38

In the equation for s_o, the maximum center-to-center spacing of the laterally supported longitudinal bars, h_x, is equal to 10.0 in.

$b_{c,1} = 24.0 - (2 \times 0.75) = 22.5$ in.

$b_{c,2} = 24.0$ in.

For confinement of the web in the direction of analysis, $b_c = b_{c,1} = 22.5$ in. (see Figure 14.73).

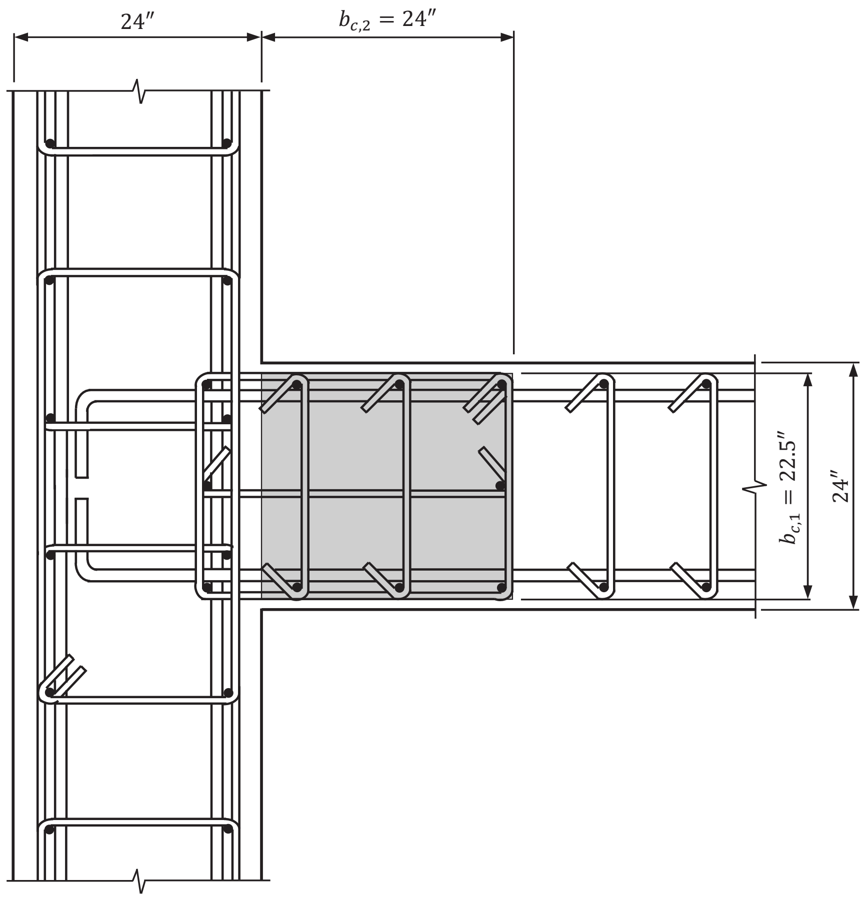


Figure 14.73 Special boundary transverse reinforcement in the web of the wall in Example 14.11.

$$A_{sh} = \text{larger of} \begin{cases} 0.3sb_c\left(\dfrac{A_g}{A_{ch}} - 1\right)\dfrac{f'_c}{f_{yt}} = 0.3 \times 4.0 \times 22.5 \times \left(\dfrac{24.0 \times 24.0}{22.5 \times 24.0} - 1\right) \times \dfrac{6}{80} = 0.14 \text{ in.}^2 \\ \\ 0.09sb_c\dfrac{f'_c}{f_{yt}} = 0.09 \times 4.0 \times 22.5 \times \dfrac{6}{80} = 0.61 \text{ in.}^2 \end{cases}$$

ACI Table 18.10.6.4(g)

where a vertical spacing of 4.0 in. is used in the web to match the vertical spacing of the transverse reinforcement in the flanges.

With a #5 hoop and one #5 crosstie, provided $A_{sh} = 3 \times 0.31 = 0.93 \text{ in.}^2 > 0.61 \text{ in.}^2$

In the direction perpendicular to the direction of analysis, $b_c = b_{c,2} = 24.0$ in.

$$A_{sh} = \text{larger of} \begin{cases} 0.3sb_c\left(\dfrac{A_g}{A_{ch}} - 1\right)\dfrac{f'_c}{f_{yt}} = 0.3 \times 4.0 \times 24.0 \times \left(\dfrac{24.0 \times 24.0}{22.5 \times 24.0} - 1\right) \times \dfrac{6}{80} = 0.14 \text{ in.}^2 \\ \\ 0.09sb_c\dfrac{f'_c}{f_{yt}} = 0.09 \times 4.0 \times 24.0 \times \dfrac{6}{80} = 0.65 \text{ in.}^2 \end{cases}$$

With a #5 hoop and two #5 crossties, provided $A_{sh} = 3 \times 0.31 = 0.93 \text{ in.}^2 > 0.65 \text{ in.}^2$ (one of the hoop legs is in the flange, so the area of that leg is not included in the provided area of transverse reinforcement).

Use a #5 hoop and #5 crossties spaced at 4.0 in. on center in the web.

Step 12 – Determine if transverse reinforcement required by ACI 18.10.6.5 is needed at the ends of the wall

It can be determined that a reinforcement ratio greater than $400 / f_y = 0.005$ must be provided in the flanges in stories 24 and above. Therefore, transverse reinforcement in accordance with ACI 18.10.6.5 is required in the flanges in these stories. The size of the transverse reinforcement must satisfy the requirements of ACI 25.7.2.2.

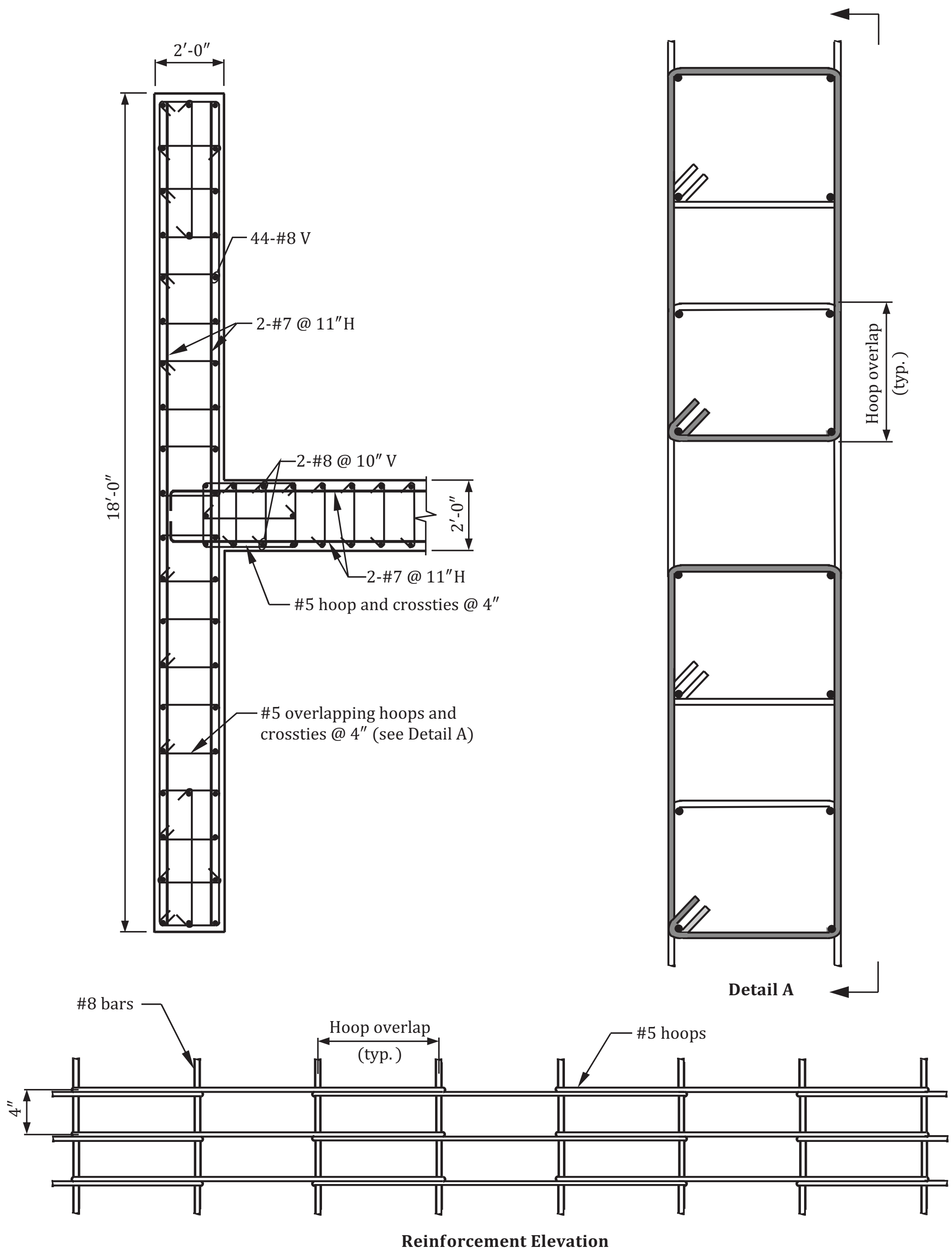


Figure 14.74 Reinforcement details for the wall in Example 14.11.

Reinforcement details for this wall are given in Figure 14.74. The special transverse reinforcement in the flanges consists of a series of overlapping hoops and supplemental 135-degree crossties, similar to Detail (b) depicted in ACI Figure R18.10.6.4a. The geometry of the overlapping hoops depicted in Detail A of Figure 14.74 satisfies the requirements in ACI 18.10.6.4(f).

Comments. For a construction sequence where (1) a wall is constructed to the underside of the slab, (2) the slab is constructed, and (3) the wall above the slab is constructed, the concrete within the thickness of the slab at the location of the special boundary elements must have a specified compressive strength of at least 0.7 times the specified compressive strength of the wall [ACI 18.10.6.4(h)]. In this example, the slab in floor levels 1 through 10 must have a minimum compressive strength of $0.7 \times 6{,}000 = 4{,}200$ psi. For a construction sequence where the wall is constructed ahead of the slab (for example, where slip forms are used to construct the wall), this requirement is not applicable.

This wall is also part of the SFRS for earthquake forces in the north-south direction. Because the axial force due to seismic forces acting along either of the principal plan axes is negligible, orthogonal load effects in accordance with ASCE/SEI 1.5.4 need not be considered. The provided reinforcement must satisfy the effects from axial forces, bending moments, and shear forces in both orthogonal directions independently.

The special moment frames on column lines A and G must be designed to independently resist at least 25 percent of the prescribed seismic forces. Design and detailing requirements for the beams, columns, and joints are given in Sects. 14.2, 14.3, and 14.4 of this publication, respectively.

14.9.12 Example 14.12 – Design of a Coupling Beam (Dual System): Building #4, SDC D

Design the coupling beam on column line 2 between column lines C and D in Building #4 at the eleventh-floor level (see Figure 1.4). The width of the beam is equal to the wall thickness, which is 24.0 in., and the depth of the beam is 42.0 in. Assume normalweight concrete with $f_c' = 6{,}000$ psi and ASTM A706 Grade 80 reinforcement.

Step 1 – Determine the factored load combinations

A three-dimensional analysis was performed for seismic forces in the north-south direction where the walls on column lines 2 and 3 and the moment frames on column lines 1 and 4 are part of the SFRS. In the east-west direction, the walls on column lines C, D, and E and the moment frames on column lines A and G are part of the SFRS. Thus, in both directions the SFRS is a dual system with special structural walls and special moment frames capable of independently resisting at least 25 percent of the prescribed seismic forces. Rigid diaphragms are assigned at each level and to account for cracking, 35 percent of the gross moment of inertia is used for the walls and beams and 70 percent for the columns [see ACI Table 6.6.3.1.1(a)].

The gravity and earthquake shear forces in the beam at the eleventh-floor level are given in Table 14.20 (the maximum shear force in the coupling beam occurs at this floor level). The effects due to wind loads are smaller than those due to the earthquake loads and are not considered in this example.

Table 14.20 Design Shear Forces for the Coupling Beam in Example 14.12

Load Case	Shear Force (kips)
Dead (D)	31.4
Live (L)	11.2
Seismic (Q_E)	$\pm$227.6

(table continued on next page)

Table 14.20 Design Shear Forces for the Coupling Beam in Example 14.12 (cont.)

Load Case			Shear Force (kips)
Load Combination			
1	$1.4D$		44.0
2	$1.2D + 1.6L$		55.6
3	$1.3D + 0.5L + 1.3Q_E$	SSR	342.3
		SSL	−249.5
4	$0.8D + 1.3Q_E$	SSR	321.0
		SSL	−270.8

Further explanation is needed on how the load factors in the load combinations that include E are obtained. In ACI Equation (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = 1.3Q_E + (0.2 \times 0.517)D = 1.3Q_E + 0.10D$ where $\rho = 1.3$ in accordance with ASCE/SEI 12.3.4.2. Therefore, this load combination becomes the following:

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + (1.3Q_E + 0.10D) = 1.3D + 0.5L + 1.3Q_E$$

Similarly, in ACI Equation (5.3.1g), the effects of gravity and seismic ground motion counteract, so $E = \rho Q_E + 0.2S_{DS}D = 1.3Q_E - (0.2 \times 0.517)D = 1.3Q_E - 0.10D$, and this load combination becomes the following:

$$0.9D + 1.0E = 0.9D + (1.3Q_E - 0.10D) = 0.8D + 1.3Q_E$$

Step 2 – Determine the clear span to height ratio

Clear span $\ell_n = 25.0 - 9.0 - 8.0 = 8.0$ ft

Height $h = 42.0$ in. $= 3.5$ ft

$\ell_n / h = 8.0 / 3.5 = 2.3$

Because $2 < \ell_n / h = 2.3 < 4$, the coupling beam is permitted to be reinforced either with two intersecting groups of diagonally placed bars symmetrical about the midspan of the beam or in accordance with ACI 18.6.3 through 18.6.5 for beams in special moment frames (ACI 18.10.7.3). The former of the two options is selected in this example.

Step 3 – Determine the required area of the diagonal reinforcing bars ACI 18.10.7.4(a)

The required area of the diagonal bars, A_{vd}, is determined by the following equation:

$$V_u = \phi V_n = 2\phi A_{vd} f_y \sin\alpha \le 10\phi\sqrt{f_c'}A_{cw}$$

where $\phi = 0.85$ for diagonally reinforced coupling beams. ACI 21.2.4.4

In order to determine the angle between the diagonal bars and the longitudinal axis of the coupling beam, α, the centroid of the diagonal bar bundles must be located with respect to the centroid of the beam. The minimum out-to-out dimensions of the transverse reinforcement are $b_w / 2 = 24.0 / 2 = 12.0$ in. for the dimension parallel to b_w and $b_w / 5 = 24.0 / 5 = 4.8$ in. along the other sides. Assume 12.0 in. by 12.0 in. out-to-out dimensions of the transverse reinforcement around the diagonal bars and a 1.5-in. cover to the transverse reinforcement near the ends of the beam (see Figure 14.75).

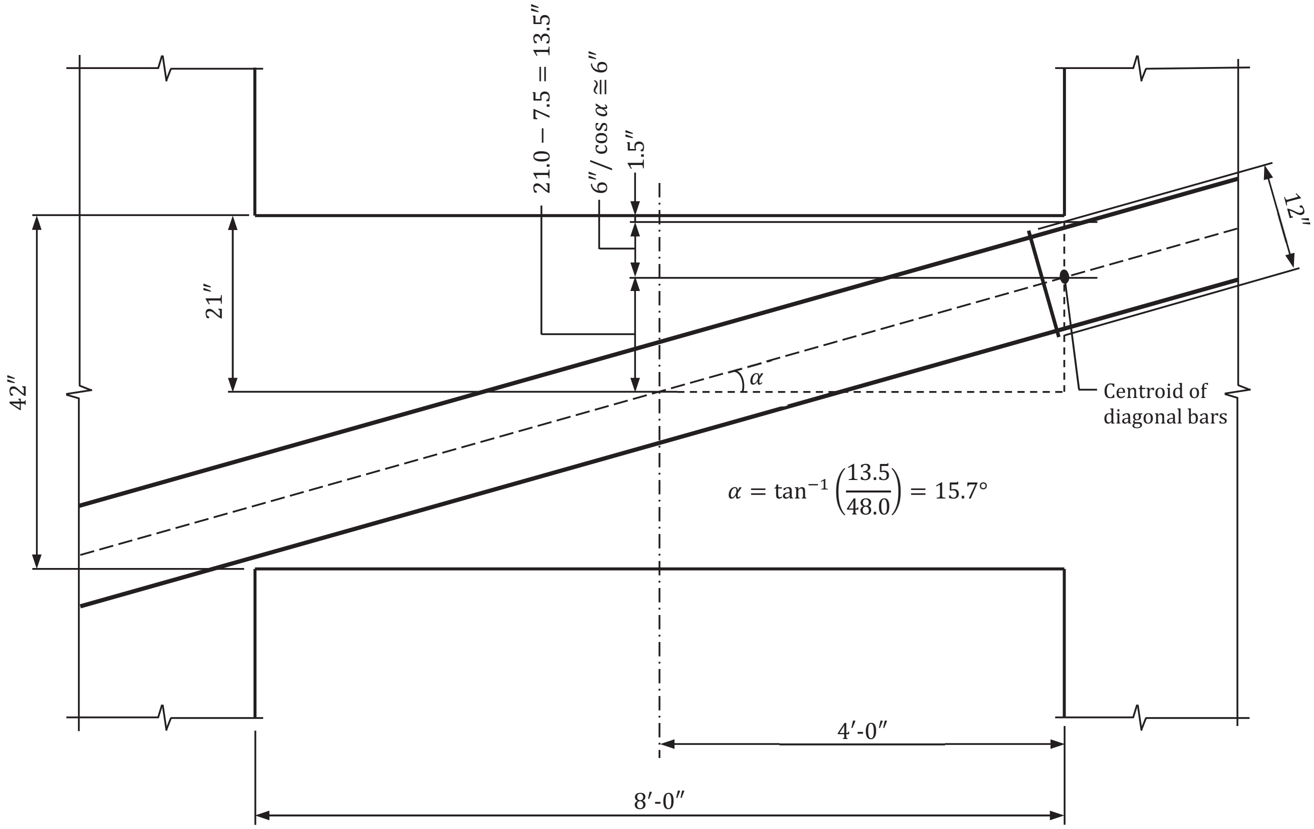


Figure 14.75 Determination of the angle between the diagonal bars and the longitudinal axis of the coupling beam in Example 14.12.

The distance from the top or bottom of the section to the centroid of the diagonal bar bundles at the face of the coupling beam is equal to $1.5 \text{ in.} + (6.0 \text{ in.} / \cos\alpha)$, which is approximately equal to $1.5 + 6.0 = 7.5$ in. for this beam. Thus, the distance from the centroid of the beam to the centroid of the diagonal bars is $(42.0 / 2) - 7.5 = 13.5$ in. Therefore,

$$\alpha = \tan^{-1}\left(\frac{13.5}{48.0}\right) = 15.7 \text{ deg}$$

The required area of diagonal reinforcement is equal to the following:

$$A_{vd} = \frac{V_u}{2\phi f_y \sin\alpha} = \frac{342.3}{2 \times 0.85 \times 80.0 \times \sin 15.7} = 9.3 \text{ in.}^2$$

Provide 8-#10 bars in each group of diagonal bars ($A_{s,provided} = 10.2 \text{ in.}^2$).

Check the upper limit on the design shear strength: ACI Eq. (18.10.7.4)

$$\phi V_n = 2\phi A_{vd} f_y \sin\alpha = 2 \times 0.85 \times 10.2 \times 80 \times \sin 15.7 = 375.4 \text{ kips}$$

$$< 10\phi\sqrt{f_c'}A_{cw} = 10 \times 0.85 \times \sqrt{6{,}000} \times (24.0 \times 42.0) / 1{,}000 = 663.7 \text{ kips}$$

Step 4 – Determine the required area of transverse reinforcement

The required transverse reinforcement is determined using both options in ACI 18.10.7.4.

- Option 1: Confinement of individual diagonals — ACI 18.10.7.4(c)

  Area A_g is determined assuming a 0.75-in. cover around the diagonal bar group:

  $$A_g = (12.0 + 1.5) \times (12.0 + 1.5) = 182.3 \text{ in.}^2$$

  Also, $A_{ch} = 12.0 \times 12.0 = 144.0 \text{ in.}^2$

  Assuming #4 bars for transverse reinforcement around the diagonal bars, the required spacing is equal to the following:

  $$s = \text{lesser of} \begin{cases} s_o = 4 + \left(\dfrac{14 - h_x}{3}\right) = 4 + \left(\dfrac{14 - 4.9}{3}\right) = 7.0 \text{ in.} > 6.0 \text{ in., use } 6.0 \text{ in.} \\ 6d_b = 6 \times 1.27 = 7.6 \text{ in.} \end{cases}$$

  Figure 14.43

  In the equation for s_o, the maximum center-to-center spacing of the laterally supported longitudinal bars, h_x, is equal to $\{13.5 - [2 \times (0.75 + 0.5)] - 1.27\} / 2 = 4.9$ in.

  The required area of transverse reinforcement is equal to the following assuming a 6.0-in. spacing:

  $$A_{sh} = \text{greater of} \begin{cases} 0.09 s b_c \dfrac{f'_c}{f_{yt}} = 0.09 \times 6.0 \times 12.0 \times \dfrac{6}{80} = 0.49 \text{ in.}^2 \\ 0.3 s b_c \left(\dfrac{A_g}{A_{ch}} - 1\right) \dfrac{f'_c}{f_{yt}} = 0.3 \times 6.0 \times 12.0 \times \left(\dfrac{182.3}{144.0} - 1\right) \times \dfrac{6}{80} = 0.43 \text{ in.}^2 \end{cases}$$

  Use #4 hoops and crossties at a spacing of 6.0 in. on center $(A_{s,provided} = 3 \times 0.20 = 0.60 \text{ in.}^2)$.

  Additional longitudinal and transverse reinforcement must be provided around the perimeter of the beam with a spacing less than or equal to 12.0 in. [ACI 18.10.7.4(c)(ii)].

  Required area of reinforcement $= 0.002 b_w s = 0.002 \times 24.0 \times 12.0 = 0.58 \text{ in.}^2$

  For the longitudinal reinforcement, provide 12-#3 bars $(A_{s,provided} = 1.32 \text{ in.}^2)$ uniformly distributed around the perimeter of the beam at a center-to-center spacing less than 12 in. For the transverse reinforcement, use #4 bars spaced at 12.0 in. on center.

  Reinforcement details for this option are given in Figure 14.76.

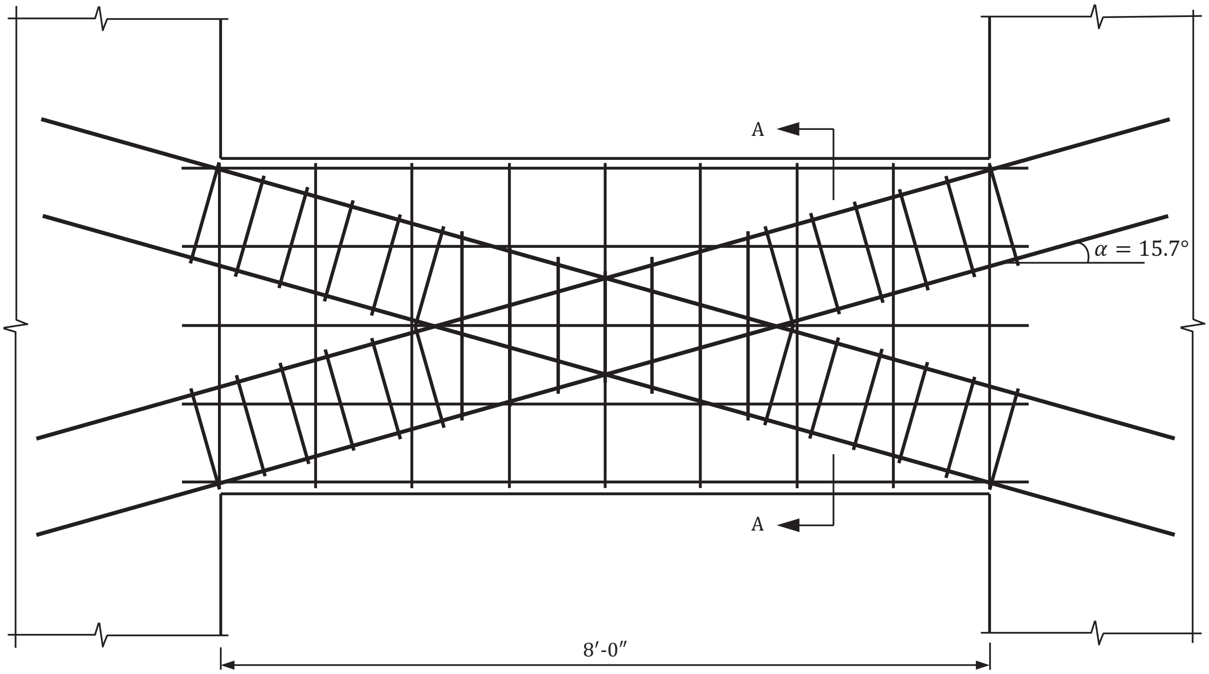
- Option 2: Confinement of the entire cross-section — ACI 18.10.7.4(d)

  The required spacing of the transverse reinforcement is equal to the following:

  $$s = \text{lesser of} \begin{cases} 6.0 \text{ in.} \\ 6d_b = 6 \times 1.27 = 7.6 \text{ in.} \end{cases}$$

  Figure 14.44

  Assuming a 1.5-in. cover, the required area of transverse reinforcement perpendicular to the long face of the beam is equal to the following assuming a 4.0-in. spacing (the 4-in. spacing is used instead of a 6.0-in. spacing to decrease the required area of transverse reinforcement):



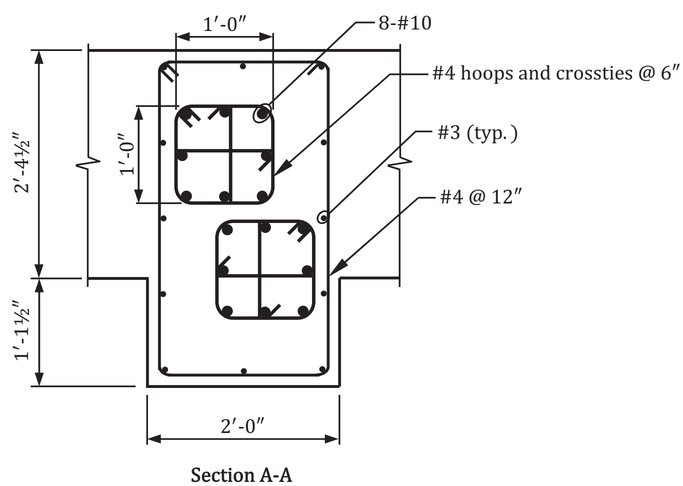


Figure 14.76 Reinforcement details for the coupling beam in Example 14.12 – Confinement of individual diagonals

$$A_{sh} = \text{greater of} \begin{cases} 0.09 s b_c \dfrac{f'_c}{f_{yt}} = 0.09 \times 4.0 \times (42.0 - 3.0) \times \dfrac{6}{80} = 1.05 \text{ in.}^2 \\ \\ 0.3 s b_c \left(\dfrac{A_g}{A_{ch}} - 1 \right) \dfrac{f'_c}{f_{yt}} = 0.3 \times 4.0 \times (42.0 - 3.0) \times \left(\dfrac{42.0 \times 24.0}{39.0 \times 21.0} - 1 \right) \times \dfrac{6}{80} = 0.81 \text{ in.}^2 \end{cases}$$

In order to satisfy the 8.0-in. maximum spacing requirement in ACI 18.10.7.4(d), 6 longitudinal bars are required on each of the long faces of the beam.

With a #4 hoop and 4-#4 crossties, provided $A_{sh} = 6 \times 0.20 = 1.20 \text{ in.}^2 > 1.05 \text{ in.}^2$

The required area of transverse reinforcement perpendicular to the short face of the beam is equal to the following:

$$A_{sh} = \text{greater of} \begin{cases} 0.09sb_c \dfrac{f'_c}{f_{yt}} = 0.09 \times 4.0 \times 24.0 \times \dfrac{6}{80} = 0.65 \text{ in.}^2 \\ 0.3sb_c \left(\dfrac{A_g}{A_{ch}} - 1 \right) \dfrac{f'_c}{f_{yt}} = 0.3 \times 4.0 \times 24.0 \times \left(\dfrac{42.0 \times 24.0}{39.0 \times 21.0} - 1 \right) \times \dfrac{6}{80} = 0.50 \text{ in.}^2 \end{cases}$$

In order to satisfy the 8.0-in. maximum spacing requirement in ACI 18.10.7.4(d), 4 longitudinal bars are required on each of the short faces of the beam.

With a #4 hoop and 2-#4 crossties, provided $A_{sh} = 4 \times 0.20 = 0.80 \text{ in.}^2 > 0.65 \text{ in.}^2$

Reinforcement details for this beam are given in Figure 14.77.

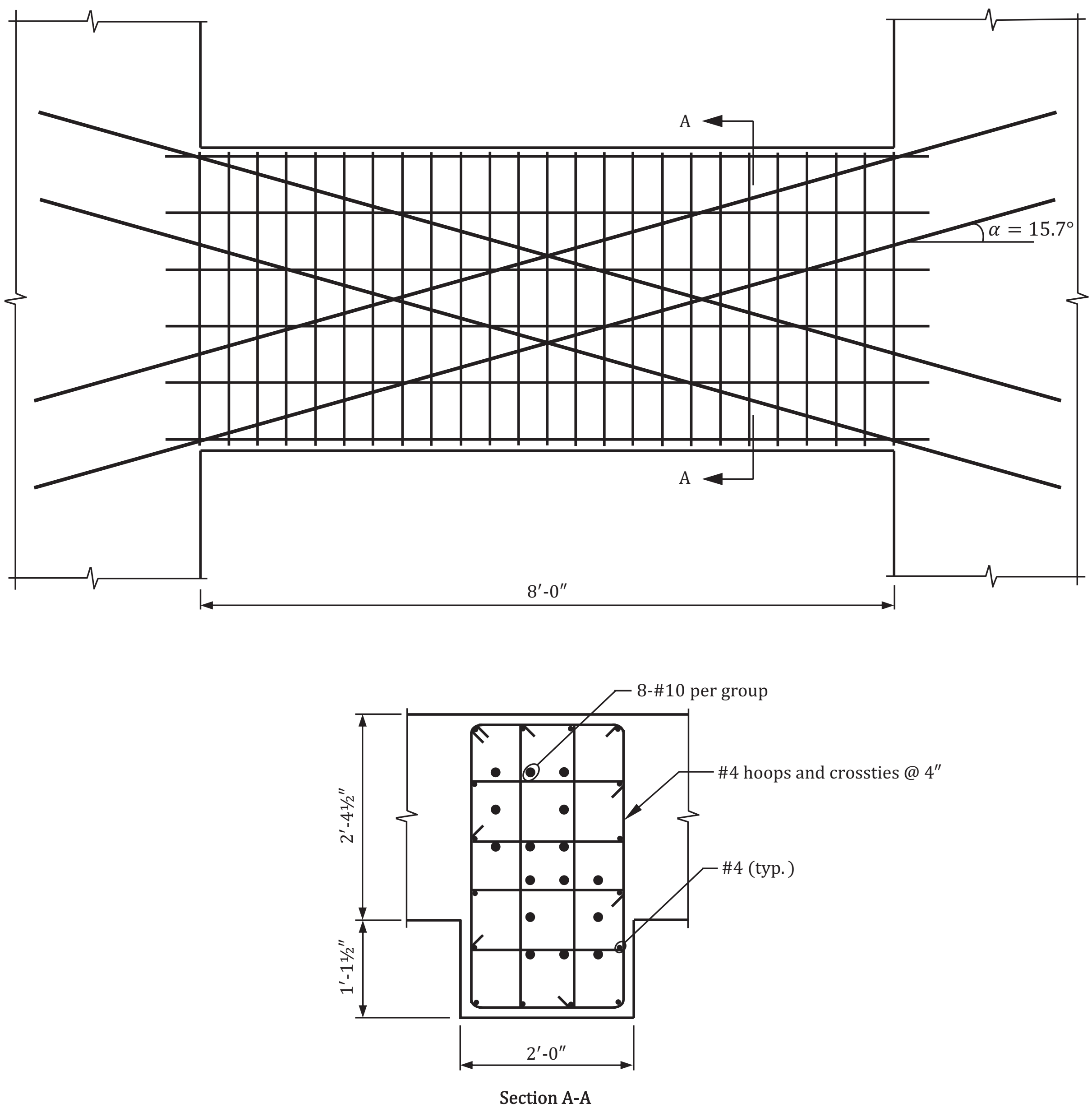


Figure 14.77 Reinforcement details for the coupling beam in Example 14.12 – Confinement of the entire cross-section

14.9.13 Example 14.13 – Determination of Diaphragm Reinforcement: Building #4, SDC D

Design the required diaphragm reinforcement at the second-floor level in Building #4 for seismic forces in the east-west direction (see Figure 1.4). The thickness of the slab (which is part of the wide-module joist floor system) is equal to 4.5 in. Assume normalweight concrete with $f_c' = 4{,}000\ \text{psi}$ and Grade 60 reinforcement.

Design data are given in Sect. 1.2.4. The building is assigned to SDC D based on the following:

- Site Class D (default)
- $S_S = 0.583,\ S_1 = 0.192$
- $S_{DS} = 0.517,\ S_{D1} = 0.284$

Step 1 – Determine the diaphragm in-plane forces

Wind and seismic in-plane forces in the east-west direction are given in Examples 3.4 and 3.16, respectively. It is evident from Tables 3.25 and 3.33 that the seismic diaphragm design forces are significantly larger than those due to wind. Therefore, wind forces are not considered in this example.

Step 2 – Determine the diaphragm classification

The information in Section 9.3.3 of this publication is used to determine the classification of this diaphragm.

In the east-west direction, the maximum span-to-depth ratio is $50.0 / 110.0 = 0.45$. In the north-south direction, the maximum span-to-depth ratio is $40.0 / 150.0 = 0.27$. Because the span-to-depth ratios are less than 3.0 and the structure does not have any of the horizontal irregularities in ASCE/SEI Table 12.3-1, the reinforced concrete floor system can be classified as a rigid diaphragm when subjected to seismic forces in both directions of analysis (ASCE/SEI 12.3.1.2).

Step 3 – Check the minimum diaphragm thickness

According to ACI 18.12.6.1, the minimum diaphragm thickness required to transmit earthquake forces is 2.0 in. In this example, a 4.5-in. slab thickness is provided.

Step 4 – Select the diaphragm model

An equivalent beam model is selected for this diaphragm. The forces in the walls and frames are determined using a three-dimensional model of the building (see Step 6 below where information is provided on how the diaphragm forces are obtained from the analysis).

Step 5 – Determine the location of the center of mass and the center of rigidity

From symmetry, $x_{cm} = x_{cr} = 55.0\ \text{ft}$ and $y_{cm} = y_{cr} = 75.0\ \text{ft}$.

There is no eccentricity between the CM and the CR in both directions, which means no inherent torsional moments are generated.

Step 6 – Determine the seismic forces in the special structural walls and the special moment frames

A three-dimensional model of the building was constructed using Reference 14. In the east-west direction, the walls along column lines C, D, and E and the moment frames along column lines A and G are assumed to be part of the SFRS. In the model, the columns and walls are fixed at the base (ASCE/SEI 12.7.1), rigid diaphragms are assigned at all levels in the building, and the following reduced moments of inertia are used, which account for the effects of cracked sections [see ASCE/SEI 12.7.3 and ACI Table 6.6.3.1.1(a)]:

- Columns: $I = 0.70I_g$
- Beams: $I = 0.35I_g$
- Walls: $I = 0.35I_g$
- Slabs (out-of-plane): $I = 0.25I_g$
- Slabs (in-plane): $I = 0.35I_g$

Seismic forces are applied at the CM in both directions. Accidental torsion in accordance with ASCE/SEI 12.8.4.2 need not be applied in the analysis for strength design or when checking the story drift limits prescribed in ASCE/SEI 12.12 because the structure, which is assigned to SDC D, does not have a Type 1a or Type 1b horizontal structural irregularity. The diaphragm force, F_{px}, at the second-floor level is equal to 421.5 kips in both directions (see Table 3.33).

Because the building does not have a Type 5 horizontal structural irregularity as defined in ASCE/SEI Table 12.3-1, the design seismic forces in the north-south and east-west directions are permitted to be applied independently in each of the two orthogonal directions and orthogonal interaction effects are permitted to be neglected (ASCE/SEI 12.5.4).

The vertical seismic force distribution in a multistory building for use in the determination of diaphragm forces is given in Figure 14.78. The forces F_x over the height of the building are the code-prescribed seismic forces, which can be determined using the ELF Procedure in ASCE/SEI 12.8. Assume the in-plane forces in the diaphragm are required at level i. The code-prescribed force, F_i, at this level is replaced in the model with F_{pi}, which is the diaphragm force determined in accordance with ASCE/SEI 12.10.1.1. The reactions in the vertical elements of the SFRS at this level are determined based on this vertical force distribution.

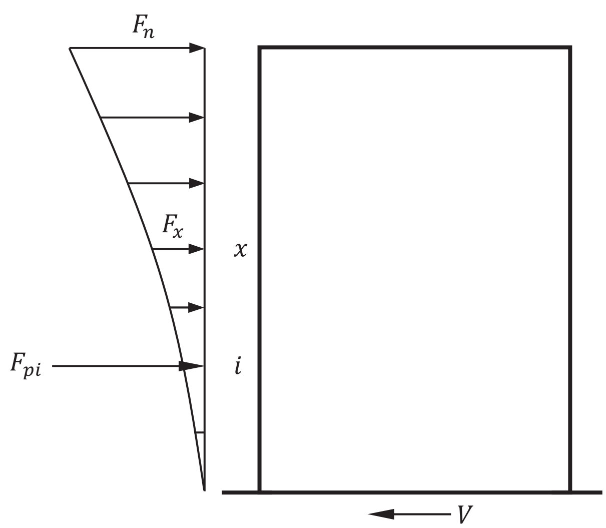


Figure 14.78 Vertical seismic force distribution in a multistory building for use in the determination of diaphragm forces.

The diaphragm forces at the second-floor level are obtained from the three-dimensional model of the building, which is subjected to the forces, F_x, from the ELF Procedure where $F_x = 1.0$ kip at the second-floor level is replaced with $F_{px} = 421.5$ kips at this level (see Table 3.33 of this publication). Therefore, for purposes of analysis, an additional force equal to $F_{px} - F_x = 421.5 - 1.0 = 420.5$ kips is applied at the CM of the second-floor level in the model.

Because rigid diaphragms are specified in the model, the force, V_u, in the diaphragm is determined using the shear forces in the vertical members of the SFRS immediately above and below the second-floor level (see Figure 14.79). For example, consider the special wall along column line D, which resists a portion of the east-west seismic force based on its relative rigidity. To determine the diaphragm force at this location, section cuts are made in the wall immediately above and below the second-floor level. The shear forces obtained from analysis are equal to 942.7 kips above the second-floor level and 1,078.2 kips below the second-floor level, so the force in the diaphragm at this location is equal to $1{,}078.2 - 942.7 = 135.5$ kips. The 135.5-kip force is equal to the reaction at this support. The forces in the walls along column lines C and E and in the moment frames along column lines A and G can be obtained in a similar fashion. The reactions at these supports are equal to the following:

Reactions at column lines C and E $= 124.0$ kips

Reactions at column lines A and G $= 19.0$ kips

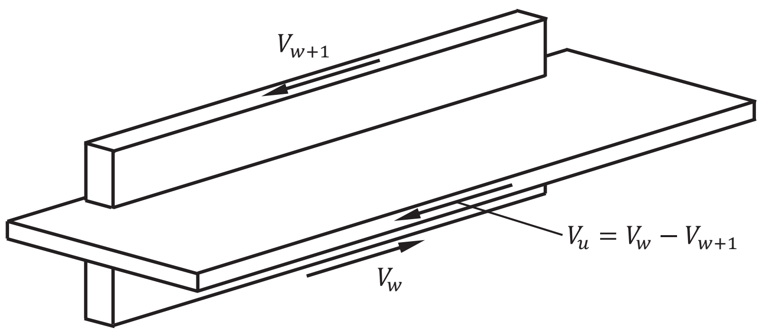


Figure 14.79 Force transferred between a diaphragm and a vertical element of the SFRS.

The total force at the second-floor level is equal to $135.5 + (2 \times 124.0) + (2 \times 19.0) = 421.5$ kips, which as expected, matches the diaphragm force applied at this level.

Step 7 – Construct the shear and moment diagrams for the diaphragm

The equivalent in-plane distributed load for seismic forces in the east-west direction is equal to the sum of the reactions divided by the width of the diaphragm:

$w_W = 421.5 / 150.0 = 2.81$ kips/ft

This load is uniformly distributed over the width of the diaphragm because there are no torsional moments. The corresponding shear and moment diagrams are given in Figure 14.80.

Step 8 – Determine the chord forces

Chord forces must be determined in both directions considering the openings in the diaphragm. Based on the results from the analysis, these openings have a nominal effect on the overall behavior of the diaphragm. As such, chord forces are determined disregarding the openings, which means special chord and collector reinforcement at the edges of the openings need not be determined.

Typically, the moment arm used to determine the tension chord force is equal to 95 percent of the depth of the diaphragm in the direction of analysis. If that depth is used in this example, the chord reinforcement will be located within the 4.5-in. slab outside of the 36.0-in. width of the beams in the special moment frames along column lines 1 and 4, which is a suitable location for this reinforcement.

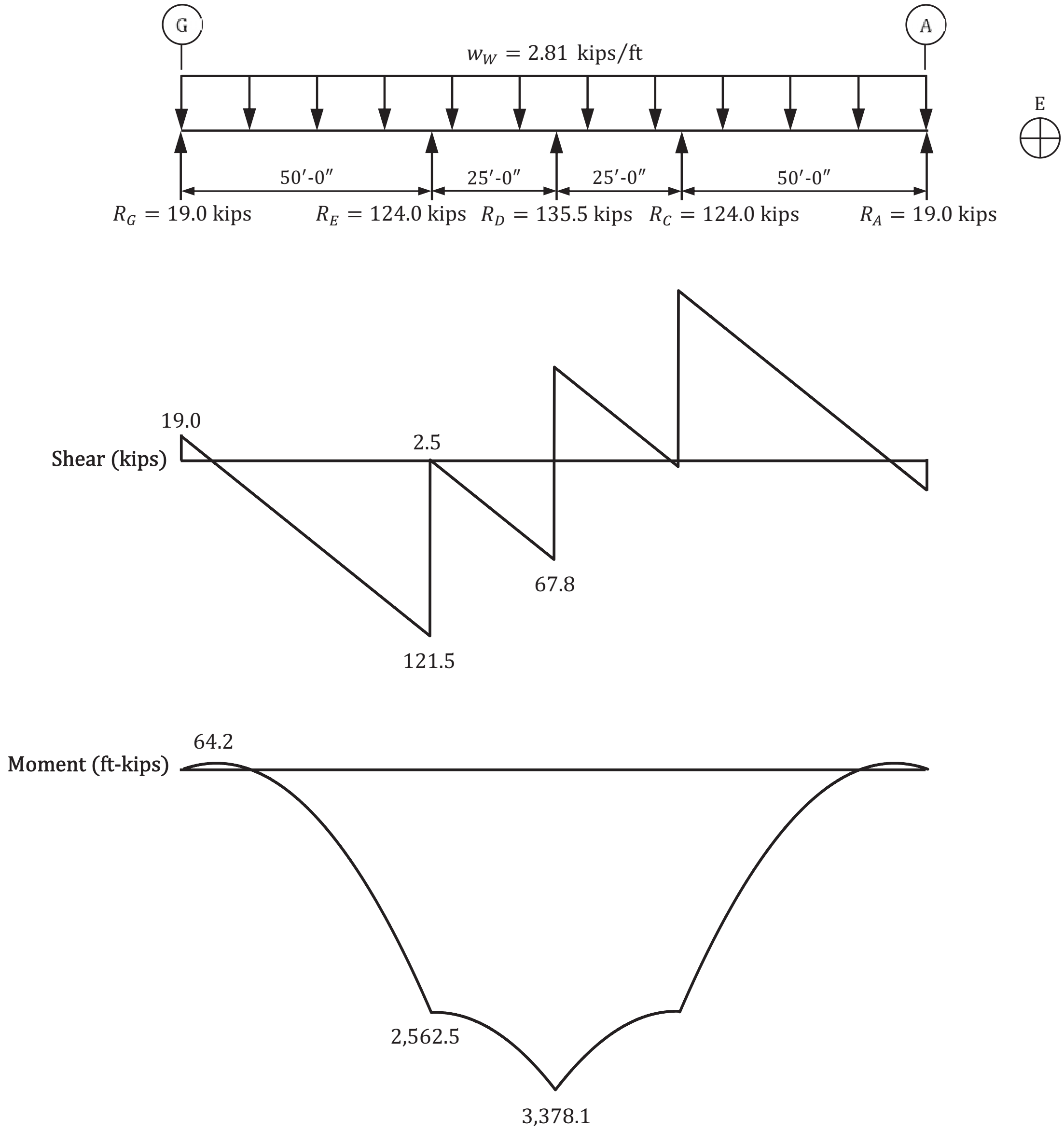


Figure 14.80 Equivalent distributed load, shear diagram, and moment diagram for seismic forces in the west direction.

The maximum moment in the diaphragm is equal to 3,378.1 ft-kips, which occurs at column line D (see Figure 14.80). Therefore, using Equation (9.1) of this publication, the maximum tension chord force is equal to the following:

$$T_u = \frac{M_u}{d} = \frac{3{,}378.1}{0.95 \times 110.0} = 32.3 \text{ kips}$$

Step 9 – Determine the unit shear forces, net shear forces, and collector forces

The unit shear forces in the diaphragm are equal to the following:

- Along column lines A and G: $(v_u)_A = (v_u)_G = 19.0 \,/\, 110.0 = 0.17$ kips/ft
- Along column lines C and E: $(v_u)_C = (v_u)_E = 121.5 \,/\, 110.0 = 1.11$ kips/ft
- Along column line D: $(v_u)_D = 67.8 \,/\, 110.0 = 0.62$ kips/ft

The unit shear forces in the walls along column lines C and E and in the wall along column line D are equal to the following:

- Along column lines C and E: $(v_u)_C = (v_u)_E = 124.0 / 30.0 = 4.13$ kips/ft
- Along column line D: $(v_u)_D = 135.5 / 30.0 = 4.52$ kips/ft

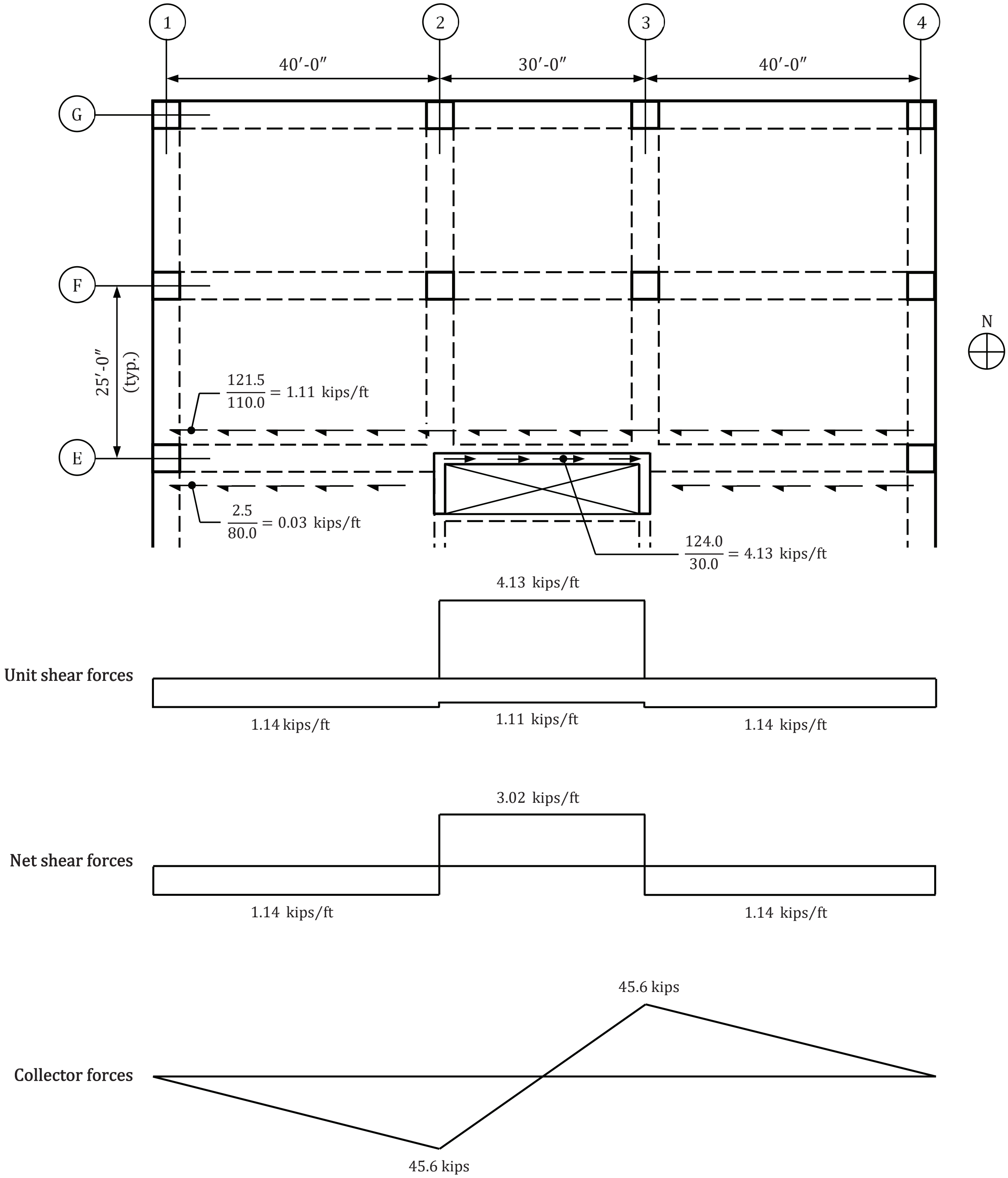


Figure 14.81 Unit shear forces, net shear forces, and collector forces along column line E for the diaphragm in Example 14.13.

The unit shear forces, net shear forces, and collector forces along column E are given in Figure 14.81 (the forces are the same along column line C). Forces along column line D can be obtained in a similar fashion.

Collectors are not required along column lines A and G because the special moment frames extend the entire depth of the diaphragm in the east-west direction.

Collectors are required along column lines C, D, and E. It is assumed the collector beams along these column lines transfer the entire axial force directly into the web of the walls. The beams must be designed for combined flexure due to gravity loads and axial tension and compression forces due to in-plane seismic forces (see Step 14 below).

Step 10 – Determine combined load effects

Combined load effects are determined using the applicable strength design load combinations in ACI 5.3 (see Table 14.1 of this publication). The governing in-plane load effects are due to design seismic forces. In ACI Eq. (5.3.1e), the effects of gravity and seismic ground motion are additive, so $E = \rho Q_E + 0.2S_{DS}D = 1.3Q_E + (0.2 \times 0.517)D = 1.3Q_E + 0.10D$ where $\rho = 1.3$ in accordance with ASCE/SEI 12.3.4.2. Therefore, this load combination becomes the following:

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + (1.3Q_E + 0.10D) = 1.3D + 0.5L + 1.3Q_E$$

In buildings assigned to SDC D, collectors and their connections must be designed for the maximum of the three forces given in ASCE/SEI 12.10.2.1:

1. Forces calculated using the seismic load effects including overstrength of ASCE/SEI 12.4.3 with the seismic forces determined by the ELF Procedure of ASCE/SEI 12.8 or the modal response spectrum analysis procedure of ASCE/SEI 12.9.1.

   For dual systems with special reinforced concrete shear walls and special reinforced concrete moment frames capable of resisting at least 25 percent of the prescribed seismic forces, the overstrength factor, Ω_o, is equal to 2.5 from ASCE/SEI Table 12.2-1. The required in-plane diaphragm force at the second-floor level based on this requirement is equal to $2.5 \times 1.0 = 2.5$ kips (see Table 3.33 of this publication).
2. Forces calculated using the seismic load effects including overstrength of ASCE/SEI 12.4.3 with the seismic forces determined by ASCE/SEI Eq. (12.10-1).

   Using the information in Table 3.33, the diaphragm force, F_{px}, at the second-floor level determined by ASCE/SEI Eq. (12.10-1) is equal to the following:

   $$F_{p2} = \left(\sum_{i=2}^{R} F_i \Big/ \sum_{i=2}^{R} w_i \right) w_{p2} = (2{,}528.9 / 109{,}954) \times 4{,}092 = 0.0230 \times 4{,}092 = 94.1 \text{ kips}$$

   The required diaphragm in-plane force including overstrength is equal to $2.5 \times 94.1 = 235.3$ kips based on this requirement.
3. Forces calculated using the load combinations of ASCE/SEI 2.3.6 with the seismic forces determined by ASCE/SEI Eq. (12.10-2).

   The diaphragm force, F_{px}, at the second-floor level determined by ASCE/SEI Eq. (12.10-2) is equal to 421.5 kips (see Table 3.33). The required diaphragm in-plane force based on this requirement is equal to ρ times this value: $1.3 \times 421.5 = 548.0$ kips.

Therefore, the collectors and their connections to the vertical elements of the SFRS must be designed for the effects due to the 548.0-kip in-plane diaphragm force stipulated in the third requirement. Thus, the axial tension and compression forces determined in Step 9 above for the 421.5-kip diaphragm force must be increased by 1.3 (see Figure 14.81): $1.3 \times 45.6 = 59.3$ kips. The collector beams must be designed for an 59.3-kip axial compression and

tension forces in combination with the effects due to the tributary factored gravity loads along the length of the member (see Step 14 below).

Step 11 – Determine the chord reinforcement

The required area of chord reinforcement for seismic forces in the east-west direction is equal to the following (see Step 8):

$$A_{s(\text{chord})} = \frac{T_u}{\phi f_y} = \frac{32.3}{0.9 \times 60} = 0.60 \text{ in.}^2$$

At column lines 1 and 4, provide 2-#5 bars ($A_{s,provided} = 0.62$ in.^{2}); these bars are located just outside the cross-section of the beams in the special moment frames along these lines (see Figure 14.82).

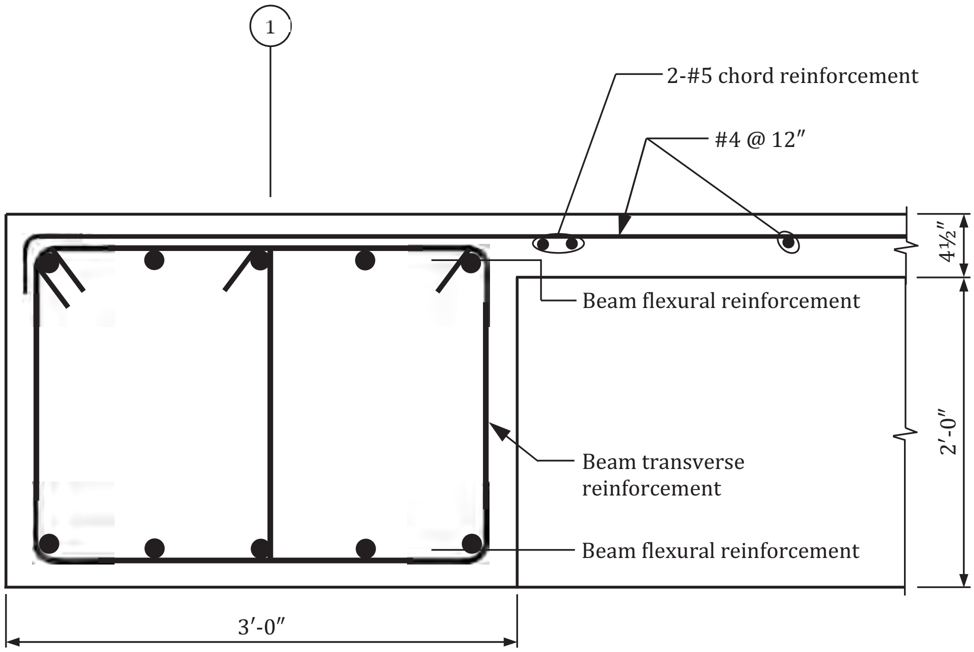


Figure 14.82 Location of chord reinforcement in the diaphragm of Example 14.13 for seismic forces in the east-west direction.

Step 12 – Determine the diaphragm shear reinforcement

The largest factored unit shear force in the diaphragm in the direction of analysis is equal to 1.11 kips/ft along column lines C and E (see Step 9). The strength reduction factor, ϕ, for shear in a diaphragm must not exceed the least value of ϕ for shear used for the vertical elements of the SFRS (ACI 21.2.4.2). For the shear design of the beams and columns in the special moment frames, ϕ is permitted to be taken as 0.75 (ACI 21.2.4). It is determined in Example 14.11 that $\phi = 0.75$ for the shear design of the special structural walls. Therefore, use $\phi = 0.75$ for the shear design of the diaphragm.

Determine ϕV_n from Eq. (14.35) assuming the shear reinforcement $\rho_t = 0$:

$$\phi V_n = \phi 2 A_{cv} \lambda \sqrt{f'_c} = 0.75 \times 2 \times (4.5 \times 12.0) \times 1.0\sqrt{4{,}000} \, / \, 1{,}000 = 5.12 \text{ kips/ft} > 1.11 \text{ kips/ft}$$

$$< \phi 8 \sqrt{f'_c} A_{cv} = 20.5 \text{ kips/ft}$$

Therefore, the shear strength of the diaphragm is adequate without shear reinforcement.

Step 13 – Determine the shear transfer reinforcement

- Special moment frames along column lines A and G

  The required shear-friction reinforcement between the diaphragm and the special moment frames along column lines A and G is equal to the following (see Step 9):

$$A_{vf} = \frac{(v_u)_A}{\phi \mu f_y} = \frac{0.17}{0.75 \times 1.4 \times 1.0 \times 60} = 0.003 \text{ in.}^2/\text{ft} \qquad \text{Eq. (9.11)}$$

  where $\mu = 1.4\lambda$ because the slab and moment frames are placed monolithically (see ACI Table 22.9.4.2).

  Because the required area of shear-friction reinforcement is very small, it is safe to assume that the one layer of flexural reinforcement in the 4.5-in. slab can also be used as the shear-friction reinforcement between the diaphragm and the special moment frames.

  Provide #4 bars spaced at 12 in. on center oriented in the north-south direction in the 4.5-in. slab over the length of the special moment frames ($A_{s,provided} = 0.20 \text{ in.}^2/\text{ft} > 0.0018A_g = 0.0018 \times 4.5 \times 12.0 = 0.10 \text{ in.}^2/\text{ft}$; see ACI 7.6.1.1 and Figure 14.82). Although #3 bars spaced at 12 in. on center are adequate, #4 bars are provided to match the required shear transfer reinforcement for the special structural walls (see below).

- Special structural walls along column lines C and E:

  The required shear-friction reinforcement between the diaphragm and the walls along column line C and E is equal to the following (see Step 9):

$$A_{vf} = \frac{(v_u)_C}{\phi \mu f_y} = \frac{4.13}{0.75 \times 1.0 \times 1.0 \times 60} = 0.09 \text{ in.}^2/\text{ft}$$

  where $\mu = 1.0\lambda$ in accordance with ACI 18.12.10.1 [the walls and slabs are not cast monolithically and the contact surface must be roughened consistent with condition (b) in ACI Table 22.9.4.2].

  The total area of reinforcement that must be provided for flexure and shear transfer is equal to $0.10 + 0.09 = 0.19 \text{ in.}^2/\text{ft}$.

  Provide #4 bars spaced at 12 in. on center oriented in the north-south direction in the 4.5-in. slab over the lengths of these walls ($A_{s,provided} = 0.20 \text{ in.}^2/\text{ft} > 0.19 \text{ in.}^2/\text{ft}$;see Figure 14.83).

- Special structural wall along column line D:

  The required shear-friction reinforcement between the diaphragm and the wall along column line D is equal to the following (see Step 9):

$$A_{vf} = \frac{(v_u)_D}{\phi \mu f_y} = \frac{4.52}{0.75 \times 1.0 \times 1.0 \times 60} = 0.10 \text{ in.}^2/\text{ft}$$

  where $\mu = 1.0\lambda$ in accordance with ACI 18.12.10.1 [the walls and slabs are not cast monolithically and the contact surface must be roughened consistent with condition (b) in ACI Table 22.9.4.2].

  The total area of reinforcement that must be provided for flexure and shear transfer $0.10 + 0.10 = 0.20 \text{ in.}^2/\text{ft}$.

  Provide #4 bars spaced at 12 in. on center oriented in the north-south direction in the 4.5-in. slab over the length of this wall ($A_{s,provided} = 0.20 \text{ in.}^2/\text{ft}$).

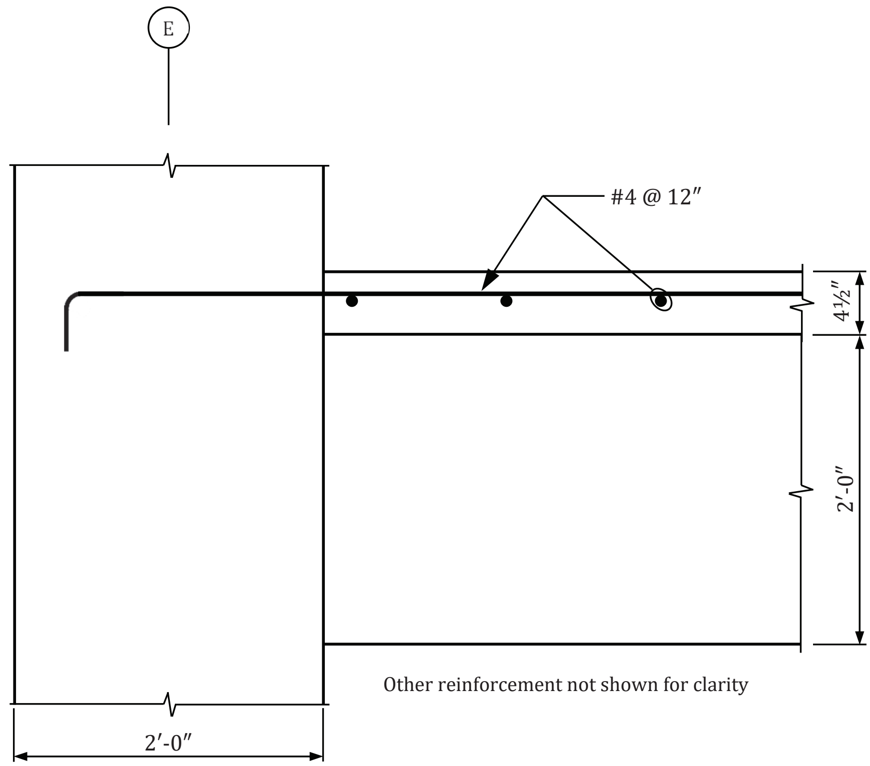


Figure 14.83 Shear transfer reinforcement between the diaphragm and the special structural wall along column line E in Example 14.13 for seismic forces in the east-west direction.

- Diaphragms and the collectors

  According to ASCE/SEI 12.10.2.1, the shear transfer reinforcement providing the connection between the diaphragm and the collectors must be based on the largest of the three forces determined in accordance with that section. From Step 10, the required diaphragm force is equal to 548.0 kips.

  The unit shear force between the diaphragm and the collector beams at column lines C and E is equal to 1.54 kips/ft based on a diaphragm force of 421.5 kips (see Figure 14.81). Therefore, the required shear-friction reinforcement at these locations is equal to the following:

$$A_{vf} = \frac{(v_u)_C}{\phi \mu f_y} = \frac{1.14 \times (548.0 \,/\, 421.5)}{0.75 \times 1.4 \times 1.0 \times 60} = 0.02 \text{ in.}^2/\text{ft}$$

  The #4 bars spaced at 12 in. on center in the slab oriented in the north-south direction are adequate for combined flexure and shear transfer $A_{s,provided} = 0.20 \text{ in.}^2/\text{ft} > 0.10 + 0.02 = 0.12 \text{ in.}^2/\text{ft}$.

  Because the unit shear force along column line D is less than that along column lines C and E, #4 bars spaced at 12 in. are also adequate for combined flexure and shear transfer at this location.

Step 14 – Determine the required collector reinforcement

Reinforcement is determined for the 36.0 in. by 28.5 in. collector beams along column lines C and E (determination of the reinforcement for the collector beam along column line D is similar). These beams must be designed for the combined effects from gravity loads (bending moments and shear forces) and seismic forces (axial compression and tension). A summary of the axial forces, bending moments, and shear forces is given in Table 14.21. The 59.3-kip axial compression and tension force in the collectors is determined in Step 10.

Table 14.21 Summary of Design Axial Forces, Bending Moments, and Shear Forces for the Collector Beams Along Column Lines C and E in Example 14.13

Load Case		Axial Force (kips)	Bending Moment (ft-kips)		Shear Force (kips)
			Negative	Positive	
Dead (D)		0	−121.8	57.4	16.5
Live (L)		0	−36.5	18.3	4.1
Seismic (Q_E)		±59.3	0	0	0
Load Combination					
ACI Eq. (5.3.1a)	$1.4D$	0	−170.5	80.4	23.1
ACI Eq. (5.3.1b)	$1.2D + 1.6L$	0	−204.6	98.2	26.4
ACI Eq. (5.3.1e)	$1.3D + 0.5L + Q_E$	±59.3	−176.6	83.8	23.5
ACI Eq. (5.3.1g)	$0.8D + Q_E$	±59.3	−97.4	45.9	13.2

The design strength interaction diagram for the collector beam with 5-#8 top bars, 5-#8 bottom bars, and 1-#5 bar on each side face is given in Figure 14.84. It is evident from the figure that the longitudinal reinforcement is adequate for the load combinations in Table 14.21.

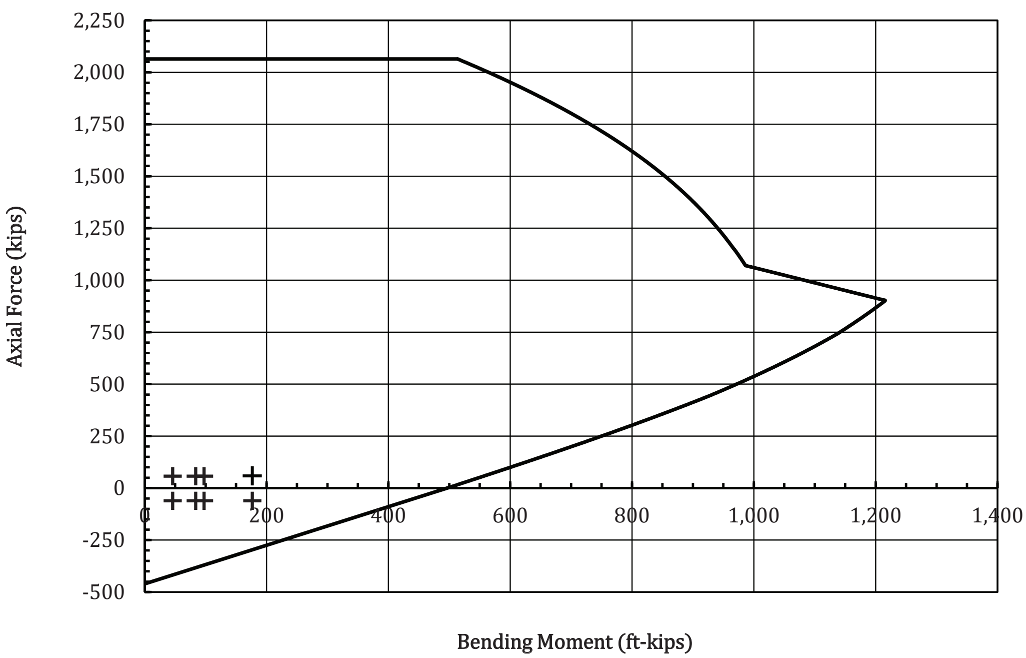


Figure 14.84 Design strength interaction diagram for the collector beams along column lines C and E in Example 14.13.

Calculate the average tensile stress in the reinforcing bars over the length of the collector and check the requirement in ACI 18.12.7.5 (note: this calculation is not necessary if the collector is designed using $f_y = 60{,}000$ psi; it is provided here for illustration purposes only):

$$\text{Average tensile stress} = \frac{59,300}{(10 \times 0.79) + (2 \times 0.31)} = 6,960 \text{ psi} < \phi f_y = 0.9 \times 60,000 = 54,000 \text{ psi}$$

Check if confinement reinforcement in accordance with ACI 18.12.7.6 must be provided.

$$\text{Maximum compressive stress} = \frac{59,300}{36.0 \times 28.5} = 57.8 \text{ psi} < 0.2 f_c' = 800 \text{ psi}$$

Thus, transverse reinforcement satisfying ACI 18.12.7.6 need not be provided. The limit of $0.2 f_c'$ is used because the maximum axial force in the collector is based on ASCE/SEI Eq. (12.10-2) with the load combinations determined in accordance with ASCE/SEI 2.3.6 (see Step 10) and not on the overstrength factor, Ω_o.

The maximum shear force in the collector is equal to 23.5 kips when the beam is subjected to combined gravity and seismic effects (see Table 14.21). The design shear strength of the concrete, ϕV_c, is determined from ACI Table 22.5.5.1 assuming at least minimum shear reinforcement is provided in the section:

$$\phi V_c = \phi \left(2\lambda \sqrt{f_c'} + \frac{N_u}{6A_g} \right) b_w d$$

$$\frac{N_u}{6A_g} = \frac{59,300}{6 \times 36.0 \times 28.5} = 9.6 \text{ psi} < 0.05 f_c' = 200.0 \text{ psi}$$

Thus,

$$\phi V_c = 0.75 \times \left[\left(2 \times 1.0\sqrt{4,000} \right) - 9.6 \right] \times 36.0 \times 26.0 / 1,000 = 82.1 \text{ kips}$$

The required spacing of #3 ties with one crosstie is equal to the following (the #3 crosstie is required to satisfy the maximum spacing of legs of shear reinforcement across the width of the member; see ACI Table 9.7.6.2.2):

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (3 \times 0.11) \times 60 \times 26.0}{23.5 - 82.1} < 0$$

$$\leq \frac{d}{2} = \frac{26.0}{2} = 13.0 \text{ in.}$$

$$\leq \frac{A_v f_{yt}}{0.75\sqrt{f_c'} b_w} = 11.6 \text{ in.}$$

$$\leq \frac{A_v f_{yt}}{50 b_w} = 11.0 \text{ in.}$$

Provide #3 ties and crossties spaced at 11.0 in. on center over the entire length of the collector beams. Note that this transverse reinforcement satisfies the detailing requirement of ACI 18.12.7.7(b) at splices and anchorage zones:

$$\text{Provided } A_v = 0.33 \text{ in.}^2 \geq \begin{cases} 0.75\sqrt{f_c'} b_w s / f_{yt} = 0.31 \text{ in.}^2 \\ 50 b_w s / f_{yt} = 0.33 \text{ in.}^2 \end{cases}$$

Reinforcement details for the collector beams are given in Figure 14.85.

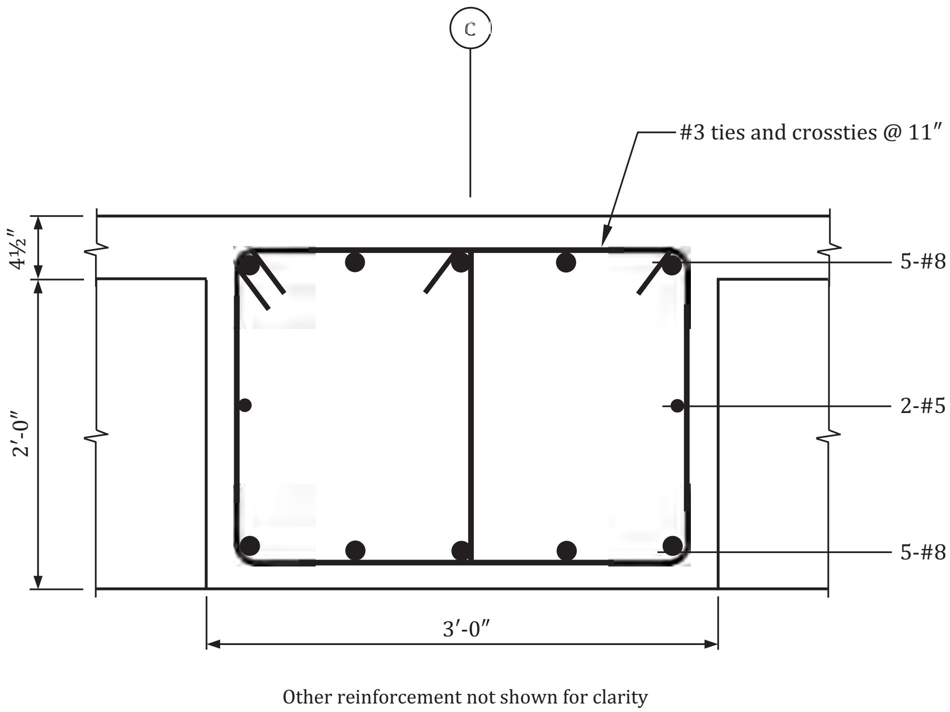


Figure 14.85 Reinforcement details for the collector beams along column lines C and E in Example 14.13.

14.9.14 Example 14.14 – Design of Foundation Seismic Tie: Building #1 (Framing Option C), SDC D

Design a foundation seismic tie for the spread footing supporting column C2 in Building #1, Framing Option C (see Figure 1.1). The column dimensions are 28 in. by 28 in. Assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement. Also assume a grade beam is used as the seismic tie.

Design data are given in Sect. 1.2.1. The building is assigned to SDC D where $S_{DS} = 0.924$.

Step 1 – Determine the seismic tie force ACI 18.13.4.3

The seismic tie must have a design strength in tension and compression of at least $0.1S_{DS}$ times the factored dead load plus factored live load in accordance with the load combination on column C2 that includes earthquake effects (assuming the axial forces on the adjacent columns are less than or equal to that on column C2).

From Table 14.16 in Example 14.4, the largest axial force corresponds to the load combination in ACI Eq. (5.3.1e) for sidesway to the left:

$$P_u = 1.39D + 0.5L + Q_E = 715.5 \text{ kips}$$

Therefore, the required axial tension and compression force in the seismic tie $= 0.1 \times 0.924 \times 715.5 = 66.1$ kips

Step 2 – Determine the required reinforcement in the seismic tie ACI 18.13.4.3

Assume the seismic tie is an 18 in. by 24 in. reinforced concrete grade beam and the grade beam is not subjected to any gravity or seismic load effects transmitted by the column or any other structural members.

Check the dimensional requirements of ACI 18.13.4.4(a):

Maximum clear spacing between columns adjacent to column C2 $= (25.0 \times 12) - 28.0 = 272.0$ in.

Smallest cross-sectional dimension $= 18.0 \text{ in.} \geq 272.0 / 20 = 13.6 \text{ in. and } 18.0 \text{ in.}$

Also assume the grade beam has a longitudinal reinforcement ratio $= 0.005$.

Thus, $A_{st} = 0.005 \times 18.0 \times 24.0 = 2.16 \text{ in.}^2$

Try 4-#7 longitudinal bars ($A_{st,provided} = 2.40 \text{ in.}^2$).

Required longitudinal reinforcement for the axial tension force in the seismic tie $= \dfrac{66.1}{0.9 \times 60} = 1.22 \text{ in.}^2 < 2.40 \text{ in.}^2$

Assuming the grade beam has transverse reinforcement consisting of ties conforming to ACI 22.4.2.4, the design axial strength, $\phi P_{n,max}$, of the section is determined using ACI Table 22.4.2.1 and ACI Eq. (22.4.2.2):

$$\phi P_{n,max} = \phi 0.80[0.85 f_c'(A_g - A_{st}) + f_y A_{st}] = 834.4 \text{ kips} > 66.1 \text{ kips}$$

Required tie spacing:

$$s = \text{lesser of} \begin{cases} h / 2 = 24.0 / 2 = 12.0 \text{ in.} \\ b_w / 2 = 18.0 / 2 = 9.0 \text{ in.} \\ 12.0 \text{ in.} \end{cases} \qquad \text{Figure 14.47}$$

Use 4-#7 longitudinal bars with #3 ties spaced 9 in. on center over the length of the grade beam.

Reinforcement details for the grade beam are similar to those in Figure 14.47. The #7 longitudinal bars are developed for tension and compression in the footing.

14.9.15 Example 14.15 – Determination of Required Reinforcement: Beam in Building #4, Beam is Not Part of the SFRS, SDC D

Determine the required reinforcement for the beam on column line B between column lines 2 and 3 at the second-floor level in Building #4. The beam is 36.0 in. wide and 28.5 in. deep, and is not part of the SFRS. Assume normal-weight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.4. The building is assigned to SDC D where $S_{DS} = 0.517$.

Step 1 – Determine the factored load combinations

All members that are not part of the SFRS must satisfy the deformation compatibility requirements in ASCE/SEI 12.12.5. This beam must be designed for the combined effects from gravity and the design displacements, δ_u, caused by the design-level earthquake in the east-west direction [see Eq. (14.39)].

A structural analysis was performed to determine the bending moments and shear forces in this beam due to δ_u, which are applied over the height of the building. Only the members that are not part of the SFRS were included in the analysis.

According to ACI 18.14.2.1, the effects caused by δ_u and the vertical ground motion are to be combined with the factored gravity load effects determined by the load combinations in ACI 5.3. A summary of the factored bending moments and shear forces for this beam at the second-floor level is given in Table 14.22.

Table 14.22 Design Bending Moments and Shear Forces at the Second-Floor Level for the Beam in Example 14.15

Load Case		Bending Moment (ft-kips) Negative	Bending Moment (ft-kips) Positive	Shear Force (kips)
Dead (D)		−128.7	70.0	35.3
Live (L)		−45.4	25.1	12.3
Seismic (Q_E)		−79.3	0	5.3
Load Combination				
ACI Eq. (5.3.1a)	$1.4D$	−180.2	98.0	49.4
ACI Eq. (5.3.1b)	$1.2D+1.6L$	−227.1	124.2	62.0
ACI Eq. (5.3.1e)	$1.3D+0.5L+Q_E$	−269.3	103.6	57.3
ACI Eq. (5.3.1g)	$0.8D+Q_E$	−182.3	56.0	33.5

Step 2 – Determine the required flexural reinforcement at the critical sections

The required area of flexural reinforcement, A_s, at the critical sections is determined by the following equations, which are given in Section 6.5.1 of this publication for rectangular sections with a single layer of tension reinforcement (an effective slab width at positive moment critical sections is not considered in this example):

$$R_n = \frac{M_u}{\phi b d^2}$$

$$A_s = \frac{0.85 f'_c b d}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right]$$

A summary of the required flexural reinforcement is given in Table 14.23. The beam is designed for $M_u \le \phi M_n$, so the requirements of ACI 18.14.3.2 are applicable. It is evident from Table 14.23 that all sections are tension-controlled because $A_s < A_{s,t}$. Also, the required reinforcement is less than the maximum amount prescribed in ACI 18.6.3.1 for Grade 60 reinforcement, which is based on $\rho = 0.025$ [see ACI 18.14.3.2(a) and Figure 14.50].

Table 14.23 Required Flexural Reinforcement for the Beam in Example 14.15

Location	M_u (ft-kips)	R_n (psi)	A_s (in.^{2})*
Negative	−269.3	148	3.12
Positive	124.2	68	3.12

* *Min.* $A_s = 200 b_w d / f_y = 200 \times 36.0 \times 26.0 / 60{,}000 = 3.12$ in.^{2}
Max. $A_s = 0.025 b_w d = 0.025 \times 36.0 \times 26.0 = 23.40$ in.^{2} for Grade 60 reinforcement
Max. $A_{s,t} = 0.018 b_w d = 0.018 \times 36.0 \times 26.0 = 16.85$ in.^{2}

Select the size and number of reinforcing bars based on the maximum and minimum spacing requirements in ACI 24.3 and 25.2, respectively.

Use 5-#8 bars for both the top and bottom reinforcement ($A_{s,provided} = 3.95$ in.$^2 > 3.12$ in.^{2}; maximum and minimum number of longitudinal bars in a single layer for a 36.0-in.-wide beam are equal to 14 and 5 from Tables 6.8 and 6.9 of this publication, respectively).

Step 3 – Determine the required transverse reinforcement

According to ACI 18.14.3.2(a), stirrups are permitted to be used because the factored axial force on the beam is less than $A_g f_c' / 10$ (see Figure 14.50).

The size and spacing of the stirrups can be determined by the following equation:

$$\frac{A_v}{s} \geq \frac{(V_u - \phi V_c)}{\phi f_{yt} d} \qquad \text{Eq. (14.11)}$$

The design shear strength of the concrete, ϕV_c, is determined from ACI Table 22.5.5.1 assuming at least minimum shear reinforcement is provided in the section:

$$\phi V_c = \phi 2\lambda\sqrt{f_c'}b_w d = 0.75 \times 2 \times 1.0\sqrt{4{,}000} \times 36.0 \times 26.0 / 1{,}000 = 88.8 \text{ kips}$$

It is evident that $\phi V_c > V_u = 62.0$ kips (see Table 14.22).

Therefore, determine the stirrup spacing based on minimum shear reinforcement requirements:

$$s \leq \frac{d}{2} = \frac{26.0}{2} = 13.0 \text{ in.}$$

$$\leq \frac{A_v f_{yt}}{0.75\sqrt{f_c'}b_w} = \frac{(3 \times 0.11) \times 60{,}000}{0.75 \times \sqrt{4{,}000} \times 36.0} = 11.6 \text{ in.}$$

$$\leq \frac{A_v f_{yt}}{50 b_w} = \frac{(3 \times 0.11) \times 60{,}000}{50 \times 36.0} = 11.0 \text{ in.}$$

Provide #3 stirrups and crossties spaced at 11.0 in. over the full length of the beam.

Reinforcement details for this beam are given in Figure 14.86.

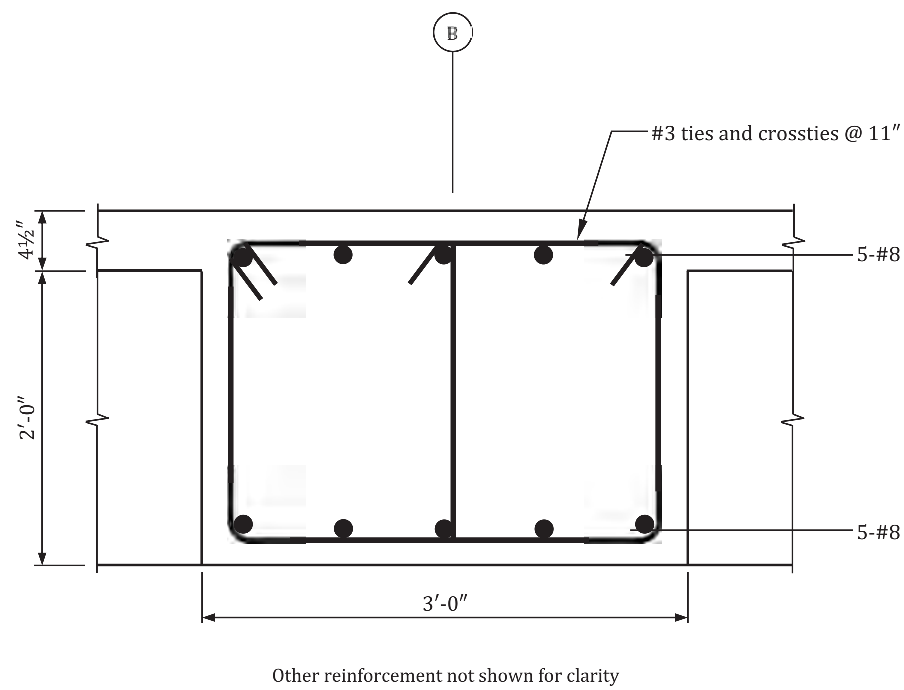


Figure 14.86 Reinforcement details for the beam in Example 14.15.

14.9.16 Example 14.16 – Determination of Required Reinforcement: Column in Building #4, Column is Not Part of the SFRS, SDC D

Determine the required reinforcement for column B2 in the first story of Building #4. Assume the column is 48.0 in. by 48.0 in. and is not part of the SFRS. Also assume normalweight concrete with $f_c' = 6{,}000 \text{ psi}$ and Grade 60 reinforcement.

Design data are given in Sect. 1.2.4. The building is assigned to SDC D where $S_{DS} = 0.517$.

Step 1 – Determine the factored load combinations

All members that are not part of the SFRS must satisfy the deformation compatibility requirements in ASCE/SEI 12.12.5. This column must be designed for the combined effects from gravity and the design displacements, δ_u, caused by the design-level earthquake in the east-west direction [see Eq. (14.39)].

A structural analysis was performed to determine the axial forces, bending moments, and shear forces in this column due to δ_u, which are applied over the height of the building. Only the members that are not part of the SFRS were included in the analysis.

According to ACI 18.14.2.1, the effects caused by δ_u and the vertical ground motion are to be combined with the factored gravity load effects determined by the load combinations in ACI 5.3. A summary of the factored axial forces, bending moments, and shear forces for this column in the first story is given in Table 14.24.

Table 14.24 Design Axial Forces, Bending Moments, and Shear Forces for Column B2 in Example 14.16

Load Case			Axial Force (kips)	Bending Moment (ft-kips)	Shear Force (kips)
Dead (D)			4,632.7	25.5	7.5
Live (L)			1,602.3	9.2	2.7
Roof live load (L_r)			24.0		
Seismic (Q_E)			±307.0	±311.0	±28.5
Load Combination					
ACI Eq. (5.3.1a)	$1.4D$		6,485.8	35.7	10.5
ACI Eq. (5.3.1b)	$1.2D + 1.6L + 0.5L_r$		8,134.9	45.3	13.3
ACI Eq. (5.3.1c)	$1.2D + 1.6L_r + 0.5L$		6,398.8	35.2	10.4
ACI Eq. (5.3.1e)	$1.3D + 0.5L + Q_E$	SSR	6,516.7	348.8	39.6
		SSL	7,130.7	−273.3	−17.4
ACI Eq. (5.3.1g)	$0.8D + Q_E$	SSR	3,399.2	331.4	34.5
		SSL	4,013.2	−290.6	−22.5

SSR = sidesway to the right, SSL = sidesway to the left

Step 2 – Determine the required longitudinal reinforcement

The column is designed so that the required strength is less than or equal to the design strength. Thus, the requirements of ACI 18.14.3.2 are applicable (see Figure 14.52).

The design strength interaction diagram for this column reinforced with 28-#11 bars $(0.010 < \rho_{st} = 0.019 < 0.060)$ is given in Figure 14.87. All load combinations fall within the boundary of the design strength interaction diagram for this column.

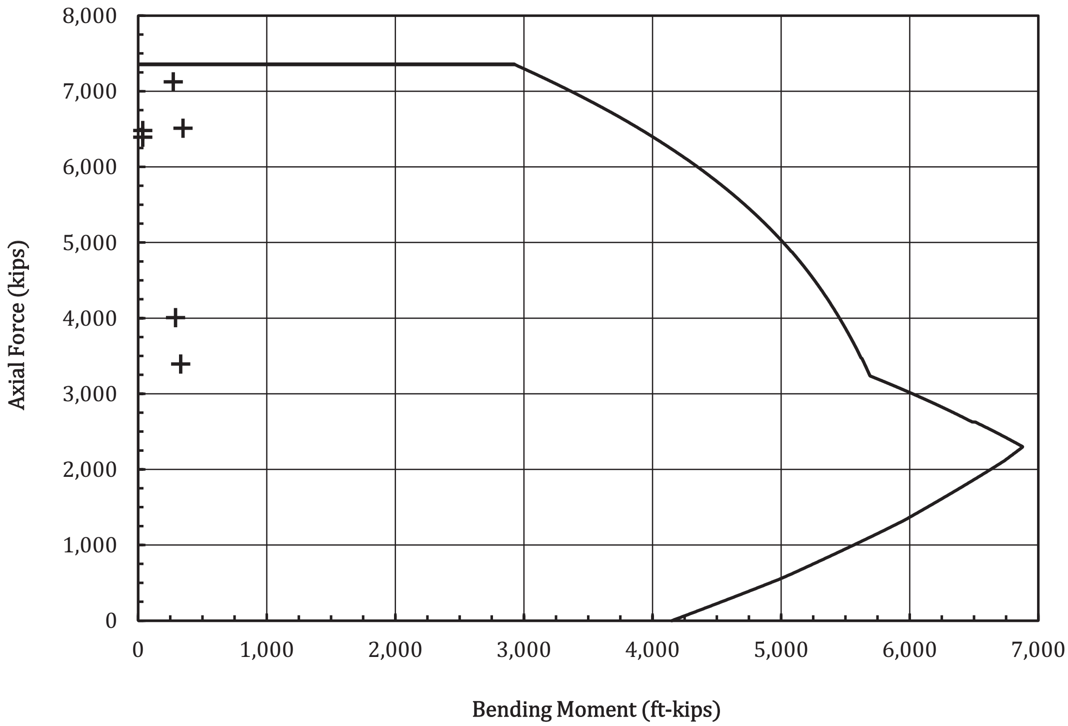


Figure 14.87 Design strength interaction diagram for column B2 in Example 14.16.

Use 28-#11 bars.

Step 3 – Determine the required transverse reinforcement

The required transverse reinforcement depends on the magnitude of the largest factored gravity axial force with respect to $0.35P_o$ where the nominal axial strength at zero eccentricity, P_o, is determined as follows:

$$P_o = 0.85f_c'(A_g - A_{st}) + f_yA_{st} = [0.85 \times 6 \times (48.0^2 - 43.7)] + (60 \times 43.7) = 14{,}149.5 \text{ kips} \qquad \text{ACI Eq. (22.4.2.2)}$$

The largest factored gravity force is equal to 8,134.9 kips from Table 14.24, which is greater than $0.35P_o = 4{,}952.3$ kips. Therefore, the transverse reinforcement must satisfy the requirements of ACI 18.14.3.2(c) [see Figure 14.52].

Determine the length over which flexural yielding is likely to occur:

$$\ell_o \geq \text{greatest of} \begin{cases} \text{Maximum cross-sectional dimension of the column} = 48.0 \text{ in.} \\ \text{Clear span of the column}/6 = [(11.0 \times 12) - (28.5 / 2)] / 6 = 19.6 \text{ in.} \\ 18 \text{ in.} \end{cases} \qquad \text{Eq. (14.15)}$$

Maximum hoop spacing is equal to the following:

$$s = \text{smaller of} \begin{cases} 6d_b = 6 \times 1.41 = 8.5 \text{ in.} \\ \\ 6.0 \text{ in.} \end{cases} \qquad \text{Figure 14.52}$$

In both directions, $b_c = 48.0 - (2 \times 1.5) = 45.0$ in.

$$A_{ch} = 45.0 \times 45.0 = 2{,}025.0 \text{ in.}^2$$

$$A_{sh} = \text{larger of} \begin{cases} 0.15 s b_c \left(\dfrac{A_g}{A_{ch}} - 1 \right) \dfrac{f'_c}{f_{yt}} = 0.15 \times 6.0 \times 45.0 \times \left(\dfrac{48.0^2}{2{,}025.0} - 1 \right) \times \dfrac{6}{60} = 0.56 \text{ in.}^2 \\ \\ 0.045 s b_c \dfrac{f'_c}{f_{yt}} = 0.045 \times 6.0 \times 45.0 \times \dfrac{6}{60} = 1.22 \text{ in.}^2 \end{cases}$$

#5 hoops with four #5 crossties spaced at 6.0 in. provides $A_{sh} = 6 \times 0.31 = 1.86 \text{ in.}^2 > 1.22 \text{ in.}^2$

Check if this transverse reinforcement is adequate for shear.

According to ACI 18.14.3.2(b), the design shear force, V_u, must be determined in accordance with ACI 18.7.6. The probable flexural strengths, M_{pr}, at the ends of the column associated with the range of factored axial forces are obtained from the design strength interaction diagram with $f_y = 1.25 \times 60 = 75$ ksi and $\phi = 1.0$ (see Figure 14.88). It is evident from the figure that the largest M_{pr} is equal to 9,185.4 ft-kips, which corresponds to an axial force of 4,013.2 kips for sidesway to the left.

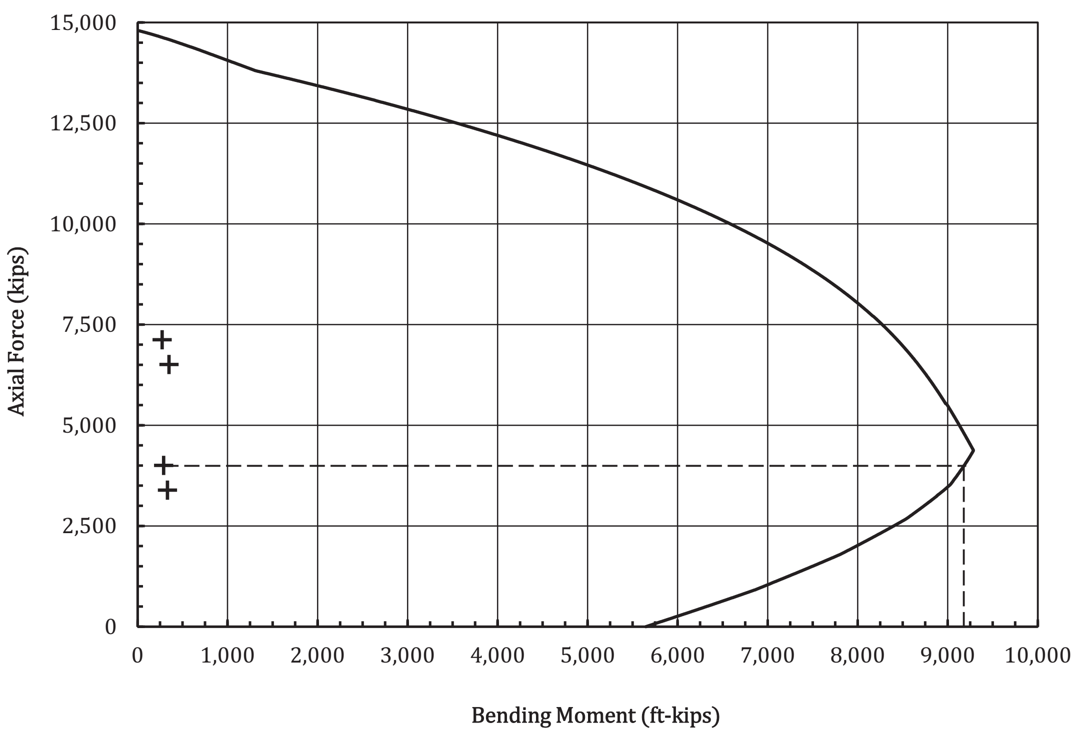


Figure 14.88 Nominal strength interaction diagram for column B2 with f_y = **75 ksi** and ϕ = **1.0**.

Therefore,

$$V_u = \frac{2M_{pr}}{\ell_u} = \frac{2 \times 9{,}185.4}{11.0 - (28.5 / 24)} = 1{,}872.2 \text{ kips}$$ Figure 14.15

This shear force is greater than the maximum shear force of 39.6 kips determined from analysis (see Table 14.24).

The earthquake-induced shear force is the total shear force on the column. Also, the minimum factored axial force including earthquake effects is equal to 3,399.2 kips, which is greater than $A_g f_c' / 20 = 691.2$ kips. Therefore, V_c is permitted to be included when determining the required shear reinforcement (ACI 18.7.6.2.1):

$$\frac{N_u}{6A_g} = \frac{4{,}013.2}{6 \times 48.0^2} = 0.29 \text{ ksi} < 0.05 f_c' = 0.30 \text{ ksi}$$

$$V_c = \left(2\lambda\sqrt{f_c'} + \frac{N_u}{6A_g}\right) b_w d = \left(\frac{2 \times 1.0 \times \sqrt{6{,}000}}{1{,}000} + 0.29\right) \times 48.0 \times 40.0 = 854.3 \text{ kips}$$

Eq. (14.17)

$$> 5\lambda\sqrt{f_c'} b_w d = 743.6 \text{ kips, use } V_c = 743.6 \text{ kips}$$

In this equation, d is determined from a strain compatibility analysis of the section for the axial compressive force $N_u = 4{,}013.2$ kips, which corresponds to the M_{pr} used to determine V_u.

The required spacing of the #5 hoops and crossties in the direction of analysis is equal to the following:

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (6 \times 0.31) \times 60 \times 40.0}{1{,}872.2 - (0.75 \times 743.6)} = 2.6 \text{ in.} < 6.0 \text{ in.}$$

Thus, the size and spacing of the hoops determined previously are not adequate for shear.

Use #6 hoops with six crossties spaced at 4.0 in. on center over a distance of at least $\ell_o = 48.0$ in. from each end of the column $(\phi V_n = 2{,}141.7 \text{ kips})$.

Outside of ℓ_o, #6 hoops with four crossties spaced at 6.0 in. on center are permitted to be used. However, given the extent of ℓ_o from each end of the column (that is, only $9.8 - 8.0 = 1.8$ ft of the center portion of the column is outside of ℓ_o), use #6 hoops and crossties spaced at 4.0 in. on center over the entire height of the column and within the joints.

Determine the tension development length, ℓ_d, of the #11 longitudinal bars:

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s\psi_g}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right) d_b \geq 12.0 \text{ in.}$$ ACI Eq. (25.4.2.4a)

The modification factors are as follows: ACI Table 25.4.2.5

For normalweight concrete, $\lambda = 1.0$

For Grade 60 reinforcement, $\psi_g = 1.0$

For uncoated reinforcing bars, $\psi_e = 1.0$

For #11 reinforcing bars, $\psi_s = 1.0$

$\psi_t = 1.0$

For #6 hoops:

$$c_b = \text{ lesser of } \begin{cases} c_c + (d_b)_{hoop} + 0.5(d_b)_{long.} = 1.5 + 0.75 + (0.5 \times 1.41) = 3.0 \text{ in.} \\ \dfrac{s}{2} = \dfrac{48.0 - [2 \times (1.5 + 0.75)] - 1.41}{2 \times 7} = 3.0 \text{ in.} \end{cases}$$

Set $K_{tr} = 0$. ACI 25.4.2.4

$(c_b + K_{tr}) / d_b = (3.0 + 0) / 1.41 = 2.1 < 2.5$

Therefore,

$$\ell_d = \left(\frac{3}{40} \frac{60{,}000}{1.0\sqrt{6{,}000}} \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.1} \right) \times 1.41 = 39.0 \text{ in.} = 3.3 \text{ ft}$$

Class B tension lap splice length $= 1.3 \times 3.3 = 4.3$ ft

Use a 4 ft-4 in. lap splice length

Reinforcement details for this column are given in Figure 14.89.

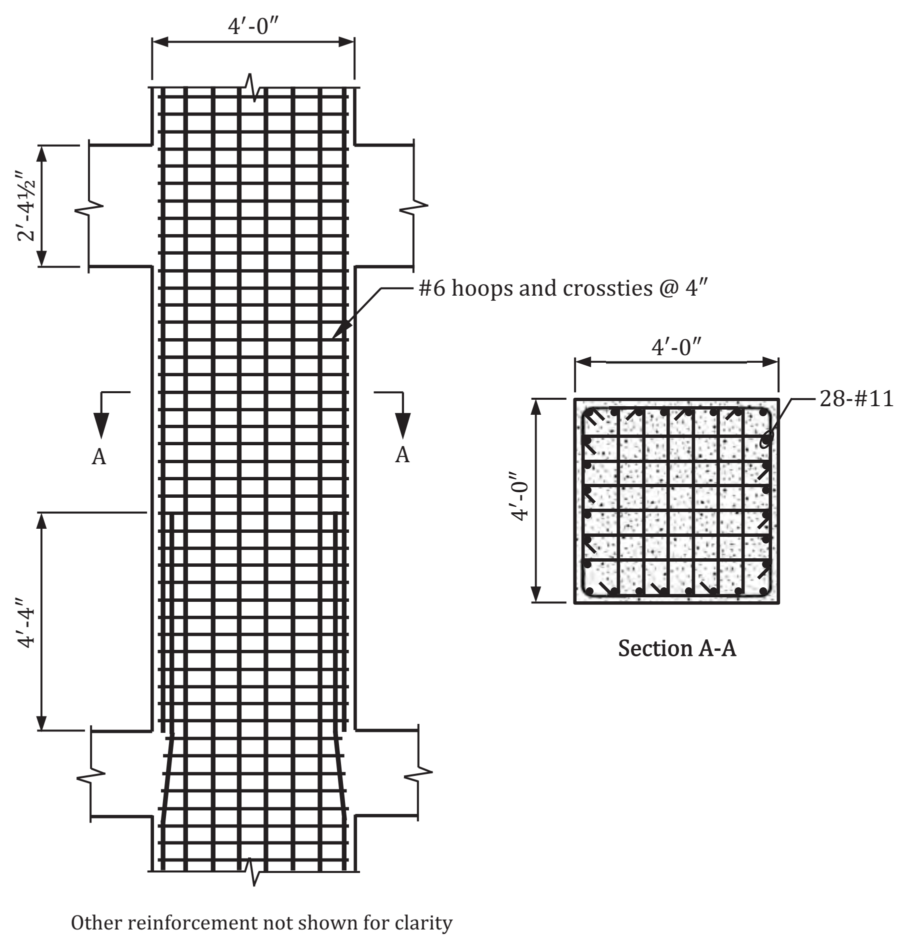


Figure 14.89 Reinforcement details for column B2 in Example 14.16.

Comments. It may be possible to reduce the shear demand on the column (thereby reducing the required area of transverse reinforcement and/or increasing the required spacing of the transverse reinforcement) by performing a more refined analysis to determine the joint strengths at the second-floor level (top of the column) based on M_{pr} of the beams framing into the joint, which is permitted in ACI 18.7.6.1.1. At the bottom of the column, M_{pr} of the column must still be used to determine the shear force.

14.9.17 Example 14.17 – Check of Slab-Column Connection: Column in Building #3, Column is Not Part of the SFRS, SDC D

Check the slab-column connection at column B3 at the sixteenth-floor level of Building #3. The column is 24.0 in. by 24.0 in. and is not part of the SFRS. The slab is 8.0 in. thick. Assume normalweight concrete with $f_c' = 4{,}000$ psi and Grade 60 reinforcement.

Design data are given in Sect. 1.2.3. The building is assigned to SDC D where $S_{DS} = 0.924$.

Step 1 – Determine if slab shear reinforcement is required

Slab shear reinforcement is required where the following equation is satisfied:

$$\frac{\Delta_x}{h_{sx}} \geq 0.035 - \frac{1}{20}\left(\frac{v_{uv}}{\phi v_c}\right) \qquad \text{Eq. (14.42)}$$

In the sixteenth and fifteenth stories, the design story drifts, Δ_x, are equal to 1.85 in. and 1.90 in. respectively. Thus, use $\Delta_x = 1.90$ in.

Slab weight $= \frac{8.0}{12} \times 150 = 100 \text{ lb/ft}^2$

Superimposed dead load $= 10 \text{ lb/ft}^2$

Live load $= 40 \text{ lb/ft}^2$

In accordance with ACI 18.14.5.1, use only the load combinations that include E to determine v_{uv}.

ACI Eq. (5.3.1e): $U = 1.2D + E + 0.5L$

$E = \rho Q_E + 0.2 S_{DS} D = (1.0 \times Q_E) + (0.2 \times 0.924 \times D) = Q_E + 0.19D$

Therefore,

$U = (1.2 + 0.19)D + 0 + 0.5L = 1.39D + 0.5L$ (earthquake effects, Q_E, are equal to zero because the two-way slab and column are not part of the SFRS).

$q_u = (1.39 \times 110) + (0.5 \times 40) = 173 \text{ lb/ft}^2$

$d = 8.0 - 1.25 = 6.75$ in.

$b_1 = b_2 = 24.0 + 6.75 = 30.75$ in.

$$V_u = 173 \times \left[\left(22.0 \times \frac{20.5 + 19.0}{2}\right) - \frac{30.75^2}{144}\right] = 74{,}033 \text{ lb}$$

$b_o = 4 \times 30.75 = 123.0$ in.

$$v_{uv} = \frac{V_u}{b_o d} = \frac{74{,}033}{123.0 \times 6.75} = 89.2 \text{ psi}$$

$$\phi v_c = \phi 4\lambda_s \lambda \sqrt{f_c'} = 0.75 \times 4 \times 1.0 \times 1.0 \times \sqrt{4{,}000} = 189.7 \text{ psi} \qquad \text{ACI Table 22.6.5.2}$$

where $\lambda_s = 1.0$ in accordance with ACI 22.5.5.1.3 and $\lambda = 1.0$ for normalweight concrete.

$$\frac{\Delta_x}{h_{sx}} = \frac{1.90}{9.5 \times 12} = 0.017 > 0.035 - \frac{1}{20}\left(\frac{v_{uv}}{\phi v_c}\right) = 0.035 - \left[\frac{1}{20} \times \left(\frac{89.2}{189.7}\right)\right] = 0.012$$

Therefore, shear reinforcement in accordance with ACI 18.14.5.3 and either ACI 8.7.6 or 8.7.7 must be provided.

Step 2 – Determine the required slab shear reinforcement

Headed shear stud reinforcement in accordance with ACI 18.14.5.3 and 8.7.7 is used in this example.

Required shear reinforcement must provide $v_s \geq 3.5\sqrt{f_c'} = 221.4$ psi.

Try ¾-in. diameter headed shear studs $(A_b = 0.442 \text{ in.}^2)$ with $f_{yt} = 51{,}000$ psi.

In the direction parallel to the column face, the maximum spacing between adjacent lines of headed shear studs $= 2d = 13.5$ in. (ACI Table 8.7.7.1.2).

For a 24-in.-wide column, use 3 lines of headed shear studs on each face of the column (maximum spacing $\cong 24.0 / 2 = 12.0$ in. < 13.5 in.).

In the direction perpendicular to the column face, maximum spacing $= 0.75d = 5.1$ in. (ACI Table 8.7.7.1.2).

Therefore,

$$v_s = \frac{A_b f_{yt}}{b_o s} = \frac{(12 \times 0.442) \times 51{,}000}{123.0 \times 5.0} = 439.8 \text{ psi} > 3.5\sqrt{f_c'} = 221.4 \text{ psi}$$

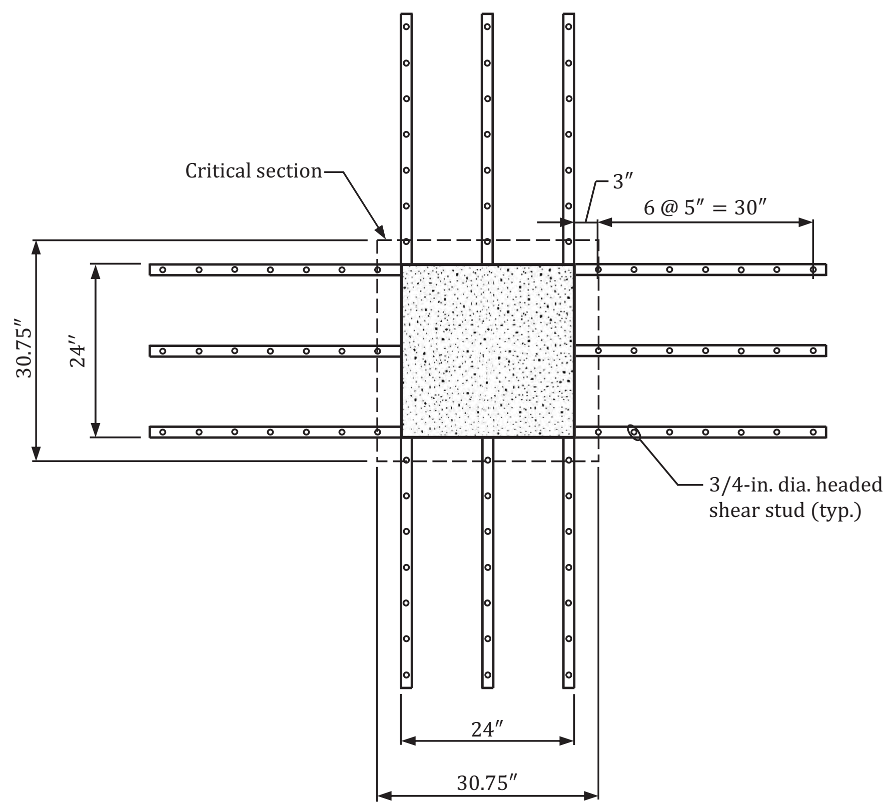


Figure 14.90 Headed shear stud reinforcement at column B3 in Example 14.17.

In accordance with ACI 18.14.5.3, the headed shear studs must extend at least $4h = 32.0 \text{ in.}$ from the face of the column. Provide 7 headed shear studs in each line on each face of the column with the first peripheral line of headed shear studs located $3.0 \text{ in.} < d\,/\,2 = 3.4 \text{ in.}$ from the face of the column (see ACI Table 8.7.7.1.2).

Headed shear stud reinforcement details for column B3 is given in Figure 14.90.

Appendix A
REFERENCES

1. American Concrete Institute (ACI). 2019. *Building Code Requirements for Structural Concrete and Commentary*, ACI 318-19, Farmington Hills, MI.
2. International Code Council (ICC). 2018. *International Building Code*, Washington, D.C.
3. American Society of Civil Engineers (ASCE). 2017. *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*, ASCE/SEI 7-16, Reston, VA.
4. American Society of Civil Engineers (ASCE). ASCE 7 Hazard Tool. https://asce7hazardtool.online/.
5. Applied Technology Council (ATC). ATC Hazards by Location. https://hazards.atcouncil.org/.
6. Structural Engineers Association of California (SEAOC) and Office of Statewide Health Planning and Development (OSHPD). Seismic Design Maps. https://seismicmaps.org/.
7. Concrete Reinforcing Steel Institute (CRSI). 2016. *Design Guide for Economical Reinforced Concrete Structures*, Schaumburg, IL.
8. Concrete Reinforcing Steel Institute (CRSI). 2018. *Manual of Standard Practice*, Schaumburg, IL.
9. Concrete Reinforcing Steel Institute (CRSI). 2017. *Reinforcing Bars: Anchorages and Splices*, Schaumburg, IL.
10. Concrete Reinforcing Steel Institute (CRSI). 2017. *Design and Detailing of Low-Rise Reinforced Concrete Buildings*, Schaumburg, IL.
11. Concrete Reinforcing Steel Institute (CRSI). 2014. *Design Guide for Voided Concrete Slabs*, Schaumburg, IL.
12. American Concrete Institute (ACI). 2008. *Guide to Shear Reinforcement for Slabs*, ACI 421.1R, Farmington Hills, MI.
13. ASTM International. 2019. *Standard Specification for Steel Stud Assemblies for Shear Reinforcement of Concrete*, ASTM A1044/A1044M-16ae1, West Conshohocken, PA.
14. Computers and Structures, Inc. (CSI). 2019. ETABS – Integrated Analysis, Design and Drafting of Building Systems, Version 16.2.1, Walnut Creek, CA.
15. American Concrete Institute (ACI). 2002. *Examples for the Design of Structural Concrete with Strut-and-Tie Models*, SP-208, Farmington Hills, MI.
16. Concrete Reinforcing Steel Institute (CRSI). 2018. “Design Guide for Reinforced Concrete Columns.” Schaumburg, IL.
17. Bresler, B. 1960. *Design Criteria for Reinforced Concrete Columns under Axial Load and Biaxial Bending*, ACI Journal, Vol. 57, No. 5, pp. 481-490.
18. Portland Cement Association (PCA). 2008. *Notes on ACI 318-08 Building Code Requirements for Structural Concrete*, Skokie, IL.
19. Furlong, R.W., Hsu, C.-T.T., and Mirza, S.A. *Analysis and Design of Concrete Columns for Biaxial Bending – Overview*, ACI Structural Journal, Vol. 101, No. 3, pp. 413-423.
20. Concrete Reinforcing Steel Institute (CRSI). 2014. *Design Guide for Cantilevered Retaining Walls*, Schaumburg, IL.
21. Concrete Reinforcing Steel Institute (CRSI). 2019. *Design Guide for Reinforced Concrete Diaphragms*, Schaumburg, IL.
22. Structural Engineers Association of California (SEAOC). 2005. *Using a Concrete Slab as a Seismic Collector*, Structural Engineers Association of California, Sacramento, CA.
23. Concrete Reinforcing Steel Institute (CRSI). 2015. *Design Guide for Pile Caps*, Schaumburg, IL.

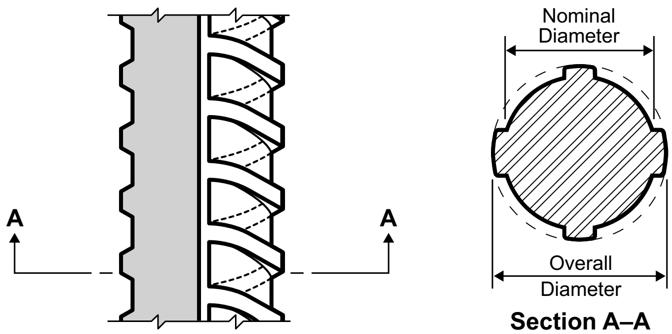
24. American Concrete Institute (ACI). 2014. *Report on Design and Construction of Drilled Piers*, ACI 336.3R, Farmington Hills, MI.
25. Concrete Reinforcing Steel Institute (CRSI). 2016. *Design Guide for Drilled Piers*, Schaumburg, IL.
26. American Concrete Institute (ACI). 2002. *Recommendations for Design of Beam-Column Connections in Monolithic Reinforced Concrete Structures*, ACI 352R (Reapproved 2010), Farmington Hills, MI.
27. Moehle, J. (2015). Seismic Design of Reinforced Concrete Buildings. McGraw-Hill Education, New York, NY.

Appendix B
REINFORCING BAR DATA

Table B.1 ASTM Standard Reinforcing Bars

Bar Size	Nominal Diameter (in.)	Nominal Area (in.2)	Nominal weight (lb/ft)
#3	0.375	0.11	0.376
#4	0.500	0.20	0.668
#5	0.625	0.31	1.043
#6	0.750	0.44	1.502
#7	0.875	0.60	2.044
#8	1.000	0.79	2.670
#9	1.128	1.00	3.400
#10	1.270	1.27	4.303
#11	1.410	1.56	5.313
#14	1.693	2.25	7.650
#18	2.257	4.00	13.600

Table B.2 Overall Reinforcing Bar Diameters



Bar Size	Approximate Diameter to the Outside Deformations (in.)
#3	7/16
#4	9/16
#5	1-1/16
#6	7/8
#7	1
#8	1-1/8
#9	1-1/4
#10	1-7/16
#11	1-5/8
#14	1-7/8
#18	2-1/2

Appendix C
SECTION INDEX

ACI Section Number		Design Guide Chapter/Page Number(s)
General	1.10	One-way slabs/4-16
		Beams/6-43
Terminology	2.2	Earthquake-resistant structures (SDC D, E, and F)/14-10
	2.3	Walls/8-1
		Earthquake-resistant structures (SDC D, E, and F)/14-12, 14-36, 14-37
Structural System Requirements	4.2.1	Material requirements and strength reduction factors/2-1
	4.2.2	Material requirements and strength reduction factors/2-1
	4.3	Design loads and load combinations/3-1, 3-11
	4.4.6.1	Design loads and load combinations/3-1
		Earthquake-resistant structures (overview)/12-1
	4.4.6.2	Design loads and load combinations/3-8
	4.4.6.3	Design loads and load combinations/3-2
	4.4.6.4	Design loads and load combinations/3-2
	4.4.6.5	Design loads and load combinations/3-2
	4.4.7.1	Design loads and load combinations/3-11
	4.4.7.5	Design loads and load combinations/3-16
	4.4.7.6	Design loads and load combinations/3-16
	4.11.1	One-way slabs/4-2
		Two-way slabs/5-5, 5-7, 5-8
		Beams/6-2
	4.11.2	One-way slabs/4-2, 4-8
Loads	5.2.1	Design loads and load combinations/3-1
	5.2.2	Design loads and load combinations/3-1
	5.2.3	Design loads and load combinations/3-3
	5.3	Foundations/10-25, 10-27
	5.3.1	Foundations/10-50
	5.3.2	Design loads and load combinations/3-8
	5.3.3	Design loads and load combinations/3-4
		Earthquake-resistant structures (SDC B and C)/13-5
		Earthquake-resistant structures (SDC D, E, and F)/14-11
	5.3.4	Design loads and load combinations/3-6
	5.3.5	Design loads and load combinations/3-6
	5.3.6	Design loads and load combinations/3-7
	5.3.7	Design loads and load combinations/3-7
	5.3.8	Design loads and load combinations/3-7
	5.3.9	Design loads and load combinations/3-7
	5.3.10	Design loads and load combinations/3-7

ACI Section Number		Design Guide Chapter/Page Number(s)
Structural Analysis	6.2	Earthquake-resistant structures (SDC B and C)/13-1, 13-4
		Earthquake-resistant structures (SDC D, E, and F)/14-1
	6.2.3	One-way slabs/4-2, 4-4
		Two-way slabs/5-11
		Beams/6-5
		Columns/7-1
		Walls/8-1
	6.2.4	Two-way slabs/5-11
	6.2.4.1	Two-way slabs/5-11, 5-26
	6.2.5	Columns/7-5, 7-7
		Walls/8-4, 8-5
	6.2.5.1	Columns/7-5, 7-6, 7-8
	6.2.5.3	Columns/7-1, 7-3
	6.2.6	Columns/7-103
	6.3.2	Beams/6-34
		Earthquake-resistant structures (SDC D, E, and F)/14-18, 14-87, 14-99
	6.3.2.1	Beams/6-14, 6-65, 6-80, 6-98, 6-112
	6.5	One-way slabs/4-2, 4-22, 4-28
		Beams/6-5, 6-9, 6-51
	6.5.1	One-way slabs/4-2, 4-4, 4-25, 4-27
		Beams/6-5, 6-63, 6-64, 6-73, 6-79, 6-97, 6-110, 6-111
	6.5.2	One-way slabs/4-3
		Beams/6-5
	6.5.3	One-way slabs/4-4
		Beams/6-5
	6.5.4	One-way slabs/4-3, 4-25, 4-27
		Beams/6-5
	6.6	Two-way slabs/5-11, 5-35, 5-70, 5-92, 5-98, 5-106, 5-116
	6.6.1	Diaphragms/9-6
	6.6.2	Diaphragms/9-6
	6.6.3	Diaphragms/9-6
	6.6.3.1	Two-way slabs/5-36
		Columns/7-4
		Walls/8-3
	6.6.3.1.1	Two-way slabs/5-35, 5-36
		Columns/7-3
		Walls/8-1
	6.6.3.1.2	Two-way slabs/5-35, 5-36
		Columns/7-4
		Walls/8-3
		Diaphragms/9-8
	6.6.3.1.3	Two-way slabs/5-36
	6.6.3.2.2	Two-way slabs/5-36
		Columns/7-3
		Walls/8-1
	6.6.4	Columns/7-2, 7-10
		Walls/8-5, 8-40
	6.6.4.3	Columns/7-5, 7-6, 7-88, 7-106
		Walls/8-3
	6.6.4.4.4	Columns/7-2, 7-11, 7-13

	ACI Section Number	Design Guide Chapter/Page Number(s)
Structural Analysis (cont.)	6.6.4.5	Columns/7-2, 7-10, 7-13, 7-103
		Foundations/10-1
	6.6.4.5.3	Walls/8-40
	6.6.4.5.4	Columns/7-10, 7-11
	6.6.4.6	Columns/7-12, 7-108
	6.6.4.6.2	Columns/7-13
	6.6.4.6.3	Columns/7-13
	6.6.4.6.4	Columns/7-13, 7-110
	6.6.5	Beams/6-9
	6.7	Two-way slabs/5-11, 5-36
		Columns/7-13
		Walls/8-5
	6.7.1.1	Columns/7-2
	6.7.1.2	Columns/7-2
	6.7.2.1.1	Two-way slabs/5-36
	6.7.2.2.2	Two-way slabs/5-36
	6.8	Two-way slabs/5-11
		Beams/6-9
		Walls/8-5
	6.8.1.1	Columns/7-2
	6.8.1.2	Columns/7-2
	6.8.1.3	Columns/7-2
	6.9	Two-way slabs/5-11
		Columns/7-2
One-way Slabs	7.3.1.1	One-way slabs/4-1, 4-25, 4-26
		Diaphragms/9-1
	7.3.1.1.1	One-way slabs/4-1
	7.3.1.1.2	One-way slabs/4-1
	7.3.2	One-way slabs/4-1
	7.3.3.1	One-way slabs/4-5
	7.4.1.1	One-way slabs/4-2
	7.4.1.2	One-way slabs/4-2, 4-28
	7.4.2.1	One-way slabs/4-3
	7.4.3.1	One-way slabs/4-3
	7.4.3.2	One-way slabs/4-3, 4-4, 4-25, 4-27
	7.5.1.1	One-way slabs/4-4, 4-25, 4-26, 4-27
	7.5.2.1	One-way slabs/4-5
	7.5.3	Diaphragms/9-22
	7.5.3.1	One-way slabs/4-6
	7.6.1	Earthquake-resistant structures (SDC D, E, and F)/14-63
	7.6.1.1	One-way slabs/4-8, 4-29
		Diaphragms/9-22
		Foundations/10-30, 10-34, 10-39, 10-46, 10-48, 10-52, 10-55
		Earthquake-resistant structures (SDC D, E, and F)/14-136
	7.6.4.1	One-way slabs/4-8, 4-30
	7.7.1.1	One-way slabs/4-8
	7.7.1.2	One-way slabs/4-10
	7.7.1.3	One-way slabs/4-20
	7.7.2.1	One-way slabs/4-8
	7.7.2.2	One-way slabs/4-9
	7.7.2.3	One-way slabs/4-9
		Foundations/10-30

	ACI Section Number	Design Guide Chapter/Page Number(s)
One-way Slabs (cont.)	7.7.3.1	One-way slabs/4-17
	7.7.3.2	One-way slabs/4-17
	7.7.3.3	One-way slabs/4-18, 4-19
	7.7.3.4	One-way slabs/4-18
	7.7.3.5	One-way slabs/4-18, 4-19
	7.7.3.8.2	One-way slabs/4-19
	7.7.3.8.3	One-way slabs/4-19
	7.7.3.8.4	One-way slabs/4-18
	7.7.7	One-way slabs/4-19, 4-22, 4-31
Two-way Slabs	8.2.2	Two-way slabs/5-45
	8.2.4	Two-way slabs/5-5, 5-37, 5-62, 5-99
	8.2.5	Two-way slabs/5-15
	8.3.1.1	Two-way slabs/5-2, 5-3, 5-5, 5-6, 5-56, 5-57, 5-62, 5-65, 5-67
		Diaphragms/9-1
	8.3.1.2	Two-way slabs/5-2, 5-7, 5-59, 5-61, 6-78
	8.3.1.2.1	Two-way slabs/5-8
	8.3.1.3	Two-way slabs/5-2
	8.3.1.4	Two-way slabs/5-2
	8.3.2	Two-way slabs/5-10
	8.3.2.1	Two-way slabs/5-2
	8.3.3.1	Two-way slabs/5-36, 5-72, 5-84, 5-93, 5-99, 5-118
	8.4.1.1	Two-way slabs/5-11
	8.4.1.2	Two-way slabs/5-11
	8.4.1.5	Two-way slabs/5-11, 5-68, 5-80, 5-88, 5-95, 5-104, 5-115
	8.4.1.6	Two-way slabs/5-11, 5-68, 5-80, 5-88, 5-95, 5-104, 5-115
	8.4.1.7	Two-way slabs/5-11
	8.4.1.8	Two-way slabs/5-4, 5-7
	8.4.1.9	Two-way slabs/5-11, 5-72, 5-107, 5-117
		Earthquake-resistant structures (SDC B and C)/13-23
	8.4.2.1	Two-way slabs/5-12
	8.4.2.2	Two-way slabs/5-12, 5-73, 5-85, 5-94, 5-100, 5-109, 5-119
		Foundations/10-63
		Earthquake-resistant structures (SDC B and C)/13-50
	8.4.2.2.2	Two-way slabs/5-12
	8.4.2.2.3	Two-way slabs/5-12
		Diaphragms/9-22
	8.4.2.2.4	Two-way slabs/5-13
	8.4.2.2.5	Two-way slabs/5-14
	8.4.3	Two-way slabs/5-126
		Earthquake-resistant structures (SDC B and C)/13-25
	8.4.3.1	Two-way slabs/5-14
	8.4.3.2	Two-way slabs/5-14
	8.4.4	Two-way slabs/5-127, 5-133, 5-138, 5-141, 5-144, 5-150
		Earthquake-resistant structures (SDC B and C)/13-25, 13-57
	8.4.4.1.1	Two-way slabs/5-14
	8.4.4.1.2	Two-way slabs/5-20
	8.4.4.2	Two-way slabs/5-12
	8.4.4.2.1	Two-way slabs/5-20
	8.4.4.2.2	Two-way slabs/5-20
	8.4.4.2.3	Two-way slabs/5-20
	8.5.1.1	Two-way slabs/5-36
	8.5.2.1	Two-way slabs/5-36

	ACI Section Number	Design Guide Chapter/Page Number(s)
Two-way Slabs (cont.)	8.5.2.2	Two-way slabs/5-37
	8.5.3	Diaphragms/9-22
	8.5.3.1.1	Two-way slabs/5-38
		Foundations/10-10
	8.5.3.1.2	Two-way slabs/5-39
		Foundations/10-10
	8.5.4.1	Two-way slabs/5-45
	8.5.4.2	Two-way slabs/5-45, 5-46, 5-136, 5-141
	8.6.1.1	Two-way slabs/5-47, 5-72, 5-84, 5-93, 5-99, 5-108, 5-118
	8.6.1.2	Two-way slabs/ 5-47, 5-74, 5-77, 5-85, 5-101, 5-110, 5-112, 5-120, 5-122
		Diaphragms/9-22
		Foundations/10-63
		Earthquake-resistant structures (SDC B and C)/13-51, 13-52, 13-53
	8.7	Two-way slabs/5-79, 5-86, 5-94, 5-102, 5-113, 5-124
	8.7.1	Earthquake-resistant structures (SDC B and C)/13-24
	8.7.1.1	Two-way slabs/5-49
	8.7.1.2	Two-way slabs/5-51
	8.7.1.3	Two-way slabs/5-53
	8.7.2	Earthquake-resistant structures (SDC B and C)/13-24
	8.7.2.1	Two-way slabs/5-49
	8.7.2.2	Two-way slabs/5-49, 5-72, 5-84, 5-93, 5-99, 5-118
	8.7.3	Two-way slabs/5-50
		Earthquake-resistant structures (SDC B and C)/13-24
	8.7.3.1.3	Two-way slabs/5-50
	8.7.4	Earthquake-resistant structures (SDC B and C)/13-24
	8.7.4.1	Two-way slabs/5-51
	8.7.4.1.1	Two-way slabs/5-51
	8.7.4.1.2	Two-way slabs/5-51
	8.7.4.1.3	Two-way slabs/5-51
	8.7.4.2	Two-way slabs/5-53, 5-79, 5-87, 5-94, 5-103, 5-124
		Earthquake-resistant structures (SDC B and C)/13-55
	8.7.6	Two-way slabs/5-41, 5-53, 5-146
		Earthquake-resistant structures (SDC D, E, and F)/14-76, 14-150
	8.7.6.3	Two-way slabs/5-42
	8.7.7	Two-way slabs/5-41, 5-42, 5-53, 5-152
		Earthquake-resistant structures (SDC D, E, and F)/14-76, 14-150
	8.7.7.1.1	Two-way slabs/5-42
	8.7.7.1.2	Two-way slabs/5-43
	8.8	Two-way slabs/5-8
	8.8.1.1	Two-way slabs/5-10
	8.8.1.2	Two-way slabs/5-10
	8.8.1.3	Two-way slabs/5-10
	8.8.1.4	Two-way slabs/5-10
	8.8.1.6	Two-way slabs/5-113
	8.8.1.8	Two-way slabs/5-10
Beams	9.2.4.4	Beams/6-7, 6-116
		Earthquake-resistant structures (SDC B and C)/13-33
	9.3.1	Beams/6-2, 6-51
	9.3.1.1	Beams/6-2, 6-63, 6-96, 6-110
	9.3.1.1.1	Beams/6-21
	9.3.1.1.2	Beams/6-21
	9.3.2	Beams/6-53

	ACI Section Number	Design Guide Chapter/Page Number(s)
Beams (cont.)	9.3.2.1	Beams/6-2
	9.3.3.1	Beams/6-1, 6-3, 6-4, 6-10
	9.4.1.1	Beams/6-5
	9.4.1.2	Beams/6-5
	9.4.2.1	Beams/6-8
	9.4.3.1	Beams/6-8
	9.4.3.2	Beams/6-8, 6-26, 6-69, 6-83, 6-100, 6-113
	9.4.4.1	Beams/6-6
	9.4.4.2	Beams/6-8
	9.4.4.3	Beams/6-9, 6-115
	9.4.4.4	Beams/6-8
	9.5.1.1	Beams/6-10
	9.5.2.1	Beams/6-11
	9.5.3	Diaphragms/9-22
	9.5.3.1	Beams/6-16
	9.5.4	Diaphragms/9-22
	9.5.4.1	Beams/6-6
	9.5.4.2	Beams/6-19
	9.5.4.3	Beams/6-29
		Earthquake-resistant structures (SDC B and C)/13-37
	9.5.4.5	Beams/6-29
	9.5.4.6	Beams/6-21
	9.6.1	Beams/6-3
		Earthquake-resistant structures (SDC D, E, and F)/14-63
	9.6.1.2	Two-way slabs/5-108
		Beams/6-22, 6-24, 6-25
		Earthquake-resistant structures (SDC D, E, and F)/14-3
	9.6.1.3	Beams/6-22
	9.6.3	Beams/6-25
	9.6.3.4	Beams/6-17, 6-25
	9.6.4	Beams/6-21
	9.6.4.2	Beams/6-17, 6-30
		Earthquake-resistant structures (SDC B and C)/13-35
	9.6.4.3	Beams/6-29
		Earthquake-resistant structures (SDC B and C)/13-35
	9.7.1.1	Beams/6-30, 6-35
	9.7.1.3	Beams/6-46
	9.7.1.4	Beams/6-37
	9.7.2.1	Beams/6-30
	9.7.2.2	Beams/6-33
	9.7.2.3	Beams/6-34
		Earthquake-resistant structures (SDC D, E, and F)/14-15
	9.7.3	Earthquake-resistant structures (SDC D, E, and F)/14-9
	9.7.3.1	Beams/6-35, 6-43
	9.7.3.2	Beams/6-43
	9.7.3.3	Beams/6-44, 6-45
		Earthquake-resistant structures (SDC D, E, and F)/14-10, 14-84
	9.7.3.4	Beams/6-44
		Earthquake-resistant structures (SDC D, E, and F)/14-84
	9.7.3.5	Beams/6-45
		Earthquake-resistant structures (SDC D, E, and F)/14-10, 14-84
	9.7.3.8.1	Beams/6-45

	ACI Section Number	Design Guide Chapter/Page Number(s)
Beams (cont.)	9.7.3.8.2	Beams/6-45
	9.7.3.8.3	Beams/6-45
	9.7.3.8.4	Beams/6-44, 6-71
		Foundations/10-65
	9.7.5	Beams/6-21, 6-47
	9.7.5.1	Beams/6-30, 6-47
		Earthquake-resistant structures (SDC B and C)/13-31, 13-37
	9.7.5.2	Beams/6-47, 6-122
		Earthquake-resistant structures (SDC B and C)/13-37
	9.7.5.3	Beams/6-47, 6-123
	9.7.5.4	Beams/6-48
	9.7.6	Beams/6-25, 6-48
	9.7.6.1.1	Beams/6-48
	9.7.6.1.2	Beams/6-48
	9.7.6.3	Beams/6-21
	9.7.6.3.2	Beams/6-50, 6-122
	9.7.6.3.3	Beams/6-29
		Earthquake-resistant structures (SDC B and C)/13-36
	9.7.7	Beams/6-50, 6-51, 6-72, 6-87, 6-102, 6-124
	9.8	Beams/6-1, 6-18
	9.8.1.2	Beams/6-1
	9.8.1.3	Beams/6-1
	9.8.1.4	Beams/6-1
	9.8.1.5	Beams/6-1
	9.9.1.1	Beams/6-1
	9.9.1.3	Beams/6-1
Columns	10.3.1.2	Columns/7-1, 7-31, 7-90
		Foundations/10-28
	10.3.1.3	Columns/7-1
	10.3.1.5	Columns/7-1
	10.4.1.1	Columns/7-1
	10.4.1.2	Columns/7-1
	10.4.2.1	Columns/7-4
	10.5	Foundations/10-25
	10.5.1.1	Columns/7-14
	10.5.3.1	Columns/7-27
	10.5.4	Columns/7-30
	10.6.1.1	Columns/7-30, 7-31, 7-38
	10.6.2	Columns/7-31
		Earthquake-resistant structures (SDC D, E, and F)/14-23
	10.6.2.1	Columns/7-29, 7-92, 7-112
	10.6.2.2	Columns/7-27
		Earthquake-resistant structures (SDC B and C)/13-43, 13-44
	10.7.1.1	Columns/7-33
	10.7.2.1	Columns/7-34
	10.7.3.1	Columns/7-34
	10.7.4.1	Columns/7-41
	10.7.4.2	Columns/7-41
	10.7.5	Columns/7-41
	10.7.5.1.3	Columns/7-41
	10.7.5.2.1	Columns/7-43, 7-73

	ACI Section Number	Design Guide Chapter/Page Number(s)
Columns (cont.)	10.7.5.2.2	Columns/7-43
	10.7.5.3	Columns/7-48
	10.7.5.3.1	Columns/7-48
	10.7.6	Columns/7-29, 7-48
	10.7.6.2	Columns/7-50
	10.7.6.3	Columns/7-50, 7-53
	10.7.6.5.2	Columns/7-33
		Earthquake-resistant structures (SDC B and C)/13-11, 13-44
Walls	11.2.2.2	Walls/8-30
	11.2.4.2	Walls/8-1
	11.3.1.1	Walls/8-1
	11.4.1.1	Walls/8-1
	11.4.1.2	Walls/8-1
	11.4.1.3	Walls/8-5
	11.4.1.4	Walls/8-3
	11.4.2	Walls/8-3
	11.4.3	Walls/8-3
	11.5.1.1	Walls/8-9
	11.5.2	Walls/8-1
	11.5.2.1	Walls/8-10
	11.5.2.2	Walls/8-10
	11.5.3	Walls/8-10, 8-13, 8-19, 8-37
	11.5.4.1	Walls/8-14, 8-15
	11.5.4.2	Walls/8-14
	11.5.4.3	Walls/8-14
	11.5.4.4	Walls/8-14
	11.5.5.1	Walls/8-15
	11.6.1	Walls/8-17, 8-19, 8-20, 8-31, 8-54
	11.6.2	Walls/8-17, 8-19, 8-20
	11.7.1.1	Walls/8-21
	11.7.1.3	Walls/8-21
	11.7.2	Walls/8-27
	11.7.2.3	Walls/8-19
	11.7.3	Walls/8-27
	11.7.4.1	Walls/8-27
	11.7.5	Walls/8-29
	11.8	Walls/8-1, 8-5, 8-6
	11.8.1.1	Walls/8-6, 8-8, 8-46, 8-47, 8-48
	11.8.2	Walls/8-6
	11.8.3	Walls/8-7
	11.8.3.1	Walls/8-45
Diaphragms	12.2.1	Diaphragms/9-2
	12.2.2	Diaphragms/9-16
	12.3.1.1	Diaphragms/9-1
	12.3.1.2	Diaphragms/9-1
	12.4.1.1	Diaphragms/9-2
	12.4.1.2	Diaphragms/9-2
	12.4.2	Diaphragms/9-6
	12.4.2.2	Diaphragms/9-6
	12.4.2.3	Diaphragms/9-6
	12.4.2.4	Diaphragms/9-6, 9-8

	ACI Section Number	Design Guide Chapter/Page Number(s)
Diaphragms (cont.)	12.5.1.1	Diaphragms/9-19
	12.5.1.3	Diaphragms/9-19
	12.5.2.1	Diaphragms/9-20
	12.5.2.3	Diaphragms/9-20
	12.5.3	Diaphragms/9-23
	12.5.3.3	Diaphragms/9-21
	12.5.3.7	Diaphragms/9-21
	12.5.4.2	Diaphragms/9-27
	12.5.4.3	Diaphragms/9-29, 9-52
	12.6.1	Diaphragms/9-22
	12.6.2	Diaphragms/9-22
	12.6.3	Diaphragms/9-22
		Earthquake-resistant structures (SDC D, E, and F)/14-62
	12.7	Diaphragms/9-28
	12.7.3.2	Diaphragms/9-29
	12.7.3.3	Diaphragms/9-29
Foundations	13.2.2	Foundations/10-17
	13.2.6.1	Foundations/10-1
	13.2.6.2	Foundations/10-10
	13.2.6.3	Foundations/10-1
	13.2.6.4	Foundations/10-1
	13.2.6.5	Foundations/10-1
	13.2.7.1	Foundations/10-14
	13.2.7.2	Foundations/10-10
	13.2.8.1	Foundations/10-14
	13.2.8.3	Foundations/10-14
	13.3.1.2	Foundations/10-8, 10-30
	13.3.2.1	Foundations/10-9
	13.3.2.2	Foundations/10-13
	13.3.3.1	Foundations/10-9
	13.3.3.2	Foundations/10-13
	13.3.3.3	Foundations/10-13
	13.4.1.2	Foundations/10-25
	13.4.2	Foundations/10-1, 10-25
	13.4.2.1	Foundations/10-67
	13.4.2.2	Foundations/10-25
	13.4.2.3	Foundations/10-25
	13.4.3	Foundations/10-25
	13.4.3.1	Foundations/10-25
	13.4.3.2	Foundations/10-26
	13.4.4.1	Foundations/10-26
	13.4.4.2	Foundations/10-26
Beam-Column and Slab-Column Joints	15.2	Beam-column and slab-column joints/11-1
	15.2.1	Beam-column and slab-column joints/11-1
	15.2.2	Beam-column and slab-column joints/11-1
	15.2.3	Beam-column and slab-column joints/11-1
	15.2.4	Beam-column and slab-column joints/11-1
	15.2.5	Beam-column and slab-column joints/11-1
	15.2.6	Beam-column and slab-column joints/11-1, 11-13
		Earthquake-resistant structures (SDC B and C)/13-19, 13-21
		Earthquake-resistant structures (SDC D, E, and F)/14-33

ACI Section Number		Design Guide Chapter/Page Number(s)
Beam-Column and Slab-Column Joints (cont.)	15.2.7	Beam-column and slab-column joints/11-1, 11-13, 11-16
		Earthquake-resistant structures (SDC B and C)/13-19, 13-21
		Earthquake-resistant structures (SDC D, E, and F)/14-33
	15.2.8	Beam-column and slab-column joints/11-1, 11-2, 11-13, 11-17, 11-19
		Earthquake-resistant structures (SDC B and C)/13-19, 13-21
		Earthquake-resistant structures (SDC D, E, and F)/14-33
	15.2.9	Beam-column and slab-column joints/11-1
	15.3	Beam-column and slab-column joints/11-1
	15.3.1.1	Beam-column and slab-column joints/11-2
	15.3.1.2	Beam-column and slab-column joints/11-1, 11-3, 11-22
		Earthquake-resistant structures (SDC B and C)/13-13, 13-48
	15.3.1.3	Beam-column and slab-column joints/11-2, 11-3, 11-22
		Earthquake-resistant structures (SDC B and C)/13-13, 13-48
	15.3.1.4	Beam-column and slab-column joints/11-2, 11-3, 11-22
	15.3.2	Beam-column and slab-column joints/11-1
		Earthquake-resistant structures (SDC B and C)/13-19
	15.3.2.1	Beam-column and slab-column joints/11-3
	15.3.3.1	Beam-column and slab-column joints/11-3
	15.3.3.2	Beam-column and slab-column joints/11-3
	15.4	Beam-column and slab-column joints/11-1
	15.4.1.1	Beam-column and slab-column joints/11-7, 11-17, 11-19, 11-20, 11-21
		Earthquake-resistant structures (SDC B and C)/13-46
	15.4.2	Beam-column and slab-column joints/11-1
		Earthquake-resistant structures (SDC B and C)/13-13
	15.4.2.1	Beam-column and slab-column joints/11-12
	15.4.2.4	Beam-column and slab-column joints/11-13
		Earthquake-resistant structures (SDC B and C)/13-21
		Earthquake-resistant structures (SDC D, E, and F)/14-35
	15.5	Beam-column and slab-column joints/11-1, 11-15
	15.5.1	Beam-column and slab-column joints/11-15, 11-23
Connections Between Members	16.3	Columns/7-53
		Walls/8-30
		Foundations/10-17
	16.3.1.2	Columns/7-57
		Walls/8-32
		Foundations/10-20
	16.3.3.4	Columns/7-54, 7-64, 7-71, 7-93, 7-112
		Walls/8-30, 8-53
		Foundations/10-17
	16.3.3.5	Columns/7-59
		Walls/8-34
		Foundations/10-21
	16.3.4.1	Columns/7-55, 7-64, 7-71, 7-94, 7-113
		Walls/8-54
		Foundations/10-18
	16.3.4.2	Walls/8-31, 8-54
		Foundations/10-18
	16.3.5.2	Columns/7-57
		Walls/8-32
		Foundations/10-20

ACI Section Number		Design Guide Chapter/Page Number(s)
Connections (cont.)	16.3.5.4	Columns/7-57
		Walls/8-32
		Foundations/10-20
	16.4	Two-way slabs/5-2
Earthquake-Resistant Structures	18.2.1	Earthquake-resistant structures (overview)/12-1
	18.2.1.3	Earthquake-resistant structures (overview)/12-1
	18.2.1.4	Earthquake-resistant structures (overview)/12-1
	18.2.1.5	Earthquake-resistant structures (overview)/12-1
	18.2.1.6	Earthquake-resistant structures (overview)/12-2, 12-7
	18.2.1.7	Earthquake-resistant structures (overview)/12-2
	18.2.2	Earthquake-resistant structures (overview)/12-1, 12-5, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1
	18.2.3	Earthquake-resistant structures (overview)/12-1, 12-5, 12-6
	18.2.4	Earthquake-resistant structures (overview)/12-1, 12-5, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1
	18.2.5	Earthquake-resistant structures (overview)/12-1, 12-5, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1
	18.2.6	Earthquake-resistant structures (overview)/12-1, 12-5, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1
	18.2.7	Earthquake-resistant structures (overview)/12-1, 12-5, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1, 14-11, 14-20, 14-41
	18.2.7.1	Earthquake-resistant structures (SDC D, E, and F)/14-61
	18.2.8	Earthquake-resistant structures (overview)/12-1, 12-5, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1, 14-11, 14-20, 14-41
	18.2.8.2	Earthquake-resistant structures (SDC D, E, and F)/14-11, 14-41
	18.2.10	Earthquake-resistant structures (overview)/12-5
	18.3	Earthquake-resistant structures (overview)/12-6, 12-7
		Earthquake-resistant structures (SDC B and C)/13-1
	18.3.2	Beams/6-51
		Earthquake-resistant structures (SDC B and C)/13-1, 13-2
	18.3.3	Columns/7-13
		Earthquake-resistant structures (SDC B and C)/13-3
	18.3.4	Earthquake-resistant structures (SDC B and C)/13-3, 13-19
	18.4	Earthquake-resistant structures (overview)/12-6
		Earthquake-resistant structures (SDC B and C)/13-1, 13-4
	18.4.2.2	Earthquake-resistant structures (SDC B and C)/13-4, 13-31, 13-37
	18.4.2.3	Earthquake-resistant structures (SDC B and C)/13-4, 13-5, 13-7, 13-32
	18.4.2.4	Earthquake-resistant structures (SDC B and C)/13-4, 13-7, 13-36
	18.4.2.5	Earthquake-resistant structures (SDC B and C)/13-4, 13-8, 13-36
	18.4.2.6	Earthquake-resistant structures (SDC B and C)/13-4, 13-8
	18.4.3.1	Earthquake-resistant structures (SDC B and C)/13-9, 13-11, 13-42
	18.4.3.2	Earthquake-resistant structures (SDC B and C)/13-8
	18.4.3.3	Earthquake-resistant structures (SDC B and C)/13-8, 13-9, 13-11, 13-12, 13-43
	18.4.3.4	Earthquake-resistant structures (SDC B and C)/13-8, 13-44
	18.4.3.5	Earthquake-resistant structures (SDC B and C)/13-8, 13-11, 13-44
	18.4.3.6	Earthquake-resistant structures (SDC B and C)/13-8, 13-12
	18.4.4.1	Earthquake-resistant structures (SDC B and C)/13-13, 13-48
		Earthquake-resistant structures (SDC D, E, and F)/14-73
	18.4.4.2	Earthquake-resistant structures (SDC B and C)/13-13, 13-48

	ACI Section Number	Design Guide Chapter/Page Number(s)
Earthquake-Resistant Structures (cont.)	18.4.4.3	Earthquake-resistant structures (SDC B and C)/13-13, 13-37, 13-48
	18.4.4.4	Earthquake-resistant structures (SDC B and C)/13-13, 13-18, 13-48
	18.4.4.5	Earthquake-resistant structures (SDC B and C)/13-13, 13-18, 13-48
		Earthquake-resistant structures (SDC D, E, and F)/14-7
	18.4.4.6	Earthquake-resistant structures (SDC B and C)/13-19
	18.4.4.7	Earthquake-resistant structures (SDC B and C)/13-13
	18.4.4.7.1	Earthquake-resistant structures (SDC B and C)/13-19
	18.4.4.7.2	Earthquake-resistant structures (SDC B and C)/13-19
	18.4.4.7.3	Earthquake-resistant structures (SDC B and C)/13-19
	18.4.4.7.4	Earthquake-resistant structures (SDC B and C)/13-19
	18.4.5	Earthquake-resistant structures (SDC B and C)/13-25, 13-59
	18.4.5.1	Earthquake-resistant structures (SDC B and C)/13-23
	18.4.5.2	Earthquake-resistant structures (SDC B and C)/13-23, 13-51
	18.4.5.3	Earthquake-resistant structures (SDC B and C)/13-23, 13-24, 13-55
	18.4.5.4	Earthquake-resistant structures (SDC B and C)/13-24, 13-55, 13-57
	18.4.5.5	Earthquake-resistant structures (SDC B and C)/13-24, 13-57
	18.4.5.6	Earthquake-resistant structures (SDC B and C)/13-24, 13-57
	18.4.5.7	Earthquake-resistant structures (SDC B and C)/13-24, 13-57
	18.4.5.8	Earthquake-resistant structures (SDC B and C)/13-25, 13-59
	18.6	Earthquake-resistant structures (overview)/12-1, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1, 14-2, 14-56, 14-64
	18.6.1.1	Earthquake-resistant structures (SDC D, E, and F)/14-2
	18.6.1.2	Earthquake-resistant structures (SDC D, E, and F)/14-2
	18.6.2	Earthquake-resistant structures (SDC D, E, and F)/14-76
	18.6.2.1	Earthquake-resistant structures (SDC D, E, and F)/14-2, 14-56
	18.6.3	Earthquake-resistant structures (SDC D, E, and F)/14-57, 14-124
	18.6.3.1	Earthquake-resistant structures (SDC D, E, and F)/14-3, 14-4, 14-42, 14-78, 14-142
	18.6.3.2	Earthquake-resistant structures (SDC D, E, and F)/14-3, 14-79
	18.6.3.3	Earthquake-resistant structures (SDC D, E, and F)/14-10
	18.6.4	Earthquake-resistant structures (SDC D, E, and F)/14-124
	18.6.4.1	Earthquake-resistant structures (SDC D, E, and F)/14-12, 14-13, 14-14, 14-15, 14-81
	18.6.4.2	Earthquake-resistant structures (SDC D, E, and F)/14-15, 14-30, 14-82
	18.6.4.3	Earthquake-resistant structures (SDC D, E, and F)/14-30
	18.6.4.4	Earthquake-resistant structures (SDC D, E, and F)/14-14, 14-30, 14-82
	18.6.4.6	Earthquake-resistant structures (SDC D, E, and F)/14-15, 14-82, 14-83
	18.6.4.7	Earthquake-resistant structures (SDC D, E, and F)/14-15
	18.6.5	Earthquake-resistant structures (SDC D, E, and F)/14-80, 14-124
	18.6.5.1	Earthquake-resistant structures (SDC D, E, and F)/14-11
	18.6.5.2	Earthquake-resistant structures (SDC D, E, and F)/14-13, 14-82
	18.7	Earthquake-resistant structures (overview)/12-2, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1, 14-2, 14-15
	18.7.2	Earthquake-resistant structures (SDC D, E, and F)/14-16, 14-17, 14-86, 14-96
	18.7.2.1	Earthquake-resistant structures (SDC D, E, and F)/14-4
	18.7.3	Earthquake-resistant structures (SDC D, E, and F)/14-16, 14-17, 14-19, 14-87, 14-89, 14-98, 14-100
	18.7.3.2	Earthquake-resistant structures (SDC D, E, and F)/14-18, 14-87, 14-98
	18.7.3.3	Earthquake-resistant structures (SDC D, E, and F)/14-17, 14-18
	18.7.4	Earthquake-resistant structures (SDC D, E, and F)/14-16, 14-20, 14-59
	18.7.4.1	Earthquake-resistant structures (SDC D, E, and F)/14-19
	18.7.4.2	Earthquake-resistant structures (SDC D, E, and F)/14-20

ACI Section Number		Design Guide Chapter/Page Number(s)
Earthquake-Resistant Structures (cont.)	18.7.4.3	Earthquake-resistant structures (SDC D, E, and F)/14-20, 14-90, 14-100
	18.7.4.4	Earthquake-resistant structures (SDC D, E, and F)/14-20
	18.7.5	Earthquake-resistant structures (SDC D, E, and F)/14-16, 14-24, 14-59
	18.7.5.1	Earthquake-resistant structures (SDC D, E, and F)/14-21, 14-24, 14-91, 14-101
	18.7.5.2	Earthquake-resistant structures (SDC D, E, and F)/14-15, 14-20, 14-21, 14-24, 14-29, 14-30, 14-50, 14-51, 14-61, 14-91, 14-101, 14-110
	18.7.5.3	Earthquake-resistant structures (SDC D, E, and F)/14-15, 14-20, 14-21, 14-24, 14-29, 14-30, 14-50, 14-61, 14-69, 14-91, 14-102
	18.7.5.4	Earthquake-resistant structures (SDC D, E, and F)/14-15, 14-21, 14-24, 14-25, 14-28, 14-29, 14-30, 14-91, 14-96, 14-102
	18.7.5.5	Earthquake-resistant structures (SDC D, E, and F)/14-24, 14-25
	18.7.5.6	Earthquake-resistant structures (SDC B and C)/13-12
		Earthquake-resistant structures (SDC D, E, and F)/14-29, 14-60
	18.7.5.7	Earthquake-resistant structures (SDC D, E, and F)/14-30
	18.7.6	Earthquake-resistant structures (SDC D, E, and F)/14-16, 14-24, 14-59, 14-146, 14-146
	18.7.6.1	Earthquake-resistant structures (SDC D, E, and F)/14-76, 14-92, 14-102
	18.7.6.1.1	Earthquake-resistant structures (SDC D, E, and F)/14-23, 14-149
	18.7.6.2.1	Earthquake-resistant structures (SDC D, E, and F)/14-23, 14-92, 14-102, 14-147
	18.8	Earthquake-resistant structures (overview)/12-2, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1, 14-29
	18.8.2.1	Earthquake-resistant structures (SDC D, E, and F)/14-29, 14-30, 14-32
	18.8.2.2	Earthquake-resistant structures (SDC D, E, and F)/14-4, 14-30, 14-79
	18.8.2.3	Earthquake-resistant structures (SDC D, E, and F)/14-4, 14-16, 14-30, 14-79
	18.8.2.3.1	Earthquake-resistant structures (SDC D, E, and F)/14-4, 14-30
	18.8.3.1	Earthquake-resistant structures (SDC D, E, and F)/14-30, 14-104
	18.8.3.2	Earthquake-resistant structures (SDC D, E, and F)/14-30, 14-96
	18.8.3.3	Earthquake-resistant structures (SDC D, E, and F)/14-30
	18.8.4	Earthquake-resistant structures (SDC D, E, and F)/14-30
	18.8.4.1	Earthquake-resistant structures (SDC D, E, and F)/14-94, 14-104
	18.8.4.2	Earthquake-resistant structures (SDC D, E, and F)/14-33
	18.8.4.3	Earthquake-resistant structures (SDC B and C)/13-19, 14-33
	18.8.5	Earthquake-resistant structures (SDC B and C)/13-13, 13-37
		Earthquake-resistant structures (SDC D, E, and F)/14-4, 14-29, 14-35, 14-56, 14-79
	18.8.5.1	Earthquake-resistant structures (SDC B and C)/13-13, 13-15
		Earthquake-resistant structures (SDC D, E, and F)/14-4, 14-6
	18.8.5.2	Earthquake-resistant structures (SDC B and C)/13-15
		Earthquake-resistant structures (SDC D, E, and F)/14-6
	18.8.5.3	Earthquake-resistant structures (SDC B and C)/13-15
		Earthquake-resistant structures (SDC D, E, and F)/14-5
	18.8.5.4	Earthquake-resistant structures (SDC B and C)/13-15
		Earthquake-resistant structures (SDC D, E, and F)/14-5, 14-6
	18.9	Earthquake-resistant structures (overview)/12-2
		Earthquake-resistant structures (SDC D, E, and F)/14-1
	18.10	Earthquake-resistant structures (overview)/12-2
		Earthquake-resistant structures (SDC D, E, and F)/14-1, 14-36, 14-59
	18.10.2.1	Earthquake-resistant structures (SDC D, E, and F)/14-37, 14-106, 14-115
	18.10.2.2	Earthquake-resistant structures (SDC D, E, and F)/14-59, 14-106, 14-115
	18.10.2.3	Earthquake-resistant structures (SDC D, E, and F)/14-38, 14-39, 14-41, 14-111
	18.10.2.4	Earthquake-resistant structures (SDC D, E, and F)/14-41, 14-106, 14-116
	18.10.2.5	Earthquake-resistant structures (SDC D, E, and F)/14-41, 14-60

	ACI Section Number	Design Guide Chapter/Page Number(s)
Earthquake-Resistant Structures (cont.)	18.10.3	Earthquake-resistant structures (SDC D, E, and F)/14-107, 14-117
	18.10.3.1	Earthquake-resistant structures (SDC D, E, and F)/14-43, 14-107, 14-117
	18.10.3.1.3	Earthquake-resistant structures (SDC D, E, and F)/14-44
	18.10.4	Earthquake-resistant structures (SDC D, E, and F)/14-59, 14-107, 14-118
	18.10.4.1	Earthquake-resistant structures (SDC D, E, and F)/14-44
	18.10.4.3	Earthquake-resistant structures (SDC D, E, and F)/14-45
	18.10.4.4	Earthquake-resistant structures (SDC D, E, and F)/14-45, 14-108, 14-118
	18.10.4.5	Earthquake-resistant structures (SDC D, E, and F)/14-45, 14-46
	18.10.4.6	Earthquake-resistant structures (SDC D, E, and F)/14-46
	18.10.5	Earthquake-resistant structures (SDC D, E, and F)/14-106, 14-116
	18.10.5.1	Earthquake-resistant structures (SDC D, E, and F)/14-46
	18.10.5.2	Walls/8-13
		Earthquake-resistant structures (SDC D, E, and F)/14-46, 14-49, 14-114
	18.10.6	Earthquake-resistant structures (SDC D, E, and F)/14-47
	18.10.6.2	Earthquake-resistant structures (SDC D, E, and F)/14-46, 14-47, 14-50, 14-51, 14-54, 14-108, 14-109
	18.10.6.3	Earthquake-resistant structures (SDC D, E, and F)/14-48, 14-50, 14-55, 14-59, 14-119
	18.10.6.4	Earthquake-resistant structures (SDC D, E, and F)/14-39, 14-50, 14-51, 14-53, 14-54, 14-109, 14-110, 14-112, 14-116, 14-119, 14-120, 14-123
	18.10.6.5	Earthquake-resistant structures (SDC D, E, and F)/14-51, 14-54, 14-110, 14-122
	18.10.7	Earthquake-resistant structures (SDC D, E, and F)/14-56
	18.10.7.1	Earthquake-resistant structures (SDC D, E, and F)/14-56
	18.10.7.2	Earthquake-resistant structures (SDC D, E, and F)/14-56
	18.10.7.3	Earthquake-resistant structures (SDC D, E, and F)/14-57, 14-124
	18.10.7.4	Earthquake-resistant structures (SDC D, E, and F)/14-42, 14-56, 14-57, 14-58, 14-59, 14-124, 14-126, 14-127, 14-128
	18.10.8	Earthquake-resistant structures (SDC D, E, and F)/14-76
	18.10.8.1	Earthquake-resistant structures (SDC D, E, and F)/14-59
	18.10.9	Earthquake-resistant structures (SDC D, E, and F)/14-59
	18.10.10.1	Earthquake-resistant structures (SDC D, E, and F)/14-60
	18.10.11.1	Earthquake-resistant structures (SDC D, E, and F)/14-60
	18.12	Earthquake-resistant structures (overview)/12-2
		Earthquake-resistant structures (SDC D, E, and F)/14-1, 14-60, 14-64
	18.12.3.2	Earthquake-resistant structures (SDC D, E, and F)/14-61
	18.12.6	Earthquake-resistant structures (SDC D, E, and F)/14-60
	18.12.6.1	Earthquake-resistant structures (SDC D, E, and F)/14-129
	18.12.7.1	Earthquake-resistant structures (SDC D, E, and F)/14-60
	18.12.7.4	Earthquake-resistant structures (SDC D, E, and F)/14-61
	18.12.7.5	Earthquake-resistant structures (SDC D, E, and F)/14-61, 14-138
	18.12.7.6	Earthquake-resistant structures (SDC D, E, and F)/14-61, 14-139
	18.12.7.7	Earthquake-resistant structures (SDC D, E, and F)/14-61, 14-139
	18.12.8.1	Earthquake-resistant structures (SDC D, E, and F)/14-61
	18.12.10.1	Earthquake-resistant structures (SDC D, E, and F)/14-63, 14-136
	18.13	Earthquake-resistant structures (overview)/12-2
		Earthquake-resistant structures (SDC B and C)/13-1, 13-25
		Earthquake-resistant structures (SDC D, E, and F)/14-1, 14-63
	18.13.2.2	Earthquake-resistant structures (SDC D, E, and F)/14-63
	18.13.2.3	Earthquake-resistant structures (SDC D, E, and F)/14-63
	18.13.2.4	Earthquake-resistant structures (SDC D, E, and F)/14-51, 14-63
	18.13.2.5	Earthquake-resistant structures (SDC D, E, and F)/14-63

ACI Section Number		Design Guide Chapter/Page Number(s)
Earthquake-Resistant Structures (cont.)	18.13.3.1	Earthquake-resistant structures (SDC D, E, and F)/14-64
	18.13.3.2	Earthquake-resistant structures (SDC B and C)/13-25, 14-64
	18.13.4.1	Earthquake-resistant structures (SDC B and C)/13-27
		Earthquake-resistant structures (SDC D, E, and F)/14-64
	18.13.4.2	Earthquake-resistant structures (SDC D, E, and F)/14-64
	18.13.4.3	Earthquake-resistant structures (SDC B and C)/13-27, 13-28, 13-60
		Earthquake-resistant structures (SDC D, E, and F)/14-64, 14-140
	18.13.4.4	Earthquake-resistant structures (SDC D, E, and F)/14-65, 14-140
	18.13.5	Earthquake-resistant structures (SDC D, E, and F)/14-65
	18.13.5.2	Earthquake-resistant structures (SDC B and C)/13-27, 13-29
		Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.13.5.3	Earthquake-resistant structures (SDC B and C)/13-27, 13-29
		Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.13.5.4	Earthquake-resistant structures (SDC B and C)/13-27
		Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.13.5.5	Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.13.5.7	Earthquake-resistant structures (SDC B and C)/13-27
		Earthquake-resistant structures (SDC D, E, and F)/14-65
	18.13.5.7.1	Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.13.5.7.2	Earthquake-resistant structures (SDC B and C)/13-27
		Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.13.5.7.3	Earthquake-resistant structures (SDC B and C)/13-27
		Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.13.6.1	Earthquake-resistant structures (SDC B and C)/13-27, 13-29
		Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.13.6.2	Earthquake-resistant structures (SDC B and C)/13-27, 13-29
		Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.13.6.3	Earthquake-resistant structures (SDC D, E, and F)/14-66
	18.14	Columns/7-1
		Earthquake-resistant structures (overview)/12-1, 12-6
		Earthquake-resistant structures (SDC D, E, and F)/14-1, 14-18, 14-69
	18.14.2.1	Earthquake-resistant structures (SDC D, E, and F)/14-70, 14-141, 14-144
	18.14.3.2	Earthquake-resistant structures (SDC D, E, and F)/14-70, 14-71, 14-73, 14-142, 14-143, 14-144, 14-145, 14-146, 14-145
	18.14.3.3	Earthquake-resistant structures (SDC D, E, and F)/14-71, 14-73
	18.14.5	Earthquake-resistant structures (SDC B and C)/13-58
		Earthquake-resistant structures (SDC D, E, and F)/14-75
	18.14.5.1	Earthquake-resistant structures (SDC B and C)/13-23
		Earthquake-resistant structures (SDC D, E, and F)/14-76, 14-149
	18.14.5.2	Earthquake-resistant structures (SDC D, E, and F)/14-76
	18.14.5.3	Earthquake-resistant structures (SDC B and C)/13-59, 13-60
		Earthquake-resistant structures (SDC D, E, and F)/14-76, 14-150, 14-151
	18.14.6	Earthquake-resistant structures (SDC D, E, and F)/14-76
	18.14.6.1	Earthquake-resistant structures (SDC D, E, and F)/14-76
Concrete Design and Durability	19.2.1.1	Material requirements and strength reduction factors/2-1, 2-2
	19.2.1.2	Material requirements and strength reduction factors/2-2
	19.2.1.3	Material requirements and strength reduction factors/2-2
	19.2.2	Beams/6-53
		Columns/7-7
		Walls/8-5
	19.2.2.1	Material requirements and strength reduction factors/2-2
		Two-way slabs/5-3

ACI Section Number		Design Guide Chapter/Page Number(s)
Concrete Design and Durability (cont.)	19.2.2.2	Material requirements and strength reduction factors/2-3
		Two-way slabs/5-3
	19.2.3	Walls/8-6
	19.2.3.1	Material requirements and strength reduction factors/2-3
		Two-way slabs/5-2
	19.2.4	Material requirements and strength reduction factors/2-3
		Two-way slabs/5-38
		Beams/6-6
		Columns/7-27
		Walls/8-15
		Diaphragms/9-20
		Foundations/10-11
		Earthquake-resistant structures (SDC B and C)/13-7
	19.2.4.1	Material requirements and strength reduction factors/2-3
		One-way slabs/4-6
		Two-way slabs/5-38
		Beams/6-6
	19.2.4.2	Material requirements and strength reduction factors/2-3
		One-way slabs/4-6
		Two-way slabs/5-38
		Beams/6-6
	19.2.4.3	Material requirements and strength reduction factors/2-3
		One-way slabs/4-6, 4-26
		Two-way slabs/5-38
		Beams/6-6
	19.3.2.1	Material requirements and strength reduction factors/2-2
Steel Reinforcement Properties	20.2.1.2	Material requirements and strength reduction factors/2-4, 2-5
	20.2.1.3	Material requirements and strength reduction factors/2-5, 2-10
	20.2.1.4	Material requirements and strength reduction factors/2-7
	20.2.1.6	Material requirements and strength reduction factors/2-7
		One-way slabs/4-15, 4-16
		Beams/6-41, 6-43
		Beam-column and slab-column joints/11-6
		Earthquake-resistant structures (SDC B and C)/13-15
		Earthquake-resistant structures (SDC D, E, and F)/14-6
	20.2.2.1	Material requirements and strength reduction factors/2-7
	20.2.2.2	Material requirements and strength reduction factors/2-4, 2-7
		One-way slabs/4-5
		Two-way slabs/5-36
		Beams/6-11, 6-53, 6-74, 6-88, 6-104
		Columns/7-14
		Walls/8-10, 8-45
		Diaphragms/9-19
	20.2.2.3	Material requirements and strength reduction factors/2-7
	20.2.2.4	Material requirements and strength reduction factors/2-7
	20.2.2.5	Material requirements and strength reduction factors/2-8
	20.4	Two-way slabs/5-49
	20.4.1	Material requirements and strength reduction factors/2-9
	20.5	Two-way slabs/5-42

ACI Section Number		Design Guide Chapter/Page Number(s)
Steel Reinforcement Properties (cont.)	20.5.1	One-way slabs/4-8
		Two-way slabs/5-49
		Beams/6-30
		Columns/7-1, 7-33
		Walls/8-21
	20.5.1.1	Material requirements and strength reduction factors/2-9
	20.5.1.2	Material requirements and strength reduction factors/2-9
	20.5.1.3	Material requirements and strength reduction factors/2-9, 2-10
	20.5.1.3.1	Material requirements and strength reduction factors/2-9
	20.5.1.3.4	Material requirements and strength reduction factors/2-9
	20.5.1.3.5	Material requirements and strength reduction factors/2-10
	20.5.1.3.6	Material requirements and strength reduction factors/2-10
		Two-way slabs/5-42
	20.5.1.4	Material requirements and strength reduction factors/2-9
	20.5.1.4.1	Material requirements and strength reduction factors/2-10
	20.5.2.1	Material requirements and strength reduction factors/2-10
Strength Reduction Factors	21.2	One-way slabs/4-5
		Two-way slabs/5-36
		Beams/6-10
		Columns/7-14, 7-17
		Walls/8-10
		Diaphragms/9-19
	21.2.1	Material requirements and strength reduction factors/2-11
	21.2.2	Material requirements and strength reduction factors/2-11
	21.2.2.1	Material requirements and strength reduction factors/2-13
		One-way slabs/4-5
		Two-way slabs/5-36
		Beams/6-11
		Columns/7-74, 7-82
	21.2.4	Material requirements and strength reduction factors/2-11, 2-13
		Earthquake-resistant structures (SDC D, E, and F)/14-13, 14-44, 14-135
	21.2.4.1	Material requirements and strength reduction factors/2-13
		Earthquake-resistant structures (SDC D, E, and F)/14-46
	21.2.4.2	Material requirements and strength reduction factors/2-13
		Earthquake-resistant structures (SDC D, E, and F)/14-63, 14-135
	21.2.4.3	Material requirements and strength reduction factors/2-13
	21.2.4.4	Material requirements and strength reduction factors/2-13
		Earthquake-resistant structures (SDC D, E, and F)/14-33, 14-58, 14-124
Sectional Strength	22.2	Columns/7-16
		Walls/8-11, 8-52
	22.2.1.2	One-way slabs/4-8
		Beams/6-22
	22.2.2.1	Material requirements and strength reduction factors/2-12
		One-way slabs/4-6
		Two-way slabs/5-37
		Beams/6-11
	22.2.2.3	Columns/7-16
		Walls/8-11

ACI Section Number		Design Guide Chapter/Page Number(s)
Sectional Strength (cont.)	22.2.2.4.1	One-way slabs/4-5
		Two-way slabs/5-37
		Beams/6-11
		Columns/7-16
		Walls/8-11
	22.2.2.4.3	One-way slabs/4-6
		Two-way slabs/5-38
		Beams/6-4
		Columns/7-16
		Walls/8-11
	22.3	One-way slabs/4-5
		Two-way slabs/5-36
		Beams/6-11
		Walls/8-10
		Diaphragms/9-9, 9-20
	22.4	Walls/8-42
		Diaphragms/9-9, 9-22, 9-22, 9-27
		Earthquake-resistant structures (SDC D, E, and F)/14-46
	22.4.2	Columns/7-15
		Walls/8-10
		Earthquake-resistant structures (SDC D, E, and F)/14-96
	22.4.2.1	Columns/7-15, 7-77
		Walls/8-10
	22.4.2.4	Columns/7-15, 7-31
		Foundations/10-26
		Earthquake-resistant structures (SDC B and C)/13-61
		Earthquake-resistant structures (SDC D, E, and F)/14-141
	22.4.2.5	Columns/7-15, 7-31
		Foundations/10-26
	22.5	One-way slabs/4-6
		Two-way slabs/5-38
		Beams/6-1, 6-16, 6-25
		Columns/7-27, 7-91, 7-100, 7-111
		Walls/8-15, 8-42, 8-48
		Foundations/10-10
	22.5.1.2	One-way slabs/4-7
		Two-way slabs/5-39
		Beams/6-17, 6-26
		Columns/7-28
		Walls/8-16
		Earthquake-resistant structures (SDC D, E, and F)/14-14
	22.5.1.5	One-way slabs/4-6
		Two-way slabs/5-38
	22.5.1.6	Beams/6-17
	22.5.1.10	Beams/6-17
		Columns/7-29, 7-101
	22.5.1.11	Beams/6-17
	22.5.2.2	Columns/7-28

ACI Section Number		Design Guide Chapter/Page Number(s)
Sectional Strength (cont.)	22.5.3.1	One-way slabs/4-7
		Two-way slabs/5-39
		Beams/6-17
		Columns/7-28
		Walls/8-16
	22.5.3.2	One-way slabs/4-7
		Two-way slabs/5-39
		Beams/6-17
		Columns/7-28
		Walls/8-16
	22.5.5	Earthquake-resistant structures (SDC D, E, and F)/14-13, 14-23
	22.5.5.1	One-way slabs/4-6
		Two-way slabs/5-38
		Beams/6-16
		Columns/7-27
		Foundations/10-10
		Earthquake-resistant structures (SDC B and C)/13-7, 13-11
	22.5.5.1.1	One-way slabs/4-6
		Two-way slabs/5-38
		Beams/6-16
		Columns/7-27
	22.5.5.1.2	Columns/7-92, 7-100, 7-112
		Walls/8-16
		Earthquake-resistant structures (SDC D, E, and F)/14-14, 14-23
	22.5.5.1.3	One-way slabs/4-6
		Two-way slabs/5-38, 5-40
		Beams/6-17
		Columns/7-28
		Walls/8-16
	22.5.8	Beams/6-17
	22.5.8.5	Earthquake-resistant structures (SDC D, E, and F)/14-13
	22.5.8.5.1	Beams/6-17
		Earthquake-resistant structures (SDC B and C)/13-7, 13-11
		Earthquake-resistant structures (SDC D, E, and F)/14-23
	22.5.8.5.2	Beams/6-18
	22.5.8.5.3	Beams/6-18
		Earthquake-resistant structures (SDC B and C)/13-33, 13-36, 13-44
	22.5.8.5.5	Beams/6-18
		Columns/7-29, 7-31, 7-33
	22.5.8.5.6	Columns/7-29, 7-31, 7-33
		Earthquake-resistant structures (SDC B and C)/13-11
		Earthquake-resistant structures (SDC D, E, and F)/14-13, 14-23
	22.5.8.6	Beams/6-18
	22.6	Two-way slabs/5-39
		Foundations/10-10
		Beam-column and slab-column joints/11-1
	22.6.4.1	Two-way slabs/5-12, 5-14, 5-18, 5-44, 5-48
	22.6.4.1.1	Two-way slabs/5-12, 5-15
	22.6.4.1.2	Two-way slabs/5-12, 5-19, 5-25, 5-141
	22.6.4.2	Two-way slabs/5-20, 5-44
	22.6.4.3	Two-way slabs/5-46, 5-137

ACI Section Number		Design Guide Chapter/Page Number(s)
Sectional Strength (cont.)	22.6.5	Two-way slabs/5-13, 5-39
		Foundations/10-11
		Earthquake-resistant structures (SDC B and C)/13-25
		Earthquake-resistant structures (SDC D, E, and F)/14-75
	22.6.6	Two-way slabs/5-40
	22.6.6.2	Two-way slabs/5-41
	22.6.6.3	Two-way slabs/5-43
	22.6.7.1	Two-way slabs/5-2, 5-41, 5-146
	22.6.7.2	Two-way slabs/5-42, 5-146
	22.6.8	Two-way slabs/5-152
	22.6.8.2	Two-way slabs/5-43
	22.6.8.3	Two-way slabs/5-43, 5-152
	22.7	Beams/6-19
	22.7.1.1	Earthquake-resistant structures (SDC B and C)/13-33
	22.7.1.3	Beams/6-6
	22.7.3	Beams/6-8
	22.7.3.1	Beams/6-8
	22.7.3.2	Beams/6-8, 6-116
		Earthquake-resistant structures (SDC B and C)/13-34
	22.7.3.3	Beams/6-8, 6-100, 6-101, 6-116
	22.7.5.1	Beams/6-8
	22.7.6.1	Beams/6-19
		Earthquake-resistant structures (SDC B and C)/13-35, 13-36
	22.7.6.1.1	Beams/6-19
	22.7.6.1.2	Beams/6-20
	22.7.7.1	Beams/6-21
		Earthquake-resistant structures (SDC B and C)/13-34
	22.7.7.1.2	Beams/6-21
	22.7.7.2	Beams/6-21
	22.8	Columns/7-54
		Walls/8-30
		Foundations/10-17
	22.9	Columns/7-53, 7-59
		Walls/8-30, 8-34
		Diaphragms/9-21
		Foundations/10-17, 10-21
	22.9.4.2	Columns/7-59
		Walls/8-34
		Foundations/10-21
	22.9.4.4	Columns/7-59
		Walls/8-35
		Foundations/10-21
	22.9.4.5	Columns/7-59
	22.9.4.6	Columns/7-59
		Foundations/10-22

ACI Section Number		Design Guide Chapter/Page Number(s)
Serviceability	24.2	One-way slabs/4-1
		Two-way slabs/5-2
		Beams/6-2, 6-3, 6-53, 6-63, 6-96, 6-110
	24.2.1	Beams/6-57
	24.2.2	One-way slabs/4-1
		Two-way slabs/5-2
		Beams/6-2, 6-53
	24.2.3.6	Beams/6-56, 6-77, 6-92, 6-95, 6-108
	24.2.3.7	Beams/6-56
	24.2.4.1.1	Beams/6-56
	24.2.4.1.2	Beams/6-56, 6-96
	24.3	One-way slabs/4-9
		Beams/6-33, 6-35, 6-70, 6-81, 6-85, 6-99, 6-101, 6-123
		Earthquake-resistant structures (SDC B and C)/13-31
		Earthquake-resistant structures (SDC D, E, and F)/14-78, 14-142
	24.3.2	One-way slabs/4-9
		Beams/6-33, 6-34
	24.3.2.1	One-way slabs/4-9
		Beams/6-33
	24.3.4	Beams/6-33, 6-66, 6-81, 6-99
	24.3.5	Beams/6-30
	24.4	One-way slabs/4-8
		Diaphragms/9-22
	24.4.3.1	Beams/6-34, 6-66, 6-81
	24.4.3.2	One-way slabs/4-8, 4-30
		Foundations/10-31
		Earthquake-resistant structures (SDC D, E, and F)/14-60
	24.4.3.3	One-way slabs/4-8, 4-30
	24.4.3.4	One-way slabs/4-8
Reinforcement Details	25.2	One-way slabs/4-8
		Two-way slabs/5-49
		Beams/6-30, 6-35, 6-70, 6-81, 6-85, 6-99, 6-101, 6-123
		Columns/7-1, 7-34
		Earthquake-resistant structures (SDC B and C)/13-31
		Earthquake-resistant structures (SDC D, E, and F)/14-78, 14-142
	25.2.1	Beams/6-11, 6-32
	25.2.2	Beams/6-31
	25.2.3	Columns/7-34, 7-38
	25.3.1	One-way slabs/4-13
		Beams/6-39
	25.3.2	Two-way slabs/5-2
		Columns/7-53
	25.3.4	Earthquake-resistant structures (SDC B and C)/13-7
		Earthquake-resistant structures (SDC D, E, and F)/14-12
	25.3.5	Earthquake-resistant structures (SDC B and C)/13-7
		Earthquake-resistant structures (SDC D, E, and F)/14-12
	25.4	One-way slabs/4-10, 4-12
		Two-way slabs/5-51
		Beams/6-35
		Beam-column and slab-column joints/11-3
		Earthquake-resistant structures (SDC D, E, and F)/14-41
	25.4.1.1	Beams/6-35

ACI Section Number		Design Guide Chapter/Page Number(s)
Reinforcement Details (cont.)	25.4.1.2	Beams/6-35
		Columns/7-56
		Walls/8-31
		Foundations/10-19
	25.4.1.3	Material requirements and strength reduction factors/2-13
		Beams/6-35
	25.4.1.4	One-way slabs/4-12
		Beams/6-35
	25.4.2	One-way slabs/4-10, 4-18
		Beams/6-35, 6-44, 6-49
		Columns/7-44
		Walls/8-24
		Diaphragms/9-29
		Foundations/10-14
		Earthquake-resistant structures (SDC D, E, and F)/14-10, 14-20, 14-38
	25.4.2.1	One-way slabs/4-20
		Beams/6-46
	25.4.2.2	One-way slabs/4-12
		Beams/6-37
	25.4.2.3	One-way slabs/4-10, 4-12, 4-13
		Beams/6-35, 6-38, 6-39
		Columns/7-44, 7-46, 7-47
		Walls/8-24, 8-26
		Foundations/10-14, 10-16
	25.4.2.4	One-way slabs/4-10, 4-12, 4-13, 4-30
		Two-way slabs/5-79, 5-87, 5-94, 5-103, 5-113, 5-124
		Beams/6-35, 6-37, 6-38, 6-71, 6-85, 6-102, 6-123
		Columns/7-44, 7-45, 7-46, 7-95, 7-114
		Walls/8-24, 8-25, 8-26, 8-56
		Foundations/10-14, 10-16, 10-49, 10-55
		Earthquake-resistant structures (SDC B and C)/13-45, 13-56
		Earthquake-resistant structures (SDC D, E, and F)/14-84, 14-90, 14-101, 14-111
	25.4.2.5	One-way slabs/4-10
		Beams/6-35
		Columns/7-44
		Walls/8-24
		Foundations/10-14
	25.4.3	Columns/7-57
		Walls/8-32
		Foundations/10-20
	25.4.3.1	One-way slabs/4-14
		Beams/6-40
		Beam-column and slab-column joints/11-4
	25.4.3.3	Beams/6-40
		Beam-column and slab-column joints/11-6, 11-16, 11-18, 11-20
	25.4.3.4	One-way slabs/4-14
		Beams/6-40
	25.4.4	One-way slabs/4-15, 4-16
		Beams/6-41, 6-43
		Earthquake-resistant structures (SDC D, E, and F)/14-6

ACI Section Number		Design Guide Chapter/Page Number(s)
Reinforcement Details (cont.)	25.4.4.1	One-way slabs/4-15
		Beams/6-41
		Beam-column and slab-column joints/11-6
		Earthquake-resistant structures (SDC B and C)/13-15
		Earthquake-resistant structures (SDC D, E, and F)/14-6
	25.4.4.2	One-way slabs/4-15
		Beams/6-41
		Beam-column and slab-column joints/11-6
		Earthquake-resistant structures (SDC B and C)/13-15
	25.4.4.4	One-way slabs/4-16
		Beam-column and slab-column joints/11-7
	25.4.4.5	One-way slabs/4-16
		Beams/6-42
	25.4.4.6	One-way slabs/4-16
		Beams/6-42
		Beam-column and slab-column joints/11-7
	25.4.5	One-way slabs/4-16
		Beams/6-43
		Earthquake-resistant structures (SDC B and C)/13-13, 13-17, 13-37
		Earthquake-resistant structures (SDC D, E, and F)/14-4
	25.4.9	Earthquake-resistant structures (SDC D, E, and F)/14-8, 14-30, 14-79
	25.4.9.1	Foundations/10-20
	25.4.9.2	Columns/7-55
		Walls/8-31
		Foundations/10-18
	25.4.10.1	One-way slabs/4-12, 4-20
		Beams/6-37, 6-46
		Foundations/10-16
		Earthquake-resistant structures (SDC D, E, and F)/14-60
	25.4.10.2	One-way slabs/4-12
		Beams/6-37
		Foundations/10-16
		Earthquake-resistant structures (SDC D, E, and F)/14-60
	25.5	One-way slabs/4-20
		Two-way slabs/5-53
		Beams/6-46
		Columns/7-41
		Walls/8-21
		Earthquake-resistant structures (SDC D, E, and F)/14-41
	25.5.1.1	One-way slabs/4-20
		Beams/6-46
		Columns/7-42
		Walls/8-22
	25.5.1.2	One-way slabs/4-20
		Beams/6-46
	25.5.1.3	One-way slabs/4-20
		Beams/6-46
	25.5.1.4	One-way slabs/4-20
		Beams/6-46
	25.5.1.5	One-way slabs/4-20
		Beams/6-46

ACI Section Number		Design Guide Chapter/Page Number(s)
Reinforcement Details (cont.)	25.5.2	One-way slabs/4-22
		Two-way slabs/5-53
		Beams/6-51
		Columns/7-58
		Walls/8-34
		Foundations/10-21
		Earthquake-resistant structures (SDC D, E, and F)/14-20
	25.5.2.1	One-way slabs/4-20, 4-31
		Beams/6-46
		Columns/7-43
		Walls/8-23
	25.5.2.2	One-way slabs/4-21
		Beams/6-46, 6-85
		Earthquake-resistant structures (SDC D, E, and F)/14-111
	25.5.5.1	Columns/7-43
		Walls/8-23
		Foundations/10-19, 10-20
	25.5.5.3	One-way slabs/4-20
		Beams/6-46
		Columns/7-42, 7-57
		Walls/8-22, 8-32
	25.5.5.4	Columns/7-57
		Walls/8-32
		Foundations/10-19
	25.5.6.2	Columns/7-48
	25.5.6.3	Columns/7-48
	25.5.7	One-way slabs/4-22
		Two-way slabs/5-53
		Beams/6-47, 6-51
		Columns/7-48, 7-58
		Walls/8-26, 8-34
		Foundations/10-21
		Earthquake-resistant structures (SDC D, E, and F)/14-11
	25.5.7.1	One-way slabs/4-21
		Beams/6-46
	25.5.7.3	Beams/6-47
		Columns/7-48
	25.5.7.4	Beams/6-47
	25.7	Beams/6-48
	25.7.1	Beams/6-48
	25.7.1.1	Beams/6-48
	25.7.1.2	Beams/6-48
	25.7.1.3	Beams/6-18, 6-48, 6-50
	25.7.1.5	Beams/6-50
	25.7.1.6	Beams/6-50, 6-51
	25.7.1.6.1	Beams/6-50
	25.7.1.7	Beams/6-49
	25.7.2	Columns/7-29, 7-48
		Earthquake-resistant structures (SDC B and C)/13-11
		Earthquake-resistant structures (SDC D, E, and F)/14-8, 14-24
	25.7.2.1	Columns/7-48, 7-64, 7-91, 7-99, 7-111

ACI Section Number		Design Guide Chapter/Page Number(s)
Reinforcement Details (cont.)	25.7.2.2	Columns/7-48, 7-63, 7-91, 7-99, 7-111
		Earthquake-resistant structures (SDC B and C)/13-8, 13-44
		Earthquake-resistant structures (SDC D, E, and F)/14-110, 14-122
	25.7.2.3	Columns/7-48, 7-50
		Earthquake-resistant structures (SDC B and C)/13-8, 13-44
		Earthquake-resistant structures (SDC D, E, and F)/14-15, 14-82
	25.7.2.4	Columns/7-50
		Earthquake-resistant structures (SDC B and C)/13-8
		Earthquake-resistant structures (SDC D, E, and F)/14-15
	25.7.2.4.1	Columns/7-50
	25.7.3	Columns/7-29, 7-48, 7-50
		Earthquake-resistant structures (SDC B and C)/13-11
	25.7.3.1	Columns/7-52, 7-70
		Earthquake-resistant structures (SDC B and C)/13-11
	25.7.3.2	Columns/7-70
	25.7.3.3	Columns/7-70
	25.7.3.4	Columns/7-53
	25.7.3.5	Columns/7-53
	25.7.3.6	Columns/7-53
	25.7.4	Earthquake-resistant structures (SDC B and C)/13-7
		Earthquake-resistant structures (SDC D, E, and F)/14-12, 14-24
	25.7.4.2	Earthquake-resistant structures (SDC B and C)/13-7
		Earthquake-resistant structures (SDC D, E, and F)/14-12
Construction Documents	26.4.3	Material requirements and strength reduction factors/2-2
	26.5.6	Earthquake-resistant structures (SDC D, E, and F)/14-60, 14-63
	26.6.4	One-way slabs/4-22
		Beams/6-47
		Columns/7-48
		Walls/8-27
	26.12.3	Material requirements and strength reduction factors/2-2

TPS 201356

Description of Manual

The purpose of this Design Guide is to assist in the proper application of the design and detailing requirements in Building Code Requirements for Structural Concrete (ACI 318-19) for cast-in-place concrete buildings with nonprestressed steel reinforcement. Many design aids and worked-out examples are provided to make designing and detailing reinforced concrete members simpler and faster. The goal is to acquire an understanding of the code requirements and to apply them properly and efficiently.

In addition to structural engineers, this Design Guide is a valuable resource for educators, students, individuals studying for licensing exams, and building officials.

CRSI Concrete Reinforcing Steel Institute

933 North Plum Grove Road | Schaumburg, IL 60173 | Tel. 847.517.1200 | **www.crsi.org**

ISBN 9781943961528

9 781943 961528

DG-ACI-318-19